Unit 2: Day 9: Summative Assessment

MCV4U

Minds On: 5 Action: 60 Consolidate: 10 Total = 75 min	**Learning Goals**: • Identify increasing and decreasing rates of change using graphical and numerical representations of polynomial functions. • Investigate connections graphically and numerically between the graph of a polynomial function and its derivative using technology. • Determine, through investigation using technology, the graph of the derivative $f'(x)$ of a given exponential function and make connections between the graphs of $f(x)$ and $f'(x)$. • Explore the ratio $f'(x)/f(x)$ and explain how to use this ratio to determine the slopes of the tangents to $f(x)$. • Investigate connections between an exponential function whose graph is the same as its derivative using technology. • Make connections between the natural logarithm function and the function $f(x)=e^x$. • Verify, using technology, that the derivative of the exponential function $f(x) = a^x$ is $f'(x) = a^x \ln a$ for various values of a.	**Materials** • Graphing calculators • BLM 2.9.1 • BLM 2.9.2 •

			Assessment Opportunities
	Minds On…	**Whole Class → Discussion** Distribute BLM 2.9.1 and discuss the criteria and qualifiers to clarify expectations for the assessment.	The data on BLM 2.9.2 can be put into a program that placed it into lists to save valuable time keying data and to avoid entry errors. Data for the population of Canada was taken from the website: http://www.statcan.ca/english/freepub/98-187-XIE/pop.htm or http://estat.statcan.ca/cgi-win/cnsmcgi.exe
	Action!	**Individual → Assessment** Distribute BLM 2.9.2 and a graphing calculator to each student. Students individually complete BLM 2.9.2. **Curriculum Expectations/Oral Questions/Rubric** Circulate as students complete the task to ask them clarifying questions. Evaluate their work using BLM 2.9.1. **Mathematical Process Focus: Communicating, Proving**	
	Consolidate Debrief	**Whole Class → Collection of Materials** Collect student work. Have students attach BLM 2.9.1 to their work. **Small Groups → Class Sharing** In heterogeneous groups of three students share strategies they found effective for preparing for and completing the assessment. Call on each group to share observations with the class.	
	Journal	**Home Activity or Further Classroom Consolidation** Respond to the following prompts in your journal: In preparing for this summative assessment, I …. Things I would consider changing for next time include ….	

2.9.1 Exploring Derivatives – Summative Assessment

- This is a one-day assessment.
- Ensure that your solution is fully justified and verified.
- Your grade on this assignment will be a combination of your written solution and your oral answers to my questions throughout the solution process.

	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
Identify increasing and decreasing rates of change using graphical and numerical representations of polynomial functions (A2.1)	Identifies increasing and decreasing rates of change using graphical and numerical representations of polynomial functions with limited understanding	Identifies increasing and decreasing rates of change using graphical and numerical representations of polynomial functions with some understanding	Identifies increasing and decreasing rates of change using graphical and numerical representations of polynomial functions with considerable understanding	Identifies increasing and decreasing rates of change using graphical and numerical representations of polynomial functions with a high degree of understanding
Investigate connections graphically and numerically between the graph of a polynomial function and its derivative (A2.2)	Investigates connections graphically and numerically between the graph of a polynomial function and its derivative with limited understanding	Investigates connections graphically and numerically between the graph of a polynomial function and its derivative with some understanding	Investigates connections graphically and numerically between the graph of a polynomial function and its derivative with considerable understanding	Investigates connections graphically and numerically between the graph of a polynomial function and its derivative with a high degree of understanding
Application				
Demonstration of understanding of relationship between original graph and graph of derivative (A2.5)	Student is able to draw the derivative graph given the original graph with limited success	Student is able to draw the derivative graph given the original with some success	Student is able to draw the derivative graph given the original with considerable success	Student clearly is able to draw the derivative graph given the original with a high degree of success
Determines the derivative of $f(x) = e^x$ (A2.6)	Determines the derivative of $f(x) = e^x$ with limited understanding	Determines the derivative of $f(x) = e^x$ with some understanding	Determines the derivative of $f(x) = e^x$ with considerable understanding	Determines the derivative of $f(x) = e^x$ with a high degree of understanding
Makes connections between $f(x) = ln(x)$ and $f(x) = e^x$ (A2.7)	Tool selected is only partially appropriate to the context of the question and used with some inaccuracies	Tool selected is generally appropriate to the question, but contains minor errors	Tool selected is appropriate and relates to the context of the question and is used correctly	Tool selected is appropriate, used correctly and efficient and relates to the broadest context of the question.
Thinking				
Creation of a model for Canadian Population	A model was used that addresses few aspects of the problem Reasoning is evident but lacks consistency	A model was used that addresses some aspects of the problem Reasoning is evident and logical in parts of the problem	An appropriate model was created Reasoning is logical and consistent within context	An appropriate model was created that integrates all aspects of the problem/context Reasoning is logical and consistent and relates to broader context
Making Conclusions and Inferences	Conclusions are somewhat connected with work but are not supported	Conclusions are consistent with work but are not fully supported	Conclusions are supported by evidence and integrate aspects of the problem	Conclusions are supported, integrate all aspects of the problem, and are convincing
Solve a variety of problems with $f(x) = a^x$, using the techniques of differential calculus (2.8)	The student selects a few appropriate tools and uses them to make a limited attempt to solve the problem	The student selects some appropriate tools and uses them to solve some of the problem	The student selects appropriate tools and uses them to solve most of the problem	The student selects appropriate tools and incorporates them in the execution of a plan to solve the problem and verify solution
Communication				
Describes key features of the graph using appropriate terminology	Correctly identifies few of the key features of the graphs	Correctly identifies some of the key features of the graphs	Correctly identifies most significant key features of the graphs	Correctly identifies all key features of the graphs and interprets them fully
Degree of clarity in explanations and justifications	Explanations and justifications lack clarity	Explanations and justifications are understandable but lack clarity	Explanations and justifications are clear	Explanations and justifications are clear and thorough

2.9.2 Exploring Derivatives – Summative Assessment

Name: _____ **Date:** _____

1) Modelling Population:

Every year Statistics Canada prepares estimates and projections of Canada's population. The estimated population of Canada since 1901 is given in the table.

Year	Years since 1901	Canadian Population (estimated)
1901	0	5 371 000
1906	5	6 097 000
1911	10	7 207 000
1916	15	8 001 000
1921	20	8 788 000
1926	25	9 451 000
1931	30	10 377 000
1936	35	10 950 000
1941	40	11 507 000
1946	45	12 292 000
1951	50	14 009 000
1956	55	16 081 000
1961	60	18 238 000
1966	65	20 015 000
1971	70	21 961 999
1976	75	23 449 791
1981	80	24 820 393
1986	85	26 101 155
1991	90	28 031 394
1996	95	29 610 757
2001	100	31 021 251
2006	105	32 623 490

http://www.statcan.ca/english/freepub/98-187-XIE/pop.htm

2.9.2 Exploring Derivatives – Summative Assessment (Continued)

Respond to the following questions on a separate sheet of paper.

1. **Modelling**

Answer the following questions about the Canadian population data.

a) Use the data from the table to create a scatter plot on the graphing calculator.

b) Create a curve of best fit. What type of function is this? Explain.

c) Sketch the function and curve of best fit.

d) Describe properties of the graph and relate its properties to what you have learned in this unit.

e) Determine the equation of a function that could be used to model Canada's population.

f) Use the data to estimate the average growth rate for a 5 year period.

g) Predict the population of Canada in 2020. What assumptions do you need to make?

h) Use your equation from e) to estimate the population of Canada in 1973.

i) Determine the rate at which the population of Canada is increasing in 1911.

j) Determine the rate at which the population of Canada is increasing in 1996.

k) How long will it take Canada to reach a population of 50, 000, 000?

l) Rewrite your equation in part d) using base 'e'.

m) Rewrite your derivative equation using base 'e'.

n) Sketch the derivative of the function. Discuss the properties of the derivative function in relation to the original function.

2) **Solving Tangent Lines**:

State conditions for a and b such that the graph of $f(x) = -ae^x - bx + c$ has NO tangent line parallel to the x axis (horizontal tangent).

3) **Relating $f(x) = e^x$ and $f(x) = ln(x)$:**

Re-write each equation in terms of x:

a) $2.5e^{3x} = 7.5$ b) $\ln x = \sqrt{2}$