

Unit 5: Day 6: What's your Vector, Victor?		MCT 4C
Minds On: 15	<b>Learning Goal:</b> <ul style="list-style-type: none"> <li>Investigate the differences between vector and scalar quantities.</li> <li>Interpret information about real-world applications of vectors.</li> <li>Discover careers that use vectors.</li> </ul>	<b>Materials</b> <ul style="list-style-type: none"> <li>BLM 5.6.1-5.6.5</li> <li>PPT 5.6.1</li> <li>pencil crayons</li> <li>Measuring tapes</li> <li>Centimetre-graph paper</li> <li>Rulers</li> </ul>
Action: 40		
Consolidate:20		
Total=75 min		
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Whole Class → Activity Instructions</b> Set context by reading BLM 5.6.1 (first half) to the class  <b>Groups of 3 → Jigsaw (Home Group)</b> Students number off 1 to 3 in their groups. Assign each number a reading from BLM 5.6.1.  <b>Groups → Jigsaw (Expert Group)</b> Students complete BLM 5.6.2 with information from their reading.  <b>Groups of 3 → Jigsaw (Home Group)</b> Share their information in round robin fashion and complete the organizer	Literacy Strategies: Structured Overview- -modified Think, Pair, Share Frayer Model  <b>Word Wall:</b> Scalar Vector Distance Displacement magnitude  If internet is available in the classroom, use <a href="http://www.resources.elearningontario.ca">www.resources.elearningontario.ca</a> . You will want ELO10739000. This is a great animated visual of distance and displacement.  Questions and PowerPoint modified from <a href="http://www.resources.elearningontario.ca">www.resources.elearningontario.ca</a> ELO1073900
<b>Action!</b>	<b>Whole Class → Presentation</b> Introduce the idea of vector, distance and displacement with PowerPoint (PPT 5.6.1).  <b>Think/Pair/Share → Summarizing</b> Students complete BLM 5.6.3. They then complete BLM 5.6.4 for 'distance' and 'displacement'.  <b>Groups of Four → Presentation</b> Pairs join another pair and share their completed BLM 5.6.4 with each other and fill-in any additional information  <b>Pairs → Activity</b> Using BLM 5.6.5, create a map from the classroom to the cafeteria or library.  <b>Learning Skills/Teamwork/Checkbric:</b> Assess students' group work skills throughout the Action portion.  <b>Mathematical Process Focus:</b> Reasoning and Proving, Reflecting, Connecting – Students <b>connect</b> mathematics to a context outside mathematics.	
<b>Consolidate Debrief</b>	<b>Small Groups → Discussion</b> Record the distance measurements on the blackboard Discuss the discrepancies in the numbers if they exist For the discrepancies, how did those students get to the cafeteria? Is this a vector quantity or a scalar quantity?	
<i>Exploration Application</i>	<b>Home Activity or Further Classroom Consolidation</b>  Complete BLM 5.6.6.	

## 5.6.1: What's your Vector, Victor?

(source: [www.mathscareers.org](http://www.mathscareers.org))

Imagine having to describe something — anything at all — to another person only using a pen and paper. If there's any movement at all involved in what you have to describe, chances are you'll soon find yourself drawing arrows. And what characterises an arrow? Well, it's the direction it's pointing in and its length. This, in a nutshell, is a vector: an object that has a direction and a magnitude.

It's not hard to come up with some precise examples of the uses of vectors. The movement of a speeding car is described by its direction and its speed — a direction and a magnitude, in other words a vector. To understand how a force like gravity acts on an object you need to know the direction and the intensity of the force, so again you have the two bits of information that form a vector. And when you watch the weather report you'll be told which way and how strongly the wind will blow tomorrow, again a direction and a magnitude together making up a vector.

Mathematics gives you a way of formalising all the information contained in the visual concept of an arrow. Using mathematical machinery, like algebra and arithmetic, you can go far beyond your doodlings and gain information that you wouldn't be able to see by simply looking at a bunch of arrows. And once you get used to thinking of vectors mathematically, you will stop seeing them simply as arrows and so be able to apply vector maths to many situations you wouldn't initially have thought of.

Here is a selection of examples of how vectors are used in real life that you will discover and discuss in your groups.

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### Vectors and language

Our highly developed language is one thing that separates us from all the other animals on our planet. For this reason many people believe that language and the way we use it can tell us a lot about who we, as humans, really are. The study of language — called linguistics — has become an important field within psychology.

But there are also more practical reasons for trying to understand the way language works. Search engines and word processors work by picking up on certain structures within texts to find the websites most relevant to your search and to weed out grammatical mistakes in your texts. The more they understand language, the more efficient they'll get. The same goes of course for

automated speech recognition systems like those that sometimes answer the phone when you ring up a company or information line.

Both psychologists and people involved in computing want to understand the structures within language. Mathematics is a great tool for capturing structure and vectors seem to be especially useful for understanding language. Words or bits of text can be represented by vectors and vector mathematics can help you see how the different components of a text interact, helping you to find structures within a text that you might not see otherwise.

### And much more ...

These are only three examples of how vectors are used. Because they are so essential in physics and convey visual information, many, many people, from engineers, architects and designers to meteorologists and oceanographers, use them at work every day.

## 5.6.1: What's your Vector, Victor? (continued)

(source: [www.mathscareers.org](http://www.mathscareers.org))

### Vectors in sports



In the 1950s a group of talented Brazilian footballers invented the swerving free kick. By kicking the ball in just the right place, they managed to make it curl around the wall of defending players and, quite often, go straight into the back of the net. Some people might think that such skill is pure magic, but really it's just physics. When a ball is in flight it's acted upon by various forces and some of these depend on the way the ball is spinning around its own axis. If you manage to give it just the right spin, the forces will interact in just the right way to deflect the ball while it's flying, resulting in a curved flight path.

The forces at work here can be described by vectors. Understanding their interaction requires vector maths. The footballers themselves rarely think about the mathematics of course, but today science is increasingly used to improve the performance of athletes' equipment. The exact shape of a football can have important implications on how it moves through the air and teams of scientists are employed to work out how to make the perfect football.

When the equipment is more complicated than a football then the use of science is even more important. Formula One teams, for example, always employ physicists and mathematicians to help build perfect cars. Tiny differences in the shape of the car can make a difference to its speed that can determine the outcome of a race.

Both the science of footballs and of race cars are really just examples of the same thing: aerodynamics, the study of how air moves. This is pure physics — and since vectors can describe movement and forces, they lie at the very heart of it.

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### Vectors and visuals



Vector mathematics is used extensively in computer graphics. Suppose you want to create an image on a computer screen. One way of doing this is to tell the computer the exact colour of each pixel on the screen. This requires a lot of memory and has another disadvantage: if you'd like the image to move, for example to give the viewer the impression that he or she is moving around a scene, you need to constantly renew the information of the pixel colours from scratch.

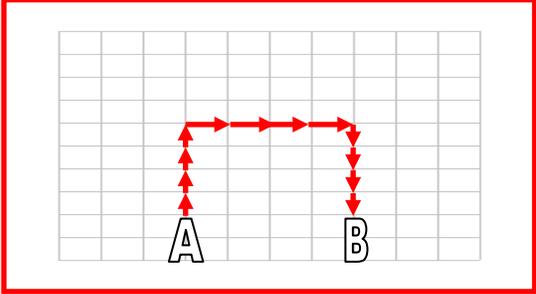
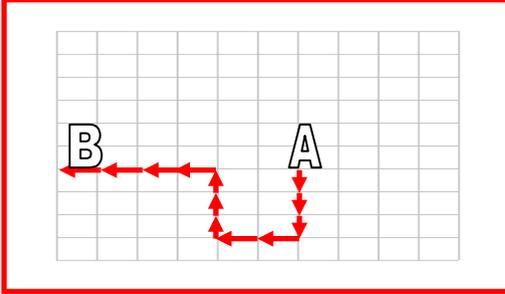
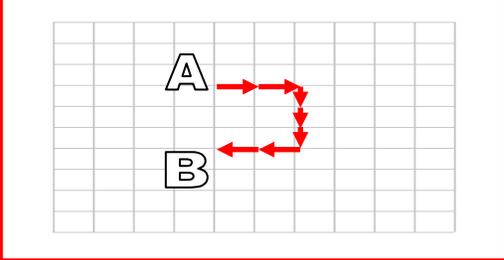
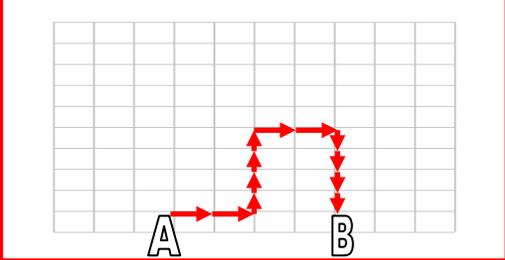
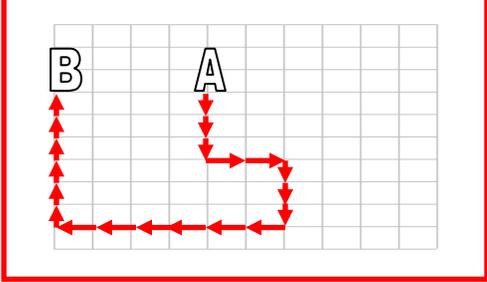
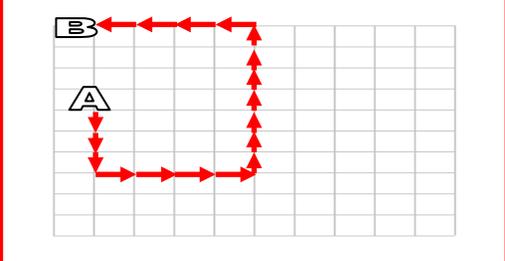
It's much easier to describe your set-up mathematically. Vectors are very useful here. Say, for example, that you're creating a scene lit by sunlight and ruffled by a strong wind. The sunlight and wind both come from a specific direction and have a certain intensity — so both can be represented by vectors. Using these vectors you can create a program that calculates exactly how an object in the scene should be coloured and move to give a realistic impression of lighting and wind. Even better, you can write your program so that the vectors representing sun and wind constantly change their direction and magnitude — thus you can create gusts of wind and clouds passing overhead.

## 5.6.2: What's your vector, Victor? Summary

Vectors and Language	Vectors in Sports	Vectors and Visuals
<p>Name the career where vectors are used?</p> <p>How are the vectors used in this career?</p>	<p>Name the career where vectors are used?</p> <p>How are the vectors used in this career?</p>	<p>Name the career where vectors are used?</p> <p>How are the vectors used in this career?</p>
<p>After listening and discussing the various careers, why would it be necessary for you to learn about vectors?</p>		

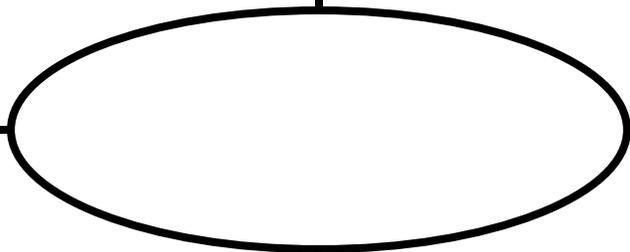
### 5.6.3: Distance and Displacement Discovered

Complete your questions by indicating the distance and displacement for each graphic. Work with your partner to verify your results.

Partner A:	Partner B:
 <p>Distance: _____</p> <p>Displacement: _____</p>	 <p>Distance: _____</p> <p>Displacement: _____</p>
 <p>Distance: _____</p> <p>Displacement: _____</p>	 <p>Distance: _____</p> <p>Displacement: _____</p>
 <p>Distance: _____</p> <p>Displacement: _____</p>	 <p>Distance: _____</p> <p>Displacement: _____</p>

## 5.6.4: Frayer Model

<b>Definition:</b>	<b>Facts/Characteristics:</b>
<b>Examples:</b>	<b>Non-examples:</b>



## 5.6.5: Vectors in the School

### Directions:

- You have been assigned to find how far it is from your math classroom to the cafeteria
- You will need to provide a vector diagram of the path that person would need to take in order to get to the cafeteria.
- Remember: A person who is not familiar with the school should be able to follow your map.

### Drawing Vectors: The Rules:

- Decide on an appropriate scale for the map.  
(1cm on the graph paper = 1 m on the floor)
- Draw a compass to indicate directions NORTH, SOUTH, EAST and WEST.
- Draw arrow heads at the end of your vectors and make sure you have the arrows pointing in the right direction.

### Measurement recording sheet:

- You will use the length of your foot to determine the magnitude of your vectors. Length of your foot in metres: \_\_\_\_\_
- Make sure to use heel-to-toe when counting your steps.
- Start making your map.



### Reflecting:

- How far is it to the cafeteria?
- Is this a vector quantity or a scalar quantity? What are your reasons for this answer?

### Class Recording:

- Record your measurement on the board.
- Looking at the class data, are all the measurements the same?
- Can you give some possible explanations for why the measurements are different?

## 5.6.6: Vector Analysis

The Grade 12 Leadership class has decided to go backpacking in Algonquin Provincial Park. There are two groups of students and each group is starting at the same access point. John's group starts at access point A and travels 3 km north, 2 km east, 1 km south, 4 km east and 5 km north to reach the campsite. Cathy's group decides to take another trail to the campsite. Her group starts at access point A and travels 4 km east, to start. Both groups are to meet at the same campsite for dinner. The last group to the campsite makes dinner and sets up the camp.

1. Draw a vector diagram to show John's journey. The diagram will need to have a scale and a compass to denote direction.
2. Since Cathy's group started out in an easterly direction, how far and in which direction would her group need to travel to reach the campsite? Give clear reasons to support your answer.
3. Draw a separate vector diagram to show Cathy's journey to the campsite. Remember to include the scale and compass.
4. Compare the distances the two groups travelled.
5. Both groups were travelling at 3 km/hour, how long did it take each group to reach the campsite. Which group is making dinner and setting up the camp? Show your calculations to justify your reasoning.

Questions modified from [www.resources.elearningontario.ca](http://www.resources.elearningontario.ca) ELO1073900

<b>Unit 5: Day 7: “Vector”y is ours!</b>		
Minds On: 10	<b>Learning Goal:</b> <ul style="list-style-type: none"> <li>Understand the equality of vectors</li> <li>Represent a vector as directed line segment with directions expressed in different ways</li> <li>Resolving a vector into horizontal and vertical components in context using Pythagorean theorem where appropriate</li> </ul>	<b>Materials</b> <ul style="list-style-type: none"> <li>BLM 5.7.1-5.7.5</li> <li>Protractors, rulers, graph paper</li> <li>MCT_U5L7GSP1.gsp</li> <li>MCT_U5L7GSP2.gsp</li> <li>MCT_U5L7AVI1.av (optional)</li> </ul>
Action: 50		
Consolidate: 15		
Total=75 min		
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Pairs → Activity</b> Develop the concept of equal vectors using BLM 5.7.1 and revisiting the ideas of magnitude and direction from the previous lesson.  Ensure that students have discerned that the conditions for equal vectors as same magnitude and direction.	  Students will require two copies of the map.
<b>Action!</b>	<b>Pairs → Exploration</b> Distribute BLM 5.7.2 to 5.7.4. Students may wish to explore using MCT_U5L7GSP1.gsp. Note: This portion of the lesson could be set up as stations (though 5.7.4 requires some knowledge of direction).  <b>Learning Skills/Observation/Checkbric</b> Use this opportunity to collect data about students’ independent learning skills.  <b>Mathematical Process Focus: Connecting</b> –Students will <b>connect</b> the mathematics to situations drawn from other contexts.	
<b>Consolidate Debrief</b>	<b>Whole Class → Discussion</b> Have students summarize their learning by creating a class note. Distribute BLM 5.7.5. The overview of the assignment should lead to a discussion of the concepts covered during the period. The map is available in MCT_U5L7GSP2.gsp.	
<i>Concept Practice Application</i>	<b>Home Activity or Further Classroom Consolidation</b> Complete BLM 5.7.5.	

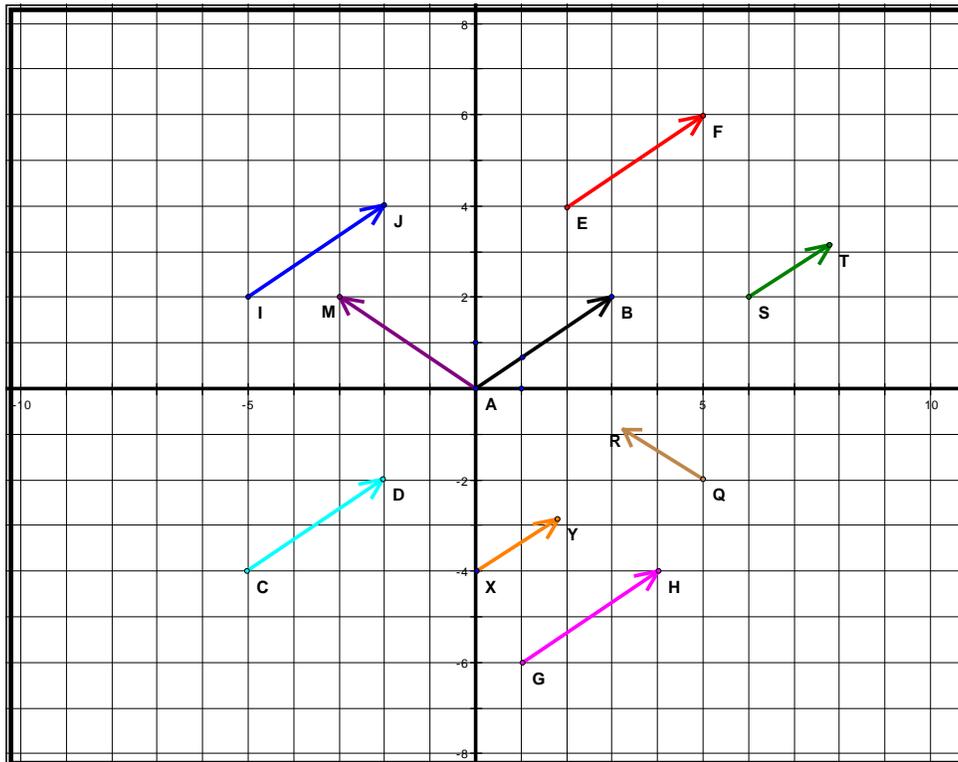
## 5.7.1: All Vectors are not Created Equal

1. Complete the following statement.

A vector is a quantity which has \_\_\_\_\_ and \_\_\_\_\_.

2. Examine the vectors in the diagram. Classify the vectors in the table below using appropriate vector notation (e.g.,  $\overrightarrow{AB}$ ).

Same Magnitude Only	Same Direction Only	Same Magnitude And Direction	No Similarities



3. On the grid, construct a vector with the same magnitude and direction as  $\overrightarrow{AM}$
4. Without measuring, how would you determine that  $\overrightarrow{AB}$  and  $\overrightarrow{EF}$  have equal magnitudes?
5.  $\overrightarrow{AB}$  and  $\overrightarrow{EF}$  are an example of equal vectors. What conditions are necessary for equal vectors?

## 5.7.2: Vector Components

1. Sketch the vector  $\overrightarrow{EF}$  with tail at E(0, 0) and head at F(3, 2).

The directed line segment  $\overrightarrow{EF}$  represents a displacement of some magnitude in the direction indicated by the arrow. We can describe  $\overrightarrow{EF}$  by giving its horizontal and vertical components.

2. Create a right triangle with the rise and run of the line segment that represents the vector.

3. Determine the horizontal component of  $\overrightarrow{EF}$  by subtracting the  $x$ -coordinates of the endpoints of the vector.

$$x = x_F - x_E$$

4. Determine the vertical component of  $\overrightarrow{EF}$  by subtracting the  $y$ -coordinates of the endpoints of the vector.

$$y = y_F - y_E$$

5. We can also define a vector by stating its components as an ordered pair of numbers. In this example,  $\overrightarrow{EF} = [\text{horizontal component}, \text{vertical component}]$ .

State the components of vector  $\overrightarrow{EF}$ .

**Note:** Square brackets are used to avoid confusion between the coordinates of a point and the components of a vector.

6. Use the above information and your knowledge of the Pythagorean theorem to determine the magnitude of vector  $\overrightarrow{EF}$ .

### Further Practice Questions

7. Sketch a diagram and determine the magnitude of the following vectors given their components.

$$\overrightarrow{AB} = [2, 5]$$

$$\overrightarrow{CD} = [-2, 5]$$

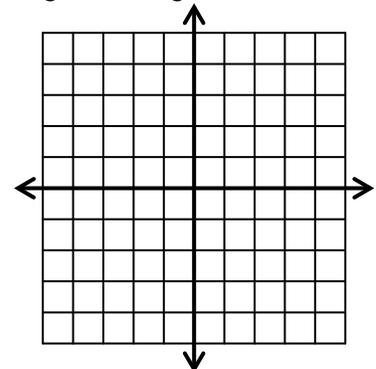
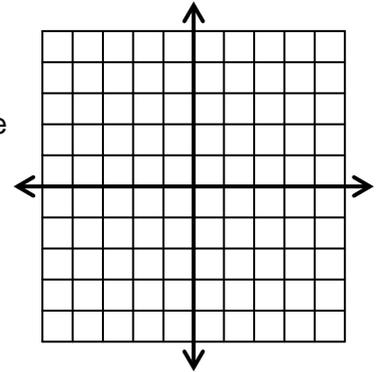
$$\overrightarrow{EF} = [3, 2]$$

$$\overrightarrow{GH} = [-2, 3]$$

8. Consider the following statement:

"If  $\overrightarrow{AB} = [2, 5]$  and  $\overrightarrow{ST} = [2, 5]$ , then  $\overrightarrow{AB}$  and  $\overrightarrow{ST}$  are equal."

Is this  always true     sometimes true     never true?  
Justify your choice.



### 5.7.3: Seeking Direction

Scaled vector diagrams are used in navigation to represent the motion of ships and aircraft. The direction is often stated as a bearing.

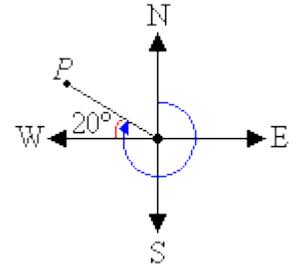
A **true bearing** to a point is the angle between due north and the line of travel of an object measured in degrees in a clockwise direction. We will refer to this as **bearing**.

A **conventional bearing** of a point is stated as the number of degrees east or west of the north-south line. We will refer to this as **direction**.

In the diagram below, the bearing of point P is  $290^\circ$ .

The direction method can be stated in two ways:

- $W20^\circ N$  (point P is  $20^\circ$  north of west)
- $N70^\circ W$  (point P is  $70^\circ$  west of north)



1. Complete the following table with reference to point P.

Diagram	Bearing	Direction	Diagram	Bearing	Direction
			Provide a sketch here.	$235^\circ$	

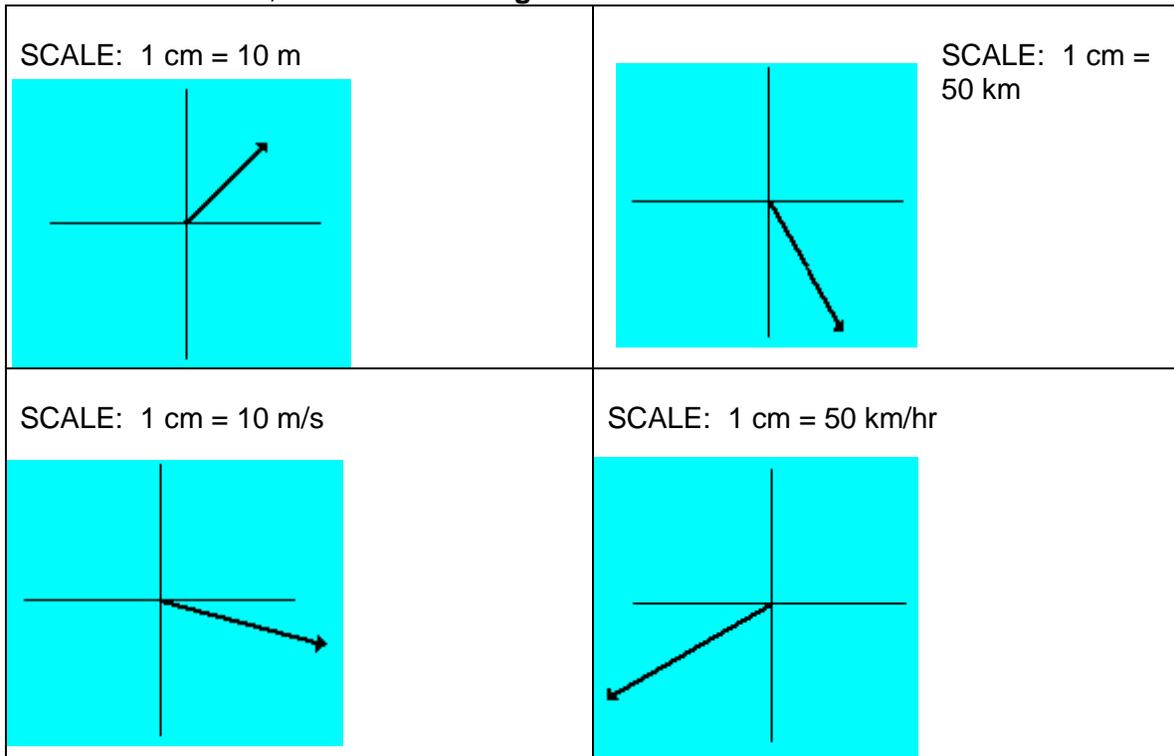
2. Describe each of the following bearings as directions in two different ways.

- $073^\circ$
- $145^\circ$
- $219^\circ$
- $305^\circ$

## 5.7.4: “Scalars” NOT Scalars

We know that a vector requires both magnitude and direction.  
Scale diagrams are useful in presenting geometric vectors.

1. Given the scale, determine the **magnitude** of each vector.



2. Use an accurately-drawn scaled vector diagram to represent the magnitude and direction for the following vectors.
- Given the SCALE: 1 cm = 10 m, represent the vector 50 m, with a bearing of  $60^\circ$  by a scaled vector diagram.
  - Given the SCALE: 1 cm = 10 m, represent the vector 60 m,  $W30^\circ N$  by a scaled vector diagram.
  - Given the SCALE: 1 cm = 15 m/s, represent the vector 120 m/s,  $S30^\circ W$  by a scaled vector diagram.
  - Given the SCALE: 1 cm = 20 m/s, represent the vector 140 m/s,  $N30^\circ E$  by a scaled vector diagram

## 5.7.5: Treasure Hunt

Examine the Treasure Map of Mathland provided. The starting point is the origin and your job is to determine a place to bury the treasure. You will provide instructions for locating the treasure using the vector ideas discussed in class.

### **INSTRUCTION CRITERIA**

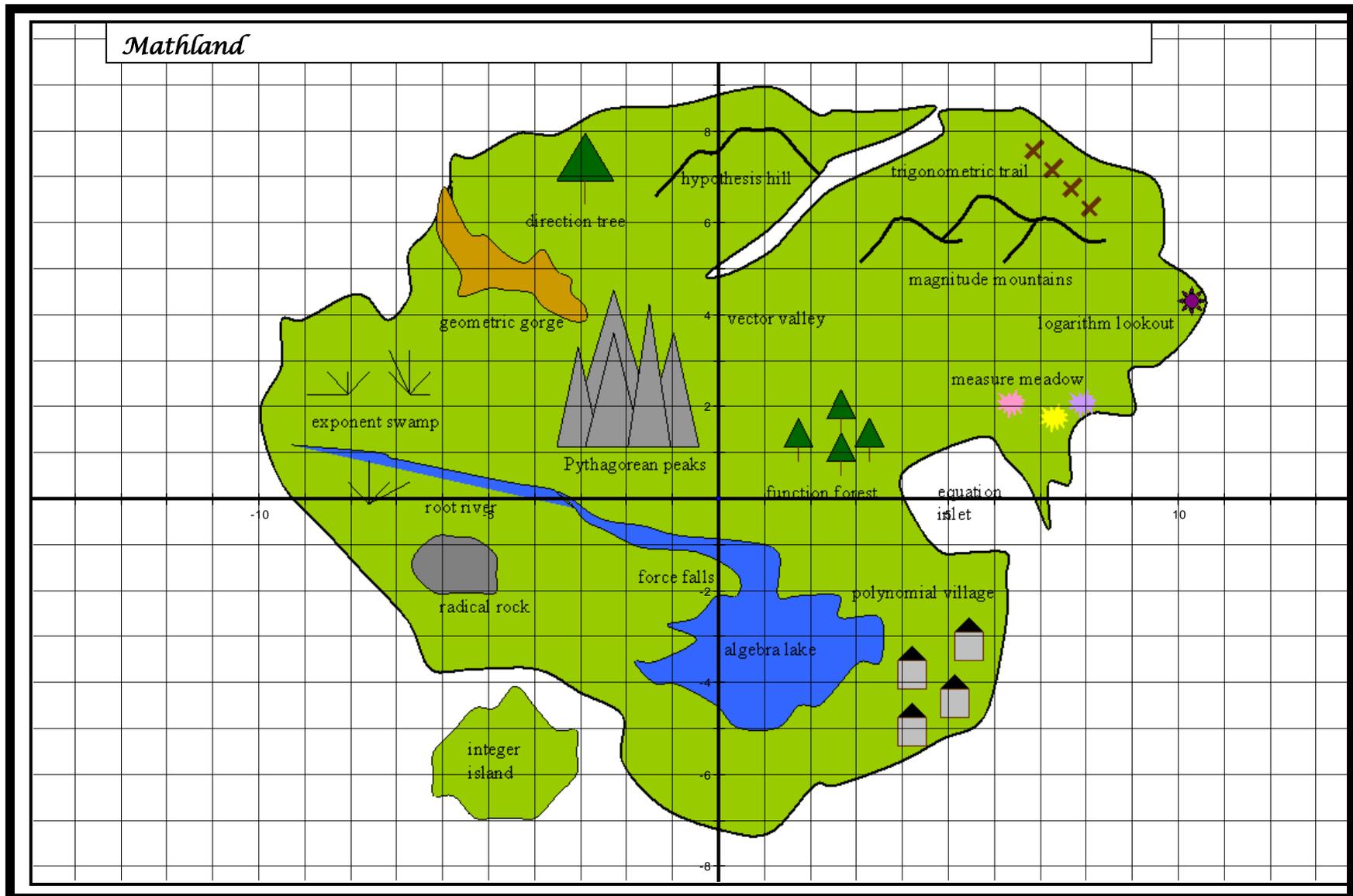
- Each map must consist of a minimum of six different vectors to find the treasure. No equal vectors may be used.
- The information to locate the treasure must include:
  - 2 vectors expressed as components (e.g.,  $\overrightarrow{AB} = [-3, 4]$ ) with at least one negative component
  - 2 vectors constructed using the scale provided and direction (e.g., bearing and/or direction angle)
  - 2 vectors described by horizontal and vertical components (e.g., 5 km east and 4 km south)
- A scale must be included with your map and all vectors must be drawn to scale.
- Instructions should be varied and include mathematical language and conventions (e.g., components, magnitude and direction, different ways of expressing direction)

### **SOLUTION CRITERIA**

To accompany your instructions, a copy of the solutions is required.

- Show all constructions on the solutions copy of your map
- Provide any calculations for determining the magnitude of each vector (e.g., given horizontal and vertical components)
- Determine the total distance travelled to locate the treasure
- Determine the total displacement from the start to the treasure

## 5.7.5: Treasure Hunt (continued)



<b>Unit 5: Day 8: Let the Force be with You</b>		
Minds On: 10	<b>Learning Goals:</b> <ul style="list-style-type: none"> <li>• Represent a vector as a directed line segments given its vertical and horizontal components.</li> <li>• Use trigonometric ratios to determine the horizontal component and/or vertical component of a vector.</li> </ul>	<b>Materials</b> <ul style="list-style-type: none"> <li>• BLM 5.8.1</li> <li>• Protractors, ruler and graph paper</li> </ul>
Action: 55		
Consolidate:10		
Total=75 min		
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Whole Class → Exploration</b> Engage students in the concept of force using an overhead cart (or similar object with wheels). Post directions N, E, W, S in the room. Create a starting position for the overhead cart at the front of the room that will allow for movement as directed in the following instructions.  Ask for two student volunteers. One will push the overhead cart to the east (relate to horizontal component) at a constant velocity and another student will push the overhead to the north (relate to vertical component) at a constant velocity.  Lead a discussion on the different forces and relate to components. Suggested questions for discussion: <ul style="list-style-type: none"> <li>• How would you alter the final position of the overhead?</li> <li>• What would need to be altered to increase the horizontal component and decrease the vertical component?</li> <li>• How would this be represented on the Cartesian plane?</li> </ul> <b>Curriculum Expectations/Oral Questions/Mental Note</b> As students respond to the questions and participate in the discussion, determine their readiness for the lesson.	  Students may struggle with the concept that the components may be negative, but their magnitude is positive.
<b>Action!</b>	<b>Pairs → Exploration</b> Students work through BLM 5.8.1 with a partner.  <b>Mathematical Process Focus:</b> Representing – Students <b>represent</b> forces using vectors.	
<b>Consolidate Debrief</b>	<b>Whole Class → Discussion</b> Discuss solutions to Part A of BLM 1.7.1. Provide a scenario where students would need to determine the direction of a vector given its horizontal and vertical components or the magnitude of the vector and one of the components.	
<i>Exploration Application</i>	<b>Home Activity or Further Classroom Consolidation</b> Continue to work on map assignment. Complete practice questions.	See MMW4 chapter 10.7 for good practice questions.

## 5.8.1: Let the Force be with You

### Warm up:

Draw and label the vectors. The tail of each vector should be at the origin. Complete the information in the table.

vector	x-component and its magnitude	y-component and its magnitude
$\vec{u} = [2,3]$	$u_x = 2$ $ \vec{u}_x  = 2$	$u_y = 3$ $ \vec{u}_y  = 3$
$\vec{v} = [-2,3]$		
$\vec{w} = [3,-2]$		
$\vec{z} = [-3,-2]$		

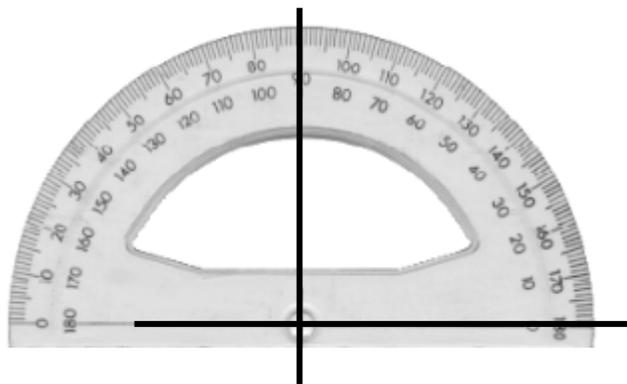
A common use of vectors involves force. Finding the resulting force on an object depends on several separate forces acting on the same object.

This symbol means to discuss your answer(s) with a partner.

### Part A - The Billiard Ball

1. Suppose you hit a billiard ball with a force of 6 Newtons (N) and direction of E40°N.

- Draw a diagram for this force vector using a scale 1 cm = 2 N.
- Label the vector  $F = 6$  N. Include the angle in your vector diagram.



2. Every vector can be broken down into two parts:

- One vector with magnitude in the  $x$ -direction (e.g.,  $F_x$  read “F sub  $x$ ”)
- One vector with magnitude in the  $y$ -direction (e.g.,  $F_y$  read “F sub  $y$ ”)

Label these two components,  $F_x$  and  $F_y$ , for the force vector,  $F$ .

3. Circle the **correct** trigonometric ratio to calculate the  $F_x$  component of the vector representing the force of the billiard ball.

A)  $\sin 40^\circ = \frac{F_x}{6}$     B)  $\sin 40^\circ = \frac{6}{F_x}$     C)  $\sin 60^\circ = \frac{F_x}{6}$     D)  $\sin 60^\circ = \frac{6}{F_x}$

### 5.8.1: Let the Force be with You (continued)

4. Use the trigonometric ratio selected in #3 to determine the magnitude of the horizontal component. Show all of your work.

∴ The horizontal force applied to the billiard ball is \_\_\_\_\_ N.

5. What other trigonometric ratio could you have used to determine the  $F_x$  component?



6. Circle the **correct** trigonometric ratio to calculate the  $F_y$  component of the vector representing the force of the billiard ball.

$$A) \sin 40^\circ = \frac{F_x}{6} \quad B) \sin 40^\circ = \frac{6}{F_x} \quad C) \sin 60^\circ = \frac{F_x}{6} \quad D) \sin 60^\circ = \frac{F_x}{6}$$

7. Use the trigonometric ratio selected in #6 to determine the magnitude of the vertical component. Show all of your work and include correct units.

∴ The vertical force applied to the billiard ball is \_\_\_\_\_ N.

8. What other trigonometric ratio could you have used to determine the  $F_y$  component?

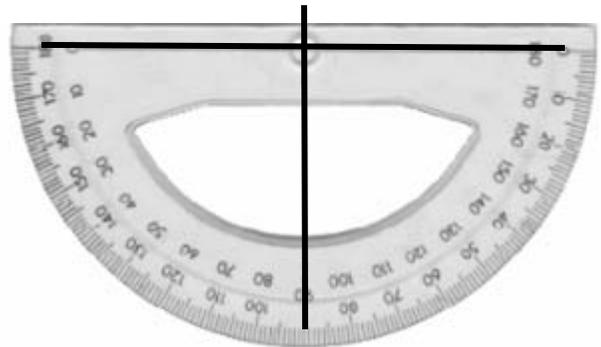


9. Show work to verify that this ratio will produce the same result.

## 5.8.1: Let the Force be with You (Continued)

10. Construct a force vector that has direction W30°S and a force greater acting vertically than horizontally.

- Label the force vector.
- Label the horizontal and vertical components of the vector.
- On a separate piece of paper, determine the magnitude of the vector and its components.



11. Is the following statement (check one):

- always true     sometimes true     never true?

“Given a force vector,  $\vec{F}$ , if  $\vec{F}_x < 0$  then  $|\vec{F}_y| > 0$ , then  $\vec{F}_x > 0$  and  $\vec{F}_y > 0$ .”

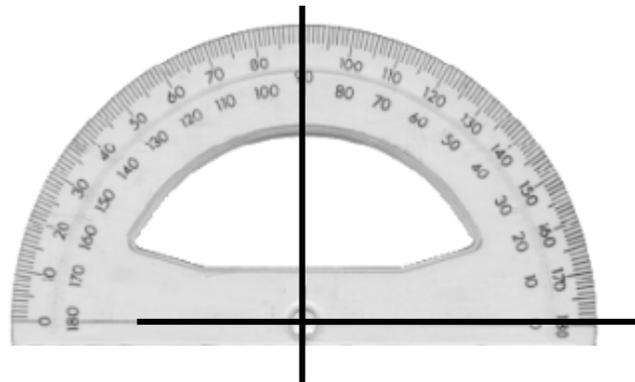
Provide justification for your choice.



### Part B - “Forcing” you to cut the lawn

A snow blower is pushed with a force of 1200 N directed along its handle. The angle with the ground made by the handle is 50°.

- Construct a scale diagram to represent this scenario.  
Provide a scale for your diagram.
- On a separate piece of paper, use your knowledge of trigonometric ratios to calculate the vertical and horizontal components of the force required for the snow blower to maintain a constant velocity.  
Show your work.



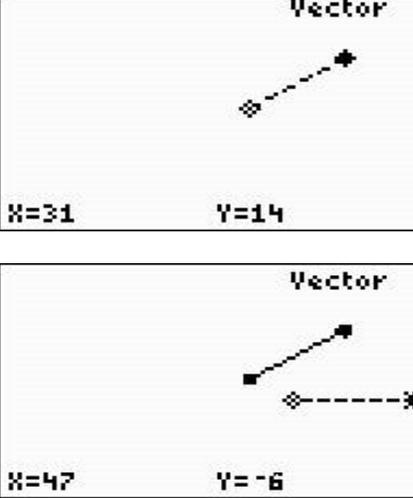
### Part C – Net Force

Two people each pull a rope that is connected to a boat. One pulls with a force of 450 N at an angle of 70° from the horizontal. The other pulls from the other side of the boat with a force of 670 N 50° from the horizontal. Determine the net force on the boat.

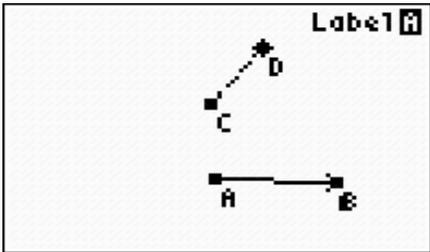
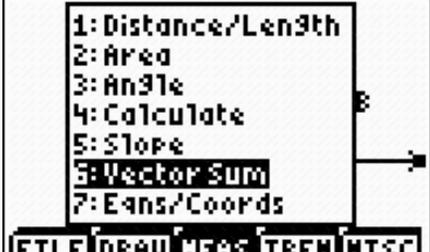
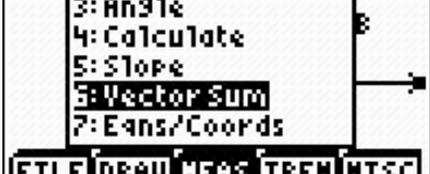
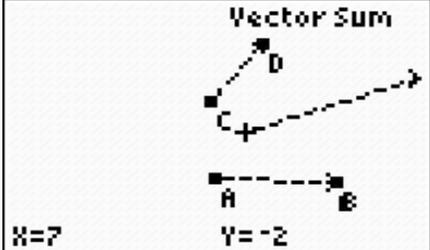
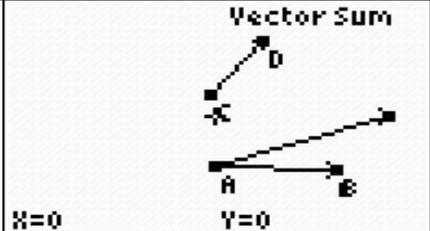
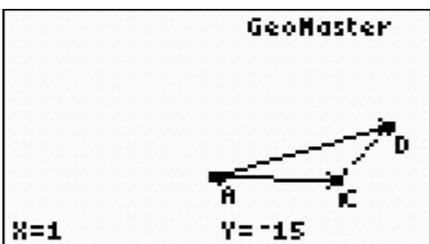
**Hint:** construct a diagram with the boat at the origin.

<b>Unit 5: Day 9: Add it up!</b>		
Minds On: 10	<b>Learning Goal:</b> <ul style="list-style-type: none"> <li>Investigate how to add and subtract two vectors</li> <li>Develop methods for adding and subtracting vectors</li> </ul>	<b>Materials</b> <ul style="list-style-type: none"> <li>BLM 5.9.1 and 5.9.2</li> <li>PPT 5.9.1</li> <li>Graphing calculator</li> <li>Graph paper</li> </ul>
Action: 45		
Consolidate:20		
Total=75 min		
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Whole Class → Activate prior knowledge</b> Use the first four slides of the PowerPoint. Have students hypothesize what they think vector addition will look like. Ask: why might we need vector addition?	Graphing calculators will need the application GeoMaster™. GeoMaster™ can be downloaded from TI website: <a href="http://www.educationti.com">www.educationti.com</a> . Go to Downloads.  Investigation modified from Aki Margaritis Brookview School Benton Harbor, MI 49022  Images from: <a href="http://www.nasaexplores.com">www.nasaexplores.com</a> <a href="http://www.physicsclassroom.com">www.physicsclassroom.com</a>
<b>Action!</b>	<b>Pairs → Investigation</b> Students work through BLM 5.9.1 and BLM 5.9.2 with a partner.  <b>Learning Skills/Observation/Checkbric</b>  <b>Mathematical Process Focus: Reasoning and Proving</b> – Students use their reasoning skills to develop different strategies for adding and subtracting vectors.	
<b>Consolidate Debrief</b>	<b>Whole Class → Debrief</b> Review questions from <i>Confirming Your Thinking</i> of BLM 5.9.2. Complete PowerPoint viewing and discuss triangle addition. Show parallelogram addition and how it will also allow students to find the resultant vector.  Use part C from lesson 8 to model solution and the idea of vector addition using parallelogram method: Two people each pull a rope that is connected to a boat. One pulls with a force of 450 N at an angle of 70° from the horizontal. The other pulls from the other side of the boat with a force of 670 N 50° from the horizontal. Determine the net force on the boat.	
<i>Concept Practice Application</i>	<b>Home Activity or Further Classroom Consolidation</b> Finish BLM 5.9.2 for next day.	

## 5.9.1: Add it Up!

Written instructions	Screen Shots
<p><b>Step 1: Activating the Application</b></p> <ul style="list-style-type: none"> <li>• Press A</li> <li>• Cursor down and hi-light GeoMaster™</li> <li>• Press e</li> </ul>	
<p><b>Step 2: Getting started</b></p> <ul style="list-style-type: none"> <li>• Press enter—home screen will be displayed</li> </ul>	
<p><b>Step 3: Finding the menu bar</b></p> <ul style="list-style-type: none"> <li>• Press % to display the menu bar</li> <li>• Press @ to have the ð menu appear</li> <li>• Select 9:Vector</li> </ul>	
<p><b>Step 4: Drawing the vectors</b></p> <ul style="list-style-type: none"> <li>• Position the cursor anywhere on the screen</li> <li>• Press e—the tail of the vector has been set</li> <li>• Move the cursor and you will begin to see the magnitude of the vector increasing</li> <li>• Move the cursor to the location for the head</li> <li>• Press e</li> <li>• Create another vector anywhere on the screen</li> </ul>	

## 5.9.1: Add it Up! (continued)

<p><b>Step 5: Labelling the vectors</b></p> <ul style="list-style-type: none"> <li>Press % to activate the menu</li> <li>Press % to activate the MISC</li> <li>Select 2: Label</li> <li>Cursor over to the tail of one of the vectors</li> <li>Press e</li> <li>Press the m button on the calculator—you are in a mode—the letter A should appear</li> <li>Cursor over to the head of the same vector</li> <li>Press e</li> <li>Press the A button on the calculator—the letter B should appear</li> <li>Repeat the above steps for the other vector BUT label its tail "C" and head "D"</li> </ul>	 
<p><b>Step 6: Finding the Vector Sum</b></p> <ul style="list-style-type: none"> <li>Press % to display the menu bar</li> <li>Press # to hi-light the MEAS (measure) menu</li> <li>Select 6: Vector Sum</li> </ul>	 
<p><b>Step 7: The resultant vector</b></p> <ul style="list-style-type: none"> <li>Place cursor on vector <math>\overrightarrow{AB}</math> and press e</li> <li>Repeat this for CD</li> <li>The vector sum should be displayed</li> <li>The cursor is on the vector sum and this vector can move</li> </ul>	
<p><b>Step 8: Bringing it all together</b></p> <ul style="list-style-type: none"> <li>Place the head of the vector sum onto the head of vector <math>\overrightarrow{AB}</math></li> <li>Press e—the vector sum vector is locked into place</li> <li>Press C and return to home screen</li> <li>Move cursor to vector <math>\overrightarrow{CD}</math></li> <li>Press e twice</li> <li>You can now move it</li> <li>Place CD so that that the head of C is on the tail of AB</li> </ul>	 

## 5.9.2: Add it Up!

Questions to consolidate your thinking:

Addition:

1. By moving vector  $\overrightarrow{CD}$ , what do you observe about point D?

---

2. What do you observe about the sum of the two vectors?

---

3. Determine a relationship for the vector sum of  $\overrightarrow{AB}$ ,  $\overrightarrow{AD}$ ,  $\overrightarrow{CD}$

---

4. Use the arrow keys to move the cursor. Begin by placing it on point A then point B and so forth. Each time record the coordinates of each point: (x y).

A = (\_\_\_\_, \_\_\_\_)  
C = (\_\_\_\_, \_\_\_\_)

B = (\_\_\_\_, \_\_\_\_)  
D = (\_\_\_\_, \_\_\_\_)

5. Because a vector shows direction as well as magnitude we need to change vectors to their component form. Recall the formula from lesson #7:

$$\overrightarrow{AB} = [x_2 - x_1, y_2 - y_1] = [v_1, v_2] \square$$

Write each of the following vectors in their component form:

$\overrightarrow{AB}$  : [\_\_\_\_, \_\_\_\_]

$\overrightarrow{CD}$  : [\_\_\_\_, \_\_\_\_]

6. Express vector AD in component form.

$\overrightarrow{AD}$  : [\_\_\_\_, \_\_\_\_]

7. Test the relationship you developed in #2 for the vector sum

---

8. Write a general formula for the sum of two vectors:

---

## 5.9.2: Add it Up! (continued)

Subtraction:

9. What do you think would be the formula for finding the difference of two vectors?

---

10 Draw your own two vectors and find their difference using GeoMaster™. Does this difference confirm your formula from question #9?

---

### Confirming your thinking

1. State the vector components:

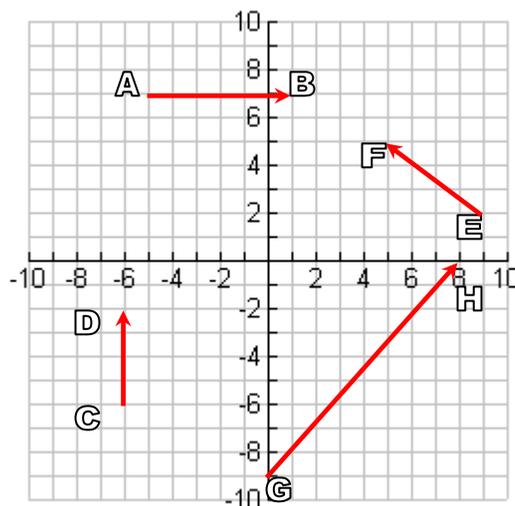
$$\vec{AB} = [ \quad, \quad ] \quad \vec{CD} = [ \quad, \quad ]$$

$$\vec{EF} = [ \quad, \quad ] \quad \vec{GH} = [ \quad, \quad ]$$

$$\vec{AB} + \vec{CD} = [ \quad, \quad ] \text{ and}$$

$$\vec{EF} + \vec{GH} = [ \quad, \quad ]$$

$$\vec{AB} + \vec{CD} = [ \quad, \quad ] \text{ and } \vec{EF} - \vec{GH} = [ \quad, \quad ]$$



### Making other connections

2. Determine the sum or difference.

$$\vec{CD} + \vec{AB} = [ \quad, \quad ] \text{ and } \vec{GH} + \vec{EF} = [ \quad, \quad ]$$

$$\vec{CD} - \vec{AB} = [ \quad, \quad ] \text{ and } \vec{GH} - \vec{EF} = [ \quad, \quad ]$$

## 5.9.2: Add it Up! (continued)

3. Is the following statement (check one):

- always true     sometimes true     never true?

For any two vectors  $\overrightarrow{PQ}$  and  $\overrightarrow{ST}$  :  $\overrightarrow{PQ} + \overrightarrow{ST} = \overrightarrow{ST} + \overrightarrow{PQ}$

Provide justification to support your thinking.

4. Is the following statement (check one):

- always true     sometimes true     never true?

For any two vectors  $\overrightarrow{PQ}$  and  $\overrightarrow{ST}$  :  $\overrightarrow{PQ} - \overrightarrow{ST} = \overrightarrow{ST} - \overrightarrow{PQ}$

Provide justification to support your thinking.

5. State a general rule for the addition of vectors and a general rule for the subtraction of vectors.

### Further Practice

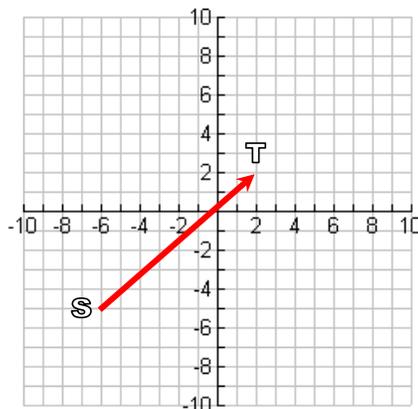
1. The resultant vector  $\overrightarrow{ST}$  is given on the graph.

a) Provide an example of 2 vectors that could be added to give this resultant

vector  $\overrightarrow{ST}$  .

b) Provide an example of 2 vectors that could be subtracted to give this resultant

vector  $\overrightarrow{ST}$  .



<b>Unit 5: Day 10: So What's Your Problem?</b>		
Minds On: 15	<b>Learning Goal:</b> <ul style="list-style-type: none"> <li>Solve various vector addition and subtraction problems using a variety of methods</li> </ul>	<b>Materials</b> <ul style="list-style-type: none"> <li>BLM 5.10.1 and 5.10.2</li> <li>Overhead</li> <li>Overhead pens</li> <li>Overhead film</li> </ul>
Action: 50		
Consolidate:10		
Total=75 min		
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Whole Class → Practise</b> Engage the students in completing the solution to the following question, using the opportunity to model proper use of vocabulary and form.  An airplane is flying at a heading of N30° W at a constant speed of 200 km/h. If the wind is blowing at constant speed of 50 km/h east, what is the actual speed and direction of the airplane with respect to the ground?	
<b>Action!</b>	<b>Groups of 3 → Practice</b> Students complete BLM 5.10.1. Groups can check solutions another group. When all solutions have been completed by a group, they are ready to create their own question.  <b>Groups of 3 → Activity</b> Groups complete BLM 5.10.2. All group members need to have a copy of the question and solution. These solutions will be shared and discussed in a future class.  <b>Learning Skills/Observation/Checkbric</b> Assess group work skills.  <b>Mathematical Process Focus: Connecting</b> – Students <b>connect</b> the mathematics to contexts outside mathematics.	
<b>Consolidate Debrief</b>	<b>Whole Class → Discussion</b> Discuss the challenges associated with solving today's problems and the effectiveness of various solution strategies.	
<i>Application Reflection</i>	<b>Home Activity or Further Classroom Consolidation</b> Ensure that your group problem is ready for copying for your presentation on _____ and that all participants are ready to present their solution.  Complete a journal entry by indicating what was challenging about creating your own problem.	

## 5.10.1: So What's Your Problem?

<p>Problem 1:</p> <p>A canoeist is travelling with a velocity of 2 m/s across a river her campsite. The current in the in the river is moving at velocity of 5 m/s and is carrying the canoeist down the river. What is the actual velocity of the canoe?</p>	<p>Problem 2:</p> <p>In order to widen a road, a tree stump needs to be removed. Two backhoes are brought to the site. One backhoe pulls with a force of 2500 N east while the second backhoe pulls with a force of 3000 N northeast. What is the resultant force and its angle with respect to the first backhoe?</p>
<p>Problem 3:</p> <p>A light airplane is travelling at a speed of 150km/h with a heading <math>N50^{\circ}W</math>. The wind is blowing due south onto the airplane. What is the ground speed of the airplane?</p>	<p>Problem 4:</p> <p>The Leadership Club has decided to go cross-country skiing and the group has decided to stop at a rest cabin located on the trail. What is the distance from the parking lot to the rest cabin, if they skied 3 km west, 2 km northwest, 3 km east and 4 km north to get to the cabin?</p>

## 5.10.2: So What's Your Problem?

Now it's your turn! Your group is to create a problem that can be modelled by vectors. Be creative!

Criteria for the presentation:

- A written problem with the necessary information.
- A vector diagram used to model the problem (including an appropriate scale and accurate angle measures).
- A solution to the problem using your diagram.
- A solution to the problem using the components of the vectors (i.e., without the diagram)
- A comment on the reasonableness of your solution.
- A concluding statement.

Ensure that all group members have roles in the presentation of your problem. Use mathematical language and reference to vector concepts in your presentation. You may want to access the internet for ideas and reasonable magnitudes for velocity, force and distance. You can use the overhead or LCD projector presentation of the problem.

### Evaluation Rubric

Your problem will be evaluated using the following rubric.

Achievement Category	Level R	Level 1	Level 2	Level 3	Level 4
Knowledge/ Understanding	No evidence	Shows a limited understanding of vectors	Shows some understanding of vectors	Shows an understanding of vectors	Shows a high degree of understanding of vectors
Application	No evidence	Shows limited connection between vectors and the context. The numbers used are not realistic to the context	Shows some connection between vectors and the context. The numbers used are somewhat realistic to the context	Shows a connection between vectors and the context. The numbers use are realistic to the context	Shows a strong connection between vectors and the context. The numbers use are realistic to the context
Communication	No evidence	Question and solution shows limited clarity	Question and solution shows some clarity	Question and solution shows clarity	Question and solution shows a high degree of clarity