Ontario

A Guide to Effective Instruction in Mathematics

Kindergarten to Grade 6

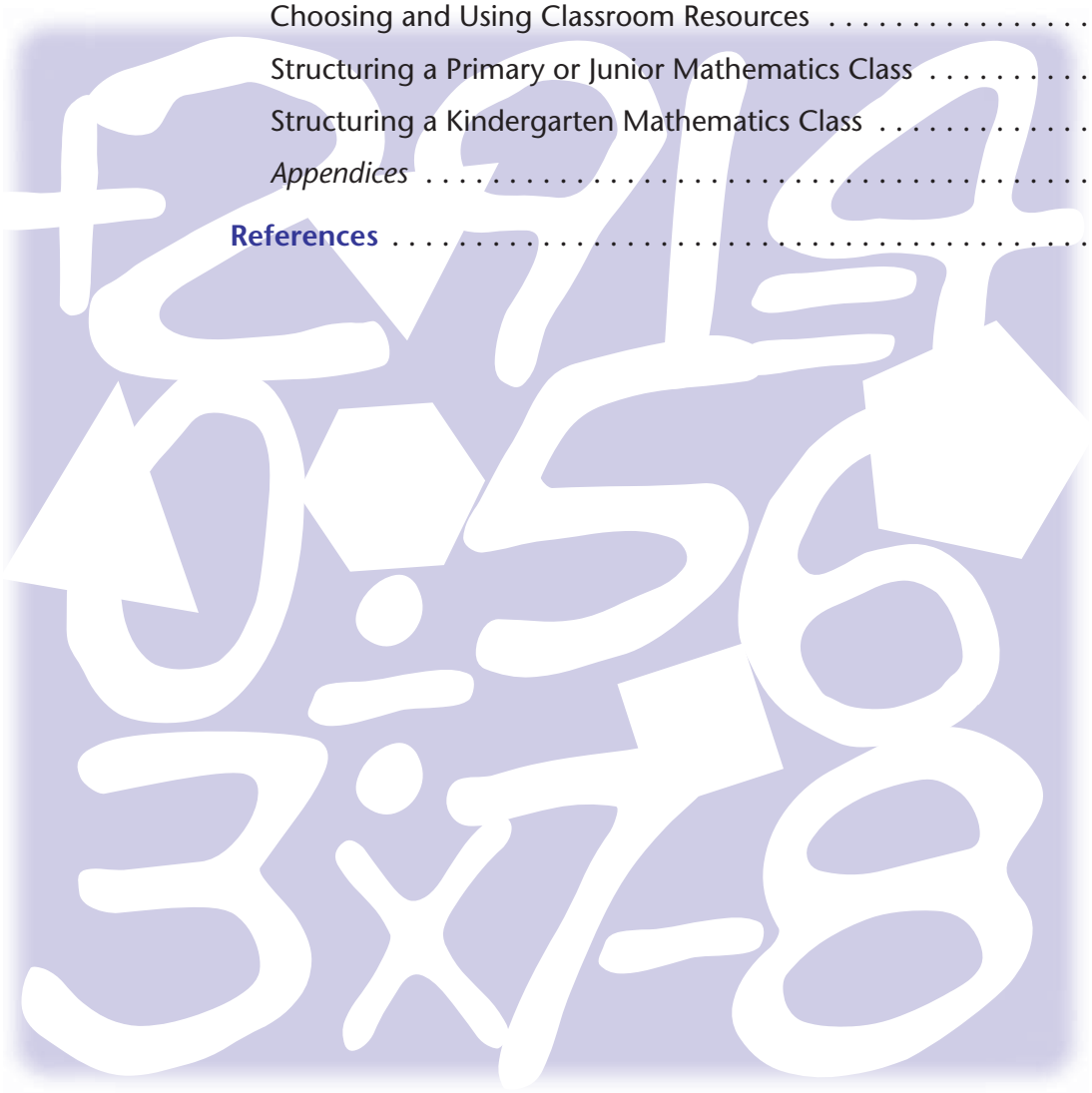
A Resource in Five Volumes
from the Ministry of Education

Volume Three
Classrooom Resources and Management

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

Une publication équivalente est disponible en français sous le titre suivant :
Guide d'enseignement efficace des mathématiques, de la maternelle à la 6e année.

Contents



Introduction

This is Volume Three of the five-volume reference guide *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6*. This volume contains Chapter 7. Chapter 7: Classroom Resources and Management provides an overview of the components of an effective learning environment, including the development of a community of mathematical learners, effective timetabling, a physical arrangement of the classroom that supports various instructional strategies, and the effective management of manipulatives. (See the Introduction of Volume One for a summary of the organization and contents of the complete five-volume guide.)

A list of suggested professional resources for teachers and administrators is included in Volume One. It is meant to provide useful suggestions, but should not be considered comprehensive. A glossary of terms used throughout the guide is also provided at the end of Volume One. References are listed at the end of each individual volume.

This guide contains a wide variety of forms and blackline masters, often provided in appendices, that teachers can use in the classroom. Electronic versions of all of these materials can be found at www.eworkshop.on.ca. These electronic forms and blackline masters are in a Word format that can be modified by teachers to accommodate the needs of their students.

Go to www.eworkshop.on.ca for electronic versions of the forms and blackline masters provided throughout this guide. They can be modified to meet classroom needs.

Locating Information Specific to Kindergarten, Primary, and Junior Students in This Guide

An important feature of this guide is the inclusion of grade-related information and examples that help clarify the principles articulated. Such information is identified in the margins of this guide by means of icons referring to the relevant grades – K for Kindergarten, Grades 1–3 for primary, Grades 4–6 for junior. Examples and other materials that are appropriate for use at more than one level or are applicable to more than one level are identified by the appropriate combination of icons.

K – Kindergarten
1–3 – Primary
4–6 – Junior

Look for the following icons in the margins of this document:

K 1-3 4-6

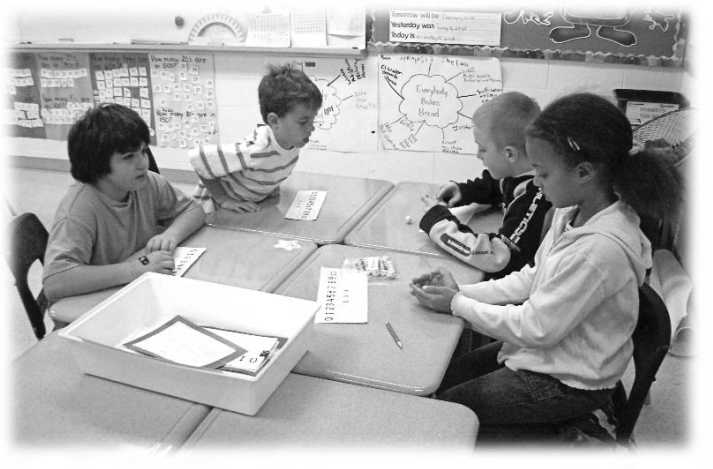
7. Classroom Resources and Management

Chapter Contents

Classroom Resources and Management

Rich learning experiences result from organized, systematic, and creative planning by a knowledgeable and effective teacher. The teacher needs to set the stage for learning by considering the diverse needs of students, creating a warm and inviting climate for learning within the classroom, and ensuring that there is a coherent and comprehensive program of mathematics instruction that develops logically over the course of the day, week, and year. This section will focus on guidelines for "setting the stage" for learning. The types of decisions that need to be made about planning and organization as well as the resources that help in that decision making will be described. The following areas are discussed as they relate to planning, organization, and managing effective mathematics experiences:

- developing a mathematical community
- arranging and organizing the classroom
- choosing and using resources
- structuring a primary/junior mathematics class



Developing a Mathematical Community

Young children, even infants, are developing mathematicians. They have an innate understanding of quantitative measures of more and less and of big and small, and they are naturally inclined to seek new learning through solving problems. Ginsburg, Inoue, and Seo (1999) found that young children naturally deal with and calculate quantities in their play. Students mathematize by making sense of their world in quantitative as well as qualitative ways. They hypothesize, make conjectures, contemplate different scenarios, review conclusions, and communicate understanding as they interact with the environment.

Teachers have the opportunity to extend students' positive tendency for, and disposition towards, mathematizing. To do this, they must create a community of mathematics learners in an environment of respect, responsibility, engagement, and high expectations. If an environment of this type is not established, then students will not feel safe enough to take the risks necessary to deepen their mathematical understanding. They will not ask the questions needed to clarify understanding, extend knowledge, and develop an interest in and curiosity about mathematics.

CONDITIONS FOR A POSITIVE LEARNING ENVIRONMENT

K 1-3 4-6

Teachers create a positive learning environment when they see to it that certain conditions prevail in their classrooms. The following are descriptions, based on the widely recognized conditions for learning in the elementary classroom articulated by Brian Cambourne (1988), of students participating in a genuine community of mathematics learners. After each description is a list of actions that the teacher can take to ensure that the condition is fulfilled.

In a community of mathematics learners, students:

- **are respected for their individual learning styles, cultural background, and previous knowledge and skills in mathematics**

  The teacher:

  - finds out what students already know and can do in mathematics;
  - uses this information to provide a range of activities and open-ended problems that have more than one solution, in order to meet the varying needs of students;
  - helps students make connections between previous knowledge, new concepts, and the world around them;
  - monitors the growth of each student's mathematical knowledge and provides or makes arrangements for support where it is needed;
  - changes from large-group discussion to small-group work to individual work, as needed, in order to meet the needs of students;
  - considers using Gardner's multiple intelligences (1993) as a guide to planning activities, in order to address the various learning styles;
  - provides developmentally appropriate tasks and problems;
  - ensures that students have the opportunity to construct their own learning.

IDEA

To gauge the prior knowledge of younger learners, try using riddles – for example, "I have three coins in my pocket that total 12 cents. What are they?" Watch for students' reactions. Which students respond right away? Which students seem confused? Which students use their fingers to count?

- **are immersed throughout the day in the language of mathematics and a rich array of mathematical activities and resources**

  The teacher:

  - reads books and poems and sings songs that relate to mathematical ideas;
  - creates visual displays, cue cards, charts, and word and strategy walls that support mathematical learning;
  - provides enough manipulative materials so that each student can become actively engaged in solving problems;
  - provides ample time for students to explore the manipulatives and their use;
  - uses a variety of manipulatives to explore the same concept;
  - thoughtfully prepares lessons in which manipulatives are used, to ensure that there is support for the mathematical concept being explored;
  - stores manipulatives in an accessible location to encourage independence and the use of materials to solve problems;
  - integrates media and technology and other curriculum areas into the mathematics program where appropriate;
  - interacts with students as they are working, in order to introduce and reinforce mathematical language and terms.

IDEA

Use think-pair-share(-write) as a model when helping students improve language and communication skills in mathematics.

- **see models of mathematics being used by other students and the teacher**

  The teacher:

  - incorporates formal and spontaneous lessons into the mathematics period;
  - uses everyday classroom experiences and problems to model mathematical thinking;
  - models "think-aloud" strategies when solving problems – for example, "First, I have to think about what the problem is asking. Then, I'm going to make a plan for how to solve it. I think I will try drawing a picture";
  - models the solving of problems as a process that takes time, and promotes the idea that solutions are not always immediately apparent;
  - resists prematurely modelling a finished product (i.e., does not try to show students what the final solution should look like or how to solve the problem), and allows students to persevere in coming to their own strategies and solutions;
  - looks for "mathematical moments" throughout the day, and uses these to reinforce mathematical ideas.

IDEA

In learning activities, include examples and problem contexts that reflect the cultural diversity of the classroom, school, and local commmunity.

- **have many opportunities to work with others to apply and use what they know about mathematics**

  The teacher:

  - plans problems that are conceptually based, meaningful, and connected with previous learning;
  - designs learning experiences that encourage students to use their reasoning and develop persistence in solving problems;
  - provides repeated experiences to solidify understanding of concepts;
  - provides large blocks of time to allow students to explore materials properly, to make plans, and to find and share solutions;
  - ensures that students work cooperatively and respectfully with a variety of their peers;
  - develops guidelines for working in cooperative groups; for example:
    1. You are responsible for your own work and behaviour.
    2. You must be willing to help any group member who asks.
    3. You may ask the teacher for help only when everyone in your group has the same question. (Burns, 1992, p. 31)

- **are expected to accept responsibility for their own learning and to use time and materials efficiently**

  The teacher:

  - introduces activities and reinforces routines in a consistent manner, ensuring that all students understand what is expected;
  - encourages students to ask peers for help when they are unclear about routines or about what is expected of them in their work;
  - provides clear criteria for problems, tasks, and behaviour;
  - provides a variety of interesting learning resources that students can explore and use to enhance understanding;
  - helps students develop time-management skills by offering prompts – for example, "We have fifteen minutes left until we begin to clean up our materials. Please make sure that . . . ";
  - expects that students will care for materials, and develops routines for storing, using, and replacing learning materials.

IDEA

Use number cards to organize groups of students. Give one card to each student, then tell all the 5's to meet in one area, all the 4's in another, and so on.

- **share responsibility for the classroom learning environment, helping one another and demonstrating respect for others and respect for materials and routines**

  The teacher:

  - works with students to establish classroom routines, norms, and agreements;
  - organizes the room in a manner conducive to various learning needs, providing areas for large-group discussions, small-group work, and individual work;
  - provides large blocks of time during which students can be helping one another to learn;
  - encourages students to talk about and record what they are learning in mathematics;
  - expects students to accept responsibility for managing math materials;
  - involves students in making decisions that affect the whole class (e.g., about dividing materials, tallying personal choices, or voting on topics).

- **are given regular and ongoing feedback by peers and adults**

  The teacher:

  - interacts with students one to one or in small groups on a regular basis;
  - responds to students throughout the learning process;
  - reinforces the use of mathematical language – for example, by saying, "You're right. A butterfly is symmetrical";
  - invites students to reflect orally or in writing on what they have learned, on how they feel about a particular activity, or on how their group worked that day;
  - provides specific feedback that is related to the math concepts as well as to the problem-solving process and that is designed to promote growth (e.g., "I liked the way you used a diagram to help explain your ideas" as opposed to "Good work!");
  - models how to provide positive and constructive feedback, and expects students to give the same kind of feedback when they are sharing and discussing ideas among themselves;
  - encourages the sharing of mathematical ideas, and models appropriate ways to respond to the ideas of others;
  - provides opportunities for feedback from parents, administrators, and other adults, where appropriate;

IDEA

Take advantage of "mathematical moments" to reinforce the use of mathematical language and concepts – for example, you might say, "Madison, you used a math word when you said you were standing *between* Marc and Rowan. What else can we say about how we are lined up?"

- sets high expectations and encourages students to develop persistence when solving problems – by saying, for example, "I think you can solve this problem on your own." "Where could you look to find a strategy?" "What can you do if you get stuck?"

- **know that all their mathematical ideas are accepted when they are learning new concepts and/or skills**

  The teacher:

  - provides a range of activities that allow students to succeed;
  - focuses on important mathematical concepts and big ideas;
  - offers open-ended problems that require the use of reasoning, that have more than one solution, or that can be solved in a variety of ways;
  - expects and encourages students to find their own strategies for solving problems;
  - listens openly to students to understand their reasoning and their solutions to problems;
  - values students' attempts as indicators of their current understanding about a particular task or activity – understanding that can be used as a basis for further learning;
  - reinforces the idea that risk taking is part of developing knowledge and skills, and develops an atmosphere in which students feel safe taking risks as mathematics learners.

- **are fully engaged and interested in learning mathematics**

  The teacher:

  - promotes positive attitudes about mathematics;
  - provides ample time for students to explore and to investigate concepts;
  - allows students to make choices regarding the materials they will use to solve a problem;
  - allows students to decide how they will share and record information;
  - invites students to share their problems and questions with the class;
  - plans activities that promote curiosity, enjoyment, and perseverance;
  - believes that all students are capable of learning mathematics, and showcases their successes;
  - helps students recognize and value the importance of mathematics.

IDEA

To promote positive attitudes about mathematics, have parents and/or members of the community come in and share how they use mathematics every day.

FIRST STEPS

K 1-3 4-6

From the first day of school, the teacher's role is to create an inviting mathematics environment, where students feel comfortable sharing their ideas, challenging others, explaining their thinking, seeking advice from others, and taking risks. In such a learning community, teachers value and build on students' experiences and ideas; students are active participants in their own learning; and the prevailing belief is that all students can learn. Interactions are based on the respect that students and teachers have for one another, on their ability to listen attentively to one another, and on their ability to disagree in an agreeable manner (Van de Walle, 2001).

To create this community of learners, teachers begin immediately to establish a respectful environment. They engage students in creating and agreeing to a set of classroom routines that encourage responsible learning. They spend time at the beginning of the year establishing what these routines look like and sound like, and then have these routines displayed on posters that students can refer to as necessary throughout the year. Teachers promote the view that learning is a shared responsibility – that students are accountable not just to themselves but also to one another. They give students time, often at the beginning of the day, to share personal thoughts or viewpoints. The teacher ensures that an atmosphere of respectful exchange is maintained throughout the day by asking for students' input, acknowledging their efforts, and seeing that their contributions are shared with others.

Teachers can begin on the first day of school to create the conditions under which optimal learning can occur, as shown in the list of teacher actions that follows each of the five conditions given below.

IDEA

To help students recognize appropriate classroom behaviours, create T-charts and have students describe what an activity such as "Group Work" might LOOK and SOUND like (people leaning in to listen, students sitting together, one person talking at a time, quiet voices, questions being asked).

In a classroom that provides the most favourable atmosphere for learning, teachers set the tone by ensuring from the start that students:

- **know that they will be treated with respect, and therefore that they will be free to learn**

  The teacher:

  - has students work in cooperative groups with a common goal to achieve;
  - has students perform a variety of team-building activities (e.g., from the Tribes program – see Gibbs, 2000) that promote learning about and accepting one another's differences;
  - encourages all students to share their thoughts, ideas, interests, and abilities;
  - actively seeks information about each student that will help the teacher in making decisions about instructional approaches to use with the student, about types of learning activities suitable for the student, and about general ways of dealing with the student.

IDEA

Invite students to bring in and share something of interest to them, and give others in the class a chance to ask questions about it. The item could be mathematical in some aspect (e.g., could have a pattern, be symmetrical, or represent a three-dimensional object).

- **know that they are valued**

  The teacher:

  - makes many opportunities to speak one on one with each student both informally (while circulating around the room) and formally (during a planned conference/interview);
  - discovers the interests of each student – both in and out of school;
  - obtains information from parents where necessary;
  - determines strengths and areas of concern for each student;
  - surveys students to gain information about their interests and past experiences in mathematics (see Appendices 7-1 and 7-2).

- **develop relationships with one another**

  The teacher:

  - has students play games that help them learn each other's names;
  - has students pair up and interview each other, asking each other about family, interests, feelings about math and school, and so forth. Then each student presents his or her partner to the rest of the class;
  - observes each student in different situations (working independently, in groups, and in the whole class) and makes notes on these observations.

- **develop problem-solving and decision-making skills**

  The teacher:

  - uses class meetings to help build an inclusive mathematics community (see Appendix 7-3);
  - allows students to meet each day to discuss their ideas and concerns or to share information;
  - reinforces the skills connected with active listening during discussion;
  - uses class meetings to deal with specific issues, to reinforce good behaviour, or to allow students to get to know one another a little better;
  - helps students develop responsibility by allowing them to make decisions.

IDEA

Have students play a fill-in-the-blanks game (e.g., "My name is Jennifer and I like jelly beans") to learn names and to find out a little about classmates at the same time.

IDEA

To help students develop decision-making skills, open a class discussion by saying, "I have noticed that the math materials are not getting put back as they should. Who has an idea of what we can do?" Students work together to find a solution and thus take ownership of the decision.

- **know what to expect**

  The teacher:

  - has a predictable structure in place for how the classroom operates;
  - establishes clear and consistent rules of conduct that promote self-respect, respect for others, and respect for classroom materials;

  - establishes routines early in the year, to accustom students to the idea that the normal approach to problem solving involves working in groups, helping one another, and choosing tools (e.g., manipulatives, calculators, visual prompts, supplies, organizational structures, strategy wall, word wall);
  - stores materials in an easy-to-reach location, where they are easy to retrieve and replace, so that students develop independence;
  - labels materials clearly, with a picture on the label for those who cannot yet read, so that all students can retrieve and replace materials.

IDEA

Introduce the idea of a "30-cm voice" for use in group activities. Only one person can speak at a time, and the speaker's voice can travel only 30 cm. Have students practise this often, so that they can hear what a "30-cm voice" sounds like.

Arranging and Organizing the Classroom

The first impression that students receive when they walk into a classroom is created by the physical set-up. If the classroom is well designed to stimulate learning and to affect attitudes towards learning, it will convey a positive message to students about learning mathematics from the moment they arrive. The arrangement of furniture, the types and the quality of visual displays, and the organization of materials should reassure students that theirs will be a safe and interesting learning environment – one in which mathematics is valued and visible.

IDEA

If students already have places (e.g., cubbyholes, bins) in which to keep their books and personal belongings, then group desks with the openings turned inward, to avoid having the desks filled with waste, scrap paper, and the like.

One meeting area can be designated as the math forum, where students get together to share ideas.

FURNITURE PLACEMENT

It is important that the classroom be organized to serve a variety of purposes. Having a large meeting area allows students to share ideas, make presentations, and feel part of the community. Primary classrooms often have large meeting areas, but junior classrooms, with their larger desks and larger class size, might not have the space. If space is available, this meeting area should be:

- well-defined and large, so that students can come together for large-group demonstrations, discussions, and community sharing;
- comfortable (e.g., there should be a large carpet, or several carpet samples [one for each student], so that students are cushioned while sitting on the floor);

- large enough that students can move or fidget a little without distracting others;
- away from shelves containing manipulatives, toys, games, and so forth, which could cause distractions during the meeting.

Regardless of whether or not there is space for a large meeting area, a few things should be considered in both primary and junior classrooms. Meeting time, presentations, discussions, and the sharing of ideas or strategies should be part of every math classroom. When the meeting/discussion/sharing is taking place, the students should be:

- near displays that will be referred to regularly (e.g., a calendar, hundreds chart or carpet, strategy wall, word wall, or problem-solving model);
- near the device that will be used for recording (e.g., a chart stand, a whiteboard, a chalkboard, an overhead projector);
- able to see the teacher and the recording device (e.g., a chart stand, a chalkboard) without obstruction.

If a large meeting area is not possible, the teacher might consider a "meeting position". Students asked to take their meeting position would know to arrange their desks or chairs in a particular manner (e.g., a semicircle of desks, or all chairs turned towards the front).

The set-up of the classroom should also allow for the organization of students into smaller groups. Many benefits accrue to students who work together in small groups. They experience mathematics as a social activity and strengthen relationships with their peers. In this less-threatening situation, they can build confidence and independence by sharing thoughts or asking questions. In the context of the small group, they can participate fully, provide regular feedback to the other members of the group, and listen to the strategies that other students are using.

In a classroom set up for small-group work:

- the tables or desks are arranged in clusters of four to six, so that students can work in small groups;
- groupings are placed near the math manipulatives, so that students can have easy access to them;
- students change groups frequently, so that they learn to appreciate each of their classmates;
- rules for working in groups are established, discussed, modelled, and posted for reference.

IDEA

Give each small group a mathematical name, such as the name of a three-dimensional figure (e.g., tetrahedron, cylinder). Each member of the group would have a name tag showing the name of the figure and a corresponding picture. Students become familiar with each term and its related image.

Other factors have an influence on optimal classroom arrangement. In every classroom there must be areas where students can work independently or can speak individually with the teacher. Students must be visible to the teacher. Students must have the proper setting in which to complete their tasks.

A classroom that provides for these factors has:

- tables or desks placed around the room, so that students can work independently when they need to;
- a conference table set apart from any of the groups;
- furniture that is easily rearranged to allow for a variety of work areas and for open spaces when these are required;
- sufficient space, so that students can move freely and safely around the room without disturbing others;
- minimal visual barriers, so that the teacher has a view of the entire classroom;
- "active" areas (e.g., for building) that are not placed next to "quiet" areas (e.g., for reading).

VISUAL DISPLAYS

IDEA

Useful visual displays include hundreds charts, geometric shapes, problem-solving strategies, number lines, coins with words and numbers, a math word wall, or a strategy wall. Useful junior visual displays also include sample journal entries, a multiplication chart, sample problems and their solutions, or a list of good questions to ask when having a conference with a peer before completing a journal entry.

The visual displays on classroom walls, tables, or shelves are key in setting the tone for the year. They should be attractive, colourful, thought-provoking, and stimulating, but also relevant to the mathematics that students will be learning. They should provide students with immediate and accessible information appropriate to their level of development. They should not be so numerous as to overwhelm, distract, or confuse students. Space should always be available in the classroom to display a variety of student work, charts developed during discussions with the class, or solutions to problems that members of small groups have recorded. A Numbers Are Everywhere chart, for example, would include pictures that show numbers used in the students' environment (e.g., on licence plates). A graph would summarize students' findings from a classroom survey (e.g., How We Get to School, Favourite Authors, or Most Popular Bands).

Criteria to consider in evaluating the impact of a visual display would include the following. The display:

- reflects mathematics topics currently being taught in the classroom;
- enhances mathematical understanding;
- uses pictures, diagrams, and vocabulary appropriately;
- stimulates student interest in mathematics;
- helps students to locate manipulatives equipment;
- gives information about where mathematics can be found in everyday life;
- indicates classroom routines.

IDEA

Try an interactive math bulletin board to stimulate student interest. Have a name card made up for each student. Place the student name cards near the bulletin board. Each student finds his or her name in the morning and builds a bar graph by putting the name on the "present" bar. Another student places the names of students who are absent on the "absent" bar. This activity helps students learn the names of other students and helps the teacher take attendance.

Change the topic of the graph regularly. For example, each student could place his or her name on the bar that indicates the month of his or her birthday.

WORD WALLS

Maintaining a mathematics word wall similar to the word walls created for high-frequency words in language is an excellent way to develop and reinforce the language of mathematics. A word wall can be organized in a variety of ways. Words can be listed alphabetically, by strand, or by the concept with which they are associated. They may also be classified according to subject areas – students will come to realize that some words have several meanings and that language is an important component of all subjects. Pictures or diagrams explaining the words may be included.

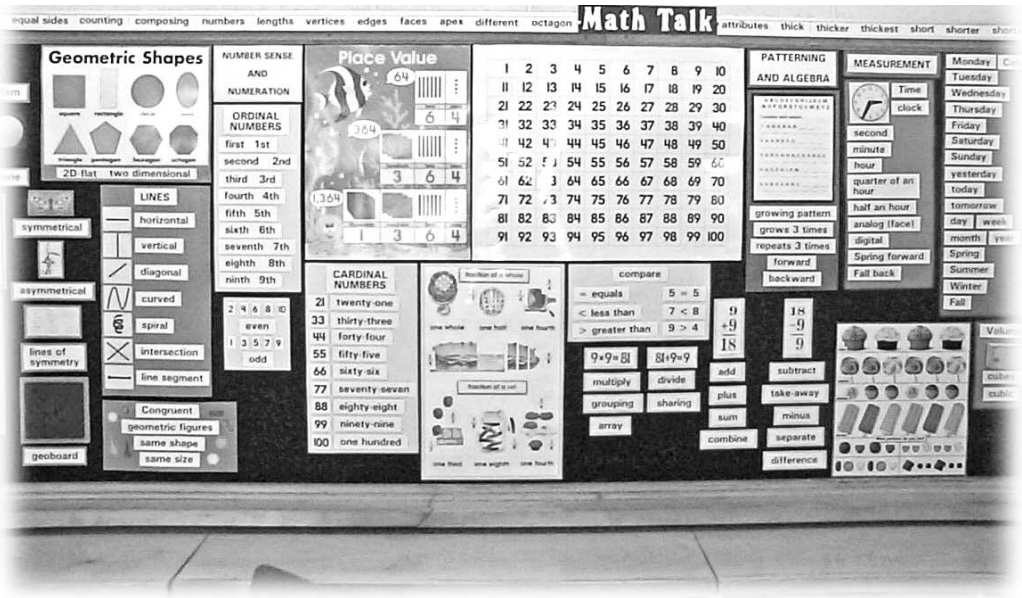
The teacher should refer regularly to the mathematics word wall and should help students develop ease in finding the relevant or required word. The use of short word-wall riddles is one way to develop this facility (e.g., "I am thinking of a word to describe a shape that is like a ball"). See Appendix 7-4 for a partial list of words for a primary-level word wall and Appendix 7-5 for a partial list of words for a junior-level word wall.

The teacher may want to consider the following when using a math word wall:

- The word wall should be accessible and unobstructed: Students must be able to see the words clearly.
- The word-wall list should grow over time: Students add words to the list when they discover them. Also, students are involved in lessons, songs, rhymes, and so forth, that help them learn new words and their meanings.
- The teacher should refer to the vocabulary that students should use when writing a journal entry.
- When helping students reflect on a day's lesson, the teacher might ask what math words students found themselves using while they were doing the activity. Students can add these words to the word wall by themselves.

- The word wall might include words that students frequently misspell in their math journals, or words whose meaning they seem to misunderstand.
- The teacher should clarify math wall words that have other meanings (e.g., *row*, *face*) or that can be confused with a homonym (e.g., *whole*, *hole*). These clarifications are particularly helpful for English language learners.
- Students could keep a "Math Word Wall Dictionary", a small notebook in which to record new words. They can use the "dictionary" to find words from previous lists that have been removed from the word wall and can refer to it when communicating about mathematics.
- The word wall is a learning tool, not just a display. Students are encouraged to refer to the word wall when they are writing in their math journals or communicating with others about mathematics.
- If wall space is an issue, the teacher might use a new word wall for each concept or unit being studied – that is, use a new piece of chart paper and have students add words to it. On completion of the unit, the teacher would take down the particular list and place it with all the other lists from completed units in a "Big Book of Math Words" to which students can have access. The teacher would then put up a new piece of chart paper for the next unit or concept.

TIP: Put pictures beside each word in a math word wall wherever possible, in order to help students who are learning to read.



STRATEGY WALLS

A strategy wall is similar to a word wall and is developed in the same way. The teacher, in helping students to reflect on a day's lesson, asks students what strategies they used or what approaches they tried in solving the problem. If they say, "We used interlocking cubes to help us," the teacher lets them know that that is a strategy called using objects or materials and puts this strategy on the strategy wall. This wall is kept visible throughout the year. Each time students are going to solve a problem,

the teacher refers them to the strategy wall and asks them what strategies might help them solve the problem. As the list grows, students will feel that they have more options and more knowledge about how to solve problems. There may be different types of strategy walls in the classroom. One strategy wall might include various strategies for solving problems. Another might display strategies discovered for addition, subtraction, multiplication, or division. Whatever the topic, the teacher and students should refer to the strategy walls frequently, as part of the routine in the mathematics classroom.

TIP: A strategy wall can simply be a piece of chart paper on which strategies are listed. It does not need to be elaborate – it just needs to be used regularly.

MATERIALS STORAGE AND DISPLAY

In the mathematics classroom, manipulatives and other mathematical materials are visible and accessible – within students' reach. Materials are stored in an unobstructed part of the classroom, in order that students can retrieve and return them without disturbing others. At the same time, they are close to the area where they will be used. As students become familiar with the manipulatives, they become more confident in selecting the most appropriate material to help them develop an understanding of a concept or help them solve a problem.

The most commonly used commercial manipulatives include pattern blocks, attribute blocks, geoboards, tangrams, Cuisenaire rods (coloured relational rods), number cubes (dice), number cards, number lines, hundreds charts, base ten materials, counters, colour tiles, measuring tools, and interlocking cubes (see Appendix 7-6 for a list of some of the materials used in primary mathematics classrooms and Appendix 7-7 for a list of some of the materials used in junior mathematics classrooms). The most commonly used teacher-prepared materials include dot plates, five frames, ten frames, and spinners. (See pp. 22 to 25 for information about five frames and ten frames. Dot plates, which are useful in pattern-recognition activities, are paper plates on which the teacher has affixed peel-off dots – from 1 to 10 dots – in various configurations or arrays. See BLM30 in Appendix 10-3 of Chapter 10: Approaches to Teaching Basic Facts and Multidigit Computations, in Volume Five.)

IDEA

Label each tub of manipulatives with words and pictures. Have a corresponding label on the shelf, so that it is easy for students to put materials back and for the teacher to see what has not yet been returned.

If possible, these manipulatives and materials should be readily available in each classroom; however, a central storage area with a sign-out system in place should work as well for some of the manipulatives that are not necessarily available in every classroom. A library book pocket can be attached to each storage container of manipulatives and a card indicating the name of the manipulative can be placed inside the pocket. Then a chart can be set up to hold a pocket for each teacher who will be using the manipulatives. When a teacher takes a manipulative, he or she places the card in his or her pocket on the chart. Another teacher needing to borrow the manipulative will know where it is and can find out when it might be returned.

In addition to commercial manipulative materials and teacher-prepared materials, collections of found objects are useful for sorting, counting, classifying that is based on common characteristics, and solving problems. Such collections might include buttons, shells, keys, bottle caps, or beads (see Appendix 7-8 for a list of found or collected materials).

The following are ideas for organizing materials and helping students use materials:

- Containers for materials can include bins, trays, freezer bags, math carts, tool kits (for each group, a kit containing a set number of appropriate manipulatives).
- Some teachers give each student a felt square, rubber mat, or small tray for containing manipulatives on individual desks during their use (rubber and felt help deaden the sound of objects such as number cubes).
- Manipulatives (especially things like calculators) can be colour coded or numbered and each number or colour allocated to a specific group.
- Good practices should be established (e.g., to ensure that each group double-checks the materials before returning them: "Put interlocking cubes in towers of 10, and each bag must contain 10 towers").
- The students in each group can be numbered, for example, from 1 to 4. The "numbered students" pick up, repackage, and replace the materials. For example, the teacher might say, "Number ones, you are responsible for getting the correct bin for your group today; number twos, for getting everything back into the containers correctly; number threes, for returning the bins to the correct place on the shelves; number fours, for making sure that everyone does his or her job." Storage bins for each round of activities can also be numbered, to help students find the bin they will be working with on any given day.
- Consider having a Math Challenge centre to which students can go when they have finished work early. At this centre students would find a problem that they would need to solve by using a manipulative of some sort. This kind of problem would help the students deepen their understanding of how manipulatives can be useful in solving problems. An example of a problem at such a centre might be the following: "Use Polydron pieces to make nets for a cube. How many different nets can you make? Record each net on the graph paper provided."

It is important that a supply of pencils, coloured pencils, graph paper, dot paper, lined and unlined paper, tape, string, and rulers be on hand, so that students can record their findings in a variety of ways. Each group could have a supply of writing tools stored in a labelled bucket at the group's table. This assignment of supplies helps promote responsibility, as the group members assume the care of their materials.

Also included in mathematical materials are games. Students of all ages love to play games, and games, when chosen correctly, can reinforce mathematical concepts and provide engaging ways for students to practise basic facts or operations. In addition to using commercial games, students enjoy making their own games. For this reason, students should have available materials such as index cards, cardboard or Bristol board, markers, stickers, and so forth, for creating these games.

Choosing and Using Classroom Resources

K 1-3 4-6

An effective mathematics program integrates a variety of resources as a way to enhance student understanding, learning, and engagement. Such resources include manipulatives, children's literature, computer software, and calculators.

MANIPULATIVES

Students who make their own models of mathematical ideas gain a powerful means of building understanding and explaining their thinking to others. Constructing such representations allows students to see relationships and to make connections between the concrete and the abstract, helps students to remember how they solved a problem, and helps students to communicate their reasoning. Students may solve a problem correctly and, when asked how they did it, answer that they "just knew". If they use a self-designed model to explain their thinking, they are better able to articulate their reasoning.

I hear and I forget,
I see and I remember,
I do and I understand.

Chinese proverb

Moreover, if students use multiple representations for concepts, they develop flexibility in their thinking about such concepts. They are not inclined to perceive any single concrete representation as "the math" but will see it rather as just one of many representations that help them understand a concept. For example, students who have seen only one manipulative, such as base ten materials, used to represent two-digit numbers may not have as strong a conceptual understanding of place value as students who have also bundled craft sticks into tens and hundreds and used an abacus. Manipulatives are also a valuable aid to teachers. By analysing students' representations

of mathematical concepts and by listening carefully to students' reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance that thinking.

Reasons for Using Manipulatives

As part of the problem-solving process, students use concrete materials as aids in modelling concepts, formulating relationships, and improving understanding. Manipulatives:

- make students' mathematical thinking visible, so that everyone can see it, talk about it, and learn it;
- provide a context for developing mathematical concepts;
- help students explore, think about, and talk about mathematics;
- help students construct meaning and see patterns and relationships;
- allow students more easily to test, revise, and confirm their reasoning;
- help students make connections between concepts and symbols;
- help students talk about the math, with the result that teachers have a basis for assessing students' understanding and can make programming decisions based on their observations.

Guidelines for Using Manipulatives

Because manipulatives are the tools that help students reason about mathematical ideas in a concrete way, they are an important part of the mathematics classroom. They also help to engage students' interest. Students will not automatically know, however, how or in what context to use specific manipulatives. Hence, teachers need

to model the use of a variety of manipulatives. They need to help students see that often more than one manipulative can be used to model a mathematical idea or concept, but sometimes a certain manipulative will be more appropriate than others.

When using manipulatives in the mathematics classroom, the teacher should:

- **select materials that are appropriate for the developmental level of students**

  In the report of the Expert Panel on Early Math in Ontario (2003, pp. 21–24) is a table showing connections between manipulatives and mathematical concepts and skills. This table is reproduced, with a few minor changes, as Appendix 7-6. A similar table for students in the junior grades was included in the report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario (2004, pp. 61–63). This table is reproduced, with a few changes and with the addition of a column showing the connections between the manipulatives and mathematical concepts and skills, as Appendix 7-7. For a list of found or collected objects that can be used as manipulatives, see Appendix 7-8.

- **introduce manipulatives explicitly and refer to them by name**

  It is important that students use the correct mathematical terminology right from the beginning. In the primary division the teacher should record the names of manipulatives on chart paper and have students draw or trace pictures of each manipulative to put beside the name. Students can refer to the chart during class discussions and can play games that reinforce the names of the manipulative materials. Teachers in the junior division need to determine the extent of their students' familiarity with various manipulatives in order to plan appropriately for introducing manipulatives.

- **establish rules for distributing/replacing manipulatives as well as routines to ensure that students are not interfering with others and are using the materials in a respectful manner**

  The teacher can discuss routines that students have at home for getting toys and putting them away, talk about classroom routines (e.g., for getting paper or for using the art centre), and then ask students for input on how to handle getting math manipulatives and how to organize putting them away. Sufficient manipulatives should be provided to allow students to work individually, with a partner, or, at times, with a small group of students. Students who are actively engaged in representing their ideas with the manipulatives are less likely to be distracted and off task.

IDEA

Choose a "material handler" from each group. This person is responsible for getting and replacing materials for the whole period. Choose a new person from the group next time.

- **provide time for students to explore the manipulative that will be used**

  Students love to play, and they will play with the manipulatives. The teacher should allow time for this kind of exploration before planned classroom activity.

- **ask students what they discovered about the manipulative**

  The teacher records students' observations and has students demonstrate, asking, "What math idea could you show using this manipulative?" and summarizing students' responses (e.g., "We could use these interlocking cubes to show how to count, or how to model a pattern"). On a piece of posted chart paper, under the name of the manipulative used as a title, the teacher lists students' ideas and adds to them throughout the year. The teacher follows the same procedure for other manipulatives.

IDEA

Have manipulatives sorted into required quantities and put into resealable plastic bags ahead of time in order to save time organizing materials during the math class.

- **start with just a few manipulatives and do several activities that help students see the wide range of uses each manipulative has**

  Seeing how a simple manipulative can serve several purposes helps students acquire skills in thinking flexibly when solving problems. For example, they can use interlocking cubes to learn about addition or subtraction, congruence, counting, equivalence, fractions, patterns, data organization, surface area, three-dimensional figures, transformational geometry, or volume.

- **think aloud when using manipulatives, so that students will learn to verbalize what they are doing**

  For example, when using interlocking cubes to collect data, the teacher might say, "Each student made a cube chain to represent the number of pets he or she has. To find out how many pets we have altogether, I am going to join all of the cube chains together. Now, to help me count faster, I am going to break the chains into groups of 10."

- **question and prompt students as they are working with manipulatives to assess understanding, encourage talk, extend thinking, or consolidate learning**

  Students should be able to talk about why they chose a specific manipulative to help solve a problem. The teacher encourages them to talk about what they are doing, to explain how they are using the manipulatives, to bring forth any questions they might have, and to tell how they think they could find the solution.

- **ensure that manipulatives are always available for students to use when solving problems and exploring concepts, and encourage their use**

  Sometimes the teacher will select the manipulative to use to explore a concept. At other times, students will want to choose what they feel will best help them solve a problem. The teacher encourages this independence but also monitors it (e.g., by suggesting a manipulative that could help if a student seems to be having trouble solving a problem). The teacher asks for students' input as well.

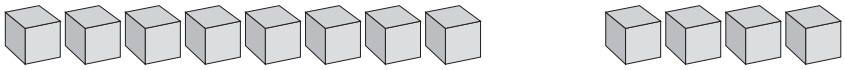
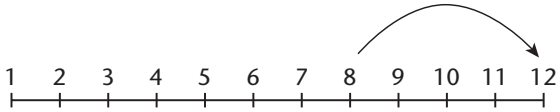
IDEA

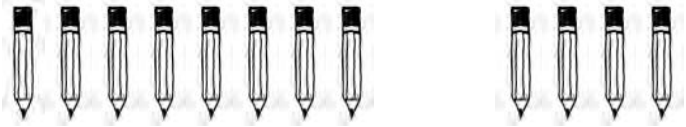
Virtual manipulatives and associated learning activities can be found on many websites. You may want to consider having your students explore math concepts using some of the excellent virtual manipulative websites that are available on the Internet.

It is important that manipulatives be linked with other representations of mathematical problems or concepts. The ultimate goal of using manipulatives is, first, to make a concept more accessible to the student, and then to help the student understand the link between such concrete representations and their symbolic or numerical counterparts. For example, students could be given the following problem:

Ruth had 8 pencils. She bought 4 more. How many did she have altogether?

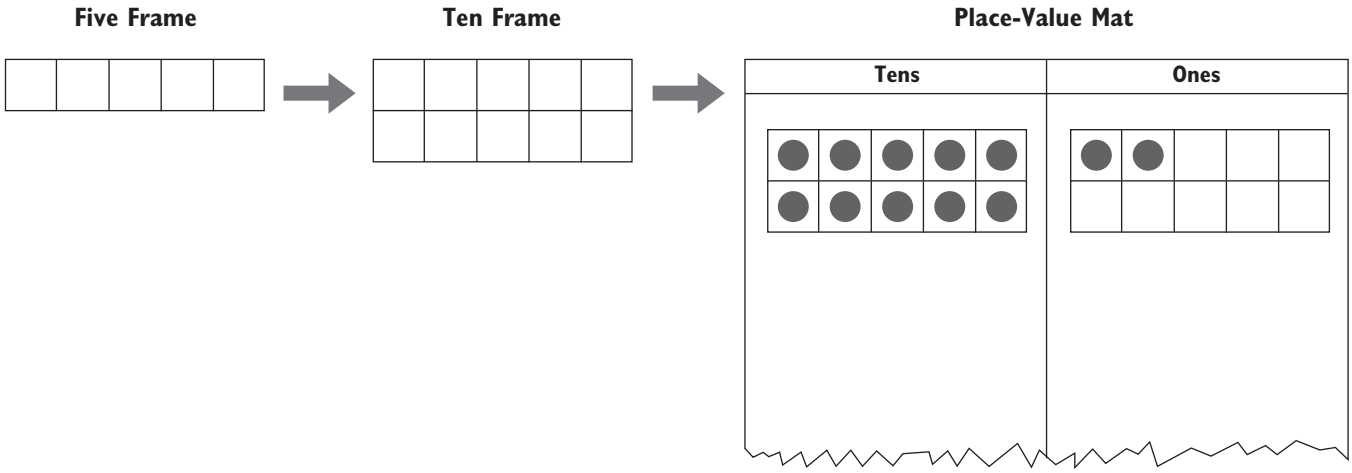
1-3

- **cubes:** 
- **a number line:** 

- **a drawing:** 
- **a tally:** 

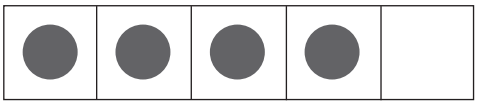
It should be noted that some manipulatives are more appropriately used in connection with specific levels of mathematics concept development. Base ten blocks, although very valuable for students who have consolidated an understanding of 10,

are not appropriate for students who are still struggling with the quantity of 5 as a concept. An appropriate progression of manipulative use in the primary grades might be from five frames to ten frames to base ten place-value mats.

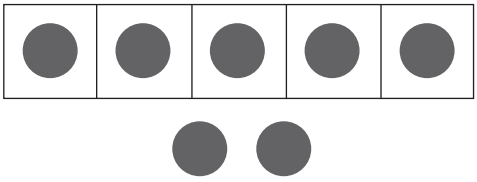


The following four points illustrate the progression in manipulative use:

1. Students use five frames in games and problem-solving activities. (See Appendix 7-9 for a blackline master of five frames.) The frames can serve as models to help develop an understanding of 5 as an anchor for other numbers between 1 and 10. Five frames should be used in a horizontal position and filled from left to right, as shown below. Students recognize 4 counters on a five frame as a quantity of 4 because they know if the frame were full, the quantity would be 5, but since the frame is not full and has 1 less, the quantity is 4.



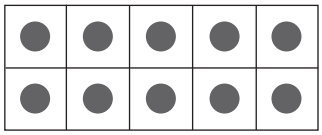
Similarly, students may quickly know that 7 is a full five frame plus 2 extra counters because they know that the frame contains 5 without having to recount the 5 counters, and they know that the 2 more counters make 7 by counting on from 5 to 7.

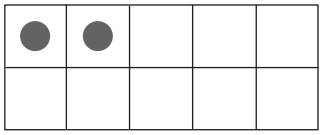


2. Students use ten frames in games and problem-solving activities. (See Appendix 7-10 for a blackline master of ten frames.) These frames can serve as models to help develop an understanding of 10 as an anchor for numbers less than and greater than 10. Ten frames should be used in a horizontal position and filled from left to right on the top row and then from left to right on the bottom row, as shown in the

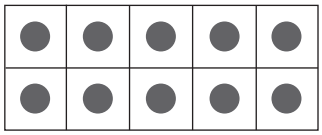
example below. Students may know that a full ten frame plus 2 more is 12 (see the left-hand column of Example 1), a full ten frame plus 6 more is 16 (see the left-hand column of Example 2), and so forth. Students may also know that such frames are linked with placement on a number line (see the right-hand column of Example 1) or on a hundreds chart (see the right-hand column of Example 2).

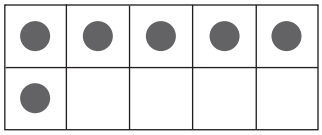
Example 1





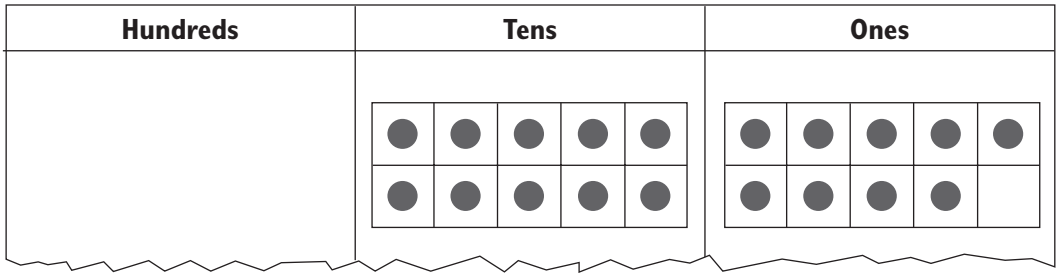
Example 2



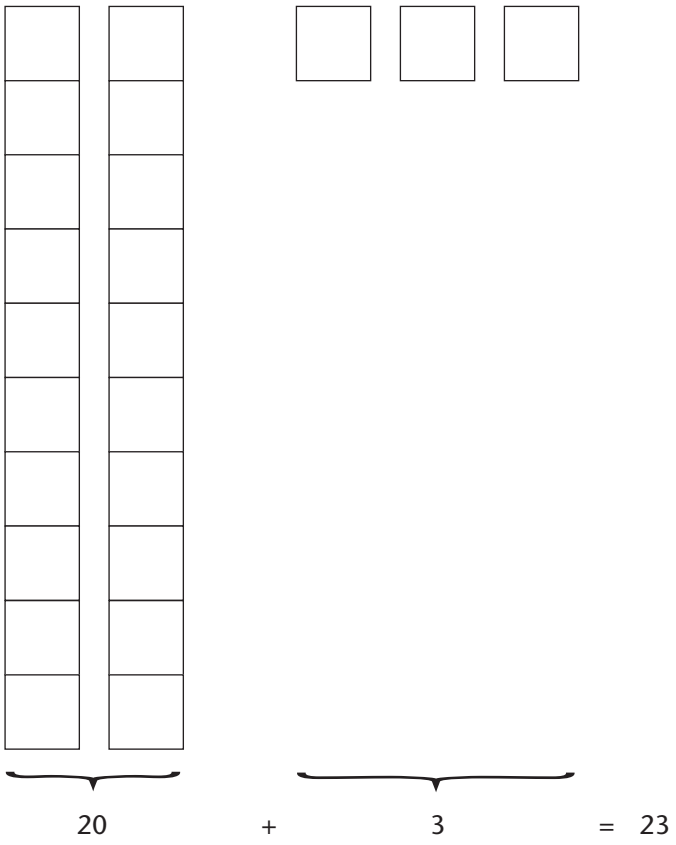


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. Students use ten frames and base ten place-value mats to further link their understanding of anchors of 10 with the place value of 10. Students fill in the ten frame until it gets to 10 (full) and then move it to the tens place as one unit of 10. Base ten place-value mats should extend down far enough to accommodate ten horizontally positioned ten frames.



4. Students use base ten materials of units (single blocks) and rods in games and problem-solving activities to develop an understanding of two-digit numbers. They learn, for example, that in a number such as 23 the digit in the tens place represents two groups of 10, to make 20, plus the 3 ones to make 23.



The sequence given in items 1–4 above is not meant to be prescriptive. The examples show how manipulatives may be utilized by students working at different levels of the concept of place value.

A variety of manipulatives can often be incorporated to help students develop their own understanding of specific concepts. For example, throughout the junior years, fraction strips, pattern blocks, 10 x 10 grids, and base ten materials can be integrated into various learning experiences that help students make sense of concepts related to fractions and decimals. The following ideas illustrate a few of the ways in which manipulatives can be used:

4-6

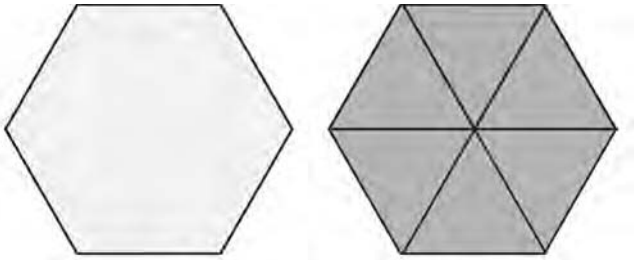
1. By developing their own fraction kits, students can explore a measurement model that illustrates the relationship of various fractional parts to a whole. Teachers provide students with strips of different-coloured construction paper measuring approximately 2 cm by 20 cm. Students then create their own representations for a whole (by taking a strip of paper and leaving it uncut) and for halves, fourths,

eighths, and sixteenths (by taking a strip of one colour and cutting it into two halves, taking a second strip of a different colour and cutting it into fourths, and so on). Students can use these strips for a variety of fraction games and problem-solving activities that focus on relationships between the fractional pieces and a whole. For example, they can try to find as many combinations as possible to create a strip the same length as one whole.

Later, students can add to their fraction kit by making strips for thirds, sixths, and twelfths, as well as for fifths and tenths.

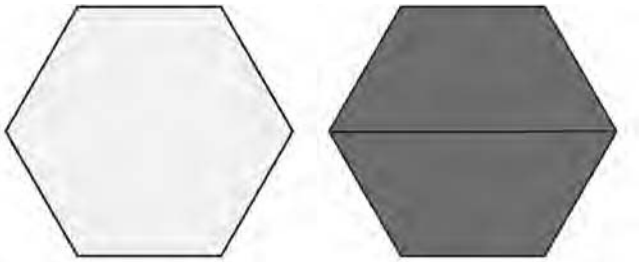
2. Students can explore area models of fractions by using pattern blocks. Students explore the various fractional relationships between pattern blocks, using the yellow hexagon as the whole. Students can determine a variety of ways to fill the whole (hexagon) with different blocks and can explain the fractional relationships.

Example 1



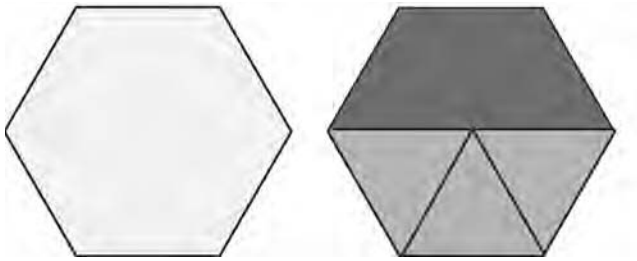
$1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 1$

Example 2



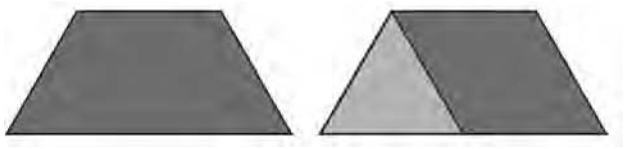
$1/2 + 1/2 = 1$

Example 3

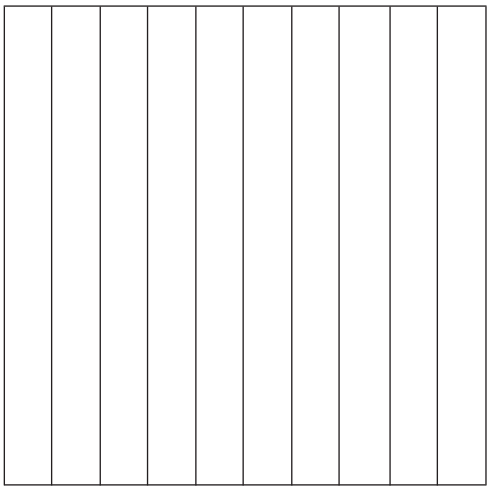


$1/2 + 3/6 = 1$

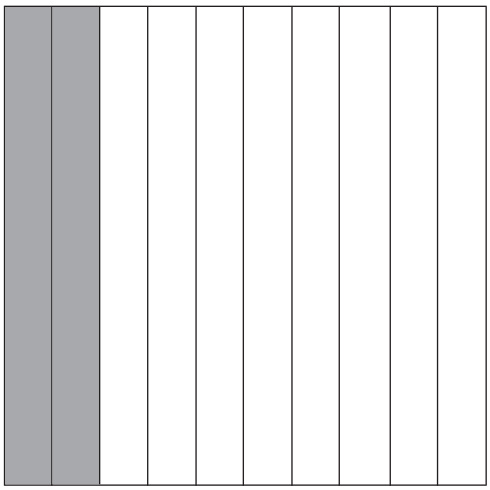
3. Later in their development of fraction concepts, students can explore more fractional relationships by using pattern blocks, with different blocks representing the whole. For example, if the red trapezoid is a whole, what fraction is the green triangle?



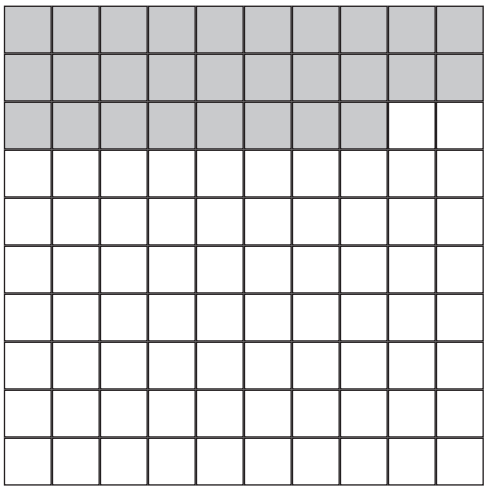
4. Students can explore the relationship between fractions and decimals by using a tenths grid, a hundredths grid, or base ten materials. A tenths grid is a 10 x 10 grid that is divided into ten equal strips, each strip representing $^{1}/_{10}$ of the whole.



A hundreds grid is a 10 x 10 grid that is divided into ten equal columns and ten equal rows. Students use the tenths and hundredths grids to represent fractions and decimals such as $^{2}/_{10}$ or 0.2, $^{28}/_{100}$ or 0.28.



$^{2}/_{10}$ or 0.2



$^{28}/_{100}$ or 0.28

Students can also use base ten materials of single cubes (hundredths), rods (tenths), and flats (wholes) in games and problem-solving activities to develop an understanding of decimal numbers.

The suggestions provided in items 1–4 above are not sequential and do not provide the full range of learning that students need in exploring fractions and decimals. Instead, the suggestions provide examples of how different manipulatives and materials can be integrated into mathematics learning to enhance the learning experience for students in the junior grades.

Other materials are introduced as appropriate when students are exploring particular mathematical concepts. These materials include:

- literature that incorporates mathematical concepts;
- pictures that relate to a concept or mathematical idea;
- computer software;
- calculators;
- objects that are relevant to the topic that students are exploring (e.g., shells for studying patterning);
- materials relating to mathematics that students have brought from home (e.g., containers such as boxes or cans that represent three-dimensional figures).

LITERATURE

K 1-3 4-6

The foundation of mathematics is set during the early years. In these years, teachers use manipulatives to help students understand mathematical concepts in a way that accommodates students' need to explore with their hands and body in order to learn. Another way in which teachers meet the needs of all learners and accommodate a range of learning styles is to use literature. This use of manipulatives and literature should continue throughout the junior years, because the same benefits exist. Stories help bring mathematics to life. Student of all ages love stories and love to talk about stories. When teachers use literature with a mathematical connection, they help students learn to "talk mathematics" and thus prepare them for the eventual task of writing about mathematics. Good stories provide good opportunities for math talk. (Each of the books referred to on pages 28–33 is listed, along with the author(s), the strand(s) of the mathematics curriculum that the book supports, and the division in which it is most suitably used, in the table in Appendix 7-11.)

The following are types of books that effectively link language and mathematics:

- books in which the storyline has obvious mathematical potential (e.g., *The Button Box*; *Moira's Birthday*; *Alexander, Who Used to Be Rich Last Sunday*; *Caps for Sale*) as well as books that are explicitly mathematical (e.g., *Ten Black Dots; 12 Ways to Get to 11*; *Sir Cumference and the First Round Table*);
- for young students, books with repetitive patterns and rhyme, in which the pictures match clearly with the words and help to make the mathematics more

explicit (e.g., *1 Hunter; 1, 2, 3 to the Zoo*) and books with numerals that are placed consistently on each page and match clearly with pictures;

- books that challenge and inspire, in which the story, the subject matter, and the topic are of consequence to students (e.g., *10 Minutes Till Bedtime; Ten, Nine, Eight; Counting My Friends; Ten Flashing Fireflies*; *Anno's Magic Seeds*);
- books that use elements of surprise and humour effectively (e.g., *1 Hunter*; *My Little Sister Ate One Hare*; *Hippos Go Berserk*; *The Greedy Triangle*);
- books that are aesthetically pleasing and represent high-quality children's literature (e.g., *Anno's Counting Book*; *Two by Two*; *The Grouchy Ladybug*; *A Cloak for a Dreamer*).

The following are things for the teacher to consider when integrating literature into the mathematics classroom:

- The story is engaging in itself as a story, and the link with mathematics is genuine.
- The reading level of the story is a match for its use; some books are great for reading aloud, others for shared reading, and still others for independent reading at a learning centre.
- Students' interests and previous knowledge guide the discussion that follows (e.g., *The Greedy Triangle*, although about polygons, might yield discussion of other mathematical areas, or connections with students' lives).
- Students have time to enjoy, think about, and talk about the story.
- Students are given a problem to solve that relates to the mathematical concept in the book.
- Students pose mathematical problems and work in groups to solve them.
- Parents are involved when math books are sent home for parents and children to read together and mathematical problems or activities are included for the family to do together.

Books that incorporate the big ideas of the Number Sense and Numeration strand of the Ontario mathematics curriculum for the primary division fall into various categories and have a variety of uses: K 1-3

- Books that combine the use of numbers with highly predictable text are especially good in Junior/Senior Kindergarten and Grade 1, because they provide high levels of support in both mathematics and literature. In these texts, the use of numerals, rather than number words, helps students develop the big ideas of counting and quantity. At the same time the books provide extra clues for students who are just beginning to understand the reading process and may be more comfortable with number symbols than letters and words (e.g., *Big Fat Hen*; *One Gray Mouse; 1, 2, 3, to the Zoo*; *Ten out of Bed*; *I Spy Two Eyes: Numbers in Art; and 1 Hunter*).

- Books that present number in a range of contexts and arrays help students develop a flexible sense of quantity (e.g., *What Comes in 2's, 3's, and 4's?*; *Ten Black Dots*; *Anno's Counting Book*; *The M&M's Brand Counting Book*; *I Spy Two Eyes: Numbers in Art*; *How Many How Many How Many*).
- Books that present not only counting up from 1 to 10 but also counting back down from 10 to 1 help to establish a conceptual basis for both addition and subtraction (e.g., *Ten, Nine, Eight*; *10 Minutes Till Bedtime*; *Ten out of Bed*; *Ten Flashing Fireflies*; *1 Hunter*).
- Books that provide pictorial representations for numbers beyond 10 help students to establish counting skills into the teens and beyond (e.g., *One Hundred Hungry Ants*; *Twenty Is Too Many*; *Miss Bindergarten Celebrates the 100th Day of Kindergarten*; *From One to One Hundred*; *One Hundred Ways to Get to 100*; and books that count candies, cereal, birds, bugs, and so forth).
- Some books focus on problem solving and operational sense (e.g., *One Guinea Pig Is Not Enough*; *Twenty Is Too Many*; *The Doorbell Rang*; *12 Ways to Get to 11*; *Each Orange Had 8 Slices*; *How Many Feet in the Bed?*; *Ten Flashing Fireflies*).
- Some books reflect diverse perspectives (e.g., *A Caribbean Counting Book*; *Feast for 10*; *Two Ways to Count to Ten: A Liberian Folktale*).
- Some books link with other subject areas (e.g., *Anno's Magic Seeds*; *Jim and the Beanstalk*; *One Watermelon Seed*; *I Spy Two Eyes: Numbers in Art*).

A list of excellent books that support all strands of the mathematics curriculum is provided in Appendix 7-11.

Books that incorporate the big ideas of the Number Sense and Numeration strand of the Ontario mathematics curriculum for the junior division fall into various categories and have a variety of uses:

4-6

- Books that present numbers in a range of contexts and arrays help students develop a flexible sense of quantity. This sense of quantity includes an understanding of the "howmuchness" of whole numbers, fractions, decimals, and percents and is the basis of the ability to compare and order numbers and to estimate the reasonableness of solutions (e.g., *How Much Is a Million?*; *If You Made a Million*; *A Million Fish . . . More or Less*; *On Beyond a Million*; *The King's Commissioners*; *The Hershey's Milk Chocolate Fractions Book*; *The Hershey's Milk Chocolate Multiplication Book*; *Twizzlers Percentages Book*).
- Books that pose problems and involve the reader in the solutions help students develop their understanding of the operations (addition, subtraction, multiplication, and division), of the properties of the operations, and of the relationships between operations (e.g., *The King's Commissioners*; *A Remainder of One*; *Anno's Mysterious Multiplying Jar*; *Pigs Will Be Pigs*; *The Doorbell Rang*; *Fractals, Googols and Other Mathematical Tales*).

- Books that use the language of mathematics help students develop their ability to use mathematical symbols and language appropriately in a variety of contexts and disciplines (e.g., *Twizzlers Percentages Book*; *The Hershey's Milk Chocolate Fractions Book*; *Fractals, Googols and Other Mathematical Tales*; *G Is for Googol*).
- Books that ask students to make comparisons, to answer "what if" questions, and to make estimates help them improve their proportional reasoning skills, including their ability to make comparisons based on multiplication rather than on addition and to develop informal strategies for comparing ratios that parallel the strategies used for comparing fractions (e.g., *How Much Is a Million?*; *If You Made a Million*; *If You Hopped Like a Frog*; *One Grain of Rice*; *Anno's Mysterious Multiplying Jar*; *Is a Blue Whale the Biggest Thing There Is?*; *What's Smaller Than a Pygmy Shrew?*; *The King's Chessboard*; *Amanda Bean's Amazing Dream*).

A list of excellent books that support all strands of the mathematics curriculum is provided in Appendix 7-11.

Literature can be used in a variety of ways in a math lesson. The big ideas and key concepts can be used as a filter to help the teacher decide which book to select, what features to emphasize, and how to present and respond to the book. Knowledge of the developmental levels of students in both mathematics and literature will also help in making these decisions.

1-3

If the focus in the primary classroom is on developing the concept of quantity, the teacher could provide a variety of books, to present quantity in a range of contexts (e.g., *What Comes in 2's, 3's, and 4's?; Ten Black Dots; Anno's Counting Book; How Many How Many How Many*). After a story has been read several times, students could write their own book as a class big book similar to/based on/adapted from the book they have read. They could do this as a whole class or in small groups. For example, they could write a book called "What Comes in 5's, 6's, 7's, 8's, 9's, and 10's?" after reading *What Comes in 2's, 3's, and 4's?* or write a book called "Ten Red Triangles" (in which they use triangles from 1 to 10 to create pictures and storylines to match) after reading *Ten Black Dots*; or write their own versions of *How Many How Many How Many*, in which they look for numbers from 1 to 12 in familiar everyday events and objects (e.g., how many heads or feet in the class, meals in a day, corners on a square, fingers on a hand, legs on an insect, days in a week, legs on a spider, innings in a baseball game, toes on both feet, players on a soccer team, months in a year).

A focus on problem solving and operational sense could be based on *12 Ways to Get to 11*, *Ten Flashing Fireflies*, or *How Many Feet in the Bed?* Depending on their age and ability, students could work in groups to develop and represent 11 ways to get to 10, 10 ways to get to 9, 9 ways to get to 8, and so forth. They could create number stories to match each page in *Ten Flashing Fireflies* (all the number combinations for the

quantity of 10), or they could adapt *How Many Feet in the Bed?* to answer how many feet (fingers, fingers and toes) are in the classroom.

The use of books to promote both emergent literacy and mathematics simultaneously is a particularly exciting application for teachers who struggle with time constraints in the primary day. A book such as *One Gray Mouse* can be used both to establish basic number concepts from 1 to 10 and to promote knowledge of rhyming pairs in early primary classrooms (e.g., mouse-house, cat-mat, fish-dish, snake-lake). Students could predict which rhyming family would generate the most words and then work in pairs or small groups to develop the lists of words to see how closely the final tallies match with their predictions. Upper-primary students could also do the prediction–rhyme generation activity but at a more sophisticated level. They could also create their own number books on the basis of the rhyming pattern in *One Gray Mouse*, but their books could be based on counting by 2′s, instead of by 1′s.

A book such as *I Spy Two Eyes: Numbers in Art* provides young students with both mathematical and literary challenges as students find in a work of art the objects that match both the number and the words in the sentence on each page. Given the sentence "I spy three puppies", students must select from a range of different sets, some of which have more than 3 objects and several of which begin with *p* (pears, puppies, plums). The sentence "I spy ten hens" is associated with a picture that has four sets of approximately 10 objects (hens, ducks, flowers, doves); students must count the numbers of objects and then match the appropriate picture with the words on the page. Students could follow up this activity by writing "I spy" number sentences of their own, using works of art that the teacher has selected or using their own artistic creations.

4-6

In the junior classroom the teacher could have students explore the characteristics of shapes by first reading one of three books by Catherine Sheldrick Ross – *Squares, Triangles,* or *Circles*. These books discuss the uses of shapes in the world around us and provide a range of activities that enable students to explore the characteristics of the shapes (e.g., paper folding, solving puzzles, building three-dimensional shapes). After the reading of the book, students could work in groups to write and illustrate minibooks that describe another shape (e.g., a hexagon, a rectangle). Students could go on a community walk and look for examples of their chosen shape to use in their books.

Junior students could be led through *Anno's Hat Tricks*. This book introduces binary logic and deductive reasoning. Each character in the book must use the clues given by the narrator to determine the colour of the hat that he or she is wearing, and the reader, having the same viewpoint as the narrator, must also determine the colours of the hats. As the teacher reads each new scenario, the students could work together in small groups or pairs to determine the colours of the hats of the characters and to

explain how they did so. Students could then create their own hat problems for others to solve.

At the junior level, the teacher can choose poems not just for their literary qualities but also for their mathematical potential. Many poems can serve as springboards for students to use in posing mathematical problems. For example, the poem "Overdues" from *A Light in the Attic* by Shel Silverstein (New York: Harper Collins, 1981) focuses on the narrator's dilemma: A book is 42 years overdue – what should the narrator do? Students can research the overdue rate at their local library and find out what the outstanding fine for a book 42 years overdue would be. They could share their strategies with one another and could extend the problem by asking "what if" questions – for example, "What if the rate was higher?" or "What if the librarian gave a choice of payment plans?" For the latter question, students would make up the choices.

Using literature is a wonderful way to get students involved in the mathematics of everyday life. Literature provides students with opportunities to make connections with their own lives, provides a context in which students can practise mathematics, and enriches students' view of the world of mathematics. Once teachers begin to link literature and mathematics, they see endless possibilities and cross-curricular applications that provide rich, authentic mathematics in the primary or junior classroom.

COMPUTER SOFTWARE

For computer software to be considered instructionally effective, it must involve the use of problem-solving strategies and must reinforce the beliefs and practices of the mathematics classroom. The software chosen for use in Ontario classrooms should promote mathematical communication, understanding, and reasoning; encourage collaboration and social interaction; provide for the varying needs of each individual student; and be linked with the Ontario curriculum. Software that students use merely to fill time (e.g., by playing mathematical games) or perform drills has little or no instructional value.

A variety of software programs are available through the Ontario Ministry of Education. Four of these programs – Kid Pix, AppleWorks, Graphers, and HyperStudio – have many applications in the primary math program. Some examples follow.

1-3

In using Kid Pix, the student can:

- stamp numbers and the corresponding number of pictures;
- practise making patterns by having one student start a stamp pattern and another student finish the pattern;
- use stamps to show addition facts;

- show results of data collection with stamps;
- create domino sets that show different ways to make a number (e.g., 12);
- use different stamps to measure the perimeter and/or area of a shape;
- use the dice tool to generate addition and subtraction number sentences;
- practise counting by creating a dot-to-dot picture and having another student count while connecting the dots.

In using AppleWorks, the student can:

- use various polygons in the drawing option to create pictures;
- practise making transformations by creating half an image using the drawing option and having another student use reflection to finish the image;
- use transformational geometry in the drawing option to create tessellations or patterns;
- use the spreadsheet option to create a hundreds chart, a two-hundreds chart, a multiplication chart, and so forth, and use the shading tool to show patterns on the charts;
- organize a data set using a graphic organizer such as a web, chart, or diagram;
- create symmetrical shapes;
- explore flips, slides, and turns;
- show the sequence of a set of objects from lightest to heaviest;
- show things that have a mass of about one gram.

In using Graphers, the student can:

- create a data set and display it in a variety of ways;
- sort a data set manually;
- sort a data set using a Venn diagram;
- select different ways to display a data set;
- select a data set, generate a survey question, collect data, and organize the data in various ways;
- sort sets using two or three attributes.

In using HyperStudio, the student can:

- make spinners to illustrate his or her understanding of probability;
- make wallpaper patterns;
- create symmetrical shapes;
- explore flips, slides, and turns;

- use a teacher-created hundreds template to fill in patterns on the grid (e.g., odd numbers, counting by 5′s);
- create a slide show demonstrating his or her understanding of the classification of polygons on the basis of their geometric properties.

Two of the software programs described above, AppleWorks and HyperStudio, have many applications in the junior mathematics program. In addition, The Geometer's Sketchpad and TinkerPlots can be used in the junior mathematics classroom. Some examples of applications of these programs follow.

4-6

In using AppleWorks draw tools, the student can:

- create and name regular and irregular three-dimensional figures;
- make a graphic organizer (e.g., T-chart, Venn diagram);
- construct tiling patterns;
- create a series of fraction strips;
- develop a series of arrays that represent and explain multiplication equations;
- draw illustrations that demonstrate the relationships between metric measures;
- draw the nets for a variety of solids.

In using AppleWorks spreadsheets, the student can:

- create a tool that converts fractions into decimals;
- create graphs of collected data and select the graph that is most appropriate for displaying the data;
- find the range of a set of data and analyse it for central tendencies;
- construct labelled graphs;
- organize data according to various criteria;
- create and use simple formulas in working with data.

In using HyperStudio, the student can:

- create a number or pattern picture book for primary students;
- represent fractions and/or decimals in a variety of ways;
- publish the results of an investigation based on a problem posed from literature;
- create a stack that shows the outcomes for a series of events;
- demonstrate how two-dimensional figures can have the same area but different perimeters or the same perimeter but different areas;
- create a two-dimensional shape that tiles a plane;
- create a stack that shows both the nets and the related solids of a variety of regular polyhedra.

In using The Geometer's Sketchpad, the student can:

- explore transformations (reflections, translations, and rotations);
- construct polygons in a variety of ways;
- make tiling patterns;
- explore the relationship between perimeter and area;
- create simple animations of transformations;
- construct regular polygons given the perimeter or area.

In using TinkerPlots, the student can:

- create a wide variety of graphs;
- find the mean, median, mode, and range of data sets;
- import data sets from selected Internet sources;
- pose problems and create stories related to the data;
- produce reports with graphs.

Good mathematics software should engage students as active learners, pose meaningful problems, and encourage collaboration with others (Ross, Hogaboam-Gray, McDougall, & Bruce, 2002).

CALCULATORS

The calculator is a valuable tool that helps students to develop number sense – particularly as they work with large numbers in problems and patterning – and to explore mathematical ideas such as estimation and counting.

TIP: Allow the use of calculators when the goal of the activity is not to assess the computation skills of students. Do not use calculators when the goal of the activity is to assess students' ability to perform mathematical computations.

There are two ways in which the calculator helps students to develop number sense – as a computational tool and as a teaching and learning aid.

- Used as a computational tool, the calculator allows students to work with numbers in ways that they could not with pencil and paper. Both primary and junior mathematics learners may struggle with problem solving if a calculation involves larger numbers or operations with which they do not feel confident. Students who use calculators in a problem-solving situation can use their reasoning and practise their problem-solving strategies. Because they can use the calculator as a computational tool to deal easily and quickly with large numbers and complex data, they are able to focus on the mathematical concept underlying the computational task.

  Teachers should also encourage students to use calculators to verify their answers. This use of a calculator as a computational tool helps to develop the student's ability to review his or her calculations and assess their reasonableness. This skill needs to be fostered from an early stage of mathematics development. If a student generates a calculation that does not make sense, he or she must be able to make that determination.

- Used as a teaching and learning aid, the calculator helps students to think about numbers in different ways. It thus becomes a tool for learning, not unlike mathematics manipulatives. This is the typical way for calculator use to be incorporated into the classroom. Activities reflecting this type of use are usually highly defined and less likely to be viewed as allowing students to rely too heavily on calculators to do the math for them. The use of calculators as teaching and learning aids is most commonly seen in mathematical games and in explorations of numerical patterns.

Both primary and junior students can enjoy playing an addition game in which using a calculator provides a quick and easy alternative to using pencil and paper or finding concrete items to serve as counters. In playing this game, each student works with a partner. The first student enters a 1 or a 2 and presses the addition key. The second player takes the calculator and enters a 1 or a 2 and presses the addition key before passing the calculator back to the first player. The object of the game is to be the player who enters either a 1 or a 2 to reach 10 or some other designated number. This activity enhances students' ability to count and develops their operational sense. As with most calculator games and problems, this game may be easily adapted for older students by using a higher designated number and different addends. A variation involves subtracting only 1, 2, 5, or 10 from 101 to reach 0.

A simple but useful calculator activity that develops students' understanding of number sense and of estimation in particular is the target game. Before playing this game, teachers may need to investigate how to use the constant feature of the classroom calculators. A target number and an operation are selected. Using the constant feature of the calculator, students take turns attempting to reach the target or to come as close to it as possible. For example, students could have 298 as the target, multiplication as the operation, and 12 as the factor. The teacher enters x 12 into the constant feature, and students take turns multiplying 12 by another factor that will bring the product as close to 298 as possible. This activity is easily modifiable to include decimals and fractions. In a variation of this game, students attempt to reach a number range rather than a specific target.

Calculators help promote in students an understanding of the connections, relationships, and patterns inherent in mathematics. An important aspect of any mathematics teaching is helping students to understand that mathematics is concerned with relationships and patterns. Primary and junior mathematics learners are naturally curious about how numbers work and about the relationships between numbers. Using the calculator to investigate number patterns engages the learner in a dynamic exploration that does not require time-consuming and laborious paper-and-pencil tasks. Exploring skip counting using the calculator, for example, allows the student to quickly and efficiently discover patterns. The teacher can begin by pressing 2 + 2 = = = to show students how to use the = key for repeated addition, and can

then have students discuss what they notice. Students should notice that the numbers increase by 2 each time the = key is pressed. This use of the calculator develops students' facility with skip counting by 2's but also reinforces the patterns discovered in this activity. The teacher then tries different starting number expressions (e.g., 4 + 5 = = =) and again has students tell about the patterns that they see and the generalizations they can make.

Calculators permit the junior student to explore number patterns, and patterns involving fractions and decimals are very appealing. Finding the decimal equivalents for $^1/_7$, $^2/_7$, $^3/_7$, and so on, will reveal an interesting pattern not readily identifiable. Students can use the calculator to discover patterns in decimal equivalents for sequences of fractions with denominators other than 7 (e.g., sequences starting with $^1/_9$ or $^1/_{11}$).

4-6

Calculators do not provide students with answers if students do not know how and when to use them appropriately. And just as students must learn to choose an appropriate manipulative material, they must also learn when calculator use is appropriate.

The teacher decides how best to include the use of calculators in the mathematics program. The teacher models the use of calculators, makes decisions regarding whether or not to use calculators during a particular lesson or activity, and provides activities involving calculator use. As a rule, allowing mathematics learners access to calculators during classroom activities and problem-solving situations removes the novelty factor often experienced with calculators in both primary and junior classrooms. Readily accessible technology becomes yet another tool for discovery in the elementary classroom.

Students can have access to calculators for use in the classroom during the early years, but they must also develop the skills needed to solve computations mentally as well as with paper and pencil. In a mathematics program, there needs to be a balance between activities that strengthen and promote mental mathematics and activities that provide opportunities for the appropriate use of calculators as both computational tools and teaching and learning tools. Mathematics learners who are given varied mathematics experiences, including mental math activities as well as appropriate opportunities to use the calculator, develop a stronger sense of number and a greater ability to recognize the patterns and relationships of mathematics.

It is recommended that calculators with a two-line display be used in the junior classroom, as such calculators permit students to keep track of their earlier entries. These calculators may also have additional features – for example, integer division, which provides quotients with remainders instead of the decimal that is seen in a one-line display. Rounding, estimating, and place-value features are additional benefits for students in their quest to make sense of mathematics through the use of a calculator.

Structuring a Primary or Junior Mathematics Class

1-3 4-6

The structure of mathematics class should reflect the needs and preferences of individual teachers and their students, and the scheduling requirements of individual schools. When planning a mathematics program, teachers in the primary and junior grades include time for students to experience a balance of different types of activities and to work in a variety of groupings.

TIME CONSIDERATIONS

In the mathematics classroom, teachers must devote sufficient time to focusing on students' understanding and helping students as they build bridges from the concrete to the abstract, make connections between mathematics and the real world, develop problem-solving and communication skills, and develop their ability to reason mathematically.

Learning mathematics is a natural process by which students come to make sense of the world. When planning mathematics experiences, teachers must ensure that the activities are relevant to the real-life situations of students and that students are actively involved. Many math activities derive from the spontaneous interests and themes of students (e.g., measuring a pumpkin and finding its mass, estimating the number of seeds it has, graphing students' estimates of the number of seeds, comparing the graph with the actual results after counting, investigating the symmetry of a pumpkin). Mathematical topics and ideas should be addressed over a time period long enough for students to consolidate their learning and begin to develop facility and fluency. Continuity is important; changing quickly from topic to topic does not enhance student learning. The most effective mathematics programs are delivered in uninterrupted blocks of time that provide a balance and range of teaching strategies.

Substantial amounts of time and resources are needed to create an effective learning environment. Teachers need extended periods of mathematics in order to provide time for exploration, guided instruction, shared learning, student discussion, and reflection. Students also need sufficient time, free of distractions and interruptions, to learn. Finally, students require enough time to reflect properly and to share their ideas with one another. Scheduling must be guided by the needs of students and what they are expected to learn.

In making scheduling decisions, teachers should ensure that students have sufficient blocks of time every day for mathematics and that mathematics is threaded throughout the day:

- In Junior and Senior Kindergarten, there should be focused time (approximately 20 minutes) every day (40 minutes every other day for students attending Kindergarten on alternate days – see pages 52–55 for details) for mathematics. This may

take the form of a guided or shared experience. In addition to this time, students should be engaged in consolidating their mathematical learning at centres within the classroom (e.g., a sand-table centre, literacy centre, water table, or measuring centre).

- In Grades 1 to 6, a minimum of one hour per day should be allocated to mathematics.
- Mathematics should also be integrated into other subject areas as appropriate. Mathematics concepts will also arise throughout the day (e.g., in lining up students from tallest to shortest, in discussing the number of minutes until recess). Teachers should make use of these teachable mathematical moments (Expert Panel on Early Math in Ontario, 2003).

INSTRUCTIONAL GROUPINGS

The teacher in any program, including mathematics, provides a variety of learning situations to accommodate the various learning styles of the students, the need to change the activity level, and the need to keep things interesting. Students can be arranged for learning activities in a variety of ways:

- in large groups
- in small groups
- as partners
- as individuals, working independently

Large and small groups of students constitute appropriate settings for introducing new materials and presenting and modelling mathematics concepts and skills. Small-group, partner, and independent activities provide opportunities for students to investigate math concepts. (See Chapter 4: Instructional Approaches, in Volume One, for more information on groupings in shared and guided learning experiences.)

The Value of Group Work

New understandings about the links between social interactions and learning suggest that classrooms should include a significant amount of small-group and partner work (Cobb, Wood, & Yackel, 1991; Linchevski & Kutscher, 1998). Such small-group and partner work is particularly important in mathematics, where students benefit from hearing a variety of strategies and processes for problem solving. The activities that teachers plan for small-group work must be mathematically relevant and must foster social interaction and discussion. For example, the teacher may assign both an activity and a discussion focus for each group task ("This is the tub that I want you to work on . . . and this is what I want you to talk about"). The teacher's role during group time is to interact with students, encouraging them to share their learning; to ask appropriate questions and make prompts that move students to a deeper level of understanding; and to celebrate students' learning and strategies and see that they are shared with peers. For example, the teacher may say, "Look at the pattern that

Adam just made! Who knows what kind of a pattern this is?" Young students, or those with limited experience of working in groups, will benefit from working with partners to establish active listening and comfort in a shared-learning environment before moving to small-group work. At the primary and junior levels, groups of four or fewer usually work best.

Familiar Versus Flexible Groupings

Teachers manage groups in a variety of ways but ordinarily strive for a balance between permanent (or familiar) and flexible groupings. Familiar groupings are best used to establish community within a classroom; students will identify with a group and look to its members for encouragement and support. Sometimes, familiar groupings can also become learning groups. Most often, the composition of learning groups is more flexible; the make-up of the small groups in a classroom will change according to the task and the learning goals.

At different times, groups may be based on friendship, a mix of abilities, concepts needing practice, interest, or random selection. Regardless of the type of group formed, students must have time to reflect and to share their discoveries, either in a small group, with a partner, or with the whole class.

The Uses of Groupings

Math classes, and parts of math classes, can be structured in many ways. At times the teacher will want to bring the whole class together; at other times students will be working in small groups, with partners, or individually. Suggestions for using each of these types of groupings follow.

The teacher can use *whole-class instruction* to:

- introduce and investigate a topic or concept;
- explain or model an activity;
- clarify vocabulary;
- guide students through a concept or procedure;
- give students opportunities to share ideas, strategies, and solutions;
- allow students to report discoveries;
- promote the development in students of the skills they use in collaborating, especially their language and social skills;
- introduce the proper use of a manipulative;
- introduce assessment criteria for a specific task;
- ensure that all students receive the same message;
- summarize findings and make generalizations;
- establish classroom norms;

- make connections between mathematics and other subjects and between mathematics and the world around the students;
- defend conjectures;
- discuss issues and solve problems.

The teacher can use *small-group* and *partner work* to:

- maximize student participation;
- give students opportunities to learn from one another;
- provide a structure that encourages students to do a great deal of talking and sharing;
- provide students with immediate feedback from their peers;
- promote risk taking in students as their comfort level increases;
- provide students with opportunities to develop independence and confidence as they help one another;
- provide English language learners with opportunities to work with other students who speak the same first langauge;
- expose students to varying viewpoints;
- reinforce students' skills in cooperating with others (e.g., in listening actively to others, providing constructive feedback to others, and building acceptance and tolerance of others' ideas);
- assess students' learning skills (e.g., group participation, cooperation, abilities in conflict resolution);
- give students the opportunity to consolidate their understanding of a mathematical concept (e.g., by having them play a game as a group);
- observe and make notes;
- have students investigate and explore a mathematical concept;
- encourage students to share and compare strategies;
- foster a sense of community in the classroom;
- encourage students to formulate and defend conjectures.

The teacher can use *independent work* to:

- find out what individual students know;
- help students develop, consolidate, or apply individual strategies or ideas;
- allow students to work at their own pace;
- provide students with the opportunity to develop independence, perseverance, and self-confidence;
- provide students with opportunities to apply what they know;
- provide students with opportunities to make choices;
- improve students' time-management skills;
- work with individuals.

IDEA

To provide students with an opportunity to make choices, have an Early Bird Specials bin filled with enjoyable math activities, games, and puzzles. The students can explore these activities only after their work has been completed.

The teacher can base decisions about the kind of grouping to use on the following factors:

- the type of activity or experience (e.g., whether the use of a manipulative is best explained to the whole class, or discussed by students as they work with a partner, in a small group, or independently);
- the materials available (e.g., some materials lend themselves better to partners or groups);
- the dynamics of the class (e.g., whether students work best in small groups, with partners, or individually);
- the time of day (e.g., students may need to exercise personal choice about how they will spend class time near the end of the day in order to settle);
- the learning goal(s) for the student or group.

THE THREE-PART LESSON FORMAT, GRADES 1–6

(For more information on long-range, unit, and lesson planning, see Chapter 3: Planning the Mathematics Program, in Volume One.)

In the mathematics classroom, students need time to explore; time to meet with one another to share thoughts or ideas and receive feedback; and "workshop" time, in which they focus on a specific task or topic, either in small groups, with partners, or independently.

Most teachers already use some kind of workshop approach when dealing with students' literacy development. The three-part lesson format for math is similar to a writing workshop, in that students have time to practise, to work with peers, to receive and give feedback, to think, to make plans, and to confer with the teacher.

In preparing for the three-part lesson, the teacher decides what task will be performed (or what choices of tasks there will be). Everyone in the class might be engaged in the same investigation, or small groups of students might be involved in different learning centres. The lesson experience has three stages: "Getting Started", "Working on It", and "Reflecting and Connecting".

> *"Children do not acquire an important mathematical idea in a single workshop. Such ideas build day after day and must then be revisited throughout the year to make sure that they continue to flourish."*
>
> (Dacey & Eston, 1999)

Getting Started (10–15 minutes): This part of the lesson usually involves the whole class, but it may involve just a small group of students who need reinforcement of a specific concept, or need to extend their knowledge or skills in a particular area. One goal of this part of the lesson is to ensure that students have a practical understanding of what will be happening (e.g., the routine they will follow, the materials they will be using, the way the groups will be set up, where the groups will work). In this phase also, the teacher might review or present a concept (e.g., even or odd numbers, prime numbers) before having students

solve a problem related to the concept. Another goal is to ensure that students understand the problem that they will be working on, or the investigation that they will be performing. For example, if students will be working in small groups to solve a problem, the teacher might pose part of the problem, or pose a similar but smaller problem. Students would discuss strategies and then try the smaller problem before joining their group to work on the larger problem.

Working on It (30–40 minutes): In this part of the lesson, students are actively engaged in a mathematical task that has a clear connection with the curriculum expectations. Working on the task may take the form of shared, guided, or independent learning. Students work in small groups, with partners, or individually to solve a problem or to perform an activity, and record the results. While the students are working on their own, the teacher circulates throughout the room, making observations and providing guidance as needed. The teacher observes how students are interacting, takes note of the mathematical language students are using, and provides an extension to those who finish the task early. Students develop independence and confidence by choosing the strategies and materials that they will use, as well as methods of recording solutions. Students are given adequate time to solve the problem, so that they learn to develop perseverance and come to see that problems take time to solve – their solutions are never immediately apparent.

TIP: Over a period of time (one to two weeks), have students work together to solve various problems related to a topic. Then, use learning centres to review and consolidate students' learning.

Reflecting and Connecting (10–15 minutes): In this phase of the three-part lesson, students share ideas, solidify understandings, and make generalizations. The teacher pulls the whole class together at the meeting area and facilitates the discussion, inviting students to share the strategies they used, asking if others used the same or different strategies, and adding to the strategy and word walls. During this phase, the teacher must question and probe to assess students' understanding, to bring clarity to the math, and to identify misconceptions. What the teacher learns from students about their understanding is directly related to the types of questions that are asked. What the teacher learns from this discussion will guide the direction of future lessons or activities.

IDEA

Put each student's name on a card. When it is time to share, draw three or four names from the container of cards (do not replace any cards until all have been removed). This ensures that each student has a turn to share with the class at some point during the week.

The following are examples of questions that teachers can use as they try to assess students' understanding and elicit the strategies that students use to solve problems.

- "Who can explain in his or her own words what _____ said?"
- "How did you count the number of _____?"
- "Did you notice any patterns in your data?"
- "How did you make your estimate?"
- "What part of this pattern-block design is growing?"

- "What did you discover about _____ (shapes, area, perimeter, arrays)?"
- "How can you sort these _____ (shapes, numbers)?"
- "Why did you use that strategy?"
- "Did anyone use a different strategy?"
- "How did you organize your work?"
- "What materials did you use? Why? How do they show the math that we are learning?"
- "What does this problem remind you of?"
- "When might you use this information?"
- "If you had a problem while you were working, what steps did you take to solve it?"

Giving students the chance to share their thoughts or ideas has many benefits. First, when students reflect and connect, they make sense of the mathematics they are studying. They are able to articulate their thinking and to receive feedback. Second, when students are exposed to the ideas and strategies of others, which might not always be the same as their own, they learn to develop different ways of approaching problems and begin to integrate their ideas with those of their peers and the teacher. Third, the teacher may facilitate a discussion that helps students see the connections between the mathematics and their world. Fourth, students learn to talk mathematically. Last, they learn that the mathematics they have been working on is important enough to discuss.

For an example of how the teacher might proceed in leading students through the "Getting Started", "Working on It", and "Reflecting and Connecting" stages of a lesson, see pages 10–25 of Chapter 5: Problem Solving, in Volume Two. The problem-solving vignettes are examples of a three-stage math lesson. For various other examples, see the learning activities provided in the companion documents to this guide that focus on the individual strands.

IDEA

Post a large class list with names of students down the left side and names of centres across the top. When students complete a centre activity (and show it to the teacher), they can place a check mark beside their name and under the centre name.

MATHEMATICS LEARNING CENTRES, GRADES 1–6

A learning centre is an area in the classroom where students can take part in small-group or independent activities that will help them to fulfil a specific curriculum expectation. At a properly organized mathematics learning centre, students engage in useful problem-solving tasks and consolidate previously taught skills.

At each mathematics learning centre, relevant materials are provided, and routines are established for their proper use. Instructions for the centre are clearly written and placed at the centre or posted nearby. The teacher accommodates those who are not yet reading by explaining orally what students are required to do at the centre. To help students keep a record of what they have been working on in the mathematics learning centre, the teacher can give each student a tracking sheet (see Appendix 7-12).

The following are examples of mathematics learning centres that could be used to help primary students consolidate their learning about symmetry:

1-3

Centre 1: Students work in small groups or as partners using pattern blocks to create different shapes that have reflective symmetry. They create designs that have vertical symmetry, horizontal symmetry, and both vertical and horizontal symmetry. They must take turns adding pattern blocks one at a time. Students record their designs on triangle dot paper.

Focus question: How do you know your shape has symmetry?

Centre 2: Students use geoboards and elastic bands to create different symmetrical shapes with vertical, horizontal, and diagonal lines of symmetry. Students record these shapes on dot paper.

Focus question: How does folding the dot paper help show that the design is symmetrical?

Centre 3: Students use toothpicks, glue, and construction paper to make all the letters of the alphabet that are symmetrical. Students form each letter from toothpicks, glue it onto the piece of paper, and then show its line of symmetry.

Focus question: How did you check that the letters were symmetrical?

For an example of student instructions for primary students at a mathematics learning centre (Centre 2 above), see Appendix 7-13.

The following are examples of mathematics learning centres that could be used to help junior students consolidate their learning about area and perimeter.

4-6

Centre 1: Each pair of students is given ten to fifteen pattern blocks. Working as partners, they use the blocks to create different shapes. Each time they make a shape, they must use all the pattern blocks they have been given, and the shape they make must differ from the other shape(s) they have made. After creating each shape, they record it on pattern block triangle paper and determine the perimeter of the shape, using the side length of the triangle as the unit of measure.

Focus questions: Do shapes that have the same area have the same perimeter? What types of shapes have the largest perimeters?

Centre 2: Students use geoboards and elastic bands to make shapes having different areas but the same perimeter. In the shapes they make, they must find the shape with the largest area. Students record each solution on dot paper, labelling the area and the perimeter of each shape on the paper as well.

Focus question: What strategies did you use to find the shape with the largest area?

Centre 3: Students use interlocking cubes to estimate the area of irregular shapes. First, students take twenty cubes and scatter them on a sheet of paper. Then they draw a shape that captures all the cubes within it – that is, they create an irregular shape. They work together to estimate and then determine the area of the shape.

Focus questions: How did you estimate the area of the shape? How did you determine the area of the shape? Was your estimate close? Explain.

For an example of instructions for junior students at a mathematics learning centre (Centre 2 above), see Appendix 7-14.

OTHER INSTRUCTIONAL OPPORTUNITIES

Mathematics learning centres are one way to consolidate mathematical learning. During everyday routines, other opportunities occur that can help teachers reinforce mathematical ideas in a meaningful context. Among these opportunities are "Calendar Time", "Math Moments", and "Math-aerobics".

Calendar Time in the Primary Grades

1-3

Students can experience the best of times or the worst of times in learning about the days, weeks, and months through calendar-based activities. They experience the worst of times when they passively sit through rote counting and rote verbal routines that focus on giving the daily date, weather, and temperature or when they simply repeat memorized verbal exercises (e.g., "Today is Wednesday, yesterday was Tuesday, tomorrow will be Thursday") rather than participate in thoughtful, creative learning that helps them achieve significant mathematical understanding.

Students can experience the best of times when they move beyond the rote learning of mathematics. Indeed, the calendar can be used as a basis for rich learning opportunities through which students can develop new skills and concepts within the secure familiarity of calendar time. For example, the teacher can use counting activities in Kindergarten to emphasize number recognition and representation (particularly by sorting out the frequently confused numerals of 2 and 5, 6 and 9, 12 and 21, 13 and 31) and the "tricky teens". If these counting activities are combined with clapping, chanting, and marching in place, and emphasis is kept on the "tricky teens" and the decade changes, students will establish and reinforce their knowledge of higher numbers, and of the consistencies of the base ten counting system.

The following are some ideas for using the calendar to teach and reinforce mathematical concepts:

- During a "Number of the Day" activity (corresponding to the date), students tell how many ways they can make that number or show that number, or what they know about that number.
- The teacher keeps some numbers on the calendar hidden, so that students can make predictions about which number will come next.
- The teacher marks special events on the calendar. Students figure out how many more days are left until someone's birthday, a field trip, and so forth.
- For each day, students figure out how to make the number with coins (e.g., for May 7, two pennies and one nickel, or seven pennies).
- The teacher creates a weather graph, so that students can keep track of the weather each day. Students build a pictograph to represent the weather that month.
- The teacher can use a variety of materials to teach place value by counting to "Hundreds Day", a common classroom celebration of the hundredth day of school (e.g., bundle craft sticks with elastic bands, group pop-can tabs or pipe cleaners, stack interlocking cubes, or even count pennies and exchange them for other coins, asking, "What's a fair exchange for 10 pennies? for 20 pennies?").
- The teacher creates patterns on a calendar and uses them to teach counting by 2's and 5's, and in later grades, by 3's and 4's.

The teacher need not use the calendar every day for calendar-based instruction to be effective. In fact, when the calendar is used less often, but more purposefully, it can become a very effective tool for addressing several areas of the curriculum.

Math Moments

1-3 4-6

Teachers need to show students how they are using mathematics in their daily lives, so that students will grow to appreciate the importance of learning mathematics and will come to develop confidence when they discover the extent to which they are already using math skills every day. Teachers look for math moments and use them to reinforce and further develop the math skills that students are already using.

The following are a few types of math moments and the questions that the teacher might ask in connection with them or the activities that the teacher might propose:

- **language moments**
  - "Make your body as *long* as you can, as *short* as you can, as *wide* as you can, as *narrow* as you can."
  - "Find a straight line to walk on."
  - "Find the longest straight line in the gym and walk on it. Now walk from one straight line to another on a diagonal."

- "Walk in a circle."
- "Walk in a zigzag line."
- "Walk an angle." (90°, 45°)
- "Stretch out to take up the largest area possible."
- "Shrink to take up the least area possible."
- "Find a regular polygon in the classroom."
- "Estimate the distance that you kick a soccer ball [throw a football] in gym class. I'll use a benchmark – the length of my pace [length of my foot] – to judge the reasonableness of your estimate."
- "Let's say that today our clock is a 24-h clock. At what time will recess begin?"
- "Estimate how many times bigger [smaller] your hands [feet] are than those of your reading buddy."

- **transition times (e.g., standing in line, waiting for a bell to ring)**
  - "Estimate how high you can count before the bell rings [or how high by 1's, 2's, 5's, 10's]. Then choose one method of counting and check your estimate to see if it is correct."
  - "See how many 'twos' you can find on your body." (eyes, ears, hands, eyebrows, elbows)
  - "See how many 'ones' you can find." (nose, face, chest)
  - "What other quantities do you have on your body?" (ten fingers, ten toes, hundreds of freckles, thousands of hairs, millions of cells)
  - "How many shapes [circles, ovals] can you find on your body?"
  - "Who has a pattern on his or her shirt? Describe the pattern."
  - "Let's line up in a pattern. Who has an idea for the pattern?"
  - "Make a 'human graph' when you line up. Let's line up by eye colour [hair colour] today. First, guess which line will be longest, and then line up and check."
  - "Line up in 10's. Then find out how many even ways there are to line up."
  - "Line up according to shoe size. Can you see whether a person's height is somehow related to that person's shoe size?"
  - "Estimate how many buttons are on all the articles of clothing that the class members are wearing today."
  - "Estimate how long it will take for all of you in the class to be changed and ready for gym class. I'll time you with a stopwatch, and you can determine the difference between your estimate and the exact amount of time."
  - "Let's line up in pairs. Discuss with your partner different polygons or angles that you see."
  - "How many seconds will you be out for recess? Use mental math to find your answer."
  - "What is the ratio of sneakers to dress shoes [girls to boys, long sleeves to short sleeves]?"

It is important for teachers to keep looking for these mathematical moments throughout the day and to capitalize on the mathematics involved. In no time, students will also be finding mathematical moments to share with one another.

"Math-aerobics"

1-3 4-6

A "math-aerobic" is a more structured version of a math moment. It is an opportunity for students to discuss math in an enjoyable way. Math-aerobic activities can require some planning and preparation on the part of the teacher, but they are meant to be done quickly by students. They reinforce and revisit concepts from earlier in the year, or concepts related to the area of focus at the time. They can be used at the beginning of a math session just to get students thinking, or at the end of a group discussion when a few minutes remain before dismissal. They can be used first thing in the morning or first thing in the afternoon, as a means of awakening the brain and providing more math time. The following are examples of "math-aerobics":

1. **Where's the Math?** The teacher takes a large newsprint scrapbook and glues a variety of pictures from children's magazines inside the book, one picture per two-page spread. The teacher opens the book to a page and asks students to explain where in the picture they could find some mathematics to do. The teacher encourages students to think about math in its entire scope and really stretch their thinking. For example, students may say, "Measure the height of the tree," "Take the temperature of the water," "Find the patterns in the quilt squares," "Sort and count the animals in the picture," "Find examples of shapes with different lines of symmetry," "Identify as many quadrilaterals as you can," "Estimate and find the size of angles," or "Describe how many times smaller an object in the picture is compared to its real-life size."

   This activity encourages the students to:
   - think abstractly and look for the mathematical concepts, not the obvious facts;
   - begin to take ownership of their own learning and to be able to recognize all strands of mathematics;
   - recognize mathematical terminology and vocabulary;
   - explain their thinking through reasoning.

2. **Math Maze.** Using a large hundreds chart located in the meeting area, the teacher takes students through a mathematical maze. The students follow the maze path on the large chart and/or on individual hundreds charts at their desks. The teacher chooses a starting number on the chart and then guides students to another number by using the words *up*, *down*, *left*, and *right* (e.g., begins at 25, moves two spaces to the right [26, 27], moves one space up [17], moves four spaces to the left [16, 15, 14, 13], moves three spaces to the right [14, 15, 16]). In conclusion, the teacher asks, "What number did the math maze take us to?" In Grades 4–6, the teacher could use a similar activity. The teacher chooses a starting number on

the chart and has students perform a series of operations (e.g., multiply by 2, add 55), ending at a "mystery number".

Activities that involve the hundreds chart encourage students to:

- use directionality;
- become familiar with a hundreds chart;
- see patterns in a hundreds chart;
- create their own math mazes;
- see the hierarchy in the hundreds chart – the single digits are at the top of the chart, the 50's are in the middle, and the 90's are at the bottom.

3. **Number of the Day.** This activity can be used in conjunction with calendar activities. It is a simple activity that gets students thinking about how numbers are put together and about the different ways in which numbers can be represented. For example, if it is May 12, then the number of the day is 12. Ask students to tell as many things about the number 12 as possible (e.g., one dozen, 6 + 6, three groups of 4, age of someone's brother, edges on a cube). For junior students the number of the day would be more complex and could include fractions, decimals, or integers.

   Activities that involve the number of the day encourage students to:

   - make connections between the strands of mathematics;
   - discuss complex or simple facts about a number;
   - develop problem-solving and reasoning skills.

4. **Puzzle/Game Time.** This activity is a good way to get students thinking about relationships between numbers and about using numbers to solve problems. For example, in making magic squares students manipulate a set of numbers so that the sums of all the rows, columns, and diagonals of a 3 by 3 square are equal. Commercial games of backgammon, chess, and battleship allow students to develop logical-thinking, problem-solving, and reasoning skills.

Math Connections

It is important that students experience and see mathematical connections during mathematics time and throughout the rest of the school day, as well as in their daily lives. As students make connections between mathematical concepts and ideas and make connections within and across strands, their understanding is enhanced and learning mathematics becomes more meaningful. Connections that should be highlighted include:

- connections between mathematical strands and concepts. For example, when students are skip counting (Number Sense and Numeration), they notice that they are also creating a pattern (Patterning and Algebra), and when they are exploring the properties of geometric figures, they notice that they are considering the type and size of angles;

- connections with other subject areas. For example, students see that they are applying skills and understanding of measurement when they measure distance and time in physical education, and they apply their data management skills when they create organizers for information collected in a science experiment;
- connections with real-world contexts and problems. For example, students use their estimation skills when determining the cost of food for the whole class on a school trip, or they use their understanding of geometric shapes when they are on a walk in the community.

Cathcart, Pothier, and Vance (1997) remind us that "connections are not automatic. Teachers must provide experiences where the connections are 'obvious' or at least where they can be made explicit" (p. 26). Because young students in particular may not recognize mathematics connections as they arise in other subjects or in their daily lives, it is important that teachers highlight such connections – for example, by saying, "Look at how we are using our graphing skills from math when we compare the heights to which our plants grew this month."

While integration is often considered an effective way of highlighting the connections between mathematics and other subjects, "a separate math time is necessary each day. Dedicated math time provides your students with time to investigate and develop their understanding of math concepts" (Elementary Teachers' Federation of Ontario, 2004). Integration provides opportunities for students to apply the understanding and skills that they have previously explored and developed during a separate mathematics time.

Structuring a Kindergarten Mathematics Class

K

TIME CONSIDERATIONS

> *"On the basis of what we know about young children's learning, mathematics in Kindergarten must be active, hands-on, child-centred, and problem-based. Concrete materials provide children with tactile experiences to help them explore and describe mathematical problems and solutions."*
>
> (Ontario Ministry of Education, 2006, p. 41)

The Kindergarten timetable, like the timetable for Grades 1–6, should allow time for developmentally appropriate teacher-guided activities, for student-selected tasks, and for partner/small-group interaction that centres on problem solving, critical thinking, and reasoning. As indicated in the report of the Expert Panel on Early Math in Ontario (2003), there should be approximately 20 minutes of focused mathematics time every day. For students attending Kindergarten on alternate days, there should be 40 minutes every day. To ensure that students can focus long enough and remain actively

engaged, the 40 minutes should be broken up. At least 20 minutes should be focused mathematics time, and the remaining mathematics time should be embedded in the regular Kindergarten routines.

In Kindergarten, it is important that students have time to explore freely. When they are given responsibility for making choices, they become more confident learners, they gain independence, and they develop skills in making decisions. The Kindergarten math program should therefore allow for both focused math activity time and student-selected exploration time.

Focused Mathematics Time

There are several ways in which mathematics could be the focus for approximately 20 minutes. One way is to follow the three-part lesson format described for use in Grades 1–6, but in a much shorter time frame. The following are a few examples of what the focused mathematics time could look like.

Sample One: Working With Patterns

Getting Started (5 minutes): If the class is working on patterns, this time could be used to clap, snap, and tap different patterns (e.g., the teacher gives the pattern – clap, clap, snap, clap, clap, snap, clap, clap, snap – and asks, "What is the pattern? What would come next? How do you know? What is a pattern?"). Alternatively, students could help create patterns with their shoes (e.g., running shoe, dress shoe, . . .) or other objects in the classroom (e.g., pencil, pencil, eraser, . . .).

Working on It (10–15 minutes): After warming up by making and talking about several patterns, students are sent to tables to work with different manipulative materials and are directed to use the materials to create a pattern. Students might use interlocking cubes, pattern blocks, shells, buttons – anything that can be used to create repeating patterns will do. The teacher reminds students that they will have to share their pattern and tell what type of pattern they have made. Knowing that they are expected to communicate about their work will help students focus on the pattern they are creating.

Reflecting and Connecting (5 minutes): At this time the teacher selects a few students to share the pattern they have created and to tell what type of pattern it is. The teacher involves other students by asking questions (e.g., "Who else created a pattern like this?").

Sample Two: Counting

Getting Started (5 minutes): As a whole class, students practise counting different things (e.g., the number of boys or girls in the classroom, the number of steps to the gym). The teacher could count and leave out a number. Students must state which number is missing. Or, the students play a game of Simon Says, with instructions like this: "Simon says pat your head 3 times." Students count while performing the task.

Working on It (10–15 minutes): Students return to their seats to work with a partner or small group. Each pair or group has a bin of materials (commercial or found objects). The teacher gives each pair or group a card with a number on it and instructs the students to make a pile of objects equal in number to the number they received on the card. When students have made their pile of objects, they trade piles with another pair or group and check for the correct number of objects.

Reflecting and Connecting (5 minutes): The teacher brings students back to the meeting area and has students discuss strategies they used for counting the objects (e.g., "We moved each spider over here as we counted spiders"). To follow up and keep students engaged and active after a period of working at their desks, the teacher could lead a song about counting or read a story about counting. Using literature is a great way to get a lesson started and is useful in helping students to reflect and connect.

For various other examples of the three-part lesson format, see the learning activities provided in the companion documents to this guide that focus on the individual strands.

Embedded Mathematics Time

When students are on an alternate-day schedule, the additional 20 minutes of mathematics time is embedded in the regular Kindergarten program, for example, through learning centre activities, which offer many possibilities for exploring mathematics; or through math moments, which give teachers the opportunity to show students how they are using mathematics in their everyday lives.

The Kindergarten teacher takes advantage of calendar time and math moments (including math-aerobics) to reinforce mathematical concepts, to review strategies and skills used for solving problems, and to keep alive the curiosity and love of mathematics that young students naturally possess.

The additional 20 minutes of mathematics can also be embedded in the Kindergarten program or in student-selected activities, which can generate many mathematical discussions and can lead to more focused mathematical lessons and activities.

The teacher gives students time to explore manipulatives before they use them in a mathematics activity, to ensure that students understand how to work with them and are ready to focus on the activity as opposed to wanting to play with the manipulatives. At other times, the teacher gives students a choice of activities, and they make decisions about the activity they will explore. Making decisions such as these helps students develop confidence and independence.

Not all student-selected choices will necessarily have a math emphasis (e.g., the dress-up centre may not lend itself to math, although even here students will discuss such terms as *too big, smaller, larger*), but many will. The following are examples of such activities that have a math focus:

- Blocks – students learn concepts such as "how high", "how much higher", "how many".
- Computers – students explore problem-solving strategies.
- Sorting bins – students explore and sort various manipulatives and found objects, and explain their reasoning.
- Board games – students develop one-to-one counting skills, skip-counting skills, cooperation skills.
- Store – students learn about money and three-dimensional geometry (e.g., by using soup cans, cereal boxes).
- Chalkboard – students practise writing and representing numerals.
- Magnetic numerals – students practise ordering and representing numerals.
- Puzzles – students develop spatial sense and the beginnings of transformational geometry (turns, flips, slides).
- Shelf activities – students work on a specific concept (e.g., making tens) by engaging in a teacher-developed activity.

INTEGRATED LEARNING EXPERIENCES IN KINDERGARTEN*

> *"Problem solving and reasoning that involve the "big ideas" of mathematics are the foundations of mathematics in the Kindergarten program. Rich mathematical problems involve important mathematical ideas and arise out of real-life situations, and can be approached in a variety of ways so that all children can be involved in exploring solutions. Solving such mathematical problems requires persistence, since they do not have one easy-to-find correct answer. Through active participation in mathematics investigations, including problem solving and discussions, children develop their ability to use mathematics as a way of making sense out of their daily experiences."*
>
> (Ontario Ministry of Education, 2006, p. 40)

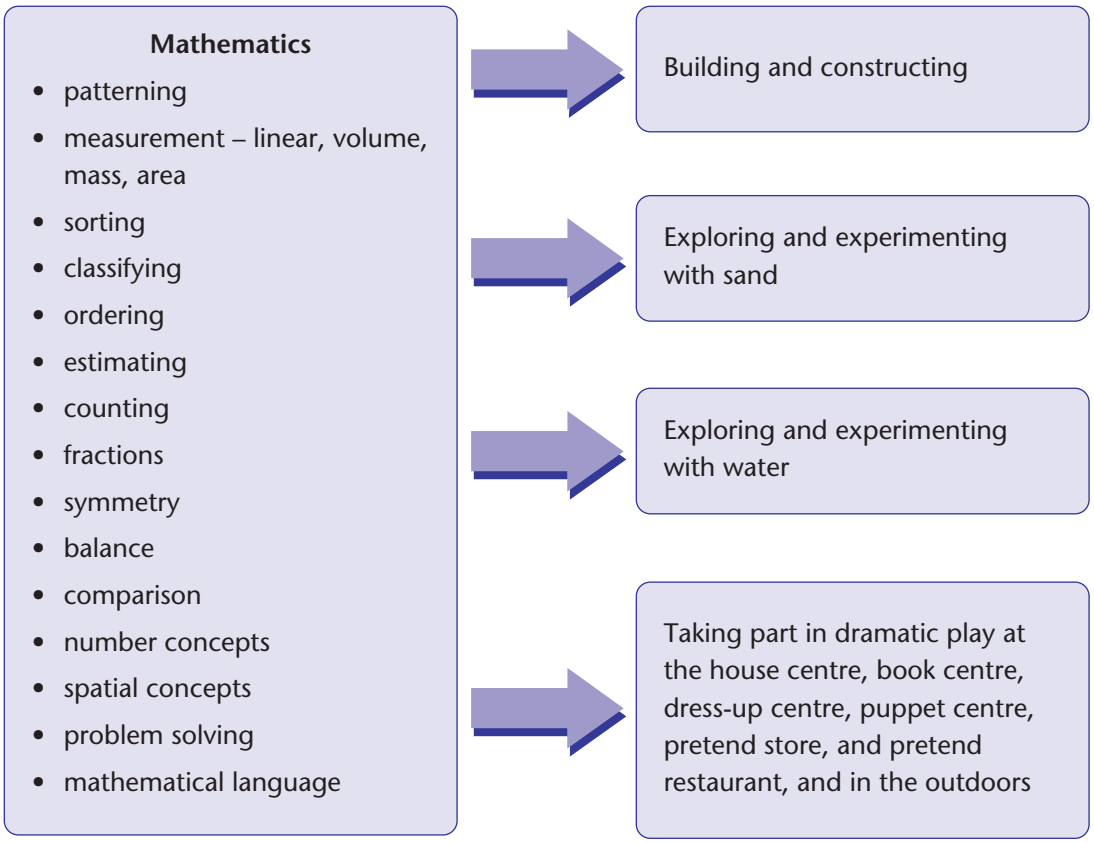
* This section (pp. 55–60) is adapted, with permission, from Toronto District School Board, *Kindergarten Documents*, 2002.

To maximize intellectual, social, and personal development, students need opportunities to:

- learn in a safe, organized, and stimulating environment;
- participate in appropriate, planned experiences that encourage risk taking, build confidence, and ensure success;
- interact with interested adults;
- explore, investigate, discover, and repeat experiences;
- engage in an activity for an extended period of time;
- develop responsibility and independence;
- use quality materials that promote inquiry, discovery, and problem solving;
- work with others in a variety of learning situations;
- participate in decisions related to the learning environment;
- initiate activities and make their own decisions.

In a developmentally appropriate program, young students will engage in mathematics many times during the day, in many areas of the classroom. The following chart lists concepts and skills in mathematics that students at play in various learning centres in the classroom can explore and engage in.

Mathematics at Learning Centres



Planning and the Role of the Teacher

Integrated learning opportunities need to be planned to encourage spontaneity and to give students the freedom to plan, organize, and produce independently. The teacher's role is to set up an environment that stimulates students to create and problem solve; that encourages them to incorporate new ideas and learning into their play; and that provides the necessary scaffolds to extend the learning.

> "Initially, children engage in exploring and 'messing about' with materials that have been selected by the teacher. This phase is pure inquiry as children make observations about the materials and processes, ask questions and become motivated to find out more. Although this phase is extremely important, children need to move beyond this phase to develop knowledge and skills. It is one of the artistic dimensions of teaching to know when to engage children further. At the next level, through interactive questioning, discussion and co-constructing information with the child, the teacher guides/leads the children towards an understanding of concepts and processes. Lastly, children need opportunities to apply what they have learned in a variety of ways through activities selected or co-selected by teacher and child to consolidate knowledge and skills. This is the place where there are challenges, e.g., to make a tall tower that will stand. The engagement at this level should signal the beginning of the inquiry phase again as new ideas and the need for exploration emerge."
>
> (Weeks, 1997, pp. 53–54)

Materials

The variety and scope of the materials at each learning centre are factors in determining the nature and extent of the play, creativity, and learning possible at the centre. The materials alone, however, do not ensure learning. The teacher must step in to help students choose materials, to support students by making appropriate comments, and to pose problems.

In giving more direction to students' choice of materials, the teacher narrows down the concepts or skills that the students can explore. For example, some materials lend themselves to experimentation, investigation, and problem solving; some more readily to patterning; some to creativity and dramatic play. ***Note:*** **Materials should be introduced gradually, to allow students to become familiar with the possibilities of a few materials at a time and to avoid overwhelming students initially with too many choices. Materials need to be changed regularly, to reflect changing interests and increased ability levels.**

A list of materials to use at each of the four types of learning centres is given in the following table.

Materials for Use at the Four Types of Learning Centres

K

Building and Constructing Materials

- large wooden blocks, coloured cubes, base ten blocks, Cuisenaire rods (coloured relational rods), marble runs, colour tiles, dominoes, Polydron sets, straws, interlocking cubes, pattern blocks, geoboards, attribute blocks, commercial building sets
- film canisters, spools, cylinders, corks, plastic planting pots (different sizes), plastic or paper cups, craft sticks, lids, bottle caps, durable boxes (different sizes)
- plastic animals (e.g., zoo, farm, prehistoric, story characters, bear counters), golf tees, people, familiar signs (e.g., STOP, railway), vehicles (e.g., cars, trucks, airplanes), beads, shells, dollhouse furniture

Sand Materials

- plastic measuring spoons, medicine measures, graduated containers of different shapes, litre containers (e.g., empty, clean milk cartons; empty sugar bags)
- different-sized plastic containers (e.g., yogurt, cottage cheese), plastic planting pots, different kinds of balance scales, various sand timers, funnels
- shovels, sieves, jelly moulds, cake pans, muffin tins, cookie cutters, craft sticks, animals, people, vehicles

Water Station Materials

- plastic measuring spoons, medicine measures, graduated containers of different shapes, litre containers (e.g., empty, clean plastic milk jugs; empty, clean plastic juice jugs)
- different-sized plastic containers (e.g., yogurt, cottage cheese), plastic planting pots, different kinds of balance scales, funnels
- shovels, sieves, jelly moulds, cake pans, muffin tins, craft sticks, animals, people, vehicles

Dramatic-Play Materials

Note: **Themes for centres should vary throughout the year and change according to classroom themes and the interests of students. The following are examples of possible centres:**

- restaurant – menu, cardboard for signs, order pads, paper for bills, money
- airport – paper for signs, maps, money, used tickets, luggage, luggage tags
- home – message pad, magnetic letters, numbers, telephone book, books, magazines, catalogues, clock, calendar, telephones, photo album, cookbook

Assessment

While students are involved in integrated learning experiences, the teacher has the opportunity to observe their social interaction, development of self-awareness, independence, language levels, problem-solving strategies, and knowledge and application of specific mathematics content.

Observation involves:

- watching what students do;
- listening to what students say;
- recording what students do and say;
- observing how students use the available materials;
- analysing information;
- reflecting on information to plan future programming.

Observation allows the teacher to:

- see growth over time;
- identify patterns of behaviour;
- determine the scaffolding required to extend each student's learning;
- make necessary changes to materials and equipment;
- plan developmentally appropriate activities based on curriculum expectations;
- provide opportunities for students to explore diverse activities.

Observation needs to be planned. While daily informal observations are ongoing, at the point of occurrence, specific observations need to be planned. Teacher time needs to be set aside to observe selected students for previously identified purposes, and, over a specific time period, to observe all students.

These purposes will vary according to the time of year and students' identified needs. **The interpretation of data collected through observation helps the teacher plan a program that meets students' identified needs and interests in relation to curriculum expectations.**

Reflecting and Celebrating

All learners need opportunities to reflect on their experiences in a variety of ways in order to make personal connections and deepen understandings.

The teacher needs to schedule time for sharing understandings and celebrating accomplishments. Sharing may occur with a partner, in a small group, or in a large group.

To help students share and celebrate, the teacher may:

- have a walkabout, leaving structures up for a period of time, so that students see what others have built and view structures from other perspectives;
- during sharing time, have students talk about their activity, any problems they encountered, and how they solved the problems, allowing time for others to ask questions;
- invite other classes to visit, so that students can share their work;
- invite students to make pictures or write about their experiences;
- make a class book with photographs of students' experiences, constructions, and so forth;
- display photos of students' work in the halls for others to see;
- make a video of students working, to share with parents on curriculum night.

Appendix 7-1: Math Survey for Kindergarten and Grade 1

K 1-3

Name: _____ Date: _____

What I Think About Math **Draw a picture or write words to complete the phrase in each box.**	
Math is like ...	Math makes me feel ...
When I do math, I use ...	When I solve problems, I ...

Appendix 7-2: Math Survey for Grades 2 to 6

1-3 4-6

Name: _____ Date: _____

To help me get to know you better, and to help make math class the best it can be, please answer the following questions as completely as you can.

1. Tell me about yourself. What do you like?

2. How do you feel about math?

3. What have you done in math before that you liked?

4. What do you know about math? What does math involve? What types of things are done in math?

5. What do you want to know about math? What would you like to learn?

6. What types of tools or manipulatives have you used in math class before? (Circle the ones you have used.)

calculators	computers	base ten blocks
geoboards	three-dimensional models	pattern blocks
tangrams	Miras (red plastic transparent tools)	number cubes (dice)
Polydron pieces	protractors	interlocking cubes
fraction pieces	polygon set	

Others: _____

_____

7. Do you like to solve problems in math? Why or why not?

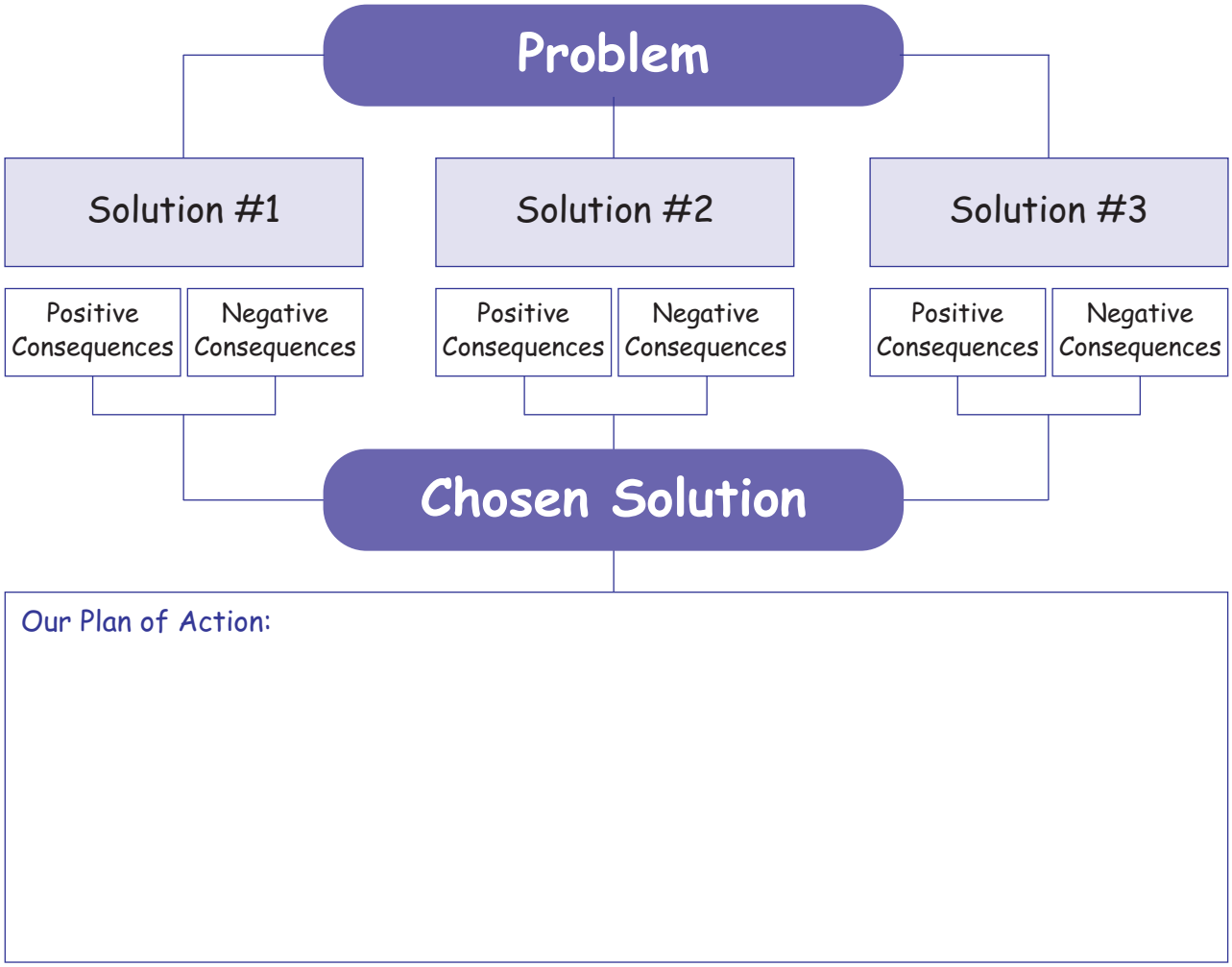
8. Have you ever worked in a math group? Tell me about a time when you solved a problem with others.

Thank you for this information.

Appendix 7-3: Checklist for Class Meetings

1-3 4-6

1. Have I set aside enough time to hold a class meeting? (20–30 minutes)
2. Have I phrased the issue/topic in a question format?
3. Have I anticipated student responses/solutions and made decisions regarding what I am prepared to accept/allow?
4. Have I taught/reviewed the rules for brainstorming with my class (e.g., All ideas accepted without judgement)?
5. Have I followed a decision-making process: identify the problem; brainstorm possible solutions; discuss the positive and negative consequences of each possible solution; agree on a solution (with the understanding that the issue can be revisited at a later date); and make a plan regarding how to carry out the solution?



Appendix 7-4: Sample Primary Word Wall List

1–3

The following is a partial list of words used at the primary level:

about
above
add
addition
always
area
bar graph
base
below
beside
between
calculator
centimetre
circle
circle graph
combination
comparing
congruent
corner
cube
curve
curved
data
day
different
dime
divide
division
dollar
edge
eight
eighth
empty
equal
estimate
even
face
far
fifth
first
five
flat
flip
four
fourth
full
gram
groups
growing pattern
heavy
hexagon
hour
hundred
hundredth
impossible
in
kilogram
kilometre
large
largest
less than
light
likely
line of symmetry
long
lots
many
mass
maybe
measurement
metre
millimetre
minute
month
more than
multiplication
multiply
near
never
nickel
nine
ninth
number
number line
octagon
odd
one
one-half
one-third
out
oval
over
parallelogram
pattern
penny
pentagon
perimeter
pictograph
possible
prism
probable
probably
pyramid
quarter
rectangle
rectangular
rhombus
roll
round
same
scale
second
seven
seventh
shape
short
side
six
sixth
slide
smallest
sometimes
sort
space
sphere
spinner
square
straight
subtract
subtraction
survey
symmetrical
table
tall
tally
ten
tenth
tetrahedron
thick
thin
third
three
transformation
trapezoid
triangle
triangular prism
turn
two
under
unlikely
Venn diagram
vertex
vertices
week
weight
year
zero

Appendix 7-5: Sample Junior Word Wall List

4-6

The following is a partial list of words used at the junior level:

acute
addition
angle
area
attribute
bar graph
base
bisect
calculator
capacity
centimetre
chance
characteristic
circle graph
circumference
column
composite number
congruent
coordinate
cubic units
data
decametre
decimetre
degree Celsius
denominator
diagonal
diagram
difference
dimension
divide
dividend
division
divisor
equal
equation
equilateral
equivalent
estimate
flip
gram
growing pattern
hectametre
heptagon
hexagon
hundredth
intersect
isosceles
kilogram
kilometre
line graph
litre
mass
mean
measurement
median
metre
midpoint
milligram
millilitre
millimetre
mode
multiple
multiplication
multiply
nonagon
numerator
obtuse
octagon
outcome
parallel
parallelogram
pentagon
percent
perimeter
perpendicular
pictograph
polygon
prime number
probability
proportion
quadrant
quadrilateral
quotient
random
range
ratio
rectangle
reflection
rhombus
right triangle
rotation
row
scale
scalene
scatterplot
similar
slide
spreadsheet
square units
stem-and-leaf plot
subtract
subtraction
sum
symmetry
translation
triangle
turn
Venn diagram
vertex
vertices
volume

Appendix 7-6: Materials in the Primary Mathematics Classroom* 1-3

The following concrete materials should be part of an effective primary mathematics classroom.

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Abacus		X	problem-solving/thinking skills, algebra/patterns, counting/skip counting/one-to-one correspondence, relationships/connections, reasoning, place value, number concepts/operations, estimation
Attribute blocks	X		classification/sorting/making sets, symmetry, reasoning, patterns, data collection/management/graphing/interpretation, number concepts/operations, problem-solving/thinking skills
Balance and weights	X		measurement/scale, money, counting/skip counting/one-to-one correspondence, reasoning, decimals, estimation, data collection/management/graphing/interpretation, reasoning, number concepts/number sense/number systems/whole numbers, classification/sorting/making sets, weight/mass
Base 10 materials	X		place value, money, measurement/scale, fractions, patterns, area, similarity/congruence, classification/sorting/making sets, number concepts/operations, perimeter, relationships/connections, problem-solving/thinking skills
Connecting plastic shapes to build 2-D shapes and 3-D nets (e.g., Polydron sets)		X	classification/sorting/making sets, number concepts/operations, perimeter, counting/skip counting, angles, reasoning, data collection/management/graphing/interpretation, one-to-one correspondence, similarity/congruence, area, problem-solving/thinking skills, spatial visualization, tessellations/tiling, fractions, transformational geometry, measurement/scale

* From *Supporting Leaders in Mathematics Education: A Source Book of Essential Information*, by the National Council of Supervisors of Mathematics, 2000. Adapted with permission.

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Clocks	X Instructional clock	X Clocks for student use	time, fractions, measurement/scale, number concepts/operations, relationships/ connections
Colour tiles		X	patterns, estimation, counting/skip counting/ one-to-one correspondence, number concepts/operations, reasoning, place value, classification/sorting/making sets, fractions, problem-solving/thinking skills, probability/chance, measurement/scale, area, perimeter, odd/even numbers, data collection/management/graphing/ interpretation, spatial visualization, similarity/ congruence, relationships/connections
Cuisenaire rods (coloured relational rods)	X	X	classification/sorting/making sets, counting/ skip counting/one-to-one correspondence, number concepts/operations, similarity/ congruence, fractions, symmetry, place value, patterns, odd/even numbers, reasoning, estimation, problem-solving/thinking skills, relationships/connections
Dice/Number cubes	X	X (JK to Grade 1)	counting/skip counting/one-to-one corre- spondence, number concepts/operations, mental math, fractions, probability/chance, decimals, problem-solving/thinking skills, classification/sorting/making sets, reasoning, data collection/management/graphing/ interpretation
Geoboards (5 x 5 and 11 x 11) and geobands	X (Grades 2 and 3)		size, shape, counting, area, perimeter, symmetry, fractions, coordinate geometry, angles, estimation, similarity, congruence, rotations, reflections, translations, classification, sorting, polygons, spatial visualization, reasoning
Graduated beakers	X		measurement/scale, volume, estimation, fractions, mental math, problem-solving/ thinking skills, number concepts/operations, counting/skip counting/one-to-one correspondence, spatial visualization, data collection/management/graphing/ interpretation, similarity/congruence, reasoning

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Hundreds chart, hundreds carpet	X		place value, counting/skip counting/ one-to-one correspondence, estimation, patterns, number concepts/operations, fractions, probability/chance, odd/even numbers, spatial visualization, mental math, decimals, money, measurement/scale, volume, problem solving, relationships/ connections, decimals, reasoning
Interlocking cubes (1 cm, 2 cm, 2.5 cm)	X		number concepts/operations, counting/ skip counting/one-to-one correspondence, place value, classification/sorting/making sets, patterns, reasoning, symmetry, weight/ mass, spatial visualization, probability/ chance, area, perimeter, volume, quantity, transformational geometry, fractions, estimation, mental math, problem-solving/ thinking skills, money, measurement/scale, relationships/connections
Materials for counting and sorting	X		measurement/scale, patterns, estimation, relationships/connections, place value, counting/skip counting/one-to-one correspondence, estimation, problem-solving/thinking skills, volume, fractions, number concepts/operations, classification/ sorting/making sets, probability/chance, spatial visualization, odd/even numbers, data collection/management/graphing/ interpretation, reasoning
Measuring spoons	X		measurement/scale, estimation, number concepts/operations, counting/skip counting/ one-to-one correspondence, data collection/ management/graphing/interpretation
Measuring tapes		X	measurement/scale, estimation, number concepts/operations, counting/skip counting/ one-to-one correspondence, data collection/ management/graphing/interpretation
Miras (red plastic transparent tools)	X	X	symmetry, transformational geometry, angles, mental math, problem-solving/thinking skills, spatial visualization

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Money	X One set	X Multiple sets	money, counting/skip counting/one-to-one correspondence, classification/sorting/making sets, fractions, probability/chance, problem-solving/thinking skills, estimation, mental math, place value, relationships/connections, data collection/management/graphing/interpretation, reasoning, measurement/scale, decimals, number concepts/operations
Number lines	X		place value, counting/skip counting/one-to-one correspondence, estimation, patterns, number concepts/operations, fractions, probability/chance, odd/even numbers, spatial visualization, mental math, decimals, money, measurement/scale, problem solving, similarity/congruence
Pattern blocks	X		patterns, angles, counting/skip counting/one-to-one correspondence, classification/sorting/making sets, tessellations/tiling, symmetry, area, perimeter, transformational geometry, problem-solving/thinking skills, reasoning, fractions, spatial visualization, data collection/management/graphing/interpretation, measurement/scale, number concepts/operations
Pentominoes		X	geometry, spatial visualization, problem-solving/thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, angles, classification/sorting/making sets, symmetry, transformational geometry, number concepts/operations, area, tessellations/tiling, counting/skip counting/one-to-one correspondence
Playing cards	X		counting, estimation, sorting, number concepts/operations, mental math, problem-solving/thinking skills

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Stackable blocks		X	number concepts/operations, counting/ skip counting/one-to-one correspondence, classification/sorting/making sets, patterns, symmetry, fractions, spatial visualization, perimeter, volume, area, problem-solving/ thinking skills, estimation, transformational geometry, reasoning, money, probability/ chance, measurement/scale
Stamps of various mathematical manipulatives (e.g., pattern blocks, tangrams, clocks, base 10 materials)	X		
Tangrams		X	spatial visualization, problem-solving/ thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, angles, classification/sorting/making sets, symmetry, transformational geometry, number concepts/ operations, area, tessellations/tiling, counting/skip counting/one-to-one correspondence
Thermometers		X	measurement/scale, estimation, number concepts/operations, counting/skip counting/ one-to-one correspondence, data collection/ management/graphing/interpretation
3-D geometric solids	X		area, volume, classification/sorting/making sets, angles, reasoning, measurement/scale, symmetry, fractions, spatial visualization, perimeter, counting/skip counting/ one-to-one correspondence, geometry, area, problem-solving/thinking skills, number concepts/operations, weight/mass, relationships/connections, transformational geometry, tessellations/tiling

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Trundle wheel		X	measurement/scale, estimation, number concepts/operations, counting/skip counting/one-to-one correspondence, data collection/management/graphing/ interpretation
Two-colour counters	X		measurement/scale, patterns, estimation, similarity/congruence, place value, counting/ skip counting/one-to-one correspondence, estimation, problem solving/thinking skills, fractions, number concepts/operations, classification/sorting/making sets, spatial visualization

Appendix 7-7: Materials in the Junior Mathematics Classroom* 4-6

The following concrete materials should be part of an effective junior mathematics classroom.

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Abacus	X		problem-solving/thinking skills, algebra/patterns, counting, equality/equivalence, relationships/connections, reasoning, place value, number concepts/operations, estimation
Attribute blocks	X		classification, symmetry, reasoning, patterns, data collection/management/graphing/interpretation, number concepts/operations, problem-solving/thinking skills, similarity/congruence, angles
Balance and weights	X		measurement/scale, money, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence
Base 10 materials, with transparent overhead set	X		place value, money, measurement/scale, fractions, decimals, ratio/proportion/percent, patterns, area, similarity/congruence, classification, number concepts/operations, perimeter, relationships/connections, problem-solving/thinking skills, equality/equivalence
Calculators with two-line display and overhead calculator	X		problem-solving/thinking skills, algebra/patterns, counting, equality/equivalence, measurement/scale, reasoning, decimals, fractions, money, number concepts/operations, estimation, mental math, place value

* From *Supporting Leaders in Mathematics Education: A Source Book of Essential Information*, by the National Council of Supervisors of Mathematics, 2000. Adapted with permission.

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Connecting plastic shapes to build 2-D shapes and 3-D nets (e.g., Polydron sets)		X	classification, perimeter, angles, reasoning, data collection/management/graphing/ interpretation, similarity/congruence, area, problem-solving/thinking skills, spatial visualization, tessellations/tiling, fractions, transformational geometry, measurement/ scale
Colour tiles		X	patterns, estimation, number concepts/ operations, reasoning, equality/equivalence, place value, classification/sorting, fractions, problem-solving/thinking skills, probability/ chance, measurement/scale, area, perimeter, data collection/management/graphing/ interpretation, spatial visualization, similarity/ congruence, relationships/connections, ratio/proportion/percent
Cuisenaire rods (coloured relational rods)		X	classification/sorting, number concepts/ operations, similarity/congruence, fractions, symmetry, ratio/proportion/percent, place value, patterns, reasoning, estimation, problem-solving/thinking skills, relationships/ connections
Counters (transparent)	X		measurement/scale, patterns, estimation, similarity/congruence, relationships/ connections, place value, counting, estimation, problem-solving/thinking skills, volume, fractions, number concepts/ operations, equality/equivalence, ratio/ proportion/percent, reasoning
Dice/Number cubes	X		number concepts/operations, mental math, fractions, decimals, probability/chance, problem-solving/thinking skills, reasoning, data collection/management/graphing/ interpretation, equality/equivalence

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Fraction kit	X		fractions, classification, spatial visualization, similarity/congruence, mental math, number concepts/operations, perimeter, area, ratio/proportion/percent, relationships/connections, problem-solving/thinking skills, equality/equivalence, estimation, reasoning
Geoboards (transparent 5 x 5 and 11 x 11) and geobands	X		classification, area, perimeter, symmetry, fractions, coordinate geometry, angles, estimation, similarity/congruence, rotations, reflections, translations, polygons, spatial visualization, reasoning
Graduated beakers		X	measurement/scale, volume, estimation, problem-solving/thinking skills, spatial visualization, data collection/management/graphing/interpretation, reasoning
Hundreds chart, hundreds board, ninety-nine chart	X		place value, counting, estimation, patterns, number concepts/operations, fractions, probability/chance, spatial visualization, ratio/proportion/percent, mental math, decimals, money, problem solving, relationships/connections, reasoning
Interlocking cubes (1 cm, 1.8 cm, 2 cm, 2.5 cm)	X		number concepts/operations, place value, patterns, reasoning, ratio/proportion/percent, equality/equivalence, symmetry, weight/mass, spatial visualization, probability/chance, area, perimeter, volume, quantity, transformational geometry, fractions, estimation, mental math, problem-solving/thinking skills, money, measurement/scale, relationships/connections
Measuring cups		X	measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation
Measuring spoons		X	measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Measuring tapes		X	measurement/scale, area, perimeter, constructions, fractions, estimation, number concepts/operations, data collection/ management/graphing/interpretation
Metre stick	X		measurement/scale, area, perimeter, constructions, fractions, estimation, number concepts/operations, data collection/ management/graphing/interpretation
Miras (red plastic transparent tools)		X	symmetry, similarity/congruence, transformational geometry, angles, mental math, problem-solving/thinking skills, spatial visualization
Mirror	X		symmetry, similarity/congruence, transformational geometry, angles, mental math, problem-solving/thinking skills, spatial visualization
Money	X		money, ratio/proportion/percent, counting, fractions, probability/chance, problem-solving/thinking skills, estimation, mental math, place value, relationships/connections, data collection/management/graphing/ interpretation, reasoning, measurement/ scale, decimals, number concepts/operations
Number lines	X		place value, counting, estimation, patterns, number concepts/operations, fractions, probability/chance, spatial visualization, mental math, decimals, money, measurement/scale, problem solving, similarity/ congruence
Pattern blocks, with transparent overhead set	X		patterns, angles, classification, ratio/ proportion/percent, tessellations/tiling, symmetry, similarity/congruence, equality/ equivalence, area, perimeter, transformational geometry, problem-solving/thinking skills, reasoning, fractions, decimals, spatial visualization, data collection/management/ graphing/interpretation, measurement/scale, number concepts/operations

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Pentominoes		X	geometry, spatial visualization, problem-solving/thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, angles, classification, symmetry, transformational geometry, number concepts/operations, area, tessellations/tiling
Plastic polygons (wide variety of triangles and regular and irregular quadrilaterals)	X		classification, ratio/proportion/percent, number concepts/operations, perimeter, angles, reasoning, data collection/management/graphing/interpretation, similarity/congruence, area, problem-solving/thinking skills, spatial visualization, tessellations/tiling, fractions, transformational geometry, measurement/scale
Playing cards	X		estimation, number concepts/operations, mental math, problem-solving/thinking skills
Protractors, including full-circle protractors	X		constructions, angles, measurement/scale
Safety compass		X	constructions, angles, measurement/scale
Scales	X		measurement/scale, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence
Spinners (number, colour)	X		number concepts/operations, decimals, fractions, probability/chance, mental math, reasoning, problem-solving/thinking skills
Stamps of various mathematical manipulatives (e.g., pattern blocks, tangrams, base 10 materials)		X	

Manipulative	Recommended for Every Classroom	Recommended for Every Grade/Division	Connections to Mathematical Concepts and Skills
Standard masses	X		measurement/scale, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence
Stopwatch		X	fractions, measurement/scale, number concepts/operations, relationships/connections
Tangrams, with transparent overhead set		X	spatial visualization, problem-solving/thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, ratio/proportion/percent, angles, classification, symmetry, transformational geometry, constructions, relationships/connections, number concepts/operations, area, tessellations/tiling
Thermometers		X	measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation
3-D geometric solids	X		area, volume, classification, angles, reasoning, measurement/scale, symmetry, fractions, spatial visualization, perimeter, similarity/congruence, area, problem-solving/thinking skills, weight/mass, relationships/connections, transformational geometry, tessellations/tiling
Two-colour counters	X		measurement/scale, patterns, estimation, similarity/congruence, place value, counting, estimation, problem-solving/thinking skills, fractions, number concepts/operations, classification, spatial visualization
Trundle wheel		X	measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation

Appendix 7-8: Found or Collected Objects for the Kindergarten or Primary Mathematics Classroom

K 1-3

Students should have access to a range of other materials that can be integrated into a mathematics environment. The following is a partial list of materials that can be collected for use in the mathematics classroom:

activity cards

adding machine tape

beads

bingo cards

bingo dabbers

bottle caps

buttons

cards and game boards

cash register (play)

containers (e.g., boxes, baskets, egg cartons, jars, tubs)

craft sticks

date stamps

dominoes

egg timers, kitchen timers

finger puppets, regular puppets (for acting out number chants and rhymes)

keys

magnetic, rubber, plastic, foam, or wooden numbers

mathematical puzzles

measuring spoons, cups, tapes

modelling clay

number lines

paper plates, cups, cutlery

pegboards and pegs

pipe cleaners

resealable plastic bags

rice, dried beans, dried pasta pieces

rulers

sand

shells

spinners

stamps

stickers

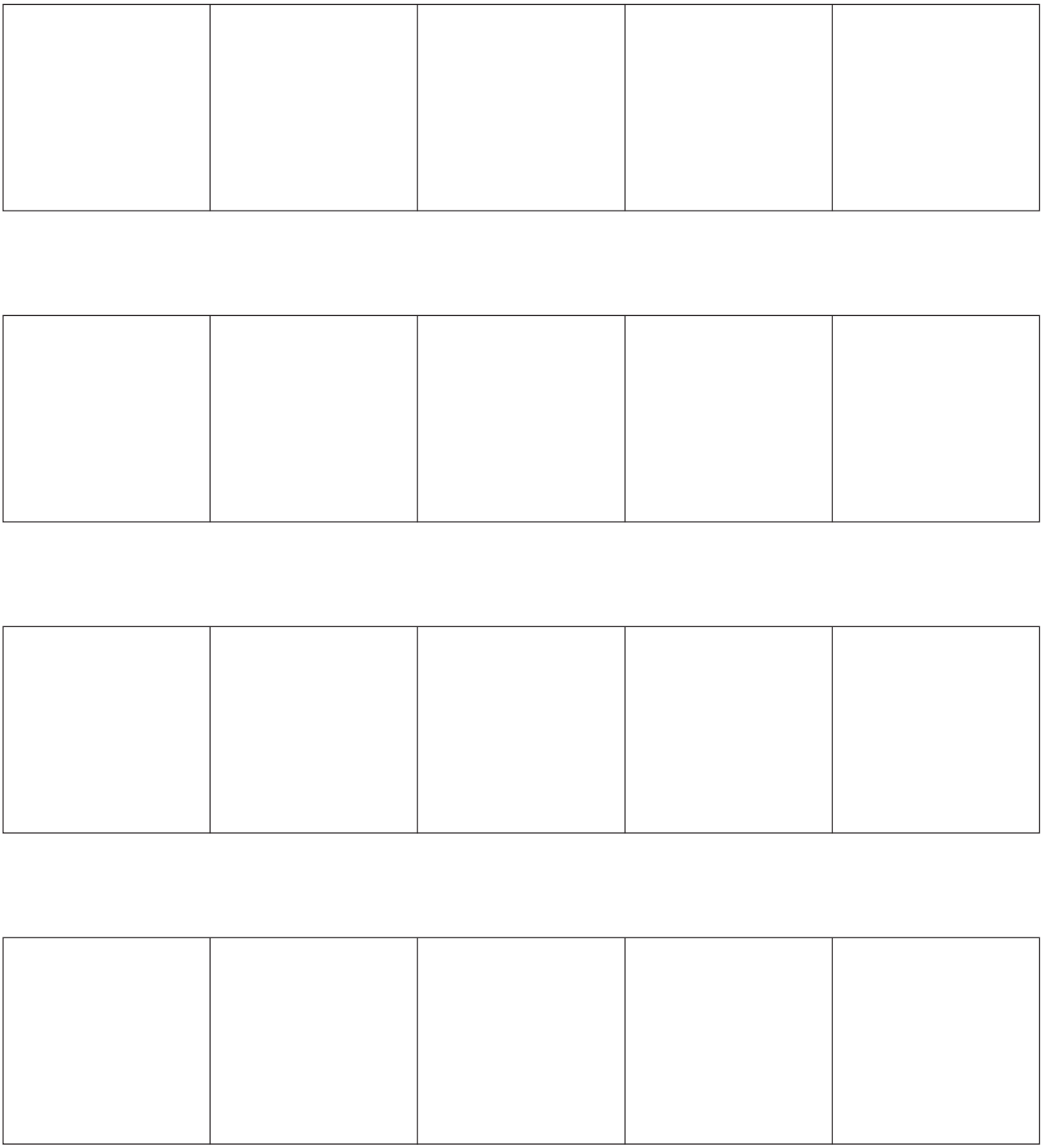
straws

string, yarn

toothpicks

Appendix 7-9: Five Frames

1-3



Appendix 7-10: Ten Frames

1–3

Appendix 7-I I: Literature for Mathematics*

1-3 4-6

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Adding Animals	Hawkins, Colin	•					•	
Alexander, Who Used to Be Rich Last Sunday	Viorst, Judith	•	•				•	
Amanda Bean's Amazing Dream	Neuschwander, Cindy	•					•	•
Among the Odds and the Evens	Turner, Priscilla	•					•	
Anno's Counting Book	Anno, Mitsumasa	•					•	
Anno's Hat Tricks	Nozaki, Akihiro, & Anno, Mitsumasa							•
Anno's Journey	Anno, Mitsumasa		•				•	•
Anno's Magic Seeds	Anno, Mitsumasa	•			•		•	•
Anno's Math Games	Anno, Mitsumasa	•	•	•	•	•	•	•
Anno's Mysterious Multiplying Jar	Anno, Mitsumasa	•					•	•
Anno's Three Little Pigs	Anno, Mitsumasa, & Mori,Tuyosi	•			•		•	
Arthur's Funny Money	Hoban, Lillian	•					•	
Baseball Counting Book, The	McGrath, Barbara B.	•					•	•
Bat Jamboree	Appelt, Kathi	•			•		•	
Bats Around the Clock	Appelt, Kathi		•				•	
Bats on Parade	Appelt, Kathi	•					•	
Benny's Pennies	Brisson, Pat	•					•	
Best of Times, The: Math Strategies That Multiply	Tang, Greg	•					•	
Big Fat Hen	Baker, Keith	•					•	
Biggest, Strongest, Fastest	Jenkins, Steve	•	•					•
Bunches and Bunches of Bunnies	Mathews, Louise	•					•	
Bunny Day: Telling Time From Breakfast to Bedtime	Walton, Rick		•				•	
Bunny Money	Wells, Rosemary	•	•				•	
Button Box, The	Reid, Margarette S.				•		•	
Caps for Sale	Slobodkina, Esphyr	•			•		•	•

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Caribbean Counting Book, A	Charles, Faustin	•					•	
Cats Add Up!	Ochiltree, Dianne	•					•	
Changes, Changes	Hutchins, Pat			•			•	
Chicken Soup With Rice	Sendak, Maurice		•				•	
Circles	Ross, Catherine Sheldrick			•				•
Cloak for a Dreamer, A	Friedmann, Aileen			•			•	•
Counting Crocodiles	Sierra, Judy	•					•	
Counting My Friends	Hooge, Selma	•					•	
Counting on Frank	Clement, Rod	•	•				•	•
Count to a Million: 1,000,000	Pallotta, Jerry	•						•
Domino Addition	Long, Lynette	•					•	
Doorbell Rang, The	Hutchins, Pat	•					•	•
Dots, Spots, Speckles, and Stripes	Hoban, Tana				•		•	
Each Orange Had 8 Slices	Giganti, Paul	•			•		•	•
Eating Fractions	McMillan, Bruce	•					•	•
Eight Hands Round: A Patchwork Alphabet	Paul, Ann Whitford				•		•	
Eight O'Cluck	Creighton, Jill		•				•	
Emily's First 100 Days of School	Wells, Rosemary	•					•	
Emily's House	Scharer, Niko				•		•	
Even Steven and Odd Todd	Cristaldi, Kathryn	•					•	
Feast for 10	Falwell, Cathryn	•					•	
Fish Eyes: A Book You Can Count On	Ehlert, Lois	•					•	
500 Hats of Bartholomew Cubbins, The	Seuss, Dr.	•					•	•
Five Little Monkeys Jumping on the Bed	Christelow, Eileen	•					•	
Five Little Monkeys Sitting on a Tree	Christelow, Eileen	•					•	
Five Little Monkeys Wash the Car	Christelow, Eileen	•					•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Five Little Monkeys With Nothing to Do	Christelow, Eileen	•					•	
Fractals, Googols and Other Mathematical Tales	Pappas, Theoni		•					•
Fraction Action	Leedy, Loreen	•					•	•
Fraction Fun	Adler, David	•					•	•
Frog and Toad Are Friends	Lobel, Arnold					•	•	
Froggy Gets Dressed	London, Jonathan	•					•	
Frogs Jump: A Counting Book	Brooks, Alan	•					•	
From One to One Hundred	Sloat, Teri	•					•	
G Is for Googol	Schwartz, David M.	•					•	•
Go-Around Dollar, The	Adams, B.J.	•					•	
Grandfather Tang's Story	Tompert, Ann			•			•	•
Grapes of Math, The: Mind-Stretching Math Riddles	Tang, Greg	•					•	
Gray Rabbit's Odd One Out	Baker, Alan	•					•	
Greedy Triangle, The	Burns, Marilyn			•			•	•
Grouchy Ladybug, The	Carle, Eric	•	•				•	
Gummy Candy Counting Book, The	Hutchings, Amy & Richard	•					•	
Harriet's Halloween Candy	Carlson, Nancy				•		•	
Hershey's Kisses Addition Book, The	Pallotta, Jerry	•					•	
Hershey's Kisses Subtraction Book, The	Pallotta, Jerry	•					•	
Hershey's Milk Chocolate Fractions Book, The	Pallotta, Jerry	•					•	•
Hershey's Milk Chocolate Multiplication Book, The	Pallotta, Jerry	•					•	•
Hershey's Milk Chocolate Weights and Measures	Pallotta, Jerry		•				•	
Hickory, Dickory, Dock	Muller, Robin		•				•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Hippos Go Berserk	Boynton, Sandra	•					•	
How Big Is a Foot?	Myller, Rolf		•				•	
How Do You Say It Today, Jesse Bear?	Carlstrom, Nancy		•				•	
How Many Bugs in the Box?	Carter, David	•					•	
How Many Feet in the Bed?	Hamm, Diane	•					•	
How Many How Many How Many	Walton, Rick	•					•	
How Many Snails?	Giganti, Paul	•					•	
How Much Is a Million?	Schwartz, David M.	•	•				•	•
How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1000?	Nolan, Helen	•	•				•	•
How Tall, How Short, How Faraway?	Adler, David A.		•				•	•
How the Second Grade Got $8,205.50 to Visit the Statue of Liberty	Zimelman, Nathan	•					•	•
I Spy Two Eyes: Numbers in Art	Micklethwait, Lucy	•					•	
If You Hopped Like a Frog	Schwartz, David M.		•				•	•
If You Made a Million	Schwartz, David M.	•	•					•
Important Book,The	Brown, Margaret Wise			•			•	•
Inch by Inch	Lionni, Leo		•				•	
Is a Blue Whale the Biggest Thing There Is?	Wells, Robert	•	•				•	•
Is It Larger? Is It Smaller?	Hoban, Tana		•				•	
Is There Room on the Bus? An Around-the-World Counting Story	Piers, Helen	•	•				•	
Jim and the Beanstalk	Briggs, Raymond	•	•				•	
Jumanji	Van Allsburg, Chris					•	•	•
Jump, Frog, Jump	Kalan, Robert						•	
Just a Minute	Slater, Teddy		•				•	
King's Chessboard, The	Birch, David	•		•	•			•

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
King's Commissioners, The	Friedman, Aileen	•					•	•
Let's Find Out About Money	Barabas, Kathy	•					•	
Little Rabbit's First Number Book	Baker, Alan	•					•	
Little Rabbit's First Time Book	Baker, Alan		•				•	
Look Whooo's Counting	MacDonald, Suse	•					•	
Lots and Lots of Zebra Stripes: Patterns in Nature	Swinburne, Stephen				•		•	
M&M's Brand Counting Book, The	McGrath, Barbara B.	•		•			•	•
M&M's Chocolate Candies Counting Board Book	McGrath, Barbara B.	•					•	
Magic Money Box, The	Williams, Rozanne L.	•	•				•	
Man Who Counted, The: A Collection of Mathematical Adventures	Tahan, Malba	•			•			•
Math Curse	Scieszka, Jon, & Smith, Lane	•						•
Math for All Seasons: Mind-Stretching Math Riddles	Tang, Greg	•					•	•
Measuring Penny	Leedy, Loreen		•				•	•
Melody Mooner Stayed Up All Night	Edwards, Frank		•				•	
Melody Mooner Takes Lessons	Edwards, Frank		•				•	
Mice Twice	Low, Joseph	•					•	
Million Fish . . . More or Less, A	McKissack, Patricia C.	•					•	•
Millions of Cats	Gag, Wanda	•					•	•
Miss Bindergarten Celebrates the 100th Day of Kindergarten	Slate, Joseph	•					•	
Missing Piece, The	Silverstein, Shel			•			•	
Moira's Birthday	Munsch, Robert	•					•	•
Moja Means One	Feelings, Muriel	•					•	•
More M&M's Brand Chocolate Candies Math	McGrath, Barbara B.	•				•	•	•
More Than One	Schlein, Marion	•					•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Mr. Archimedes' Bath	Allen, Pamela		•				•	
Much Bigger Than Martin	Kellogg, Steven		•				•	
My Little Sister Ate One Hare	Crossman, Bill	•					•	
Number Devil, The	Enzensberger, Hans Magnus	•						•
On Beyond a Million	Schwartz, David M.	•						•
One Duck Stuck	Root, Phyllis	•					•	
One Grain of Rice	Demi	•			•			•
One Gray Mouse	Burton, Katherine	•					•	
One Guinea Pig Is Not Enough	Duke, Kate	•					•	
One Hundred Hungry Ants	Pinczes, Elinor	•			•		•	•
One Hundred Ways to Get to 100	Pallotta, Jerry	•			•		•	•
100th Day of School, The	Medearis, Angela S.	•					•	
100th Day Worries	Cuyler, Margery	•					•	
1 Hunter	Hutchins, Pat	•					•	
One Moose, Twenty Mice	Beaton, Clare	•					•	
One Tiger Growls: A Counting Book of Animal Sounds	Wadsworth, Ginger	•					•	
1, 2, 3, to the Zoo: A Counting Book	Carle, Eric	•					•	
One Watermelon Seed	Lottridge, Celia	•					•	
Pablo's Tree	Mora, Pat		•					•
P. Bear's New Year's Party	Lewis, Paul	•					•	
Phantom Tollbooth, The	Juster, Norton	•	•	•				•
Pigs Go to Market: Fun With Math and Shopping	Axelrod, Amy	•					•	
Pigs in the Pantry: Fun With Math and Cooking	Axelrod, Amy		•				•	
Pigs on a Blanket: Fun With Math and Time	Axelrod, Amy		•				•	
Pigs on the Ball: Fun With Math and Sports	Axelrod, Amy			•			•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Pigs Will Be Pigs: Fun With Math and Money	Axelrod, Amy	•					•	•
Pocket for Corduroy, A	Freeman, Don	•					•	
Principal's New Clothes, The	Calmenson, Stephanie		•					•
Quack and Count	Baker, Keith	•					•	
Quarter From the Tooth Fairy, A	Holtzman, Caren	•	•				•	
Quilt, The	Jonas, Ann				•		•	
Quilt Story, The	Johnston, Tony				•		•	
Reese's Pieces: Count by Fives	Pallotta, Jerry	•			•		•	•
Reflections	Jonas, Ann			•			•	•
Remainder of One, A	Pinczes, Elinor	•					•	•
Right Number of Elephants, The	Sheppard, Jeff	•					•	
Rooster's Off to See the World	Carle, Eric				•		•	
Round Trip	Jonas, Ann			•			•	•
Sam Johnson and the Blue Ribbon Quilt	Ernst, Lisa Campbell				•		•	•
School Bus Comes at Eight O'Clock, The	McKee, David		•				•	
Sea Shapes	MacDonald, Suse			•			•	•
Sea Squares	Hulme, Joy	•					•	
Secret Birthday Message, The	Carle, Eric			•			•	
Seven Blind Mice	Young, Ed	•					•	
Seven Stars More!	Mallat, Kathy	•					•	
17 Kings and 42 Elephants	Mahy, Margaret	•					•	
Sir Cumference and the First Round Table	Neuschwander, Cindy		•	•				•
Six Dinner Syd	Moore, Inga	•					•	•
Six Snowy Sheep	Enderle, Judith	•					•	
Skittles Riddles Math	McGrath, Barbara B.	•						•
Sleepy Owl, The	Pfister, Marcus		•				•	
So Many Cats	De Regniers, Beatrice S.	•					•	
So Many Circles, So Many Squares	Hoban, Tana			•			•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Spaghetti and Meatballs for All	Burns, Marilyn	•	•				•	•
Squares: Shapes in Math, Science and Nature	Ross, Catherine Sheldrick			•				•
Stone Soup	Forest, Heather					•	•	
Telling Time With Big Mama Cat	Harper, Dan		•				•	
Ten Apples up on Top!	Seuss, Dr.	•					•	
Ten Black Dots	Crews, Donald	•					•	
Ten Bright Eyes	Hindley, Judy			•			•	
Ten Crazy Caterpillars	Tuer, Judy	•					•	
Ten Flashing Fireflies	Sturges, Philemon	•					•	
10 For Dinner	Bogart, Jo Ellen	•					•	
Ten Little Angels	Simmons, Andrea	•					•	
10 Minutes Till Bedtime	Rathmann, Peggy		•				•	
Ten out of Bed	Dale, Penny	•					•	
Ten Seeds	Brown, Ruth	•					•	
Ten Sly Piranhas: A Counting Story in Reverse	Wise, William	•					•	
Ten Terrible Dinosaurs	Strickland, Paul	•					•	
Ten, Nine, Eight	Bang, Molly	•					•	
13 Days of Halloween, The	Greene, Carol	•			•			
Three Hat Day, A	Geringer, Laura					•	•	
Three Pigs, One Wolf, and Seven Magic Shapes	Maccarone, Grace	•		•			•	•
Tiger Math: Learning to Graph From a Baby Tiger	Nagda, Ann Whitehead					•		•
Today Is Monday	Carle, Eric		•				•	
Triangles: Shapes in Math, Science and Nature	Ross, Catherine Sheldrick			•				•
Twelve Circus Rings, The	Chwast, Seymour	•					•	•
12 Ways to Get to 11	Merriam, Eve	•					•	
Twenty Is Too Many	Duke, Kate	•					•	
26 Letters and 99 Cents	Hoban, Tana	•					•	

* The material in this chart is used by permission of the Halton District School Board.

Title	Author	Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability	Primary	Junior
Twizzlers Percentage Book	Pallotta, Jerry	•						•
Two by Two	Reid, Barbara	•					•	
Two Ways to Count to Ten: A Liberian Folktale	Dee, Ruby	•				•	•	
Very Hungry Caterpillar, The	Carle, Eric	•	•		•		•	
Village of Round and Square Houses, The	Grifalconi, Ann			•			•	•
What Comes in 2's, 3's, and 4's?	Aker, Suzanne	•					•	•
What's Smaller Than a Pygmy Shrew?	Wells, Robert E.	•	•					•
What's the Time, Mr. Wolf?	Jones, Carol		•				•	
When This Box Is Full	Lillie, Patricia		•				•	
Wolf's Chicken Stew, The	Kasza, Keiko	•	•				•	

* The material in this chart is used by permission of the Halton District School Board.

Appendix 7-12: Sample Tracking Sheet

1-3 4-6

Focus: All About Numbers Name: _____

Date	Centre	Complete	Not Complete
	Pattern blocks: Using pattern blocks to explore growth patterns		
	Interlocking cubes: Using cubes to collect data		
	Exploring number through games		
	The King's Commissioners by Aileen Friedman (New York: Scholastic, 1995) (story): Making arrays		
	Creating number books		

Appendix 7-13: Sample Student Instructions for a Primary Mathematics Learning Centre

1–3

SYMMETRY CENTRE 2

Materials

- geoboard and geobands/elastic bands
- pencil
- dot paper

Instructions

1. Use a geoband to make a vertical (up and down) line of symmetry.
2. Create a symmetrical shape using that line of symmetry.
3. Draw your shape on the dot paper. Be sure to label the line of symmetry.
4. Repeat for lines of symmetry that are horizontal (left to right) and diagonal.

Focus question: How does folding the dot paper help show that the design is symmetrical?

Appendix 7-14: Sample Student Instructions for a Junior Mathematics Learning Centre

4-6

AREA/PERIMETER CENTRE 2

Materials

- geoboard and geobands or elastic bands
- pencil
- dot paper

Instructions

1. Use geobands to make shapes having different areas. The perimeter must remain at 20 units.
2. Record each shape on dot paper, and record the area and perimeter of the shapes on the paper.
3. Remember that the geobands must follow a vertical or horizontal path (no diagonals), so that the perimeter remains constant.

Focus question: What strategies did you use to find the shape with the largest area?

References

Adams, L., Waters, J., Chapple, N., & Onslow, B. (2002). *Esso family math.* London, ON: Esso Family Math Centre, University of Western Ontario.

Baroody, A.J. (1998). *Fostering children's mathematical power.* Mahwah, NJ: Erlbaum.

Baroody, A.J. (2004). The developmental bases for early childhood number and operations standards. In D.H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 173–220). Mahwah, NJ: Erlbaum.

Baskwill, J. (1992). Ask me about: A newsletter with a difference. *Teaching PreK–8, 22*(3), 44–48.

Beavers, D. (2001). Professional development: Outside the workshop box. *Principal Leadership, 1*(9), 43–46.

Bennett, B., & Rolheiser, C. (2001). *Beyond Monet*. Toronto: Bookstation.

Burns, M. (1992). *Math and literature (K–3)*. Sausalito, CA: Math Solutions Publications.

Burns, M. (1995). *Writing in the math class.* Sausalito, CA: Math Solutions Publications.

Burns, M. (2000). *About teaching mathematics: A K–8 resource* (2nd ed.). Sausalito, CA: Math Solutions Publications.

Burns, M., & Silbey, R. (2000). *So you have to teach math? Sound advice for K–6 teachers.* Sausalito, CA: Math Solutions Publications.

Cambourne, B. (1988). *The whole story: Natural learning and the acquisition of literacy in the classroom.* New York: Ashton-Scholastic.

Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, E.P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Education Research Journal*, 26, 499–531.

Carpenter, T.P., Franke, M.L., Jacobs, V.R., Fennema, E., & Empson, S.B. (1998). A longitudinal study of invention and understanding of children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1), 3–20.

Cathcart, W.G., Pothier, Y.M., & Vance, J.H. (1997). *Learning mathematics in elementary and middle school* (2nd ed.). Scarborough, ON: Prentice-Hall Canada.

Clements, D.H., & Callahan, L.G. (1983). Number or prenumber foundational experiences for young children: Must we choose? *Arithmetic Teacher, 31*(3), 34–37.

Clements, D.H., Sarama, J., & DiBiase, A.-M. (Eds.) (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum.

Coates, G.D., & Stenmark, J.K. (1997). *Family math for young children: Comparing.* Berkeley: University of California.

Cobb, P., Wood, T., & Yackel, E. (1991). Assessment of a problem-centered second-grade mathematics project. *Journal for Research in Education, 22*(1), 3–29.

Copley, J.V. (2000). *The young child and mathematics.* Washington, DC: National Association for the Education of Young Children.

Dacey, L., & Eston, R. (1999). *Growing mathematical ideas in kindergarten.* Sausalito, CA: Math Solutions Publications.

Dacey, L., & Eston, R. (2002). *Show and tell.* Sausalito, CA: Math Solutions Publications.

Elementary Teachers' Federation of Ontario. (2004). *Making math happen in the junior years*. Toronto: Author.

Epstein, J.L. (1991). Paths to partnership: What we can learn from federal, state, district, and school initiatives. *Phi Delta Kappan, 72*(5), 344–349.

Expert Panel on Early Math in Ontario. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto: Ontario Ministry of Education.

Expert Panel on Early Reading in Ontario. (2003). *Early reading strategy: The report of the Expert Panel on Early Reading in Ontario.* Toronto: Ontario Ministry of Education.

Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs. (2005). *Education for all: The report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Needs, Kindergarten to Grade 6*. Toronto: Ontario Ministry of Education.

Expert Panel on Literacy in Grades 4 to 6 in Ontario. (2004). *Literacy for learning: The report of the Expert Panel on Literacy in Grades 4 to 6 in Ontario*. Toronto: Ontario Ministry of Education.

Expert Panel on Mathematics in Grades 4 to 6 in Ontario. (2004). *Teaching and learning mathematics: The report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario*. Toronto: Ontario Ministry of Education.

Fosnot, C.T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction.* Portsmouth, NH: Heinemann.

Fullan, Michael. (2001). *Leading in a culture of change.* San Francisco: Jossey-Bass.

Fullan, Michael. (2003). *The moral imperative of school leadership.* Thousand Oaks, CA: Corwin Press.

Gardner, H. (1993). *Multiple intelligences: The theory in practice.* New York: Basic.

Gibbs, J. (2000). *Tribes: A new way of learning and being together.* Sausalito, CA: Center Source Systems.

Ginsberg, H.P., Inoue, N., & Seo, K.-H. (1999). Young children doing mathematics. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 88–99). Reston, VA: National Council of Teachers of Mathematics.

Glanfield, F., Bush, W.S., & Stenmark, J.K. (Eds.). (2003). *Mathematics assessment: A practical handbook for Grades K–2*. Reston, VA: National Council of Teachers of Mathematics.

Greenes, C. (1999). Ready to learn: Developing young children's mathematical powers. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 399–447). Reston, VA: National Council of Teachers of Mathematics.

Halton District School Board. (2001). *Junior math and information technology*. Burlington, ON: Author.

Halton District School Board. (2001). *Primary math and information technology.* Burlington, ON: Author.

Halton District School Board. (2003). *Literature for mathematics.* Burlington, ON: Author.

Halton District School Board Numeracy Team. (2001). *Home connections: Primary grades.* Burlington, ON: Author.

Henderson, A.T. (1988). Parents are a school's best friends. *Phi Delta Kappan, 70*(2), 148–153.

Henderson, A.T., & Berla, N. (1994). *A new generation of evidence: The family is critical to student achievement.* Washington, DC: Center for Law and Education.

Hiebert, J.C., & Carpenter, T.P. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Hill, P., and Crévola, C. (1997). The literacy challenge in Australian primary schools. *IARTV Seminar Series, No. 69*.

Jensen, E. (1998). *Teaching with the brain in mind*. Alexandria, VA: Association for Supervision and Curriculum Development.

Kamii, C. (1985). *Young children reinvent arithmetic.* New York: Teachers College Press, Columbia University.

Kawartha Pine Ridge District School Board. (2000). *Teaching and learning mathematics: A resource document for early years and formative years teachers (JK–3)*. Peterborough, ON: Author.

Kilpatrick, J., & Swafford, J. (2003). *Helping children learn mathematics*. Washington, DC: National Academy Press.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education, 29*(5), 533–554.

Litton, N. (1998). *Getting your math message out to parents: A K–6 resource*. Sausalito, CA: Math Solutions Publications.

Lyons, C.A., & Pinnell, G.S. (2001). *Systems for change in literacy education: A guide to professional development.* Portsmouth, NH: Heinemann.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Erlbaum.

McCain, M.N., & Mustard, J.F. (1999). *Reversing the real brain drain: Early years study. Final report.* Toronto: Publications Ontario.

Mokros, J. (1996). *Beyond facts and flashcards.* Portsmouth, NH: Heinemann.

Morrow, L.J., & Kenney, M.J. (Eds.). (1998). *The teaching and learning of algorithms in school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Association of Elementary School Principals (NAESP). (2000). *Leading learning communities: Standards for what principals should know and be able to do.* Washington, DC: Author.

National Council of Supervisors of Mathematics. (2000). *Supporting leaders in mathematics education: A source book of essential information.* Retrieved April 14, 2006, from http://www.mathforum.org/ncsm/NCSMPublications/2000/sourcebook2000.html

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Onslow, B., & Chapple, N. (2004). *Esso family math: Resources for Grades 2–5 children and their parents* (3rd ed.). London, ON: Esso Family Math Centre, University of Western Ontario.

Ontario College of Teachers. (1999). *Standards of practice for the teaching profession.* Toronto: Author.

Ontario Ministry of Education. (2003a). *A guide to effective instruction in mathematics, Kindergarten to Grade 3 – Number sense and numeration.* Toronto: Author.

Ontario Ministry of Education. (2003b). *Helping your child learn math: A parent's guide.* Toronto: Author.

Ontario Ministry of Education. (2004). *The Individual Education Plan (IEP): A resource guide.* Toronto: Author.

Ontario Ministry of Education. (2005). *The Ontario curriculum, Grades 1–8: Mathematics.* Toronto: Author.

Ontario Ministry of Education. (2006). *The Kindergarten program.* Toronto: Author.

Payne, J.N. (Ed.). (1990). *Mathematics for the young child.* Reston, VA: National Council of Teachers of Mathematics.

Piaget, J. (1995). *Sociological studies.* L. Smith (Ed.). New York: Routledge.

Polya, G. (1945). *How to solve it.* Garden City, NY: Doubleday.

Reys, R.E., Lindquist, M.M., Lambdin, D.V., Suydam, M.N., & Smith, N.L. (2001). *Helping children learn mathematics.* (6th ed.). New York: Wiley.

Rodgers, C.R. (2002). Seeing student learning: Teacher change and the role of reflection. *Harvard Educational Review*, Summer, 230–253.

Ross, J., Hogaboam-Gray, A., McDougall, D., & Bruce, C. (2002, April). *The contribution of technology to mathematics education reform.* Paper presented at the American Educational Research Association Conference, New Orleans, LA.

Sandholtz, J.H. (2000). Interdisciplinary team teaching as a form of professional development. *Teacher Education Quarterly, 27*(3), 39–54.

Schifter, D., & Fosnot, C.T. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York: Teachers College Press, Columbia University.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–379). Reston, VA: National Council of Teachers of Mathematics.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(34), 9–15.

Sophian, C. (2004). A prospective developmental perspective on early mathematics instruction. In D.H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 253–266). Mahwah, NJ: Erlbaum.

Steen, L.A. (1990, Spring). Numeracy. *Daedalus, 119*(2), 211–231.

Stenmark, J.K., & Bush, W.S. (Eds.). (2001). *Mathematics assessment: A practical handbook.* Reston, VA: National Council of Teachers of Mathematics.

Stenmark, J.K., Thompson, V., & Cossey, R. (1986). *Family math*. Berkeley: University of California.

Stiggins, R.J. (2001). *Student-involved classroom assessment.* Upper Saddle River, NJ: Prentice-Hall.

Sutton, J., and Krueger, A. (Eds.). (2002). *EDThoughts: What we know about mathematics teaching and learning.* Aurora, CO: Mid-continent Research for Education and Learning.

Thompson, V., & Mayfield-Ingram, K. (1998). *Family math: The middle school years – Algebraic reasoning and number sense* (3rd ed.). Berkeley: University of California.

Toronto Catholic District School Board. (2001). *Assessment of student achievement in Catholic schools resource document.* Toronto: Author.

Toronto District School Board. (2002). *Kindergarten documents.* Toronto: Author.

Trafton, P.R., & Thiessen, D. (1999). *Learning through problems.* Portsmouth, NH: Heinemann.

Van de Walle, J.A. (2001). *Elementary and middle school mathematics: Teaching developmentally* (4th ed.). New York: Addison Wesley Longman.

Van de Walle, J.A. (2004). *Elementary and middle school mathematics: Teaching developmentally.* (5th ed.). New York: Pearson Education.

Vygotsky, L. (1980). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Waterloo County Board of Education. (1992). *Addition and subtraction of whole numbers: The formative years.* Waterloo, ON: Author.

Waterloo County Board of Education. (1993). *Multiplication and division of whole numbers*. Waterloo, ON: Author.

Weeks, Ronald C. (1997). *The child's world of science and technology: A book for teachers. Teaching and learning science and technology in the elementary school.* Scarborough, ON: Prentice-Hall Allyn and Bacon Canada.

Whitin, P., & Whitin, D.J. (2000). *Math is language too.* Urbana, IL: National Council of Teachers of English.

Wright, R.J., Martland, J., Stafford, A.K., & Stanger, G. (2002). *Teaching number: Advancing children's skills and strategies.* London: Paul Chapman.

The Ministry of Education wishes to acknowledge the contribution of the many individuals, groups, and organizations that participated in the development and refinement of this resource document.