# Grade 11: Functions and Applications – MCF3M

The MCF3M course is comprised of three strands: **Quadratic Functions, Exponential Functions** and **Trigonometric Functions**. In relation to the previous MCF3M course, some new expectations have been added, some of the old ones have been removed, and there have also been modifications made to some of the specific expectations. In the listing that follows, the following coding has been used directly below the Expectation Number: The symbol  $\rightarrow$  is used to identify expectations that have been changed slightly from the current MCF3M, and  $\rightarrow$  is used to identify new expectations that were not in the current MCF3M. If neither symbol appears, then the expectation is currently in the MCF3M course and has not changed significantly in the revised course.

In the columns marked 10P and 10D, you will see an N to represent that this idea is NEW to students or a C to represent that this idea is a CONTINUATION of something started in the grade 10 course or an R to indicate that this idea was developed in the grade 10 course and is a REVIEW for the students in MCF3M. **\* indicates that the lesson(s) will be included in the work being produced by the writing team** 

## Unit 1: Introduction Unit (4 days + 0 jazz days + 0 summative evaluation days)

## **BIG Ideas:**

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#### Notes to writers (delete in final version)

- Give students the big picture before they get into the course....where are we headed...what functions will we be exploring
- Review of linear, quadratic and very general intro to exponential and trig
- If you zoom in to parts of each of these functions (quadratic, exponential, trig) they can look very similar....how do we know when to use each model? (this is something we will have to explore throughout the course)
- explore these relations with CBR....how do you make each graph by walking in front of the motion detector?
- is there something they could also explore with sketchpad?
- This gives students the big picture of where they are headed in the course and something to connect back to
- Maybe start Frayer models on each type of function which they can add to throughout course as they learn more about the functions

DAY	10 P	10 D	Expec	tations	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References
	Ν	Ν	EF1.06 → *	distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth)	<b>Sample problem:</b> Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$ .	
	Ν	N	TF3.01	<u>collect data</u> that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g. websites such as Statistics Canada, E-STAT), and graph the data	<b>Sample problem:</b> Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.	

## Unit 2: Functions (6 days + 1 jazz day + 1 summative evaluation day)

### **BIG Ideas:**

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#### Notes to writers (delete in final version)

- Look at real world examples of quadratics...reviewing ideas from grade 10
- Introduce the idea of functions and function notation through functions they are already familiar with (linear and quadratic)
- Function machines act out? Sketchpad?
   Talk about domain and range introducing the idea of necessary restrictions when modeling real-life situations

DAY	10 P	10 D	Expecta	tions	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References
	С	С	QF3.01 → *	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	<b>Sample problem:</b> When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.	Nelson 10 (4.8) McGraw 10 (4.8) AW 10 (4.2, 4.3) Nelson 11(4.1-4.3)
	Ν	Ζ	QF2.01	explain the meaning of the term <i>function</i> , and <u>distinguish a function from a relation that is</u> <u>not a function, through investigation of linear</u> <u>and quadratic relations using a variety of</u> <u>representations</u> (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., using the vertical line test)	<b>Sample problem:</b> Investigate, using numeric and graphical representations, whether the relation $x = y^2$ is a function, and justify your reasoning.);	Harcourt 11 (1.1, 1.2) Nelson 11 (3.2) McGraw 11 (3.1) AW11 (7.1page 392, 393) AW10 (1.2)
	N	N	QF2.02	substitute into and <u>evaluate linear and</u> <u>quadratic functions</u> represented using function notation [e.g., evaluate $f(\frac{1}{2})$ , given $f(x) = 2x^2 + 3x - 1$ ], including functions arising from real-world applications	<b>Sample problem:</b> The relationship between the selling price of a sleeping bag, <i>s</i> dollars, and the revenue at that selling price, <i>r</i> ( <i>s</i> ) dollars, is represented by the function $r(s) = -10s^2 + 1500s$ . Evaluate, interpret, and compare <i>r</i> (29.95), <i>r</i> (60.00), <i>r</i> (75.00), <i>r</i> (90.00), and <i>r</i> (130.00).	Nelson 11 (3.2) McGraw 11(3.1) AW11 (7.2) AW10 (4.1)

N	N	QF2.03	explain the meanings of the terms <i>domain</i> and <i>range</i> , through investigation using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately (e.g., for $y = x^2 + 1$ , the domain is the set of real numbers, and the range is $y \ge 1$ );		Nelson 11(3.2) McGraw 11(3.1) AW10 (1.3)
N	Ν	QF2.04 →	explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications	<b>Sample problem:</b> A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?	Nelson 11 (3.6) AW10 (4.1)
С	С	QF1.01	pose and solve problems involving quadratic relations arising from real-world applications and represented by tables of values and graphs (e.g.,"From the graph of the height of a ball versus time, can you tell me how high the ball was thrown and the time when it hit the ground?");		Nelson 10 (page 246-249, 256-259, 408) McGraw 10 (page 190+) AW10 (4.1)
С	С	QF3.03 →	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement)	<b>Sample problem:</b> In a DC electrical circuit, the relationship between the power used by a device, <i>P</i> (in watts,W), the electric potential difference (voltage), <i>V</i> (in volts,V), the current, <i>I</i> (in amperes, A), and the resistance, <i>R</i> (in ohms, $\Omega$ ), is represented by the formula $P = IV - I^2 R$ . Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 $\Omega$ . Determine the current needed in order for the device to use the maximum amount of power.	Nelson 10 (embedded in chap 3, 4) McGraw 10 (4.2-4.4) McGraw 11 (2.3) AW10 (6.5) AW11 (4.1, 4.2

## Unit 3: Investigating Quadratics (9 days + 1 jazz day + 1 summative evaluation day)

#### **BIG Ideas:**

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- Notes to writers (delete in final version)
  Make use of algebra tiles and/or virtual tiles
- Focus on contexts where they will need to find the zeroes of the quadratic •

DAY	10 P	10 D	Expecta	tions	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References	
	С	R	QF1.05 → ★	determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the <i>x</i> -intercepts of the corresponding quadratic relation	<b>Sample problem:</b> The profit, <i>P</i> , of a video company, in thousands of dollars, is given by $P = -5x^2 + 550x - 5000$ , where <i>x</i> is the amount spent on advertising, in thousands of dollars. Determine, by factoring and by graphing, the amount spent on advertising that will result in a profit of \$0. Describe the connection between the two strategies.	Nelson 10 (3.4, 4.7) Nelson 11 (4.3) McGraw FM11 (6.8)	
	С	С	QF1.02 →	represent situations (e.g., the area of a picture frame of variable width) using quadratic expressions in one variable, and expand and simplify quadratic expressions in one variable [e.g., $2x(x + 4)-(x+3)^2$ ];*	*The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper.	Nelson 10 (3.9, 4.7, pg 297-298, 301) Nelson 11 (4.12) McGraw 10 (3.1-3.3) McGraw 11 (1.4)	
	Have only done simple trinomials	С	QF1.03 → ★	factor quadratic expressions in one variable, including those for which $a \neq 1$ (e.g., $3x^2 + 13x - 10$ ), differences of squares (e.g., $4x^2 - 25$ ), and perfect square trinomials (e.g., $9x^2 + 24x + 16$ ), by selecting and applying an appropriate strategy (	<b>Sample problem:</b> Factor $2x^2 - 12x + 10$ .); The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper.	Nelson 10 (3.8, page 298) McGraw 10 (3.4-3.7) AW10 (5.1, 5.3-5.5) Nelson 11 (pg 303 # 7) McGraw 11 (pg 3)	
	N	R	QF1.04	solve quadratic equations by selecting and applying a factoring strategy;		Nelson 10 (3.9) McGraw 10 (5.2) AW10 (5.6) Nelson 11 (4.3) McGraw 11 (2.3 + )	

С	С	QF3.03 → *	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement) [	<b>Sample problem:</b> In a DC electrical circuit, the relationship between the power used by a device, <i>P</i> (in watts,W), the electric potential difference (voltage), <i>V</i> (in volts,V), the current, <i>I</i> (in amperes, A), and the resistance, <i>R</i> (in ohms, $\Omega$ ), is represented by the formula <i>P</i> = <i>IV</i> - <i>I</i> <sup>2</sup> <i>R</i> . Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 $\Omega$ . Determine the current needed in order for the device to use the maximum amount of power.].	Nelson 10 (4.6, 4.7 + embedded in chp 3, 4) McGraw 10 (4.2-4.4) AW 10 (6.5) McGraw 11 (2.3) AW11 (4.1, 4.2)
С	С	QF3.01 →	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data ();	<b>Sample problem:</b> When a $3 \times 3 \times 3$ cube made up of $1 \times 1 \times 1$ cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.	Nelson 10 (4.8) McGraw 10 (4.8) AW 10 (4.2, 4.3) Nelson 11 (4.1-4.3)
Ν	Ν	QF3.02 → *	determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the x-intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator);	SuggestionBouncing ballstudents can find the equation using the x-intercepts and 1 point	Nelson 10 (3.7, page 292)Balloon catapult data Also, Nelson 10, 3.8, 3.9 -check old grade 10 profiles

## Unit 4: Quadratic - Highs and Lows (14 days + 1 jazz + 2 midterm summative evaluation days)

## **BIG Ideas:**

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#### Notes to writers (delete in final version)

• It is important for students to see the need for working with vertex form. Perhaps use a problem where both roots and vertex are interesting. Could take sample problem on profit from previous unit – in previous unit students would find break even point (roots) and now we want to see where the maximum profit is (vertex)

• Transformations are review for 10 academic but new for 10 applied – it will be important to keep it fresh for the 10 academics but not go too quickly for the 10 applied

Can complete the square using algebra tiles (if you keep the numbers nice)

DAY	10 P	10 D	Expecta	tions	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References
	C (no "a" )	С	QF2.09 → ★	sketch graphs of quadratic functions in the factored form $f(x) = a(x - r)(x - s)$ by using the x- intercepts to determine the vertex;	Use this expectation to talk about how to get the vertex from the factored form – is there another way to find this vertex?	Nelson 10 (3.2 – 3.4) McGraw 10 (4.4 + ) AW 10 (6.2) Nelson 11 (4.3)
	N	С	QF2.05 → *	determine, through investigation using technology, and describe the roles of <i>a</i> , <i>h</i> , and <i>k</i> in quadratic functions of the form <i>f</i> $(x) = a(x - h)^2 + k$ in terms of transformations on the graph of $f(x) = x^2$ (i.e., translations; reflections in the <i>x</i> -axis; vertical stretches and compressions)	<b>Sample problem:</b> Investigate the graph $f(x) = 3(x - h)^2 + 5$ for various values of $h$ , using technology, and describe the effects of changing $h$ in terms of a transformation.	Nelson 10 (4.3) McGraw 10 (4.2, 4.3) AW 10 .1) Nelson 11 (3.7) McGraw 11 (2.2) AW 11 (7.3-7.4)
	N	С	QF2.06 → *	sketch graphs of $g(x) = a(x - h)^2 + k$ by applying one or more transformations to the graph of $f(x) = x^2$	<b>Sample problem:</b> Transform the graph of $f(x) = x^2$ to sketch the graphs of $g(x) = x^2 - 4$ and $h(x) = -2(x + 1)^2$	Same as above, plus AW10 (6.3)
	N	N	QF2.07	express the equation of a quadratic function in the standard form $f(x) = ax^2 + bx + c$ , given the vertex form $f(x) = a(x - h)^2 + k$ , and verify, using graphing technology, that these forms are equivalent representations	<b>Sample problem:</b> Given the vertex form $f(x) = 3(x - 1)^2 + 4$ , express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.	Nelson 10 (4.3) AW10 (6.4)

Ν	С	QF2.08	express the equation of a quadratic function in the vertex form $f(x) = a(x - h)^2$ + $k$ , given the standard form $f(x) = ax^2 + bx + c$ by completing the square (e.g., using algebra tiles or diagrams; algebraically), including cases where $\frac{b}{a}$ is a simple rational number (e.g., $\frac{1}{2}$ , 0.75), and <u>verify</u> , using graphing technology, that these forms are equivalent representations;		Nelson 10 (4.6) McGraw 10 (4.4) AW10 (6.4) Nelson 11 (4.1) McGraw 11 (2.3) AW 11 (pg 198-199)
Ν	N	QF2.10 →	describe the information (e.g., maximum, intercepts) that can be obtained by inspecting the standard form $f(x) = ax^2 + bx + c$ , the vertex form $f(x) = a(x - h)^2 + k$ , and the factored form $f(x) = a(x - r)(x - s)$ of a quadratic function;		Nelson 10 (3.4, 3.7, 4.2) McGraw 10 (4.3, 4.4) AW 10 (6.2, 6.4) Nelson 11 (4.3) AW 11 (4.1)
Z	R	QF2.11 →	sketch the graph of a quadratic function whose equation is given in the standard form $f(x) = ax^2 + bx + c$ by using a suitable strategy (e.g., completing the square and finding the vertex; factoring, if possible, to locate the <i>x</i> -intercepts),and identify the key features of the graph (e.g., the vertex, the <i>x</i> - and <i>y</i> -intercepts, the equation of the axis of symmetry, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing).		Nelson 10 (4.3) McGraw 10 (4.3, 4.4) AW10 (6.4, 6.5) Nelson 11 (4.3)
Ν	R	QF1.06 → *	explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development, with technology, such as computer algebra systems, or without technology [student reproduction of the development of the general case is not required]), and apply the formula to solve quadratic equations, using technology;	- numerical example and show algebraic side by side	Nelson 10 (3.4, 4.7) McGraw 10 (5.4) AW10 (5.8) Nelson 11 (4.3) McGraw 11 (2.3) AW11 (4.2, pg 219)
N	N	QF1.07 →	relate the real roots of a quadratic equation to the x-intercept(s) of the corresponding graph, and connect the number of real roots to the value of the discriminant (e.g., there are no real roots and no x-intercepts if $b^2 - 4ac < 0$ );		Nelson 10 (3.4, 4.7page 316-317) Nelson 11 (4.3) McGraw FM11 (6.8)

N	С	QF1.08 →	determine the real roots of a variety of quadratic equations (e.g., $100x^2 = 115x + 35$ ), and describe the advantages and disadvantages of each strategy (i.e., graphing; factoring; using the quadratic formula)	<b>Sample problem:</b> Generate 10 quadratic equations by randomly selecting integer values for <i>a</i> , <i>b</i> , and <i>c</i> in $ax^2 + bx + c = 0$ . Solve the equations using the quadratic formula. How many of the equations could you solve by factoring?).	Nelson 10 (3.4, 3.7) Nelson 11 (4.3) McGraw 11 (2.3)
С	С	QF3.01 → *	collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	<b>Sample problem:</b> When a 3 x 3 x 3 cube made up of 1 x 1 x 1 cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.)	Nelson 10 (4.8) McGraw 10 (4.8) AW10 (4.2, 4.3)
N	N	QF3.02 → *	determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the <i>x</i> -intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator);		Nelson 10 (3.4, 3.7, 4.2) McGraw 10 (4.8) AW10 (4.2, 4.3)
С	С	QF3.03 →	solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement)	<b>Sample problem:</b> In a DC electrical circuit, the relationship between the power used by a device, <i>P</i> (in watts,W), the electric potential difference (voltage), <i>V</i> (in volts,V), the current, <i>I</i> (in amperes, A), and the resistance, <i>R</i> (in ohms, $\Omega$ ), is represented by the formula <i>P</i> = $IV - I^2 R$ . Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 $\Omega$ . Determine the current needed in order for the device to use the maximum amount of power.].	Nelson 10 (embedded in chp 3, 4) McGraw 10 (4.2-4.4) AW10 (6.5) McGraw 11 (2.3) AW11 (4.1, 4.2)

Unit	Unit 5: Exponential Functions (9 days + 1 jazz day + 1 summative evaluation day)								
BIG Ideas: • • • • • • • • • • • • •									
DAY	10 P	10 D	Expectat	tions	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References			
	N	Ν	EF1.06 → ★	distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth)	<b>Sample problem:</b> Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$ .	Nelson 10 (3.2) McGraw – 11C book (6.3) Peter Taylor document - Growth and Change			
	N	N	EF2.01 → *	collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data	<b>Sample problem:</b> Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.)	-spilling dice experiment from ministry grade 11 training -m&m's - Peter Taylor :Growth and Change -cooling curve ? (Yeager) -check STATS can for lessons/data			
	N	N	EF1.03 → *	graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ ( $a > 0, a \neq 1$ ), define this relation as the function $f(x) = a^x$ , and explain why it is a function;	Suggestion: may want to define exponential as $y = a^*b^x$ so that students are always seeing "a" as the vertical stretch. You may want to graph $y = 2^x$ , $y = 3^*(2^x)$ and then $y = 6^x$ , to show that "3" is a multiplier which results in a vertical stretch and to show that $3^*2^x$ is not the same as $6^x$ .	McGraw FM12 (7.1) AW – MCT 12 book			

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N	Ν	EF1.04 → ★	determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x$ ( $a > 0, a \neq 1$ ), function machines]	<b>Sample problem:</b> Graph $f(x) = 2^x$ , $g(x) = 3^x$ , and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the <i>y</i> -intercepts.	McGraw FM12 book – 7.1 AW MCT 12 book
N	C (some in grade 9)	EF1.05 ➡ ★	determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g., $(\frac{1}{2})^3 \times (\frac{1}{2})^2$ ], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g., $(5^3)^2$ ], and use the rules to simplify numerical expressions containing integer exponents [e.g., $(2^3)(2^5) = 2^8$ ];	Note: students don't actually solve exponential equations in this course so the main use of these exponent rules would likely be to help develop an understanding of rational exponents (see sample problem below) and to understand the compound interest formula	Nelson 11 (1.10) McGraw 11 (1.1, 1.2) AW 11 (1.6)
N	Ν	EF1.01 ₽ ★	determine, through investigation using a <u>variety of tools</u> (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$ , where $x > 0$ and $m$ and $n$ are integers)	<b>Sample problem:</b> The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{1}$ . What value would you assign to $4^{\frac{1}{2}}$ ? What value would you assign to $27^{\frac{1}{3}}$ ? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$ , where $x > 0$ and $n$ is a natural number.	Nelson 11 (1.10) McGrw 11 (1.1, 1.2) AW 11 (1.6) Harcourt 11 AW MCT 12 book
N	C (have seen zero and integer)	EF1.02	evaluate, with and without technology, numerical expressions containing integer and rational exponents and rational bases [e.g., 2 <sup>-3</sup> , (-6) <sup>3</sup> , $4^{\frac{1}{2}}$ , 1.01 <sup>120</sup> ];	- may want to explore on sketchpad. Students can graph $y = 4^x$ and then examine the y-value when $x = \frac{1}{2}$ and then graph $y = 9^x$ and examine the y-value when $x = \frac{1}{2}$	Nelson 11 (1.9) McGraw11 (1.1, 1.2) AW 11 (1.5) Harcourt 11 AW MCT 12 book
N	Ζ	EF2.02 → ★	identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve);		Nelson 11 (1.11) McGraw FM12 (7.2) McGraw 11C book (6.4) AW 11 (1.2) -Peter Taylor: Growth and Change

	N	N	EF2.03 → ★	solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations	<b>Sample problem:</b> The temperature of a cooling liquid over time can be modelled by the exponential function $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$ , where $T(x)$ is the temperature, in degrees Celsius, and $x$ is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28°C.	AW 11 (3.1) McGraw FM12 (7.2) McGraw 11C book (6.4) AW – MCA 12U book uses a graphical approach to solving exponentials
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## Unit 6: Financial Applications of Exponential Functions (9 days + 1 jazz day + 1 summative evaluation day)

## **BIG Ideas:**

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#### Notes to writers (delete in final version)

- Important to make the connection to exponentials by graphingGreater focus on using technology than in the current course

DAY	10 P	10 D	Expect	tations	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References
	N	N	EF3.01 →	compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time (;	<b>Sample problem:</b> Compare, using tables of values and graphs, the amounts after each of the first five years for a \$1000 investment at 5% simple interest per annum and a \$1000 investment at 5% interest per annum, compounded annually.	Nelson 11 (1.6, 1.8) McGraw 11 (7.1-7.1) AW 11 (Pg 128 plus 3.1, 3.2) McGraw 11C book
	N	N	EF3.02	solve problems, using a scientific calculator, that involve the calculation of the amount, <i>A</i> (also referred to as future value, <i>FV</i> ), and the principal, <i>P</i> (also referred to as present value, <i>PV</i> ), using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV=PV(1+i)^n$ ]	<b>Sample problem:</b> Calculate the amount if \$1000 is invested for three years at 6% per annum, compounded quarterly.	Nelson 11 (1.8) McGraw 11 (7.1-7.7) AW 11 (3.1, 3.2) McGraw 11C book – chapter 3, 5
	N	N	EF3.03 → *	determine, through investigation (e.g., using spreadsheets and graphs), that compound interest is an example of exponential growth [e.g., the formulas for compound interest, $A = P(1 + i)^n$ , and present value, $PV = A(1 + i)^{-n}$ , are exponential functions, where the number of compounding periods, $n$ , varies]	<b>Sample problem:</b> Describe an investment that could be represented by the function $f(x) = 500(1.01)^{x}$ .	Nelson 11 (1.8) McGraw 11 (7.1-7.7) AW 11 (3.6) McGraw 11C book – Chapter 3,5
	N	N	EF3.04	solve problems, using a TVM Solver in a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period, <i>i</i> , or the number of compounding periods, <i>n</i> , in the compound interest formula $A = P(1 + i)^n$ [or $FV=PV(1+i)^n$ ]	<b>Sample problem:</b> Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.	Nelson 11 (1.8) McGraw 11 (7.1-7.7) AW 11 (3.6) McGraw 11C book – Chapter 3,5

Ν	Ν	EF3.05 →	explain the meaning of the term <i>annuity</i> , through investigation of numerical and graphical representations using technology;		Nelson 11 (1.8) McGraw 11 (7.1-7.7) AW 11 (3.3, 3.4) McGraw 11C book – chapter 3,5
Ν	Ν	EF3.07	solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity <u>in situations where the</u> <u>compounding period and the payment period</u> <u>are the same</u> (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).		Nelson 11 (2.4, 2.5, 2.9, 2.11, 2.12) McGraw 11 (7.1-7.7) AW 11 (3.6, 3.7) McGraw 11C book – chapter 3,5
Ν	Ν	EF3.06 ₽ <b>*</b>	determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities <u>in situations where the compounding period and the payment period</u> <u>are the same</u> (e.g., long-term savings plans, loans)	<b>Sample problem:</b> Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?);	Nelson 11 (2.4, 2.5, 2.7, 2.8) McGraw 11 (7.1-7.7) AW 11 (3.6) McGraw 11C book – chapter 3,5

Unit 7: Acute Triangle Trigonometry (5 days + 1 jazz day + 1 summative evaluation day)								
BIG Ideas: • •								
DAY	10 P	10 D	Expectat	ions	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References		
	R	R	TF1.01 →	solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;		Nelson 10 (5.7) McGraw 10 (6.6, 6.7) AW 10 (8.3) Nelson 11 McGraw11 (4.1) AW 11 (pg. 246-248)		
	Ν	Ν	TF1.02 (no 3D) ➡	solve problems involving two right triangles in <u>two dimensions</u>	<b>Sample problem:</b> A helicopter hovers 500 m above a long straight road. Ahead of the helicopter on the road are two trucks. The angles of depression of the two trucks from the helicopter are 60° and 20°. How far apart are the two trucks?	Nelson 10 (5.8, 6.2) McGraw 10 (6.6, 6.7) AW 10 (8.5, 8.6) Nelson 11 McGraw 11 (4.1)		
	Ν	R	TF1.03 →	verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios , $\frac{a}{\sin A}$ , $\frac{b}{\sin B}$ , and $\frac{c}{\sin C}$ , and in triangle ABC while dragging one of the vertices);	- possibly provide differentiated instructions here – pose a very general investigation for classes that have experience with sketchpad and provide more guided instruction for students who have less experience with the program	AW 10 (8.7) AW 11 Grade 10 academic course profile		
	Ν	Ν	TF1.04 ➔	describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;		Nelson 10 (6.3, 6.4, 6.6, 6.7) McGraw 10 (6.9, 6.10) AW10 (8.,8, 8.9) Nelson 11 (6.1) McGraw 11 (4.3) AW 11 (pg 248-251)		

	Ν	С	TF1.05 (no amb. Case) ➡	solve problems that require <u>the use of the</u> <u>sine law or the cosine law in acute</u> <u>triangles</u> , including problems arising from real-world applications (e.g., surveying; navigation; building construction).		Nelson 10 (6.5) McGraw 10 (6.9, 6.10) AW 10 (8.8, 8.9) Nelson 11 (6.2) McGraw 11 (4.3)
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## Unit 8: Trigonometric Functions (9 days + 1 jazz day + 1 summative evaluation day)

## **BIG Ideas:**

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#### Notes to writers (delete in final version)

tides, ferris wheel, hoola hoop with CBR, sunrise/sunset, biorythms, swing, pendulum

DAY	10 P	10 D	Expec	tations	Teaching/Assessment Notes and Curriculum Sample Problems	Resource References
	N	N	TF3.01 → ★	collect data that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g.,websites such as Statistics Canada, E- STAT), and graph the data	<b>Sample problem:</b> Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.	Nelson 11 (5.1) AW 11 (6.1)
	N	N	TF2.01 *	describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numerical or graphical representation;		Nelson 11 (5.1) McGraw 11 (5.3)
	N	N	TF2.02 ➔	predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural-gas consumption in Ontario from previous consumption);		Nelson 11 McGraw 11 (5.3) AW 11(6.1)

N	N	TF2.03 → ★	make connections between the sine ratio and the sine function by graphing the relationship between angles from 0° to 360° and the corresponding sine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$ , and explaining why it is a function;	Note: these students will not have seen trig ratios with angles greater than 90° so this will take some work Should just be creating a table of values using the calculator	***** <i>this lesson will need to be developed """""</i> Nelson 11 (5.5, 5.6) McGraw 11 (5.4) AW 11 (6.2)
N	N	TF2.04	sketch the graph of $f(x) = \sin x$ for angle measures expressed in degrees, and determine and describe its key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals);		Nelson 11 (5.5, 5.6) McGraw 11 (5.4) AW 11 (6.2)
Ν	N	TF2.05 ➔	make connections, through investigation with technology, between changes in a real-world situation that can be modelled using a periodic function and transformations of the corresponding graph (e.g., investigating the connection between variables for a swimmer swimming lengths of a pool and transformations of the graph of distance from the starting point versus time)	<b>Sample problem:</b> Generate a sine curve by walking a circle of two-metre diameter in front of a motion sensor. Describe how the following changes in the motion change the graph: starting at a different point on the circle; starting a greater distance from the motion sensor; changing direction; increasing the radius of the circle; and increasing the speed	Ferris Wheel activity (Yeager) Hoola Hoop (GUM project) McGraw 11
Ν	N	TF2.06	determine, through investigation using technology, and describe the roles of the parameters <i>a</i> , <i>c</i> , and <i>d</i> in functions in the form $f(x) = a \sin x$ , $f(x) = \sin x + c$ , and $f(x)$ $= \sin(x - d)$ in terms of transformations on the graph of $f(x) = \sin x$ with angles expressed in degrees (i.e., translations; reflections in the <i>x</i> -axis; vertical stretches and compressions);		Nelson 11 (5.5, 5.6) McGraw 11(5.5, 5.6) AW11 (6.3, 6.4)
N	N	TF2.07	sketch graphs of $f(x) = a \sin x$ , $f(x) = \sin x + c$ , and $f(x) = \sin(x - d)$ by applying transformations to the graph of $f(x) = \sin x$ , and state the domain and range of the transformed functions (note: only 1 transformation at a time)	<b>Sample problem:</b> Transform the graph of $f(x)$ = sinx to sketch the graphs of $g(x) = -2\sin x$ and $h(x) = \sin(x - 180^\circ)$ , and state the domain and range of each function	Nelson 11 (5.5, 5.6) McGraw 11(5.5, 5.6) AW11 (6.3, 6.4)
N	N	TF3.02 → *	identify sine functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range;		Nelson 11 (5.7) McGraw 11 (5.6) AW 11 (6.5)

	Ν	Ν	TF3.03 ₽ *	pose and solve problems based on applications involving a sine function by using a <u>given graph or a graph generated</u> with technology from its equation	<b>Sample problem:</b> The height above the ground of a rider on a Ferris wheel can be modelled by the sine function $h(x) = 25 \sin(x - 90^{\circ}) + 27$ , where $h(x)$ is the height, in metres, and $x$ is the angle, in degrees, that the radius to the rider makes with the horizontal. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, and the measures of the angle when the height of the rider is 40 m.	Nelson 11 (5.7) McGraw 11 (5.6) AW 11 (6.5) Ferris Wheel (Yeager)
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Summative - REVIEW and TASK (4 days)			
	Course Review	2 additional days	
	Course Summative Performance Task	2 additional days	
	Examination	As per school exam schedule	