Ontario

A Guide to Effective Instruction in Mathematics

Grades 1 to 3

Number Sense and Numeration

Revised

2016

Ontario

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Grades 1 to 3

Number Sense and Numeration

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

Ministry of Education

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Une publication équivalente est disponible en français sous le titre suivant :
Guide d'enseignement efficace des mathématiques, de la 1re à la 3e année – Numération et sens du nombre.

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Introduction

This document is a practical guide that teachers will find useful in helping students to achieve the curriculum expectations for mathematics outlined in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*.

The expectations outlined in the curriculum documents describe the knowledge and skills that students are expected to acquire by the end of each grade. In *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario* (Expert Panel on Early Math, 2003), effective instruction is identified as critical to the successful learning of mathematical knowledge and skills, and the components of an effective program are described. As part of the process of implementing the panel's vision of effective mathematics instruction for Ontario, *A Guide to Effective Instruction in Mathematics, Grades 1 to 3* provides a framework for teaching mathematics. This framework includes specific strategies for developing an effective program and for creating a community of learners in which students' mathematical thinking is nurtured. The strategies focus on the "big ideas" inherent in the expectations; on problem solving as the main context for mathematical activity; and on communication, especially student talk, as the conduit for sharing and developing mathematical thinking. The guide provides strategies for assessment, the use of manipulatives, and home connections.

Purpose and Features of This Document

This document provides:

- an overview of each of the big ideas in the Number Sense and Numeration strand;
- three appendices (Appendices A–C), one for each of Grades 1 to 3, which provide learning activities that introduce, develop, or help to consolidate some aspect of each big idea;
- an appendix (Appendix D) that lists the curriculum expectations in the Number Sense and Numeration strand under the big idea(s) to which they correspond. This clustering of expectations around each of the five big ideas allows teachers to concentrate their programming on the big ideas of the strand while remaining confident that the full range of curriculum expectations is being addressed;
- a glossary of terms used in this document.

"Big Ideas" in the Curriculum for Grades 1 to 3

In developing a mathematics program, it is essential to concentrate on important mathematical concepts, or "big ideas", and the knowledge and skills that go with those concepts. Programs that are organized around big ideas and focus on problem solving provide cohesive learning opportunities that allow students to explore concepts in depth.

> All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas", or key principles, of mathematics, such as pattern or relationship. This understanding of key principles will enable and encourage students to use mathematical reasoning throughout their lives. (Ontario Ministry of Education, 2005, p. 25)

Students are better able to see the connections in mathematics and thus to learn mathematics when it is organized in big, coherent "chunks". In organizing a mathematics program, teachers should concentrate on the big ideas in mathematics and view the expectations in the curriculum policy documents for Grades 1 to 3 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a

big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers to see that the concepts represented in the curriculum expectations should not be taught as isolated bits of information but rather as a connected network of interrelated concepts. In building a program, teachers need a sound understanding of the key mathematical concepts for their students' grade levels, as well as an understanding of how those concepts connect with students' prior and future learning (Ma, 1999). Such knowledge includes the "conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum" (p. xxiv) and how best to teach the concepts to children. Concentrating on developing this knowledge will enhance effective teaching.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a lens for:

- making instructional decisions (e.g., deciding on an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students' thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the strategies a child uses to count a set);
- collecting observations and making anecdotal records;
- providing feedback to students;
- determining next steps;
- communicating concepts and providing feedback on students' achievement to parents[1] (e.g., in report card comments).

Teachers are encouraged to focus their instruction on the big ideas of mathematics. By clustering expectations around a few big ideas, teachers can teach for depth of understanding. This document provides models for clustering the expectations around a few major concepts and includes activities that foster an understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other lessons in Number Sense and Numeration, as well as lessons in the other strands of mathematics.

1. In this document, "parent(s)" refers to parent(s) and guardian(s).

The "Big Ideas" in Number Sense and Numeration

Number sense refers to a general understanding of number as well as operations and the ability to apply this understanding in flexible ways to make mathematical judgements and to develop useful strategies for solving problems. In this strand, students develop their understanding of number by learning about different ways of representing numbers and about the relationships among numbers. They learn how to count in various ways, developing a sense of magnitude. They also develop a solid understanding of the four basic operations and learn to compute fluently, using a variety of tools and strategies.

A well-developed understanding of number includes a grasp of more-and-less relationships, part-whole relationships, the role of special numbers such as five and ten, connections between numbers and real quantities and measures in the environment, and much more.

(Ontario Ministry of Education, 2005, p. 8)

Overview

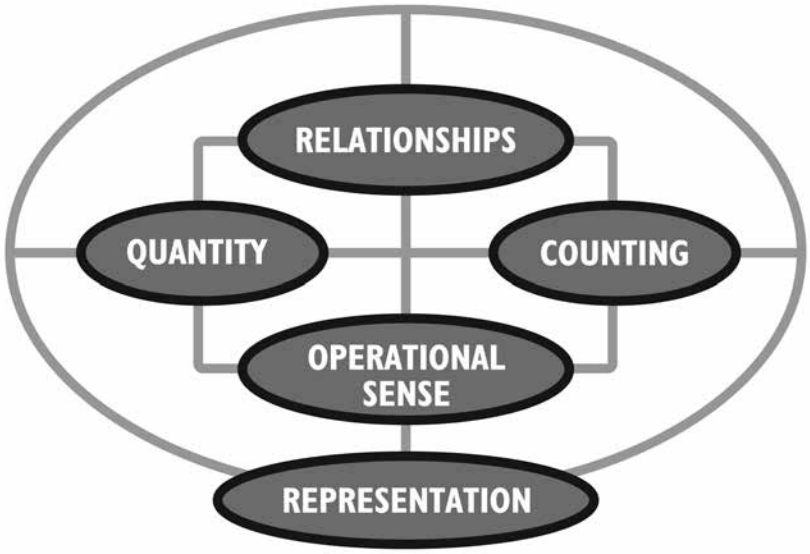
To assist teachers in becoming familiar with using the "big ideas" of mathematics in their instruction and assessment, this section focuses on Number Sense and Numeration, one of the strands of the Ontario mathematics curriculum for Grades 1 to 3. This section identifies the five big ideas that form the basis of the curriculum expectations in Number Sense and Numeration during the primary years and elaborates on the key concepts embedded within each big idea.

The big ideas or major concepts in Number Sense and Numeration are the following:

- counting
- operational sense
- quantity

- relationships
- representation

These big ideas are conceptually interdependent, equally significant, and overlapping. For example, meaningful counting includes an understanding that there is a quantity represented by the numbers in the count. Being able to link this knowledge with the relationships that permeate the base ten number system gives students a strong basis for their developing number sense. And all three of these ideas – counting, quantity, relationships – have an impact on operational sense, which incorporates the actions of mathematics. Present in all four big ideas are the representations that are used in mathematics, namely, the symbols for numbers, the algorithms, and other notation, such as the notation used for decimals and fractions.



In this section, the five big ideas of Number Sense and Numeration are described and explained; examined in the light of what students are doing; discussed in terms of teaching strategies; and finally, in Appendices A–C, addressed through appropriate grade-specific learning activities.

For each big idea in this section, there is:

- an **overview**, which includes a general discussion of the development of the big idea in the primary grades, a delineation of some of the key concepts inherent in the big idea, and in some instances additional background information on the concept for the teacher;
- **grade-specific descriptions** of (1) characteristics of learning evident in students who have been introduced to the concepts addressed in the big idea under consideration, and (2) instructional strategies that will support those learning characteristics in the specific grade.

General Principles of Instruction

In this section, specific instructional strategies are provided for each big idea in Number Sense and Numeration in each primary grade. However, many principles of instruction apply in all the primary grades and in all the strands, and are relevant in teaching all the big ideas of mathematics. It is essential that teachers incorporate these principles into their teaching. Some of the most important of these principles are as follows:

- **Student talk is important across all grade levels.** Students need to talk about and talk through mathematical concepts, with one another and with the teacher.
- **Representations of concepts promote understanding and communication.** Representations of concepts can take a variety of forms (e.g., manipulatives, pictures, diagrams, gestures, or symbols). Children who use manipulatives or pictorial materials to represent a mathematical concept are more likely to understand the concept. Children's attitudes towards mathematics are improved when teachers effectively use manipulatives to teach difficult concepts (Sowell, 1989; Thomson & Lambdin, 1994). However, students need to be guided in their experiences with concrete and visual representations so that they make the appropriate links between the mathematical concept and the symbols and language with which it is represented.
- **Problem solving should be the basis for most mathematical learning.** Problem-solving situations provide students with interesting contexts for learning mathematics and give students an understanding of the relevancy of mathematics. Even very young children benefit from learning in problem-solving contexts. Learning basic facts through a problem-solving format, in relevant and meaningful contexts, is much more significant to children than memorizing facts without purpose.
- **Students need frequent experiences using a variety of resources and learning strategies (e.g., number lines, hundreds charts or carpets, base ten blocks, interlocking cubes, ten frames, calculators, math games, math songs, physical movement, math stories).** Some strategies (e.g., using math songs, using movement) may not overtly involve children in problem solving; nevertheless, they should be used in instruction because they address the learning styles of many children, especially in the primary grades.
- **As students confront increasingly more complex concepts, they need to be encouraged to use their reasoning skills.** It is important for students to realize that math "makes sense" and that they have the

skills to navigate through mathematical problems and computations. Students should be encouraged to use reasoning skills such as looking for patterns and making estimates:

- **looking for patterns.** Students benefit from experiences in which they are helped to recognize that the base ten number system and the actions placed upon numbers (the operations) are pattern based.
- **making estimates.** Students who learn to make estimates can determine whether their responses are reasonable. In learning to make estimates, students benefit from experiences with using benchmarks, or known quantities, as points of reference (e.g., "This is what a jar of 10 cubes and a jar of 50 cubes look like. How many cubes do you think are in this jar?").

Kindergarten Mathematics: Characteristics of Learning and Instructional Strategies

When making connections across Kindergarten to Grade 3 (e.g., during professional learning conversations; across the learning continuum), the following references can be referred to from *The Kindergarten Program* (2016):

- Pedagogy in a play- and inquiry-based learning environment (pp. 75–85);
- Overall expectations, conceptual understandings, and specific expectations related to the concepts (pp. 216–248);
- How Kindergarten children demonstrate their understanding through what they say, do, and represent;
- Educators' Intentional Interactions – noticing and naming, responding, extending, and challenging;
- Educator reflections in professional learning conversations.

Counting

> Competent counting requires mastery of a symbolic system, facility with a complicated set of procedures that require pointing at objects and designating them with symbols, and understanding that some aspects of counting are merely conventional, while others lie at the heart of its mathematical usefulness.
>
> (Kilpatrick, Swafford, & Findell, 2001, p. 159)

Overview

Many of the mathematical concepts that students learn in the first few years of school are closely tied to counting. The variety and accuracy of children's counting strategies and the level of their skill development in counting are valuable indicators of their growth in mathematical understanding in the primary years.

The following key points can be made about counting in the primary years:

- Counting includes both the recitation of a series of numbers and the conceptualization of a symbol as representative of a quantity.
- In their first experiences with counting, children do not initially understand the connection between a quantity and the number name and symbol that represent it.
- Counting is a powerful early tool intricately connected with the future development of students' conceptual understanding of quantity, place value, and the operations.

Counting as the recitation of a series of numbers and conceptualization of number as quantity

Children usually enter school with some counting strategies, and some children may be able to count to large numbers. Much of children's earliest counting is usually done as a memory task in one continuous stream similar to the chant used for the alphabet. But if asked what the number after 5 is, children may recount from 1 with little demonstration of knowing what is meant by the question. Young children may not realize that the count stays consistent. At one time they may count 1, 2, 3, 4, 5, 6, . . . and at another time 1, 2, 3, 5, 4, 6, 8, . . . , with little concern about their inconsistency. If asked to count objects, they may not tag each item as they count, with the consequence that they count different amounts each time they count the objects, or they may count two items as they say the word "sev-en". If asked the total number of objects at the end of a count, they may recount with little understanding that the final tag or count is actually the total number of objects. They also may not yet understand that they can count any objects in the same count (even very unlike objects, such as cookies and apples) and that they can start the count from any object in the group and still get the same total.

Learning to count to high numbers is a valuable experience. At the same time, however, children need to be learning the quantities and relationships in lower numbers. Children may be able to count high and still have only a rudimentary knowledge of the quantity represented by a count. Even children who recognize that 4 and 1 more is 5 may not be able to extrapolate from that knowledge to recognize that 4 and 2 more is 6. Young children often have difficulty producing counters to represent numbers that they can easily count to. For example, children may be able to count to 30 but be unable to count out 30 objects from a larger group of objects.

Making the connection between counting and quantity

It is essential that the quantitative value of a number and the role of the number in the counting sequence be connected in children's minds. Some of the complexity in counting comes from having to make a connection between a number name, a number symbol, and a quantity, and children do not at first grasp that connection. Counting also involves synchronizing the action of increasing the quantity with the making of an oral representation, and then recognizing that the last word stated is not just part of the sequence of counted objects but is also the total of the objects. Students need multiple opportunities to make the connection between the number name, the symbol, and the quantity represented.

Counting as it relates to developing understanding of quantity, place value, and the operations

Through authentic counting experiences, students develop basic concepts and strategies that help them to understand and describe number, to see the patterns between numbers (e.g., the relationships between the numbers to 9, the teens, and then the decades), to count with accuracy, and to use counting strategies in problem-solving situations.

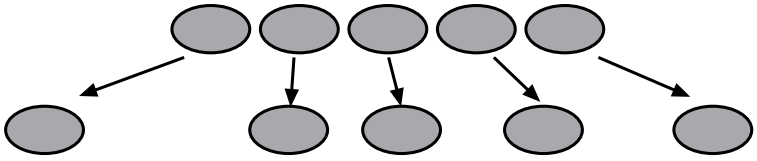
Place value is developed as students begin to count to numbers greater than 9. Students can run into difficulties with some of these numbers. For example, the teen numbers are particularly difficult in the English language, as they do not follow the logical pattern, which would name 12 as "one ten and two" or "twoteen" (as is done in some languages). The decades also produce difficulties, especially in the changes in pattern that occur between groups of numbers, as between 19 and 20 or 29 and 30.

Counting is the first strategy that students use to determine answers to questions involving the operations. For example, in addition, students learn to count all to determine the total of two collections of counters. Later, they learn to count on from the collection with the larger amount.

Key Concepts of Counting

The purpose of this section is to help teachers understand some of the basic concepts embedded in the early understanding of counting. These concepts do not necessarily occur in a linear order. For example, some students learn parts of one concept, move on to another concept, and then move back again to the first concept. The list of concepts that follows is not meant to represent a lockstep continuum that students follow faithfully but is provided to help teachers understand the components embedded in the skill of counting:

- **Stable order** – the idea that the counting sequence stays consistent; it is always 1, 2, 3, 4, 5, 6, 7, 8, . . . , not 1, 2, 3, 5, 6, 8.
- **Order irrelevance** – the idea that the counting of objects can begin with any object in a set and the total will still be the same.
- **Conservation** – the idea that the count for a set group of objects stays the same no matter whether the objects are spread out or are close together (see also "Quantity").



- **Abstraction** – the idea that a quantity can be represented by different things (e.g., 5 can be represented by 5 like objects, by 5 different objects, by 5 invisible things [5 ideas], or by 5 points on a line). Abstraction is a complex concept but one that most students come to understand quite easily. Some students, however, struggle with such complexity, and teachers may need to provide additional support to help them grasp the concept.
- **One-to-one correspondence** – the idea that each object being counted must be given one count and only one count. In the early stages, it is useful for students to tag each item as they count it and to move the item out of the way as it is counted.
- **Cardinality** – the idea that the last count of a group of objects represents the total number of objects in the group. A child who recounts when asked how many candies are in the set that he or she has just counted does not understand cardinality (see also "Quantity").
- **Movement is magnitude** – the idea that, as one moves up the counting sequence, the quantity increases by 1 (or by whatever number is being counted by), and as one moves down or backwards in the sequence, the quantity decreases by 1 (or by whatever number is being counting by) (e.g., in skip counting by 10's, the amount goes up by 10 each time).
- **Unitizing** – the idea that, in the base ten system, objects are grouped into tens once the count exceeds 9 (and into tens of tens when it exceeds 99) and that this grouping of objects is indicated by a 1 in the tens place of a number once the count exceeds 9 (and by a 1 in the hundreds place once the count exceeds 99) (see also "Relationships" and "Representation").

It is not necessary for students in the primary years to know the names of these concepts. The names are provided as background information for teachers.

Instruction in Counting

Specific grade-level descriptions of instructional strategies for counting will be given in the subsequent pages. The following are general strategies for teaching counting. Teachers should:

- link the counting sequence with objects (especially fingers) or movement on a number line so that students attach the counting number to an increase in quantity or, when counting backwards, to a decrease in quantity;
- model strategies that help students to keep track of their count (e.g., touching each object and moving it as it is counted);

- provide activities that promote opportunities for counting both inside and outside the classroom (e.g., using a hopscotch grid with numbers on it at recess; playing hide-and-seek and counting to 12 before "seeking"; counting students as they line up for recess);
- continue to focus on traditional games and songs that encourage counting skills for the earliest grades, but also adapt those games and songs so that students gain experience in counting from anywhere within the sequence (e.g., counting from 4 to 15 instead of 1 to 10), and gain experience with the teen numbers;
- link the teen words with the word "ten" and the words "one" to "nine" (e.g., link eleven with the words "ten" and "one"; link "twelve" with "ten" and "two") to help students recognize the patterns to the teen words, which are exceptions to the patterns for number words in the base ten number system;
- help students to identify the patterns in the numbers themselves (using a hundreds chart). These patterns in the numbers include the following:
  - The teen numbers (except 11 and 12) combine the number term and "teen" (e.g., 13, 14, 15).
  - The number 9 always ends a decade (e.g., 29, 39, 49).
  - The pattern of 10, 20, 30, . . . follows the same pattern as 1, 2, 3, ...
  - The decades follow the pattern of 1, 2, 3, . . . within their decade; hence, 20 combines with 1 to become 21, then with 2 to become 22, and so on.
  - The pattern in the hundreds chart is reiterated in the count from 100 to 200, 200 to 300, and so on, and again in the count from 1000 to 2000, 2000 to 3000, and so on.

Characteristics of Student Learning and Instructional Strategies by Grade

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- may have some difficulty counting though the teen numbers and the transition between such numbers as 19 and 20 or 29 and 30. Students may also say something like "twoteen" for "twelve" or "oneteen" for "eleven". Such mistakes are attributable to the nature of the English teen-number words, which look, for example, like 10 and 1 or 10 and 2 but do not follow that pattern when spoken. Students often have less difficulty with the numbers from 20 to 29;

- develop skill in orally counting by 1′s, 2′s, 5′s, and 10′s to 100, with or without a number line, but may lack the skill required to coordinate the oral count sequence with the physical counting of objects;
- count to 10 by 1′s, beginning at different points in the sequence of 1 to 10;
- consolidate their skill in one-to-one correspondence while counting by 1′s to larger numbers or producing objects to represent the larger numbers. Students may have difficulty in keeping track of the count of a large group of items (e.g., 25) and may not have an understanding of how the objects can be grouped into sets of 10′s to be counted. They may have more difficulty with correspondence when skip counting by 2′s, 5′s, and 10′s;
- are able to count backwards from 10, although beginning the backwards count at numbers other than 10 (e.g., 8) may be more problematic;
- may move away from counting-all strategies (e.g., counting from 1 to determine the quantity when joining two sets, even though they have already counted each set) and begin to use more efficient counting-on strategies (e.g., beginning with the larger number and counting on the remaining quantity);
- use a calculator to explore counting patterns and also to solve problems with numbers greater than 10;
- recognize the patterns in the counting sequence (e.g., how 9′s signal a change of decade – 19 to 20, 29 to 30); recognize how the decades (e.g., 10, 20, 30, . . .) follow the patterns of the 1′s (1, 2, 3, . . .); and use their knowledge of these number patterns to count on a number line or on a hundreds chart. Students can recreate a hundreds chart, using counting patterns to help them identify the numbers.

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing opportunities to experience counting to 50 in engaging and relevant situations in which the meaning of the numbers is emphasized and a link is established between the numbers and their visual representation as numerals. Especially important is the development of an understanding that the numeral in the decades place represents 10 or a multiple of 10 (e.g., 10, 20, 30, 40, . . .). For example, have the students play Ten-Chair Count. For this game, 10 chairs are placed at the front of the class, and 10 students sit in the chairs. The class count 1, 2, 3, . . . and point in sequence to the students in the chairs. As the count is being made, the class follow it on individual number lines or hundreds charts. Each time the count reaches a decade (10, 20, 30, . . .), the student being pointed to leaves his or her seat, each student moves up a seat, a new student sits in the end seat. The count continues to go

up and down the row of chairs until it reaches a previously chosen number that has been kept secret. When the count reaches that number, the student being pointed to is the winner;

- using songs, chants, and stories that emphasize the counting sequences of 1′s, 2′s, 5′s, and 10′s, both forward and backwards and from different points within the sequence, especially beginning at tricky numbers (e.g., 29);
- providing opportunities to engage in play-based problem solving that involves counting strategies (e.g., role-playing a bank; shopping for groceries for a birthday party);
- providing opportunities to participate in games that emphasize strategies for counting (e.g., games that involve moving counters along a line or a path and keeping track of the counts as one moves forward or backwards). These games should involve numbers in the decades whenever possible (e.g., games using two-digit numbers on a hundreds carpet);
- building counting activities into everyday events (e.g., lining up at the door; getting ready for home);
- using counters and other manipulatives, hundreds charts or carpets, and number lines (vertical and horizontal) in meaningful ways, on many different occasions;
- continuing to build up their understanding of 5 and 10 as anchors for thinking about all other numbers;
- providing support to help students recognize the various counting strategies. For example, for strategies such as tagging each object as it is counted or grouping items into sets that are easier to count, have them play Catch the Mistake and Make It Right, in which the teacher has to count out objects (e.g., money, books, pencils) but gets confused and does the count wrong (e.g., by missing numbers, counting objects more than once). The students have to find the mistake. Students may also take turns leading the game.

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- count by 1′s, 2′s, 5′s, 10′s, and 25′s beyond 100. Students count backwards by 1′s from 50 but may have difficulty counting down from larger numbers. They are able to produce the number word just before and just after numbers to 100, although they may sometimes need a running start (e.g., to determine the number right before 30, they may have to count up from 20). They have difficulty with the decades in counting backwards (e.g., may state the sequence

as 33, 32, 31, 20, counting backwards by 10 from the decade number in order to determine the next number). These counting skills have important implications for students' understanding of two-digit computations;

- may not yet count by 10's off the decade and have to persist with counting on (e.g., for a question such as 23 + 11, instead of being able to calculate that 23 + 10 would be 33 and then adding on the remaining single unit, they may count on the whole of the 11 single units);

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing opportunities to experience counting beyond 100 in engaging and relevant situations in which the meaning of the numbers is emphasized and a link is established between the numbers and their visual representation as numerals. For example, have students play the Counting Game. Students stand, and as the teacher points to each student, the class states the number (101, 102, 103, . . .). One student points to the numbers on the hundreds chart so that students link the count with the numeral. (Even though the hundreds chart ends at 100, students can use it as a guide for counting numbers after 100.) On the first decade (at 110), the student being counted sits down. When the next decade – student 120 – is reached, the student being counted sits down, and the student who sat down previously can stand up again. The count continues to go around the class until it reaches a previously set number, such as 200. The student sitting when the last number is reached is the winner. Students should be encouraged to look for patterns; they can be asked whom they think the next person to sit down will be when the count gets to different points ("The count is at 127. Who do you think will sit down next?"); and they can hypothesize who will be sitting when the count gets to 200. This game can be played with multiples of 2, 5, and 15, and the count can begin from anywhere;
- using songs, chants, and stories that emphasize the counting sequences of 1's, 2's, 5's, 10's, and 25's from different points within the sequence;
- providing opportunities to engage in problem solving that involves counting strategies;
- providing opportunities to participate in games that emphasize strategies for counting (e.g., games that involve the use of money);
- building counting activities into everyday events (e.g., fund-raising for a charity; preparing for a field trip);
- using counters and other manipulatives, hundreds charts or carpets, and number lines in meaningful ways, especially to identify the patterns in the counting sequence (e.g., block out the numbers from 36 to 46 in the

hundreds chart, ask the students what numbers are missing, and ask them how they know);

- providing support to help students recognize the various counting strategies for counting larger numbers (e.g., counting by 100's from 101, 201, 301, . . .).

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- use counting in different ways than in previous grades. Most of the students will have consolidated the counting concepts (see the "Counting" overview, on pp. 7–8). They will also have begun to use other strategies for calculating quantities and using the operations. At this point, counting by 10's and making tens are strategies for working with computations involving multi-digit numbers. (The strategy of making tens involves using knowledge of all the combinations for 10 to help solve computations. For example, to solve 25 + 6, students have immediate recall that 5 + 5 = 10. They use this information to determine that 25 + 5 will bring them to the next decade, namely, 30, and they add on the remaining 1 to make 31);
- begin to use grouping strategies for calculating rather than relying solely on simple counting strategies (e.g., in determining how many pennies* are in a jar, grouping the pennies into 10's and then counting by 10's to find a solution; in working out an addition problem such as 56 + 32, counting from 56 by 10's to 86 and then continuing on from 86 to add the remaining 2 single units);
- count by 1's, 2's, 5's, 10's, and 100's to 1000, using various starting points, and by 25's to 1000, using multiples of 25 as starting points. Students use their knowledge of counting patterns to count by 10's from positions off the decade (e.g., from 21 to 101);
- extend their understanding of number patterns into the 100's and are able to generalize the patterns for counting by 100's and 1000's by following the pattern of 100, 200, . . . or 1000, 2000, . . . ;
- make appropriate decisions about how to count large amounts (e.g., by grouping objects by 2's, 5's, 10's, or 100's);
- count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points. Students count backwards by 100's from any number less than 1000. They use their understanding of the counting patterns to identify

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

patterns on the number line and the hundreds chart, and to determine what number or numbers go before and after a sequence of numbers up to 50. They use their knowledge of counting and counting patterns, along with manipulatives, to help determine or estimate quantities up to 1000;

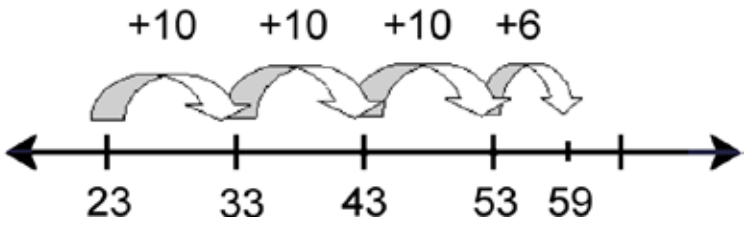
- use calculators to skip count in various increments (e.g., of 3, 6, 7), to make hypotheses (e.g., about the next number in a sequence, about the relationship between counting and the operations), and to explore large numbers and counting patterns in large numbers.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing opportunities to experience counting beyond 100 in engaging and relevant situations in which the meaning of the numbers is emphasized and a link is established between the numbers and their visual representation as numerals;
- using songs, chants, and stories that emphasize the counting sequences of 2's, 5's, 10's, and 25's from different points within the sequence;
- providing opportunities to engage in problem solving in contexts that encourage students to use grouping as a counting strategy (e.g., grouping objects into 2's, 5's, 10's, 25's);
- providing opportunities to participate in games that emphasize strategies for counting (e.g., games that involve the use of money);
- building counting activities into everyday events (e.g., fund-raising for a charity; preparing for a field trip);
- using counters and other manipulatives, hundreds charts or carpets, and number lines in meaningful ways;
- providing support to help students recognize the various counting strategies for counting larger numbers (e.g., counting by 100's from 101, 201, 301, . . .);
- providing support to help students sketch an open number line that will facilitate counting to solve a problem (e.g., to solve 23 + 36, they count 23, 33, 43, 53 on the number line and then add the remaining 6 from the 36 to make 59).

23 + 36 =



Operational Sense

> Fuson (1982) observed that in doing 8 + 5 by counting on their fingers, about a third of the six year olds in her sample got 12 as the answer because they said, "8, 9, 10, 11, 12" as they extended the first, second, third, fourth and fifth fingers. They were not using reasoning, only relying on the rote attributes of the algorithm.
>
> (Kamii, 1985, p. 68)

Overview

Students need to understand the concepts and procedures involved in operations on numbers. Research (Ma, 1999) on instructional practices related to the operations indicates that most children are taught only the surface aspects of the procedures involved in the operations and that little attention is given to the underlying concepts (e.g., the composing and decomposing of numbers, especially the understanding of how the numerals in a number increase by a rate of 10 as they move to the left and decrease by a rate of 10 as they move to the right) or to the connections between various operations, such as inverse relationships.

When teachers give attention to key pieces of knowledge that surround the operations, they help students to develop a sense of how numbers and operations work together. Students who have this sense gain a deeper understanding of the basic principles of the entire number system and are better able to make connections with more abstract concepts (e.g., rational numbers) when those concepts are introduced. To develop these key pieces of knowledge, students need multiple opportunities to model solutions to problems with manipulatives and pictures; to develop their own algorithms; and to estimate answers to addition, subtraction, multiplication, and division questions before using and memorizing a formal algorithm.

The following key points can be made about operational sense in the primary years:

- Students' effectiveness in using operations depends on the counting strategies they have available, on their ability to combine and partition numbers, and on their sense of place value.
- Students learn the patterns of the basic operations by learning effective counting strategies, working with patterns on number lines and in hundreds charts, making pictorial representations, and using manipulatives.
- The operations are related to one another in various ways (e.g., addition and subtraction are inverse operations). Students can explore these relationships to help with learning the basic facts and to help in problem solving.
- Students gain a conceptual understanding of the operations when they can work flexibly with algorithms, including those of their own devising, in real contexts and problem-solving situations.

Using knowledge of counting strategies, combining and partitioning, and place value in doing computations

Young students depend on their counting strategies to help them make sense of addition and subtraction. Moving from the counting-all stage to the counting-on stage helps them do simple computations (e.g., when adding 12 and 4, rather than counting all the counters for 12 and then for 4, they count on from 12: 13, 14, 15, 16). Students also need to develop a flexible approach to combining and partitioning numbers in order to fully understand the operations of addition and subtraction. This flexible approach to combining and partitioning numbers involves knowing that two quantities, such as 6 objects and 5 objects, can be combined by partitioning 5 into 1 and 4, combining the 4 with 6 to make one unit of 10, and adding the 1 remaining to make 11. Using a flexible approach also involves, for example, combining 26 objects and 25 objects by partitioning both numbers into their respective tens and ones, combining the tens, combining the ones into tens (if there are enough ones), and then combining the tens with the ones. Another student might solve the same problem by decomposing 26 into 25 and 1, adding the two 25's to make 50 (because he or she knows that two quarters make 50 cents), and then adding the 1 remaining to make 51.

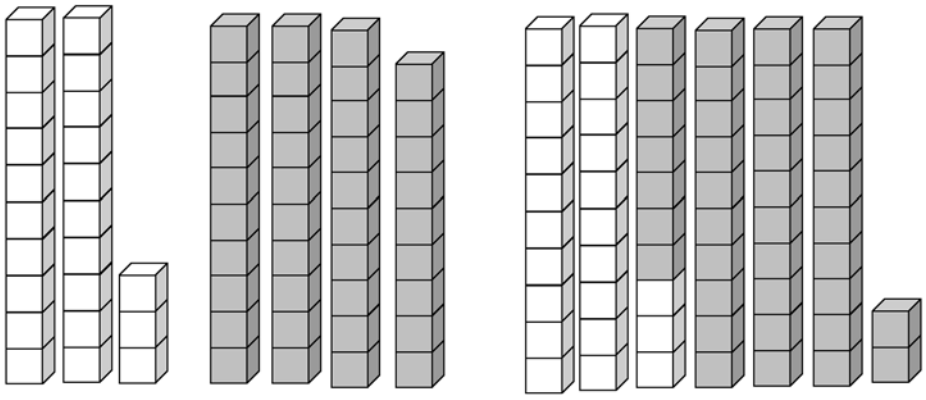
Once students know all the combinations of numbers to 10, they are prepared to work with numbers less than 20 that do not require regrouping (e.g., 10 + 4, 11 + 6, 15 – 5, 16 – 3). After this, they can work with numbers less than 20 that do require regrouping (e.g., 13 + 8, 17 – 9), and having developed this valuable regrouping concept, they can extend it to all other numbers less than 100.

Knowing that 13 + 8 is 21 (the tens and ones are regrouped) is similar to knowing that 33 + 8 = 41 (the tens and ones are again regrouped).

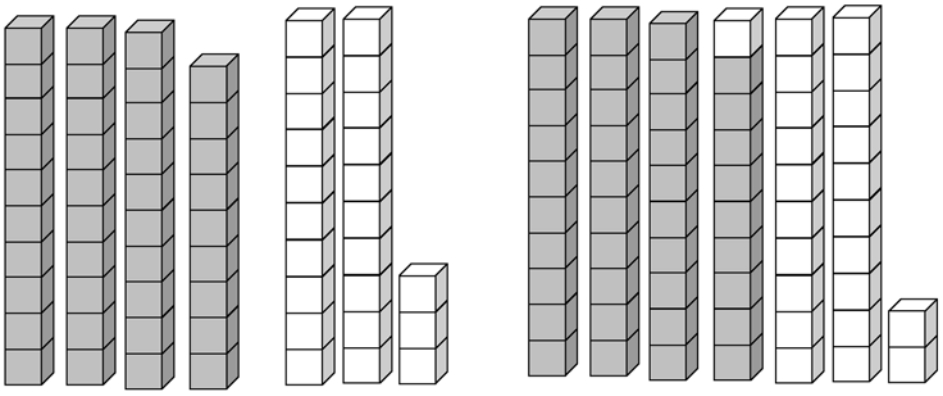
Students who have fully consolidated their understanding of combining and partitioning in addition and subtraction can be introduced to derived facts, which are strategies based on the facts that they already know. For example, they may use one-more-than facts. Students who know a fact such as 5 + 5 = 10 can use this information to solve 6 + 5 as the same as 5 + 5 but with 1 more.

There is significant evidence (Carpenter et al., 1998; Fuson et al., 1997; Kamii & Dominick, 1998) that children develop an enhanced understanding of computations and the place-value system if they are given the opportunities and support to develop their own strategies for solving number problems. When children couple their emerging computational strategies with an understanding of base ten grouping (also called unitizing), they develop very efficient ways of using their understanding of place value to mentally calculate complex computations, such as 23 + 39. Their initial strategies often involve a left-to-right orientation, in that they add the tens first, group what is left of the ones into tens, recalculate all the tens, and then add the ones.

23 + 39 = 23 + 7 + 32 = 30 + 32 = 62



OR 39 + 23 = 39 + 1 + 22 = 40 + 22 = 62



When students develop such methods, they make mental computations more easily, are less prone to making mistakes, and are more likely to recognize mistakes. For instance, students left to their own devices to figure out 29 + 33 may simply add 20 and 30 mentally, then put the 9 and one of the 3 ones together to make another 10, then add on the remaining 2 single units for a total of 62. Students without such flexibility may use the standard algorithm accurately.

But if they misuse the algorithm, they often end up with an answer such as 512 (they put down 12 for 9 + 3 and then put down 5 for 2 + 3) and have no sense that the answer is incorrect.

$$\begin{array}{r} 29 \\ +33 \\ \hline 512 \end{array}$$

Students also need many opportunities to develop models for the operations and to see and understand other students' models. For example, a student might model a question such as 45 + 69 using base ten materials as follows: 4 tens rods combined with 6 tens rods and traded for 1 hundreds flat, then 5 ones units combined with 9 ones units and traded for 1 tens rod with 4 ones units remaining, for a solution of 1 hundreds flat, 1 tens rod, and 4 ones units (114). Another student may model a solution to 43 + 37 using a hundreds chart, beginning at 43 on the chart, moving down 3 spaces to the 73 square and then moving 7 spaces to the right to the 80 square. These two different procedures are equally satisfactory as long as the students can explain their reasoning. Students who can respond flexibly to questions such as the ones just discussed will have a better understanding of how to use standard algorithms because they will know what is happening to the numbers in the algorithm. Experience with making models for the operations also aids students in being able to do two-digit computations mentally, without needing paper and pencil.

Using patterns of numbers to develop operational sense

As a first step in gaining operational sense, students need to have developed an understanding of the efficiency of counting on instead of counting all when combining quantities. They also need to recognize that addition or subtraction represents a movement on a number line or hundreds chart as well as a change in quantity. Students often use patterns and the anchors of 5 and 10 (the relationship of all the numbers from 0 to 10 with 5 and then 10; e.g., 7 is 2 more than 5 and 3 less than 10) to help with computations. They might use compensation patterns (e.g., 9 + 6 = 15 is the same as 10 + 5) to work with familiar numbers that they can easily add. Or they may focus on making tens in either

subtraction or addition (e.g., for 7 + 8, they add 2 to 8 to make 10 and then add on the remaining 5 to make 15; or for 13 – 5 they take away 3 from 13 to make 10 and then take away 2 more for an answer of 8). An understanding of tens can also help as students move to operations with larger numbers (e.g., knowing 10 – 3 helps with 30 – 3). Visual representations and manipulatives (e.g., the number line for counting strategies; the hundreds chart or base ten blocks for operations with larger numbers) are essential aids in helping students to develop an understanding of operations.

Understanding the relationships between operations

Students use the relationships between operations to enhance their computational skills. Subtraction and addition are interconnected as inverse operations, and students often use this inverse relation when first learning subtraction (e.g., solving 8 – 3 is helped by knowing that 5 + 3 = 8). Multiplication can be viewed as repeated addition, and a strategy such as using doubles (e.g., 4 + 4 = 8) is closely related to multiplying by 2. This same strategy can be used with multi-digit computations (e.g., knowing that 4 + 4 = 8 can be extended to knowing that 4 tens and 4 tens are 8 tens, which is 80).

Division can be thought of as repeated subtraction or as equal partitioning or sharing. The relationship of division to fractional sense (e.g., 4 counters divided into 2 groups represents both 4 ÷ 2 = 2 and a whole divided into two halves, each half containing 2 counters) helps students make connections when they move to an understanding of fractions. Multiplication and division are also connected as inverse operations, and this relationship aids in division computations.

Working flexibly with algorithms in problem-solving situations

Students need many experiences with using addition, subtraction, multiplication, and division in problem-solving situations and need experiences in modelling the relationships and actions inherent in these operations before the standard algorithms are introduced, while they are being introduced, and after they have been introduced. It is often in a practical context that students make sense of the operations. For example, a situation in which students make equal groups helps to develop an understanding of partitioning and hence of multiplication and division.

A real context helps to give meaning to such abstract concepts as "remainder". For example, in solving the problem "How many cars are needed to take 23 students on a field trip if each car holds 4 children?", students may give the answer 5 R3 but with probing may recognize that the remainder of 3 represents 3 children with no transportation. Students also need to be given many opportunities to develop their own strategies for working with numbers. When they create their own strategies for computations, they bring more meaning and confidence to their understanding of the standard algorithms when those algorithms are introduced.

Understanding the Properties of the Operations

When teaching students about operations, teachers must recognize the properties of the operations, which they can demonstrate through the use of examples and which students at this grade level understand intuitively. **It is not necessary for students in these grades to know the names of the properties.** Rather, these are properties that the children use naturally as they combine numbers.

The properties of addition include:

- the commutative property (e.g., $1 + 2 = 2 + 1$)
- the associative property [e.g., $(8 + 9) + 2$ is the same as $8 + (9 + 2)$]
- the identity rule (e.g., $1 + 0 = 1$)

The properties of subtraction include:

- the identity rule ($1 - 0 = 1$)

The properties of multiplication include:

- the commutative property (e.g., $2 \times 3 = 3 \times 2$)
- the associative property [e.g., $5 \times (2 \times 6)$ is the same as $(5 \times 2) \times 6$]
- the identity property of whole-number multiplication (e.g., $3 \times 1 = 3$)
- the zero property of multiplication (e.g., $2 \times 0 = 0$)
- the distributive property [e.g., $(2 + 2) \times 3 = (2 \times 3) + (2 \times 3)$]

The properties of division include:

- the identity property (e.g., $5 \div 1 = 5$)

Instruction in the Operations

Specific grade-level descriptions of instructional strategies for operational sense will be given in subsequent pages. The following are general strategies for teaching the operations. Teachers should:

- focus on problem-solving contexts that create a need for computations;
- create situations in which students can solve a variety of problems that relate to an operation (e.g., addition) in many different ways so that they can build confidence and fluency;
- encourage students to use manipulatives or pictorial representations to model the action in the problems;
- allow students to discover their own strategies for solving the problems;
- use open-ended probes and questioning to help students understand what they have done and communicate what they are thinking;
- encourage students' own reasoning strategies;
- prompt students to move to more efficient strategies (e.g., counting on, counting back, using derived facts, making tens);
- most importantly, encourage students to talk about their understandings with the teacher and with their classmates;
- use what they have learned about the mathematical thinking of individual students to "assess on their feet" in order to provide immediate, descriptive feedback to students about their misconceptions and about any of their ideas that need more exploration.

Characteristics of Student Learning and Instructional Strategies by Grade

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- use joining and partitioning strategies to solve problems involving one-digit addition facts and can model addition and subtraction, using manipulatives and drawings;
- recognize part-part-whole patterns for numbers (e.g., 7 as 3 and 4, 2 and 5, or 1 and 6) – an important prerequisite to working with addition and subtraction;
- are able to model the grouping of ones into tens and to calculate numbers on the basis of groupings of tens and ones (e.g., can represent 22 as two bundles of 10 and 2 ones units and know that if they take one bundle of 10 away, they have 12 remaining);

- may have difficulty in finding missing addends or missing subtrahends, or in making comparisons (e.g., 3 + 4 = □ + 2);
- begin to create their own strategies for addition and subtraction, using grouping techniques for tens;
- can use some of the strategies for learning the basic facts to help with mathematical computations. For example, they use the doubles strategy (e.g., 6 is the egg-carton double: 6 on each side doubled makes 12; 4 is the spider double: 4 on each side doubled makes 8) and the strategy of doubles plus one (e.g., knowing that 5 + 5 = 10 helps with knowing that 5 + 6 must be the same as 5 + 5 = 10 plus 1 more, or 11).

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing experiences with part-part-whole relationships (e.g., using counters, blocks, number lines, rekenreks);
- providing experiences with number lines and hundreds charts, and experiences with movement on the number lines and charts to represent addition and subtraction questions;
- providing opportunities to use concrete materials to represent problems that involve addition and subtraction;
- providing addition and subtraction tasks involving screened (hidden) groups;
- providing opportunities to identify subtraction as both a counting-up procedure (counting up is easier for some students) and a counting-down procedure;
- providing opportunities to use both vertical and horizontal formats for addition and subtraction so that students rely on their own mental strategies and not just the formal algorithms;
- having them use the calculator to make predictions and to self-correct as they work with the operations;
- providing opportunities to create their own strategies for adding and taking away numbers – strategies that will often involve using what they know to find out what they do not know (e.g., they may extrapolate from knowing that 8 + 2 is 10 to knowing that 8 + 3 is the same as 8 + 2 + 1 more = 11);
- supporting them in identifying, developing, and describing useful strategies for solving addition and subtraction problems – for example, using known facts, using doubles (which are often readily remembered), making tens, using compensation, counting up, counting down, using a number line or hundreds chart, using the commutative property of addition, using the inverse relationship of addition and subtraction, using 0 and 1 in

both addition (0 plus any number equals that number; 1 plus any number equals the next number in the number sequence) and subtraction (any number minus 0 equals that number; any number minus 1 equals the previous number in the number sequence);

- using everyday situations as contexts for problems (calculating milk money, taking attendance) and/or using real-life contexts for problems (e.g., "How many players are on your soccer team? How many would there be if 5 players quit?");
- providing opportunities to discuss their solutions in social contexts with their classmates and with the teacher;
- providing opportunities to write about the problems and to connect solutions with the appropriate algorithms;
- presenting traditional algorithms through guided mathematics, including a focus on the meaning behind the algorithms, the use of models to demonstrate the algorithm, and instruction that addresses students' misunderstandings of the equals symbol (e.g., in the question 2 + ☐ = 5, students may supply 7 as the missing addend);
- encouraging them to use estimating strategies (e.g., "Do you think this cookie jar would hold about 20 cookies or about 100 cookies?");
- encouraging them to use self-initiated drawings and representations of the operations.

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- understand the four basic operations of whole numbers and represent them using visuals and concrete objects;
- are able to use a range of flexible strategies to solve double-digit addition and subtraction problems, although they may have difficulty in recording such strategies. In using standard algorithms, they may have difficulty with regrouping across 0's;
- begin to use a range of "non-count-by-one" strategies (strategies other than counting all the objects one by one) – for example: using commutativity, making doubles, using compensation, using known facts, using doubles, making tens, counting up from, counting up to, counting down from) – although these strategies may not yet be fully consolidated, particularly with regard to larger numbers. In a subtraction question such as 10 – 3, students are able to count down from 10, keeping track of the 3 backwards counts. In the case of

missing addends, they may misinterpret symbols (e.g., for 7 + □ = 8, may give the missing addend incorrectly as 15);

- develop initial multiplication and division knowledge related to making equal groups, making equal shares, combining equal groups, sharing equally and finding the number in one share, making an array, or determining how many dots are in an array.

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing meaningful experiences with number lines and hundreds charts – experiences in which they use movement and patterns on the lines and charts to represent addition and subtraction questions;
- providing opportunities to identify subtraction as both a counting-up procedure (e.g., solving 15 – 11 by counting up from 11) and a counting-down procedure. Counting up is easier for some students;
- providing opportunities to use both vertical and horizontal formats for addition and subtraction so that students rely on their own mental strategies and not just on the formal algorithms;
- having them use a calculator to make predictions and to self-correct as they work with the operations;
- supporting them in identifying, developing, and describing mental strategies for solving addition and subtraction problems (e.g., using known facts, using doubles);
- providing opportunities to develop their own algorithms and to use algorithms flexibly. With experience, students are able to construct and understand their own algorithms for solving problems that require adding and subtracting two-digit numbers and to justify and explain their methods. For example, in response to a problem that involves adding the numbers 55 and 69, their method may be the following: "I added 50 and 50 to get 100, then added on the extra 10 (from the 60) to get 110; then I added another 10 (the 9 plus 1 from the 5) to get 120, and then I added on the leftover 4 to get 124." As long as students can justify and explain their methods, they should be allowed to use them. These original algorithms, although they may be longer, help students to develop good number sense, which they can apply later to more formal algorithms. Moreover, students who develop their own algorithms are much more likely to make sense of a formal algorithm, and are able to compare methods and see which method is more efficient.

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- use a range of strategies for addition and subtraction. They use doubles and near doubles to work out facts to 9, and can extend these two strategies into the teens and decades (e.g., may use the knowledge of 3 + 4 = 7 to figure out 13 + 4 = 17);
- are able to use grouping by 5's and 10's to add or subtract more efficiently. They use compensation or partitioning strategies for calculating sums and differences (e.g., they might solve 17 – 9 by taking 10 from the 17 and then adding 1 more to the difference to make 8). They make links between their understanding of single-digit number facts to calculate questions involving the decades (e.g., they might immediately know that 21 plus 8 is 29 because they know that 1 plus 8 is 9). They use combinations of 10, or all the facts that make 10 (0 + 10, 1 + 9, 2 + 8, . . .) so that, in calculating 18 + 6, they add 18 and 2 to move up to the decade of 20, and then add on the remaining 4 ones to make 24;
- can increase and decrease numbers to and from 100 by tens so that they quickly recognize that 82 + 10 is 92 or 75 – 10 is 65 without having to complete the pencil-and-paper algorithm. They can extend this strategy to the addition of three-digit numbers so that they immediately recognize that 111 + 10 is 121. They may be able to mentally subtract and add two- and three-digit numbers, particularly if they have had many opportunities to understand place value and how it can be used flexibly in mental computations;
- are able to use counting on, counting back, and counting forward by 1's, 2's, 5's, and 10's as strategies for solving problems requiring one- and two-digit addition and subtraction and one-digit multiplication;
- may have difficulty with the subtraction or addition of numbers involving 0 in the tens place in a three-digit number, particularly where regrouping across decades is required;
- are able to use a range of strategies for solving problems requiring addition and subtraction – for example, using known facts, using doubles, making tens, using compensation strategies, counting up, counting down to find unknown facts, using the inverse relationship of addition and subtraction, using the commutative (e.g., 1 + 2 = 2 + 1) and associative [e.g., 80 + 30 + 20 can be calculated as (80 + 20) + 30] properties of addition, using 0 and 1 in both addition (0 plus any number equals that number; 1 plus any number equals the next number in the number sequence) and subtraction (any number minus 0 equals that number; any number minus 1 equals the previous number in the number sequence);

- are able to multiply and divide using one-digit whole numbers and select the appropriate operation in problem-solving situations. They are able to use skip counting and repeated addition or subtraction to solve multiplication and division tasks.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- promoting the development of a conceptual understanding of multiplication and division by solving problems using models with manipulatives and pictures;
- providing experiences using 0 and 1 in both addition (0 plus any number equals that number; 1 plus any number equals the next number in the number sequence) and subtraction (any number minus 0 equals that number; any number minus 1 equals the previous number in the number sequence);
- providing experiences with multiplication and division using arrays and repeated addition or subtraction;
- providing opportunities to develop their own algorithms, using both written and mental methods to find the answers to computation questions;
- providing opportunities to discuss, develop, and explain their student-generated algorithms in social contexts and to hear the explanations of strategies that other students use;
- guiding them in monitoring their own learning, to help them identify and communicate their own strategies for solving problems;
- teaching traditional algorithms through guided mathematics, including a focus on the meaning behind the algorithms and the use of models to demonstrate the algorithms, with reference back to student-generated algorithms;
- encouraging the use of estimating strategies (e.g., ask, "Do you think this cookie jar would hold about 20 cookies or about 100 cookies?");
- using everyday situations as contexts for problems or using real-life contexts for problems;
- having them use drawings and a variety of representations to illustrate the operations;
- using known facts to derive new facts. For example, use 5 times 6 is 30 to help calculate 6 times 6 (just add one more 6 to the 30);
- frequently using and discussing mental math strategies, such as partitioning (e.g., calculate 8×9 by multiplying 8×10 and then subtracting the extra 8 from the product to make 72);

- having them link concrete and pictorial representation with the written form of an operation or equation;
- providing experiences with division that involve either fair sharing (e.g., dividing 24 candies among 4 friends) or repeated subtraction (e.g., 24 candies are to be eaten in equal numbers over 6 days. How many will be eaten per day?);
- using "chunking" strategies, such as partitioning out 25's and 50's (e.g., to calculate 332 + 227, they pull out 325 and 225 to make 550, and then add on the remaining 7 and 2 to make a total of 559);
- providing opportunities to use calculators to explore the effects of changing numerals in an operation and to identify the patterns that occur as the numerals are changed (e.g., add 10, 100, 1000, . . . to a number and identify the pattern in the answers: 10 + 3 = 13, 100 + 3 = 103, 1000 + 3 = 1003, 10 000 + 3 = 10 003, . . .);
- providing opportunities to build and use multiplication charts and to identify the patterns that occur in the charts;
- providing opportunities to identify multiplication fact patterns in hundreds charts;
- providing sufficient opportunities so that, with experience, students are able to construct and understand their own algorithms for solving problems requiring two- and three-digit addition and subtraction to justify and explain their methods. For example, in response to the problem "Jane collected 203 pop can tabs and Julie collected 318. How many did they have altogether?", students can use their own flexible algorithms to find and share a solution. One student may give this method: "I took 18 from the 318 and 3 from the 200; then I added the 200 and the 300 to get 500; then I added the 18 and 2 (of the 3 ones) to get 20, so my answer is 520 plus the extra 1 (from the original 3 ones) or 521." Another child may respond by adding 18 and 3 to make 21 and then adding the 200 + 300 = 500 to the 21 to make 521. As long as students can justify and explain their methods, they should be allowed to use them. By developing and understanding their own algorithms first, students are much more likely to make sense of more formal algorithms and be able to compare various methods and see which method is more efficient. Having students share their methods with one another is also important in fostering new insights and approaches.

Quantity

Overview

Children learn about the quantitative world long before they enter school. They know when something is bigger, smaller, more, less, the same as, and so on. Even toddlers recognize that there is more in a whole cookie than in half a cookie, and express their preference. However, having an initial sense of quantity is distinct from being able to count. In time, children make the link between the counting words and the quantities that they represent, but children do not understand intuitively that numbers and quantity are related in many different ways.

The following are key points that can be made about quantity in the primary years:

- Quantity represents the "howmuchness" of a number and is a crucial concept in developing number sense.
- Understanding the idea of quantity is an important prerequisite to understanding place value, the operations, and fractions.
- An early understanding of quantity helps students estimate and reason with numbers. It is particularly important for understanding relative magnitude (e.g., few, many) and for proportional reasoning (e.g., bigger, smaller, twice as big, half as big).

Quantity as "howmuchness"

As children learn to count, they do not at first recognize that their count of 1, 2, 3, 4, 5 actually corresponds to a quantity of 5 things. Quantity is a complex quality of number. The idea that a quantity of 2 apples is the same as a quantity of 2 loaves of bread and that 1 apple and 1 loaf of bread also represent the concept of 2 is not necessarily intuitive. Neither is the idea of how such quantitative relationships apply when units of measure are used – litres

or millilitres of water (e.g., 1 L vs 1 mL), distance in kilometres or metres (e.g., 1 km vs 1 m), and so on.

Quantity as it relates to understanding operations, place value, and fractions

An understanding of the quantity represented by a number helps students understand the meaning behind an operation. In the operation of addition, the quantities increase as numbers are added together and in subtraction the quantities decrease. Making the connection between the increase or decrease in quantity that occurs in an operation is important for making sense of mathematical problems. The concept of quantity is also important for students' understanding of what is represented in the placement of numerals in a number.

The fact that the numeral to the left of any digit in a number represents an increase by a rate of 10 and that the numeral to the right of any digit represents a decrease by a rate of 10 is an important concept for students, especially as they begin to work with larger numbers. It also has significant implications for their later work with decimals, in Grades 4 to 6. A sense of quantity is also important as students begin to work with fractions. Students often have difficulty with fractions because they view the two numerals in a fraction such as $\frac{2}{3}$ as two separate whole numbers rather than as a relationship between a part of a quantity and the whole quantity.

Quantity as it relates to estimating and reasoning with numbers, including relative magnitudes, proportional reasoning, and rational numbers

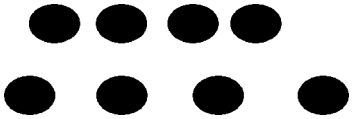
Over time, a sense of quantity becomes crucial in developing estimation skills, a sense of magnitude, a sense of proportion, and an understanding of rational numbers. Estimation skills develop as students become aware of the relationships between quantities (e.g., "Is the amount closer to 10 or to 20"?). The sense of quantity extends to a sense of magnitude – for example, in recognizing that the magnitude of a quantity is affected by movement (e.g., movement on a number line – right or left; movement on a thermometer – up or down; and cyclic movement – as on a clock, clockwise or counterclockwise). Understanding the consistent relationship between quantities (e.g., in doubling a recipe) sets the stage for proportional reasoning. Dividing quantities into parts of a whole provides a concrete introduction to rational numbers.

Understanding Quantity

The following information is provided as background information to help teachers increase their understanding of the concept of quantity.

Conservation of Number: Piaget's (1965) work with young children revealed a misconception on their part about quantity. Blocks arranged on a table were moved farther away from one another, and the children were asked if the quantity was still the same or if it had changed. The children responded that there were now more blocks. They did not understand the conservation of number – that is, that the quantity stays the same even when the items are spread out to look like more or, conversely, that the quantity can be changed only by adding to it or removing from it.

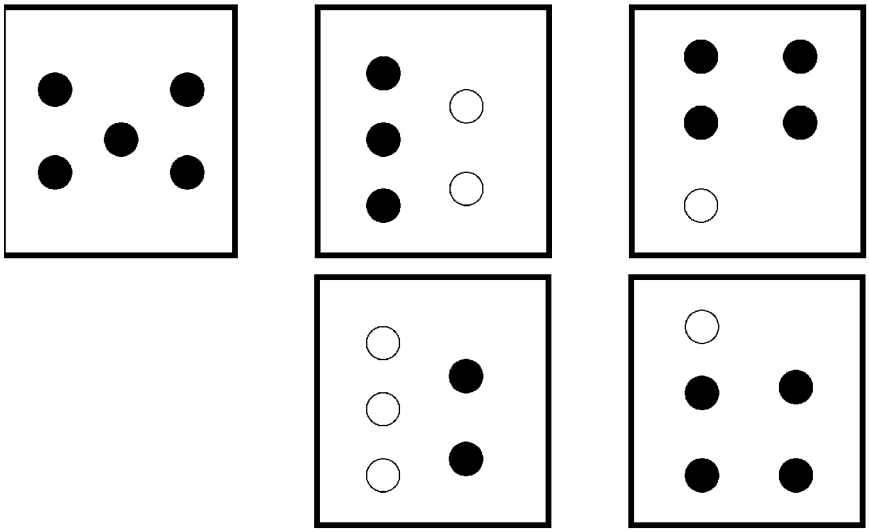
Which row has more?



Cardinality: When children develop an understanding that the last number of the count of a set of objects actually represents the number of objects in the set (cardinality of a number), they have begun to understand quantity (see also "Counting").

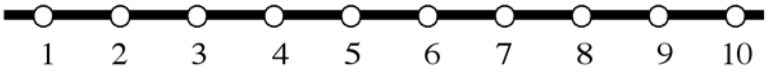
Subitizing: Another indication that children are consolidating the concept of quantity is their ability to recognize small quantities without having to count each of the objects (subitizing) – for example, knowing that a collection holds 5 beads without having to count them. Dot cards can be helpful in assessing students' ability to subitize.

Dot cards for 5



Magnitude and Quantity: Understanding of quantity is linked to the recognition that movement forward or backwards – for example, on a number line, a clock, or a scale – means an increase or a decrease in quantity.

Movement along a number line



Part-Part-Whole: Even when young children have developed the capability to count to numbers such as 10, they do not necessarily understand that the quantities represented by such numbers are made up of different parts. In fact, they will often dispute the assertion that 4 counters and 3 counters joined together to make a whole set of 7 counters is the same amount as 2 counters and 5 counters joined together to make another set of 7. Students develop an understanding that a whole – as represented by 7 counters, for example – can be separated into parts – consisting, for example, of 3 counters and 4 counters or 2 counters and 5 counters.

Anchors of 5 and 10: Students develop an understanding that numbers such as 2 can be thought of in relation to 5 (e.g., as 3 less than 5 on a five frame). Ten can be a very important anchor for students, as it helps them understand many other number relationships. Remembering the combinations that make 10 (e.g., 6 + 4, 7 + 3) and recognizing that some numbers can be a combination of 10 and another number (e.g., 12 is 10 and 2 more, 13 is 10 and 3 more) are useful skills for further development of number sense.

Decomposition and Composition of Numbers: Students develop quantitative understanding when they begin to recognize that a number such as 25 can be decomposed into 2 tens and 5 ones, or 1 ten and 15 ones, or 25 ones. Eventually students recognize that all numbers can be composed, decomposed, and recomposed. This can help them to understand how the operations (addition, subtraction, multiplication, and division) act on numbers.

Estimation Skills: Estimation skills are related in complex ways to an understanding of quantity. They help students with applying logic and reasoning in problem-solving situations. Without estimation skills, students cannot judge the appropriateness of an answer. Students who automatically make a guess of 100 for the number of objects in any large (or small) container holding many objects do not have strategies for making logical estimates. A related difficulty that students may have is recognizing that if two containers of the same size are filled, one with larger objects and one with smaller objects, the total number of objects in the container filled with larger objects will be less than the total number of objects in the container filled with smaller objects.

Proportions: The concept of quantity is also related to that of proportion and, in particular, to the use of proportion in problem solving. Knowing that 3 objects and 4 objects together make 7 objects is information that students can learn to transfer to other kinds of quantities, enabling them to recognize, for example, that 3 L plus 4 L makes 7 L or that 3 mL plus 4 mL makes 7 mL.

Rational Numbers: The concept of quantity is further developed through an understanding of the amounts represented by rational numbers. Being able to understand fractions and decimals as parts of a whole is an aspect of understanding quantity.

Characteristics of Student Learning and Instructional Strategies by Grade

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- match quantitative and numerical terms, such as "2 more" and "1 less";
- begin to reason about changes in magnitude, using the same quantities applied to different objects. For instance, they begin to recognize that the same reasoning used to solve a problem in which 4 cats plus 2 more cats provides a total of 6 cats can be applied to calculating a distance of 4 cm plus 2 cm or a time period of 4 hours plus 2 hours. They also recognize that this change in magnitude is represented by the same symbolic system: 4 + 2 = 6. This recognition is a significant milestone in their understanding (Griffin, Case, & Siegler, 1994);
- integrate their growing understanding of relative amount with their counting skill to determine whether a quantity is more or less, and by how much. For example, presented with two containers, one containing 3 blocks and the other containing 7 blocks, they can combine counting and the concept of quantity to determine which container holds more, and how many more. They automatically see (without needing to count) that the one container holds 3 blocks, and then they may count the 7 blocks in the other container to determine that it has 4 blocks more;
- build an understanding that a quantity such as 4 can be demonstrated by 4 objects or by a movement of 4 spaces along a sequence. This understanding introduces the concept of measurement;
- continue to develop facility with subitizing in the range of 1 to 7 or 8;

- build on their understanding of 5 to begin working efficiently with quantities in relation to 10 – for example, decomposing 10 or numbers less than 10 into their component parts;
- use manipulatives such as ten frames, rekenreks, groupings of counters, or base ten materials to show quantities – for example, to show 32, they might use 3 full frames and 2 loose counters;
- consolidate their understanding of quantities of 10 in relation to the teens and the decades. This understanding of 10 as a quantity and of the digit 1 in 10 as representing a bundle of 10 is crucial to understanding concepts involving larger two-digit numbers;
- estimate small numbers, although in their initial experiences they may use random or inefficient strategies to make reasonable estimates. Such randomness can be reduced by giving students opportunities to establish benchmarks for numbers, as illustrated in this example: seeing that 5 blocks fill a cup and 100 blocks fill a bucket helps students to recognize that a container that is larger than the cup and smaller than the bucket will contain between 5 and 100 blocks.

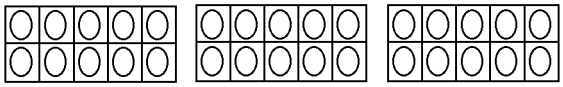
Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- continuing to provide opportunities to use their fingers and hands to build up the concepts of 5 and 10, particularly in finger plays or when doing problem-solving tasks and singing songs. Students benefit from explorations in which they make comparisons that help them to think about how many more fingers they might need to show other amounts, such as 13;
- providing experiences with quantities of up to 50 using concrete materials and pictures in real-life or socio-dramatic situations:
  - Set up a store in a socio-dramatic area of the classroom. "Sell" objects in set quantities, assigning prices such as $1, $5, and $10 for sets of 1, 5, and 10.
  - Have children calculate the cost of items for the classroom, such as pencils, erasers, or stickers.
  - Play barrier games using different quantities of materials and have students make a conjecture about the number of items their partner is holding: "Do you have 10 objects? Do you have more than 10? Less than 50?", and so on;
- providing experiences with estimating, using concrete materials and pictures, and opportunities to use benchmarks to help determine a range of numbers (or values). Teacher- or student-initiated questions that limit the range of

possibilities for determining an amount are helpful with this (e.g., "Is it less than or more than 10?"; "Is it closer to 5 or closer to 20?");

- providing experiences that repeat the same types of estimation activities so that students can build up their conceptual understanding of the amount of something familiar. For example, ask them to estimate the number of cans in the recycling bin. Give them a benchmark (e.g., "Remember that there were 10 yesterday and it was full on the bottom");
- providing experiences with the numbers 5 and 10 to consolidate an understanding of those numbers as anchors for the numbers below and above them. For example, use ten frames to show 30:



- linking instruction related to the big idea of counting with the concept of quantity so that the two ideas are developed simultaneously;
- providing experiences of "divvying up" various amounts to give students informal experiences with fractions of sets before they have to attach such experiences to formal notation.

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- use their facility with the quantities of 5 and 10 to estimate the relative size of other quantities. For example, they can say whether a quantity is closer to 10 or to 1 or whether a larger quantity is closer to 10 or to 20;
- develop a quantitative understanding of 25 and 50 when dealing with larger numbers (particularly through their experiences with money);
- continue to expand on their understanding of changes in magnitude, using the same quantities applied to different objects. For example, they recognize that the reasoning used to combine 20 cars plus 10 cars can be applied to calculating a distance of 20 km plus 10 km, and that this change in magnitude is represented in the same symbolic manner: 20 + 10;
- develop reasoning strategies for determining whether an answer makes logical sense – for example, knowing that if 10 objects are combined with more objects, a solution of more than 10 is required;
- continue to develop facility with subitizing (telling at a glance how many objects there are, without having to count them) in the range of 1 to 7 or 8. They may be able to quickly tell the quantity if objects are grouped together in ten frames or five frames;

- recognize patterns in quantities. For example, if they quickly view 4 filled ten frames and 2 more counters, which are then covered, they are able to duplicate what they saw or state that the quantity was 42;
- recognize that a number such as 42 represents a quantity of four groups of 10 and one group of 2;
- learn to make tens to help in working with quantities. For instance, they recognize the smaller numbers in 10 and can use this knowledge to add quantities such as 12 and 9. They know that 2 and 8 make 10, so 12 and 8 make 20, and they add on the remaining 1 to make a total of 21;
- identify and describe equal-sized parts of the whole (halves and fourths or quarters)
- develop an understanding that fractions are numbers that represent part of a whole and that fractional parts can be regrouped to make wholes, for example: two halves make a whole, three thirds make a whole, four fourths make a whole, and so on;
- develop an understanding that, as a consequence of splitting a whole into equal parts, there are more parts, but these parts are all smaller than the whole split into fewer parts;
- begin to understand that quantity is closely tied to the placement of numerals (e.g., in a two-digit number);
- estimate amounts using quantitative and numerical reasoning. For example, if a box holds 10 blocks, then a box that looks about twice as big will hold about 20 blocks.

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- continuing to provide opportunities to estimate, using concrete materials and pictures in problem-solving situations;
- providing experiences with estimation strategies using grouping of tens and hundreds. For example, in response to a question about how many students are in the primary grades, they use the knowledge that there are about 2 groups of 10 people in each classroom to estimate how many students there are in 4 classrooms;
- providing opportunities to use estimation in mental-math situations (e.g., estimating how many apples the class might have in their lunch bags and recognizing that the number would probably be the same as or less than the quantity of students);

- providing experiences with teacher- or student-initiated questions that limit the range of possibilities for determining an amount (e.g., "Does the amount seem closer to 5 or to 50? Could you hold this many blocks in your hands? In a cup?");
- using benchmarks such as 5, 10, 25, and 50 to make estimates (e.g., ask, "If this pile has 5 pennies,* how many do you think are in that pile? Is it less than or more than 20? Is it closer to 25 or closer to 50?");
- providing experiences with the number 10 and the bundling of tens into hundreds to consolidate an understanding of the importance of 10 in our place-value system;
- providing experiences with manipulatives and ten frames to build up an understanding of 10 as an anchor for all other numbers in our place-value system, and linking such grouped quantities with two-digit numbers.
- using "fair-sharing" problems that relate to students' prior personal knowledge – for example, how to share a chocolate bar with 12 pieces among 3 people. Have the students try to think about what they will call each of the pieces so that the pieces can be distinguished from the whole chocolate bar.
- providing experiences with games using pattern blocks, fraction blocks, and/or Cuisenaire rods to model fractions and to compare fractions as parts of a whole;
- providing experiences in informal problem-solving situations using fractions. Ask students how they would divide 8 pizzas among 6 people. Use manipulatives such as toy people or drawn people and 8 paper plates as pizzas, and let students come up with their own solutions;
- providing experiences with the incidental naming of fractional amounts, especially common fractions that students are less likely to hear in their everyday activity, such as $\frac{1}{3}$ or $\frac{1}{4}$ (e.g., say, "I am going to give each of these children $\frac{1}{4}$ of these balls").

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- continue to develop a conceptual understanding of quantities of one- and two-digit numbers and extrapolate from this to an understanding of larger three- and four-digit numbers;

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

- use the sense of quantity to further their estimation skills (e.g., estimating how high a stack of 1000 quarters would be or how many stars they could draw in one minute);
- recognize patterns in larger quantities than in Grade 2. For example, if they quickly view 4 hundreds flats and 2 more counters, which are then covered, they are able to duplicate what they saw or state that the quantity was 402;
- estimate large and small quantities by using their knowledge of ones, tens, and hundreds or making other logical assumptions about quantity. They use their understanding of number to solve problems and check the solutions for reasonableness. This understanding of quantity is important for later work in decimal notation (e.g., with money);
- estimate large numbers, although they may still experience some confusion if they do not have benchmarks to help them determine amounts;
- often do not have a good intuitive sense of what happens to the denominator in a fraction if the digit in the denominator is replaced by a larger or smaller digit (i.e., does it represent a larger or small quantity?);
- may begin to memorize information – for example, that $\frac{1}{2}$ is greater than $\frac{1}{3}$ – but have difficulty in applying it as they begin to think of fractions in terms of symbols rather than as concrete parts of objects – for example, they may forget that $\frac{1}{2}$ of a very short ribbon will be less than $\frac{1}{3}$ of a longer ribbon.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing experiences with estimating, using concrete materials (e.g., base ten blocks and money) and pictures in problem-solving situations;
- providing opportunities to use estimation in mental-math situations;
- providing experiences with teacher- or student-initiated questions that limit the range of possibilities for determining an amount (e.g., "Is it between 50 and 100 or 100 and 200?");
- using "nice numbers" such as 5, 10, 25, 50, 100, 1000 to make estimates (e.g., ask, "If this pile has 50 pennies,* how many do you think are in that pile? Is it less than or more than 50? Is it closer to 100 or closer to 50?");
- providing experiences with estimation strategies (e.g., grouping in tens and/or hundreds, rounding to tens and to hundreds);

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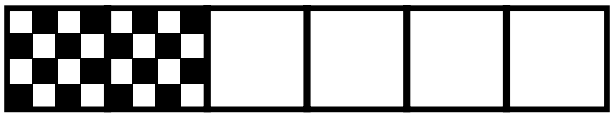
- providing experiences with the numbers 10 and 100 to consolidate an understanding of the importance of 10 and 100 in our place-value system;
- providing experiences with numerals and amounts of 10, 100, and 1000, using manipulatives and place-value charts to build up an understanding of 10, 100, and 1000 as anchors for all other numbers in our place-value system;
- using "fair-sharing" problems that relate to students' prior personal knowledge – for example, how to share a chocolate bar among 3 people. Have the students try to think about what they will call each of the pieces so that the pieces can be distinguished from the whole chocolate bar;
- providing opportunities to use fraction manipulatives – pattern blocks, Cuisenaire rods, and fraction blocks – to explore and represent fractions and mixed numbers. The overuse of any one representation (e.g., pizza, pie) may create difficulties in understanding other types of models (e.g., number-line representations, rectangles);
- using fraction problems. For example, give this problem: Ellen has 2 cupcakes that she wants to share with 3 friends. Then probe with questions (e.g., "Will each of the friends be able to get at least one whole cupcake? Why or why not? Will they be able to get $\frac{1}{2}$ or more of a cupcake? Why or why not?");
- using prompts to remind students that:
  - when the fraction represents a region (area model), then the fraction portions must be of equal size;
  - when using fractions to describe sets (set model), the objects in the sets may be different sizes (e.g., if you say $\frac{1}{2}$ of the class is boys, not all of the boys will be the same size);
  - a fraction represents a relationship, not a particular amount. It is important for students to know that $\frac{1}{2}$ of a small amount may be much smaller than $\frac{1}{3}$ of a large amount;
  - a fraction represents part of the whole. Students often make the mistake of comparing a part with the remaining parts. For example, when asked, "What fraction of the grid is chequered?", a student might reply $\frac{2}{4}$ instead of $\frac{2}{6}$;



- using labelled fractions in the classroom (e.g., labelling one of 6 windows as $\frac{1}{6}$ of the windows in the room);
- illustrate fractional amounts posted on number lines (e.g., show $\frac{1}{2}$ mark at 50 on a 0 to 100 number line);

- encouraging students to develop a mental image of fractional amounts, to help with reasoning in problem-solving activities that involve fractions (e.g., ask, "What does this sheet of paper look like when it is divided in halves? In fourths? In eighths?");
- providing experiences "divvying up" amounts between or among friends to develop the concept of parts of a set, and linking this with $\frac{1}{2}$ or $\frac{1}{3}$, and so on.

Relationships

> Since mathematics is essentially the study of systematic patterns of relationships, any activity which helps people to recognize relations is to be welcomed.
>
> (Lovell, 1971, p. 155)

Overview

Understanding the relationships between numbers helps children make powerful connections in mathematics. These relationships are most clearly manifested in the patterns in numbers. Children who know the numbers from 1 to 10 and recognize the relationship of each of these numbers to the others in that sequence are at a considerable advantage when they look at other patterns in the number system that reflect this relationship (e.g., the relationship between 1 and 5 is similar to the relationship between 21 and 25). Other patterns also become evident as children work with numbers on hundreds charts or number lines (e.g., odd and even numbers). The importance of recognizing the relationships between numbers extends throughout the elementary curriculum (e.g., in larger numbers, rational numbers, integers). A firm understanding of the relationships between whole numbers provides a strong foundation for the mathematics introduced in the later grades.

The following are key points that can be made about number relationships in the primary years:

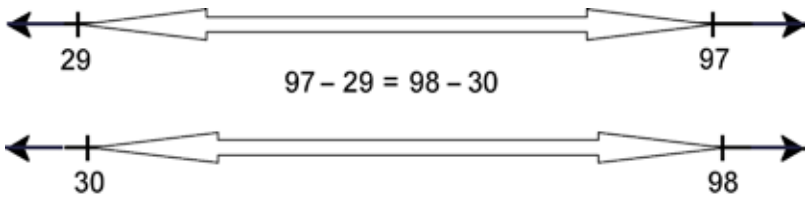
- An understanding of number depends upon a recognition of how numbers are related to one another for purposes of comparing or ordering. For instance, numbers increase as they move up (to the right) on the number line and decrease as they move down (or to the left).

- Number sense depends upon a recognition and an understanding of the patterns in numbers, particularly the pattern of the 1 to 9 counting sequence and the way in which this pattern is repeated in the decades and later in the 100's, 1000's, and so on.
- When children compose and decompose numbers and perform operations, they see how numbers relate to one another, and their understanding of number is enhanced. For example, students should recognize that 7 can be thought of as 5 and 2 more or conversely as 10 and 3 less.
- Knowing the anchors 5 and 10 and how to relate other numbers to these anchors provides the foundation for work with operations and place value.

Relationships in ordering and comparing

Evidence of an early understanding of the relationship between numbers is the ability to compare quantities in terms of "more", "less", or "the same as". This understanding is related to the later understanding that children develop of the relationship of 1 more than, 1 less than, 2 more than, or 2 less than. As well, the comparison concept of part-part-whole can be developed as students move through the primary years to understand fractions and their relationships to wholes of varying sizes or amounts.

Students who are confident at working with the relationships in ordering numbers on a number line can use this knowledge in creative ways to solve problems and do basic computations. For instance, students who know the relationships on the number line can use this information to think about 97 – 29. They can recognize that the distance between 97 and 29 is the same as that between 98 (1 more than 97) and 30 (1 more than 29) and make the easier calculation of 98 – 30 (or count up from 30 to 90 and then to 98, as follows: 40, 50, 60, 70, 80, 90, 91, 92, 93, 94, 95, 96, 97, 98).



Relationships in number patterns

Being able to identify the repetitive pattern of the count of 1 to 9 and then recognizing that this pattern extends throughout the number system helps children with counting and with understanding the actions on numbers (the operations).

Conceptual understanding of number is further developed through the use of patterns on hundreds charts and number lines. Children can learn to identify the pattern of 10's on the hundreds chart. Knowing this pattern helps them work with two-digit numbers. Other relationship patterns that children can identify are even and odd numbers; multiples of 1, 2, 3, . . . , 10; more difficult multiples (e.g., of 11 and 25); addition of two-digit numbers (e.g., adding 21 to any number involves moving vertically down two spaces from the number and horizontally to the right one space); subtraction of two-digit numbers (e.g., subtracting 21 from any number involves moving vertically up two spaces and horizontally to the left one space).

Hundreds chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The example illustrated in this hundreds chart involves adding 21 to 25.

Relationships in understanding and performing operations

Knowing the relationships between numbers helps students understand basic operations. For instance, students who know the 2 times table can use their understanding of the relationship of 2 to 3 to determine the 3 times table. In other words, if they know that $2 \times 8 = 16$, they can determine 3×8 by adding one more 8 to the 16. Similarly, the relationship between 2 and 4 can help to determine the 4 times tables (e.g., 4 is double 2, so 4×6 is the same as 2×6 doubled, or 24).

Furthermore, the operations themselves are related, and when students understand how the operations are related, they may find more efficient ways of learning and working with them. Addition and subtraction are related as inverse operations, as are multiplication and division. Also, subtraction and division are related (division can be repeated subtraction) and multiplication and addition connect in a similar way (multiplication can be repeated addition). Looking at the relationships between these operations often can provide multiple approaches to a problem.

Relationships to the anchors of 5 and 10

Children discover relationships between numbers, beginning with their first experiences of counting their fingers. This finger counting helps them to see the relationship of the numbers 1–4 to 5, and understanding that relationship (of 1–4 to 5) helps them to recognize that a five frame with counters in all but one square represents the number 4. This understanding of the relationship of 1–4 to 5 is later extended to an understanding of the relationship between all the numbers from 0 to 10 and, particularly, the relationship of the numbers from 0 to 10 with the anchors of 5 and 10. This understanding is an important foundation for an understanding of the larger numbers. If students have worked with combinations of 10, they have a better understanding of problems that involve operations with larger numbers. For example, in adding 27 and 35, if they know that 7 is 3 away from 10, they can take the 3 from 35, add it to 27 to make 30, add on the 30, and then add the 2 that remains. This understanding of the 10 relationships can be extended to an understanding of 100's, and the patterns for 1 to 9 provide a useful guidepost for learning how to count by 100's and then by 1000's.

Understanding the relationships of other numbers to these anchor numbers of 5 and 10 helps students later to understand place value (see also "Representations" for a discussion of place value). The relationships between 1's and 10's, 10's and 100's, 100's and 1000's, and so on, are important in understanding number and the operations on number. Using base ten materials or other proportional manipulatives helps students see the relationships between these quantities. The base ten materials consist of single units (ones), rods (tens), flats (hundreds), and cubes (thousands). Ten of the ones units are put together to form 1 tens rod, 10 rods are put together to form 1 hundreds flat, and 10 flats of hundreds form a thousands cube. Knowing

the relationships in place value (as the digits move to the left, numbers go up by a factor of 10; as digits move to the right, numbers decrease by a factor of 10) helps students work efficiently and effectively with numbers. It is important to note that, although proportional manipulatives help students develop their concept of place value, the concept is not inherent in the manipulatives. Students must develop the concept through interaction with the materials, using the materials as tools for building up the concept. Showing students models of a concept – for example, two-digit addition – using base ten materials without allowing students to develop the concept of two-digit addition themselves is as ineffective as making them memorize a rote procedure without any understanding.

Characteristics of Student Learning and Instructional Strategies by Grade

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- see relationships between two numbers (e.g., that 7 is 2 more than 5, 1 less than 8, or 3 away from 10);
- consolidate their understanding of part-part-whole relationships, recognizing that numbers can be separated into component parts (e.g., 5 is 1 + 4 or 2 + 3, and so on);
- develop an understanding of the relationships between numerals and the grouping or bundling of objects into tens and ones;
- recognize some of the relationships that are inherent in number lines and the hundreds chart. For example, counting by 2's involves skipping every second number, and such an action produces a consistent pattern along the number line and down the hundreds chart. Students see other patterns in the hundreds charts (e.g., odd and even numbers, counting by 5's and 10's);
- see the relationship between wholes and halves (a whole can be partitioned into two equal parts, and each part is one half) as they grow to understand part-part-whole relationships;
- begin to use their understanding of the relationships between numbers as strategies for the basic facts. For example, knowing the doubles (e.g., 2 + 2, 3 + 3) makes learning the doubles plus one easier (e.g., 2 + 3 is double 2 plus 1 more).

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing multiple experiences of composing and decomposing amounts. For example: decompose the teen and decade numbers into their many parts and look at the relationships of those parts (e.g., 20 is ten more than 10 and ten less than 30);
- providing many opportunities to use five frames, ten frames, rekenreks, hundreds charts (or carpets), number lines, and simple place-value charts to build up an understanding of relationships between numbers;
- building a hundreds chart for the purpose of exploring the patterns of the numbers and their relationships. Students can work together in groups or can work individually to build a chart.

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- build on their ability to represent, compare, and order numbers using concrete materials to develop an understanding of larger numbers. For example, bundling tens and single units to understand two-digit numbers leads into bundling hundreds, tens, and single units. These representations help students to understand three-digit numbers and recognize the magnitude of the digits in the ones, tens, and hundreds positions;
- decompose larger numbers to get a sense of the relationships of a number to other single digits and to the decades (e.g., in adding 29 and 31, they may decompose 29 into 20 and 9, add the 9 to 31 to get 40 and then add the 20 for a total of 60);
- develop a sense of the relationships between the operations, recognizing that addition is the inverse of subtraction;
- continue to use their understanding of the relationships between numbers to learn the basic and multidigit facts of addition and subtraction. For example, they may use the strategy of making tens to help with adding 18 and 6. They can add 18 to 2 of the 6 to make 20, and then add on the remaining 4. They can also use strategies such as compensation, which involves a good understanding of the proportional relationship between sets of numbers. For example, knowing that the distance on the number line between 22 and 68 is the same as that between 20 (22 less 2) and 66 (68 less 2) allows them to turn a harder question into an easier one (especially when they are making the computation mentally);

- make comparisons between numbers and know from the larger digit in the tens place that a number like 92 is significantly larger than 29. This conceptual understanding helps them begin to see relationships between larger numbers, between common fractions, and between the operations: addition, subtraction, multiplication, and division.

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing experiences of composing and decomposing larger numbers, especially numbers with a tens and/or hundreds digit (e.g., 56 can be understood as 5 tens and 6 ones or as 4 tens and 16 ones);
- providing experiences with bundling or grouping objects into 5's (e.g., in tallies) and into 10's (e.g., bundles of 10 craft sticks) to help students recognize the relationships between numbers, especially in understanding place value;
- providing many opportunities to use ten frames, hundreds charts or carpets, number lines, arrays, rekenreks, and place-value charts to build up an understanding of relationships between numbers;
- working with hundreds charts to explore patterns in number. For example, adding by 10's on a hundreds chart involves movement from one number to the one immediately below it; adding by 9's involves movement from one number to the number immediately below and to the left of it; adding by 11's involves movement from one number to the number immediately below and to the right of it.

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- extend their understanding of the base ten number system to identify relationships throughout that number system, from the decades to the 100's to the 1000's. This understanding of the relationships in number is still dependent upon concrete representations, especially for an understanding of place value;
- develop a sense of the relationships between the operations, recognizing that addition is the inverse of subtraction, multiplication is the inverse of division, and so on;
- use more abstract constructs – for example, a mental image of the hundreds chart or a number line – to help determine the relationship between such numbers as 51, 61, and 71;

- use manipulatives to develop their understanding of four-digit numbers and common fractions;
- develop a sense of the relationships between the operations, recognizing that addition is the inverse of subtraction, multiplication is the inverse of division, multiplication can be viewed as repeated addition, and division can be viewed as repeated subtraction;
- continue to use their understanding of the relationships between numbers to learn the basic and multidigit facts of addition, subtraction, multiplication, and division. For example, they may use their knowledge of the 5 times table to calculate the 6 times tables, or their knowledge of the 2 times tables to learn the 4 times tables.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing experiences of composing and decomposing larger numbers, especially numbers with a tens or hundreds digit (e.g., 456 can be understood as 4 hundreds, 5 tens, and 6 ones or 3 hundreds, 15 tens, and 6 ones, etc.);
- providing experiences with bundling or grouping objects into 5's (e.g., in tallies) and into 10's (e.g., bundles of 10 craft sticks) to help students recognize the relationships between numbers, especially in understanding place value;
- using ten frames, hundreds charts, number lines, arrays, rekenreks, and place-value charts to build up an understanding of relationships between numbers;
- working with hundreds charts to explore patterns in number. For example, adding by 10's on a hundreds chart involves movement from one number to the one immediately below it; adding by 9's involves movement from one number to the number immediately below and to the left of it;
- encouraging students to use an "invisible" or internal number line (or similar mental image) to help in developing more complex strategies for solving problems involving an understanding of the relationships between numbers. For example, in adding 50 and 22, they can move along a blank line, which represents the number line, to the decades, from 50 to 60 to 70, and then make 2 more single moves to come to 72.

Representation

Overview

A number is an abstract representation of a very complex concept. Numbers are represented by numerals, and a numeral can be used in many different ways. For instance, consider this sentence: "John, who is in Grade 1, is inviting 15 children to his 7th birthday on January 5, 2018, at 2 o'clock." Think about the different ways that "number" is used in the statement.

The following are key points that can be made about representation in the primary years:

- A numeral represents the number symbol, the number word, placement in a series of counts, placement on a number line, a place-value position, and a quantity of objects. The numeral 1, depending on its placement, can mean 1, 10, 100, 1000, and so on.
- A very important aspect of understanding number is the connection between the symbol for a number or part of a number (e.g., a fraction or a decimal) and what that symbol represents with reference to quantity, position, or magnitude or size.
- An important aspect of representation is learning how to read and write numerals and connect numerals with written and spoken words for numbers.

A numeral represents a variety of things

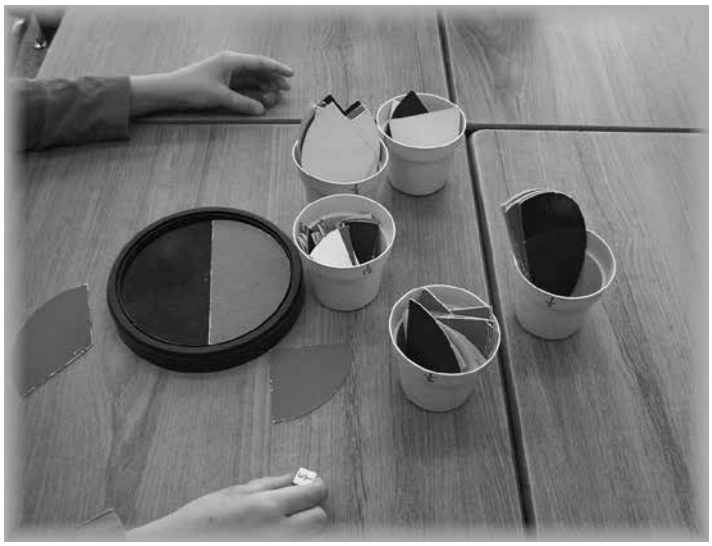
A concept such as the number 4 involves connections between the digit 4, the spoken word "four", the concrete representation of a quantity of 4 things (cardinality of number), the placement of the number on a number line or in a sequence (ordinal aspect of number), and the use of the number to simply codify or name something – for example, 4 in a social insurance number or 4 on a soccer jersey (nominal aspect of number). For adults who have long ago integrated these concepts into their understanding of numbers, it is sometimes difficult to recognize how inherently abstract these concepts are.

The concept of what the numeral represents becomes even more complex when the position of the numeral within a two-digit or larger number is considered. Placement of the digit 4 then relates to a multiple of 10 or 100, and so on, as the digit moves to the left from the ones place, or relates to decimals and fractional amounts as the digit moves to the right of the decimal. Important in this understanding is the concept that groups of 10's are represented as single entities (i.e., for the number 21, there is a single 2 but it represents 20). The use of 0 as a place holder is also an important concept in the understanding of the representation of numbers.

Use of symbols and placement in place value and fractions

Throughout the primary years, an understanding of the role of placement in determining the value of a number continues to be a difficult concept, and students need many experiences of composing and decomposing numbers into 10's and 1's to help them solidify an understanding of place value. For example, students in Grade 2 were asked to count out 26 blocks. Then they were asked to write down the number of blocks, and they were able to write the number 26. When asked to show the blocks representing the ones digit (6), they were able to do it. When asked to show the blocks representing the digit in the tens place (2), they showed 2 blocks and were unable to explain why so many of the 26 blocks were left.

Fractional sense involves an understanding that whole numbers can be partitioned into equal parts that are represented by a denominator (which tells how many parts the number is divided into) and a numerator (which indicates the number of those equal parts being considered). Fractions involve complex representations of parts of wholes that are developed over a long period of time as students construct and compare parts of wholes and sets and represent such parts with fractional names. Students' understanding of whole number often interferes with their understanding of fractions. For example, the concept that $\frac{1}{2}$ is bigger than $\frac{1}{3}$ is difficult for some students. They are misled by the appearance of the 2 and the 3 in the denominator position and confuse it with the appearance of 2 and 3 in a whole-number situation. Students need many experiences of linking the symbolic representation of fractions with concrete representations.



Baroody (1997) states that it is important to realize that common fractions are children's first encounter with a set of numbers not based on counting. He also says that instruction on fractions in the primary years is often too abstract and therefore does not provide a good conceptual foundation for future understanding.

Reading and writing numbers

In the early years of school, an important aspect of representation is learning how to read and write numerals and connect numerals with the written and spoken words for numbers and with the concepts they represent.

Young children need to develop some specific skills before they are able to identify and write numerals. In the past, it was believed that repetitive practice, often without a relevant context, was the only effective way to help children write numerals. Today, as Baroody (1997) states, a more effective way exists. This method involves helping children to recognize the characteristics of the numerals (e.g., 1 is a straight stick, 9 is a ball in the air and then a stick) and then having them practise numeral writing in engaging and relevant contexts (e.g., when playing a game of store).

Characteristics of Student Learning and Instructional Strategies by Grade

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- may visually recognize and print the numerals from 0 to 100 but may still be confused about the written position of numerals, particularly the numerals for the teens, because the oral representations of the teens are the reverse of the visual. For example, if asked to write 17, they might write the 7 first and then put the digit 1 to the left of the 7. They write the correct numerals, but they give evidence of having difficulty in matching the sound of the number with its appearance (the word "seventeen" begins with the word and sound of "seven", so intuitively it might seem to them that the 7 should be written first). Some students will reverse digits. For example, when identifying two-digit numbers, they will write 37 as 73;

- may be able to use 0 as a place holder in two-digit numbers, although their understanding of the importance of 0, based on its position, may not be fully developed;
- represent numbers to 50 using cubes, tally marks, placement on a hundreds chart or a number line, and so on, by the end of the grade. They also consolidate their understanding of ordinal numbers, such as first, second, . . . , tenth, and possibly higher;
- may have procedural knowledge that allows them to write the number 34 but not have a conceptual understanding of numbers in the tens place as representing groups of 10 objects. If you ask students which digit in the number 23 represents more, they may say 3. A significant conceptual leap is made when students move from understanding the number 23 as 23 single units to understanding 23 as 2 groups of 10 units and 3 single units;
- are able to represent fractional parts of a whole, particularly $\frac{1}{2}$, using concrete materials. They usually have more difficulty with the representation of $\frac{1}{2}$ of a group of objects (e.g., determining $\frac{1}{2}$ of 6 counters is more difficult than finding $\frac{1}{2}$ of a chocolate bar);
- continue to have a naive understanding of place value. They need many opportunities to represent the tens digits as groups of 10 objects so that they can build an understanding of 23 as 2 groups of 10 objects and 3 single units. They also need experience with numbers beyond the teens before they develop an understanding of the concept of tens. For example, they need to be able to compare the 2 tens in 21 with the 3 tens in 31 in order to make the connection between tens and place-value position.

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing many continuing experiences with numbers in real-world situations, role-playing situations, games, centre activities, and so on, so that students consolidate their understanding of the nominal, the cardinal, and the ordinal representations of number;
- providing many opportunities to construct and partition numbers and to make representations of numbers and parts of numbers using number lines, ten frames, rekenreks, hundreds charts, calculators, computer games, and manipulatives (e.g., cubes);
- providing continuing experiences of bundling or grouping objects into 5's (e.g., in tallies) and into 10's (e.g., bundles of 10 craft sticks), to help students recognize the relationships between numbers, especially in understanding place value;

- providing many opportunities to develop a consolidated understanding of 10 as a whole with a relationship to all other numbers (e.g., bundle units into tens to represent 20 days of school during calendar activities);
- providing experiences with fractions in real-life situations (e.g., cut lunch sandwiches in half; split a chocolate bar in half).

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- recognize simple fractions as representational of the parts of a whole, and compare the amounts that fractions represent (e.g., compare $\frac{1}{4}$ and $\frac{1}{2}$, and recognize that $\frac{1}{2}$ is larger) using concrete materials;
- may still be developing their understanding of place value, particularly in their recognition that a group of 10 objects must be perceived as a single item (e.g., in the number 23, the 2 represents 2 sets of 10 objects – a fairly sophisticated concept) and that the quantitative value of a digit is derived from its position. For instance, students at this age may look upon the number 33 as 33 single units, not 3 groups of tens and 3 single units or 2 sets of tens and 13 single units. Students may also have difficulty if asked to represent a number – for example, 33 – and then write the number that is 10 more or 10 less than 33;
- may not yet have identified the pattern of adding by tens in base ten numeration. When asked to add 10 mentally to 34, they may not be able to do so without counting by 1's;
- can typically recognize and print numerals to 500, but some students may still continue to reverse digits in two-digit and three-digit numbers when reading and writing numbers. They continue to develop and consolidate their understanding of ordinal numbers (e.g., talk about the 31st day of school).

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing opportunities to represent quantities to 100 in a variety of problem-solving situations;
- providing many continuing experiences with numbers in real-world situations, role-playing situations, games, books, centre activities, and so on, so that students consolidate their understanding of the representations for ones, tens, and hundreds;
- providing many experiences of writing numerals in authentic writing situations;

- providing many experiences of making groups of 10 and trading ones for tens, particularly through games such as Spinning for a Flat (see Appendix B, "Grade 2 Learning Activity: Representation", Centre 4);
- providing opportunities to represent numbers from 1 to 100 in various concrete ways (e.g., use base ten blocks, interlocking cubes, money, geoboards, ten frames) so that students can observe the relationship between numbers;
- providing many continuing opportunities to develop a consolidated understanding of 10 as a whole with a relationship to all other numbers (e.g., bundle single units into tens to represent 20 days of school during calendar activities);
- providing experiences of representing fractions concretely and pictorially;
- providing experiences of comparing fractions using concrete materials (e.g., pattern blocks, fraction blocks, Cuisenaire rods) or pictures;
- providing opportunities to use calculators to explore how adding or subtracting 10's affects the digit in the tens place (e.g., link the movement on a number line or hundreds chart with what occurs on the calculator as 10 is added to or taken away from a number).

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- perceive sets of tens as single entities and recognize that the value of a digit depends on its placement. They may have consolidated an understanding that adding 10 automatically includes a change in the tens place so that they immediately recognize 43 and 10 as 53;
- often know how to represent larger numbers but may initially still write one hundred and four as 1004. They may be able to recognize and use standard and non-standard base ten models to help them work with place value in computations and problem solving. For example, they may know the standard place-value division of 53 into 5 tens and 3 ones as well as the non-standard place value of 53 into 4 tens and 13 ones (concepts that help considerably in reasoning about numbers in problem-solving situations);
- read and print numbers to 1000. Occasional two-digit reversals may still occur. They also use and understand 0 as a place holder in three-digit numbers;
- begin to understand that fractions are based on separating a whole into equal shares; that the more parts there are, the smaller the portions; and that the size of the whole does not change even though the whole can be shared in many equal ways. They also recognize that the total number of equal shares into which a whole is divided is represented by the denominator and that the number of those equal shares being considered is represented by the numerator.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing opportunities to estimate quantities to 100 in a variety of problem-solving situations;
- providing many continuing experiences with numbers to 1000 in real-world situations, role-playing situations, games, centre activities, and so on, so that children consolidate their understanding of ones, tens, and hundreds;
- providing many experiences of writing numbers in relevant contexts;
- providing many opportunities to develop mathematics concepts and basic facts in engaging contexts (e.g., use games that focus on experiences with trading ones, tens, and hundreds);
- providing many continuing opportunities to see models of the numbers from 0 to 100 and to see charts giving the corresponding numerals and number words;
- providing many continuing opportunities to see models of the numbers from 100 to 1000 and to see charts giving the corresponding numerals;
- providing opportunities to represent numbers from 1 to 1000 in various concrete ways (e.g., using base ten blocks, interlocking cubes, money, geoboards);
- providing many opportunities to use number lines, calculators, hundreds charts, and so on, to help students develop an understanding of how the movement of a digit to the right or left significantly alters its value and to help students recognize the patterns that such movement creates;
- providing experiences with representations of fractions as both parts of a whole object and parts of a set of objects, and connecting these parts to the symbols for numerator and denominator.

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Learning Activities for Number Sense and Numeration

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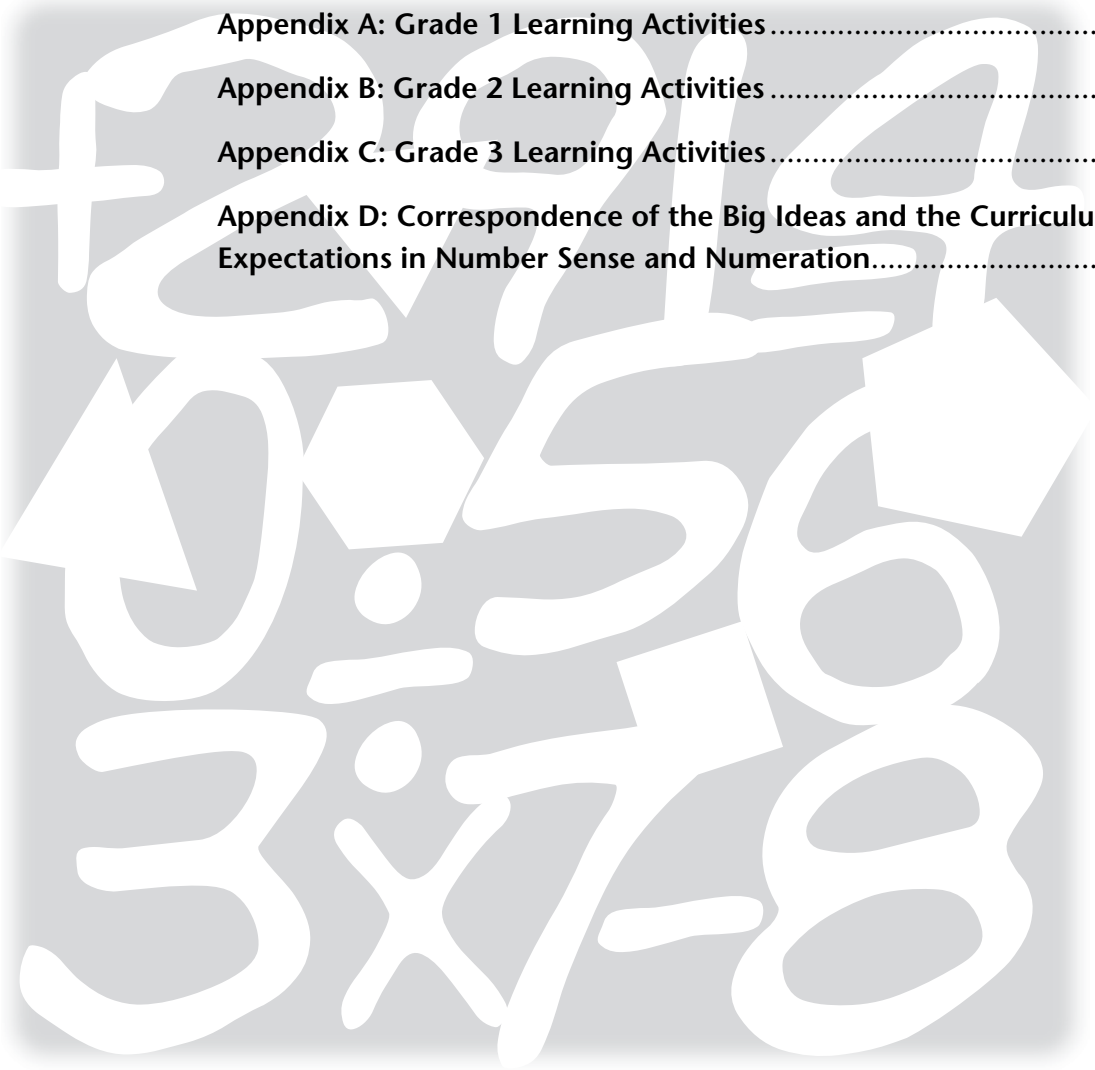

Introduction

The following three appendices (Appendices A–C) include learning activities that are practical applications of the big ideas in Number Sense and Numeration for Grades 1 to 3, respectively. For each grade, one activity is included for each of the five big ideas: counting, operational sense, quantity, relationships, and representation. The activities do not address all the key concepts for each big idea, since the big ideas cannot be addressed fully in one activity. The learning activities provide a starting point for classroom instruction related to the big ideas; however, students need multiple experiences throughout the school year to build an understanding of each big idea.

The learning activities are organized as follows:

- **CURRICULUM EXPECTATIONS:** The curriculum expectations are indicated for each activity. Some activities also include process expectations that are featured in the task
- **MATERIALS:** A materials list is included. The list applies only to the learning activity, not to the learning connections, which have their own materials lists.
- **ABOUT THE MATH:** Background mathematical information that connects to the big idea is provided. In some instances, reference is made to some of the important prior learning that should precede the activity.
- **GETTING STARTED:** This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.
- **WORKING ON IT:** In this part, students work on a mathematical task, often in small groups or with a partner. The teacher interacts with students by providing prompts, asking questions, and giving descriptive feedback.
- **REFLECTING AND CONNECTING:** This section usually includes a whole-class debriefing time that allows students to share strategies related to the mathematical processes of reflecting and connecting, as well as an opportunity to emphasize mathematical concepts.
- **ADAPTATIONS/EXTENSIONS:** These are suggestions for meeting the needs of all learners in the classroom.

- **MATH LANGUAGE:** Vocabulary that is important to this activity and to communicating the concepts presented is included under this heading. This is important to the mathematical process of communicating.
- **SAMPLE SUCCESS CRITERIA:** This section provides sample success criteria that can be used to monitor student learning and develop descriptive feedback to guide student progress.
- **HOME CONNECTION:** This section includes a sample task connected to the mathematical focus of the learning activity for students to do at home. Not all learning activities have a home connection section.
- **LEARNING CONNECTIONS:** These are suggestions for follow-up activities that either consolidate the mathematical focus of the lesson or build on other key concepts for the big idea.
- **BLACKLINE MASTERS:** These pages are referred to and used throughout the activities.

A. Grade 1 Learning Activities

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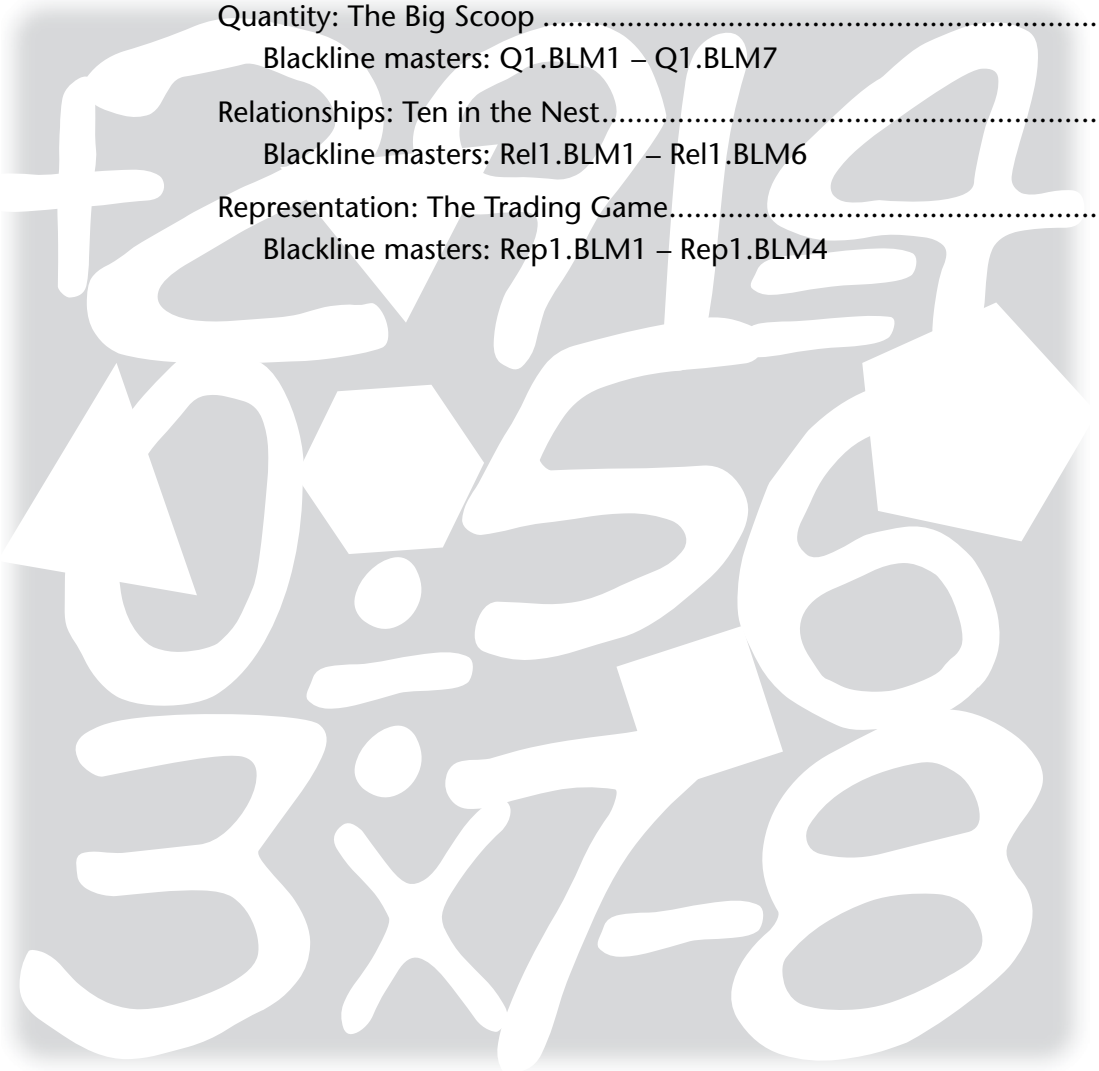


Healing Solutions

BIG IDEA ***Counting***

CURRICULUM EXPECTATIONS

Students will:

- solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines);
- compose and decompose numbers to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3);
- demonstrate, using concrete materials, the concept of one-to-one correspondence between number and objects when counting;
- count forward by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies (e.g., move with steps; skip count on a number line; place counters on a hundreds chart; connect cubes to show equal groups; count groups of pennies,* nickels, or dimes);
- estimate the number of objects in a set, and check by counting (e.g., "I guessed that there were 20 cubes in the pile. I counted them and there were only 17 cubes. 17 is close to 20.");
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

MATERIALS

- overhead projector or chart paper
- poster paper
- transparency of C1.BLM1: Bandages Cut-Out
- markers
- adhesive bandages or masking tape cut in short strips
- a copy of the poem "Band-Aids" by Shel Silverstein, from *Where the Sidewalk Ends* (New York: Harper and Row, 1974), p. 140
- C1.BLM1: Bandages Cut-Out for each student (if desired)

- C1.BLM2: Bandages Problem-Solving Worksheet for each student
- C1.BLM3: How Many?

ABOUT THE MATH

In the early grades counting is both a skill and a strategy. Using counting as a strategy helps students to solve problems. In Grade 1 students continue to practise number words by counting to 100, and they continue to explore the patterns involved in counting. Counting by 2's, 5's, and 10's begins in Grade 1. This type of counting helps students to recognize patterns and provides them with a strategy for counting large groups of items. The strategies of counting on and counting back are also used in Grade 1 to solve simple number problems. Counting on and counting back require students to be able to begin counting from any number, which is more challenging than counting up by rote from the number 1 or back from the number 10. For most students, counting backwards is more difficult than counting forward. It is common for students to hesitate and sometimes have difficulty when counting across decades, such as moving from 29 to 30, or to have difficulty with some of the numbers in the teens.

GETTING STARTED

Obtain a copy of the poem "Band-Aids" by Shel Silverstein, from *Where the Sidewalk Ends* (New York: Harper and Row, 1974), p. 140, and put it on chart paper or on an overhead to read aloud with the class.

The poem is about someone who has put many Band-Aids on his or her body, but who, in fact, has no cuts or sores. The Band-Aids are located as follows: 1 on the knee, 1 on the nose, 1 on the heel, 2 on the shoulder, 3 on the elbow, 9 on the toes, 2 on the wrist, 1 on the ankle, 1 on the chin, 1 on the thigh, 4 on the belly, 5 on the bottom, 1 on the forehead, 1 on the eye, and 1 on the neck. The person also has a box that contains 35 more Band-Aids.

Invite students to circle number words in the poem using a coloured marker. Then have other students circle body part words in the poem using a different colour of marker.

WORKING ON IT

Read the poem to the class a couple of times.

Make a large cut-out of a person by projecting a transparency of C1.BLM1: Bandages Cut-Out onto poster paper and tracing it. Use real adhesive bandages or masking tape pieces to model the poem on the cut-out. **Note:** Be sure the cut-out can be turned backwards to have the adhesive bandages affixed to both the front and the back.

Remind students of some simple methods they have used to solve number problems (i.e., counting, joining, and taking away).

Introduce the following problem: Does the boy or girl in the poem have more Band-Aids on his or her body or more left in the box?

Review the question with the class. Have several students explain the problem in their own words. Have students share some of the ways they might begin solving the problem. They may act it out, use manipulatives, or use C1.BLM1: Bandages Cut-Out.

Show students C1.BLM2: Bandages Problem-Solving Worksheet. Tell students that when they have finished solving the problem, they will need to clearly show how they solved it on the paper and be ready to share their strategy with their classmates later.

Consider giving pairs of students each a copy of the poem "Band-Aids". They may circle the number words they find in the poem. They may use this as part of their problem-solving strategy.

Have plenty of manipulatives, including adhesive bandages or strips of masking tape, available for students to use to solve the problem.

Have students work with a partner to solve the problem.

Circulate around the room as students are working to observe the process. Ask questions to keep students thinking:

- "What steps are you using to solve the problem?"
- "Can you show how you represented the Band-Aids on the cut-out?"
- "How are you going to count how many Band-Aids were used?"
- What strategy will you use to determine if there are more Band-Aids on the cut-out or in the box?

REFLECTING AND CONNECTING

Have students share with the class the strategies they used to solve the problem.

The cut-out can become the centrepiece of a bulletin board display of the poem and students' recordings of how they solved the problem.

Ask: "What made this problem difficult to solve? What were some of the ways you counted to help you solve the problem?"

Reinforce the idea that there are many different ways to count, such as by 1's or by grouping by 2's, 5's, and 10's. Emphasize that grouping can make it easier and faster to organize and count large numbers of items.

Ask: "How did you decide whether there were more Band-Aids on the person or in the box?"

ADAPTATIONS/EXTENSIONS

For students who need an extra challenge, ask them to figure out how many Band-Aids the person in the poem has altogether, on his or her body **and** left over in the box.

Follow-up Problem Scenario:

Tell the students that the boy or girl in the poem takes four bandages off every day until they are all gone. How many days will go by before all the bandages are off?

MATH LANGUAGE

- counting words

SAMPLE SUCCESS CRITERIA

- chooses an appropriate way to mathematize the problem (e.g., represents the Band-Aids with manipulatives, tally marks)
- uses an appropriate addition strategy (e.g., counting, making tens)
- uses an appropriate subtraction strategy (e.g., one-to-one matching, number line)
- demonstrates use of one-to-one correspondence in representing the problem
- organizes appropriately for and accurately counts by 1's, 2's, or 5's
- explains the strategy using appropriate mathematical language

HOME CONNECTION

Send home C1.BLM3: How Many? Students will use their counting skills to find out the number of various items they have in their home: doors, windows, spoons, forks, lamps, and clocks.

LEARNING CONNECTION 1

Counting Necklaces

Materials

- string
- beads
- interlocking cubes
- cereal shaped like zeros

At the beginning of the year, Grade 1 students benefit from making counting necklaces (i.e., necklaces on string made from beads, cereal, or interlocking cubes). Students can be assigned different numbers depending on their number awareness. Once students are successful at counting out a number of items one at a time, challenge them to make necklaces that model counting by 2's, 5's, and 10's. Students may alternate materials, colours, and so on, to make sections of 2's, 5's, and 10's, or they can put a marker between each group, such as a paper cut-out or another manipulative. After they have finished their necklaces, students can exchange them with a partner to count to find out how many. Students can save the necklaces to use as counters throughout the year.

LEARNING CONNECTION 2

Let's Take a Walk

Materials

- craft sticks

Ask students to estimate how many steps it is from one end of the classroom to the other. Check the answer by having students count aloud as they walk.

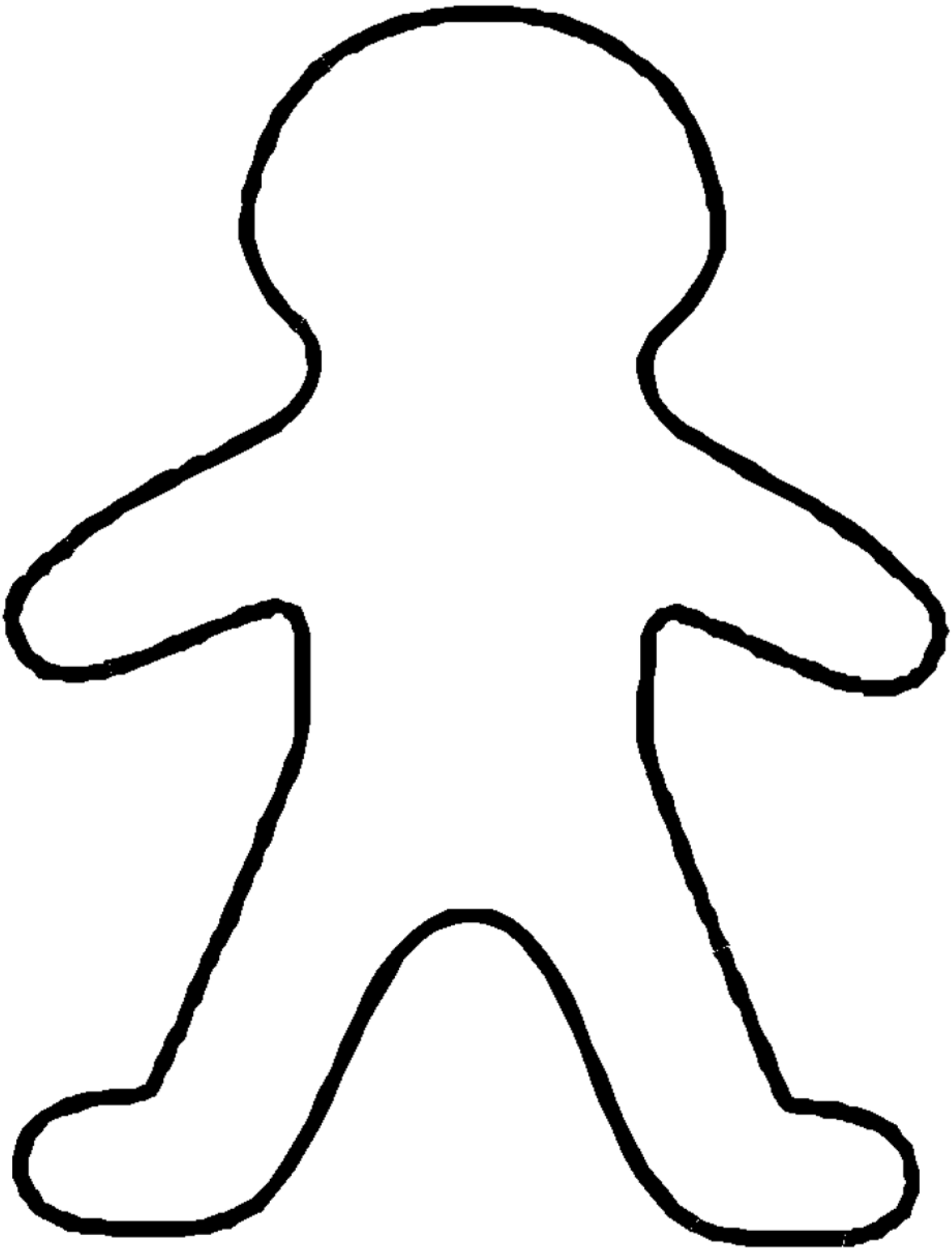
Ask students to find out how many steps it is to the school office or the gymnasium. Send students in partners to find out. Ask: "How will you keep track of how many steps as you count?"

Some students will be able to count continuously to the number of steps required; others will need a strategy.

Try offering each pair of students craft sticks to use to keep track of every 10 steps. One partner counts while the other holds the sticks. Each time they come to 10, the holder passes one stick to his or her partner. When they are finished their walk, they can count each group of 10 (one stick) to determine how many steps it took altogether.

C1.BLM1

Bandages Cut-Out



C1.BLM2

Bandages Problem-Solving Worksheet

Are there more Band-Aids on the person or in the box?

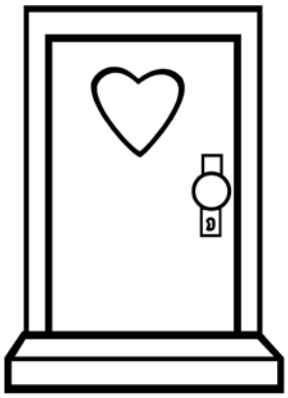
How we found our answer.

How Many?

Dear Parent/Guardian:

Counting helps children solve problems. Please help your child to count some of these things at home.

How Many? 1? 2? 3? 4? 5?



Doors _____

Windows _____

Spoons _____

Lamps _____

Forks _____

Clocks _____

Train Station

BIG IDEA ***Operational Sense***

CURRICULUM EXPECTATIONS

Students will:

- compose and decompose numbers up to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3);
- solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines);
- solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles);
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems.

MATERIALS

- interlocking cubes or coloured tiles
- OS1.BLM1: Two-Centimetre Grid Paper
- scissors
- crayons or markers
- glue
- construction paper
- OS1.BLM2: Roll and Add Game

ABOUT THE MATH

In Grade 1, students are formally introduced to both addition and subtraction. To develop number sense, students need to understand the many different but related meanings that addition and subtraction represent in real contexts. Students learn about operations by modelling problems using manipulatives or by using pictures or drawings. As much as possible, the modelling should be done by the students. In other words, models should not become the objective of the lessons but rather the tool used by students to understand situations and problems. Most problems can be modelled in different ways; encourage students to model the situation in a way that makes sense to them.

The symbols used for operations and the equal sign are conventions that must be taught to students. Each symbol has many different meanings and therefore can be confusing for students. For example, a minus sign (–) is generally associated with "take away". This definition is narrow and limiting. With experience, students will see that symbols such as the minus sign sometimes describe a quantity taken away and at other times describe a missing part of an equation, as in the number sentence 8 – □ = 3. Providing students with various problem situations and allowing them to use symbols, words, and objects to describe those situations in a way that makes sense to them is the best way to encourage students to think flexibly about operations.

The concept of part-part-whole is developed throughout the early primary grades. Conceptual understanding of this type of relationship is helpful in understanding operations, number sentences, and equations. Modelling part-part-whole situations using concrete materials helps students develop an understanding of quantity and can be used to provide students with models of addition and subtraction.

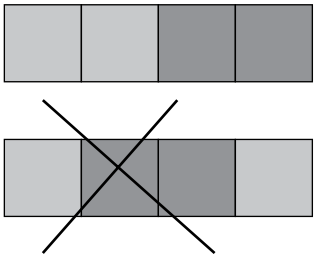
GETTING STARTED

Be sure to complete this activity only after students have been introduced to the addition symbol (+), the subtraction symbol (–), and the equal sign (=).

Tell students they are going to be making 2-colour trains using interlocking cubes or coloured tiles. Explain that the trains have to follow two rules:

1. The cubes must be in a row, and the same colours must stay together.
2. All trains must start with the same colour.

Show students an example of a 4-car train that follows the rules and an example of a 4-car train that does not follow the rules.



Follows the rules

Does not follow the rules because same colours are not together

WORKING ON IT

Have students work in pairs to create as many different seven-car trains as they can.

Provide each pair with OS1.BLM1: Two-Centimetre Grid Paper. Show students how to cut out a strip of seven squares and then how to colour the squares to represent one of their trains. Ask students to make coloured strips to represent each of their seven-car trains.

Challenge students to find a way to organize and display their paper trains on a piece of construction paper (e.g., 3 yellow + 4 blue = 7 cubes; 4 blue + 3 yellow = 7 cubes).

Finally, challenge students to write number sentences to describe each of their trains. As students work, observe the process and ask probing questions:

- "How are you organizing your trains?"
- "How will you check whether you have found all the different ways to make the trains?"

Students' work may look something like this:

$1 + 6 = 7$

$2 + 5 = 7$

$3 + 4 = 7$

$4 + 3 = 7$

$5 + 2 = 7$

$6 + 1 = 7$

REFLECTING AND CONNECTING

Have each pair of students bring their displays of coloured strips to share with the class. Ask students the following questions:

- "What was difficult about making the trains?"
- "What was easy?"
- "Did you notice any patterns when you were making your trains?"
- "Did you use a pattern to help you put your trains in order?"
- "How did you put your trains in order?"
- "How do you know you have built all the trains?"

It is valuable to have students repeat this activity for one or two more days, creating displays of all the trains for 8, 9, 10, and 11 cubes. Each time emphasize, for example, that number expressions such as 5 + 4 and 6 + 3 both equal 9, and that 5 + 4 = 9 is the same as 4 + 5 = 9. Question students about the part-part-whole relationships for the number they were working on. When looking at 2 + 4 = 6, for example, ask students to name the parts (2 and 4). Then ask students to name the whole (6).

As they move onto the next number, ask students how their trains with 7 cubes could help them to make trains with 8 cubes and so on. Ask them whether they notice something about the number of trains possible for 7, 8, 9, 10, and 11 cubes. Is there a pattern? How can they describe the pattern? Can they predict how many trains can be made for 10?

ADAPTATIONS/EXTENSIONS

Students could also complete this task using 2-colour counters.

For students who require additional challenges, have them make trains using larger numbers (12 to 20).

Follow-up Activity:

Make trains using 3 different colours of interlocking cubes.

MATH LANGUAGE

- pattern
- addition
- equals

SAMPLE SUCCESS CRITERIA

- finds and represents all the different combinations of interlocking cubes
- writes number sentences correctly
- explains strategy and patterns clearly, using mathematical language
- interprets problem situations requiring addition and subtraction
- models/represents problem situations requiring addition and subtraction
- uses a variety of mental strategies to add single digit whole numbers (e.g., doubles)
- uses a variety of mental strategies to subtract single-digit whole numbers (e.g., think-addition)

HOME CONNECTION

Roll and Add Game

Send home OS1.BLM2: Roll and Add Game. Students can teach the game to someone at home. In the Roll and Add Game, students roll two number cubes, adding the numbers and colouring one square on the grid above the sum. The game ends when one player has coloured all the squares in one number column.

Note: Be sure that students are familiar with the game and the rules before taking it home.

LEARNING CONNECTION 1

Addition and Subtraction Stories

Materials

- 1 resealable plastic bag per student
- 10 craft sticks per student
- markers, stickers, or construction paper and glue

Give each student a resealable plastic bag and 10 craft sticks. Have students decorate their sticks to make 5 stick people. Students can make their people using cut-and-paste materials or simply by using markers to draw on happy faces. Students can use their stick people to model stories told by the teacher or by other students.

Tell students that they are going to hear some stories and that they should act them out using their stick people.

Note: It is a good idea to write the number sentence 3 + 2 = 5 on the chalkboard.

Substitute students' names in the following story:

Jason, Jennifer, and Corey were thirsty and went to have a drink at the water fountain. How many people are at the water fountain?

Ruth and Jamal decided they wanted a drink, too. How many people are at the water fountain now?

Tell a few more stories for students to act out, including stories involving subtraction. Students could act out lining up, going to the book centre or the math centre, and so on. Groups of students can work together to make up addition and subtraction stick people stories to present to the class.

LEARNING CONNECTION 2

Word Problems

Materials
- OS1.BLM3a–c: Act the Problem Cards
- small manipulatives (e.g., bingo chips, cubes, buttons, bread tags)
- OS1.BLM4: Turnaround Mat

Students generally develop operational sense through experiencing, discussing, and modelling mathematical situations and problems. It is important to vary the problem types and to ensure that students can act out or demonstrate how a problem could be solved in ways that makes sense to them.

OS1.BLM3a–c: Act the Problem Cards provides various types of problems. Students can form small groups to act out one of the problems.

As each problem is acted out, the remaining students can use manipulatives to model a solution. Have students share their methods of solving the problem.

Provide students with OS1.BLM4: Turnaround Mat to use for modelling problems with manipulatives.

LEARNING CONNECTION 3

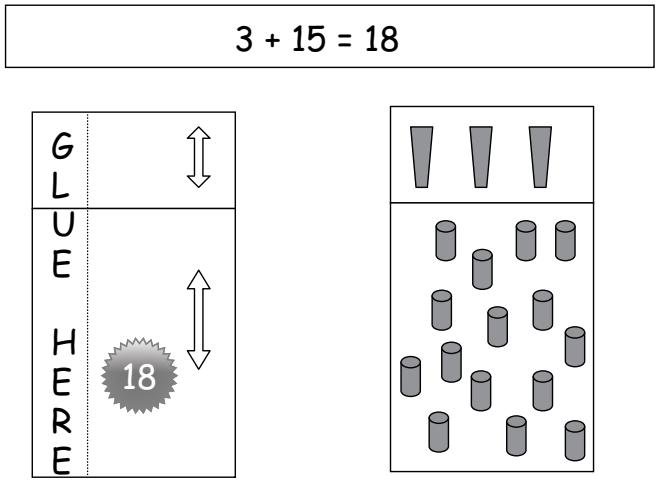
Refrigerator Booklets

Materials
- OS1.BLM5: Refrigerator Template
- construction paper
- grocery store flyers
- glue

Have students make booklets modelling the part-part-whole relationships of addition for any fact to 20 (such as 3 + 15 = 18) by using OS1.BLM5: Refrigerator Template and following these steps. As an alternative, student helpers or parent volunteers could cut the refrigerator templates before starting this activity.

1. Cut along outer edges of the refrigerator.
2. Stopping at the dotted line, cut along the solid line between the fridge and the freezer.
3. Fold along the dotted line.
4. Glue the narrow section between the dotted line and the edge of the fridge to a piece of construction paper, allowing the doors to fold open.

5. Draw a line on the construction paper to separate the fridge and freezer sections.
6. Write a number on the fridge magnet on the fridge door. (18)
7. Cut small pictures of food from grocery store flyers.
8. Create a part-part-whole relationship by gluing pictures of food on the construction paper behind the doors of the fridge and the freezer to add to the number written on the magnet (3 + 15 = 18). Write a number sentence to represent the part-part-whole relationship.



outside

inside: freezer (3)
fridge (15)

LEARNING CONNECTION 4

The Hot Dog Stand

Materials

- OS1.BLM6: Hot Dog Stand Game Board (1 per student)
- small manipulatives (cubes, counters, etc.)
- OS1.BLM7: Plus/Minus Spinner (1 per pair of students)
- number cube (1 per pair of students)

Students play this game with a partner. Give each student a copy of OS1.BLM6: Hot Dog Stand Game Board.

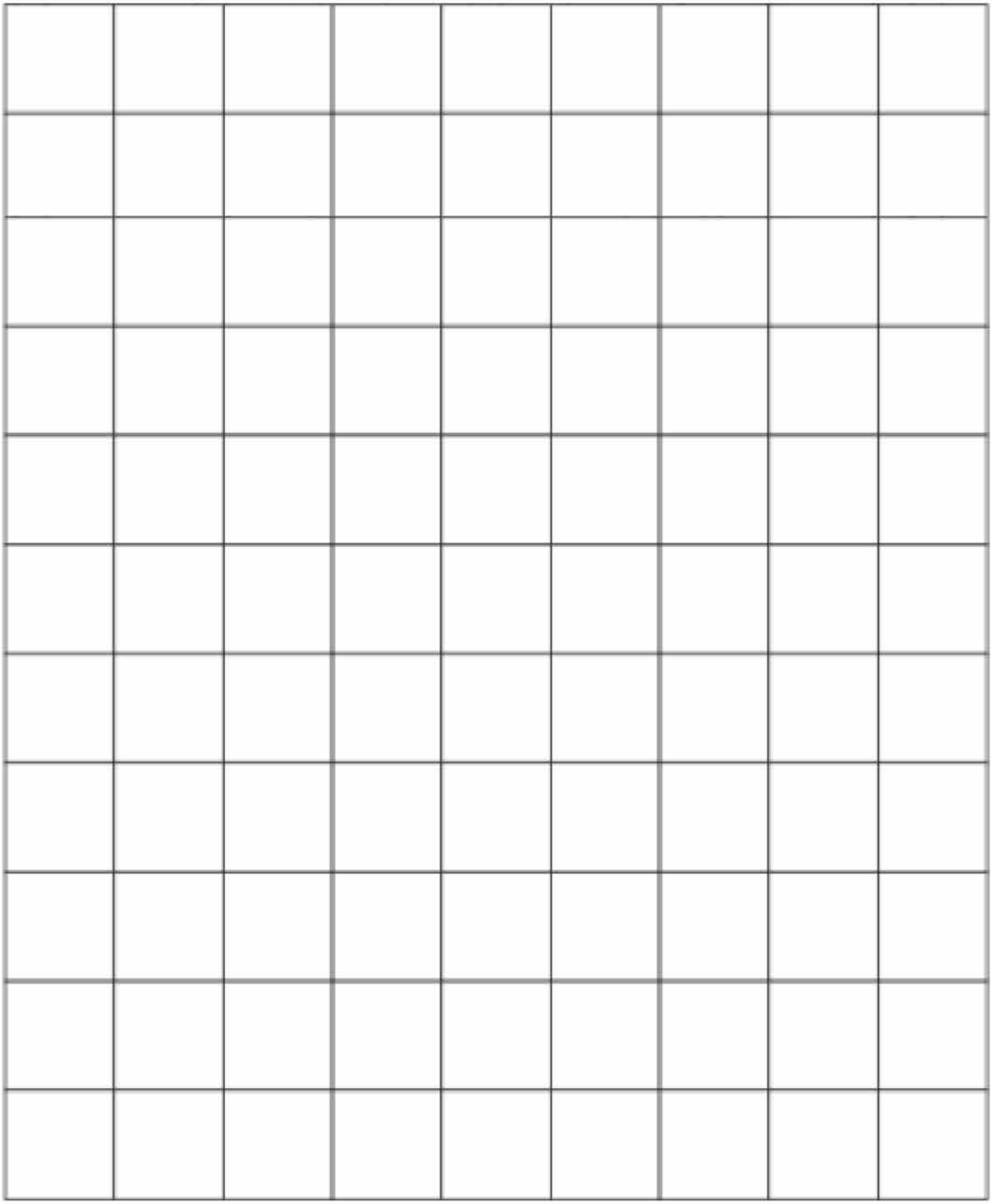
Each player begins with 15 manipulatives, which they place on OS1.BLM6: Hot Dog Stand Game Board to represent people lined up at the hot dog stand. The first player spins the plus/minus spinner to find out the operation for his or her turn. After the spin, the player rolls the number cube to find out how many people have joined the line or how many people have left the line. The player then performs the operation by modelling

with manipulatives, either adding to or removing from the line. Players take turns until one player's line has disappeared (reached zero).

After playing the game, consider having have each student write a number sentence to describe their "people" at the end of the game (e.g., 8 people in the line + 6 people left the line = 2 people in all).

OS1.BLM1

Two-Centimetre Grid Paper



OS1.BLM2

Roll and Add Game

Dear Parent/Guardian:

Playing this game at home with your child provides valuable practice in adding numbers.

1. Play with a partner. Give each player a game sheet.
2. Take turns rolling two number cubes.
3. Add the dots on the two cubes.
4. Colour one square in the column that is labelled with the correct sum.
5. Play until one player fills up a column.

Once your child is adept in working with two cubes, try playing with a third cube and sums to 18.

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Act the Problem Cards

Act the Problem Card #1

The children are getting ready to see a performance. Sammy, Alta, Mervin, and Cassi put their chairs in a row. Tunis, Kay, and Edwin put their chairs in the same row. How many chairs are in the row?

Act the Problem Card #2

The children lined up their lunches along the window. Ten lunches were along the window. Then, Mary, Sun Woo, Freddie, Ebba, Abti, Mit, and Blake took their lunches to eat them. How many lunches are left along the window?

Act the Problem Card #3

It is tidy up time. The books must be picked up and put on the shelf. Five books are already on the shelf. Deva, Myrna, and Suke put their books on the shelf. How many books are on the shelf now?

Act the Problem Card #4

The class helpers are coming to get the children ready to go outside. Maria, Sven, and Bjorn came to help. Each helper brought a friend who will also help. How many helpers are there altogether?

Act the Problem Card #5

Seven children are sitting quietly on the carpet. Besto and Ephi got up from the carpet and went to the art centre. How many children are sitting on the carpet?

Act the Problem Card #6

Savira has 12 pennies. George has 4 pennies. How many more pennies does Savira have than George?

Act the Problem Card #7

Eliz has 8 markers. She gave some to Melik. Now she has 5 markers. How many did she give to Melik?

Act the Problem Card #8

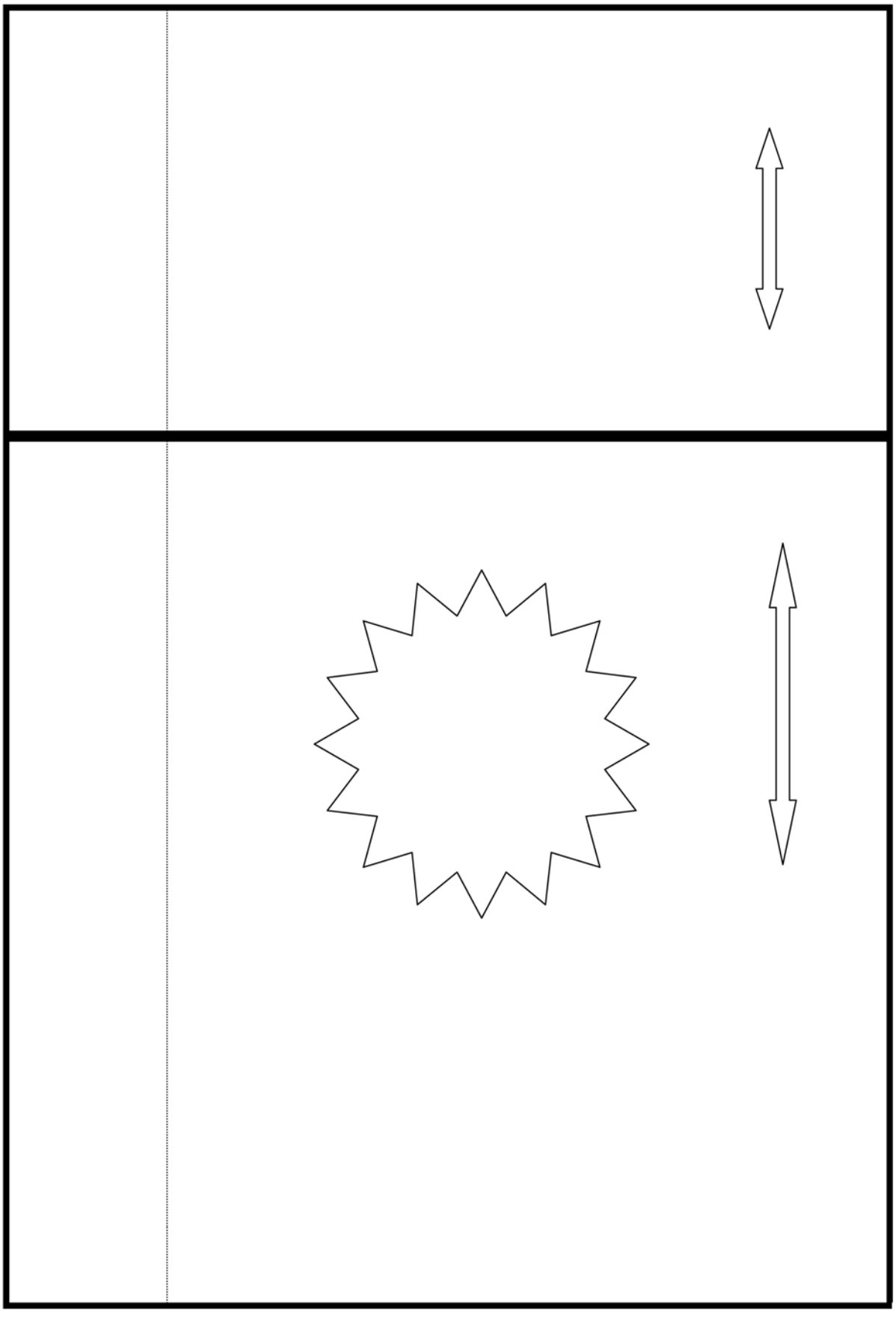
There were some boots outside the door to the classroom. The children who went outside took 6 of the boots. Now there are 4 boots outside the door. How many boots were outside the door to begin with?

Act the Problem Card #9

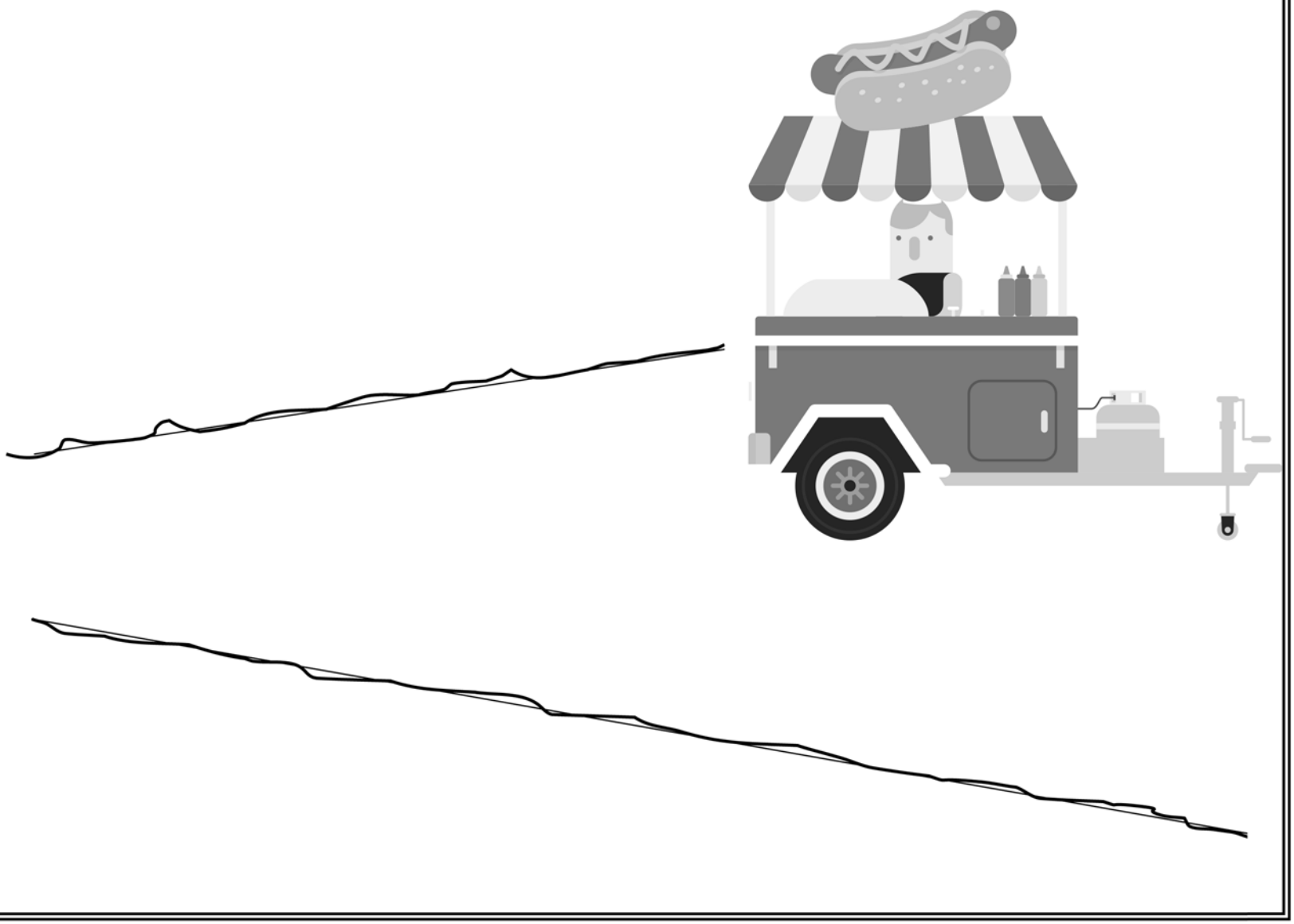
There are 5 more children at the art centre than the sand table. There are 7 children at the art centre. How many children are at the sand table?

Turnaround Mat

Refrigerator Template

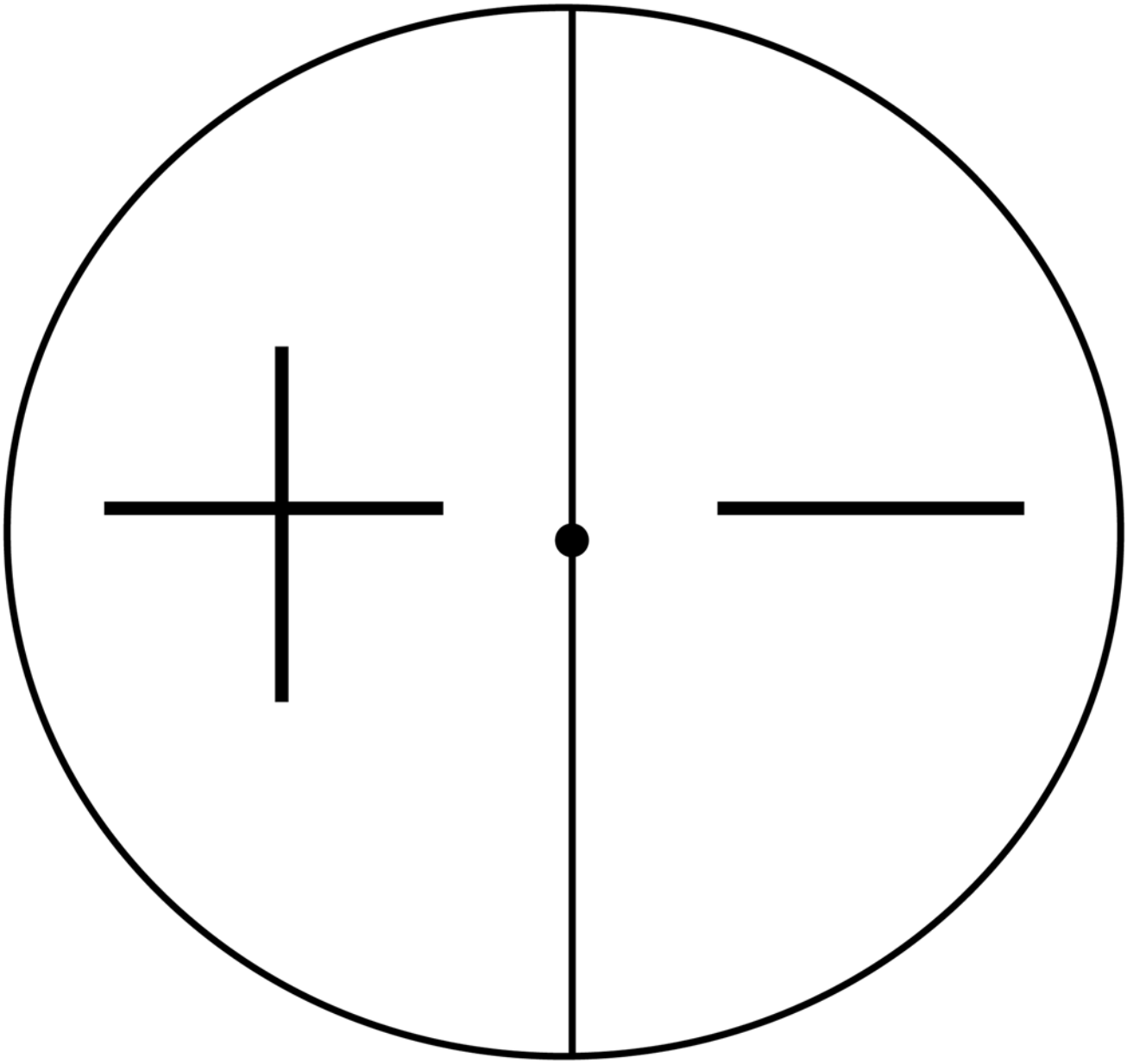


Hot Dog Stand Game Board



OS1.BLM7

Plus/Minus Spinner



The Big Scoop

BIG IDEA **Quantity**

CURRICULUM EXPECTATIONS

Students will:

- demonstrate, using concrete materials, the concept of conservation of number (e.g., 5 counters still represent the number 5, regardless whether they are close together or far apart);
- represent, compare, and order whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences and number stories);
- estimate the number of objects in a set, and check by counting (e.g., "I guessed that there were 20 cubes in the pile. I counted them and there were only 17 cubes. 17 is close to 20.");
- count forward by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies (e.g., move with steps; skip count on a number line; place counters on a hundreds chart; connect cubes to show equal groups; count groups of pennies,* nickels, or dimes);
- solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles);
- add and subtract money amounts to 10¢ using coin manipulatives and drawings;
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why their solution is correct).

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

MATERIALS

- small manipulatives (e.g., multilinks, centicubes, interlocking cubes, 2-colour counters, beans, buttons)
- chart paper
- Q1.BLM1: Target Game Recording
- Q1.BLM2: In My Home

ABOUT THE MATH

Students need to develop a sense of the quantities that they see in various visual arrangements and forms. Estimation of quantity is related to concepts in measurement and should be integrated whenever possible. In the early grades, students are more successful at estimating quantities when given a fixed choice; "Is this about 5 or 10? Is this about 20 or 30?" Such guiding questions help students who have not developed the sense of number and quantity necessary for making reasonable estimates. Comparing estimates will help students develop the concept of using a benchmark to improve their estimates.

GETTING STARTED

Show students a container of multilinks, centicubes, 2-colour counters, beans, buttons, or other small manipulatives.

Students are told that the teacher is going to try to scoop out 22 manipulatives. Record 22 on chart paper. Scoop and count aloud as a class. Record on the chart paper the amount actually scooped.

Ask: "Did I scoop more or fewer? Did I hit the target? Did I scoop too many or too few?"

Repeat the activity, trying to get closer to the target number. Again, ask whether the number scooped was too many or too few.

WORKING ON IT

Introduce Q1.BLM1: Target Game Recording.

Students choose a target number between 10 and 25. Each student records the target on an individual recording sheet.

Each student reaches into a collection of manipulatives, trying to grab the same number of manipulatives as he or she has chosen for the target number.

Each student counts and records the first try on the recording sheet and indicates whether he or she hit the target, or scooped too many or too few.

Students scoop again, trying to get closer to their target numbers. Students continue until they have completed all four target numbers.

Circulate around the classroom while students are working. Observe their process and ask probing questions:

- "How did you determine whether you had scooped too few or too many?"
- "Did you get closer to the target number the second time you scooped?"
- "Why do you think you were closer the second time?"
- "Did you get better at scooping the right amount after playing a few times?"

REFLECTING AND CONNECTING

Gather students to share their experiences playing the Target Game. Ask them the following questions:

- "What was your largest target number?"
- "What was your smallest target number?"
- "How did you determine whether you had scooped too few or too many?"
- "Did you get closer to the target number the second time you scooped?"
- "Why do you think you were closer the second time?"
- "Did you get better at scooping the right amount after playing a few times?"

Reinforce that students have used their first scoop as a benchmark for estimating their second scoop.

ADAPTATIONS/EXTENSIONS

If students are having difficulty determining whether their scoop has more or fewer, have them show their target number with manipulatives first so that they can match one to one to compare. **Note:** It may be appropriate to limit the size of the target number for some students.

MATH LANGUAGE

- more
- fewer
- same

SAMPLE SUCCESS CRITERIA

Record observations on an anecdotal record sheet.

- estimates a small quantity and verifies through counting
- determines whether an estimate is more than, fewer than, or the same as their target number
- refines estimates based on new information (e.g., benchmark number)
- uses the words "more", "fewer", and "the same" accurately
- represents a count with a number/numeral
- uses one-to-one correspondence to represent a number with objects
- uses one-to-one correspondence to represent a number in a count (tally)
- subitizes as a strategy for counting
- represents money amounts to 20¢

HOME CONNECTION

Have students take home Q1.BLM2: In My Home. Students find things in their home that they have in large and small quantities.

In my home we have: 1
2 _____
5 _____
10 _____
Around 30 _____
Around 50 _____
Can you find 100 of something?

LEARNING CONNECTION 1

A Juicy Problem

Materials
- chart paper
- Q1.BLM3: A Juicy Problem
- manipulatives (e.g., cubes or counters in the same colours as the juice boxes)

Conduct a survey in the class about juice flavours. Ask students which they prefer: apple juice, orange juice, or grape juice. Tally their choices on chart paper. (The teacher may decide to take this tally further and create a graph).

Provide students with Q1.BLM3: A Juicy Problem. Pose the following problem: Suppose we are having a party. We want each student to have at least 1 box of his or her favourite juice. How many boxes of each kind of juice should we buy?

Review Q1.BLM3: A Juicy Problem with students, pointing out that apple juice comes in packs of 4 boxes, orange juice comes in packs of 6 boxes, and grape juice comes in packs of 8 boxes.

Note: Be sure students understand that they must buy whole packs of juice boxes.

Ask students to work with a partner to solve the problem. Encourage them to use objects or to draw pictures to help them solve the problem.

Remind students that their work should show how they solved the problem. Gather students at the end of the activity to share the problem-solving strategies they used.

LEARNING CONNECTION 2

Estimation Containers

Materials
- a variety of empty clear containers
- interlocking cubes or multilinks
- centicubes

The focus of these estimation tasks is using a benchmark as a reference to improve estimates.

Show students one container that has 10 items (interlocking cubes or multilinks). Tell students that the container has 10 items.

Show students a similar container with 25 of the same items. Do not tell them how many items are in it. Ask students to use what they know about the first container to estimate how many items are in the second container.

Show students a third container with 40 of the same items. Ask students to use what they know about the first and second containers to estimate the number of items in the third one.

Repeat this activity several times, always starting with 10 in the first container and varying the number of items in the second and third. Be sure to include containers that contain fewer items than the benchmark.

Use the same container again, but this time use smaller items to fill the containers, perhaps centicubes. Repeat the activity several times.

Use the same items (centicubes, interlocking cubes) in different-sized and different-shaped containers. Repeat the procedure of showing 10 first and then estimating the quantity in the other two containers.

LEARNING CONNECTION 3

Paper Plate Dot Arrangements

Materials
- Q1.BLM4: Paper Plate Dot Arrangements
- paper plates
- circle stickers or bingo dabbers
- number cards

Students benefit from being able to identify patterns and spatial organizations of numbers. They learn dice or number cube dot patterns through playing games and eventually are able to identify the number represented without counting the dots. They begin to subitize, which means that they instantly recognize how many.

Teachers can make other arrangements of dots by using circle stickers or bingo dabbers on paper plates and using Q1.BLM4: Paper Plate Dot Arrangements as a guide.

Note: Some arrangements are one colour and some are made of two smaller arrangements using two colours. Organize the dots close together to make the patterns easier to see.

Once the plates are made, use them as flash cards during group times. Hold up a plate for one to three seconds. Take it away and ask, "How many dots did you see?" Begin with plates that have just a few dots, gradually adding more complicated patterns over the school year. Students will also enjoy using the plates to play flash card games with their peers during free exploration periods.

Adaptations/Extensions

- Have students take some plates and match them to number cards.
- Have students take some plates and order them from least to most.
- Have students take some plates and match them to number cards that are 1 more than the plates or 2 more than the plates. Or have students match the plates to cards that are 1 less or 2 less than the plates.

LEARNING CONNECTION 4

How Much Is It Worth?

Materials

- Q1.BLM5: Pennies*
- Q1.BLM6: Nickels
- Q1.BLM7: Price Tag
- glue

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

Provide students with Q1.BLM5: Pennies, Q1.BLM6: Nickels, and Q1.BLM7: Price Tag.

Ask students to cut out the pennies and nickels. Then ask them to make a price tag for an item in the class (e.g., an eraser, a pencil, a chair). The price should not be greater than 20 cents (in accordance with the expectations). Have students count out the coins they need to pay for the item they want and stick the coins to the price tag.

Ask students how they figured out which coins to use for their purchase. Which coins did they count first? Why?

Use the tags to create a bulletin board gallery. Sort tags by their value, by most expensive or least expensive, or by the most coins used or the fewest coins used. Talk about what makes a tag more or less expensive. Can students tell by looking? How can a tag with fewer coins cost more than a tag with more coins?

Q1.BLM1

Target Game Recording Sheet

Target Number

First Try | Second Try

Too Many Too Few | Too Many Too Few

Target Number

First Try | Second Try

Too Many Too Few | Too Many Too Few

Target Number

First Try | Second Try

Too Many Too Few | Too Many Too Few

Target Number

First Try | Second Try

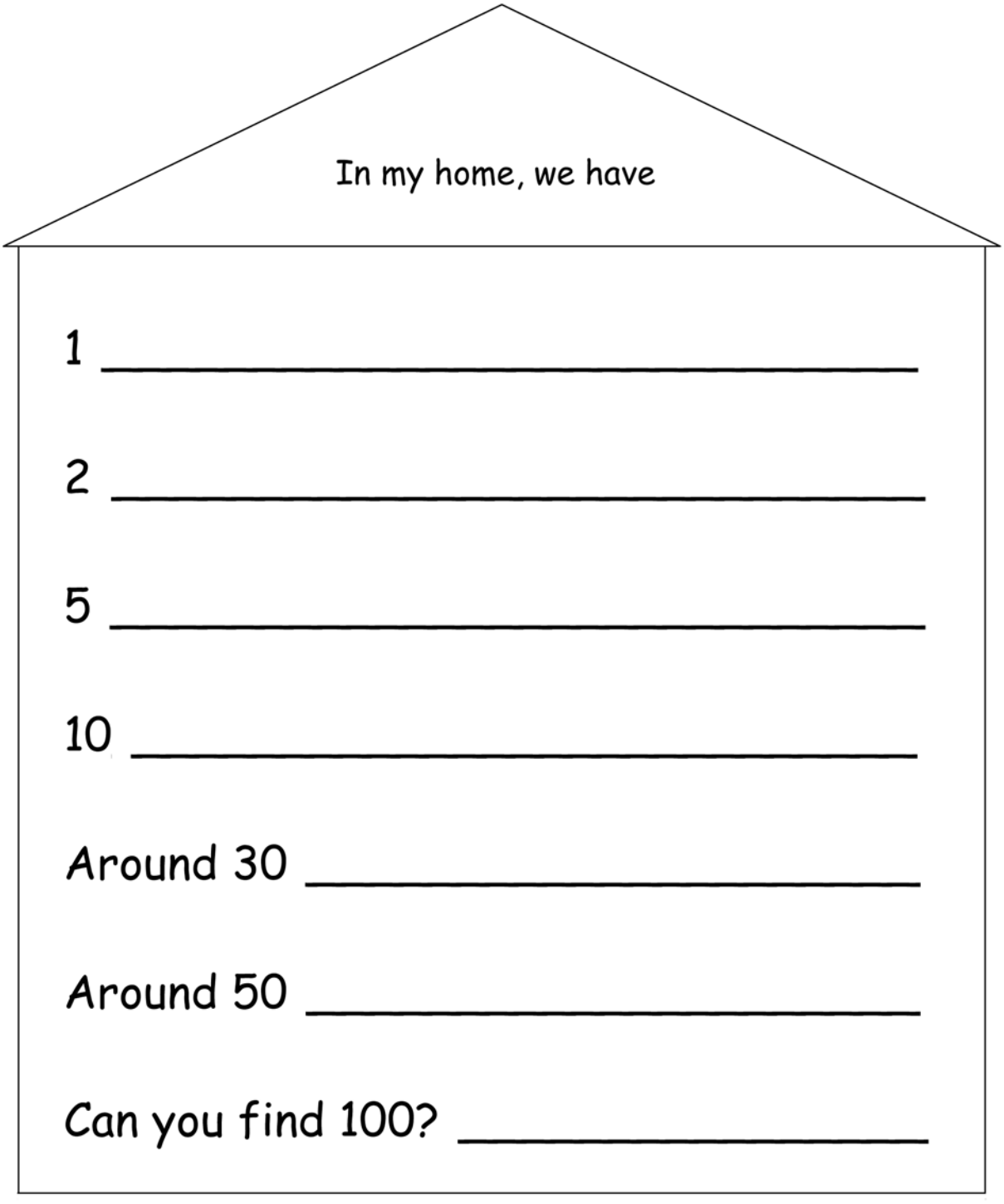
Too Many Too Few | Too Many Too Few

Q1.BLM2

In My Home

Dear Parent/Guardian:

We are learning about quantity. Please help your child to find things in your home that you have in small and large quantities.



Q1.BLM3

A Juicy Problem

Suppose our class is having a party. We want each student to have at least one box of their favourite juice. How many boxes of each type of juice should we buy? Look at our graph to help you solve the problem.

Apple	Apple
Apple	Apple

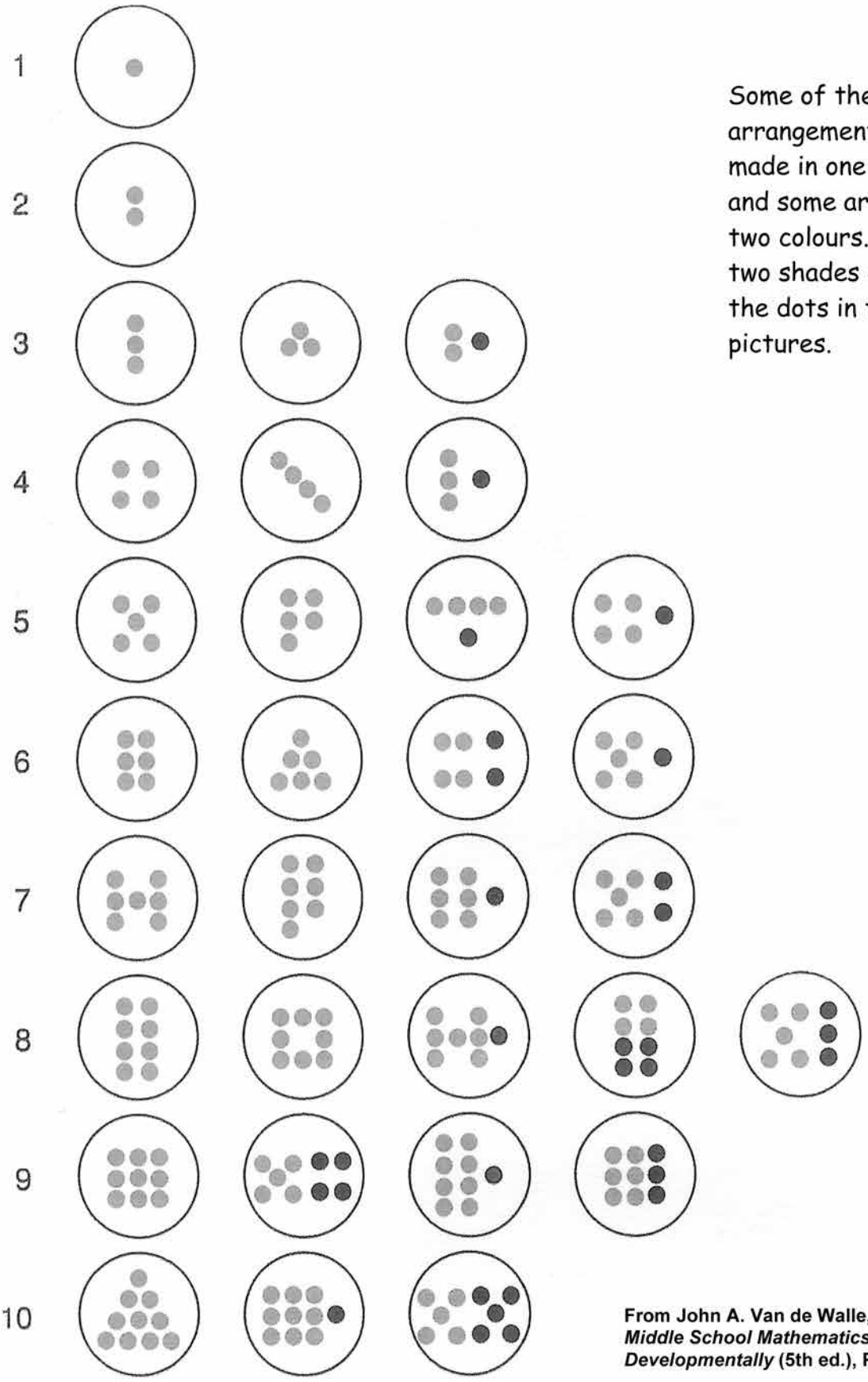
Orange	Orange	Orange
Orange	Orange	Orange

Grape	Grape	Grape	Grape
Grape	Grape	Grape	Grape

How we solved our problem.

Q1.BLM4

Paper Plate Dot Arrangements



Some of these arrangements are made in one colour and some are made in two colours. Note the two shades of grey of the dots in the pictures.

From John A. Van de Walle, 2004, *Elementary and Middle School Mathematics: Teaching Developmentally* (5th ed.), Pearson Education.

Pennies

Nickels

Price Tag

_____ cents

Ten in the Nest

BIG IDEA **Relationships**

CURRICULUM EXPECTATIONS

Students will:

- represent whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundred charts) and contexts (e.g., real-life experiences, number stories);
- relate numbers to the anchors of 5 and 10 (e.g., 7 is 2 more than 5 and 3 less than 10);
- compose and decompose numbers up to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3);
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

MATERIALS

- Rel1.BLM1: "Ten in the Nest"
- chart paper
- Rel1.BLM2: "Ten in the Nest" Recording Sheet
- a variety of manipulatives (e.g., bread tags, buttons, blocks)
- drawing of two nests
- Rel1.BLM3: Twelve in the Nest

ABOUT THE MATH

The concept of "more than or less than" is an important preliminary concept for students to understand before they begin to work with more complex relationships between numbers. Students begin to develop these concepts before entering school and should be successful at identifying sets with more items when the difference is visually obvious (e.g., when there are 25 red counters and 5 blue counters). Learning activities should help students refine this skill. This concept of "more than or less than" is also related to the symbols for greater than (>) and less than (<), which are introduced in later grades to describe the relationship between numbers.

It is important to note that understanding quantity relationships is integral to the development of number sense. Students learn about relationships of 1 more and 2 more and 1 less and 2 less when they count on and back to solve number problems. Another important relationship is the part-part-whole relationship developed when students model numbers and find out that 7 can be made up of 6 and 1, or 5 and 2, or 4 and 3.

The extremely important relationships are the ones between 5 and 10, and between 5 and 10 and the other numbers. These relationships are called anchors of 5 and anchors of 10. Using these numbers as anchors means learning other numbers by learning how they relate to 5 or 10 (e.g., 7 is 2 more than 5 and 3 less than 10). A firm understanding of the relationships of all the numbers from 1 to 10 with the anchors of 5 and 10 helps considerably when learning about larger numbers (e.g., 27 is 2 more than 25 and 3 less than 30). When adding and subtracting, students will use these relationships repeatedly to solve problems.

GETTING STARTED

Based on the song "Ten in the Bed", sing the song Rel1.BLM1: "Ten in the Nest". Act out the song, each time asking, "How many are in the nest? How many are out?"

Record the first few stages of the song on chart paper as students act it out:

10 in and 0 out

9 in and 1 out

8 in and 2 out

7 in and 3 out

Ask students to describe the pattern they see.

Stop recording and finish singing the song.

WORKING ON IT

Pose the following problem: The "Ten in the Nest" family decided it was time to get another nest. They have built a new nest and are wondering how many different ways they will be able to organize themselves using both nests.

Tell students that they may use anything in the classroom to help them solve the problem. **Note:** This is a good opportunity for kinaesthetic learning by having 10 students organize themselves into various groups.

Encourage students to talk about and share their solutions with a partner. If the methods each partner used were different, they should then work cooperatively to show more than one way to solve the problem.

Tell students that they will need to explain how they solved the problem. Provide students with Rel1.BLM2: "Ten in the Nest" Recording Sheet to record the method they used to solve the problem.

Circulate around the classroom while students are working. Observe their process and ask probing questions:

- "Why did you choose these manipulatives?"
- "What strategy are you using to solve the problem?"
- "How are you keeping track of the ways that you have found?"
- "How is this problem like acting out the song?"

REFLECTING AND CONNECTING

Gather students to share how they solved the problem. Focus students' attention on the different strategies each student used. Be sure to have students justify their answers.

Note: Some students will include 10 and 0 as one way the bird family can be arranged. This is not wrong; it is simply another way of viewing the problem. Have students explain why they included that way when counting all the possibilities.

Return to the original recording of the combination of birds in and out of the nest. Have students complete the list of number pairs to 10.

ADAPTATIONS/EXTENSIONS

Some students might find it difficult to organize materials to help them solve the problem. Provide these students with 10 manipulatives and a sketch of 2 nests. Help them to organize one combination and record it. Ask them to try to find another combination, and so on. Students may also want to work with a partner to solve the problem.

For students who need an additional challenge, ask them to investigate what would happen if the family built 2 new nests – that is, if there were 3 nests altogether. Would there be more or fewer ways to organize the bird family? Allow students to explore this possibility using manipulatives.

MATH LANGUAGE

- pairs to 10
- add

SAMPLE SUCCESS CRITERIA

- chooses an appropriate manipulative to represent the problem
- accurately models/represents the problem using manipulatives
- selects and uses an appropriate strategy to solve the problem (e.g., making an organized list, uses the anchor of 5)
- finds all combinations for the two nests
- explains how the problem was solved using appropriate mathematical language
- relates numbers to the anchors of 5 and 10
- decomposes numbers to 20 and shows part-part-whole relationships

HOME CONNECTION

Send home Rel1.BLM3: Twelve in the Nest. Have students draw some nests in the tree. Have them imagine that 12 birds came to the nests. How could they organize all those birds? Have students explain their answers.

LEARNING CONNECTION 1

Ten-Frame Activities

Materials

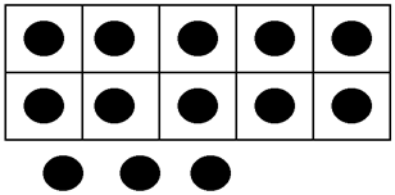
- Rel1.BLM4: Ten-Frame Mat
- bingo chips

Note: Students should have experience using a five-frame model before working with the ten frame. Introduce the ten frame to small groups of students. Students who have worked with a five frame will know that they must fill the top row with bingo chips first and then begin the second row from the left.

Ask students to represent various numbers on their ten frame using bingo chips. Have them show 6, 7, and so on. Each time ask students to tell how many more they would need to make 10. How do they know? Encourage students to talk about the empty spaces.

Play the Five-And game. The teacher calls out a number between 5 and 10. Students respond with "5" and the number needed to make 10. For example, if the teacher calls out "7", students need to respond "5 and 2".

Ask students how they would show numbers greater than 10 on their ten-frame mat. Have students describe these numbers (up to 15) as "10 and _____". For example, a student might show 13 as below and describe the number as "10 and 3".



10 and 3

LEARNING CONNECTION 2

Ten-Frame Cards

Materials
- Rel1.BLM5a–e: Ten-Frame Cards

Students benefit from being able to quickly identify numbers represented on a ten frame. Make copies of Rel1.BLM5a–e: Ten-Frame Cards. Flash the cards to the class and have students hold up number words or number cards to show they can correctly identify the number represented. Use ten-frame cards with the class for a few minutes periodically, at the beginning of a math lesson or in calendar activities.

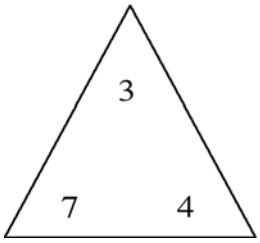
LEARNING CONNECTION 3

Part-Part-Whole Cards

Materials
- Rel1.BLM6a–f: Part-Part-Whole Cards
- manipulatives (pattern blocks, interlocking cubes, 2-colour counters)

Photocopy Rel1.BLM6a–f: Part-Part-Whole Cards and place the cards with a collection of manipulatives: pattern blocks, interlocking cubes, 2-colour counters.

Have students select a card. Their task is to determine which number represents the whole and which two numbers are the parts. Students then prove their solution by modelling with manipulatives.



examples

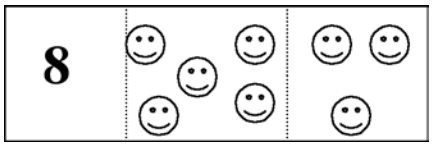
LEARNING CONNECTION 4

Missing Number Strips

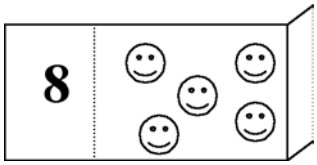
Materials

- 3 or 4 strips of paper folded to show 3 sections for each student
- stamps or stickers

Students create part-part-whole strips by using stamps or stickers, or by drawing circles or simple pictures.



After each student has made a few cards, have students play Missing Part with a partner. Students take turns folding over the last section of their strip of paper and having their partner try to figure out what is missing.



Collect all the strips created by students and use them periodically as part of whole-group warm-up activities. Show the folded strips to the whole class for ongoing practice with this concept.

"Ten in the Nest"

There were ten in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were nine in the nest and the little one said,
"Roll over, roll over." So they all rolled over and one fell out.
There were eight in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were seven in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were six in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were five in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were four in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were three in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There were two in the nest and the little bird said,
"Roll over, roll over." So they all rolled over and one fell out.
There was one in the nest and the little bird said,
"Good night!!"

"Ten in the Nest" Recording Sheet

Show all the ways you found to put 10 birds in two nests.
Show how you solved the problem.

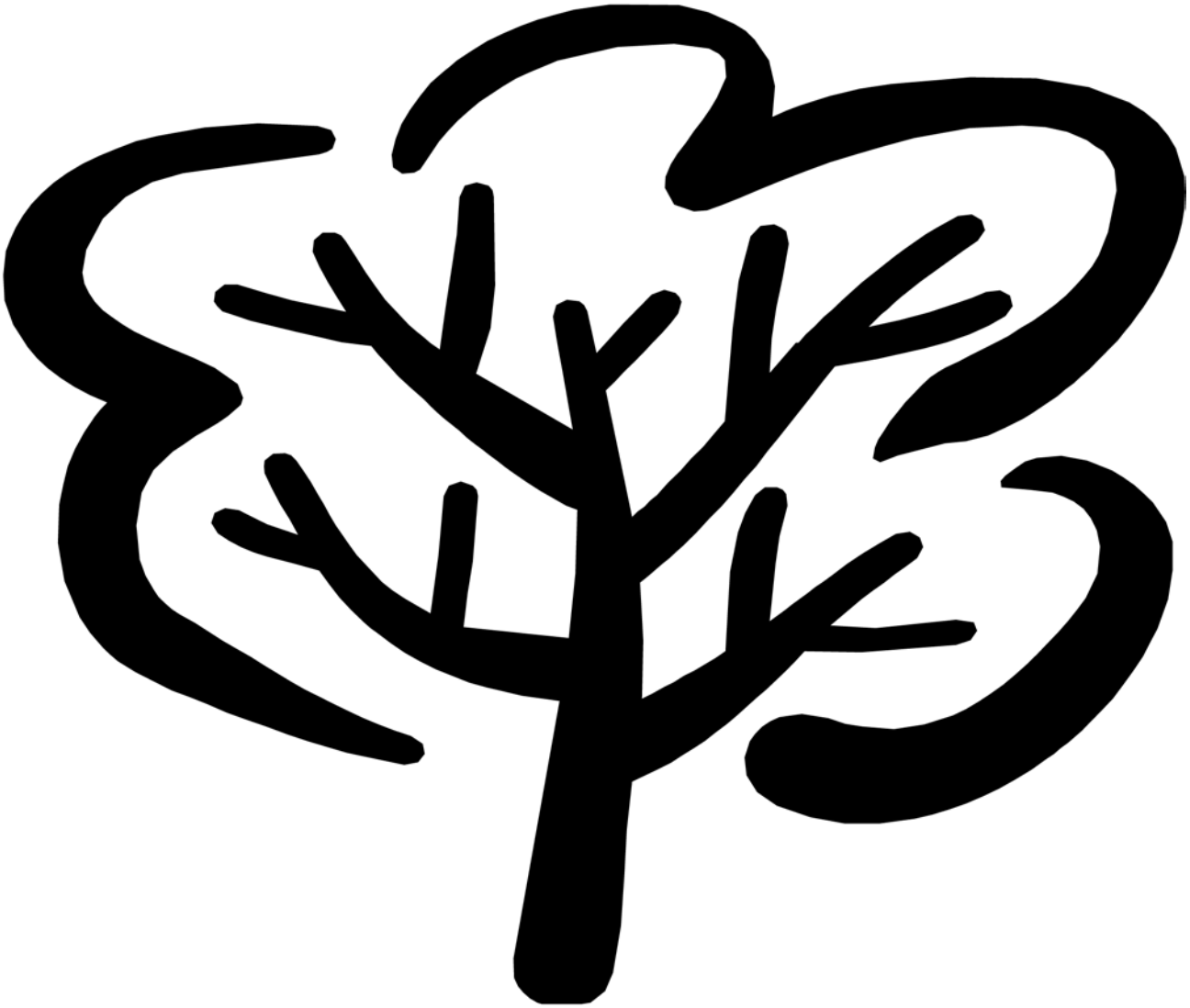
Twelve in the Nest

Dear Parent/Guardian:

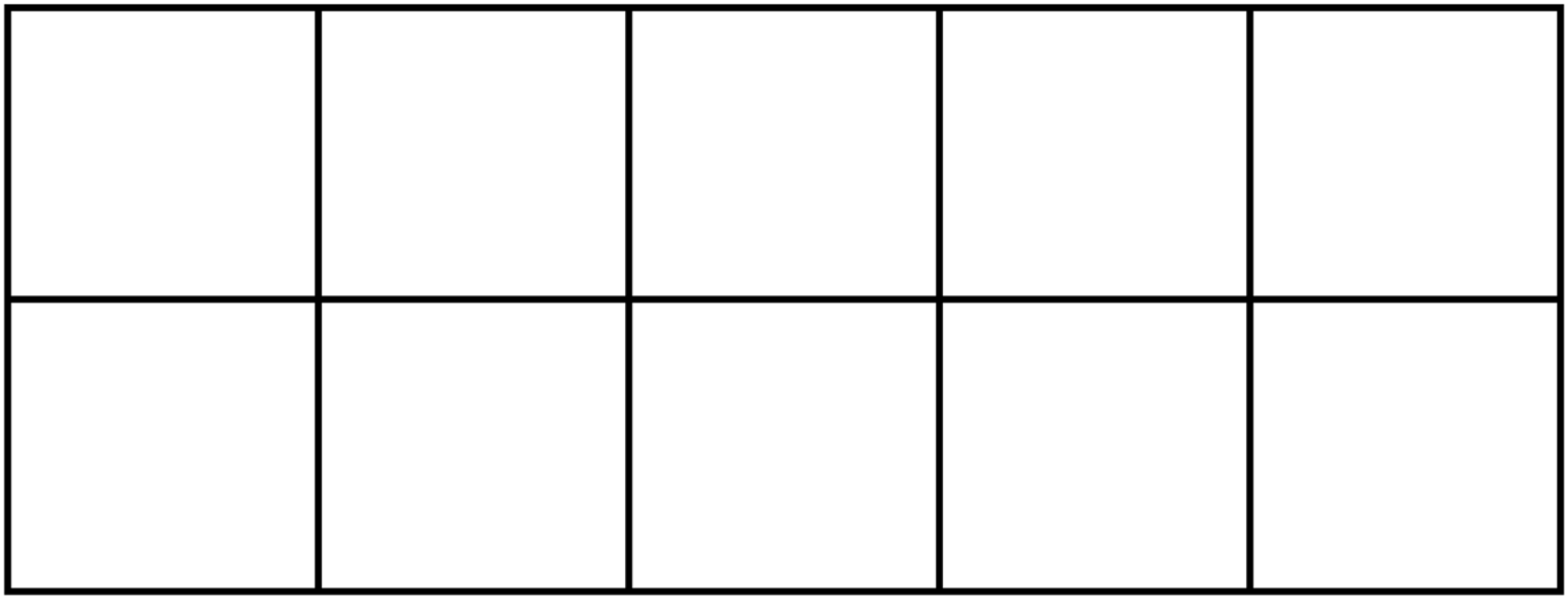
We have learned different ways to make 10: 7 + 3 = 10
6 + 4 = 10 9 + 1 = 10 2 + 8 = 10 5 + 5 = 10

Please help your child to find different ways to make 12.

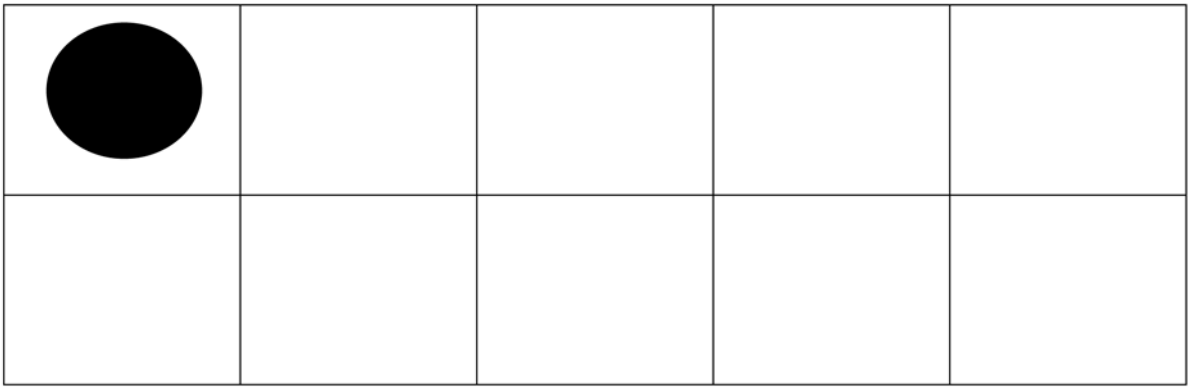
Twelve birds live in this tree. Ask your child to draw some nests in the tree. How can your child organize 12 birds in the nests? Can your child organize the birds different ways?

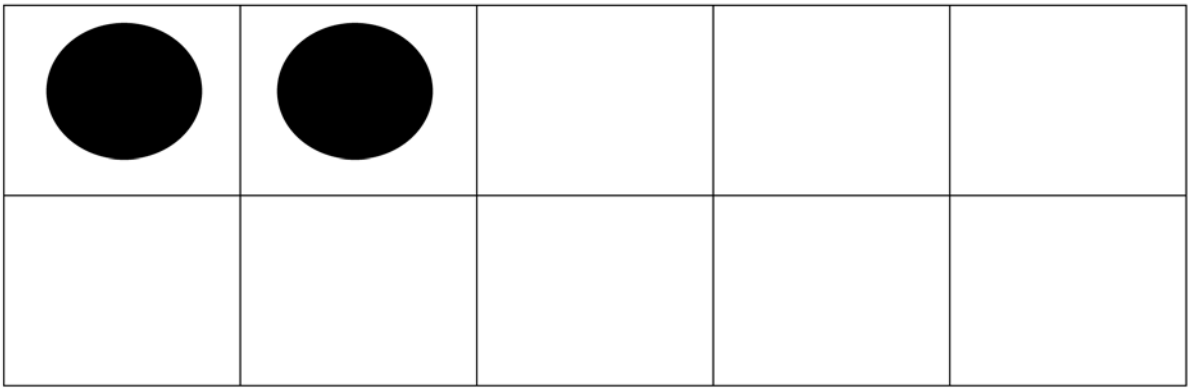


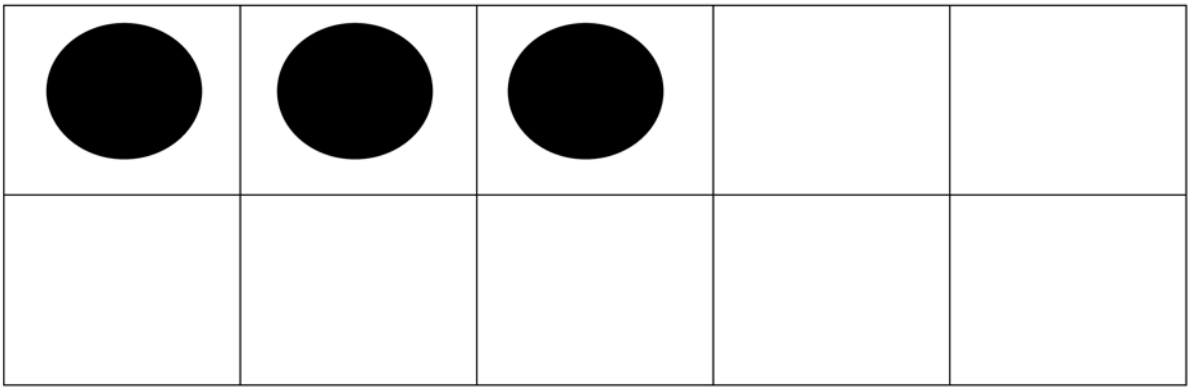
Ten-Frame Mat



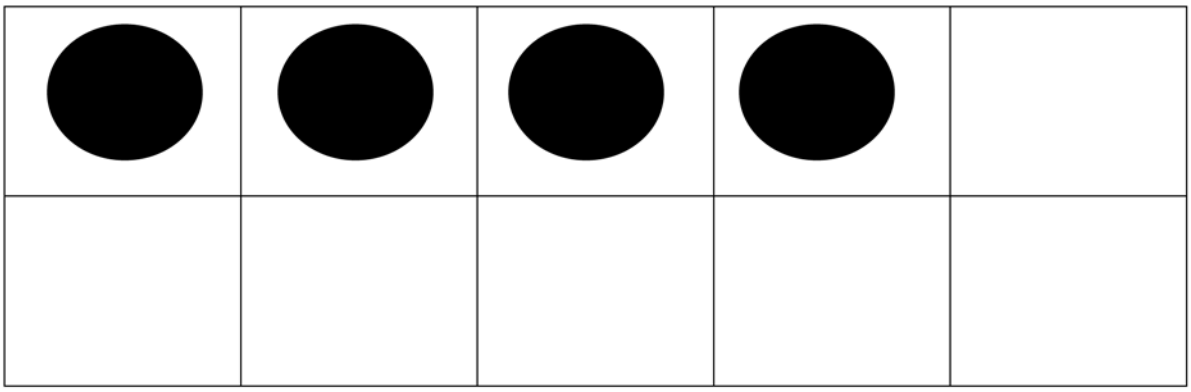
Ten-Frame Cards

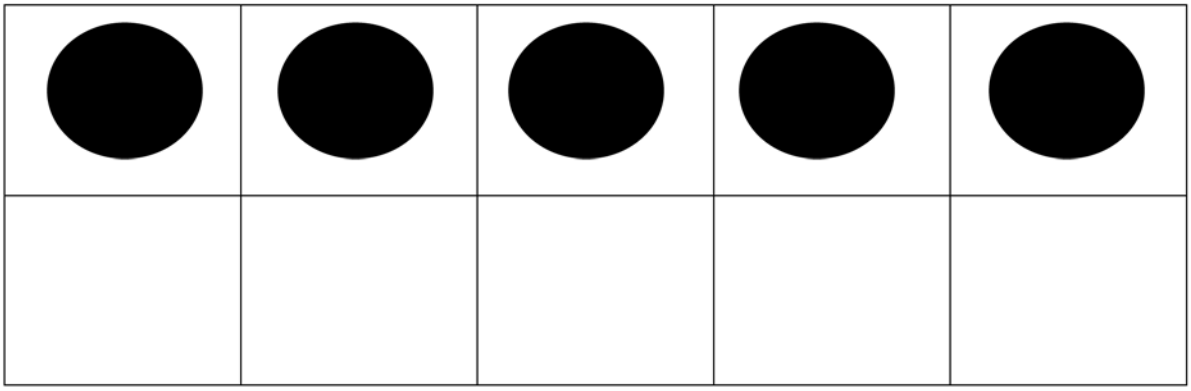


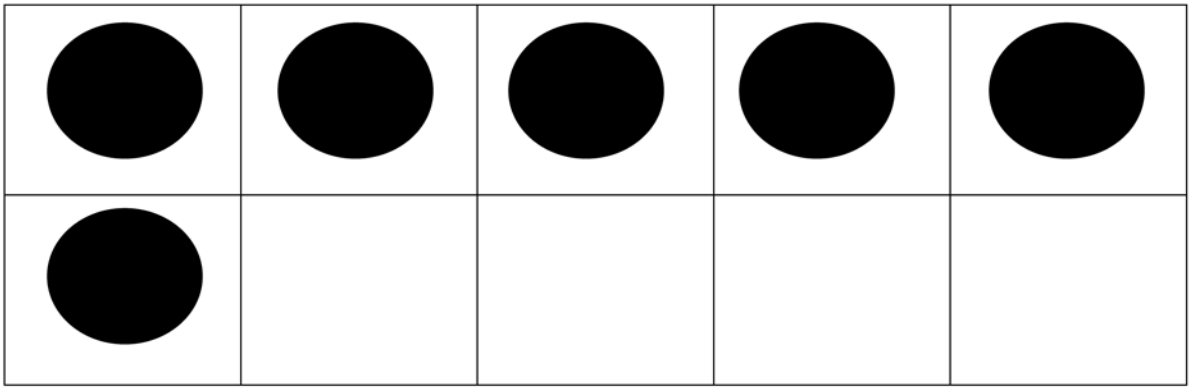




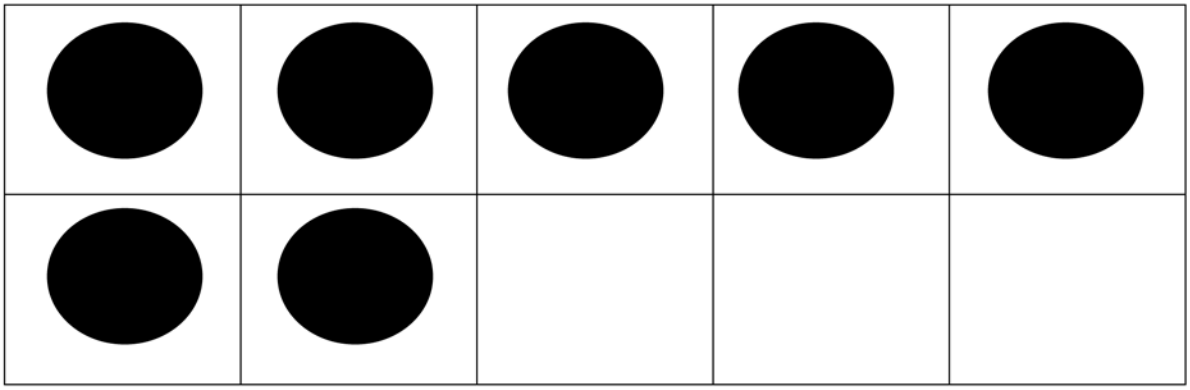
Ten-Frame Cards

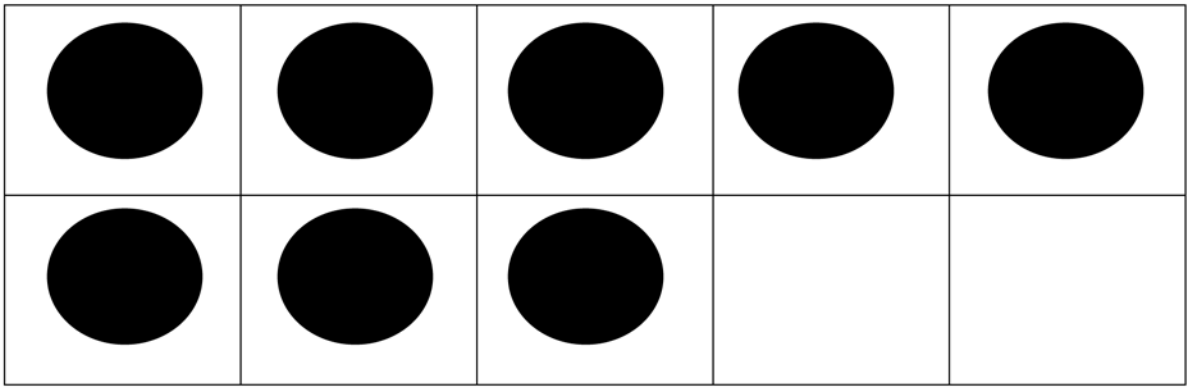


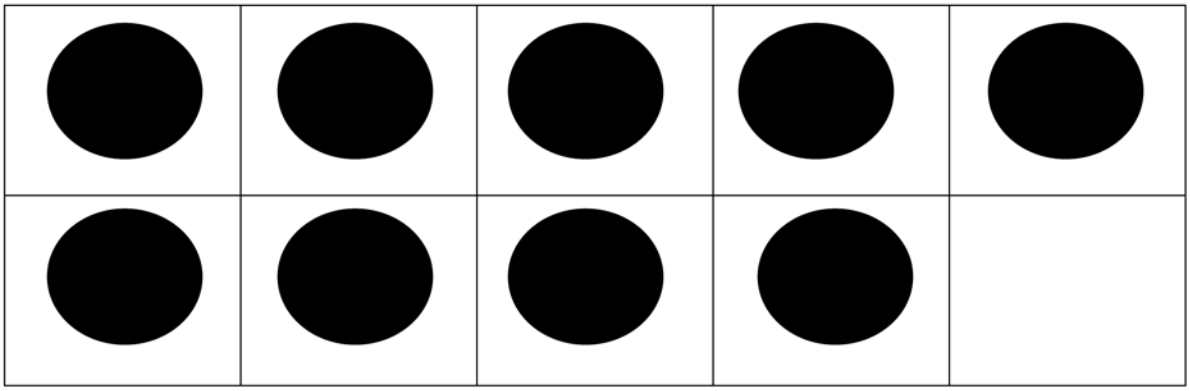




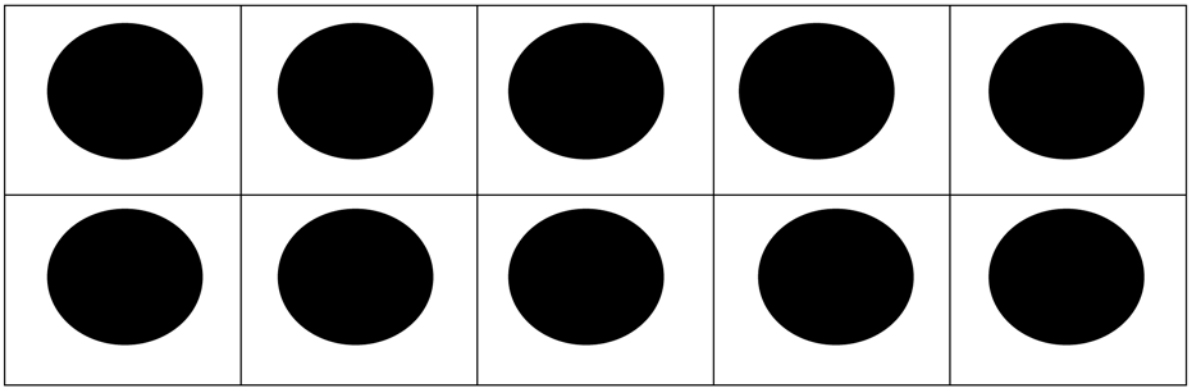
Ten-Frame Cards

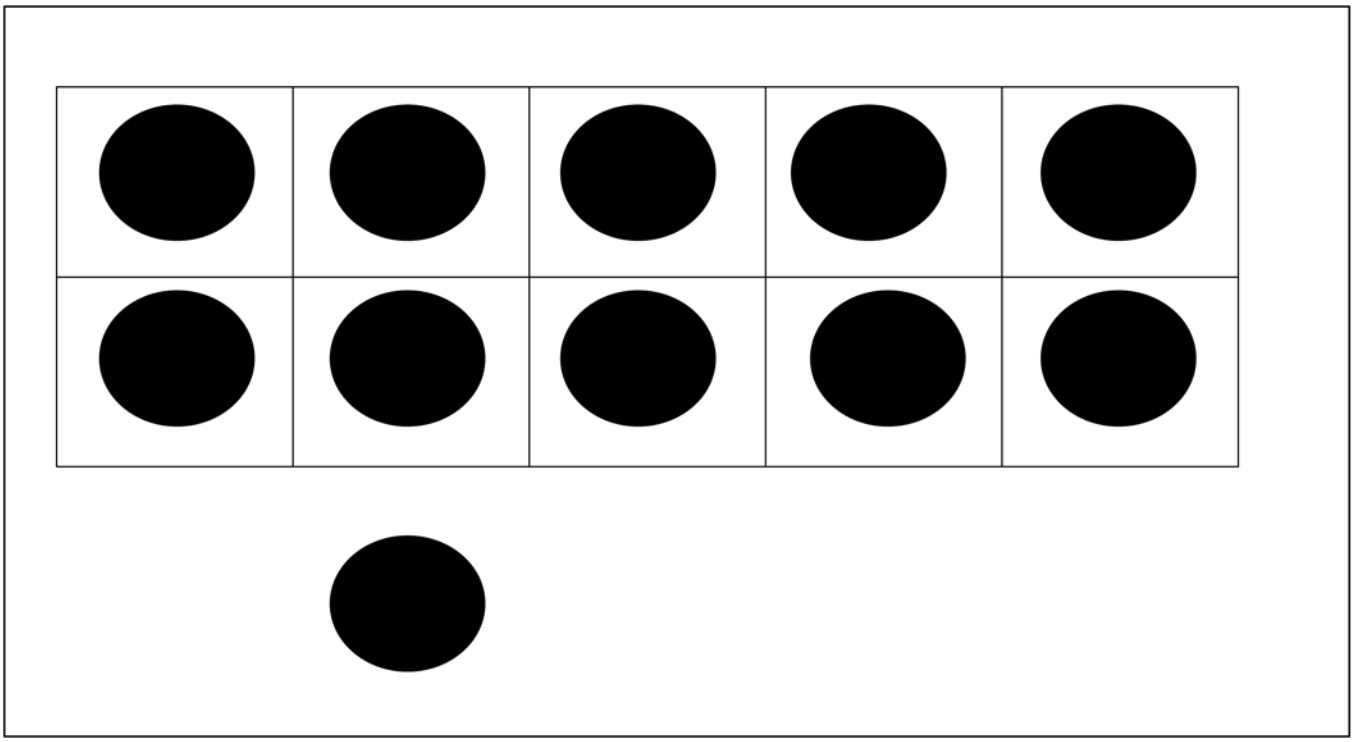




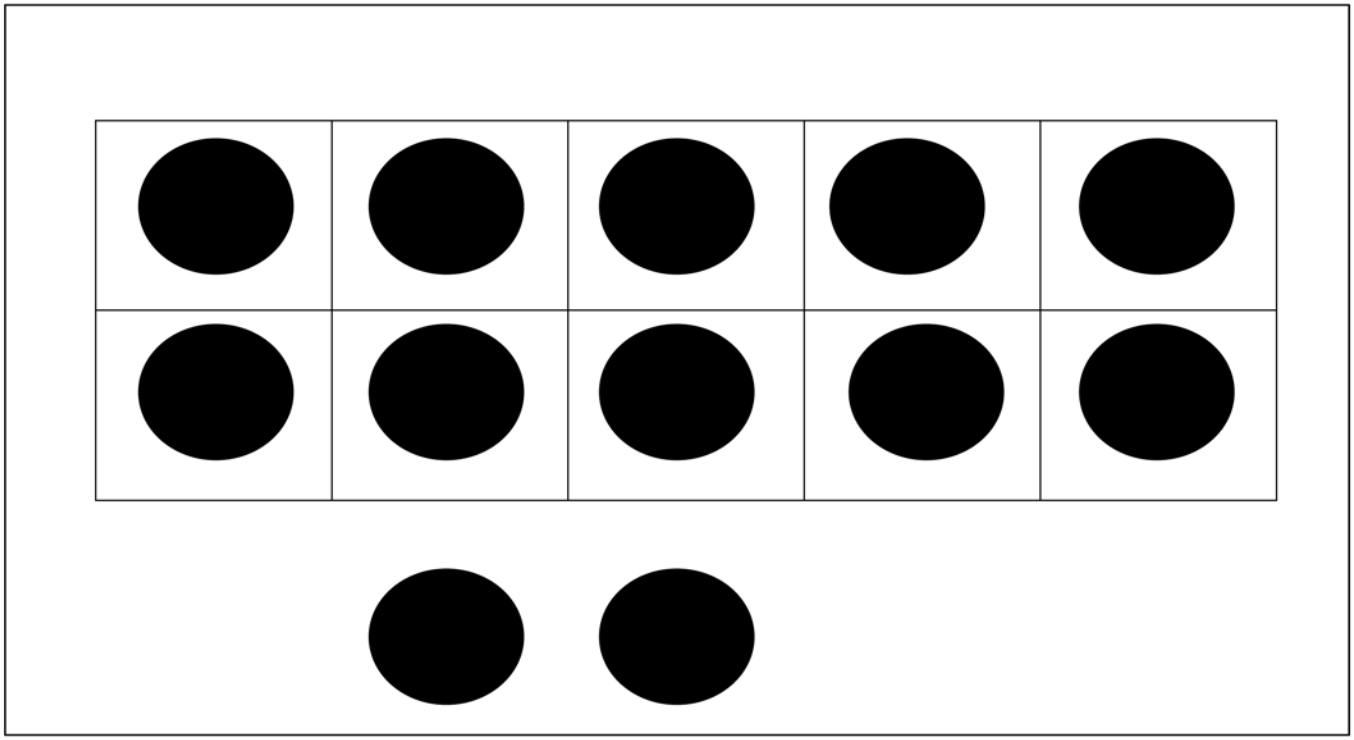


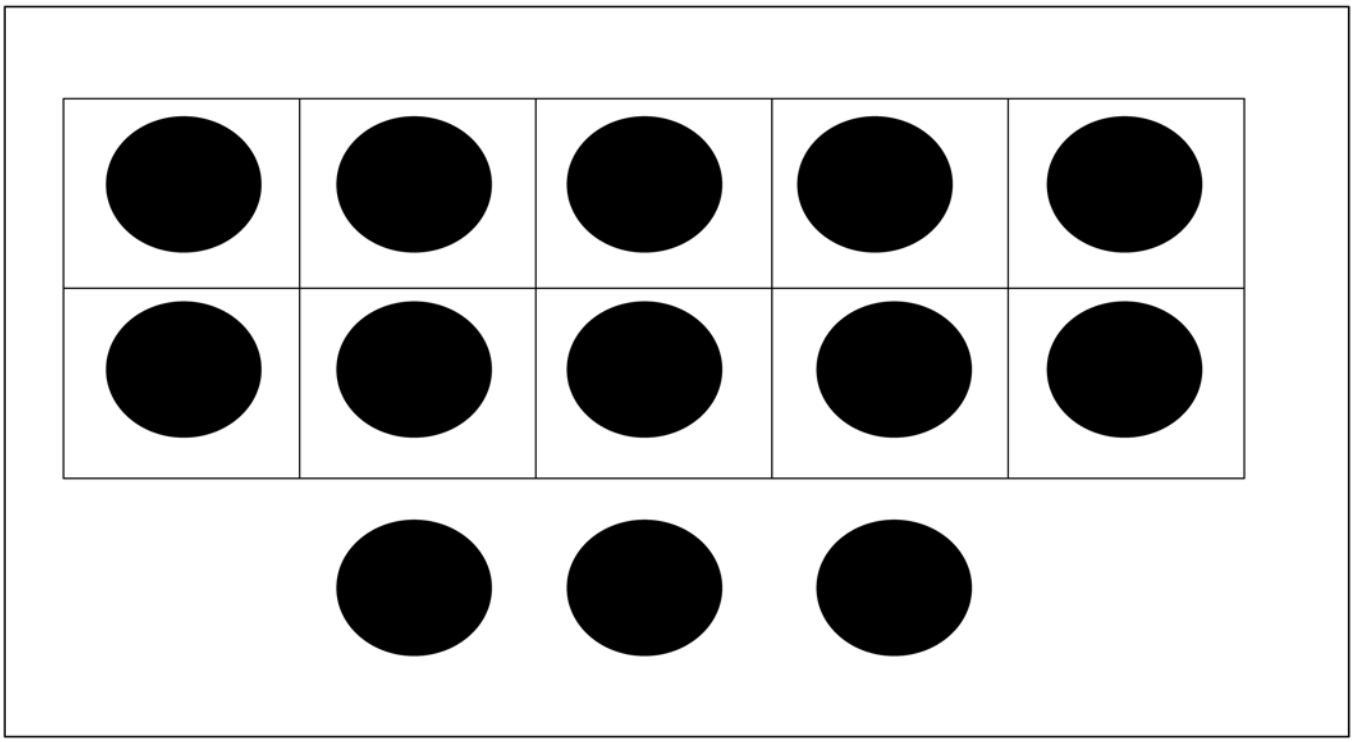
Ten-Frame Cards





Ten-Frame Cards





Part-Part-Whole Cards

1 3 2	2 2 4
1 3 4	5 3 2

Rel1.BLM6 (b)

Part-Part-Whole Cards

1 5 4	6 2 4
3 3 6	5 6 1

Part-Part-Whole Cards

6 1 7	2 7 5
7 4 3	8 7 1

Rel1.BLM6 (c)

Part-Part-Whole Cards

2 8 6	3 5 8
8 4 4	1 9 8

Rel1.BLM6 (d)

Part-Part-Whole Cards

7 2 9	3 9 6
4 5 9	10 1 9

Rel1.BLM6 (e)

Part-Part-Whole Cards

2 10 8	7 3 10
10 5 5	6 10 4

Rel1.BLM6 (f)

The Trading Game

BIG IDEA **Representation**

CURRICULUM EXPECTATIONS

Students will:

- represent, compare, and order whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences, number stories);
- count by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies;
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems.

MATERIALS

- chart paper
- 35 counters in containers per student
- 60 craft sticks or stir sticks per student
- 4 pipe cleaners per student
- 1 resealable plastic bag per student
- Rep1.BLM1: Place-Value Mat
- an overhead transparency of Rep1.BLM1: Place-Value Mat
- overhead place-value materials (craft sticks or toothpicks)
- a marker
- 1 number cube per group of students

ABOUT THE MATH

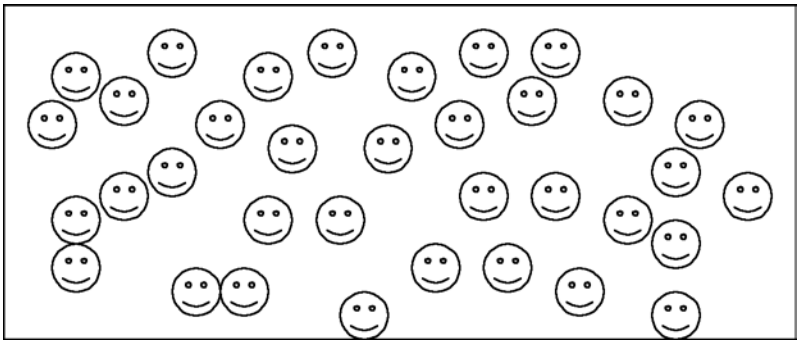
In Grade 1, students continue to develop their understanding of how numbers can be represented in many different ways. They have learned that a number can be represented with objects using a variety of different spatial and visual organizations and that numerals represent quantity. The idea of place value and how it is used to represent numbers is introduced in Grade 1. Although students in Grade 1 are capable of counting to 100 and are becoming increasingly successful at modelling numbers using concrete materials, they continue to see collections as groups of individual items. For example, a Grade 1 student asked to model the number 23 may show 3 counters to represent the 3 ones and only 2 counters to represent the 20. It is difficult for many students to grasp the idea that a group of 10 items is represented by the ones digit in

a number such as 11. Place-value models such as craft sticks and interlocking cubes are more effective than commercial materials such as base ten blocks. When students begin to learn about place value, they tend to find these traditional models too abstract.

GETTING STARTED

Before students come to class, draw 35 stars, flowers, or happy faces on chart paper. Scatter the pictures so no rows or columns are easily visible.

Place a container of 35 counters on each student's desk.

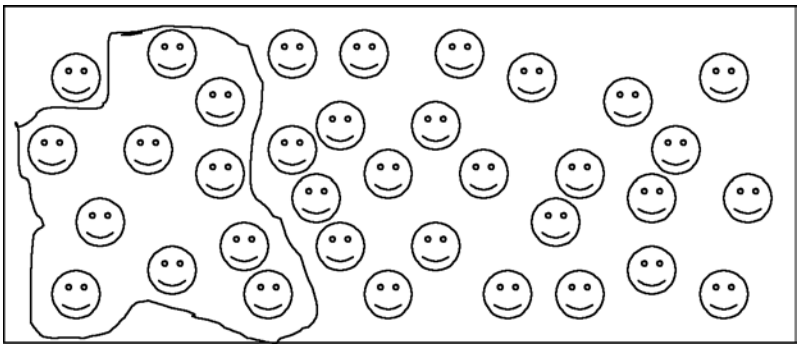


Ask students to estimate how many happy faces they see. Students may try to count by pointing at the chart, which will be difficult.

Have students go to their desks. Ask them to take out the 35 counters and to think about all the different ways they might count the counters.

Students should come up with ideas such as counting by 2's, 5's, and 10's. Have students use the manipulatives to model the different ways of counting 35 things, such as by laying out groups of 2's, 5's, and 10's on their desk.

Gather students back at the chart. Show students what it would look like if they counted the happy face picture by 10's.



Use a marker to circle groups of 10 happy faces.

Count the 10's and 1's. Write "3 tens and 5 ones". Ask: "How many faces altogether?" Write "35".

Ask: "How is 3 groups of 10 and 5 ones like the number 35?"

WORKING ON IT

Tell students they will be making place-value kits to use in math class.

To make the kits, students will need to make 4 bundles of 10 craft sticks by tying 10 sticks together with a pipe cleaner. They keep the remaining 20 single sticks to use to represent 1's. Students can store the kit in a resealable plastic bag.

Give each student Rep1.BLM1: Place-Value Mat. Have students practise representing numbers on the mat.

It is helpful to have an overhead copy of the mat and to model building the numbers on the screen for students to check their work. The teacher or a teacher's helper can make the same number as the class does by placing craft sticks or toothpicks on the overhead.

Ask students for one way to show 25 on the place-value mat. Ask them for another way, then for a third, each time modelling their suggestions on the overhead. Reinforce that 25 singles, 2 bundles of 10 and 5 ones, and 1 bundle of 10 and 15 ones all equal 25 sticks. Make a variety of 1-digit and 2-digit numbers.

Teach students how to play the Trading Game, a game for two to four players. Each player needs a place-value kit and Rep1.BLM1: Place-Value Mat. Each group needs a number cube.

Explain to students that a rule in the Trading Game is that they can never have more than 9 single sticks in the ones column. If they get more than 9 in the ones column, they must trade 10 singles for a bundle of 10. All bundles have to go in the tens column.

Have students explain why bundles go in the tens column: "Because there are 10 sticks in a bundle."

Model the game on the overhead while students play along on their own mat.

Roll the number cube. If a 6 comes up, place 6 sticks in the ones column.

Roll again. If a 2 comes up, add 2 sticks to the ones column. Ask students how many ones there are now (8).

Roll again. If a 4 turns up, add 4 sticks to the ones column. Ask students how many ones there are now (12). Say, "Oh, oh! What's the rule?" Let students see that 10 single sticks are traded for a bundle.

Talk about what the place-value mat looks like now, with 1 ten and 2 ones. Ask, "How much is that?" (12). Reinforce to students that the number has not changed, just how the number is shown has changed.

Continue rolling, modelling, and talking until the number has reached or passed 30.

Have students play the game with a partner or in small groups. Players take turns rolling the number cube and adding to their place-value mat. Every time they get more than 9 in the ones column, they must trade 10 ones for a bundle. The first player to reach 3 bundles (30) wins.

Have students play this game many times over the school year. Tubs with the place-value kits students made, number cubes, and place-value mats should be available for students to use during activity periods.

After students have played the game up to 30 several times, it is important to change the rules to play from 30 to 0. This requires students to break apart bundles to subtract the number rolled. This activity will help students develop a conceptual understanding of regrouping and of how the decomposition and composition of numbers work in 2-digit addition and subtraction (which they will learn in Grade 2).

REFLECTING AND CONNECTING

It is important to have many discussions about place-value activities. Give each student many opportunities to use words to describe the different ways to make numbers on the place-value mats. Have individual students provide more than one example. For instance, a student may say 25 can be 2 tens and 5 ones, 1 ten and 15 ones, or 25 ones. Always ask which way is most like the way that numbers are written (tens and ones). Reinforce the idea that 2 tens and 3 ones have the same value as 23 ones. The quantity stays the same; the representation is different.

ADAPTATIONS/EXTENSIONS

Spend time having students practise making numbers on the place-value mat before moving to the place-value game. In some classes, individual students' readiness to move to the Trading Game will vary. Introduce the game slowly over time to small groups of students. These students can become experts and teach the game to other students when they are ready.

MATH LANGUAGE

- tens, ones
- trade
- group/regroup
- place value

SAMPLE SUCCESS CRITERIA

- represents numbers accurately using base 10 models
- represents a number in more than one way using base 10 models
- decomposes numbers based on place value
- uses vocabulary related to place value correctly (e.g., "tens", "ones", and "regroup")
- organizes for and counts by 1's, 2's, 5's, and 10's to 100
- organizes for and counts backwards by 1's, 2's, and 5's from 20
- orders numbers to 50 on a hundreds chart using place value

HOME CONNECTION

The Trading Game is an excellent game to send home. Be sure that students are familiar with the game and the rules before taking home a place-value kit. Students can teach the game to someone at home.

LEARNING CONNECTION 1

Student-Created Place-Value Centre

Materials

- masking tape
- markers
- resealable plastic bags
- variety of small manipulatives
- Rep1.BLM2: Place-Value Bags Recording Sheet
- Rep1.BLM3: Place-Value Bags Master Recording Sheet

Use masking tape and markers to label the bags A, B, C, and so on. Have students work with a partner. Provide each pair with a plastic bag. Have each pair of students collect a quantity of manipulatives to put in their bag.

Have students record the number of items in their bag on Rep1.BLM2: Place-Value Bags Recording Sheet and then complete the boxes showing the different groups of tens and ones they can make to represent the number using those materials.

Make a master list of all bags using Rep1.BLM3: Place-Value Bags Master Recording Sheet. Place the materials at a centre for students to investigate during activity periods. Students complete a new Rep1.BLM3: Place-Value Bags Master Recording Sheet each time they go to the centre.

LEARNING CONNECTION 2

Hundreds Chart Hi!

Materials
- bingo chips
- Rep1.BLM4: Hundreds Chart

Provide students with a clue to a specific number (2 tens and 5 ones). Have students place a bingo chip on their hundreds chart to cover the number.

Continue until the final pattern spells the word "Hi" (2, 12, 22, 32, . . . , 92; 5, 15, 25, 35, . . . , 95; 8, 18, 28, 38, . . . , 98; 43, 44).

Note: Be sure clues are adapted to match the level of understanding of the particular group of students.

Sample Clues

Cover the number that is:
- 2 tens and 5 ones
- after 17
- 4 in the tens place and 4 in the ones place
- after 34
- after 47
- 10 plus 20
- between 41 and 43
- 10 less than 40
- 20 more than 15
- 10 more than 22
- 4 tens and 3 ones

Adaptations/Extensions

After students have played this game as a class, they may want to create their own puzzles, choosing to make the initial of their first name or a simple picture. Students should shade their design on a hundreds chart and then write the clues. After the

puzzles have been checked by the teacher, the clues may be placed at a centre with blank hundreds charts and bingo chips for students to try during activity periods.

LEARNING CONNECTION 3

Hundreds Chart Activities

Materials

- hundreds chart pocket chart
- number cards 1–100

Note: At the beginning of the year, use number cards between 1 and 50, gradually inserting additional numbers as students are ready.

1. **Mixed-up numbers.** Before students come to class, mix up the number cards on the hundreds chart. Say: "Something is wrong. Who can help fix it?" (See graphic suggestion below.)

1	3	2	5	6	7	8	4	9	10
11	12	23	14	16	18	15	17	19	20

2. **Continue the line.** Leave out parts of each line on the hundreds chart and have students tell which numbers are missing. (See graphic suggestion below.)

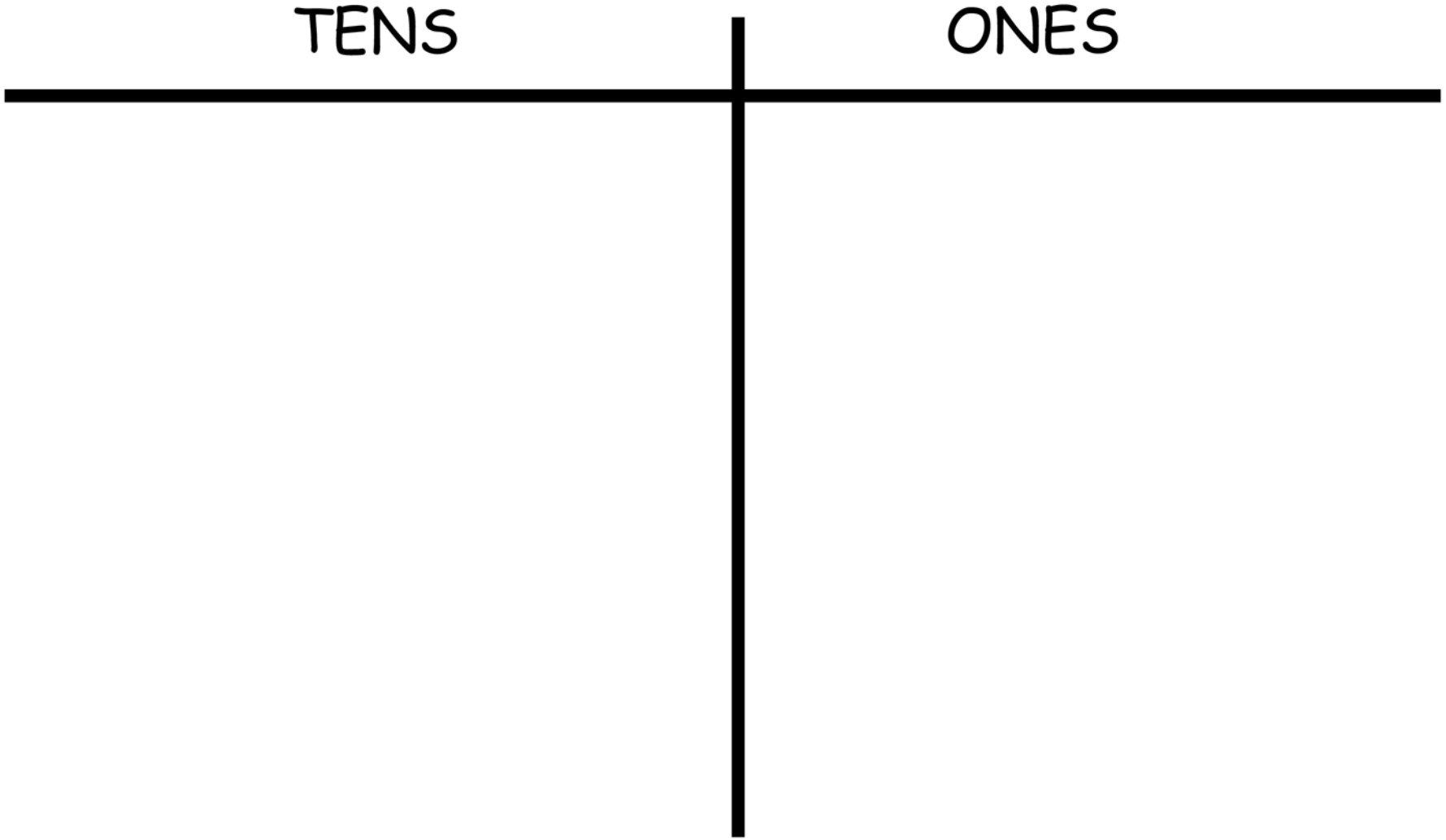
1	2	3							
11	12		14	15	16				
21	22	23	24						

3. **Find the number.** Fill the hundreds chart from 1 to 50, with the number cards face down. Ask: "Where is 10? Where is 30? Where is 25?"
4. **Before and after.** Develop the vocabulary of "before", "after", and "between" by leaving columns blank and asking for the numbers before, after, and between.

1		3		5					
11		13		15					
21		23		25					
31		33		35					

5. **Secret number or 20 questions.** The teacher thinks of a secret number. Students ask questions that the teacher can answer with only yes or no to eliminate numbers. For example, a student may ask, "Does it have a 4 in the ones place?" If the answer is no, the teacher removes all the numbers that have a 4 in the ones place from the board. Encourage students to think of clues that will remove the greatest number of cards, such as, "Is it an odd number? Is it more than 50? Is it less than 50?" Emphasize that the patterns in the rows let the teacher know which numbers to remove.
6. **Counting patterns.** Show counting patterns by turning even-numbered cards face down, odd-numbered cards face down, or every fifth number face down, and so on.
7. **Counting down.** Use the hundreds chart to have students practise counting backwards. Remove numbers one at a time while students count aloud. Start initially by counting back by ones from 20 and any number less than 20. Then explore counting backwards by 2's and 5's from 20. Help students to understand these patterns of counting backwards and extend them to higher numbers on the hundreds chart.

Place-Value Mat



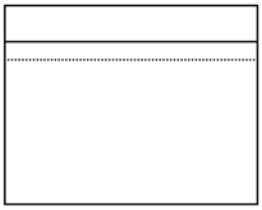
Rep1.BLM2

Place-Value Bags Recording Sheet

My Bag!

Bag # _____

How many? →



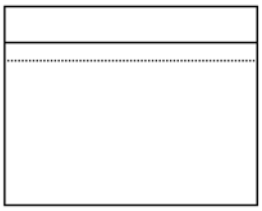
Tens _____ Ones _____

Tens _____ Ones _____

Tens _____ Ones _____

Bag # _____

How many? →



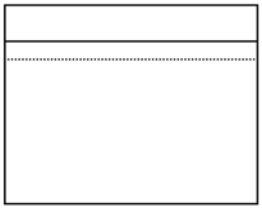
Tens _____ Ones _____

Tens _____ Ones _____

Tens _____ Ones _____

Bag # _____

How many? →



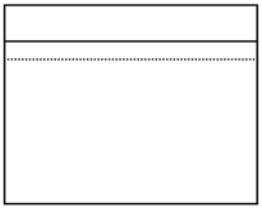
Tens _____ Ones _____

Tens _____ Ones _____

Tens _____ Ones _____

Bag # _____

How many? →



Tens _____ Ones _____

Tens _____ Ones _____

Tens _____ Ones _____

Rep1.BLM3

Place Value Bags Master Recording Sheet

Bag #	Bag #	Bag #	Bag #	Bag #
Bag #	Bag #	Bag #	Bag #	Bag #
Bag #	Bag #	Bag #	Bag #	Bag #
Bag #	Bag #	Bag #	Bag #	Bag #
Bag #	Bag #	Bag #	Bag #	Bag #
Bag #	Bag #	Bag #	Bag #	Bag #

Rep1.BLM4

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

B. Grade 2 Learning Activities

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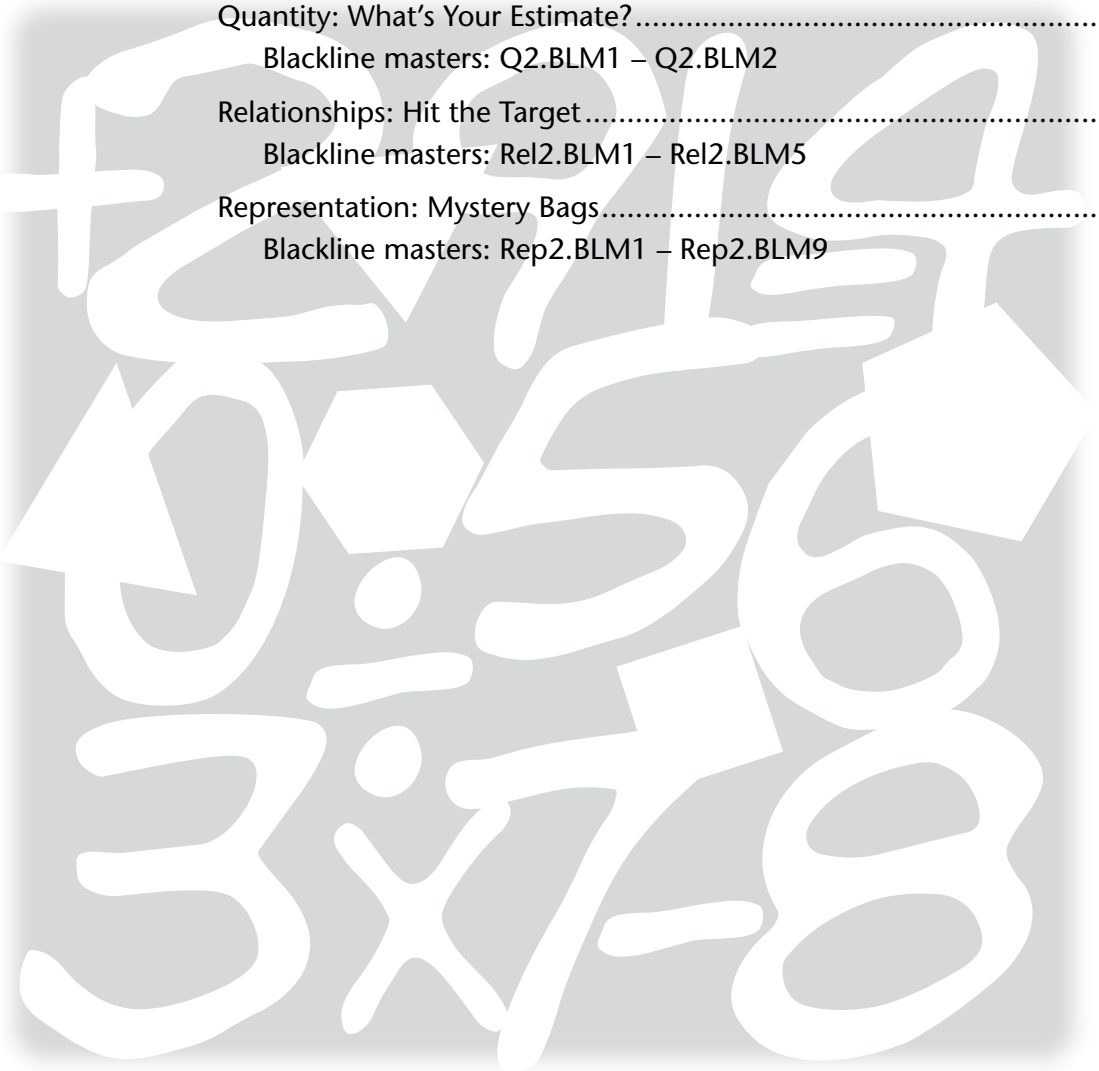


The Magician of Numbers

BIG IDEA *Counting*

CURRICULUM EXPECTATIONS

Students will:

- solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (e.g., base ten materials, counters), student-generated algorithms, and standard algorithms;
- count forward by 1's, 2's, 5's, 10's, and 25's to 200, using number lines and hundreds charts;
- represent, compare, and order whole numbers to 100, including money amounts to 1000, using a variety of tools (e.g., tens frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets);
- locate whole numbers to 100 on a number line and a partial number line (in this case, a hundreds chart);
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

MATERIALS

- blank hundreds carpet or mat
- C2.BLM1: Blank Hundreds Chart
- C2.BLM2: Hundreds Chart
- small object (such as a 2-sided counter)
- magician's wand or hat (optional)
- bingo chips or counters
- calculator
- C2.BLM3: Guess My Rule!

ABOUT THE MATH

In Grade 2, students count by 1's, 2's, 5's, 10's, and 25's to 100 and beyond. Students begin to produce the number words just before and just after numbers to 100, and this can create difficulties when they use counting to solve 2-digit computations.

Opportunities to use hundreds charts or mats will allow students to visually and kinaesthetically explore these counting patterns.

Before doing this activity, students would benefit from exploring 1-digit and 2-digit numbers in a variety of contexts. Learning experiences using the required manipulatives (calculators, base ten blocks, hundreds mat, hundreds charts, number cubes, and playing cards or number cards) would also be helpful. Experience counting by 2's, 5's, and 10's to 100 and verbalizing larger numbers to 100 will also help students with the following activities.

Management Tip: Students would benefit from a review of using base ten materials and other manipulatives to model and represent larger numbers to 100, i.e., in representing numbers using base ten materials, if there are more than 9 objects they must be traded for a tens bundle.

GETTING STARTED

In a whole class guided investigation, play the Landing Zone game. Use a blank hundreds mat or carpet or enlarge C2.BLM1: Blank Hundreds Chart. Have students sitting at the base of the mat or chart so they are all viewing it from the same perspective.

Toss any small object (e.g., a 2-sided counter) onto one of the blank squares. Students then figure out on which number square the object landed. Listen for students communicating their thinking and proving their answer. For example, if the object landed on 32, a student might reason that he or she counted along to 2 and then counted down the chart by 10's from 12 to 22 and landed on 32.

Note: C2.BLM1: Blank Hundreds Chart could be used on an overhead projector. Include either a 1 in the upper left corner or a 100 in the bottom right corner on the blank hundreds chart to orient students.

ADAPTATIONS/EXTENSIONS

As a variation of this activity, state a number and then have a student place an object on or stand on that space on the mat.

WORKING ON IT

This activity makes use of a blank hundreds mat or carpet or blank hundreds chart on the floor. If the classroom does not have any of those, use C2.BLM1: Blank Hundreds Chart.

Students will explore patterns created while they add using a game called the Magician of Numbers.

Students take turns being the Number Magician and may wear a special hat or have a wand. Their job is to choose another student to be the Number Seeker and to provide numbers for the Number Seeker to find.

The Number Magician chooses a number below 50, and the Number Seeker stands on it (if using a mat) or points to it (if using a chart). The Number Magician chooses another

number to add to the first one, and the Number Seeker must find the sum and move to or point to the sum.

The Number Seeker orally describes the steps to the rest of the group (who are the Number Masters) as he or she makes the moves on the chart or the mat.

Note: The Number Seeker must move one grid space at a time. The Number Masters can help the Number Seeker if necessary by providing clues about where and how to move.

Another student is assigned the role of Number Checker. The Number Checker uses a calculator to check the Number Seeker's progress on the hundreds chart.

Management Tip: As the Number Seeker describes the moves, the teacher may want to record them on a large blank hundreds chart. Use this record later for reflection by the class.

Example: If the Number Magician chooses 23, the Number Seeker stands on that number. The Number Magician then gives the number 34 to add to 23. The Number Seeker may move down the chart to 33, which represents a jump of 10, then 43, which is another 10, and 53, which is a further jump of 10. Then the Number Seeker would jump by 1's to 57, which is the final destination.

To play the game again, the Number Magician, the Number Seeker, and the Number Checker can choose three new students to take on these roles.

After playing the game several times as a class, provide pairs of students with C2.BLM1: Blank Hundreds Chart and bingo chips or other counters. Partners take turns playing the roles of Number Magician and Number Seeker. The Number Magician chooses the starting number and the addend. The Number Seeker places a bingo chip on the starting number and describes the steps to reach the final sum. The two students check the result of the Number Seeker's moves using a calculator.

REFLECTING AND CONNECTING

Gather students together to discuss the various strategies they used to play the game. If the moves or strategies students used as they played the game were recorded, refer to them to aid in recalling those strategies.

Ask questions such as the following:

- "What is the fastest way to find your starting point?"
- "What patterns did you notice?"
- "What made the game easier to play?"
- "How do you think this game could help someone to add 2-digit numbers?"
- "How do you think the game would be different if the Number Magician asked you to subtract a number from the starting number?"

- "Why did you choose to . . . ?"
- "How would you go to . . . ?"
- "What would be a faster way to get to . . . ?"
- "What would be a different way of getting to . . . ?"

ADAPTATIONS/EXTENSIONS

For students who find this activity difficult, use C2.BLM2: Hundreds Chart. Students can use a pencil to draw the path they took to reach the sum. Students could also use a calculator to help them to find the sum or to check their answer.

Play the Magician of Numbers using subtraction instead of addition.

MATH LANGUAGE

- hundreds chart
- hundreds mat
- calculator

SAMPLE SUCCESS CRITERIA

- finds the starting number on a (blank) hundreds chart
- demonstrates addition on the hundreds chart accurately (e.g., counts on to the starting number, moves down to add ten and to the right to add one)
- decomposes the addend into tens and ones
- describes counting patterns on a hundreds chart (e.g., counting by tens)
- orders numbers using place value
- composes numbers based on place value
- decomposes numbers based on place value

HOME CONNECTION

Guess My Rule!

Before sending home C2.BLM3: Guess My Rule! and C2.BLM2: Hundreds Chart, have students create patterns on a hundreds chart by colouring squares or placing counters on squares. Students then exchange their completed patterns and have a partner guess their pattern rule.

LEARNING CONNECTION 1

Secret Patterns

Materials

- calculators
- counters
- overhead calculator or computer calculator (optional)
- C2.BLM2: Hundreds Chart

Have students work in pairs. One partner secretly enters a number expression that uses the same number (e.g., 5 + 5) into a calculator. That student then presses the equal sign key to show the first number in the pattern, which he or she shows to a partner.

The partner places a counter on that number on C2.BLM2: Hundreds Chart. The process of one partner pressing the equal sign key and the other placing the counter on the hundreds chart continues until the partner placing the counters determines the pattern.

This activity can also be completed using an overhead calculator or a computer calculator.

Adaptation/Extensions
Having students use two different numbers in the number expression (e.g., 2 + 5) will provide more of a challenge in finding the secret pattern. This activity may also be done with subtraction.

LEARNING CONNECTION 2

Disappearing Numbers

Materials
- C2.BLM1: Blank Hundreds Chart
- counters
- C2.BLM2: Hundreds Chart (if necessary)

Students work in pairs for this activity. Each student secretly covers four or five numbers on C2.BLM1: Blank Hundreds Chart using counters. Students reveal their hundreds chart to their partner and take turns guessing each other's numbers. Students explain to their partner how they figured out the number (e.g., "It was the 4th number in the 3rd row, so I knew it was 34."). This is an excellent time for teachers to interact with students and listen to their strategies for discovering the missing numbers on the hundreds chart. For students who need more guidance, this game can be played using C2.BLM2: Hundreds Chart with numbers on it.

LEARNING CONNECTION 3

Money Madness

Materials
- plastic pennies* and dimes with hook-and-loop fasteners stuck to them
- hundreds chart

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

Stick hook-and-loop fasteners (like Velcro) on the back of 10 plastic or real pennies. Using a hundreds chart, stick the other half of the hook-and-loop fasteners on each square. Each day, have students stick a penny on the chart to represent that day of school, repeating this until the 10th day. On the 10th day, a student will trade the first 10 pennies for a dime with a fastener on it by pulling pennies off the hundreds chart. On the 20th day, a student will trade 10 more pennies for another dime, and so on.

This is an effective visual and tactile way to show trading and grouping using tens and ones.

LEARNING CONNECTION 4

Hundreds Chart Puzzles

Materials

- C2.BLM2: Hundreds Chart

The teacher cuts C2.BLM2: Hundreds Chart into strips, chunks, or individual pieces. The chart can be cut in different ways to reinforce a variety of number concepts. For example, if the teacher makes 10 horizontal cuts and asks students to piece the hundreds chart back together, students will have to look at the tens column to rebuild the chart. If the teacher makes ten vertical cuts, students will have to look at the ones columns in order to rebuild the chart.

C2.BLM1

Blank Hundreds Chart

C2.BLM2

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

GUESS MY RULE!

We have been learning about number patterns on a hundreds chart in our classroom. One of the activities that you could play with your child at home is Guess My Rule!

Ask your child to create a pattern by placing some pennies (or other counters) on the attached hundreds chart (C2.BLM1) and to think about the rule they used. Then, try to guess your child's rule.

Next, you create a different pattern and ask your child to guess the rule for your pattern.

Some examples of number patterns:

An example of a simple pattern might be to cover all the squares with even numbers. Then, you try to guess the rule (e.g., "Was your rule to cover every other one?")

An example of a more complex pattern might be to add 4 every time so that the numbers covered up would be 4, 8, 12, and so on.

An example of a difficult pattern would be a *growing* pattern where 1 is added to 1 and so the 2 is covered. Then 2 is added to the 2 and the 4 is covered. Next the 3 is added to the 4 and 7 is covered, and so on.

Two by Two

BIG IDEA ***Operational Sense***

CURRICULUM EXPECTATIONS

Students will:

- solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies;
- add and subtract two-digit numbers with and without regrouping, with sums less than 101, using concrete materials;
- compose and decompose two-digit numbers in a variety of ways, using concrete materials (e.g., place 42 counters on ten frames to show 4 tens and 2 ones; compose 37¢ using one quarter, one dime, and two pennies*);
- solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials;
- locate whole numbers to 100 on a number line and on a partial number line;
- apply developing problem-solving strategies as they pose and solve problems and conduct investigations to help deepen their mathematical understanding;
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

MATERIALS

- manipulatives
- paper
- pencils, crayons, and so on
- chart paper or an overhead projector
- OS2.BLM1: Show What You Know!

ABOUT THE MATH

Flexible approaches to algorithms (such as allowing students to develop their own algorithms) are an excellent way for students to develop their own thinking, their own strategies, and a greater awareness of number.

Traditionally, teachers have provided students with the algorithm to answer an operational question, rather than allowing students the freedom to solve a problem in a way that makes sense to them. For example, with 2-digit addition, students were traditionally taught the algorithm of adding the ones column first and then the tens column. If students are allowed to solve the problem on their own, some students start with the tens column and then move to the ones because this way makes more sense to them.

Flexible approaches to algorithms are built on student understanding. Because these strategies are so dependent on base ten concepts and are developed in a problem-solving atmosphere, students rarely use a strategy they do not understand. Students make fewer errors when they use flexible approaches.

Students need to understand operations on numbers conceptually and procedurally and in the context of problem solving. They also need to develop an understanding of the combining and partitioning of numbers. Students who need to use a traditional regrouping algorithm of borrowing and crossing out numerals to solve 2000 – 1 = ☐ do not have good number sense. They may have become so dependent on the rote use of operations that they rely less on their own reasoning skills.

By providing students with many opportunities to develop their own strategies for working with numbers, teachers allow students to bring more meaning and flexibility to their understanding of traditional algorithms. For instance, students left to their own devices to figure out 29 plus 33 may simply add 20 and 30 in their head, then put the 9 and 1 of the 3 ones together to make another 10, then add the 2 remaining units for a total of 62, as shown below:
29 + 33
20 + 30 is 50
9 + 1 is 10 (easier to add numbers to make 10's because 10's are easy to add)
50 + 10 = 60
2 more ones left over from the 3 ones in the 33
So, the answer is 60 + 2 = 62.

Students without such flexibility may use the standard algorithm and end up with an answer such as 512 (by putting down 12 for 9 + 3 and then 5 for 2 + 3) and have no sense that the answer is incorrect:

$$\begin{array}{r} 29 \\ +33 \\ \hline 512 \end{array}$$

If presented with a problem such as 27 + 56, students will use a variety of strategies to find an answer. They might begin in the tens column and add the tens (20 + 50 = 70). Then they might proceed to the ones column and add 7 + 6 = 13. They will add these two sums to find the total sum (70 + 13 = 83).

Students need many opportunities to estimate answers to addition and subtraction questions in order to view their solutions as reasonable. As in the above example, if students had estimated an answer before solving, they would realize that their answer is not reasonable.

The process of using flexible strategies is ongoing and cannot be presented to students in a single day or unit. This developmental process needs to be encouraged and supported throughout the year. Learning should be scaffolded to help students move towards more efficient strategies. For example, if they continually use counting by 1's as a strategy, suggest that they group by 2's or 10's to make their counting easier.

GETTING STARTED

Share the following problem with students. Have students use the strategy of think-pair-share to work through this problem with a partner.

"We are ordering pizza for our class for pizza day. We didn't have enough pizza on our last pizza day, so we need to order more. Last time we ordered 38 pieces of pizza. This time we want to order 13 more pieces. How many pieces of pizza should we order for this pizza day?"

Give students time to think about what they would do and how they might figure this problem out before they meet with their partner. Suggest that students work with manipulatives. They could also choose to use paper to record their ideas. Once students have had time to consider the problem, divide students into pairs. Try to pair students heterogeneously, putting those students together who have different methods for solving problems, and give them a few minutes to share their ideas about solving the problem. Have students gather any materials they want to use and quietly find a space in the classroom to work. Students cannot use calculators.

As students are working, observe the various strategies that they are using. Circulate among the pairs and ask the following questions:

- "How are you solving the problem?"
- "Why did you choose to use those manipulatives?"
- "What are you trying to find out?"
- "What do you already know?"
- "How can you use what you know to solve the problem?"
- "Can you show me what you already know?"
- "Is there another way you could solve the problem?"
- "Can you prove your solution?"

Once students have had sufficient time to work through the problem, have them come back to the group meeting area. Students who have recorded their work on paper should bring it with them. Invite pairs to share their strategies for solving the problem.

Encourage students to demonstrate their strategy by using the manipulatives they selected or by explaining their recording. As pairs share, invite others who used the same manipulative but in a different way to share how they used it.

Examples of how students may have solved the problem 38 + 13 = ?

One way:

Represent 38 and 13 with base 10 materials (making tens strategy)

30 + 10 is 40 (adding the 10's bundles)

8 + 2 is 10 (making 10 strategy; regroup to make another ten)

40 + 10 is 50

plus 1 left

50 + 1 is 51

Another way:

Using hundreds chart (adding tens, then one strategy)

Start at 38.

Move down one ten (+10).

Move to the right (+1 +1 +1).

Using a blank number line (adding by decomposing addend)

On a blank number line, write the number 38.

Represent adding 10 with a "hop" to land at 48.

Now add 2 more to get to 50, then 1 more to get to 51.

Help students to name their strategies, and record them on an anchor chart.

Use the following questions to facilitate discussion:

- "What were you thinking as you were solving the problem?"
- "How did you discover that strategy?"
- "How are some of these strategies the same?"
- "How could you explain your strategy to someone who is absent today?"

WORKING ON IT

Give students this problem to try to solve individually:

The library had 36 animal books. The librarian purchased 36 more animal books at the book sale. How many animal books does the library have now?

Give students the opportunity to think through this problem mentally and decide which strategy they would like to use. Students will work through the same process as they did in "Getting Started"; however, this time they will be working independently. Let students know that they must represent their thinking in their work.

Refer to the strategy board, where strategies that students used to solve problems have been posted. As students are solving the problem, direct them to the strategy board and encourage them to use strategies they are comfortable with. As students continue to work, use the questions in "Getting Started" to facilitate their mathematical thinking. Have a sharing circle when students are ready, highlighting the strategies they used from the strategy board and adding any new ones as they are shared. Use this process for 2-digit subtraction problems as well.

REFLECTING AND CONNECTING

As an extension of the problem above, ask, "How many more books will the librarian need to order to have 100 animal books?"

Use the following questions to facilitate discussion:

- "What were you thinking as you were solving the problem?"
- "How did you discover that strategy?"
- "How are some of these strategies the same?"
- "How could you explain your strategy to someone who is absent today?"
- "How did your understanding of place value help you solve the problem?"

ADAPTATIONS/EXTENSIONS

Students who are struggling may need to use smaller numbers.

For students who need extensions, they can use larger numbers. Students could also be challenged by using more than two numbers.

MATH LANGUAGE

- strategy
- sum
- solution
- flexible thinking
- algorithm

SAMPLE SUCCESS CRITERIA

- represents problems requiring addition and subtraction appropriately (understands that addition requires joining and subtraction requires removing or taking away)
- selects appropriate tools and/or strategy to solve addition and subtraction problem

- models addition and subtraction problems using concrete materials (e.g., moves down or to the right on the hundreds chart to show addition and moves up or to the left to show subtraction)
- adds two-digit numbers, with and without regrouping
- subtracts two-digit numbers, with and without regrouping
- decomposes an addend or subtrahend by tens and ones (see Glossary)

HOME CONNECTION

Send home OS2.BLM1: Show What You Know! This alternative form of a worksheet promotes the use of a flexible approach to algorithms. Often worksheets are of a drill nature and are not connected to what is happening in the classroom or to a particular strategy for basic fact practice.

Class strategies could also be shared in a class newsletter. Provide some sample problems for students to try solving at home, without using pencil and paper.

LEARNING CONNECTION 1

How Much?

Materials

- chart paper or note paper
- markers, pencils, or crayons

Have students work with a partner and present them with the following problem: Bev had saved 50 cents in her piggybank. Her grandma gave her some money for running an errand. Now Bev has 95 cents. How much money did Bev earn for running the errand for her grandma?

Provide each pair with chart paper and markers (or other paper and pencils) for them to record their thinking. After the pairs have had time to find an answer, gather students in a group. Allow time for each pair to present their strategy for solving the problem and to present the solution they discovered.

LEARNING CONNECTION 2

Party Planning

Materials

- chart paper or note paper
- markers, pencils, or crayons

Present students with the following problem: Suzie is planning a birthday party and would like to decorate for the party with balloons. She decides that she needs 55 balloons. She has 2 packages of 24 balloons. Does she have enough balloons?

LEARNING CONNECTION 3

What Is My Problem?

Materials

- chart paper or note paper
- markers, pencils, or crayons

Tell students, "This is the number expression in my problem: 25 – 14. What is my problem?" There are many contexts for this problem. It could be "I had 25 candies and gave 14 to my friend Navjot." It could be "I had 25 marbles and 14 rolled under the couch." It could be "I had 25 cents and I spent 14 cents on a sucker." Give students a similar problem another day. Consider having students create a book of these problems and keep it in the reading area.

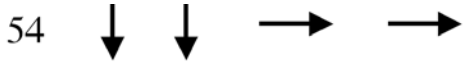
LEARNING CONNECTION 4

Secret Number

Materials

- C2.BLM1: Blank Hundreds Chart (following page 110)
- overhead transparency of C2.BLM1: Blank Hundreds Chart or hundreds carpet or mat
- counters

Provide students or pairs of students with C2.BLM1: Blank Hundreds Chart and 1 counter as a place marker. Give students a start number, such as 54, and then draw an arrow or several arrows on the overhead or on the board. The students' task is to follow the arrows to find the secret number the teacher is thinking of. For example, if 54 were the start number, students would need to find 54 and place a counter on its location on the hundreds chart as a visual aid to help them remember where they started. Then the teacher draws a series of arrows:



Students need to think about where the arrows would take them on their blank hundreds chart. In this case, the arrows tell them to go down 10 to 64, go down another 10 to 74, then go right 1 to 75 and right again to 76, which is the secret number for this example. This activity will need much modelling and would be an excellent activity to do using a hundreds mat or carpet. It can also be done on an interactive whiteboard. Students can also think of secret numbers and give the clues to their partner.

Challenge students to record a number sentence to go with the pathway to the secret number (e.g., 54 + 10 + 10 + 1 + 1 = 76).

LEARNING CONNECTION 5

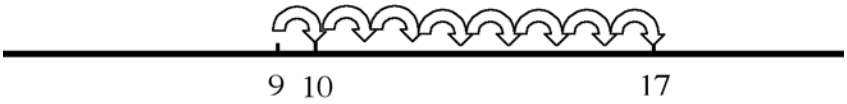
Empty Number Line

Materials

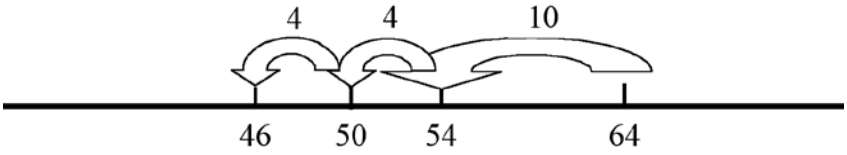
- chart paper
- markers
- notepaper
- crayons, pencils

The empty number line is an instructional tool to help students demonstrate their mental strategies. A simple line is drawn and is empty except for the mark of the start number in the number expression. Model this strategy on chart paper for the whole class.

For example, suppose the number expression is 9 + 8. A student might say, "I start with 9. I know that 9 and 1 more is 10. I counted on 7 more: 11, 12, 13, 14, 15, 16, 17. The answer is 17." The empty number line provides a visual of that student's thinking.



The process can be more complex, such as in the next example. Provide students with a number expression such as 64 – 18. Give students time to think about their answer, and when many students have an idea, ask for suggestions. One idea a student may have that could be recorded on an empty number line would be as follows:



The student might explain it this way, "I started with 64. I came back 10 from 64 and got 54. Then I went back 4 to 50 and back 4 more to 46."

Show What You Know!

Tip for parents: when adding numbers mentally, a good strategy is to break up numbers (or decompose) into tens or fives. Try these questions without any paper, pencil, or calculator. Use just your brain and see what happens! Try it with your child; you will be amazed.

35 + 28

35 + 20 = 55	35 + 5 = 40	35 + 30 = 65
55 + 5 = 60	40 + 23 = 63	65 – 2 = 63
60 + 3 = 63		

Show what you know in 3 different ways!

59 + 47

52 + 49

81 + 19

Writing the questions horizontally encourages mental math!

What's Your Estimate?

BIG IDEA **Quantity**

CURRICULUM EXPECTATIONS

Students will:

- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct).

MATERIALS

- 3 clear containers or resealable plastic bags
- collection of objects to fill the containers (Ping-Pong balls, marbles, golf balls, popped popcorn, dried beans)
- resealable plastic bags filled with manipulatives
- Q2.BLM1: Sample Estimation Chart
- sticky notes for each student
- Q2.BLM2: What's Your Estimate?

ABOUT THE MATH

Estimation is a critical skill in mathematics. It is important to integrate estimation into all areas of the curriculum and allow students to explain their estimates and their estimation strategies.

Estimation skills are intricately related to students' understanding of quantity. These skills help students to use logic and reasoning in problem-solving situations with numbers. Without estimation skills, students cannot judge the appropriateness of an answer. Students who automatically make a guess of a hundred for any large container of objects, whether the objects are small or large, do not have an effective understanding of estimation.

Students can learn to estimate more accurately if they use benchmarks as an estimation strategy. A benchmark refers to a quantity that can be known or understood. With the knowledge of a benchmark, students can extrapolate or estimate the number of items in a larger quantity. For example, students are better prepared to estimate a large number

of blocks if they count out 10 blocks first. Knowing what 10 blocks look like (a benchmark), they can look at the larger collection and judge the number of groups of 10 first and then estimate the total amount.

GETTING STARTED

Fill three identical clear containers or plastic bags with three different items for estimating (e.g., Ping-Pong balls, marbles, golf balls, popped popcorn, dried beans).

In a guided learning situation, show students the container with the largest item. Pass the container around for students to get a sense of the size and quantity of items. Ask students to estimate the number of objects in the container. Record their estimates on the board.

Empty the contents of the container and ask for suggestions about how to count the items (e.g., by 2's, 5's, 10's). Count the objects and compare the total with students' estimates. Discuss the results:

- "Was your estimate close?"
- "What strategy could you use to get closer next time?"

Show students the second container. Ask students these questions:

- "How many items do you think are in this container?"
- "Do you think there are more or fewer items than in the first container?"
- "How can the information about the number of items in the first container help us estimate the contents of this container?"

Have students explain their reasoning.

Empty the contents of the second container and begin counting the items. When 10 items are counted, stop and ask students whether they want to change their estimates based on their understanding of 10 items. Ask students who change their estimates to explain their reasoning. After counting all the items from the second container, compare the estimates with the actual count.

Ask students, "Why do you think your estimates are improving?"

Discuss the use of a benchmark in estimating: Knowing the number of items in a small collection can help to determine the number of items in a larger collection.

Finally, show the third container. Before they give their estimates, have students explain strategies that could help them estimate the contents of the container.

Encourage students to use these strategies as they estimate. Refer to Q2.BLM1: Sample Estimation Chart as a possible way to organize a class estimation chart. Emphasize that

the goal of estimation is to get within an appropriate range and that it is not necessary to provide the actual number.

Count the contents of the third container and discuss the results. Reinforce the idea that estimates improve when benchmarks are used as a strategy.

WORKING ON IT

Provide pairs of students with a copy of Q2.BLM2: What's Your Estimate?

Give each pair a resealable plastic bag filled with various manipulatives. Each bag should hold enough materials to make estimating necessary (i.e., they cannot be readily counted).

Ask students to estimate and then count the number of items in each bag. Students record their estimates and actual amounts on the Q2.BLM2: What's Your Estimate? worksheet. Pairs can trade bags after they have completed estimating and counting the contents.

Encourage students to use a benchmark strategy: count a few items before estimating the number of all the items in the bag.

Circulate throughout the room and observe students as they work. Ask probing questions about the strategies they are using to estimate:

- "What benchmark did you use to estimate the number of items in the bag?"
- "What other strategies are you using as you estimate the number of items in the bag?"
- "How can estimating one bag of items help you with estimating the number in another?"

REFLECTING AND CONNECTING

Assemble the class to discuss the estimating activity. Ask questions similar to the following:

- "What strategies did you use to estimate the number of items in the bags?"
- "Were some items more difficult to estimate? Why?"
- "What strategies did you use to count the number of items?"
- "If the items in the bag are big, is the number of items in the bag large or small?"
- "What if the items in the bag are small? Is the number of items in the bag large or small?"

Discuss times when students use estimation outside school. Share an example, such as estimating to see whether a bag holds enough caramels for a recipe that calls for 50 caramels. Explain how to use 5 caramels as a benchmark to judge whether there are about 50 caramels in the bag.

ADAPTATIONS/EXTENSIONS

If students have difficulty counting or estimating, have them work with a partner who has good counting and estimating skills.

MATH LANGUAGE

- estimate
- actual
- guess
- close to
- double
- half
- benchmark

SAMPLE SUCCESS CRITERIA

- understands that in the same size bag, there can be fewer big items than small items
- adjusts estimates to reflect new information (e.g., benchmark)
- organizes objects to count efficiently (e.g., groups of two or five)
- counts accurately
- articulates if an estimate was too high or too low

LEARNING CONNECTION 1

Big Estimates

The focus of this activity is on the discussion of estimation strategies rather than on finding a number.

Discuss estimation strategies with students by posing problems for which an exact answer would be difficult to find.

- "How many words are in a novel?"
- "How many people live on your street or in your apartment building?"
- "How many pencils are there in the school?"
- "How many pattern blocks are in the entire school?"
- "How many apples are in an orchard?"

Encourage students to think of a benchmark (a quantity of items that can be counted) and to use this benchmark to make appropriate estimates. For example, by counting the number of words on one page, we could estimate the number of words in a book.

Discussions about possible estimation strategies promote the development of students' skills.

LEARNING CONNECTION 2

Pizza Anyone?

Pose this problem to students: "The principal has put you in charge of ordering a pizza lunch for everyone in your grade at your school. Estimate how many slices of pizza you would need if each child in your grade ate 2 slices. What strategies would you use to estimate the number of pizza slices?"

Extend the problem by asking, "If you know that a pizza has 8 slices in it, how many pizzas will you need?"

Sample Estimation Chart

Create a large chart on chart paper or the blackboard similar to the one shown below. For each estimation activity, students record their name and estimate on a sticky note and post them on the appropriate section of the chart.

Nushin Shane	Jane Naman Roz Pierre	Kwan Ming Grant Heather Dan	Nori Ruth
5 - 10	11 - 15	16 - 20	21 - 25

Create sections with appropriate ranges if the estimation activity involves a large number of items.

1 - 50	51 - 100	101 - 150	151 - 200

What's Your Estimate?

Objects We Are Estimating	First Estimate	Second Estimate	Actual Number

Hit the Target

BIG IDEA **Relationships**

CURRICULUM EXPECTATIONS

Students will:

- solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies.

MATERIALS

- chart paper
- manipulatives
- 6-sided number cubes (2 per pair of students)
- 2-sided counters (12 per student)
- plastic, paper, or foam cups (1 per student or pair of students)
- Rel2.BLM1: Shake and Spill
- number cards or a deck of regular playing cards (1 deck of cards per group)
- Rel2.BLM2: Number Strip Cover-Up and Lucky Rolls

ABOUT THE MATH

To begin to understand addition and subtraction and the relationships between these two operations, students need to have a solid understanding of counting, composition of numbers (knowing how quantities can be combined), and decomposition of numbers (knowing how quantities can be broken down or separated into two or more parts).

Knowing that numbers can be composed or decomposed helps students use think-addition as a strategy for understanding subtraction and its relationship to addition. With think-addition, students think about what number can be combined with another number to make the total. For example, to find 15 – 8, a student thinks, "8 and what number make 15"? This strategy encourages students to use known addition facts to find the unknown quantity or part in a subtraction situation. An understanding of this relationship between addition and subtraction is essential for understanding operational sense and for mastering basic facts.

GETTING STARTED

Students may complete this activity individually or with the entire class as the teacher records students' responses on chart paper.

Ask students to complete the following statements:

"Addition is . . ."

"Subtraction is . . ."

After students have shared their responses, discuss situations in which addition and subtraction are used. Ask students to reflect and share their ideas about how addition and subtraction are the same and how they are different.

Record three related numbers (two addends and their sum) on the board or chart paper (e.g., 5, 7, 12). Ask students to write 2 addition sentences using these numbers. After students have come up with 5 + 7 = 12 and 7 + 5 = 12, ask them how they might use the think-addition strategy to show that 12 – 5 = 7 and 12 – 7 = 5.

Provide similar examples and encourage students to record the addition and subtraction for facts to 18.

WORKING ON IT

The following activities reinforce students' mastery of addition and subtraction facts to 18. Each activity should be modelled with the whole class before students work independently, with a partner, or in small groups.

Shake and Spill

Students should do this activity with a partner. Students roll a pair of 6-sided number cubes and find the sum of the two numbers. Students place the corresponding number of 2-coloured counters in a cup.

Students shake and spill their counters, revealing a combination of 2 colours (e.g., 11 counters: 4 red-side up and 7 yellow-side up).

Students' challenge is to record four different number sentences to describe the combination of counters (i.e., 7 + 4 = 11, 4 + 7 = 11, 11 – 4 = 7, 11 – 7 = 4).

Students use Rel2.BLM1: Shake and Spill to record the results of four spills and to explain what they learned from the activity.

Hit the Target

This game requires a deck of 52 number cards (four cards of each number from 0 to 12). A deck of 40 playing cards can be used, with some modifications. Use 2 to 9 at face value, aces as ones, and 10 to stand for 0.

Arrange students in groups of two to four players. Each player is dealt six cards. The remaining cards are placed face down in the middle of the group.

One player rolls two number cubes. The sum of the two numbers rolled establishes the target number for the round.

In turn, each player examines his or her cards and tries to find two number cards that can be added or subtracted to equal the target number. If a player is able to "hit the target" using two cards, he or she shows the cards to the other players and explains how the numbers can be added or subtracted to make the target number. If all players agree, the student places the two cards in his or her discard pile and replaces these cards by picking up two cards from the middle pile.

If a player is unable to "hit the target" with the cards in his or her hand, the turn passes to the next player.

At the end of the round, players keep their hand of six cards, but a new target number is determined by tossing the number cubes again.

The game continues until all cards from the middle pile have been used.

It may be difficult for some students to find subtraction sentences using their number cards. While observing students playing the game, ask questions to help them recognize possible combinations of cards. "You have a 5. Is there another number that you can add to 5 to make 9? Can you use the 8 and another number to make a subtraction sentence for the target number?"

REFLECTING AND CONNECTING

After the Shake and Spill and Hit the Target activities, discuss the following questions with the class:

- "Was it difficult thinking about addition and subtraction at the same time?"
- "Was it easier to think about addition or subtraction?"
- "How can addition help you to think about subtraction?"
- "As the game progressed, did the rounds get faster or slower? Why do you think this happened?"

Students may write their own reflections about the game in their math journal following the whole-class discussion.

ADAPTATIONS/EXTENSIONS

Ensure that manipulatives are available for all students. Students who experience difficulties can work with a partner. In some cases, limiting the number of 2-sided counters used for the Shake and Spill game by having students roll only one number cube to find their target number might be appropriate.

MATH LANGUAGE

- addition
- subtraction
- difference

- sum
- combination
- relationship

SAMPLE SUCCESS CRITERIA

- uses the proper conventions to write addition and subtraction facts
- recognizes that addition involves joining
- recognizes that subtraction involves taking away
- knows that for every two addition facts, there are two corresponding subtraction facts
- understands how the think-addition strategy is helpful in solving a subtraction problem

HOME CONNECTION

Play the games on Rel2.BLM2: Number Strip Cover-Up and Lucky Rolls at school before sending it home.

LEARNING CONNECTION 1

Subtraction as Think-Addition

Materials

- paper
- manipulatives

Use problems similar to the following to explore subtraction as think-addition:

Grant had 6 grapes on his plate for a snack. His dad knew he was hungry so he gave Grant more grapes. Grant then had 14 grapes on his plate. How many grapes did Grant's dad give him?

To solve this task, students might use a think-addition strategy: "6 and how many more is 14?"

LEARNING CONNECTION 2

Match the Fact

Materials

- Rel2.BLM3a–b: Match the Fact

It helps students to learn a subtraction fact if they know its matching addition fact. Make cards from Rel2.BLM3a–b: Match the Fact. Hold up a subtraction fact, such as 18 – 4 = 14. Students need to find and hold up a corresponding addition fact (14 + 4 = 18). Discuss how the two facts are related.

Adaptations/Extensions

An extension of this activity would be a version of the Concentration game. Students place all the cards face down in front of them. They turn over two cards to find a match. For example, if they turn over 19 – 5 = 14, they try to match the fact by finding 14 + 5 = 19.

LEARNING CONNECTION 3

Fact-O

Materials

- Rel2.BLM4: Fact-O

Ask students to record their own addition or subtraction expressions on Rel2.BLM4: Fact-O. Decide whether students will record addition or subtraction facts, or a combination of both. Tell students to be sure they know the answers to the expressions. Give an answer orally, and instruct students to use counters to cover all the facts that match the answer. For example, if "8" is an answer, students can cover number expressions such as 4 + 4, 3 + 5, 7 + 1, 10 – 2, or 9 – 1. At various points in the game, have students generate and list possible number expressions that match an answer.

After playing the game, encourage students to share their strategies for writing their number expressions.

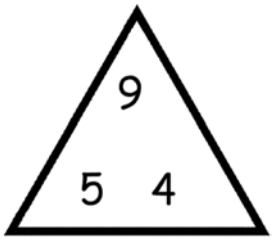
LEARNING CONNECTION 4

Combination Cards

Materials

- Rel2.BLM5: Combination Cards

Use the combination cards from Rel2.BLM5: Combination Cards or create some based on the examples provided. In each triangle, the top number is the sum of the two bottom numbers.



Students can place their thumb or a counter over a corner to hide a number and ask a partner to find the missing number.

Rel2.BLM1

Shake and Spill

First Spill:

Second Spill:

Third Spill:

Fourth Spill:

What I learned

Number Strip Cover-Up

Each player needs a number strip like this one:

0	1	2	3	4	5	6	7	8	9	10

Players take turns rolling 2 number cubes and can either add or subtract the two numbers that come up.

Each player uses a counter (button, bread tag, small piece of paper) to cover the number on the number strip that matches the answer he or she gets at each turn.

The first player to cover all the numbers on the number strip wins the game.

Lucky Rolls

One of the players rolls a number cube to decide the lucky number.

Players take turns rolling 2 number cubes and can either add or subtract the two numbers that come up.

Every time a player makes the lucky number, he or she earns a point.

The first player to get 10 points is the winner.

Match the Fact

4 + 5 = 9	9 - 4 = 5	6 + 6 = 12
7 + 8 = 15	15 - 8 = 7	12 - 6 = 6
6 + 9 = 15	15 - 6 = 9	9 + 9 = 18
3 + 8 = 11	11 - 3 = 8	18 - 9 = 9
5 + 2 = 7	7 - 5 = 2	8 + 8 = 16
8 + 1 = 9	9 - 8 = 1	16 - 8 = 8
4 + 8 = 12	12 - 4 = 8	5 + 5 = 10

Match the Fact

3 + 3 = 6	6 - 3 = 3	5 + 7 = 12
2 + 8 = 10	10 - 8 = 2	12 - 5 = 7
1 + 9 = 10	10 - 1 = 9	6 + 2 = 8
7 + 8 = 15	15 - 8 = 7	8 - 6 = 2
10 + 2 = 12	12 - 10 = 2	12 - 9 = 3
8 + 0 = 8	8 - 8 = 0	9 + 3 = 12
4 + 2 = 6	6 - 4 = 2	10 - 5 = 5

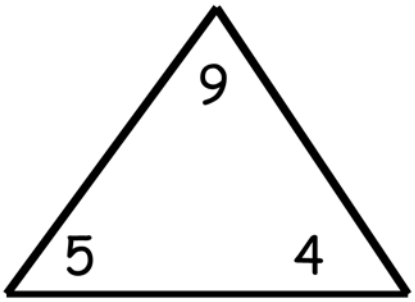
Rel2.BLM4

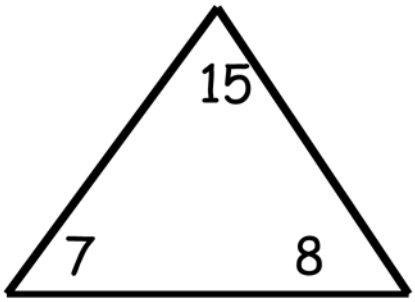
FACT-O		

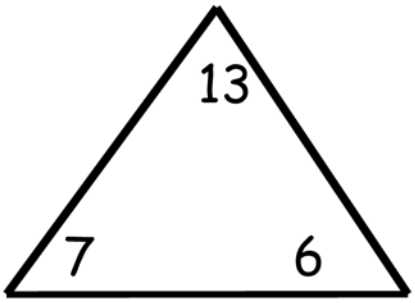
FACT-O		

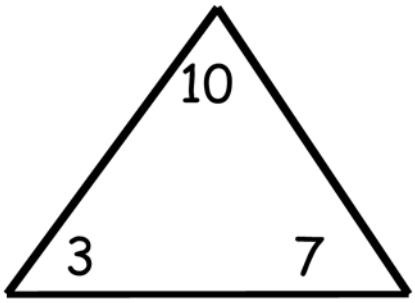
Combination Cards

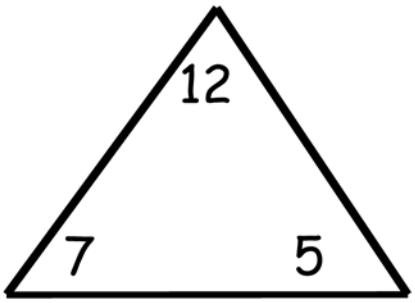
Make similar combination cards on heavy paper or cardboard. As you hold up the card for the class to see, cover one of the numerals. Challenge the students to find the hidden number and to explain their answer by providing an addition or subtraction sentence that contains all three numerals. Students may play this game with each other after you have modelled the activity.

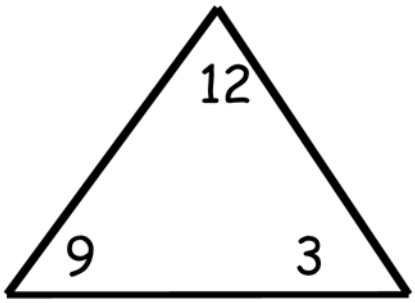












Mystery Bags

BIG IDEA **Representation**

CURRICULUM EXPECTATIONS

Students will:

- represent, compare, and order whole numbers to 100, including money amounts to 100¢, using a variety of tools (e.g., ten frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets);
- compose and decompose two-digit numbers in a variety of ways, using concrete materials (e.g., place 42 counters on ten frames to show 4 tens and 2 ones; compose 37¢ using one quarter, one dime, and two pennies*).

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

MATERIALS

- resealable plastic bags
- a variety of manipulatives for the mystery bags
- Rep2.BLM1: Mystery Bag
- chart paper
- markers
- Rep2.BLM2: Sample Centre Organizer
- Rep2.BLM3: The Teacher's Helpers
- Rep2.BLM4: Place-Value Mat
- Rep2.BLM5: Spinning for a Flat
- number cards for each pair of students
- base ten materials: flats, rods, units
- calculators
- interlocking cubes
- paper bags
- paper clips
- index cards or small squares of paper
- Rep2.BLM6: Reflection Stems
- Rep2.BLM7: Countdown to Zero – Anecdotal Record
- Rep2.BLM8: Number Detective

ABOUT THE MATH

Students at this age may still be developing their understanding of place value, particularly their recognition that:

- groups of 10 things must be perceived as a single item, as in the number 23; the 2 represents 2 sets of 10 things – a fairly sophisticated concept;
- the position of the digit gives it its quantitative value.

For instance, with students in Grade 2, an entire number such as 33 may be looked at as 33 single units, not 3 groups of tens and 3 single units. What is even more problematic is that such a number can also be thought of as 2 sets of tens and 13 single units. Students may have difficulty if asked to represent a number such as 33 and then the number that is 10 more than 33 – or 10 less than 33 – which is even more difficult. They may not yet have identified the pattern of adding by 10's in base ten numeration. When asked to mentally add 10 to 33, they may not be able to do so without counting by 1's.

GETTING STARTED

Mystery Bags

The teacher will create enough mystery bags for each set of partners to have one.

Management Tip: Ask a parent volunteer or older student to make the bags.

The mystery bags should hold between 30 and 80 manipulatives. For example, put 48 pattern blocks in one resealable plastic bag or 63 counters or 52 interlocking cubes that connect on only one side. Ensure that only one kind of manipulative is in each bag. Other manipulatives such as plastic bugs, pompoms, bread tags, and so on, may be used. By using a variety of different manipulatives and different numbers and sizes, students will have different experiences, which will help them gain a broader understanding of the activity when they share their strategies.

In a whole-group guided learning situation, introduce the mystery bag activity. Tell students that different manipulatives are in each mystery bag and that their help is needed to count them. Before estimating, students will need to know what item is in their mystery bag.

With a partner, students will use Rep2.BLM1: Mystery Bag to first estimate how many manipulatives are in their bag and record the reason they made that estimate. Students will then determine how many manipulatives are in their bag in whatever way they choose. Some students will count their manipulatives by 1's, 2's, 5's, 10's, and so on.

While students are exploring, the teacher will be moving throughout the classroom, watching the pairs of students and their strategies for counting. The following probing questions can be used during this time:

- "Can you show me how you counted?"

- "How many manipulatives are there?"
- "How can you prove it? Is there another way to count them?"
- "Can you find an easier way to count them?"

When students have had sufficient time to explore and count their manipulatives, they will come back to the large group for a discussion about what they discovered. The teacher will facilitate pairs of students sharing their counting or grouping strategies. Students can show their mystery bag and talk about their estimate, how they counted, and what was the easiest way. They may need to show their groupings on the carpet for the other students.

The teacher needs to highlight efficient ways of grouping and counting as the pairs are sharing. For example, counting by 1's is not an efficient strategy because it is easy to lose track and it takes a long time. It is essential for other students to hear a variety of different methods and to develop the understanding that grouping objects makes it easier to count larger numbers.

The following day, as an extension of the Mystery Bag activity, the teacher may read the story *The King's Commissioners* by Aileen Friedman (Toronto: Scholastic, 1995) or a similar book.

Management Tip: When using literature in a mathematics lesson, it is important to have read the complete story for enjoyment during a prior shared reading opportunity, before using it to teach the lesson.

Stop reading at various points in the story to discuss the different counting methods. Take extra time to discuss counting by grouping (e.g., 10's). Listen to students' suggestions. Model examples using different numbers such as 45.

Ask the following questions:
- "How many groups of 10 are there in the number 45?"
- "Which number shows that?" (4)
- "How many 1's are there in the number 45?"
- "Which number shows that?" (5)

WORKING ON IT

Tell students that they will be working at four different centres on activities about place value and the ways numbers are represented. To prepare the classroom for centres, here are some management tips:
- Divide students into four equal, heterogeneous groups and have a written record of the groups for students to refer to.
- Prepare an organizational chart on chart paper listing the centres and the rotation schedule for the week (see Rep2.BLM2: Sample Centre Organizer).

- Organize the materials needed for each centre into four different bins or containers, ensuring all materials that students will require are there. The bins could be labelled with the centre name.
- Have the checklist or anecdotal recording sheet with the criteria for observation listed. This sheet could be kept on a clipboard with a pencil or pen at each centre.
- Determine where each of the centres will be in the classroom each day. Places to use for centres are groupings of desks, a table, a meeting area, or a reading corner.
- Find a storage place in the classroom where students can find and return the bins.
- Before starting work at the centres, consider doing a T-chart with students about expectations during centre time. On the left side of the T-chart, record "What does it look like?" On the other side record "What does it sound like?"

Introduce the centres to students. Choose one bin to highlight first. Introduce the centre name and explain the process that students will work through at the centre. It is essential to demonstrate the activity and helpful to play the game or model the use of materials for the whole group.

Note: Review the expectations for each centre and the general expectations from the T-chart before every work session.

Suppose that the Countdown to Zero centre is the one that the teacher is assessing during this experience. Students will need to know that the teacher will be watching and listening for the criteria on the checklist or anecdotal record at this particular centre. While students are working on their centres, the teacher will spend the majority of the time at the centre being assessed.

During this time the teacher also needs to be looking at how well the centres are working and whether there is a need to make revisions or changes to individual centres.

Centre 1: The Teacher's Helpers

Materials
- Rep2.BLM3: The Teacher's Helpers
- manipulatives

Students are provided with Rep2.BLM3: The Teacher's Helpers. The problem they are presented with is as follows:

"The teacher needs more classroom helpers. Make a list of classroom helpers the teacher could choose. Record the names in the box. Suppose the teacher already has 25 helpers. How many helpers will the teacher have if he or she chooses all the new helpers you have listed? Show your strategy for counting how many helpers the teacher will have."

Students will solve this problem independently, using manipulatives if desired.

Centre 2: Countdown to Zero

Materials
- Rep2.BLM4: Place-Value Mat
- base ten materials (flats, rods, ones)
- number cards (1 to 9)

Students work with a partner and play the game with another pair of students. Each pair needs Rep2.BLM4: Place-Value Mat and a hundreds flat to begin. Each group also needs several sets of number cards with 1 to 9 on them. The cards are placed face down in between the two pairs. Students work with their partner by drawing two cards from the deck, adding them together, then taking away that number from their hundreds flat. The pairs need to trade their hundreds flat for 10 rods before they can create their new number on their place-value mat.

For example, a pair of students draws a 2 and a 6. They add those together to get 8 and then trade their hundreds flat for 10 rods. Players should notice that they still are not able to take away 8 from the rod of 10. They need to trade 1 rod for 10 ones. They are now able to take away 8 ones, which leaves them 92 represented on their place-value mat.

After their place-value mat has been updated, the players turn the cards they drew face down beside the first pile. After the original cards have been used, the drawn cards can be shuffled well and reused.

The object of the game is to count down to zero.

Centre 3: Mystery Numbers

Materials
- interlocking cubes
- paper bags
- paper clips
- index cards or small squares of paper

Students will reproduce an activity that has already been presented by the teacher and explored by the whole class in a guided situation. As a follow up, students create their own mystery 2-digit numbers.

Each student chooses a mystery number, then represents that number using interlocking cubes built into towers of 10 and units of 1. Keeping the mystery number represented in front of them, students create three or more mystery number clues on paper or index cards to help the number detectives in the class find out what the mystery number is.

For example, if a student chooses 42, the clues could be as follows:
- I have 6 pieces in my bag.
- Both of my digits are even.
- I have 2 more towers than units.
- My 10's digit and my 1's digit add up to 6.

Then students put their mystery number pieces into a paper bag, paper clipping their clues onto it. Students can exchange bags with a partner to see whether their partner can figure out the mystery number before the whole class tries to figure it out.

Centre 4: Spinning for a Flat

Materials
- Rep2.BLM5: Spinning for a Flat
- Rep2.BLM4: Place-Value Mat
- base ten materials (flats, rods, and ones)

Using Rep2.BLM5: Spinning for a Flat, students play the game in groups of two to four. Players take turns spinning the spinner and, depending on where the spinner lands, they collect either a unit of 1 or rod of 10 and place it on Rep2.BLM4: Place-Value Mat.

Students take turns spinning the spinner, adding to their place-value mat, trading for 10's when appropriate, and verbalizing their new number each time they make it. The game continues until one student reaches 100 and trades their tens rods for a flat.

REFLECTING AND CONNECTING

At the end of the work time, bring the whole class to the meeting area to have a math talk about what occurred that day. Students may share "ah ha's" and struggles that they had at their centre, and they may offer advice for students who will be at the centre the next day. The teacher will need to facilitate the discussion with questions to help students to explore place value in detail. Suggested questions are as follows:
- "What did you learn about how grouping helps you to count?"
- "Why did you group into 10's?"
- "How did the manipulatives help you?"
- "What strategies did you use to help you at your centre?"
- "How did trading help you to find the total?"

After completing their centre, students may use Rep2.BLM6: Reflection Stems to record their reflections in their math journals. They may do this independently, in pairs, or as a small group. Show students a number such as 43 and ask them to talk about or show in their journal what the 4 means and what the 3 means.

Note: An understanding of place value and representation will be built over time with repeated explorations of place-value activities, multiple representations of number, and reflections on learning.

ADAPTATIONS/EXTENSIONS

The teacher may want to partner students who have difficulties.

To extend students' thinking, challenge them to create their own place-value game, which could be based on what they have done or be a new one of their own invention.

Note: It is beneficial to have calculators available in the classroom for students who need to use them at the centres.

MATH LANGUAGE

- hundreds
- tens
- ones
- value
- represent
- place value
- placeholder
- regrouping
- trade, trading

SAMPLE SUCCESS CRITERIA

Use the Rep2.BLM7: Countdown to Zero – Anecdotal Record to collect formative assessment about students' understanding of place value. Suggested criteria to be used when observing the Countdown to Zero centre, which is the assessment centre in this case, follow:

- determines a number that meets specific characteristics or criteria
- accurately adds the two cards to determine the subtrahend (number being taken away)
- decomposes the flat and the rods when necessary
- takes away the subtrahend to represent the new number correctly
- names new number after taking away the subtrahend
- compares the size of his or her number to a partner's to determine which is larger
- composes dimes from pennies* and composes loonies from dimes
- correctly finds the difference between two numbers
- uses appropriate mathematical language

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

HOME CONNECTION

Number Detective

Ensure that students are familiar with the Number Detective game and the rules before sending home Rep2.BLM8: Number Detective.

The person who plays the game with the student thinks of a number between 0 and 100 and asks the student to guess what it is. The student has to ask questions such as the following to guess the number:

- "Is it bigger than 50?"
- "Is it an even number?"
- "Could I count by 10's to get to it?"
- "Is the tens digit bigger than the ones?"

Then the student can choose the number and have the other player ask them the questions. This is an excellent game to play anywhere and it helps to develop number sense.

LEARNING CONNECTION 1

Luck of the Draw

Materials

- two 6-sided or 10-sided number cubes or number cards for each pair of students
- Rep2.BLM4: Place-Value Mat for each pair of students
- coin for each pair of students

Students work in pairs and need two 6-sided or 10-sided number cubes or number cards 1 through 9. The first player rolls two number cubes or draws two cards, decides what 2-digit number he or she can create, represents it on Rep2.BLM4: Place-Value Mat, and says it aloud. The second student follows the same procedure, rolling the two number cubes to get a 2-digit number, representing it on the place-value mat, and saying the number aloud.

After each player has had a turn, students take turns flipping a coin to determine heads or tails. If heads turns up, the person with the higher number represented on their place-value mat gets 10 points. If tails turns up, the person with the lower number gets 10 points. Students use a recording sheet to keep a running tally of their own points, counting by 10's. The game continues until one partner reaches 100 points.

LEARNING CONNECTION 2

Let's Get Loonie

Materials

- 50 pennies,* 20 dimes, 1 loonie per pair of students
- paper bag or plastic container
- 6-sided number cube
- Rep2.BLM9: Let's Get Loonie

*Although the penny coin is no longer produced in Canada, the penny still has value in our money system. Therefore, it is important that students understand the value of the penny (i.e., there are 10 pennies in a dime and 100 pennies in 1 dollar). This relationship closely resembles the concept of place value where it takes 10 units to make 1 rod and 100 units to make 1 flat. The penny acts in the same way that a unit does with a set of base ten blocks.

Each set of partners needs 50 pennies, 20 dimes, and 1 loonie (either plastic manipulative money or real money), a paper bag or container to act as a bank, and one 6-sided number cube. The object of the game is to be the first person to get to $1.00. Students take turns rolling the number cube and taking that number of pennies from the bank, which they put on Rep2.BLM9: Let's Get Loonie. As they continue rolling the number cube and playing the game, students need to trade for dimes, which also are placed on Rep2.BLM9: Let's Get Loonie. The player must roll the right number of pennies to reach $1.00 and the player that reaches the "loonie" is the winner.

LEARNING CONNECTION 3

Let's Play Soccer!

Materials

- paper
- manipulatives

Teachers may want to present this problem to students as an extension activity. Students may work on this problem independently or in pairs to present their thinking.

Jordan's dad was coaching her soccer team for the very first time. Jordan was excited because her dad let her choose the uniforms for the team. Every team had to have black and another colour, so Jordan chose green as the other colour. She also needed everyone to sign up for the number that they wanted on the back of their shirts. Jordan's team was only allowed to use the numerals 3, 4, 5, and 7 to create the numbers for the backs of their shirts. Each shirt could have no more than two numerals on it. Jordan made a list of all the numbers that her teammates could choose from, checked it with her dad, and then let each player sign up for the number from her list. Make a list of the numbers Jordan had on her sheet.

LEARNING CONNECTION 4

Add and Subtract

Materials

- number cards 1 through 12 for each pair of students
- interlocking cubes that connect on only one side or other counters

Students can play this game in pairs. The pair needs number cards 1 through 12 and interlocking cubes that connect on only one side or other counters. One player deals six cards to each player, and the players leave the cards face down in front of them. The players determine who is player A and who is player B. Each player turns over two cards from the pile he or she was dealt. The players each add up the total of their cards and then find the difference between the two totals. In the first round, player A takes the difference between the two totals of cards in manipulatives or just records the difference. If using manipulatives, it is helpful to build towers of 10 with the interlocking cubes or create groups of 10 with other counters. In the second round, player B takes the difference. The game is finished when one player reaches 100 counters or a total of 100 if the players are keeping a running total.

Rep2.BLM1

Mystery Bag

I have a bag of _____.

I predict that there are _____ of them in my bag.

Check it out! Look how I solved the mystery.

Let me show you another way that I could prove that I solved the mystery correctly!

Sample Centre Organizer

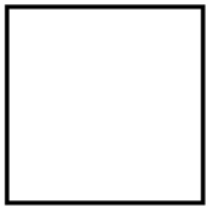
Place Value Centres	Tuesday	Wednesday	Thursday	Friday
Countdown To Zero	Group 1	Group 2	Group 3	Group 4
Spinning For A Flat	Group 4	Group 1	Group 2	Group 3
The Teacher's Helpers	Group 3	Group 4	Group 1	Group 2
Mystery Numbers	Group 2	Group 3	Group 4	Group 1

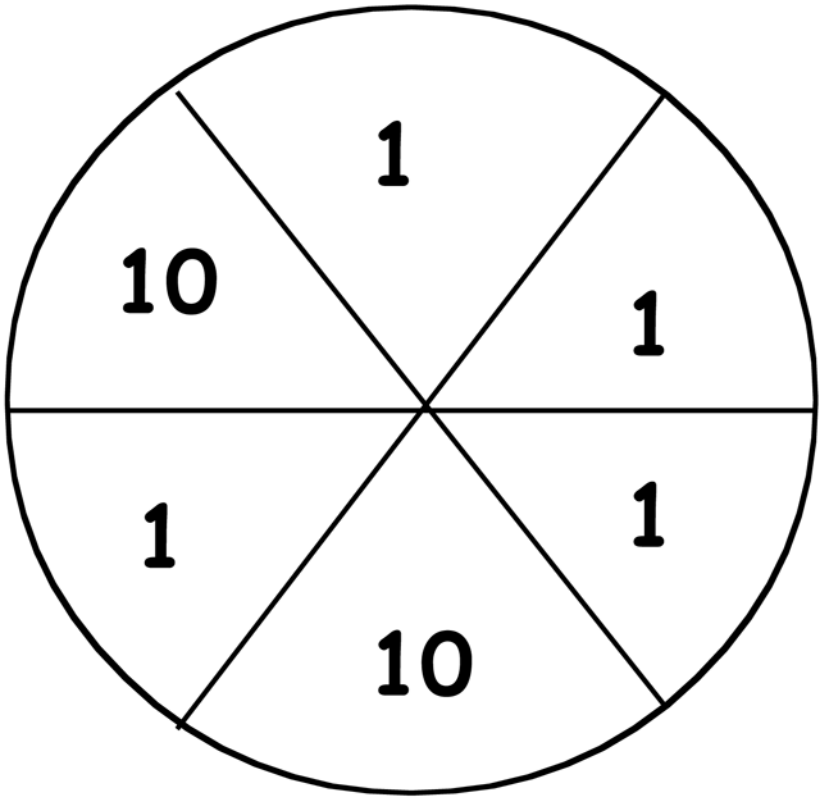
The Teacher's Helpers

The teacher needs more classroom helpers.
Make a list of the new helpers the teacher could choose.
Record the names in the box.

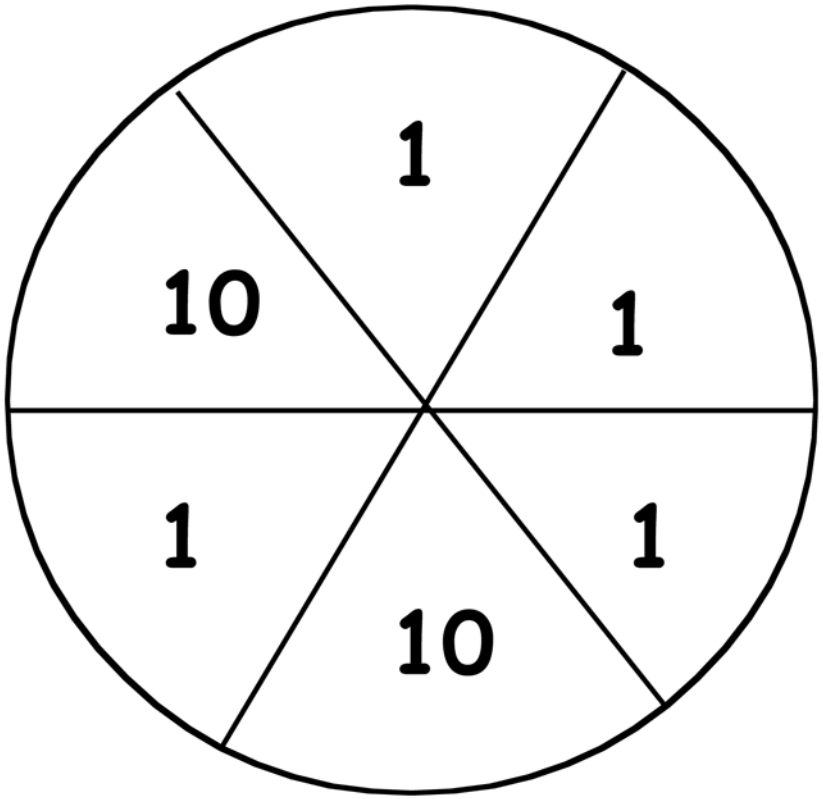
Suppose the teacher already has 25 helpers.
How many helpers will the teacher have if he or she chooses all the new helpers you have listed?
Show your strategy for counting how many helpers the teacher will have.

Place-Value Mat

Hundreds 	Tens	Ones



Spinning for
a Flat



Spinning for
a Flat

Math Journal Ideas for Everyday

* The best thing about math today was...

* Today I learned...

* I want to find out more about...

* A strategy that I used today was...

* The easiest part of today's math was...

* The hardest part of today's math was...

* The problem that I solved today was...

* I am proud of the way that I ...

* I think that I still need to work on...

* I knew my answer was right because...

* I think that I still need to work on ...

* After math today I felt...

* Tomorrow I am going to...

* I would like to try this again because...

* Today I learned that...

* Tips that I would give a friend to help solve this problem are...

* Next time I will...

* What you need to remember about this kind of problem is...

Countdown to Zero – Anecdotal Record

While observing students working, use this anecdotal record sheet to record observations relating to criteria such as the following:

- accurately adds the two cards to determine the subtrahend (number being taken away)
- decomposes the flat and the rods when necessary
- takes away the subtrahend to represent the new number correctly
- names the new number after taking away the subtrahend
- uses appropriate mathematical language

Number Detective

This is an excellent game to play anywhere, anytime, and it helps to develop children's sense of number.

How to Play

Think of a number between 0 and 100. Your child has to be a detective to discover what the number is. He or she can ask questions such as, "Is it bigger than 50? Is it an even number? Could I count by 10's to get to it? Is the tens digit bigger than the ones digit?"

Then, switch roles and have your child choose a number. This time, you ask him or her the questions to figure out the number.

Let's Get Loonie

$1.00 **Loonie**	10¢ **Dimes**	1¢ **Pennies**

C. Grade 3 Learning Activities

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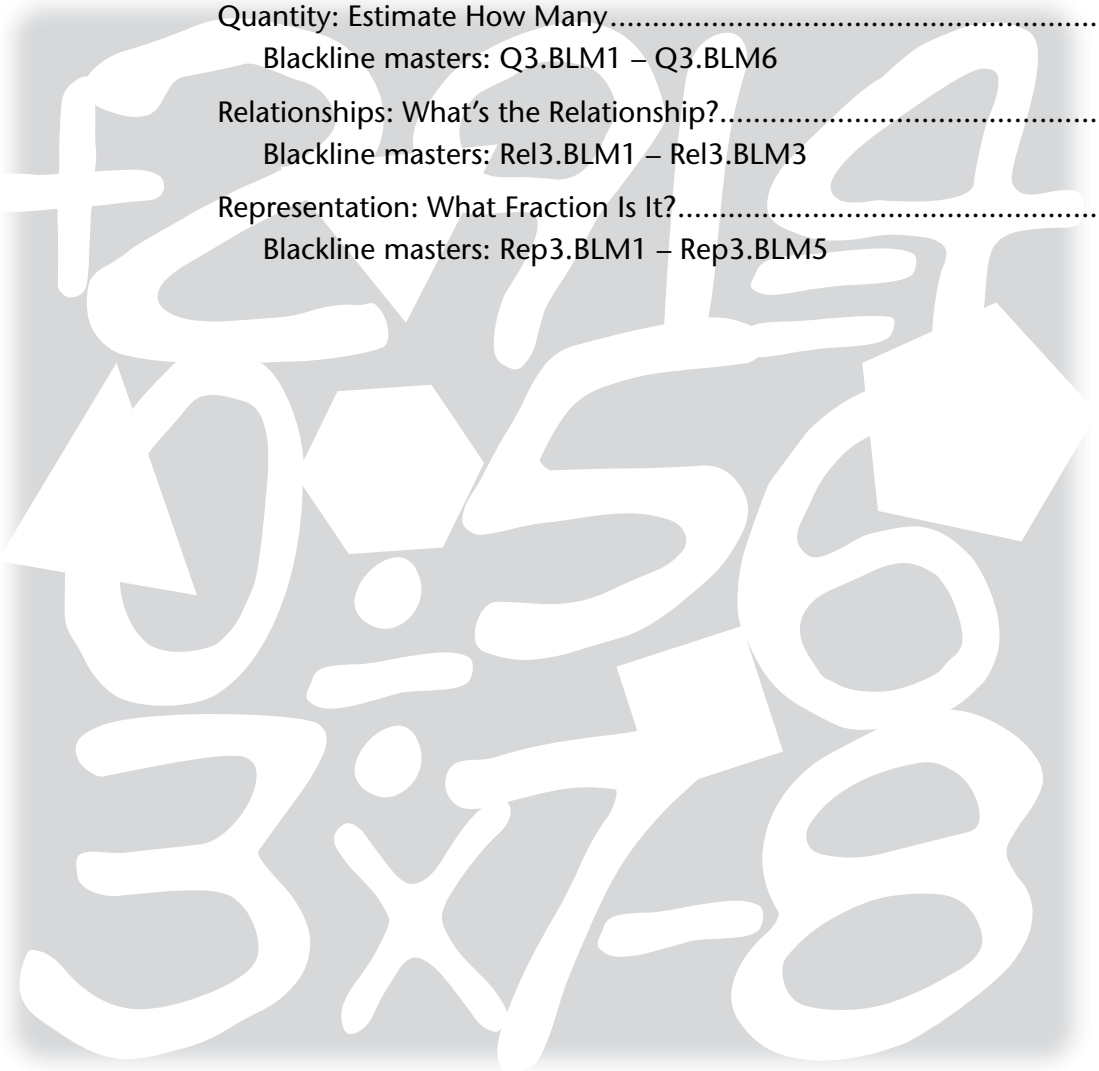


Trading up to 1000

BIG IDEA *Counting*

CURRICULUM EXPECTATIONS

Students will:

- represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts);
- count by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points and by 25's to 1000, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10's using dimes);
- count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points and by 100's from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies;
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems.

MATERIALS

- C3.BLM1: Place-Value Mat
- base ten blocks
- 6-sided number cubes per team
- C3.BLM2: Trading up to 500
- C3.BLM3: Trading up to 1000
- calculator
- C3.BLM4: Hop to 500

ABOUT THE MATH

Counting experiences in Grade 3 extend students' understanding of number patterns and relationships. In the Grade 3 program, students count:

- by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points;
- by 25's to 1000 using multiples of 25 as starting points.

Students should also have opportunities to skip count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points; and by 100's from 1000 and any number less than 1000.

Base ten materials, hundreds charts, and number lines support students in identifying counting patterns and in recognizing relationships among numbers.

In the following activities, students explore counting by 10's, 25's, and 100's from different starting points using manipulative materials, number lines, and mental math strategies.

GETTING STARTED

Trading up to 500

Divide the class into team A and team B. Each team will need a copy of C3.BLM1: Place-Value Mat, base ten blocks, and two 6-sided number cubes.

To prepare for the game, each team establishes a starting number by tossing two number cubes. The numbers on the number cubes are used as digits in the starting number. For example, a team tossing a 2 and a 4 may use either 24 or 42 as its starting number. A member of the team uses base ten blocks to represent the starting number on C3.BLM1: Place-Value Mat.

The goal of the game is to be the first team to get to 500 without surpassing it. To get to 500, each team begins at its starting number and then counts on by 1's, 10's, or 100's according to the following rules:

To begin, a member from team A rolls a number cube. The number on the number cube signifies the number of counts the team may make in each round. The team also needs to decide whether it will count by 1's, 10's, or 100's. For example, if the team's starting number is 23 and the number on the number cube is 3, the team makes 3 counts on from 23 in one of three possible ways:

- by 1's (24, 25, 26)
- by 10's (33, 43, 53)
- by 100's (123, 223, 323)

As team A counts on from its starting point, base ten blocks are added to the place-value mat. Throughout the game, appropriate trades are made (e.g., 10 ones for a rod, 10 rods for a hundreds flat).

At each turn, the team records the skip counts it makes on C3.BLM2: Trading up to 500. For example, to show 3 skip counts of 100 starting at 23, students would record this:

Or, to show 4 skip counts of 10 starting at 423 (later in the game), students would record this:

These are rough number lines. The recorded skips by 1's, 10's, and 100's do not need to be to scale.

The game proceeds with each team counting on from the number on its place-value mat. As teams get close to 500, they may find that they need to skip a turn if the number of counts given by the number cube is too high. For example, if a team is at 496 and then rolls a 6, any 6 counts by 1's, 10's, or 100's would result in a number greater than 500.

Following the game, discuss the following questions with the class during a math talk time:

- "What strategies helped you play the game?"
- "When is it best to count by 1's? by 10's? by 100's?"
- "How did the base ten blocks help you when you played the game?"
- "What patterns did you see when you counted by 10's? by 100's?"

WORKING ON IT

Play Trading up to 1000. The procedures for the game are the same as those for Trading up to 500, except that teams try to reach 1000 and "win" the thousands cube from the set of base ten blocks.

This time, divide students into groups of four. Students will work in pairs as a team to play this game with another pair. (Pairs of students could be selected by the teacher, or students may choose a partner.) Students use C3.BLM3: Trading up to 1000 as a recording sheet.

While students are playing, move throughout the classroom, listening to students, asking questions, and observing their strategies. Ask questions such as the following:

- "Why did you decide to count by 10's that time?"
- "What would have happened if you had counted by 100's?"
- "What strategies are you using in this game?"
- "Why did you need to make a trade on the place-value mat?"

REFLECTING AND CONNECTING

In a math talk time, discuss the following questions:

- "What was challenging about the game?"
- "What strategies did you use in this game?"
- "What did you learn about counting by 10's and 100's?"

- "What did you learn by using the base ten blocks?"
- "What did the number lines show you about skip counting?"

Students can write in their math journals to reflect on their learning from the game. Students may use pair journals or small group journals, depending on prior experience with math journals.

Provide sample sentence starters such as the following:
- Using base ten blocks helped me today because . . .
- During the game I . . .
- Helpful hints I would give someone playing this game for the first time would be . . .
- Patterns I noticed when I was counting by 10's and 100's were . . .
- When I add 100 to a number like 230 or 340 . . .
- When I add 10 to a number like 360 or 280 . . .

ADAPTATIONS/EXTENSIONS

For students needing a greater challenge, use 25 as the skip-counting number instead of 10 and 100. Use 0 as the starting point for students who are having difficulty. Provide a calculator for use during the game.

MATH LANGUAGE

- skip count
- counting on
- counting back
- ten
- hundred
- thousand
- trade
- represent
- regroup
- place value

SAMPLE SUCCESS CRITERIA

- represents whole numbers to 1000, using a variety of base ten materials
- selecting an appropriate increment for the count
- skip counts forward by 10's from various starting points
- skip counts forward by 100's from various starting points
- counts backwards by 10's
- counts backwards by 100's
- trades base ten materials appropriately
- uses a variety of strategies (e.g., patterns, when counting by 10's and 100's)

HOME CONNECTION

Hop to 500

Using C3.BLM4: Hop to 500, play the game at school to familiarize students with it before sending it home.

To begin, player A thinks of a starting number between 1 and 10. Player B may add either 10 or 100 to the starting number. Player A, in turn, adds either 10 or 100 to the number given by player B. Turns continue back and forth between the two players, who add either 10 or 100 each time to the other's number. The winner is the player who gets to 500 or closest to 500 without going over.

The game can also be played with 1000 as the target number.

Hop Backwards to 0

In this version of the game (also on C3.BLM4), player A thinks of a 3-digit number as the starting point. Player B may subtract either 10 or 100 from the starting number. Player A, in turn, subtracts either 10 or 100 from the number given by player B. Turns continue back and forth between the two players, who subtract either 10 or 100 each time from the other's number. The winner is the player who gets closest to 0.

LEARNING CONNECTION 1

The Skip-Counting Machine

Materials

- calculators

Explain to students that the calculator is a skip-counting machine. Have students work in pairs, and give each pair a calculator. To begin the skip-counting game, player A clears the calculator, enters a 2-digit number, and tells player B what that number is. Player A presses the + key and then enters 10. Player B needs to give the answer without looking at the calculator's display. Next, player A states the next number in the skip-counting sequence (i.e., the next number when counting by 10). Player B presses the = key and both students check this answer. (The constant function of the calculator automatically adds 10 to each number.) Players continue to take turns predicting the next number the calculator will show. The game continues until students reach or surpass a given number (e.g., 300).

The game can also be played using 100 as the skip-counting number with 1000 as the number to reach.

Note: Not all calculators have a constant function. Ensure the calculators used are able to perform this activity.

An alternative game is to skip count backwards. To begin, a player enters a 3-digit number, presses the – key, and then enters 10 (or 100). With each consecutive pressing of the = key, the calculator skip counts backwards by 10 (or 100) from the starting number.

Other numbers such as 2, 5, and 25 could be used as the skip-counting numbers.

LEARNING CONNECTION 2

Going for 1000

Materials
- C3.BLM5: Going for 1000

This game can be played with two to four players using C3.BLM5: Going for 1000 as a game board. To begin, all players place their marker on the Start space on the game board. In turn, each player rolls a number cube or chooses a numeral card (1 to 9) to determine the number of spaces to move his or her marker around the board. When a player lands on a space, he or she collects the number of points indicated on the space. Each player uses a place-value mat and base ten materials to represent their running totals as they play the game. The first student to reach 1000 is the winner.

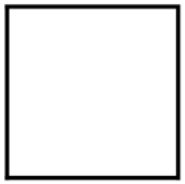
LEARNING CONNECTION 3

One Hundred Sit-Down

Students stand in a circle. The teacher chooses a volunteer to start the game. The first student begins by saying "25", the next student in the circle says "50", the third student says "75", and so on. Whenever a counting number is 100 or a multiple of 100 (200, 300, 400, etc.), the student must sit down. The counting continues with only students who remain standing. The game ends when only one student is standing.

Use different starting points each time students play the game and use multiples other than 25 (e.g., 2, 5, 10).

Place-Value Mat

Hundreds 	Tens	Ones

C3.BLM2

Trading up to 500

Starting number is

Round	Skip Counts	Running Total
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Trading up to 1000

Starting number is

Round	Skip Counts	Running Total
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Hop to 500

To begin, player A thinks of a starting number between 1 and 10. Player B may add either 10 or 100 to the starting number. Player A, in turn, adds either 10 or 100 to the number given by player B.

Turns continue back and forth between the two players, who add either 10 or 100 each time.

The winner is the player who gets to 500 or closest to 500 without going over.

The game can also be played with 1000 as the target number.

Hop Backwards to Zero

In this version of the game, player A thinks of a three-digit number as the starting point. Player B may subtract either 10 or 100 from the starting number. Player A, in turn, subtracts either 10 or 100 from the number given by player B.

Turns continue back and forth between the two players, who subtract either 10 or 100 each time.

The winner is the player who gets closest to 0.

C3.BLM5

START				
20	80	100	70	40
40				50
50				20
90				30
60				10
30				90
70				10
10	80	100	60	50

Going for 1000

What Comes in 2's, 3's, and 4's?

BIG IDEA ***Operational Sense***

CURRICULUM EXPECTATIONS

Students will:

- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- relate multiplication of one-digit numbers and division by one-digit divisors to real-life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences);
- multiply to 7 x 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting);
- make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems.

MATERIALS

- chart paper
- counters
- paper
- calculators
- manipulatives (e.g., counters)
- OS3.BLM1: Circles and Dots Game
- OS3.BLM2: What Things at Home Come in 2's, 3's, 4's, and 5's?

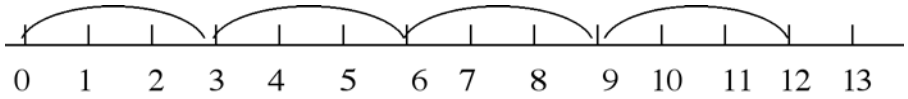
ABOUT THE MATH

Using concrete materials helps students to understand the meaning of multiplication. Students need to use concrete materials to represent and solve problems involving multiplication, and to help them verbally describe multiplication situations. The following activity, What Comes in 2's, 3's, and 4's?, emphasizes that multiplication involves the combining of equal groups and that multiplication is a form of repeated addition. For example, with a problem involving "3 groups of 6", students create equal sets of counters. Initially, some students may count by 1's to find the product. With experience, students use more sophisticated counting and reasoning strategies, such as skip counting and using known addition combinations (e.g., 3 groups of 6: "6 plus 6 is 12, and 6 more is 18").

The activity reinforces the notion of multiplication as a form of repeated addition, a concept that was introduced in Grade 2. It links this idea to the symbolic representation of multiplication (e.g., that 4 groups of 5 can be written as 4 x 5). The task also promotes an understanding of the need to learn multiplication facts to increase fluency in doing calculations; however, basic facts should be presented only after concepts about multiplication are established.

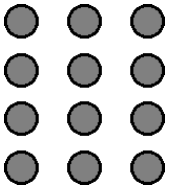
The activity is appropriate for the early stages of instruction on multiplication in Grade 3. It helps students make connections among concrete, pictorial, verbal, and symbolic representations of multiplication. The focus of this activity is on modelling multiplication using equal groups of objects, but students should engage in similar activities that introduce them to other models for representing multiplication:

Number line



4 jumps of 3

Array



4 rows of 3

GETTING STARTED

Read and discuss the book *What Comes in 2's, 3's, and 4's?* by Suzanne Aker (New York: Simon & Schuster, 1990). Ask students to work with a partner and make lists of other things that come in 2's, 3's, and 4's. Bring the class back together, and record students' ideas on a class chart.

What comes in …		
2's	3's	4's
Wheels on a bike Eyes on a face Horns on a bull	Wheels on a tricycle	Tires on a car Legs on a chair

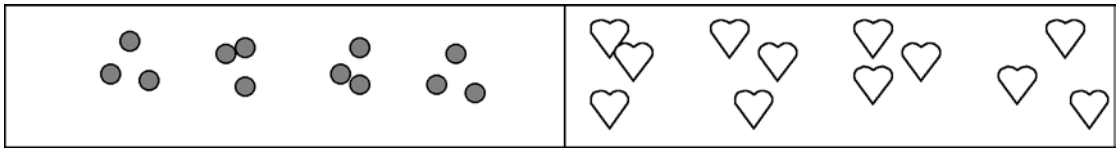
Discuss the following questions: "Was it easiest to find examples for things that come in 2's, 3's, or 4's? For which number was it most difficult to find examples?" Extend the task by having students list things that come in 5's, 6's, 7's, and 8's. Add these examples to columns added to the class chart.

Pose a problem using an example from the chart. "If a spider has 8 legs, how many legs will 3 spiders have?" Provide students with counters. Ask them to work with a partner to solve the problem and to record their work.

Invite a few students to share how they solved the problem, and discuss different approaches. Using students' ideas about creating equal groups of 8 counters, discuss how multiplication is a way to show sets of groups:

- "How many groups did you need to make? How many counters were in each group?"
- "How do the groups of counters show the problem?"
- "How could you find the total number of legs using addition?" (8 + 8 + 8)
- "How can we show 3 groups of 8 using a multiplication expression?" (3 x 8)
- "What is the product of 3 x 8?"
- "What are different ways we can find the product of 3 multiplied by 8?"

Use counters, diagrams, words, and number sentences to work through a few other examples with the class (e.g., If a tricycle has 3 wheels, how many wheels will 4 tricycles have?).



Counters | Diagram

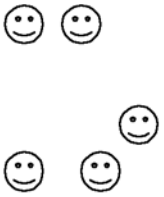
4 groups of 3 is 12	4 x 3 = 12

Words | Number sentence

WORKING ON IT

Explain to students that they will work with a partner to create other problems based on the ideas from the class chart. Encourage students to model their problem on those done by the class ("If a spider has 8 legs, how many legs will 3 spiders have?").

Ask students to use counters, draw diagrams, write words, and use number sentences to show their multiplication ideas. Have students use paper folded into four sections to record their work.

If a face has 2 eyes, how many eyes will 5 faces have?	
5 groups of 2 is 10	$5 \times 2 = 10$

REFLECTING AND CONNECTING

Bring the class together and have them bring their work with them. Choose one pair to share their work by covering one quadrant on their worksheet with a piece of paper (the problem, diagram, words, or number sentence). The other students predict what is hidden. Have each pair of students show and explain their work to the class.

Ask questions similar to the following:

- "How do the different sections of the paper show the same idea?"
- "How would you explain the meaning of 5 x 2 to someone?"
- "What diagram could you draw to show the meaning of 3 x 6?"
- "Can you think of a problem for 3 x 6?"

Extend students' understanding by posing oral problems similar to the following:

- "If a carpenter has 15 stool legs, how many stools with 3 legs can the carpenter make?"
- "If there are 14 wheels at the bike stand, how many bikes are there?"
- "If there are 16 hands playing pianos, how many people are playing piano?"
- "If there are 24 spider legs crawling up a wall, how many spiders are there?"

Ask students to explain their thinking.

Have students write in their math journal about what they have learned about multiplication.

ADAPTATIONS/EXTENSIONS

Encourage students who are experiencing difficulties to use manipulatives to represent the problem before drawing a diagram and recording words and a number sentence. If students demonstrate uncertainty in recording multiplication sentences (e.g., 4 x 5 = 20), ask them to represent the situation using addition sentences first

(e.g., 5 + 5 + 5 + 5 = 20). Some students might experience more success working with smaller numbers or numbers that are easily used for skip counting (2, 3, 5).

For students requiring a greater challenge: write a multiplication sentence on the board (e.g., 3 x 8 = 24) and ask students to write a few different problems that might go with it. Students might also create problems involving larger numbers and use base ten blocks to represent and solve the problem. Students could look at grocery flyers or catalogues for ideas (e.g., If a box of chocolates has 24 candies, how many candies will there be in 4 boxes?).

MATH LANGUAGE

- multiplication
- equal groups of
- product
- factor

SAMPLE SUCCESS CRITERIA

- understands that multiplication involves equal groups
- explains the meaning of multiplication as repeated addition of equal groups by using diagrams, number lines, or arrays
- represents multiplication symbolically in a number sentence
- explains the relationships between the problem, diagram, words, and number sentences

The focus of this activity is on the conceptual understanding of multiplication, not on students' fluency with basic facts that will be developed through subsequent learning experiences.

HOME CONNECTION

Circles and Dots Game

Introduce students to the game at school before sending home OS3.BLM1: Circles and Dots Game. The game reinforces the idea that multiplication involves equal groups.

What Things at Home Come in 2's, 3's, 4's, and 5's?

This activity is an extension of the classroom activity. Students create and solve multiplication problems based on things in the home that come in 2's, 3's, 4's, and 5's. This activity is shown on OS3.BLM2: What Things at Home Come in 2's, 3's, 4's, and 5's?

LEARNING CONNECTION 1

Finding Equal Groups

Materials

- counters
- square tiles

For this Learning Connection, be sure students understand that arrays are rows and columns with an equal number of objects in each.

Have students count out 12 counters. Challenge them to find ways of placing the counters in equal-sized groups. Ask them to write different multiplication sentences in which the product is 12. Repeat with other numbers that have several factors (e.g., 16, 18, 24, 30).

A similar activity can be used to reinforce the idea that arrays can represent multiplication. Provide students with square tiles. Ask them to create as many different rectangles as possible by arranging all 12 tiles. Discuss how an array is a way to show multiplication.

LEARNING CONNECTION 2

Graphs and Multiplication

Throughout the year, create pictographs and bar graphs with many-to-one correspondence (e.g., 1 face on the graph represents 5 students). Discuss how the graph requires the use of multiplication. (If 1 face represents 5 students, then 3 faces represent 3 x 5 or 15 students.)

Provide opportunities for students to collect data and to construct many-to-one graphs. Have students create and answer questions using information from the graph.

LEARNING CONNECTION 3

Create a Problem

Ask students to write a multiplication story that follows two rules:
- The story must end with a question.
- The question must be answered using multiplication.

Have students solve the problem in different ways. Have students exchange and solve each other's problems.

LEARNING CONNECTION 4

The Broken Calculator Key

Materials
- calculators

Challenge students to use a calculator to find the products of multiplication questions. Explain that they may use any key except the x key, because it is "broken". For example, to find 3 x 6, students can press 6 + 6 + 6 =. (Alternatively, students can use the constant function on a calculator and skip count by entering + 6 = = = .)

Circles and Dots Game

Play this game with a partner.

First, player 1 rolls a number cube and draws that many circles on a piece of paper.

Next, player 1 rolls the number cube a second time. This number indicates the number of dots to draw in each circle.

Finally, player 1 records a multiplication number sentence. For example, for 3 circles with 4 dots in each circle, he or she writes 3 × 4 = 12.

Here's what a player's paper would look like after rolling a 3 (number of circles) and then a 4 (number of dots in each circle).

3 × 4 = 12

Now it is player 2's turn.

Player 2 does the same things as player 1 did:

- Roll a number cube for the number of circles to draw.
- Roll the number cube again for the number of dots to draw in each circle.
- Write a number sentence.

The answer to a multiplication question is called a *product*. The player with the largest product earns a point.

Keep playing until one player gets 10 points.

Note to parents:

This game emphasizes the meaning of multiplication. Your child might use counting, skip counting, or addition to find products in this game.

What Things at Home Come in 2's, 3's, 4's, and 5's

Look around your home and find things that come in 2's, 3's, 4's, and 5's.
Make a list of these things.

2's	3's	4's	5's

Now, make up multiplication problems using ideas from your lists. Show the answer by using a picture, words, and a number sentence.

Here is an example.

If a table has 4 legs, how many legs will 3 tables have?

Picture:

Words: 3 groups of 4 is 12

Number sentence: $3 \times 4 = 12$

Estimate How Many

BIG IDEA **Quantity**

CURRICULUM EXPECTATIONS

Students will:

- apply developing problem-solving strategies as they pose and solve problem investigations, to help deepen their mathematical understanding;
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
- make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;
- solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1000.

MATERIALS

- Q3.BLM1a–b: Estimate How Many
- containers with at least 200 manipulatives (e.g., interlocking cubes, pattern blocks, counters) per group
- calculators
- chart paper
- Q3.BLM2: What's Cooking?

ABOUT THE MATH

In Grade 3, students continue to develop a conceptual understanding of the quantity of 1-digit and 2-digit numbers. They extend this understanding to gain a sense of the quantity of 3-digit and 4-digit numbers.

Students can estimate large numbers, although their estimates may be unreasonable if they do not use benchmarks. A benchmark is a quantity that is known or understood. With the knowledge of a benchmark, students can extrapolate or estimate the amount in a larger quantity. For example, a person might know that there are about 10 candies in a handful. They would judge that a candy jar holds about 6 handfuls and estimate that there are 60 candies in the jar.

In the following activities, students estimate quantities of objects and use these estimates to solve problems. Students should understand that an estimate is an attempt

to get as close as is reasonable to the actual number without counting each one and that the use of benchmarks along with proportional reasoning helps to determine a reasonable estimate.

GETTING STARTED

As with many primary quantity-estimating activities, the result is to count and determine the actual number. The problem with this is that by the time they reach Grade 3, students no longer estimate; they always go for the actual number as that is what they expect they are going to have to do anyway. This is a good time to restate why students need to learn to estimate: to be able to judge whether answers in other areas are reasonable.

Ask the following questions to encourage students to think about large numbers. Record students' ideas on the board or chart paper:

- "What is a large number you have seen and where did you see it?"
- "What things come in large numbers?"
- "How can we count things that come in big numbers?"

Present students with the following problem: "I need to know the number of manipulatives (interlocking cubes, pattern blocks, links, counters, etc.) we have in the classroom for some activities. I need your help in estimating the number of each manipulative."

Provide a large container (preferably a tub) of manipulatives for each group of three to four students. Explain that the task of each group is to estimate the number of manipulatives in the container. Emphasize that each group must think of a strategy that will help them make an estimate that is close to the actual quantity. (Counting the entire quantity is not an estimating strategy.)

After students have had an opportunity to think of and discuss a way to estimate the number of manipulatives, ask each group to share its strategy. Next, ask each group to apply its strategy and make an estimate. After making an estimate, groups may count the manipulatives to see how close their estimate is to the actual number.

Observe students as they estimate and count the materials. Are students using effective estimation and counting strategies?

Assemble the class for a math talk. Groups take turns showing their container of manipulatives and asking other students in the class to make an estimate of the number of manipulatives. Then, a member of the group can explain the group's estimation strategy and can tell the actual number of manipulatives.

After the groups have presented, ask students to explain good estimation strategies. Discuss the use of benchmarks and how they can help to make a reasonable estimate.

WORKING ON IT

Review the concept of a benchmark and how it helps in estimating. Relate the use of a benchmark to the problem students solved about the number of manipulatives in large containers. Emphasize with students that an estimate should be close to the actual number but that the exact number is unnecessary.

Discuss situations in which an estimate is appropriate:

- "When and why do we estimate in real life?"
- "Tell about a time when you had to estimate."
- "What would happen if you visited a bulk food store to buy ingredients to make cookies and you overestimated or underestimated the amounts to buy?"

If the book is available, read *Betcha* by Stuart J. Murphy (Toronto: HarperCollins, 1997). After reading the story, ask students to describe the different estimation strategies the boys used.

Provide students with estimation problems from Q3.BLM1a–b: Estimate How Many. Emphasize the use of benchmarks as a strategy to make appropriate estimates. Solve one problem collectively to model the use of a benchmark. Pairs or small groups of students can discuss the strategies for solving other problems.

REFLECTING AND CONNECTING

Have students share their problems and solutions with the class. It is important that students explain the strategies used. Summarize and record each strategy on the board or chart paper. Possible strategies include the following:

- using a benchmark
- working with "nice" numbers (e.g., using 20 instead of 23)
- repeating addition or counting
- rounding to the nearest 10 or 100
- using multiplication

Continue the discussion by asking questions similar to the following:

- "Could you have used another strategy to estimate?"
- "Were your estimates reasonable?"
- "Would 10 have been an acceptable estimate? What about 2000? 256? Why or why not?"

ADAPTATIONS/EXTENSIONS

Some students may require calculators to help them with larger numbers. Others may need to estimate smaller amounts, perhaps fewer than 100 items.

MATH LANGUAGE

- estimate
- estimation
- rounding
- benchmark
- strategy
- actual
- reasonable
- more than
- less than
- fewer than
- close

SAMPLE SUCCESS CRITERIA

- explains the difference between estimating and counting
- understands that estimation helps us to judge the reasonableness of an answer
- justifies the reasonableness of an estimate
- applies effective estimation strategies (clustering, "nice" numbers, front end estimating, rounding; see Glossary for more details on **estimation strategies**)
- establishes a benchmark and uses it effectively

HOME CONNECTION

What's Cooking?

While cooking with someone at home, students can estimate the quantity of ingredients needed for a recipe (e.g., the amount of pasta for a family dinner). As a benchmark, the person doing the cooking could show how much one person would need. Students could also estimate the number of cookies or pancakes they would be able to make from a bowl of batter. As well, they could estimate how many small or large marshmallows are in 6 cups, or how many pieces of elbow macaroni are in 2 cups.

Q3.BLM2: What's Cooking? can be sent home or included in a newsletter.

LEARNING CONNECTION 1

Fill It Up!

Materials

- Q3.BLM3: Fill It Up!
- overhead projector
- pattern blocks
- Q3.BLM4: The Great Cover-Up

Show a transparency of Q3.BLM3: Fill It Up! using the overhead projector. Place one square pattern block on the transparency and ask several students to estimate the number of square pattern blocks it will take to completely cover the shape. Record their estimates on the board.

Students take turns adding square pattern blocks to the shape one at a time. After a few more pattern blocks have been added, pause and ask students whether they would like to adjust their estimate. Discuss how it is possible to make more accurate estimates when more information is available.

Ask students to select one type of pattern block. Have them estimate the number of blocks it will take to cover Q3.BLM4: The Great Cover-Up. Discuss the strategies students used to make their estimates. Talk about ways that students might use a benchmark to help them estimate. Have students check their estimates. (Because of the form of some pattern blocks, students may not be able to completely cover the shape on Q3.BLM4: The Great Cover-Up.)

A similar activity could involve students estimating the number of pattern blocks it would take to cover a larger surface, such as a sheet of paper or a desktop. Encourage students to use benchmarks as an estimation strategy.

LEARNING CONNECTION 2

What Can I Buy?

Materials
- Q3.BLM5: Yummy Snack Foods
- pretend food (such as empty cereal boxes), optional

It is important for students to see how estimation strategies relate to real-life situations. Discuss the strategy of using "nice" numbers in estimation (e.g., using 80 instead of 83 or using 200 instead of 192).

Display Q3.BLM5: Yummy Snack Foods or create similar price list charts. Consider actually creating a mini-store with pretend food items (e.g., empty cereal boxes). Tell students that they have $10.00 to spend and that they are to choose what they will purchase by using estimation only. Encourage students to estimate using "nice" numbers. Emphasize that students are not to calculate exact amounts but are to use estimation strategies to determine what they could purchase.

LEARNING CONNECTION 3

Gumball Machine

Materials
- chart paper
- coloured circle stickers

Draw a large gumball machine on chart paper or the board. Cover the gumball machine with coloured circles representing gumballs, and ask students to estimate the number of gumballs. To promote the use of a benchmark in estimating, show students 5 or 10 circle stickers together and encourage them to use these quantities to estimate the number of gumballs in the machine.

Use large circle stickers the first time students do this activity. Use smaller circle stickers at subsequent times and as students become more proficient at estimating. Discuss how students need to alter their estimates based on the size of the circle stickers.

LEARNING CONNECTION 4

Creatures, Creatures

Materials
- soft modelling clay
- balance scale
- small manipulatives
- Q3.BLM6: What's Your Estimate?

Provide students with different amounts of soft modelling clay. Ask each student to create a creature. Consider giving students a time limit, as some students might want to take a long time to create their creature.

Explain to students that they will use a balance scale to find the mass of their creature. Demonstrate the use of a balance scale, if necessary.

Provide students with a light manipulative to use on the balance scale (e.g., interlocking cubes, coloured tiles, craft sticks). Students estimate the mass of their creature and record their estimate in the second column of the Q3.BLM6: What's Your Estimate?

Next, students have a partner make a second estimate of the creature's mass. The partner records his or her estimate in the Second Estimate column. Finally, students use the balance scale to find the actual mass of their creatures.

Extend this activity by having groups of five or six students compare the masses of their creatures and order them from lightest to heaviest.

Estimate How Many

Estimate the answer for each problem. You do not need to find an exact amount. Think of a benchmark that will help you estimate. Be prepared to explain the strategy you used to make your estimate.

Imagine that your teacher is going to have a meeting for parents in your classroom. If all of the furniture were cleared out of the room, about how many chairs could be set up to fill the room?

The parent's group is preparing a special lunch for your whole school. Every student will get 1 hotdog, 2 cookies, and 1 milk. How many hot dogs, cookies, and milk does the parent's group need?

Some people have one or more pets; other people have no pets at all. About how many people in your school have pets? How many pets do you think there would be if you put them all together?

It does not take a long time to write your name quickly. If you wrote your name many times it might take a long time. About how much time would it take to write your name 1000 times?

About how many teeth do the students in your classroom have altogether?

Your school is planning a skating party for all of the kids in the school and their brothers and sisters. If each child coming to the party will have a hot chocolate, about how many cups would the school need to have?

If everyone in your school lay down side by side in a straight line, about how long do you think the line would be?

The secretary gives students a pencil for their birthday. If she has blue pencils for the girls and red pencils for the boys, about how many of each should she buy?

Provide many opportunities for students to estimate quantities by having them estimate the number of objects contained in a jar each week. Fill the jar with different objects at the beginning of each week. During the week, students can record their name and their estimate on a slip of paper that they drop in a box. Do an actual count on Fridays. This activity will allow you to see whether students are progressing with their estimation of quantity over time. There could be a reward for the winner, or the winner could get the honour of choosing the item with which to fill the jar for the next week.

What's Cooking?

Cooking provides an excellent opportunity to talk about mathematics.

Children can estimate how much of something needs to be added to a recipe. It is helpful to show how much one person would eat, as a benchmark, and then estimate how much food is needed for the entire family.

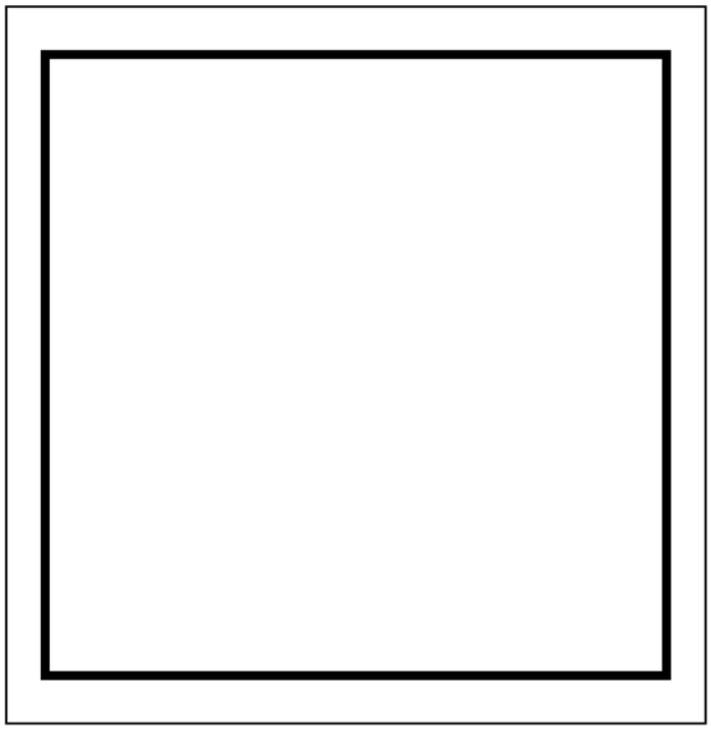
Here are some questions that will help your child estimate:

How much pasta will the family need for dinner?

How many cookies or pancakes can we make from this bowl of batter?

How many items are in the bowl (e.g., crackers, raisins, potato chips)?

Fill It Up!



Estimate the number of square pattern blocks it will take to cover this shape.

You may change your estimate as we add more blocks.

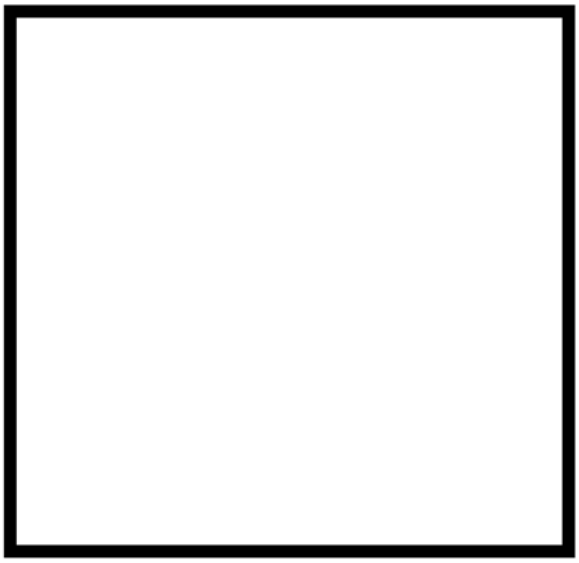
The Great Cover-Up

Choose one kind of pattern block to cover the shape.

Estimate the number of pattern blocks it will take to cover the shape.

I chose _____ to cover my shape.

I predict that I will need _____ blocks to cover the shape.



I actually needed _____ _____ to cover the shape.

Think About It

What strategy did you use to estimate the number of pattern blocks it takes to cover the shape?

Was your estimate close to the actual number of pattern blocks it took to cover your shape?

Yummy Snack Foods

Snacks

popcorn..... $1.59

chips.....$1.05

nachos & cheese.$2.75

soft pretzel....$2.29

goldfish crackers..$0.89

cheese string.....$0.65

cheese & crackers..$1.15

Candy

licorice.....$0.45

jelly beans.....$0.79

gummy bears....$0.65

chocolate bar.....$1.19

sucker.....$0.20

smarties.....$1.29

Drinks

chocolate milk.....$0.99

iced tea $1.79

pop.....$1.45

juice.....$1.60

Other Stuff

granola bar.....$0.79

apple.....$0.45

sandwich.....$2.80

big cookie.....$1.19

What's Your Estimate?

Object I am estimating	First Estimate	Second Estimate	Actual

What's the Relationship?

BIG IDEA **Relationships**

CURRICULUM EXPECTATIONS

Students will:

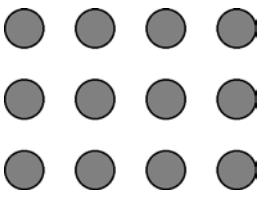
- relate multiplication of one-digit numbers and division by one-digit divisors to real- life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences);
- multiply to 7 x 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting);
- create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;
- communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

MATERIALS

- resealable plastic bag
- manipulatives to make arrays
- Rel3.BLM1: Number Cards for Arrays
- counters
- Rel3.BLM2a–b: Amazing Array Booklet
- bingo dabbers, stickers, or stamps
- counters
- drinking straws, pencils, or string
- Rel3.BLM3a–b: What's Missing?

ABOUT THE MATH

An array is an important model for teaching operational sense because of its ability to represent the relationships in multiplication and division, and their connections to repeated addition and subtraction. An array is an arrangement of objects in equal rows and columns. In the classroom, manipulatives such as counters, cubes, and square tiles can be used to make arrays.



An array can be used to show:

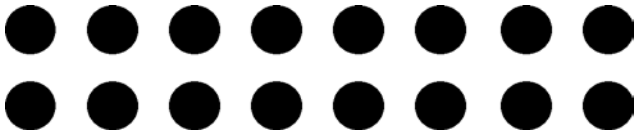
- repeated addition (3 + 3 + 3 + 3 = 12 and 4 + 4 + 4 = 12);
- multiplication (3 x 4 = 12 and 4 x 3 = 12);
- repeated subtraction by removing or covering a column or row at a time (12 – 3 – 3 – 3 – 3 = 0 and 12 – 4 – 4 – 4 = 0);
- division (12 ÷ 4 = 3 and 12 ÷ 3 = 4).

GETTING STARTED

Show students an array of objects (classroom materials or manipulatives) and explain how the objects are arranged in a rectangular shape with rows and columns. Ask students to identify places where they have noticed arrays (e.g., boxes or cans at a grocery store, chairs in the gym, concrete slabs on a patio).

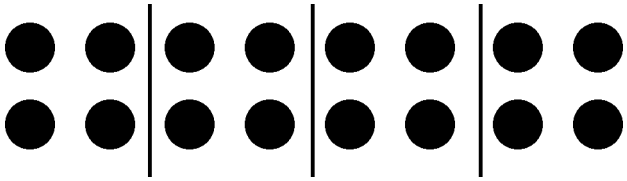
WORKING ON IT

In a whole-class guided lesson, use an overhead projector to show an array of counters or similar objects.



Ask students to describe what they notice about the array. They might observe, for example, that there are 16 counters arranged in columns and rows, that there are 2 rows and 8 columns, and that there are 2 counters in each column.

Next, ask students to find ways of dividing the counters in the array into equal groups. Ask students to place drinking straws or pencils on the array to show ways of subdividing it.



Each time students show a way of subdividing the array, generate and list different number sentences. For example,

4 + 4 + 4 + 4 = 16

4 x 4 = 16

16 – 4 – 4 – 4 – 4 = 0

16 ÷ 4 = 4

Have students work with a partner. Provide each pair with a resealable plastic bag containing number cards (Rel3.BLM1: Number Cards for Arrays) and 30 counters. Partners draw a number card from the bag and use their counters to build an array representing the number drawn from the bag. Next, they use pencils, drinking straws, or string to divide the array into equal parts. Challenge students to find more than one way to subdivide the array into equal parts.

Have each student record the arrays using a bingo dabber, stickers, or stamps in an "Amazing Array" booklet (see Rel3.BLM2a–b: Amazing Array Booklet). The student should represent one array on each page and record appropriate number sentences for the array. Students repeat this activity for other arrays they represent and subdivide.

REFLECTING AND CONNECTING

During a math talk time, each pair of students can show and explain different arrays and how they were subdivided into equal groups. Encourage discussion and reflection by asking questions similar to the following:

- "What different arrangements of arrays did we make?"
- "Who has an array for 18? Does anyone have a different array for 18?"
- "What are all the different arrays for 12?"
- "How can addition help us with subtraction? with multiplication?"
- "How can subtraction help us with addition? with division?"

ADAPTATIONS/EXTENSIONS

If available, read *Amanda Bean's Amazing Dream* by Cindy Neuschwander (New York: Scholastic, 1998). Develop a class book titled *Our Class's Amazing Dream*.

Provide access to manipulatives while students are investigating and recording arrays and number sentences. Use manipulatives to represent multiplication and division situations and story problems.

MATH LANGUAGE

- array
- row
- column
- equal groups
- multiply
- divide
- add
- subtract
- subdivide

SAMPLE SUCCESS CRITERIA

- uses an array to demonstrate and explain the relationships among the operations
- shows how multiplication is related to addition
- uses a multiplication sentence to find a division sentence
- explains how their representation shows multiplication and division

HOME CONNECTION

Students can play Rel3.BLM3a: What's Missing? with someone at home. Students should think of a number before their partner begins to guess the missing number. Examples of the What's Missing? activity (Rel3.BLM3b) could be included in the class newsletter.

LEARNING CONNECTION 1

Arrays and Arrays!

Materials

- overhead projector
- counters

Select a number that has several factors, such as 12, 18, 24, or 36. Using counters on an overhead projector, create as many arrays as possible for the number. For example, if 18 is the selected number, demonstrate that 1 x 18, 2 x 9, 3 x 6, 18 x 1, 9 x 2, and 6 x 3 are possible arrays. Discuss how the 6 x 3 and the 3 x 6 arrays are related (same arrangement, but the array is "turned").

Challenge students to use counters to find different arrays for 12. For each array they create, students should write as many addition, multiplication, and division sentences as possible. For example, for the 2 x 6 array, students might record the following number sentences:

6 + 6 = 12

2 + 2 + 2 + 2 + 2 + 2 = 12

2 x 6 = 12

6 x 2 = 12

12 ÷ 2 = 6

12 ÷ 6 = 2

After students have had plenty of time to find different arrays for 12 and have recorded corresponding number sentences, make a collective list of number sentences students have found.

LEARNING CONNECTION 2

Arrays Around Us

Materials
- manipulatives

Have students find examples of arrays in the classroom or school (e.g., art displays, book arrangements, tiles on the floor, panes of glass in a window, chairs arranged in rows). Discuss and record number sentences that represent the array.

Pose problems related to arrays at school:
- "What are all the possible ways to arrange 24 chairs in an array?"
- "How could you arrange 12 pieces of art on a bulletin board so that you create an array?"

Have students use manipulatives to solve the problem. Students should record their solutions and provide number sentences that represent the arrays.

LEARNING CONNECTION 3

Roll an Array

Materials
- two 6-sided number cubes per group
- counters

Divide students into pairs or groups of three. Students need two 6-sided number cubes and counters to play this game. Each player, in turn, tosses the number cubes. The numbers on the number cubes indicate the number of rows and columns in an array. For example, if a student tosses a 3 and 5, the student uses counters to make an array with 3 rows and 5 columns (an array with 5 rows and 3 columns is acceptable as well).

Next, the student writes three different number sentences for the array using three different operations. For example, the student could record $5 + 5 + 5 = 15$, $3 \times 5 = 15$, and $15 \div 3 = 5$. If the other students agree, the player earns a point, and the turn passes to the next player, who tosses the number cubes, creates the array, and records the number sentences.

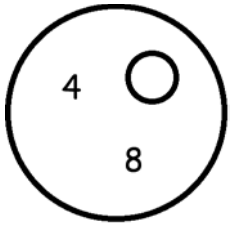
LEARNING CONNECTION 4

What's Missing

Materials

- Rel3.BLM3a: What's Missing?

Challenge students to find the missing number and to prove their choice of number. See Rel3.BLM3b: What's Missing? for other examples.



Examples of what students might say:

2 is missing because 4 x 2 = 8

4 is missing because 4 + 4 = 8

4 is missing because 8 – 4 = 4

Rel3.BLM1

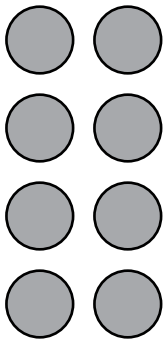
Number Cards For Arrays

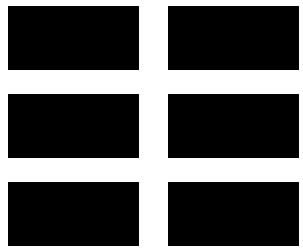
12	14	15
16	18	20
21	22	24
25	26	27
28	30	

The number I chose was _____.

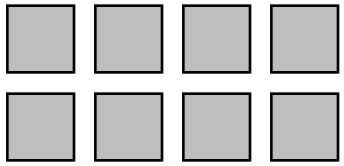
The array I created looks like this ...

Number sentences I can make from my array are ...





Amazing Array Booklet



Name _____

The number I chose was _____.

The array I created looks like this ...

Number sentences I can make from my array are ...

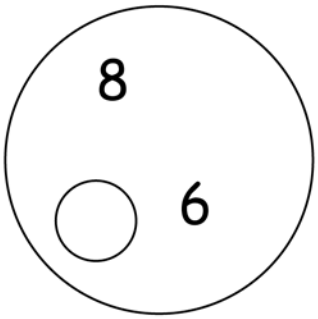
The number I chose was _____.

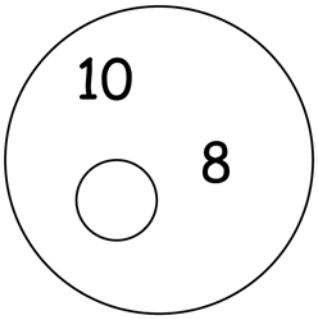
The array I created looks like this ...

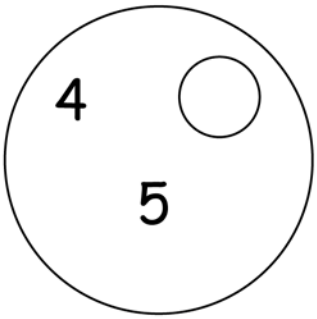
Number sentences I can make from my array are ...

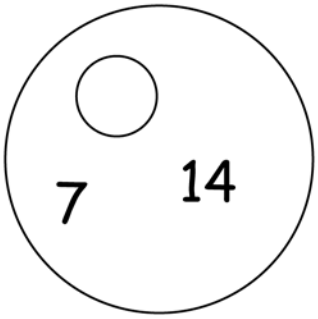
Rel3.BLM3 (a)

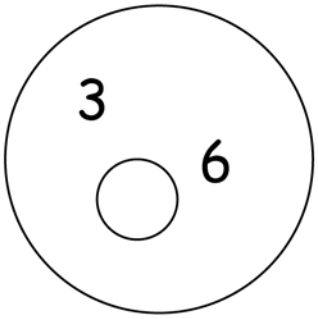
What's Missing?

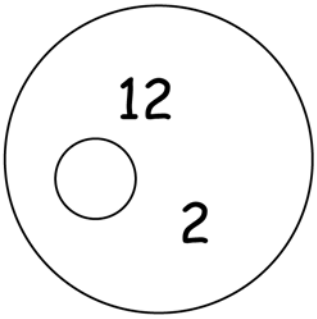


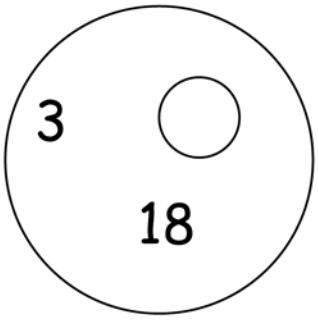


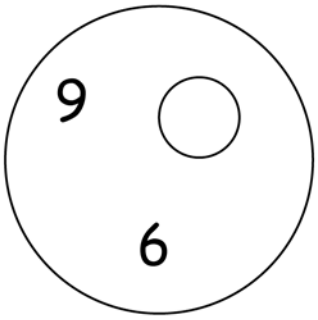


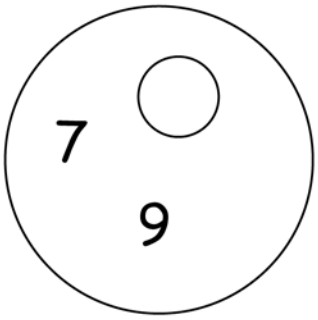


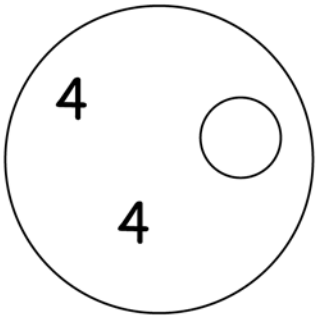


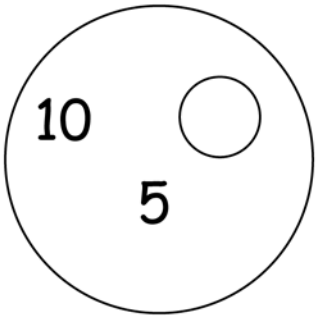


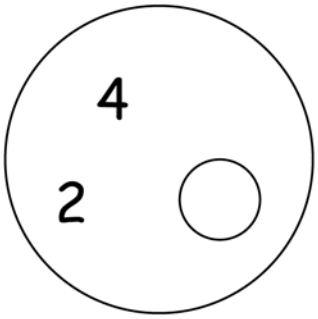




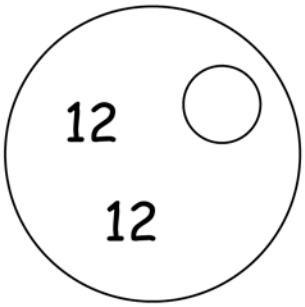




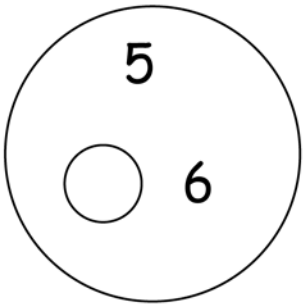




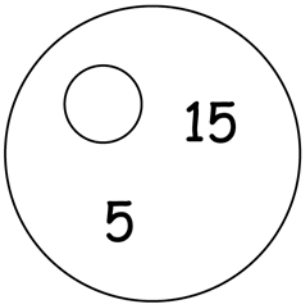
What's Missing?
(Examples)



144 is missing	12 x 12 = 144
0 is missing	12 – 12 = 0 or 12 + 0 = 12
1 is missing	12 x 1 = 12
24 is missing	12 + 12 = 24
12 is missing	12, 12, 12
2 is missing	2 dozen



4 is missing	4, 5, 6
7 is missing	5, 6, 7
1 is missing	5 + 1 = 6 or 6 – 1 = 5
30 is missing	6 x 5 = 30
11 is missing	11 – 6 = 5 or 11 – 5 = 6 or 5 + 6 = 11 or 6 + 5 = 11



10 is missing	10 + 5 = 15 or 5 + 10 = 15 or 15 – 5 = 10 or 15 – 10 = 5
3 is missing	3 x 5 = 15 or 5 x 3 = 15
10 is missing	5, 10, 15
25 is missing	5, 15, 25

What Fraction Is It?

BIG IDEA **Representation**

CURRICULUM EXPECTATIONS

Students will:

- divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation;
- apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
- apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others).

MATERIALS

- rectangular sheets of brown construction paper
- Rep3.BLM1: Sharing Chocolate Bars
- Cuisenaire rods
- coloured construction paper to create Cuisenaire rods if materials aren't available
- Rep3.BLM2: Cuisenaire Rod Relationships
- egg cartons
- counters
- Rep3.BLM3: What's in a Dozen?
- centimetre graph paper
- Rep3.BLM4: How Can You Share?

ABOUT THE MATH

A knowledge of fraction concepts involves an understanding that a whole or a set can be divided into equal parts, and that these parts are represented by a denominator (which tells the number of equal parts the whole or set is divided into) and a numerator (which indicates the number of those parts being considered).

In Grade 3, students' understanding of fractions continues to develop as they construct and compare fractional parts and represent these parts with fractional names. They recognize that a whole can be divided into equal parts in different ways (e.g., into halves, thirds, fourths/quarters). They learn that the more parts there are in a whole, the smaller the parts are.

Students' understanding of whole numbers may interfere with their understanding of fractions. For example, the concept that $\frac{1}{2}$ is bigger than $\frac{1}{3}$ is difficult for some students. They misinterpret the meaning of the 2 and the 3, thinking of these numbers as whole numbers rather than as denominators that represent parts of the whole.

To avoid these kinds of misconceptions, instruction on fractions should provide opportunities for students to visualize and represent fractions using concrete materials. Hands-on experiences, along with meaningful activities and discussion, help students develop a strong conceptual foundation for their future understanding of fraction concepts and operations.

GETTING STARTED

Show students rectangular pieces of brown paper and explain that they represent chocolate bars. Write the name of a different chocolate bar on each rectangle.

Choose a chocolate bar and ask students, "Who likes this kind of chocolate bar?" Choose two students and explain that they will share the chocolate bar. Ask students the following questions:

- "How can Jamal share the chocolate bar with Nabil so that they each get the same amount?"
- "What does 'half' mean?"
- "How do I divide the chocolate bar into halves?"
- "Where will I need to cut?"
- "How will I know if I am sharing the chocolate bar fairly?"
- "What if I cut here? Would it still be cut into halves?"

Continue the discussion:

- "What if 4 people wanted to share the chocolate bar equally? Into how many pieces would I need to cut the chocolate bar?"
- "What is the fraction that names each piece?"
- "Is a fourth bigger or smaller than a half? How do you know?"

Talk about other numbers of people sharing a chocolate bar equally, and discuss the size of pieces and fraction names.

Show students two different-sized chocolate bars, and ask: "Is one half of this chocolate bar the same size as one half of this chocolate bar?"

- "How do we know that this piece is one half of the jumbo chocolate bar?"
- "How do we know that this piece is one half of the mini chocolate bar?"
- "If all these pieces are one half, why are they different sizes?"

Use Rep3.BLM1: Sharing Chocolate Bars to demonstrate the meaning of fractions as equal parts of a whole and to review fraction names.

Have students work in pairs and give each pair Cuisenaire rods or coloured strips of paper cut to duplicate the Cuisenaire rod kits. Refer to Rep3.BLM2: Cuisenaire Rod Relationships. Tell students to imagine that the rods or paper strips are candy bars.

Ask them to explore the relationships among the rods. As examples, show students that the yellow Cuisenaire rod is one half the orange rod and that they need 4 red rods to make a brown rod. Students can record their discoveries on a piece of paper. Refer to Rep3.BLM2: Cuisenaire Rod Relationships.

During a math talk time, ask students to show and explain their discoveries about the relationships with the Cuisenaire rods. Discuss how there are different ways to show one half (e.g., white is one half of red, purple is one half of brown, red is one half of purple). Introduce the idea that fourths are sometimes referred to as "quarters" and make the connection to money (i.e., there are four quarters in one dollar).

Continue the discussion using questions similar to the following:
- "How many different ways are there to show one half?"
- "How many different ways are there to show one fourth (one quarter)?"
- "What fraction is the light green of the blue?"
- "Red is one third of the dark green. What fraction of the dark green do two reds make?"
- "If orange represents the whole, what colour is one half?"
- "If blue represents the whole, what colour is one third?"
- "If black is one whole, how could you show one half?"

Following the whole class discussion, have students work with their partner to continue to discover other relationships among the Cuisenaire rods.

WORKING ON IT

Students in Grade 3 may have some difficulty understanding that fractions can be part of a set of objects (e.g., that 4 objects are one half of a set of 8 objects). They need many opportunities to see this meaning for fractions modelled and explained in a variety of ways using real objects and manipulatives.

Give each student an egg carton with a counter in each of the 12 sections. Pose the problem, "If 12 eggs are the whole set, how many eggs are one half of the set? How can you show your thinking? Is there another way?"

Then ask, "How many eggs are one fourth of the set? How can you show your thinking? Is there another way?"

Pose similar problems, asking students to find one third and to explain their thinking and other possible solutions.

Have students complete Rep3.BLM3: What's in a Dozen? to show different ways of showing fractions of a set of 12 eggs. Discuss the different fractions shown by students.

REFLECTING AND CONNECTING

Provide problems similar to the following:

- "If 15 counters are a whole set, how many counters make one third of the set?"
- "If 4 counters are one third of a whole set, how many are in the whole set?"
- "If 10 counters are one half of a whole set, how many are in the whole set?"
- "How can you use Cuisenaire rods to show fractions? to show arrays?"
- "How can arrays of objects be used to show fractions?"

Students can record their solutions and explanations in their math journals. Encourage students to use manipulatives to solve these problems.

ADAPTATIONS/EXTENSIONS

Challenge students to use Cuisenaire rods to determine which fraction is biggest: $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$. Ask students to explain how they know which fraction is the biggest and to prove their thinking. It may help some students to record their ideas on centimetre graph paper. The graph paper helps keep their diagrams accurate and in proportion to one another.

MATH LANGUAGE

- whole
- part
- half
- quarter (fourth)
- third
- divide
- equal

SAMPLE SUCCESS CRITERIA

- represents fractions in a variety of ways, using concrete materials
- represents fractions using arrays
- explains the relationship between two simple fractions, using Cuisenaire rods (e.g., two fourths make a half)
- demonstrates and explains fractions as parts of a whole and as parts of a set, using a model
- determines the whole set when given information about part of it (e.g., 5 counters are one fourth of the set)

HOME CONNECTION

How Can You Share?

Ask students to explain how foods can be shared equally among family members. What different ways could your family share 25 grapes? 1 chocolate bar? a pizza? an apple? a watermelon? Can you divide all things equally? Why or why not?

How are fractions used in cooking or baking? In what other ways are fractions used at home?

Send home Rep3.BLM4: How Can You Share? or include it in a newsletter.

LEARNING CONNECTION 1

Pattern Block Puzzles

Materials

- pattern blocks

Provide students with pattern blocks. Challenge students to find different ways to cover the yellow hexagon using other pattern blocks.

Ask questions that encourage students to see relationships among the pattern blocks and to use fraction language:

- "How many triangles cover the hexagon?"
- "What fraction of the hexagon is covered by 1 green triangle? 2 triangles? 4 triangles? 6 triangles?"
- "What fraction of the hexagon is covered by 1 red trapezoid? 2 trapezoids? 5 trapezoids?"
- "What fraction of the hexagon is covered by 1 blue rhombus? 2 rhombi? 5 rhombi?" Ask students to describe other relationships among the pattern block shapes.

LEARNING CONNECTION 2

Full of Fractions

Materials

- Rep3.BLM5: Full of Fractions
- pattern blocks (more blue rhombi may be needed for this activity than are usually in a pattern block kit)
- 6-sided number cubes

Divide students into groups of three or four. Provide a copy of Rep3.BLM5: Full of Fractions for each student. Clarify with students that there are 6 hexagons on the game board. Show that a blue rhombus covers one third of the hexagonal shape.

The goal for each student is to completely fill his or her game board of hexagonal shapes using the blue rhombus pattern blocks. The first student tosses a 6-sided number cube and places the given number of rhombus pattern blocks on his or her game board. (The student may choose to place the pattern blocks on any of the 6 hexagons.) As the student places each pattern block, he or she needs to tell what fraction of the hexagon is covered (e.g., "This hexagon is one third covered." or "Now this hexagon is completely covered.") Other group members must agree with the student's statement.

In turn, each student tosses the number cube and places rhombi on his or her game board. If the number cube indicates a number exceeding the remaining spaces on the game board, the student loses the turn. The winner is the first player to fill his or her game board completely.

The game can also be played using red trapezoids (halves of the hexagon) or green triangles (sixths of the hexagon).

LEARNING CONNECTION 3

Fraction Towers

Materials
- interlocking cubes

Have students build several fraction towers using different colours of interlocking cubes. For example, a student could build a tower with 12 interlocking cubes, 3 blue and 9 red, to show one fourth (one quarter) and three fourths (three quarters). Provide an opportunity for students to show their discoveries. Use the fraction towers to show that one fourth (one quarter) of 12 is a different size than one fourth (one quarter) of 20, or that one third of 9 is a different size than one third of 15.

LEARNING CONNECTION 4

MMMMM Cookies

Materials
- manipulatives

Challenge students to solve the following problem:

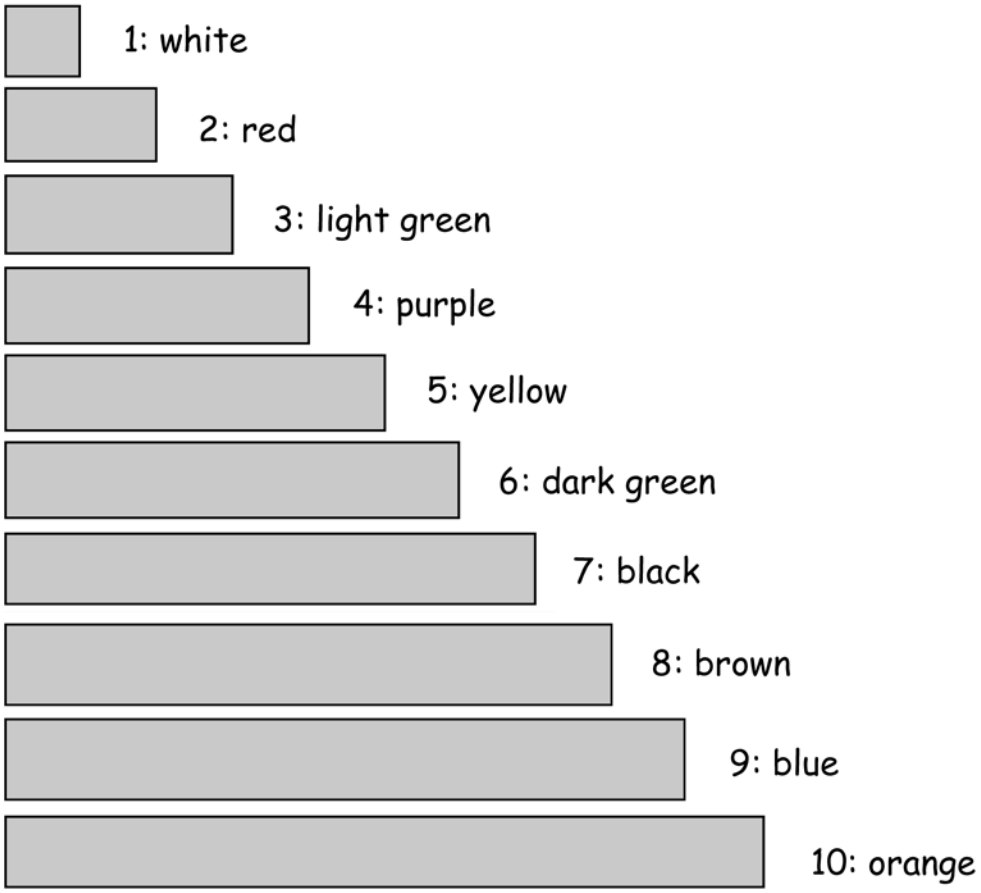
On Saturday morning, Jeremy and his mom made some chocolate chip cookies. His mom put the cookies on a plate. That evening, Jeremy could not believe it when he saw that there was only 1 cookie left. He began some investigating to see who had eaten all the cookies. Sure enough, he found out that his big sister and her friend had eaten half of the cookies that afternoon. His dad had eaten one fourth of the cookies. His mom had not eaten any cookies, but his little sister had eaten 2. Jeremy had eaten only 1 cookie that morning. How many cookies were on the plate to begin with?

Sharing Chocolate Bars

Sharing Chocolate

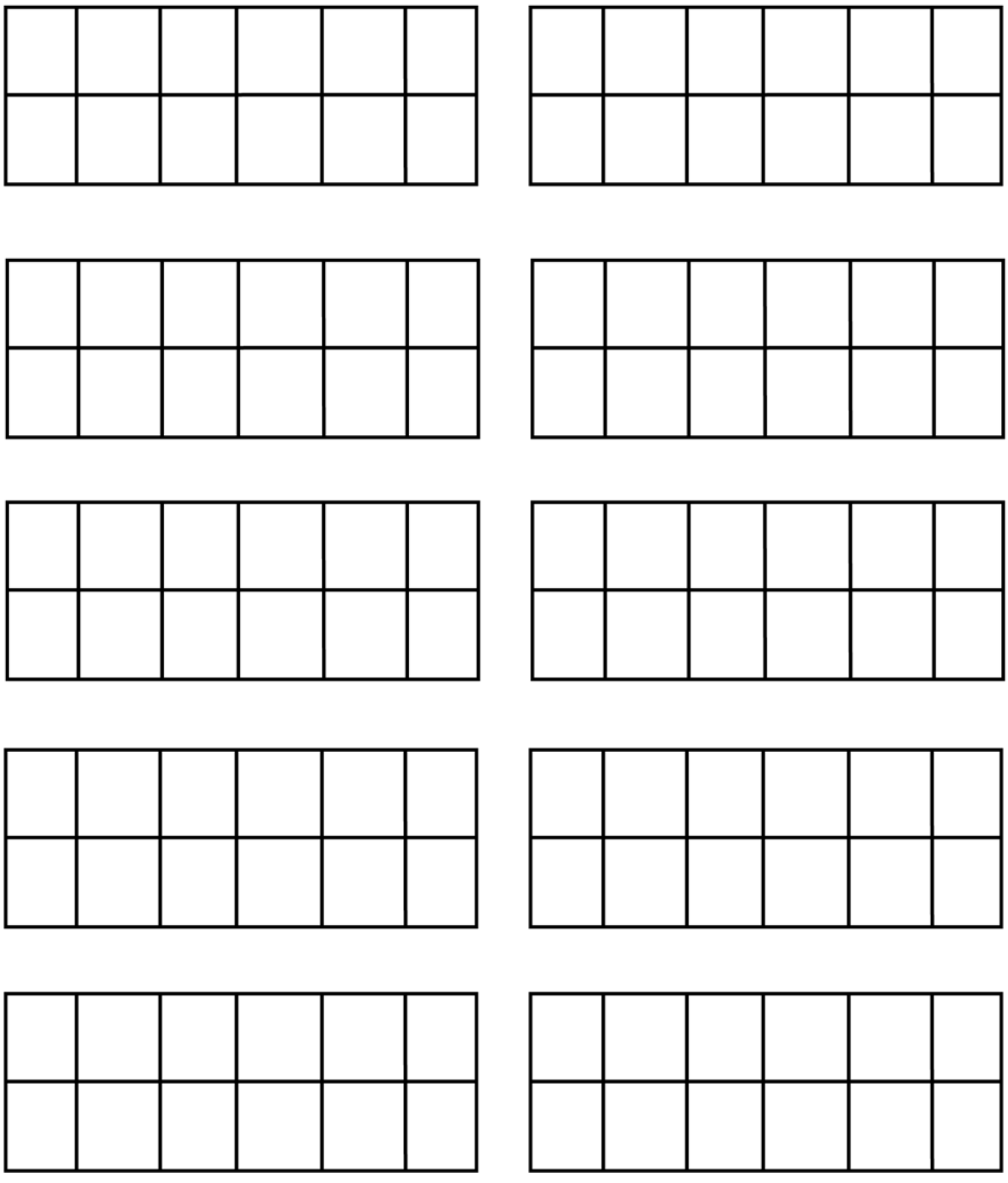
Create a similar chart with your students. Using construction paper chocolate bars, have the students explain how to share each chocolate bar (cut each piece of paper) equally among 2, 3, 4, 6, or 8 people. As you create the fractional pieces you can glue/tape the pieces on the chart and label them with fraction symbols. The chart demonstrates that the pieces get smaller as you share with more people and that the same-sized whole can be divided into various parts.

Cuisenaire Rod Relationships



Rep3.BLM3

What's in a Dozen?

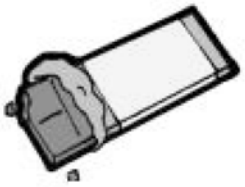


Rep3.BLM4

How Can You Share?

How would your family share these foods equally?

25 candies

one chocolate bar 

a pizza

an apple

a package of gum

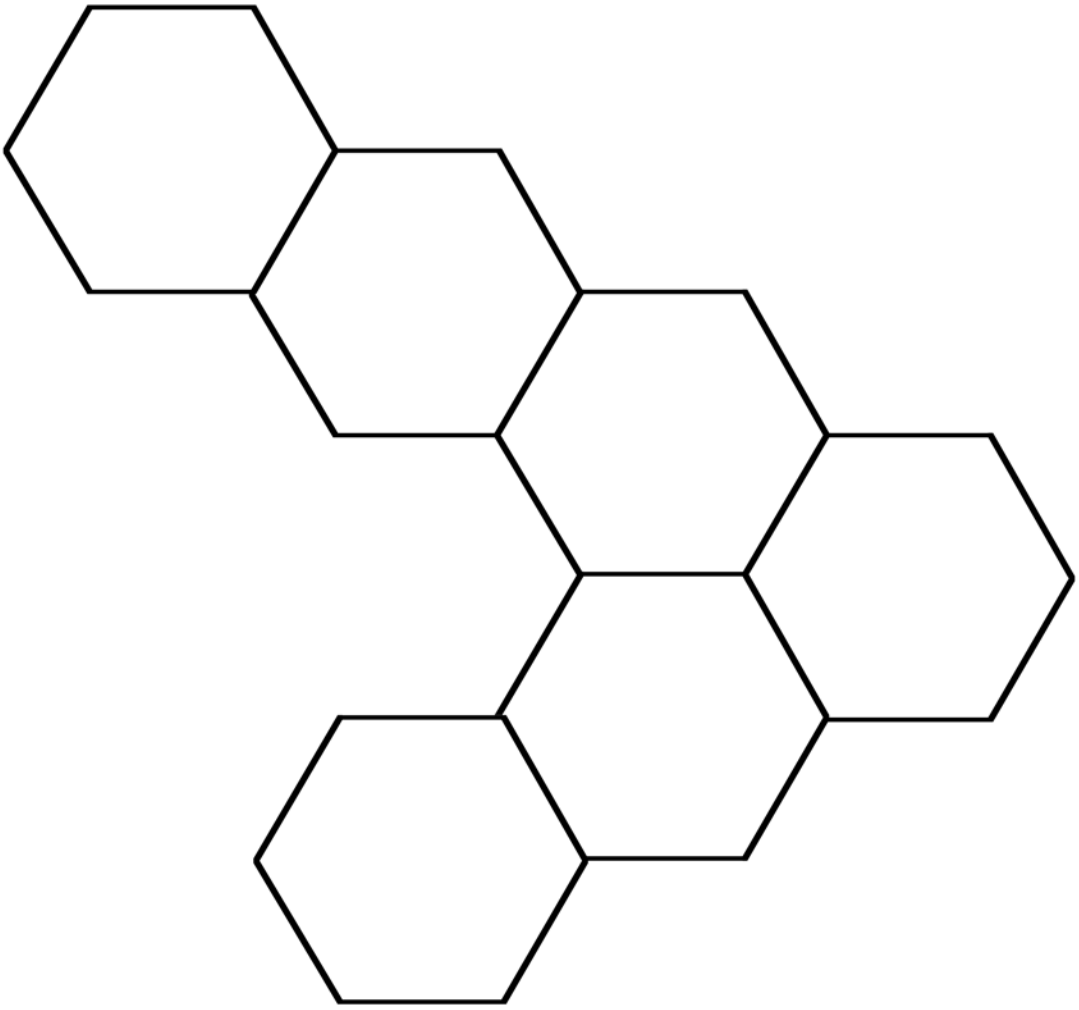
Can you share all foods equally? Why or why not?

How are fractions used in cooking?

Full of Fractions

Fill the hexagons with pattern blocks.

What fraction of each hexagon is covered?



D. Correspondence of the Big Ideas and the Curriculum Expectations in Number Sense and Numeration

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Specific Expectations in Relation to the Big Ideas 175

Overall Expectations

FDK	GRADE 1	GRADE 2	GRADE 3
Students will:	Students will:	Students will:	Students will:
• demonstrate an understanding of numbers, using concrete materials to explore and investigate counting, quantity, and number relationships	• read, represent, compare, and order whole numbers to 50, and use concrete materials to investigate fractions and money amounts	• read, represent, compare, and order whole numbers to 100, and use concrete materials to represent fractions and money amounts to 100¢	• read, represent, compare, and order whole numbers to 1000, and use concrete materials to represent fractions and money amounts to $10
• explore and communicate the function/purpose of numbers in a variety of contexts (e.g., use magnetic and sandpaper numerals to represent the number of objects in a set [to indicate quantity]; line up toys and manipulatives, and identify the first, second, and so on [to indicate ordinality]; use footsteps to discover the distance between the door and the sink [to measure]; identify a favourite sports player: "My favourite player is number twenty-four" [to label or name]	• demonstrate an understanding of magnitude by counting forward to 100 and backwards from 20	• demonstrate an understanding of magnitude by counting forward to 200 and backwards from 50, using multiples of various numbers as starting points	• demonstrate an understanding of magnitude by counting forward and backwards by various numbers and from various starting points
	• solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of strategies	• solve problems involving the addition and subtraction of one-and two-digit whole numbers, using a variety of strategies, and investigate multiplication and division	• solve problems involving the addition and subtraction of single-and multi-digit whole numbers, using a variety of strategies, and demonstrate an understanding of multiplication and division

Specific Expectations in Relation to the Big Ideas

FDK	GRADE 1	GRADE 2	GRADE 3
Counting Students will:	**Counting** Students will:	**Counting** Students will:	**Counting** Students will:
• make use of one-to-one correspondence in counting objects and matching groups of objects	• demonstrate, using concrete materials, the concept of one-to-one correspondence between number and objects when counting		

continued . . .

Specific Expectations in Relation to the Big Ideas

FDK	GRADE 1	GRADE 2	GRADE 3
Counting Students will:	**Counting** Students will:	**Counting** Students will:	**Counting** Students will:
• demonstrate an understanding of the counting concepts of stable order (i.e., the concept that the counting sequence is always the same – 1 is followed by 2, 2 by 3 and so on) and of order irrelevance (i.e., the concept that the number of objects in a set will be the same regardless of which object is used to begin the counting)	• count forward by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies (e.g., move with steps; skip count on a number line; place counters on a hundreds chart; connect cubes to show equal groups; count groups of pennies, nickels, or dimes)	• count forward by 1's, 2's, 5's, 10's, and 25's to 200, using number lines and hundreds charts, starting from multiples of 1, 2, 5, and 10 (e.g., count by 5's from 15; count by 25's from 125)	• count forward by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points, and by 25's to 1000 starting from multiples of 25, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10's using dimes)
	• count backwards by 1's from 20 and any number less than 20 (e.g., count backwards from 18 to 11), with and without the use of concrete materials and number lines	• count backwards by 1's from 50 and any number less than 50, and count backwards by 10's from 100 and any number less than 100, using number lines and hundreds charts	• count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points, and count backwards by 100's from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies
	• count backwards from 20 by 2's and 5's, using a variety of tools (e.g., number lines, hundreds charts)		
• use information to estimate the number in a small set (e.g., apply knowledge of quantity; use a common reference such as a five frame; subitize) • subitize quantities to 5 without having to count, using a variety of materials (e.g., dominoes, dot plates, dice, number of fingers) and strategies (e.g., composing or decomposing numbers)	• estimate the number of objects in a set, and check by counting (e.g., "I guessed that there were 20 cubes in the pile. I counted them and there were only 17 cubes. 17 is close to 20.")		
	• use ordinal numbers to thirty-first in meaningful contexts (e.g., identify the days of the month on a calendar)		
		• locate whole numbers to 100 on a number line and on a partial number line (e.g., locate 37 on a partial number line that goes from 34 to 41)	

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Counting Students will:	**Counting** Students will:	**Counting** Students will:	**Counting** Students will:
	• solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines)	• solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (e.g., base ten materials, counters), student-generated algorithms, and standard algorithms	
	• solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles)		
		• estimate and count the value of a collection of coins with a maximum value of one dollar	• estimate and count the value of a collection of coins and bills with a maximum value of $10
	• add and subtract money amounts to 10¢, using coin manipulatives and drawings	• add and subtract money amounts to 100¢, using a variety of tools (e.g., concrete materials, drawings) and strategies (e.g., counting on, estimating, representing using symbols)	• add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10
Quantity Students will:	**Quantity** Students will:	**Quantity** Students will:	**Quantity** Students will:
• investigate (e.g., using a number line, a hundreds carpet, a board game with numbered squares) the idea that a number's position in the counting sequence determines its magnitude (e.g., the quantity is greater when counting forward and less when counting backward) • investigate some concepts of quantity and equality through identifying and comparing sets with more, fewer, or the same number of the ideas of more, less, or the same, using concrete materials such as counters or five and ten frames; recognize that the last number	• represent, compare, and order whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences, number stories)	• represent, compare, and order whole numbers to 100, including money amounts to 100¢, using a variety of tools (e.g., ten frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets)	• represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts)

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Quantity Students will:	**Quantity** Students will:	**Quantity** Students will:	**Quantity** Students will:
	• demonstrate, using concrete materials, the concept of conservation of number (e.g., 5 counters represent the number 5, regardless whether they are close together or far apart)		
	• identify and describe various coins (i.e., penny, nickel, dime, quarter, $1 coin, $2 coin), using coin manipulatives or drawings, and state their value (e.g., the value of a penny is one cent; the value of a toonie is two dollars)		
	• estimate the number of objects in a set, and check by counting (e.g., "I guessed that there were 20 cubes in the pile. I counted them and there were only 17 cubes. 17 is close to 20.")		
	• compose and decompose numbers up to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3)	• compose and decompose two-digit numbers in a variety of ways, using concrete materials (e.g., place 42 counters on ten frames to show 4 tens and 2 ones; compose 37¢ using one quarter, one dime, and two pennies)	• compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials (e.g., use base ten materials to decompose 327 into 3 hundreds, 2 tens, and 7 ones, or into 2 hundreds, 12 tens, and 7 ones)
		• determine, using concrete materials, the ten that is nearest to a given two-digit number, and justify the answer (e.g., use counters on ten frames to determine that 47 is closer to 50 than to 40)	• round two-digit numbers to the nearest ten, in problems arising from real-life situations
			• identify and represent the value of a digit in a number according to its position in the number (e.g., use base ten materials to show that the 3 in 324 represents 3 hundreds)
			• solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1000

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Relationships Students will:	**Relationships** Students will:	**Relationships** Students will:	**Relationships** Students will:
• demonstrate an understanding of number relationships for numbers from 0 to 10, through investigation (e.g., show small quantities using fingers or manipulatives)	• represent, compare, and order whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences, number stories)	• represent, compare, and order whole numbers to 100, including money amounts to 100¢, using a variety of tools (e.g., ten frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets)	• represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts)
	• demonstrate, using concrete materials, the concept of conservation of number (e.g., 5 counters represent the number 5, regardless whether they are close together or far apart)		
	• relate numbers to the anchors of 5 and 10 (e.g., 7 is 2 more than 5 and 3 less than 10)		
• explore different Canadian coins, using coin manipulatives (e.g., role-play the purchasing of items at the store in the dramatic play area; determine which coin will purchase more – a loonie or a quarter)	• identify and describe various coins (i.e., penny, nickel, dime, quarter, $1 coin, $2 coin), using coin manipulatives or drawings, and state their value (e.g., the value of a penny is one cent; the value of a toonie is two dollars)		• represent and describe the relationships between coins and bills up to $10 (e.g., "There are eight quarters in a toonie and ten dimes in a loonie.")
	• use ordinal numbers to thirty-first in meaningful contexts (e.g., identify the days of the month on a calendar)		
	• divide whole objects into parts and identify and describe, through investigation, equal-sized parts of the whole, using fractional names (e.g., halves; fourths or quarters)	• determine, through investigation using concrete materials, the relationship between the number of fractional parts of a whole and the size of the fractional parts (e.g., a paper plate divided into fourths has larger parts than a paper plate divided into eighths)	
		• compare fractions using concrete materials, without using standard fractional notation (e.g., use fraction pieces to show that three fourths are bigger than one half, but smaller than one whole)	

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Relationships Students will:	**Relationships** Students will:	**Relationships** Students will:	**Relationships** Students will:
		• describe relationships between quantities by using whole-number addition and sub- traction (e.g., "If you ate 7 grapes and I ate 12 grapes, I can say that I ate 5 more grapes than you did, or you ate 5 fewer grapes than I did.")	
			• represent and explain, using concrete materials, the relationship among the numbers 1, 10, 100, and 1000, (e.g., use base ten materials to represent the relationship between a decade and a century, or a century and a millennium)
			• relate multiplication of one-digit numbers and division by one-digit divisors to real-life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences)
Representation Students will:	**Representation** Students will:	**Representation** Students will:	**Representation** Students will:
• use, read, and represent whole numbers to 10 in a variety of meaningful contexts (e.g., use a hundreds chart to read whole numbers; area; find and recognize numbers in the environment; write numerals on imaginary bills at the restaurant in the dramatic play area)	• represent whole numbers to 50, using a variety of tools (e.g., connecting cubes, ten frames, base ten materials, number lines, hundreds charts) and contexts (e.g., real-life experiences, number stories)	• represent whole numbers to 100, including money amounts to 100¢, using a variety of tools (e.g., ten frames, base ten materials, coin manipulatives, number lines, hundreds charts and hundreds carpets)	• represent whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts)
	• read and print in words whole numbers to ten, using meaningful contexts (e.g., story- books, posters)	• read and print in words whole numbers to twenty, using meaningful contexts (e.g., storybooks, posters, signs)	• read and print in words whole numbers to one hundred, using meaningful contexts (e.g., books, speed limit signs)
	• represent money amounts to 20¢, through investigation using coin manipulatives	• represent (using the ¢ symbol) the value of a collection of coins with a maximum value of one dollar	• represent (using the $ symbol) the value of a collection of coins and bills with a maximum value of $10 • represent and describe the relationships between coins and bills up to $10 (e.g., "There are eight quarters in a toonie and ten dimes in a loonie.")

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Representation Students will:	**Representation** Students will:	**Representation** Students will:	**Representation** Students will:
• compose and decompose quantities to 10 (e.g., make multiple representations of numbers using two or more colours of linking cubes, blocks, dot strips, and other manipulatives; play "shake and spill" games	• compose and decompose numbers up to 20 in a variety of ways, using concrete materials (e.g., 7 can be decomposed using connecting cubes into 6 and 1, or 5 and 2, or 4 and 3)	• compose and decompose two-digit numbers in a variety of ways, using concrete materials (e.g., place 42 counters on ten frames to show 4 tens and 2 ones; com- pose 37¢ using one quarter, one dime, and two pennies)	• compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials (e.g., use base ten materials to decompose 327 into 3 hundreds, 2 tens, and 7 ones, or into 2 hundreds, 12 tens, and 7 ones)
	• divide whole objects into parts and identify and describe, through investigation, equal-sized parts of the whole, using fractional names (e.g., halves; fourths or quarters)	• regroup fractional parts into wholes, using concrete materials (e.g., combine nine fourths to form two wholes and one fourth)	• divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation
			• identify and represent the value of a digit in a number according to its position in the number (e.g., use base ten materials to show that the 3 in 324 represents 3 hundreds)
			• represent and explain, using concrete materials, the relationship among the numbers 1, 10, 100, and 1000, (e.g., use base ten materials to represent the relationship between a decade and a century, or a century and a millennium)
	• count forward by 1's, 2's, 5's, and 10's to 100, using a variety of tools and strategies (e.g., move with steps; skip count on a number line; place counters on a hundreds chart; connect cubes to show equal groups; count groups of pennies, nickels, or dimes)	• count forward by 1's, 2's, 5's, 10's, and 25's to 200, using number lines and hundreds charts, starting from multiples of 1, 2, 5, and 10 (e.g., count by 5's from 15; count by 25's from 125)	• count forward by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points, and by 25's to 1000 starting from multiples of 25, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10's using dimes)

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Representation Students will:	**Representation** Students will:	**Representation** Students will:	**Representation** Students will:
	• count backwards by 1's from 20 and any number less than 20 (e.g., count back- wards from 18 to 11), with and without the use of concrete materials and number lines • count backwards from 20 by 2's and 5's, using a variety of tools (e.g., number lines, hundreds charts)	• count backwards by 1's from 50 and any number less than 50, and count backwards by 10's from 100 and any number less than 100, using number lines and hundreds charts	• count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points, and count backwards by 100's from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies
	• add and subtract money amounts to 10¢, using coin manipulatives and drawings	• add and subtract money amounts to 100¢, using a variety of tools (e.g., concrete materials, drawings) and strategies (e.g., counting on, estimating, representing using symbols)	• add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10
		• compare fractions using concrete materials, without using standard fractional notation (e.g., use fraction pieces to show that three fourths are bigger than one half, but smaller than one whole)	
		• locate whole numbers to 100 on a number line and on a partial number line (e.g., locate 37 on a partial number line that goes from 34 to 41)	
		• represent and explain, through investigation using concrete materials and drawings, multiplication as the combining of equal groups (e.g., use counters to show that 3 groups of 2 is equal to 2 + 2 + 2 and to 3 × 2)	

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Operational Sense Students will:	**Operational Sense** Students will:	**Operational Sense** Students will:	**Operational Sense** Students will:
• investigate addition and subtraction in everyday experiences and routines through the use of modelling strategies and manipulatives (e.g., join two sets of objects, one containing a greater number than the other, and count all the objects; separate out the smaller number of objects and determine how many remain) and counting strategies (e.g., use a counting sequence to determine how many objects there are altogether; count backward from the largest number to determine how many objects remain)	• solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines)	• solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (e.g., base ten materials, counters), student-generated algorithms, and standard algorithms	
	• solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles)	• solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies (e.g., "To add 6 + 8, I could double 6 and get 12 and then add 2 more to get 14.")	• solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies (e.g., to add 37 + 26, add the tens, add the ones, then combine the tens and ones, like this: 30 + 20 = 50, 7 + 6 = 13, 50 + 13 = 63)
	• add and subtract money amounts to 10¢, using coin manipulatives and drawings	• add and subtract money amounts to 100¢, using a variety of tools (e.g., concrete materials, drawings) and strategies (e.g., counting on, estimating, representing using symbols)	• add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10
		• describe relationships between quantities by using whole-number addition and subtraction (e.g., "If you ate 7 grapes and I ate 12 grapes, I can say that I ate 5 more grapes than you did, or you ate 5 fewer grapes than I did."	

continued . . .

FDK	GRADE 1	GRADE 2	GRADE 3
Operational Sense Students will:	**Operational Sense** Students will:	**Operational Sense** Students will:	**Operational Sense** Students will:
		• represent and explain, through investigation using concrete materials and drawings, multiplication as the combining of equal groups (e.g., use counters to show that 3 groups of 2 is equal to 2 + 2 + 2 and to 3 × 2)	
		• represent and explain, through investigation using concrete materials and drawings, division as the sharing of a quantity equally (e.g., "I can share 12 carrot sticks equally among 4 friends by giving each person 3 carrot sticks.")	
			• add and subtract three-digit numbers, using concrete materials, student- generated algorithms, and standard algorithms
			• use estimation when solving problems involving addition and subtraction, to help judge the reasonableness of a solution
			• relate multiplication of one-digit numbers and division by one-digit divisors to real- life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences)
			• multiply to 7 × 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting)

Glossary

Note: Words and phrases printed in boldface in the following definitions are also defined in this glossary.

abstraction. In **counting**, the idea that a **quantity** can be represented by different things. For example, 5 can be represented by 5 like objects, by 5 different objects, by 5 invisible things (5 ideas), or by 5 points on a line.

abstract level of understanding. Understanding of mathematics at a symbolic level.

accommodation. A support given to a student to assist him or her in completing a task (e.g., providing more time for task completion, reading printed instructions orally to the student, scribing for the student).

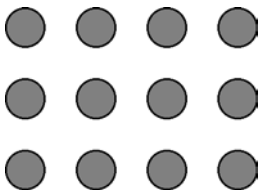
achievement level. The level at which a student is achieving the Ontario curriculum **expectations** for his or her grade level. The Ministry of Education document *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* provides an achievement chart that describes student performance at four levels of achievement in four categories of knowledge and skills: knowledge and understanding; thinking, communication, and application Teachers are expected to base their **assessment** and **evaluation** of students' work on these four levels of achievement. Level 3 is defined as the provincial standard.

algorithm. A systematic procedure for carrying out a **computation**. See also **flexible algorithm** and **standard algorithm**.

See also **diagnostic assessment**, **formative assessment**, and **summative assessment**.

anchors (of 5 and 10). Significant numbers, inasmuch as 10 is the basis of our number system, and two 5's make up 10. Relating other numbers to 5 and 10 (e.g., 7 as 2 more than 5 and 3 less than 10) helps students to develop an understanding of number **magnitude**, to learn basic addition and subtraction facts, and to acquire **number sense** and **operational sense**. See also **five frame** and **ten frame**.

array. A rectangular arrangement of objects into rows and columns, used to represent multiplication (e.g., 5 × 3 can be represented by 15 objects arranged into 5 columns and 3 rows).



assessment. The process of gathering, from a variety of sources, information that accurately reflects how well a student is achieving the curriculum expectations in a subject or course. The primary purpose of assessment is to improve student learning. Assessment for the purpose of improving student learning is seen as both "assessment for learning" and "assessment as learning". As part of assessment for learning, teachers provide students with descriptive feedback and coaching for improvement.

associative property. In an addition expression, the notion that three or more numbers can be added in any order (e.g., 3 + 5 + 7 has the same sum as 5 + 7 + 3). Likewise, in multiplication, three or more numbers can be multiplied in any order and the product will be the same (e.g., $2 \times 4 \times 5 = 4 \times 5 \times 2$). The associative property allows flexibility in **computation**. For example, $8 \times 2 \times 5$ is easier to calculate if 2×5 is done first.

attribute. A quantitative or qualitative characteristic of an object or a shape (e.g., colour, size, thickness).

automaticity. The ability to use skills or perform **mathematical procedures** with little or no mental effort. In mathematics, recall of **basic facts** and performance of computational procedures often become automatic with practice. See also **fluency**.

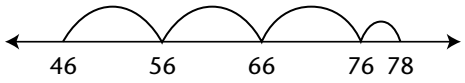
base ten blocks. Three-dimensional models designed to represent ones, tens, hundreds, and thousands proportionally. Ten **ones units** are combined to make 1 tens **rod**, 10 rods are combined to make 1 hundreds **flat**, and 10 flats are combined to make 1 thousands cube. The blocks were developed to help students understand the concept of **place value** and operations with numbers.

basic facts. (Also called "basic number combinations".) The single-digit addition and multiplication **computations** (i.e., up to 9 + 9 and 9 × 9) and their related subtraction and division facts. Students who know the basic facts and know how they are derived are more likely to have computational **fluency** than students who have learned the basic facts by rote.

benchmark. An important or memorable count or measure that can be used to help estimate quantities or other measures. For example, knowing that a cup holds 20 small marbles (a benchmark) and judging that a large container holds about 8 cups allows a person to estimate the number of marbles in the large container. In measurement, knowing that the width of the little finger is about one centimetre (the benchmark) helps to estimate the length of a book cover.

big ideas. In mathematics, the important concepts or major underlying principles. For example, the big ideas for Grades 1 to 3 in the Number Sense and Numeration **strand** of the Ontario curriculum are **counting**, **operational sense**, **quantity**, **relationships**, and **representation**.

blank number line. (Also called "empty number line" or "open number line".) A line that is drawn to represent relationships between numbers or number operations. Only the points and numbers that are significant to the situation are indicated. The placement of points between numbers is not to scale.



A blank number line showing 46 + 32.

calculation. The process of figuring out an answer using one or more **computations**.

cardinality. The idea that the last count of a set of objects represents the total number of objects in the set.

cardinal number. A number that describes how many are in a set of objects.

classifying. Making decisions about how to sort or categorize things. Classifying objects and numbers in different ways helps students recognize **attributes** and characteristics of objects and numbers, and develops flexible thinking.

cluster (of curriculum expectations). A group of curriculum **expectations** that relate to an important concept. By clustering expectations, teachers are able to design learning activities that highlight key concepts and address curriculum expectations in an integrated way, rather than planning separate instructional activities for each individual expectation. In this document, curriculum expectations are clustered around **big ideas**.

clustering. See estimation strategies.

combinations problem. A problem that involves determining the number of possible pairings or combinations between two sets. The following are the 6 possible outfit combinations, given 3 shirts – red, yellow, and green – and 2 pairs of pants – blue and black:

red shirt and blue pants
red shirt and black pants
yellow shirt and blue pants
yellow shirt and black pants
green shirt and blue pants
green shirt and black pants

combining. The act or process of joining quantities. Addition involves combining equal or unequal quantities. Multiplication involves joining groups of equal quantities. See also **partitioning**.

commutative property. See **commutativity**.

commutativity. (Also called "commutative property".) The notion that the order in which numbers are added does not affect the sum (e.g., $3 + 2 = 2 + 3$). Likewise, multiplication is commutative – the order in which numbers are multiplied does not affect the product (e.g., $4 \times 5 = 5 \times 4$).

comparison model. A representation, used in subtraction, in which two sets of items or **quantities** are set side by side and the difference between them is determined.

compensation. A mental arithmetic technique in which part of the value of one number is given to another number to facilitate **computation** (e.g., $6 + 9$ can be expressed as $5 + 10$; that is, 1 from the 6 is transferred to the 9 to make 10).

composition of numbers. The putting together of numbers (e.g., 2 tens and 6 ones can be composed to make 26). See also **decomposition of numbers** and **recomposition of numbers**.

computation. The act or process of determining an amount or a **quantity** by **calculation**.

conceptual approaches. Strategies that require understanding on the part of the student and not just rote memorization.

conceptual understanding. The ability to use knowledge flexibly and to make connections between mathematical ideas. These connections are constructed internally by the learner and can be applied appropriately, and with understanding, in various contexts. See also **procedural knowledge**.

concrete materials. See **manipulatives**.

conservation. The idea that the count for a set group of objects stays the same no matter whether the objects are spread out or are close together.

counting. The process of matching a number in an ordered sequence with every element of a set. The last number assigned is the **cardinal number** of the set.

counting all. A strategy for addition in which the student counts every item in two or more sets to find the total. See also **counting on**.

counting back. Counting from a larger to a smaller number. The first number counted is the total number in the set (**cardinal number**), and each subsequent number is less than that **quantity**. If a student counts back by 1's from 10 to 1, the sequence of numbers is 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. Young students often use counting back as a strategy for subtraction (e.g., to find 22 – 4, the student counts, "21, 20, 19, 18").

counting on. A strategy for addition in which the student starts with the number of the known **quantity**, and then continues **counting** the items of another quantity. To be efficient, students should count on from the larger addend. For example, to find 2 + 7, they should begin with 7 and then count "8" and "9".

decomposition of numbers. The taking apart of numbers. For example, the number 13 is usually taken apart as 10 and 3 but can be taken apart as 6 and 7, or 6 and 6 and 1, and so forth. Students who can decompose numbers in many different ways develop computational **fluency** and have many strategies available for solving arithmetic questions mentally. See also **composition of numbers** and **recomposition of numbers**.

denominator. In common fractions, the number written below the line. It represents the number of equal parts into which a whole or a set is divided.

derived fact. A **basic fact** to which the student finds the answer by using a known fact. For example, a student who does not know the answer to 6 × 7 might know that 3 × 7 is 21, and will then double 21 to get 42.

developmental level. The degree to which physical, intellectual, emotional, social, and moral maturation has occurred. Instructional material that is beyond a student's developmental level is difficult to comprehend and might be learned by rote, without understanding. Content that is below the student's level of development often fails to stimulate interest.

developmentally appropriate. Suitable to a student's level of maturation and cognitive development. Students need to encounter concepts that are presented at an appropriate time in their development and with a developmentally appropriate approach. The mathematics should be challenging but presented in a manner that makes it attainable for students at a given age and level of ability.

diagnostic assessment. Assessment that is used to identify a student's needs and abilities and the student's readiness to acquire the knowledge and skills outlined in the curriculum expectations. Diagnostic assessment usually takes place at the start of a school year, term, semester, or teaching unit. It is a key tool used by teachers in planning instruction and setting appropriate learning goals.

See also **formative assessment** and **summative assessment**.

distributive property. The notion that a number in a multiplication expression can be decomposed into two or more numbers. For example, 4 × 7 can be expressed as (4 × 5) + (4 × 2). Although young students should not be expected to know the name of the property or its definition, they can apply the property to derive unknown facts (e.g., to find 6 × 7, think 6 × 5 = 30 first, and then add 12 [6 × 2]). See also **derived facts**.

dot plates. Paper plates with dots applied in various arrangements to represent numbers from 1 to 10. Dot plates are useful in pattern-recognition activities.

doubles. Basic addition facts in which both addends are the same number (e.g., 4 + 4, 8 + 8). Students can apply a knowledge of doubles to learn other addition facts (e.g., if 6 + 6 = 12, then 6 + 7 = 13) and multiplication facts (e.g., if 7 + 7 = 14, then 2 × 7 = 14).

drill. Practice that involves repetition of a skill or procedure. Because drill often improves speed but not understanding, it is important that **conceptual understanding** be developed before drill activities are undertaken. See also **automaticity**.

empty number line. See **blank number line**. Also referred to as **open number line**.

equal group problem. A problem that involves sets of equal **quantities**. If both the number and the size of the groups are known, but the total is unknown, the problem can be solved using multiplication. If the total in an equal group problem is known, but either the number of groups or the size of the groups is unknown, the problem can be solved using division.

estimation. The process of arriving at an approximate answer for a **computation**, or at a reasonable guess with respect to a measurement. Teachers often provide very young students with a range of numbers within which their estimate should fall.

estimation strategies. Mental mathematics strategies used to obtain an approximate answer. Students estimate when an exact answer is not required and when they are checking the reasonableness of their mathematics work. Some estimation strategies are as follows:

- clustering. A strategy used for estimating the sum of numbers that cluster around one particular value. For example, the numbers 42, 47, 56, 55 cluster around 50. So estimate 50 + 50 + 50 + 50 = 200.
- "nice" numbers. A strategy that involves using numbers that are easy to work with. For example, to estimate the sum of 28, 67, 48, and 56, one could add 30 + 70 + 50 + 60. These nice numbers are close to the original numbers and can be easily added.
- front-end estimation. (Also called "front-end loading".) The addition of significant digits (those with the highest place value), with an adjustment of the remaining values.
  For example:
  Step 1 – Add the left-most digit in each number.
  193 + 428 + 253
  Think 100 + 400 + 200 = 700.
  Step 2 – Adjust the estimate to reflect the size of the remaining digits.
  93 + 28 + 53 is approximately
  (100 + 25 + 50 =) 175.
  Think 700 + 175 = 875.
- rounding. A process of replacing a number by an approximate value of that number. For example, 106 rounded to the nearest ten is 110.

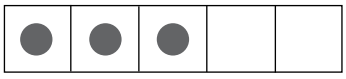
evaluation. The process of judging the quality of student learning on the basis of established criteria and assigning a value to represent that quality. Evaluation is based on assessments of learning that provide data on student achievement at strategic times throughout the grade/subject/course, often at the end of a period of learning.

expectations. The knowledge and skills that students are expected to learn and to demonstrate by the end of every grade or course, as outlined in the Ontario curriculum documents for the various subject areas.

extension. A learning activity that is related to a previous one. An extension can involve a task that reinforces, builds upon, or requires application of newly learned material.

figure. See **three-dimensional figure**.

five frame. A 1 by 5 **array** onto which counters or dots are placed, to help students relate a given number to 5 (e.g., 7 is 2 more than 5) and recognize the importance of 5 as an **anchor** in our number system. See also **ten frame**.



flat. In **base ten blocks**, the representation for 100.

flexible algorithm. (Also called "student-generated algorithm".) A non-standard **algorithm** devised by a person performing a mental **calculation**, often by decomposing and recomposing numbers. For example, to add 35 and 27, a person might add 35 and 20, and then add 7. See also **decomposition of numbers** and **recomposition of numbers**.

fluency. Proficiency in performing **mathematical procedures** quickly and accurately. Although computational fluency is a goal, students should be able to explain how they are performing **computations**, and why answers make sense. See also **automaticity**.

formative assessment. Assessment that takes place during instruction to provide direction for improvement for individual students and for adjustment to instructional programs for individual students and for a whole class. The information gathered is used for the specific purpose of helping students improve while they are still gaining knowledge and practising skills. See also **diagnostic assessment** and **summative assessment**.

fractional sense. An understanding that whole numbers can be divided into equal parts that are represented by a **denominator** (which tells how many parts the whole is divided into) and a **numerator** (which indicates the number of those equal parts being considered). Fractional sense includes an understanding of relationships between fractions, and between fractions and whole numbers (e.g., knowing that $\frac{1}{3}$ is bigger than $\frac{1}{4}$ and that $\frac{2}{3}$ is closer to 1 than $\frac{2}{4}$ is).

homework. Work that students do at home to practise skills, consolidate knowledge and skills, and/or prepare for the next class. Effective homework engages students in interesting and meaningful activities.

horizontal format. A left-to-right arrangement (e.g., of addends), often used in presenting **computation** questions to encourage students to use **flexible algorithms** (e.g., 23 + 48). By contrast, a **vertical format** or arrangement lends itself to the use of **standard algorithms**.

23 + 48	23 +48
Horizontal format	Vertical format

hundreds chart. A 10 by 10 grid that contains the numbers from 1 to 100 written in a sequence that starts at the top left corner of the chart, with the numbers from 1 to 10 forming the top row of the chart and the numbers from 91 to 100 forming the bottom row. The hundreds chart is a rich context for exploring number patterns and relationships.

identity rule. In addition, the notion that the sum of adding 0 to any number is the same number (e.g., 0 + 4 = 4). In multiplication, the notion that a number multiplied by 1 equals that number (e.g., 4 × 1 = 4).

inverse operations. The opposite effects of addition and subtraction, and of multiplication and division. Addition involves joining sets; subtraction involves separating a **quantity** into sets. Multiplication refers to joining sets of equal amounts; division is the separation of an amount into equal sets.

investigation. An instructional activity in which students pursue a problem or an exploration. Investigations help students to develop problem-solving skills, learn new concepts, and apply and deepen their understanding of previously learned concepts and skills.

journal. (Also called "learning log".) A collection of written reflections by students about learning experiences. In journals, students can describe learning activities, explain solutions to problems, respond to open-ended questions, report on **investigations**, and express their own ideas and feelings.

magnitude. The size of a number or a **quantity**. Movement forward or backwards, for example, on a **number line**, a clock, or a scale results in an increase or a decrease in number magnitude.

making tens. A strategy by which numbers are combined to make groups of 10. Students can show that 24 is the same as two groups of 10 plus 4 by placing 24 counters on **ten frames**. Making tens is a helpful strategy in learning addition facts. For example, if a student knows that 7 + 3 = 10, then the student can surmise that 7 + 5 equals 2 more than 10, or 12. As well, making tens is a useful strategy for adding a series of numbers (e.g., in adding 4 + 7 + 6 + 2 + 3, find combinations of 10 first [4 + 6, 7 + 3] and then add any remaining numbers, in this case, +2).

manipulatives. (Also called "concrete materials".) Objects that students handle and use in constructing their own understanding of **mathematical concepts** and skills and in illustrating that understanding. Some examples are **base ten blocks**, interlocking cubes, construction kits, number cubes (dice), games, geoboards, **hundreds charts**, measuring tapes, Miras (red transparent plastic tools), **number lines**, pattern blocks, spinners, **rekenreks**, and coloured tiles.

mathematical concepts. The fundamental understandings about mathematics that a student develops within problem-solving contexts.

mathematical model. (Also called "model" or "representation".) Representation of a **mathematical concept** using **manipulatives**, a diagram or picture, **symbols**, or real-world contexts or situations. Mathematical models can make math concepts easier to understand.

mathematical procedures. (Also called "procedures".) The skills, operations, mechanics, manipulations, and calculations that a student uses to solve problems.

mathematical skills. Procedures for doing mathematics. Examples of mathematical skills include performing paper-and-pencil **calculations**, using a ruler to measure length, and constructing a bar graph.

mental calculation. See **mental computation**.

mental computation. (Also called "mental calculation".) The ability to solve **computations** in one's head. Mental computation strategies are often different from those used for paper-and-pencil computations. For example, to calculate 53 – 27 mentally, one could subtract 20 from 53, and then subtract 7 from 33.

movement is magnitude. The idea that, as one moves up the **counting** sequence, the **quantity** increases by 1 (or by whatever number is being counted by), and as one moves down or backwards in the sequence, the quantity decreases by 1 (or by whatever number is being counted by) (e.g., in skip counting by 10's, the amount goes up by 10 each time).

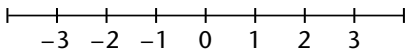
multiplicative relations. Situations in which a **quantity** is repeated a given number of times. Multiplicative relations can be represented symbolically as **repeated addition** (e.g., 5 + 5 + 5) and as multiplication (e.g., 3 × 5).

next steps. The processes that a teacher initiates to assist a student's learning following **assessment**.

"nice" numbers. See **estimation strategies**.

non-standard units. Measurement units used in the early development of measurement concepts – for example, paper clips, cubes, hand spans, and so on.

number line. A line that matches a set of numbers and a set of points one to one.



number sense. The ability to interpret numbers and use them correctly and confidently.

numeral. A word or **symbol** that represents a number.

numeration. A system of **symbols** or **numerals** representing numbers. Our number system uses 10 symbols, the digits from 0 to 9. The placement of these digits within a number determines the value of that numeral. See also **place value**.

numerator. In common fractions, the number written above the line. It represents the number of equal parts being considered.

ones unit. In **base ten blocks**, the small cube that represents 1.

one-to-one correspondence. In **counting**, the idea that each object being counted must be given one count and only one count.

operational sense. Understanding of the **mathematical concepts** and **procedures** involved in operations on numbers (addition, subtraction, multiplication, and division) and of the application of operations to solve problems.

order irrelevance. The idea that the **counting** of objects can begin with any object in a set and the total will still be the same.

ordinal number. A number that shows relative position or place – for example, first, second, third, fourth.

partitioning. One of the two meanings of division; sharing. For example, when 14 apples are partitioned (shared equally) among 4 children, each child receives 3 apples and there are 2 apples remaining (left over). A more sophisticated partitioning (or sharing) process is to partition the remaining parts so that each child, for example, receives $3\frac{1}{2}$ apples.

The other meaning of division is often referred to as "measurement". In a problem involving measurement division, the number in each group is known, but the number of groups is unknown (e.g., Some children share 15 apples equally so that each child receives 3 apples. How many children are there?).

part-part-whole. The idea that a number can be composed of two parts. For example, a set of 7 counters can be separated into parts – 1 counter and 6 counters, 2 counters and 5 counters, 3 counters and 4 counters, and so forth.

patterning. The sequencing of numbers, objects, shapes, events, actions, sounds, ideas, and so forth, in regular ways. Recognizing patterns and relationships is fundamental to understanding mathematics.

pattern structure. The order in which elements in a pattern occur, often represented by arrangements of letters (e.g., AAB AAB AAB).

place value. The value given to a digit in a number on the basis of its place within the number. For example, in the number 444, the digit 4 can equal 400, 40, or 4.

prerequisite understanding. The knowledge that students need to possess if they are to be successful in completing a task. See also **prior knowledge**.

prior knowledge. The acquired or intuitive knowledge that a student possesses before instruction.

problem solving. Engaging in a task for which the solution is not obvious or known in advance. To solve the problem, students must draw on their prior knowledge, try out different strategies, make connections, and reach conclusions. Learning by inquiry or **investigation** is very natural for young children.

problem-solving strategies. Methods used for tackling problems. The strategies most commonly used by students in the primary grades include the following: act it out, make a model using **manipulatives**, find/use a pattern, draw a diagram, guess and check, use logical thinking, make a **table**, use an organized list.

procedural knowledge. Knowledge that relates to selecting the appropriate method (procedure) for solving a problem and applying that procedure correctly. Research indicates that procedural skills are best acquired through understanding rather than rote memorization. See also **automaticity** and **conceptual understanding**.

procedures. See **mathematical procedures**.

proportional reasoning. Reasoning that involves the relation in size of one object or **quantity** compared with another. Young students express proportional reasoning using phrases like "bigger than", "twice as big as", and "half the size of".

quantity. The "howmuchness" of a number. An understanding of quantity helps students estimate and reason with numbers, and is an important prerequisite to understanding **place value**, the operations, and fractions.

recomposition of numbers. The putting back together of numbers that have been decomposed. For example, to solve 24 + 27, a student might decompose the numbers as 24 + 24 + 3, then recompose the numbers as 25 + 25 + 1 to give the answer 51. See also **composition of numbers** and **decomposition of numbers**.

regrouping. (Also called "trading".) The process of exchanging 10 in one **place-value** position for 1 in the position to the left (e.g., when 4 ones are added to 8 ones, the result is 12 ones or 1 ten and 2 ones). Regrouping can also involve exchanging 1 for 10 in the place-value position to the right (e.g., 56 can be regrouped to 4 tens and 16 ones). The terms "borrowing" and "carrying" are misleading and can hinder understanding.

relationship. In mathematics, a connection between mathematical concepts, or between a mathematical concept and an idea in another subject or in real life. As students connect ideas they already understand with new experiences and ideas, their understanding of mathematical relationships develops.

remainder. The **quantity** left when an amount has been divided equally and only whole numbers are accepted in the answer (e.g., 11 divided by 4 is 2 R3). The concept of a remainder can be quite abstract for students unless **concrete materials** are used for sharing. When concrete materials are used, very young students have little difficulty understanding that some items might be left after sharing.

rekenrek. A **manipulative** that is similar to an abacus but without place-value columns. Each row is made up of five white and five red beads so that relationships between a number, fives, and tens can easily be seen.

repeated addition. The process of adding the same number two or more times. Repeated addition can be expressed as multiplication (e.g., 3 + 3 + 3 + 3 represents 4 groups of 3, or 4 × 3).

repeated subtraction. The process of subtracting the same **subtrahend** from another number two or more times until 0 is reached. Repeated subtraction is related to division (e.g., 8 – 2 – 2 – 2 – 2 = 0 and 8 ÷ 2 = 4 express the notion that 8 can be partitioned into 4 groups of 2).

representation. See **mathematical model**.

rod. In **base ten blocks**, the representation for 10.

rubric. A scale that uses brief statements based on the criteria provided in the achievement chart and expressed in language meaningful to students to describe the levels of achievement of a process, product, or performance.

separate problem. A problem that involves decreasing an amount by removing another amount.

shape. See **two-dimensional shape**.

shared characteristics. Attributes that are common to more than one object.

stable order. The idea that the **counting** sequence stays consistent. It is always 1, 2, 3, 4, 5, 6, 7, 8, . . . , not 1, 2, 3, 5, 6, 8.

standard algorithm. An accepted step-by-step procedure for carrying out a **computation**. Over time, standard **algorithms** have proved themselves to be efficient and effective. However, when students learn standard algorithms without understanding them, they may not be able to apply the algorithms effectively in problem-solving situations. See also **flexible algorithm**.

strand. A major area of knowledge and skills. In the Ontario mathematics curriculum for Grades 1–8, there are five strands: Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability.

student-generated algorithm. See **flexible algorithm**.

subitizing. Being able to recognize the number of objects at a glance without having to count all the objects.

subtrahend. In a subtraction question, the number that is subtracted from another number. In the example 15 – 5 = 10, 5 is the subtrahend.

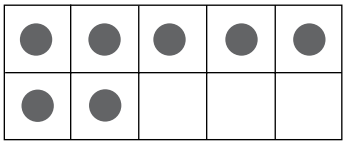
success criteria. Standards or specific descriptions of successful attainment of learning goals developed by teachers on the basis of criteria in the achievement chart, and discussed and agreed upon in collaboration with students, that are used to determine to what degree a learning goal has been achieved. Criteria describe what success "looks like", and allow the teacher and student to gather information about the quality of student learning.

summative assessment. Evaluation that occurs at the end of important segments of student learning. It is used to summarize and communicate what students know and can do with respect to curriculum expectations. See also **diagnostic assessment** and **formative assessment**.

symbol. A letter, **numeral**, or figure that represents a number, an operation, a concept, or a **relationship**. Teachers need to ensure that students make meaningful connections between symbols and the mathematical ideas that they represent.

table. An orderly arrangement of facts set out for easy reference – for example, an arrangement of numerical values in vertical or horizontal columns.

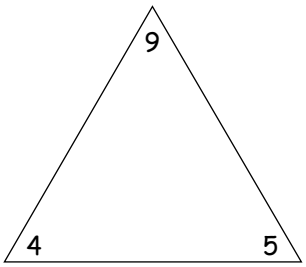
ten frame. A 2 by 5 **array** onto which counters or dots are placed to help students relate a given number to 10 (e.g., 8 is 2 less than 10) and recognize the importance of using 10 as an **anchor** when adding and subtracting. See also **five frame**.



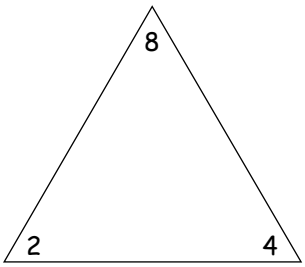
three-dimensional figure. (Also called "figure".) An object having length, width, and depth. Three-dimensional figures include cones, cubes, prisms, cylinders, and so forth. See also **two-dimensional shape**.

trading. See **regrouping**.

triangular flashcards. Flashcards in the shape of a triangle with an addend in each of two corners and the sum in the third. To practise addition and subtraction facts, one person covers one of the numbers and shows the card to a partner, who must determine the missing number. Triangular flashcards can also be made for the practice of basic multiplication and division facts.



A triangular flashcard for addition and subtraction



A triangular flashcard for multiplication and division

two-dimensional shape. (Also called "shape".) A shape having length and width but not depth. Two-dimensional shapes include circles, triangles, quadrilaterals, and so forth. See also **three-dimensional figure**.

unitizing. The idea that, in the base ten system, 10 ones form a group of 10. This group of 10 is represented by a 1 in the tens place of a written **numeral**. Likewise, 10 tens form a group of 100, indicated by a 1 in the hundreds place.

Venn diagram. A diagram consisting of overlapping circles used to show what two or more sets have in common.

vertical format. In written **computation**, a format in which numbers are arranged in columns to facilitate the application of **standard algorithms**. See also **horizontal format**.

zero property of multiplication. The notion that the product of a number multiplied by 0 is 0.