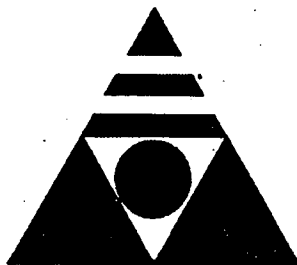




Ontario Math Olympics



THE ONTARIO ASSOCIATION
FOR MATHEMATICS EDUCATION

1996 - 1997



ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

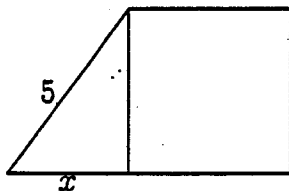
NAME _____

CHAPTER TEAM _____

Answers

1. A ball bounces $\frac{1}{3}$ of the height from which it is dropped. If it reaches a height of 3 units on the third bounce, from what height was it originally dropped? _____

2. The area of the square is 16 units². Determine the value of x .



3. Determine three whole numbers whose sum is equal to their product. _____

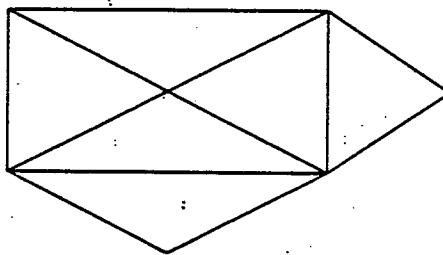
GO TO HURDLE NUMBER 1

ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

CHAPTER TEAM _____

Use the enclosed string to form the figure below without cutting the string, or retracing any line.



WHEN YOU HAVE COMPLETED THIS QUESTION RETURN TO YOUR SEATS FOR NUMBERS 4, 5, AND 6.

Task completed correctly

MARSHALL

ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

NAME _____

CHAPTER TEAM _____

Answers

4. Evaluate.

$$\sqrt{13 + \sqrt{8 + \sqrt{3 - \sqrt{4}}}}$$

5. Evaluate.

$$3(2^4 - 11) \times 2^2 - 4$$

6. A bowl contains 75 red smarties and 60 blue smarties. If 20 blue smarties are added to the bowl, determine the number of red smarties that must be added to make the ratio of red to blue equal to 3:2.

GO TO HURDLE NUMBER 2

ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

CHAPTER TEAM _____

Evaluate.

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{97 \times 98} + \frac{1}{98 \times 99}$$

Answer

WHEN YOU HAVE COMPLETED THIS QUESTION RETURN TO YOUR SEATS FOR NUMBERS 7, 8, AND 9.

ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

NAME _____

CHAPTER TEAM _____

Answers

7. There are 24 vertical fence posts enclosing a rectangular field. The posts are evenly spaced and 2 m apart. There are 8 posts along the length. Determine the number of square metres in the area of the field.

8. If $a = 6.3$ and $b = 7.2$, determine the value of $a - \left(\frac{a}{b}\right) \left(\frac{b}{a}\right)$.

9. A circus clown buys balloons at \$1.44 per dozen and sells them for \$0.25 each. Determine his profit on the day 40 dozen balloons are sold.

GO TO HURDLE NUMBER 3

ONTARIO MATHEMATICS OLYMPICS
1997

MATHALON

OBJECTIVE #1	OBJECTIVE #2
OBJECTIVE #3	OBJECTIVE #4
OBJECTIVE #5	OBJECTIVE #6
OBJECTIVE #7	OBJECTIVE #8

ONTARIO MATHEMATICS OLYMPICS
1997

HURDLES

CHAPTER TEAM _____

A dog is tied to the corner of a house that is 10 m long and 5 m wide. His leash is 7 m long. Determine, in square metres the largest area over which the dog may roam.

Answer

TIME

ONTARIO MATHEMATICS OLYMPICS
1997

MATHALON

OBJECTIVE #9	OBJECTIVE #10
OBJECTIVE #11	OBJECTIVE #12
OBJECTIVE #13	OBJECTIVE #14
OBJECTIVE #15	OBJECTIVE #16

ONTARIO MATHEMATICS OLYMPICS

1997

1500 m

NAME _____

CHAPTER TEAM _____

Note: This is a timed event.

Answers

1. A city has a large reservoir which stores water for emergency use. The reservoir is a rectangular prism 50 m long, 30 m wide and 10 m high. During an emergency the full reservoir is emptied in 9 h. Assuming the water flows out at a constant rate, determine the rate of flow of the water in litres per second. One litre of water occupies 1000 cm^3 . _____
2. On a digital clock showing hours and minutes, how many different readings between midnight and 4 a.m. contain at least two 1's? _____
3. How many more complete turns will a wheel of radius 12 cm make than a wheel of radius 15 cm in travelling 1 km? _____
4. Bill bought a number of golf balls in a store where a 5% sales tax is added to every purchase. If he did not have to pay the tax he could have bought three more golf balls for the same amount of money. How many golf balls did he buy? _____
5. The mean of 13 whole numbers is 320. Eleven of them are 300, 320, 199, 175, 325, 520, 156, 225, 326, 421 and 504. The median of these 13 numbers is 325. Determine the remaining two numbers if it is known that these two have the greatest possible difference. _____
6. The ratio of red marbles to green marbles in a bag is 3 to 5. If 150 green marbles are in the bag, what is the smallest number of marbles that you have to add to the bag so that the ratio of red marbles to green marbles becomes 1 to 2. _____
7. In a right angled triangle it is known that the lengths of the two largest sides are 48 cm and 50 cm. Determine the number of square centimetres in the area of the triangle. _____

8. Susan drove from Midhurst to Stayner at 90 km/h. She drove back from Stayner to Midhurst at 60 km/h. What was her average speed, in kilometres per hour, for the entire trip?

9. A friend has 4 different tops, 3 different pairs of shorts, and 2 different pairs of sandals. How many different outfits can your friend make, assuming that an outfit consists of a top, a pair of shorts and a pair of sandals?

10. A rectangular parking lot is 80 m by 50 m. How much less, in square metres, is its area than that of a square parking lot with the same perimeter?

11. If A represents the answer to question 9, and R represents the answer to question 10, determine

$$\frac{\sqrt{A \times R \times \frac{1}{6}}}{3}$$

12. Let B represent the answer to question 11. Determine the square root of B^B .

13. How many times will the LED display of a digital clock show the correct time on June 7, if it is turned upside down? (The digital display is of the type *number : number*).

14. What is half of 2^{30} ?

15. If x and y are positive integers, $x \leq 5$, how many solutions are there to the equation $x^y = y^x$?

ONTARIO MATHEMATICS OLYMPICS

1997

Team Problem Solving

NAME _____

CHAPTER TEAM _____

Note: This is a timed event

As a team you must remove the ring without removing the string from your wrist.

If you break the string or become entangled in your solution the marshall will stop your team and reset the problem to its original state.

Task completed correctly

MARSHALL

TIME

ONTARIO MATHEMATICS OLYMPICS
1997

RELAY

CHAPTER _____

Answers

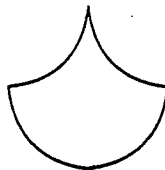
Note: This is a timed event.

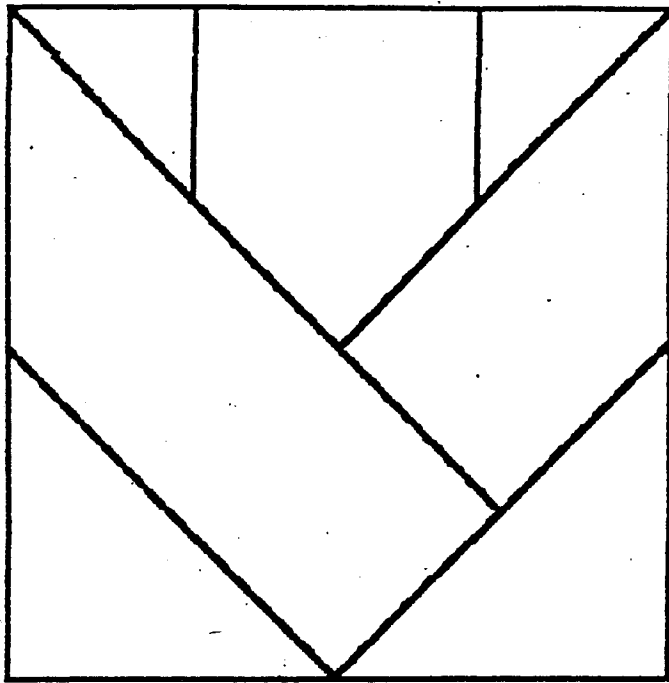
1. A merchant buys some oranges at 3 for \$0.58 and twice as many at 5 for \$1.17. At what price per dozen (to the nearest cent) must the merchant sell all the oranges to just cover the cost?

2. A ball player has had 322 "at bats" this season and has an average of 0.289. (The batting average is calculated by dividing the number of hits by the number of "at bats" and rounding the answer to 3 decimal places.) She has 53 more "at bats" to complete the season. What batting average must she attain in the remaining games, in order to achieve a 0.300 batting average for the entire season?

3. How many different combinations of stamps can you use for an exact postage on a package of \$4.95 if you only have \$0.45 and \$0.20 stamps?

4. An area is enclosed by 4 quarter circle arcs with radius 10, two of which are inverted, as shown below. Calculate the number of square units in the enclosed area. (If you wish, you may use $\pi = 3.14$.)





Used for the Math Olympics
June 1997

SOLUTIONS

Hurdles

1) 81 2) 3 3) 1,2,3 4) 4 5) 56 6) 45 Hurdle 2 98/99

7) 140 8) 5.3 9) \$62.40 Hurdle 3

1500 m

1) 463 L / s 2) 45 3) 265 4) 60 5) 325 and 364 6) add 30 to green
7) 336 8) 72 9) 24 10) 225 11) 10 12) 10 to the exponent 5
13) 6 or 7 14) 2 to the exponent 29 15) 6

Relay

1) \$2.65 2) 0.367 3) 3 4) 200



REGIONAL COMPETITIONS

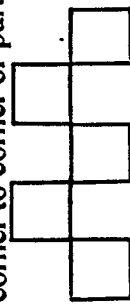


Junior Math Olympics

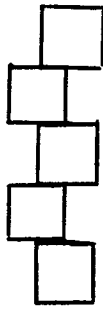
Spatial Sense & Geometry

A domino is two squares joined together. A triomino is three squares joined together. You, however, will be working with pentominos.

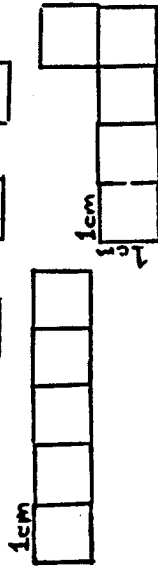
When joining five squares together, they must share complete sides and cannot be joined corner to corner or partial side to partial side.



These are not pentominos:



These are pentominos:

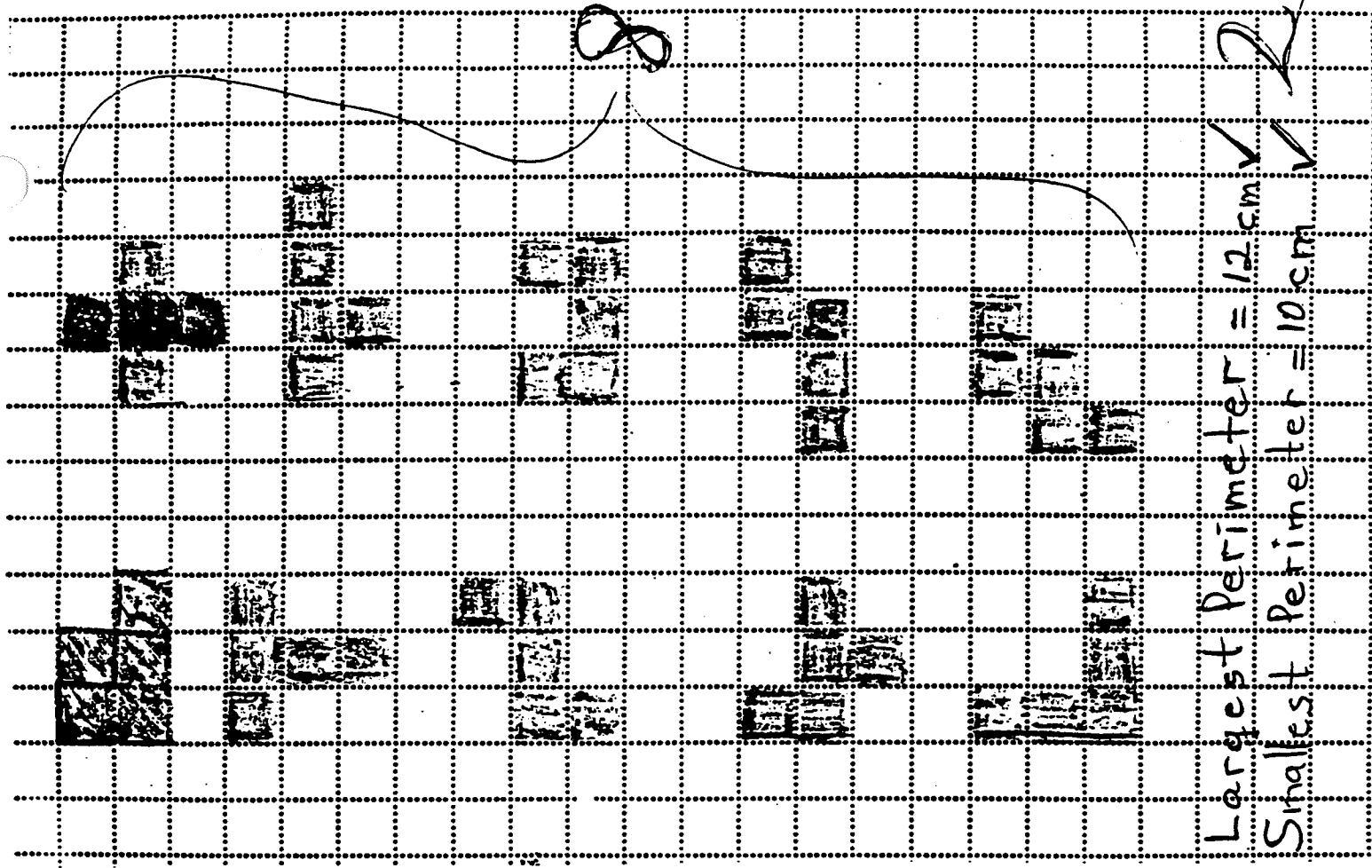


Determine all the possible pentominos. There are more than ten (including the ones shown above), but less than 15. Be careful not to be fooled by arrangements which are rotations or reflections of one another.

Determine the pentomino with the largest perimeter and the smallest perimeter. Draw all your arrangements on the grid provided.

Largest perimeter = _____

Smallest perimeter = _____



Marking Scheme

8 marks if they get the remaining ten pentominoes.

7 marks if they get 8 or 9 more.

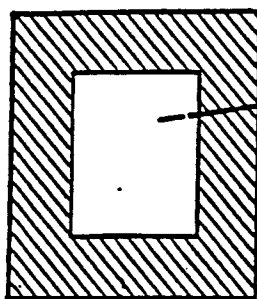
6 marks for six more.

5 marks for five more.

4 marks for four more etc.

Two marks for determining the maximum and minimum perimeter.

Total Ten marks.



MEASUREMENT

matt frame.

1. Create a picture frame matt that satisfies the following criteria:

- a) Area of the matt frame = Area of the picture opening
- b) The outside dimensions of the card must not be reduced
- c) The picture matt must be in one solid piece

2. Answer the following questions in cm to the nearest tenth:

- a) What are the inside dimensions of the frame?
- b) What is the area of the picture opening?
- c) Show two different ways to calculate the area of the frame.

3. Evaluation:

- a) Solutions to the questions in 2.
- b) Accuracy (criteria satisfied)
- c) Quality of the matt (straight edges, sharp corners)

4. Equipment:

- 3 21.5cm x 28cm cards
- 2 rulers
- 1 pair of scissors
- scrap paper

Team Name _____

MEASUREMENT

ASSESSMENT:

2.a) 2 MARKS INSIDE DIMENSIONS OF FRAME CORRECT TO NEAREST TENTH

2.b) 2 MARKS AREA OF PICTURE OPENING CORRECT TO NEAREST TENTH

2.c) 2 MARKS AREA CALCULATIONS (1 MARK EACH METHOD)

3.b) 3 MARKS ACCURACY (1 MARK EACH FOR EQUAL AREAS, OUTSIDE EDGES NOT CHANGED, ONE SOLID FRAME)

3.c) 1 MARK QUALITY (.5 MARK EACH FOR STRAIGHT EDGES, SHARP CORNERS)

TOTAL MARKS: 10

ONTARIO MATHEMATICS OLYMPICS
1997

RELAY

CHAPTER _____

Answers

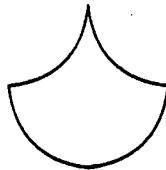
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SOLUTIONS

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1500 m

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REGIONAL COMPETITIONS

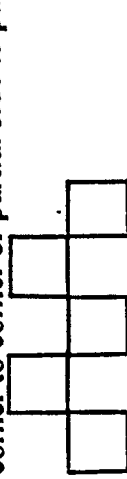


Junior Math Olympics

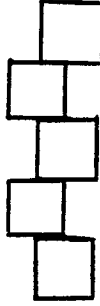
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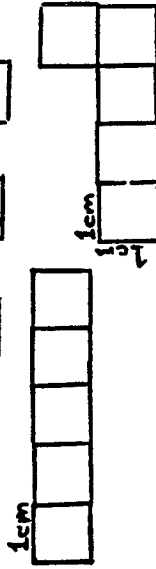
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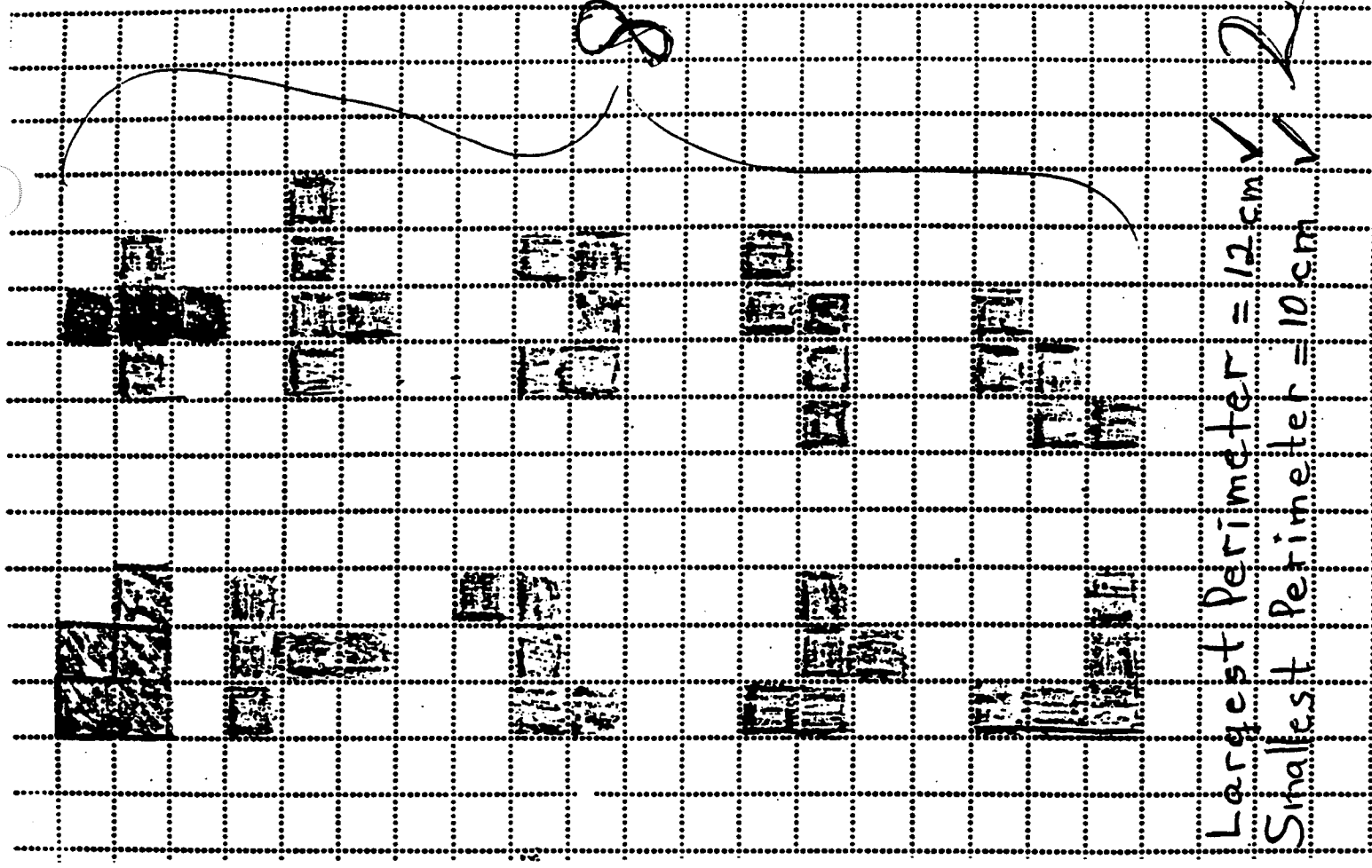


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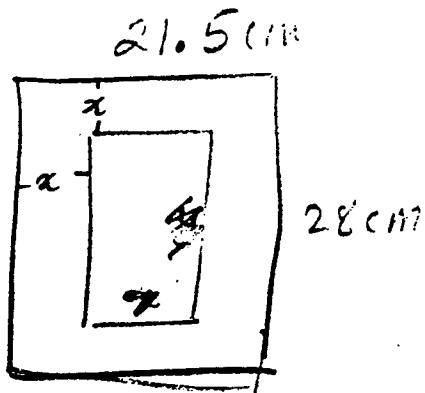
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Total Ten marks.



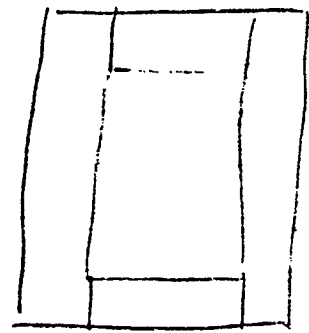
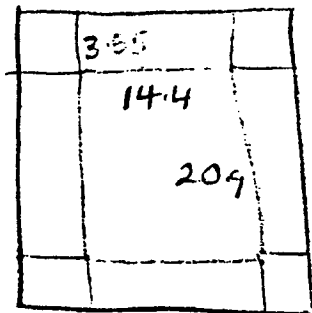
$$w = 21.5 - 2x$$

$$l = 28 - 2x$$

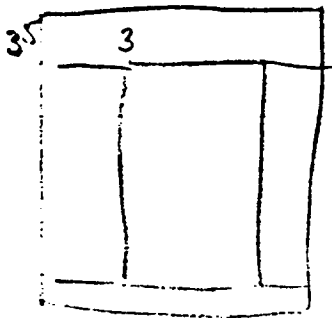
$$A = lw$$

x	w	l	Area
2	17.5	24	420
3	15.5	22	341
4	13.5	20	270
3.5	14.5	21	304.5
3.6	14.3	20.8	297.44
3.55	14.4	20.9	300.96

\therefore inside measurements should be 14.4 cm \times 20.9 cm



$$\textcircled{1} A = 4(3.55)^2 + 2(3.55)(14.4) + 2(3.55)(20.9)$$



$$A = 2 \times 28(3.55) + 2 \times 14.4(3.55)$$

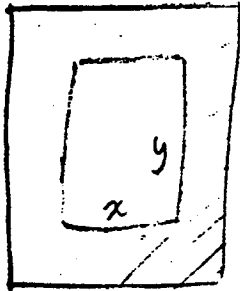
$$\textcircled{2} A = 2 \times (3.5)(21.5) + 2 \times (20.9)(3.55)$$

Measurement

Area of card = $21.5 \times 28 = 602 \text{ cm}^2$

Area of picture

$\therefore \text{opening} = 301$



$A = xy = 301$

$x = \frac{301}{y}$ ①

$\frac{x}{y} = \frac{21.5}{28}$ ② $\Rightarrow x = \frac{21.5y}{28}$

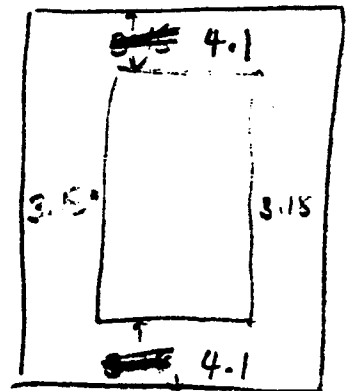
compare ① and ②

$\frac{301}{y} = \frac{21.5y}{28}$

$y^2 = \frac{301(28)}{21.5} = 392$

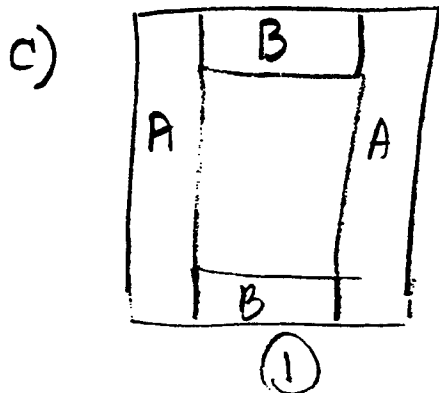
$y \approx 19.8 \text{ cm}$

$\therefore x = 15.2 \text{ cm}$



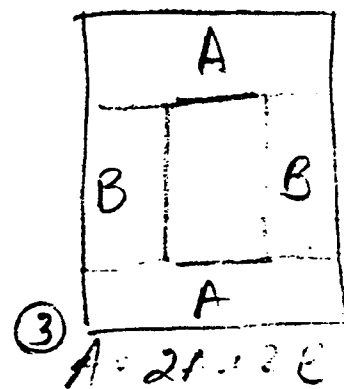
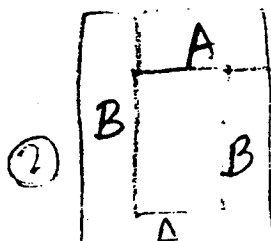
2. a) inside dimensions are $15.2 \text{ cm} \times 19.8 \text{ cm}$

b) $300.96 \text{ cm}^2 \approx 301 \text{ cm}^2$



① Area = $2B + 2A$

② $A = 2A + 2B$



③ $A = 2A + 2B$

DATA MANAGEMENT

Team Name:

THE QUALITIES OF A LEADER

The table shows the results of a survey on the most important qualities of a political leader like the Prime Minister of Canada.

Females	Males	Quality
119	114	Honesty
44	40	Intelligence
61	86	Experience
3	2	Health
5	3	Appearance
8	15	Other

TASK #1

- 1.a) What is the proper type of graph to display all of the information in the table.
Justify your choice of graph.
- b) Construct an appropriate, fully - labelled graph of the information.

TASK #2

Use your graph and/or table to answer the following problems.

2. a) How many people answered the questions from the survey?
- b) What 2 qualities that might be included in the category labelled as other?
3. Suppose you show your graph to the Prime Minister of Canada, what 2 conclusions might he or she make?

ASSIGNMENT OF MARKS QUALITIES

TASK #1

1 MARK selecting a doublebar or two separate circle graphs

1 MARK clear and precise justification of choice

4 MARKS for a **DOUBLE BAR GRAPH**

- 1 mark for the main title
- 0.5 mark for vertical axis title
- 0.5 mark for horizontal axis title
- 1 mark for labelling/naming bars
- 1 mark for bars at correct height

OR

4 MARKS for two separate **CIRCLE GRAPHS**

- 1 mark for the main title
- 1 mark for putting a number or percent on all sectors
- 1 mark for labelling all sectors
- 1 mark for sectors in correct proportion

TASK #2

1 MARK for correct response to problem 2a)

1 MARK for 2 qualities stated for problem 2b) (0.5 marks for each)

2 MARKS for 2 conclusions stated for problem 3.(1 mark each)
(if more than 2 conclusions are provided only 2 marks
are allowed for this problem)

TOTAL MARKS : 10

TEAM MARK: _____

SOLUTIONS TO THE QUALITIES OF A LEADER

1a) Double bar or two separate circle graphs.

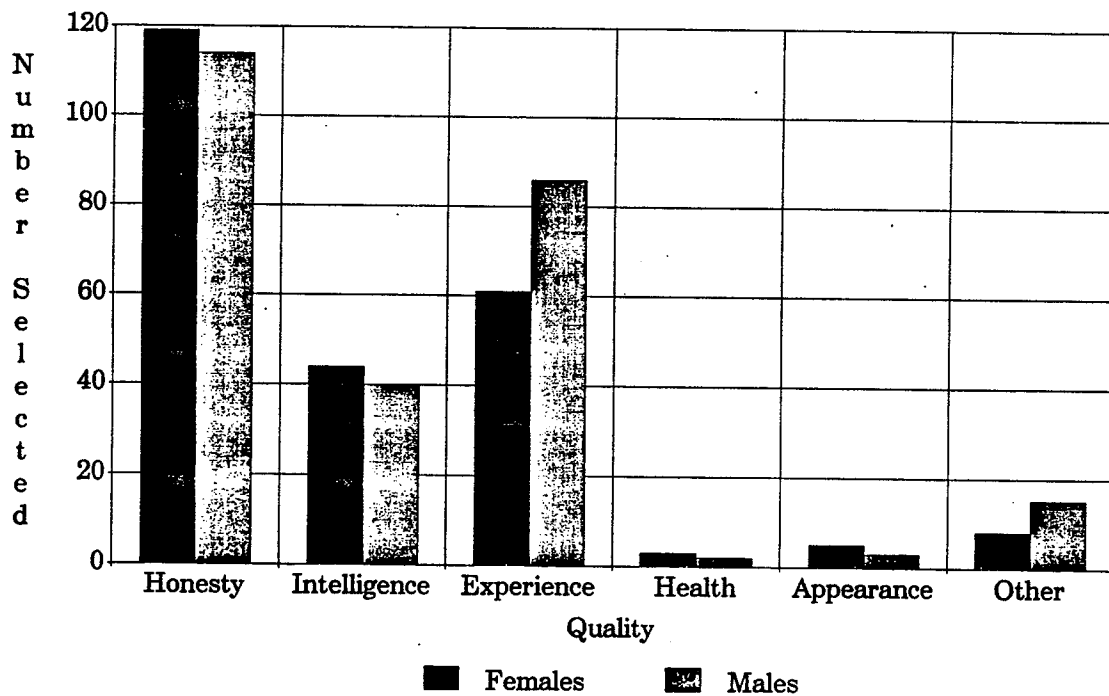
Double bar is used to show comparison of qualities for leader by males and females.

Circle graphs - 1 for females and 1 for males shows the relationship between each quality and all qualities.

1b)

DOUBLE BAR GRAPH

The Qualities of a Leader

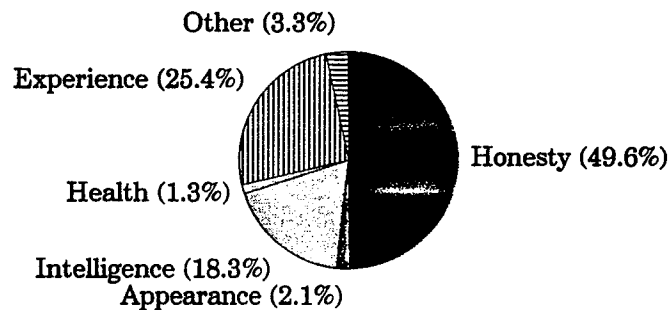


1b)

CIRCLE GRAPHS

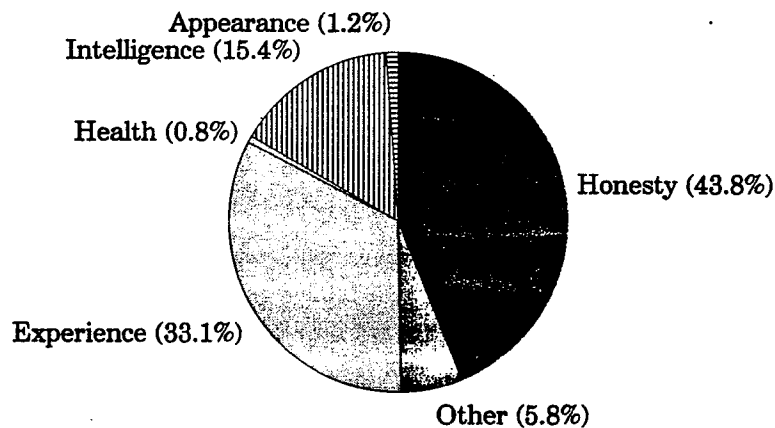
FEMALE RESPONSE

The Qualities of a Leader



MALE RESPONSE

The Qualities of a Leader



- 2a) 500 people answered the questions from the survey.
- 2b) Possible answers: high morals, nice family, well respected...
3. Males and females together feel honesty is the top quality.
Health is the lowest quality for both groups.
Males feel experience is more important of a quality than the females.

Number Sense and Numeration

Team Name:

Given the first 3 diagrams:

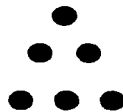
1st



2nd



3rd



(a) Draw the next four diagrams.

4th

5th

6th

7th

(b) (i) How many dots in the 15th diagram?

(ii) How many dots in the 24th diagram?

Number Sense and Numeration

- (c) In your own words describe how you could find the number of dots if you knew the number of the diagram.
- (d) Write an algebraic expression for the number of dots in the "nth" diagram.
- (e) In your own words describe how you could find the number of the diagram if you knew the number of dots.

NUMBER SENSE AND NUMERATION

TEAM NAME:

Supplies: calculator, pen or pencil.

#1 Find the product of each:

$$\begin{array}{r} (i) \quad 13 \\ \times 62 \\ \hline \end{array}$$

$$\begin{array}{r} 31 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{r} (ii) \quad 41 \\ \times 28 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 82 \\ \hline \end{array}$$

#2 What do you notice about the products in each example?

#3 What do you notice about the pairs of numbers in each of the above examples?

#4 Provide two examples of a pair of two-digit numbers that have the same property as in the above examples. Show all your work.

(NOTE: each digit in the two-digit numbers must be DIFFERENT.)
ie. CANNOT give an example such as 11 x 22

Example # 1

Example #2

<OVER>

- #5 Provide two examples of a pair of two-digit numbers that do NOT have the same property as in the examples in # 1. Show all your work.

Example # 1

Example #2

- #6 Explain why the pattern in question #1 works.

- #7 Use your pattern to find the missing digits:

$$\begin{array}{r} 1 \square 3 \\ \times 6 \square 2 \\ \hline 78\ 966 \end{array}$$

$$\begin{array}{r} 3 \square 1 \\ \times 2 \square 6 \\ \hline 78\ 966 \end{array}$$

- #8 (a) Describe the pattern used to find these middle digits in question #7.

- (b) Give one example that has the same pattern as in question #7. (Do NOT use 1, 3 and 6, 2 as the outside digits.)

MEASUREMENT

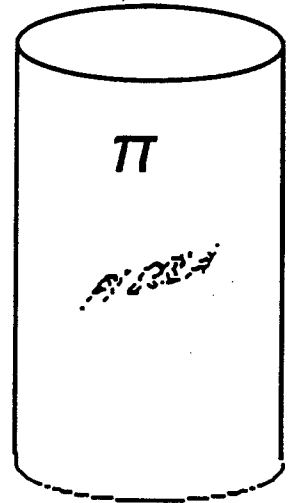
Team Name:

CANS AND CARTONS

Task: Design a carton to hold 12 soft-drink cans
(Design only; do not make)

Materials: soft drink cans, ruler, bristol board,
scissors, tape, calculator, scrap paper, string

Reminder: for full credit, show all
major steps and calculations.
Explain your reasoning!



1. What is the volume of your carton?
2. What is the "wasted" volume (how much empty space is there)?
3. How much cardboard is needed to make the carton?
4. What is the most efficient shape for such a carton? Explain.
5. If you were not restricted to the traditional numbers in a package, like 6, 12 or 24, would some other shape be more efficient?

ASSESSMENT FOR CANS AND CARTONS

1. *What is the volume of your carton?*

1 MARK - measuring dimensions of one can (approximately 6.5 cm x 12.5 cm)

1 MARK - calculating dimensions of chosen arrangement

1 MARK - calculating volume of carton

2. *What is the "wasted" volume (how much empty space is there)?*

1 MARK - calculating volume of 1 can (approx 414 ml. - no less than 355 ml. answers may vary due to irregular shape of can)

1 MARK - calculating difference of volume of carton and volume of 12 cans

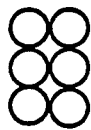
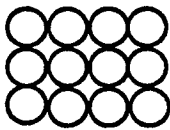
3. *How much cardboard is needed to make the carton?*

1 MARK - calculating surface area of carton

4. *What is the most efficient shape for such a carton? Explain.*

1 MARK - only one efficiency factor considered

1 MARK - two or more factors considered



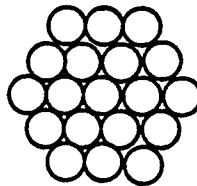
(stacked)

some factors to consider:

wasted space,
amount of cardboard needed,
difficulty of making carton,
packability of cartons, etc.

5. *If you were not restricted to the traditional numbers in a package, like 6, 12 or 24, would some other shape be more efficient?*

2 MARKS - answers will vary; for example, a space-efficient packing might nest cans more efficiently; 2 marks if a hexagonal arrangement for 7, 19 etc. cans



SPATIAL SENSE AND GEOMETRY

Team Name:

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TASK:

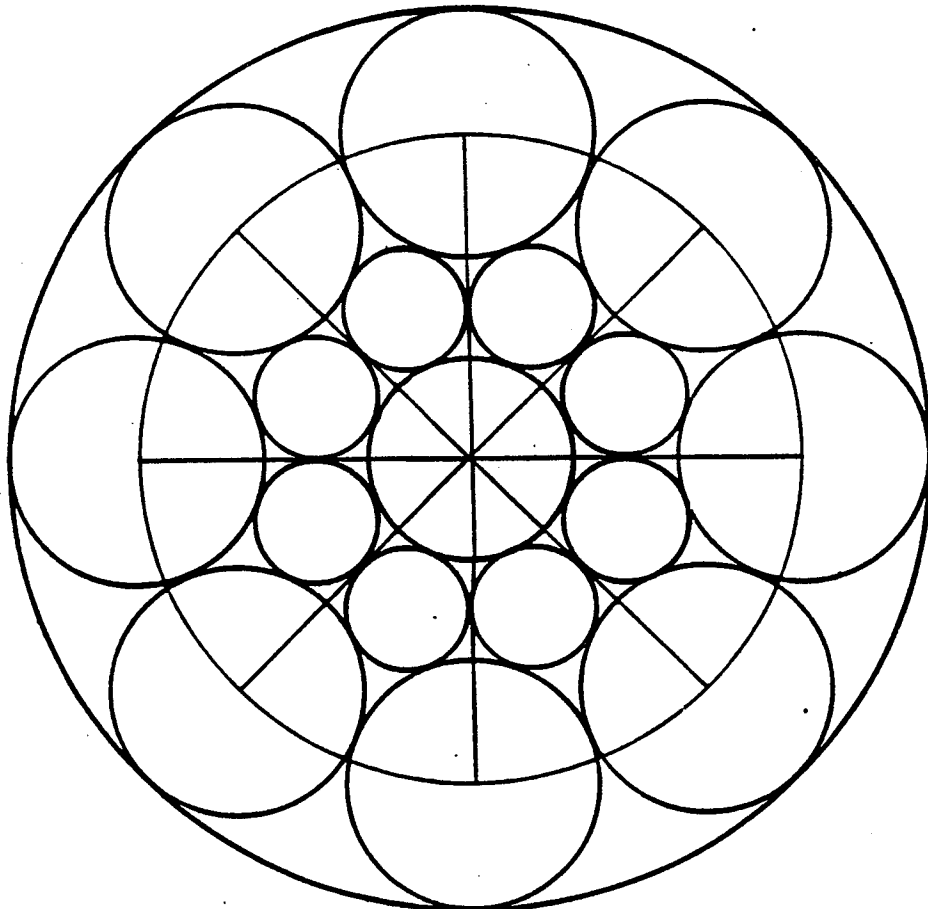
- Using the equipment provided, recreate this design on 8½ by 11 paper: your construction lines do not need to be erased. Your copy must have a different scale than the example given. (It **must not** be the same size). Hand in this sheet with your rough work and your final design.

EQUIPMENT:

Geometry set, mira, tracing paper, string, rough paper, 2 pieces of 8½ by 11 paper.

EVALUATION:

Marks will be given for changing the size, accuracy of the circles (edges touching not overlapping), neatness, scale (although different than the original it should be in the same ratio and completion (part marks will be given if you are able to recreate only some portions of the design)



SPATIAL SENSE AND GEOMETRY

EVALUATION:
MARKS

DESCRIPTOR

- | | |
|---|--|
| 1 | size has been changed |
| 3 | accuracy: the outer ring of 8 circles are tangent
the middle ring of 8 circles are tangent
the lines meet at 45° angles |
| 2 | neatness: construction lines do not need to be erased
circles have been constructed with a compass, lines with a ruler etc. |
| 2 | scale: the radius of the inner circle is the same as the space between
the outer ring of circles and the inner circle
the diameter of the circles in the middle ring is the same as the
radius of the circles in the outer ring |
| 2 | completion: whole diagram completed
part marks will be given for partially completed diagrams
(rough work will be considered) |

