Unit 4: Day 13: Sinusoidal Swing

MCT 4C

Minds On: 10 Action: 55 Consolidate:10 Total=75 min	**Learning Goal**: • Identify periodic and sinusoidal functions, including those that arise from real-world applications involving primary data. • Make connections between the change in a graph and a change in context.	**Materials** • BLM 4.13.1 • BLM 4.13.2 • CBR • Graphing calculator • "TICTOC" application • Pendulum • Stop watch

			Assessment Opportunities
Minds On…	**Small Groups → Brainstorm** Have students review concepts learned so far and work individually to write down what they know about sinusoidal functions. They will then share their thoughts with small groups and summarize the terms they came up with. **Whole Class → Discussion** Each group shares one response with the class that is different from those already presented and then posts one or more of the key words on the word wall (i.e. period, phase shift, periodic, amplitude, maximum, minimum, etc.) **Mathematical Process Focus:** Reflecting - Students will **reflect** upon their prior learning to clarify their understanding.		Circulate and assist students in remembering terms Literacy: Word Wall
Action!	**Small Groups → Investigation** Using BLM 4.13.1, students complete the investigation. **Learning Skills/Observation/Checklist** As students complete the investigation, assess their teamwork skills and initiative. **Mathematical Process Focus:** Connecting – Students will **connect** the graph of a sinusoidal function with primary data.	⇄	The "TICTOC" application needs to be installed on the calculator attached to the CBR. The "TICTOC" application can be downloaded from www.education.ti.com.
Consolidate Debrief	**Whole Class → Discussion** Discuss properties of the graphs developed from each of the changing scenarios. Encourage students to discuss "what's the same, what's different?" Students should be able to predict what properties of the sinusoidal graph would change given a change in context.		
Exploration Application	**Home Activity or Further Classroom Consolidation** Complete BLM 4.13.2. (data reference http://www.nsa.gov/teachers/hs/trig06.pdf)		

4.13.1 Pendulum



Introduction

In this experiment you will be examining the motion of a pendulum over time. You will swing a pendulum back and forth so that it is always in the line of sight of the CBR. The CBR will measure the pendulum's distance from the CBR over regular time intervals. A distance vs. time graph will be plotted.

You will be examining the effects of changing two variables independently:

1. the length of the pendulum
2. the initial displacement of the pendulum from resting position (vertical position)

Hypotheses:

1. The shorter the pendulum, the _____ period.
2. The greater the displacement from rest, the _____ amplitude.

Equipment Needed:

- CBR
- graphing calculator with a link cable and "TICTOC" program loaded
- pendulum - string, a large object to swing (e.g. pop can, or juice jug, a bucket), metre stick, retort stand
- clock with a second hand or stop watch
- observation tables

Performing the Experiment:

1. Connect the calculator to the CBR.

2. Make sure the program "TICTOC" has been loaded on the calculator.

3. Set up the pendulum (with the length indicated in table 1) and the CBR so that the motion detector is at the same level as the swinging object when at rest.

4. Position a metre stick along the table so that you can measure the distance from the motion detector to the pendulum.

5. Measure the distance between the pendulum and the CBR in cm. The distance must be at least 75 cm. Adjust your set-up as needed when changing the length of the pendulum.

6. Use the stopwatch to determine the time it takes to complete 5 cycles. Record this information in table 1 on the next page.

7. Run the program "TICTOC" and follow the instructions on your calculator.

8. Complete the Tables of Observations and Analyses Questions. Use the graphing calculator (sinusoidal regression) to determine an equation of best fit.

9. Repeat the experiment, this time keeping the pendulum length fixed at 50cm and varying the initial displacement. Record your observations in table 2.

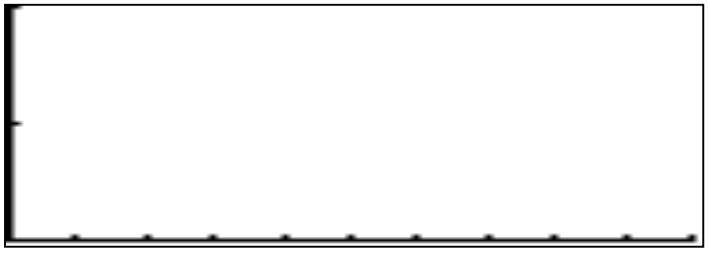
4.13.1 Pendulum (continued)

Tables of Observations

Table 1: The Effect of Changing the Length of the Pendulum

- Initial displacement is fixed at 40 cm.

Note: Label key values on your graphs by using the TRACE key to determine the value of the maximum y-value, minimum y-value, y–intercept and distance between consecutive y-max values.

Pendulum Length (cm)	Time for 5 cycles (s)	Distance-Time Graphs
50		 y = _____
40		 y = _____
30		 y = _____

4.13.1 Pendulum (continued)

Table 2: The Effect of Changing the Displacement of the Pendulum

- Pendulum length is fixed at 50 cm.

Note: Label key values on your graphs by using the TRACE key to determine the value of the maximum y-value, minimum y-value, y –intercept and distance between consecutive y-max values.

Displacement from Rest (cm)	Time for 5 cycles (s)	Distance-Time Graphs
50		y = _____
40		y = _____
30		y = _____

4.13.1 Pendulum (continued)

Analysis

Using mathematical terms, state what parameters of the distance-time graphs were affected by:

- changing the length of the pendulum, while keeping the displacement constant

- changing the displacement, while keeping the length of the pendulum constant

4.13.2 The Beat of My Heart



Sinusoidal functions can be used to represent repetitive behavior. EKGs (electrocardiograph) have been used in the medical field for the last three centuries to monitor heart activity. This instrument is able to detect the electrical changes when a person's heart beats.
The following two figures are actual data taken using a CBL and a graphing calculator. Both figures depict the heart activity for a 16-year-old male at rest. The horizontal axis measures time in seconds and on the vertical axis each peak represents a heart beat.

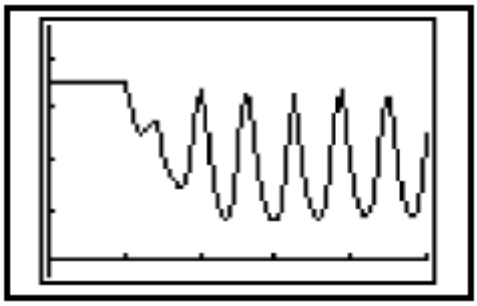


Figure 1

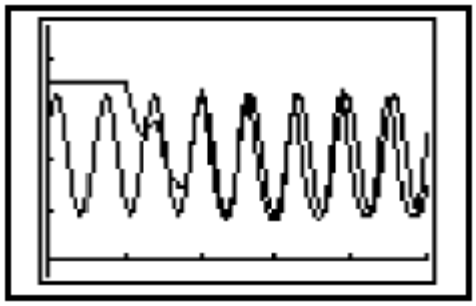


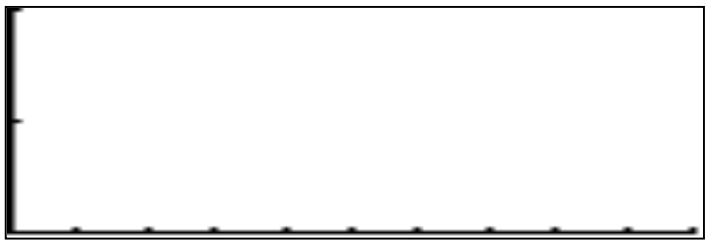
Figure 2

1. What factors would affect a person's heartbeat?

Answer Questions 2 and 3 with reference to Figures 1 and 2
(Note: Figure 2 has the graph from Figure 1 also plotted.)

2. Using mathematical terms, describe how the above graphs would change if the 16-year-old male ran quickly up ten flights of stairs?

3. Typically, while sleeping, a person's heart rate decreases. Sketch the curve that would model this situation for the 16-year-old male.



4. What changes in your heart rate might occur when additional adrenalin is introduced into the system? (i.e. fear,terror,…) How would this change the graph?

Unit 4: Day 14: Where on Earth are we?

Minds On: 10 Action: 50 Consolidate:15 Total=75 min	**Learning Goal:** • Investigate and explore properties of sinusoidal functions using real world data • Make connections between changes in a real-world situation that can be modelled using a sinusoidal function	**Materials** • Graphing calculator • World Map • BLM 4.14.1 • BLM 4.14.2 • BLM 4.14.3

				Assessment Opportunities
	Minds On…	**Small Groups → Activity** Distribute BLM 4.14.1. Students use their prior knowledge of amplitude and period to match sinusoidal graphs to various travel destinations **Whole Class → Discussion** Students explain the strategies they used in the matching exercise; draw students to connect the concepts of periodic behaviour and changes in the amount of daylight hours over time.		A world map may aid in students connecting the latitude/longitude to the shapes of the graphs.
	Action!	**Groups of 4 → Exploration** Using graphing calculators, students work in groups of four to complete BLM 4.14.2. **Process Expectations/Observation/Rubric** Observe students use of the Mathematical Processes as they complete BLM 4.14.2. **Mathematical Process Focus:** Connecting – Students **connect** sinusoidal functions to secondary data.		(BLM 4.14.2 data: http://aa.usno.navy.mil/data/docs/Dur_OneYear.html)
	Consolidate Debrief	**Whole Class → Discussion** Each group displays their results on the board underneath each of the cities Choose one group for each city and have that group explain their selection, using the Toronto data as their point of reference. Students should be encouraged to use mathematical vocabulary Guided Questions: *What features of the graph changed when the number of daylight hours decreased/increased? Is there a pattern between the geographical location and the characteristics of the sinusoidal curve?*		A cross-curricular connection can be made with mathematics and geography.
	Exploration Application	**Home Activity or Further Classroom Consolidation** Complete BLM 4.14.3		(BLM 4.14.3 data: http://www.thehopewellrocks.ca/english/documents/aug.pdf) For more discussion on the Bay of Fundy the following link is helpful http://www.thehopewellrocks.ca/english/tidetables.htm

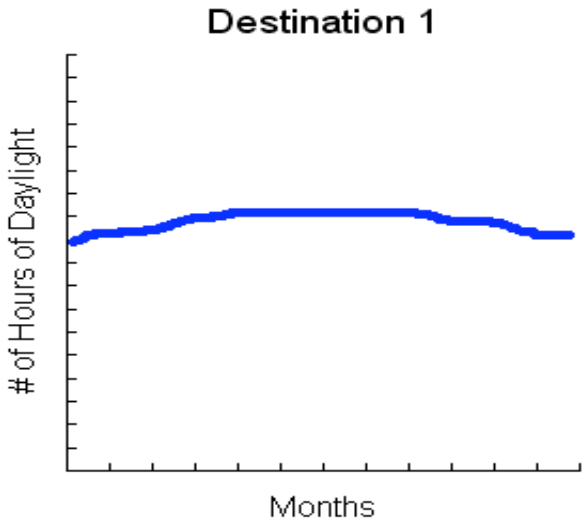


4.14.1: The Amazing "Sinusoidal" Race

Part 1

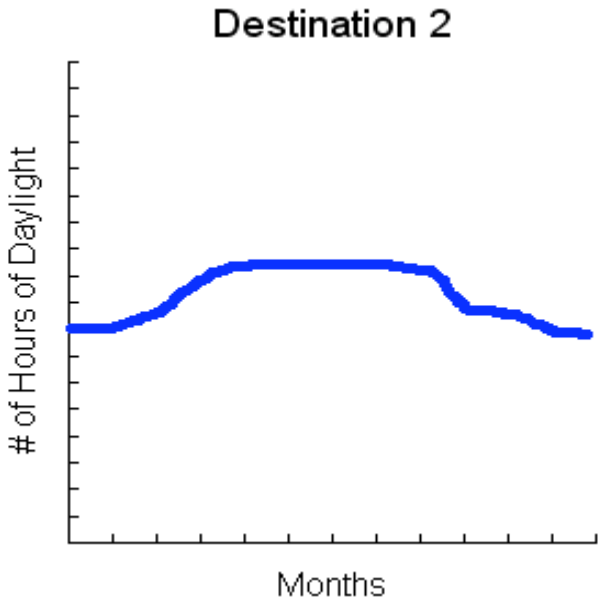
Your group has been chosen to participate in this season's *Amazing Race.* The first challenge you have is to determine the order of the three locations you will be travelling to by matching the sinusoidal function below with the correct city. The only problem is that the graphs depict the number of hours of daylight for a specific city over a one – year period. Therefore you have to use your knowledge of both geography and sinusoidal functions to correctly plan out the order of your group's race.

Locations: Ottawa (Canada), Quito (Ecuador), Anchorage (Alaska)



City:

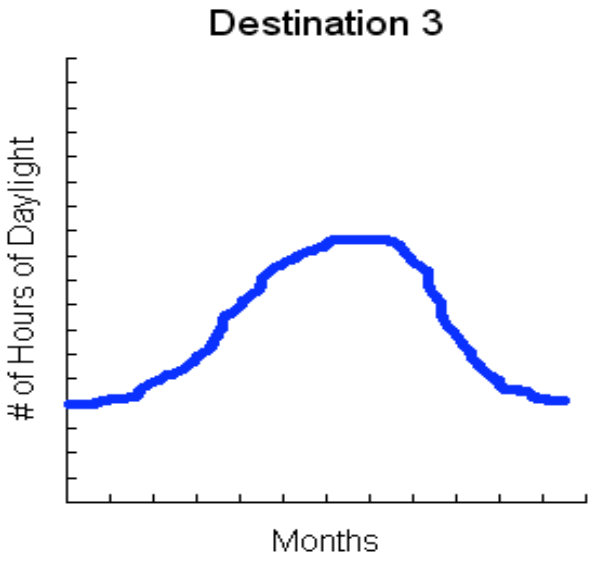
Why?



City: _____

Why?

4.14.1: The Amazing "Sinusoidal" Race (continued)

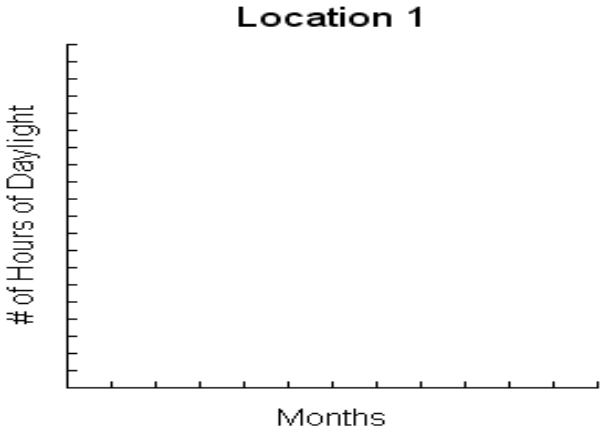


City: _____

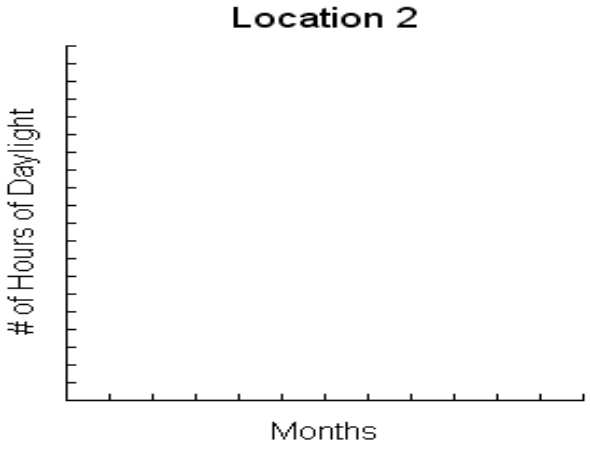
Why?

Part 2

Your group's second challenge is to sketch sinusoidal graphs that would depict the amount of daylight for a one - year period for each of the following locations.



- This location is to the south of both the equator and the tropic of Capricorn.



- This location can be found 500 km north of the equator.

4.14.2: Where on Earth are we?



The following tables give data recorded during 2006 on the number of daylight hours, recorded on the first of every month, for various cities.

In transferring the data from the website http://aa.usno.navy.mil/data , there was a mix-up and only one table was copied with the location name. The remaining tables are taken from the following locations:

- **Auckland, New Zealand**
  Location: E175 00, S37 00
- **Nairobi, Kenya**
  Location: E037 01, S00 24
- **Hong Kong, China**
  Location:
- **Buenos Aires, Argentina**
  Location: W058 40, S34 3 5
- **Whitehorse,Yukon**
  Location: W163 54, N62 32



p://images.google.com/images?um=1&tab=wi&ie=UTF-8&oe=UTF-8&rls=com.microsoft%3Aen-us%3AIE-SearchBox&rlz=1I7GGIH&q=brazil

4.14.2: Where on Earth are we? (continued)

Your task is to:

- ✓ **Use your graphing calculator to graph the data from Toronto, Canada, noting the type of graph that is occurring, and fill in details indicated in each table.**
- ✓ **Repeat the above for the remaining tables.**
- ✓ **Using the Toronto information and your general geographical knowledge, match each graph to one of the locations given on p. 1.**
- ✓ **In your group come to a consensus as to which table matches which city and provide reasoning to support your choice.**

Table 1: Toronto, Canada

Location: W043 40, N79 25

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	9.18	9.00	9.90	11.20	12.75	14.15	11.22	15.38	14.58	13.20	11.73	10.25

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

4.14.2: Where on Earth are we? (continued)

Table 2: _____

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	10.87	10.8	11.17	11.72	12.38	12.98	13.4	13.47	13.15	12.57	11.93	11.32

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

Reasoning to support location match:

_____

_____

4.14.2: Where on Earth are we? (continued)

Table 3: _____

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	12.15	12.15	12.13	12.12	12.10	12.10	12.10	12.10	12.10	12.10	12.11	12.13

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

Reasoning to support location match:

_____

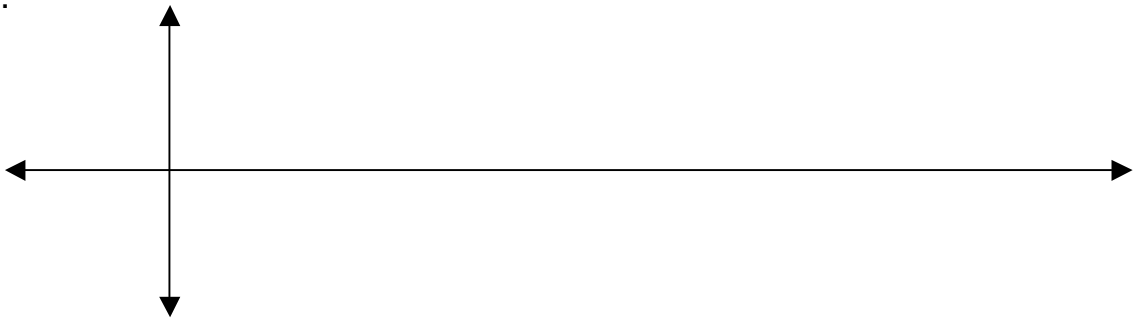
_____

4.14.2: Where on Earth are we? (continued)

Table 4: _____

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	5.68	5.13	7.48	10.28	13.43	16.47	19.84	19.53	17.48	14.42	11.42	8.32

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

Reasoning to support location match:

_____

_____

4.14.2: Where on Earth are we? (continued)

Table 5 _____

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	14.5	14.65	13.93	12.9	11.68	10.58	9.78	9.65	10.27	11.32	12.47	13.65

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

Reasoning to support location match:

_____

_____

4.14.2: Where on Earth are we? (continued)

Table 6 _____

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
Month #	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	14.3	14.42	13.77	12.83	11.70	10.70	9.98	9.87	10.42	11.38	12.43	13.52

Graph:



Maximum Value: _____

Minimum Value: _____

Average Value: _____

Amplitude: _____

Reasoning to support location match:

_____

_____

4.14.3 The Bay of Fundy



The *Bay of Fundy* is located between the provinces of Nova Scotia and New Brunswick, and is known for having the greatest difference in water level between high and low tides in the world. Tidal charts are very important when visiting the Bay as water heights and times vary according to various meteorological and astronomical effects.

You have been hired to create a graphical representation of the data below to determine whether the time of day and the height of the tide in the Bay form a sinusoidal relationship.

Each day in the table below shows three depths (low tide, high tide, low tide) in metres and feet.

Bay of Fundy Tidal Chart – August 2007

Month/Mois: August							
	Time/Heure	Ft/pi.	Meters		Time/Heure	Ft/pi.	Metres
1	8:41 AM	3.3	1.0	17	10:06 AM	5.9	1.8
	2:29 PM	42.0	12.8		3:53 PM	38.7	11.8
	9:00 PM	3.6	1.1		10:27 PM	6.9	2.1
2	9:24 AM	2.6	0.8	18	10:45 AM	7.9	2.4
	3:15 PM	42.7	13.0		4:31 PM	37.4	11.4
	9:46 PM	3.0	0.9		11:08 PM	8.5	2.6
3	10:09 AM	2.6	0.8	19	11:24 AM	9.8	3.0
	4:02 PM	43.0	13.1		5:11 PM	35.8	10.9
	10:33 PM	3.0	0.9		11:50 PM	10.2	3.1
4	10:56 AM	3.3	1.0	20	5:34 AM	33.8	10.3
	4:52 PM	42.7	13.0		12:06 PM	11.5	3.5
	11:24 PM	3.6	1.1		5:54 PM	34.4	10.5
5	5:19 AM	41.3	12.6	21	6:22 AM	32.5	9.9
	11:47 AM	4.3	1.3		12:53 PM	13.1	4.0
	5:46 PM	41.7	12.7		6:44 PM	33.8	10.3
6	6:15 AM	40.0	12.2	22	7:16 AM	31.5	9.6
	12:43 PM	5.6	1.7		1:48 PM	13.8	4.2
	6:44 PM	40.7	12.4		7:40 PM	33.1	10.1
7	7:17 AM	38.7	11.8	23	8:16 AM	31.5	9.6
	1:44 PM	6.6	2.0		2:49 PM	13.8	4.2
	7:46 PM	40.0	12.2		8:40 PM	33.5	10.2
8	8:22 AM	38.1	11.6	24	9:16 AM	32.2	9.8
	2:50 PM	7.2	2.2		3:51 PM	13.1	4.0
	8:51 PM	39.7	12.1		9:39 PM	34.8	10.6
9	9:29 AM	37.7	11.5	25	10:13 AM	33.8	10.3
	3:57 PM	6.9	2.1		4:48 PM	11.2	3.4
	9:55 PM	40.0	12.2		10:35 PM	36.4	11.1
10	10:32 AM	38.4	11.7	26	5:20 AM	8.9	2.7
	4:59 PM	6.2	1.9		11:05 AM	35.8	10.9
	10:56 PM	40.7	12.4		5:39 PM	9.2	2.8
11	11:30 AM	39.4	12.0	27	6:07 AM	6.9	2.1
	5:57 PM	5.2	1.6		11:52 AM	38.4	11.7
	11:52 PM	41.3	12.6		6:26 PM	6.6	2.0
12	6:28 AM	3.3	1.0	28	6:51 AM	4.9	1.5
	12:23 PM	40.0	12.2		12:38 PM	40.7	12.4
	6:49 PM	4.3	1.3		7:10 PM	4.6	1.4
13	7:17 AM	2.6	0.8	29	7:34 AM	3.0	0.9
	1:11 PM	40.7	12.4		1:22 PM	42.7	13.0
	7:37 PM	3.6	1.1		7:54 PM	2.6	0.8
14	8:02 AM	2.6	0.8	30	8:17 AM	2.0	0.6
	1:55 PM	41.0	12.5		2:06 PM	44.0	13.4
	8:22 PM	3.6	1.1		8:38 PM	1.6	0.5
15	8:45 AM	3.3	1.0	31	8:59 AM	1.3	0.4
	2:36 PM	40.7	12.4		2:51 PM	44.6	13.6
	9:05 PM	4.3	1.3		9:23 PM	1.3	0.4
16	9:26 AM	4.3	1.3				
	3:15 PM	39.7	12.1				
	9:46 PM	5.2	1.6				

1. Using the graph provided on the following page and the table to left, graph the three measurements (each in a different colour) for each of the 31 days in August.

2. Is this data periodic? Why or why not?

3. If the function is sinusoidal what is the period and what is the amplitude? Explain what each of these values mean, given the context.

4.14.3 The Bay of Fundy (continued)

