MCV4U
Calculus and Vectors
University Preparation
Unit 6

To be updated the week of August 27 and further edited in September 2007

Unit 6: Representing Lines and Planes

Grade 12

Lesson Outline

Big Picture

Students will:

- represent lines and planes in a variety of forms and solve problems involving distances and intersections;
- determine different geometric configurations of lines and planes in three-space;
- investigate intersections of and distances between lines and/or planes.

Day	Lesson Title	Math Learning Goals	Expectations
1		• Recognize that a linear equation in two-space forms a line and represent it geometrically and algebraically • Represent a line in two-space in a variety of forms (scalar, vector, parametric) and make connections between the forms	C3.1, C4.1
2		• Recognize that a line in three-space cannot be represented in scalar form • Represent a line in two-space in a variety of forms (vector and parametric) and make connections between the forms	C4.2
3		• Recognize that a linear equation in three-space forms a plane and represent it geometrically and algebraically • Determine through investigation geometric properties of planes including a normal to a plane • Determine using the properties of the plane the scalar, vector and parametric equations of a plane	C3.2, C4.3, C4.5
4		• Determine the equation of a plane in its scalar, vector, or parametric form given another of these forms. • Represent a line in three-space by using the scalar equations of two intersecting planes	C4.6, C4.2
Refer to Smart Ideas file (Overview.ipr) for a flowchart of the concepts covered in Lessons 5 through 10.			
5	Lots of Lines *(Sample Lesson Included)*	• Recognize that a linear equation in two-space forms a line represent it geometrically and algebraically • Recognize that the solution to a system of two linear equations in two-space determines a point in two space if the lines are not coincident or parallel • Solve and classify solutions to systems of equations in two-space in vector and parametric forms and understand the connections between the graphical and algebraic representations	C3.1, C4.1
6	Concrete Critters *(Sample Lesson Included)*	• Determine through investigation different geometric configurations of combinations of up to three lines and/or planes in three space • Classify sets of lines and planes in three space that result in a common point, common line, common plane or no intersection	C3.3
7	Interesting Intersections I *(Sample Lesson Included)*	• Determine the intersections of two lines, and a line and a plane in three space given equations in various forms and understand the connections between the geometric and algebraic representations	C3.3, C4.7

Day	Lesson Title	Math Learning Goals	Expectations
8	Interesting Intersections II *(Sample Lesson Included)*	• Determine the intersections in three-space of 2 planes and 3 planes intersecting in a unique point given equations in various forms and understand the connections between the graphical and algebraic representations of the intersection	C3.3, C4.3, C4.4, C4.7
9	Interesting Intersections III *(Sample Lesson Included)*	• Determine the intersections of 3 planes in three space given equations in various forms and understand the connections between the graphical and algebraic representation of the intersection • Recognize that if $\vec{a} \bullet \vec{b} \times \vec{c} \neq 0$ is true then the three planes intersect at a point • Solve problems involving the intersection of lines and planes in three-space represented in a variety of ways	C4.4, C4.7
10	How Far Can it Be? *(Sample Lesson Included)*	• Calculate the distance in three-space between lines and planes with no intersection • Solve problems related to lines and planes in three-space that are represented in a variety of ways involving intersections	C3.3, C4.3, C4.7
11	Jazz Day		
12–14	Summative Assessment Units 5 and 6		

75 min

Math Learning Goals

- Recognize that a linear equation in two-space forms a line represent it geometrically and algebraically
- Recognize that the solution to a system of two linear equations in two-space determines a point in two space if the lines are not coincident or parallel
- Solve and classify solutions to systems of equations in two-space in vector and parametric forms and understand the connections between the graphical and algebraic representations

Materials

- BLM 6.5.1
- chart paper and markers

Assessment Opportunities

Minds On…

Groups → Graffiti

Prepare 9 sheets of chart paper each with a system of 2 equations. See teacher BLM 6.5.1 for examples. Post these in order on the wall to make 9 stations.

Curriculum Expectation: Solving a system of 2 equations in 2-space/ Observation/Mental Note.

Each group solves two of the three types of systems and summarizes the third type.

In heterogeneous groups of 3 or 4 (total 9 groups), students visit 3 consecutive stations therefore working with systems of equations having a unique solution, representing two coincident lines, and representing parallel lines. Each group starts at a different station. At the first station each group solves the system graphically. Then each group moves clockwise one station and solves the system at this station algebraically. Finally, each group moves clockwise one station and by observing and reasoning about the graphical and algebraic work completed, students write a summary of the connections between the algebraic and graphical representations of the system.

Assessment for Learning to ensure student readiness to proceed

Refer to Smart Ideas file Overview.ipr for a flowchart of the concepts covered in lessons 5 through 10.

Groups → Gallery Walk

Groups visit the next three stations to consolidate their findings.

Action!

Whole Class → Teacher Led Instruction

Lead a discussion of algebraic solutions of systems of 2 equations in 2-space (scalar & parametric, parametric & parametric, vector & vector). See teacher BLM 6.5.1 for examples.

Mathematical Process Focus: Representation – Students represent linear systems in 2-space graphically and algebraically.

See pages 30-33 of THINK LITERACY : Cross - Curricular Approaches , Grades 7 - 12 for more information on graphic organizers.

Consolidate Debrief

Pairs → Graphic Organizer

Students summarize the possible solutions resulting from solving systems of equations in 2-D in various forms and the connections between the graphical and the three algebraic representations of systems of 2 equations in 2-space

Practice

Home Activity or Further Classroom Consolidation

Complete assigned practice questions.

Choose consolidation questions based on observations of need

6.5.1: Systems of Equations in 2-D Minds On…

For Graffiti Activity:

✂

	Coincident	Parallel
1) $2x + y = -1$ $3x - y = -4$	2) $y = 3x - 5$ $6x - 2y - 10 = 0$	3) $y = \frac{2}{5x}x - 2$ $2x - 5y = 20$
4) $3x - y = -10$ $2x + 3y = 8$	5) $y = \frac{1}{4}x + 1$ $2x - 8y = 2$	6) $y = 5$ $5y - 15 = 0$
7) $2x - 3y = 9$ $3x + 4y = 5$	8) $x - 2y = 3$ $2x - 4y - 6 = 0$	9) $6x - 2y = 8$ $y = 3x + 1$

For Teacher Led Instruction – Action

	Parametric and Parametric	Vector and Vector
L1: $x - 2y = 3$ L2: $x = \frac{t}{3}$; $y = 2 - t$	L1: $x = \frac{t}{2}$ $y = -1 - t$ L2: $x = \frac{8}{3}$ $y = s + 4$	L1: $\vec{r} = (1, -2) + t(1.3)$ L2: $\vec{r} = (0, -5) + s(1, 3)$

Unit 6: Day 6: Concrete Critters Intersecting Lines and Planes — Grade 12

75 min

Math Learning Goals

- Determine through investigation different geometric configurations of combinations of up to three lines and/or planes in three space
- Classify sets of lines and planes in three space that result in a common point, common line, common plane or no intersection

Materials

- BLM 6.6.1, 6.6.2, 6.6.3
- card stock
- straws OR pipe cleaners OR wooden skewers

Assessment Opportunities

Minds On…

Pairs Share → Review

Curriculum Expectation/Observation/Mental Note: Circulate, listen and observe for student's understanding of this concept as they complete BLM 6.5.1

Students coach each other as they complete the solutions to the systems of equations on BLM 6.6.1 (A coaches B, and B writes, then reverse)

For Pair/Share: one handout and one pencil per pair.

Teachers may wish to have students work in small groups instead of pairs.

Whole Class → Discussion

Review three possible solutions from previous day's intersection of lines in 2-space (point of intersection, parallel lines, coincident lines).

Discuss / invite suggestions on what would be the same / different / new if solving for the intersection of two lines in 3-space.

Differentiated instruction: The graphic organizer on BLM 6.6.3 can be used to provide scaffolding for students.

Action!

Pairs → Investigation/Experiment

Mathematical Process Focus: Representing

Using BLM 6.6.2 and materials (e.g., cardstock, straws) students represent geometrically lines and planes in 3-space. Students use concrete materials to model and/or construct as many different possibilities of intersections (or non-intersections) using up to 3 lines and/or planes.

Students describe each possibility briefly and sketch what it looks like on BLM 6.6.2

Consolidate Debrief

Small Groups → Graphic Organizer

Students complete their choice of a graphic organizer in small heterogeneous groups (3 or 4) to summarize the various outcomes of lines and planes that will result in a single point of intersection, a line of intersection, a plane, or no common intersection.

Whole Class → Summary

Share results of investigation and graphic organizer activity.

Application

Home Activity or Further Classroom Consolidation

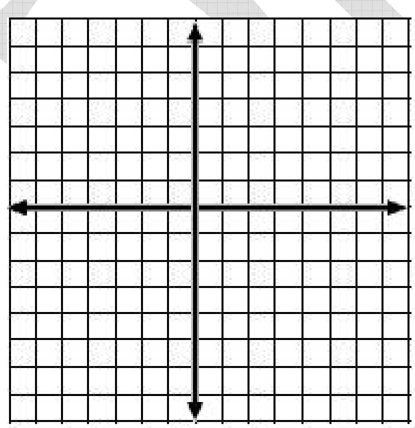
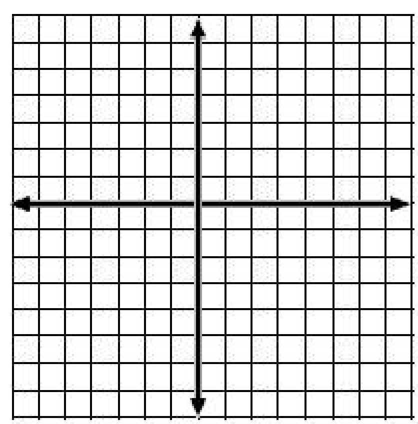
Bring to class the next day interesting visual examples (e.g., photos, newspaper clippings, physical objects) of real-life intersections of lines and planes.

Consider preparing a visual display of the examples to be used over the next several days.

6.6.1: Pair Share – Don't Double Cross the Line

Instructions

A solves question A, B coaches
B solves question B, A coaches

	Question B
$x = 5s$ $y = 7s$ $(x, y) = (1, 7) + t(3, 7)$	$(x, y) = (3, 9) + t(2, 5)$ $(x, y) = (-5, 6) + s(3, -1)$
	

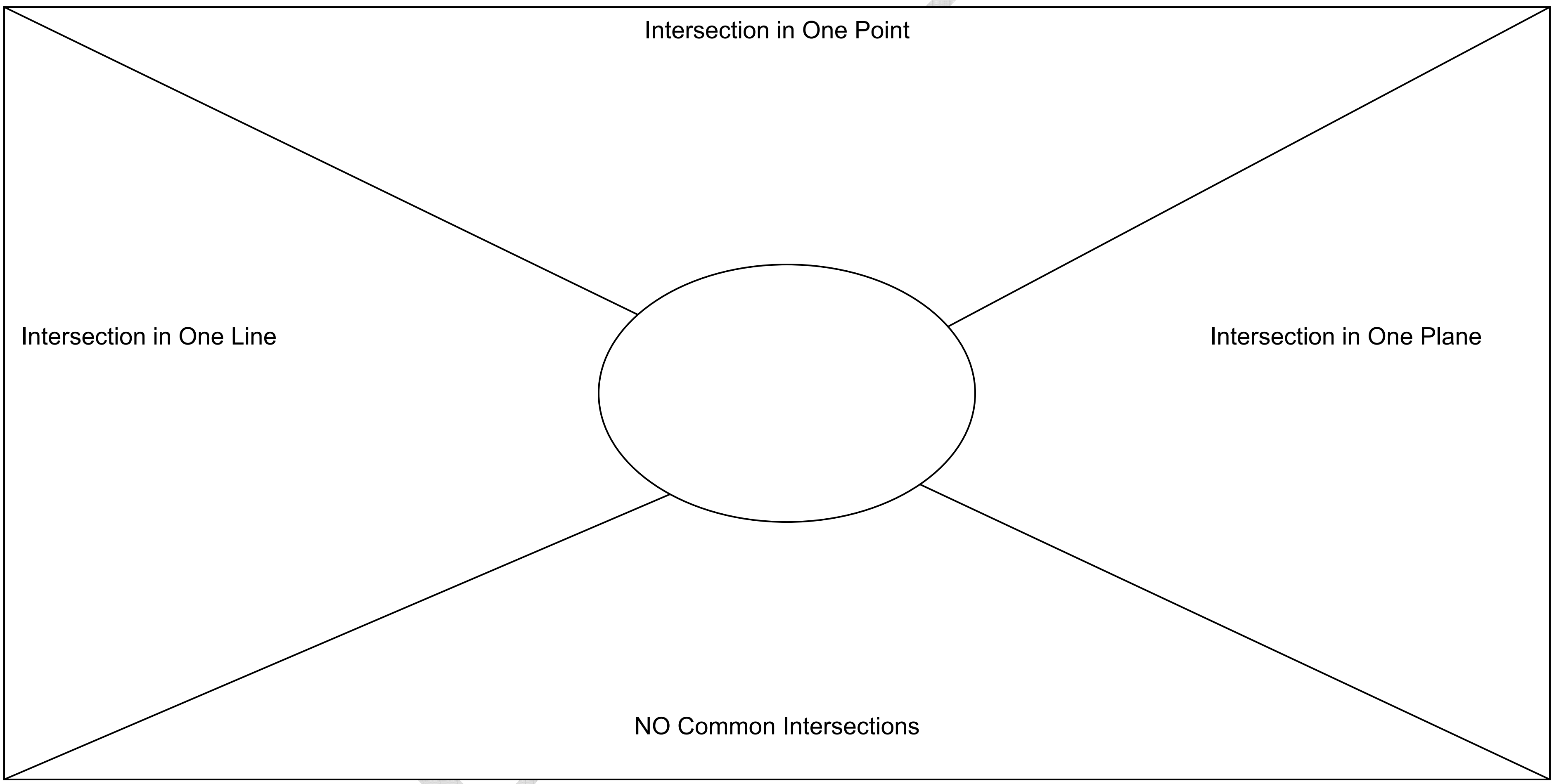
6.6.2: Intersection Investigation

Use concrete materials to model and/or construct as many different possibilities of intersections (or non-intersections) using up to 3 lines and/or planes. Make a sketch and describe what it looks like.

Combination	Sketch and description(*)			
2 Lines	*	*	*	*
3 Lines	*	*	*	*
1 Line + 1 Plane	*	*	*	*
2 Planes	*	*	*	*
3 Planes	*	*	*	*

6.6.3: Intersection Convention

Summarize the various outcomes of lines and planes that will result in an intersection of: a single point, a line, a plane, or no common intersection at all.



Unit 6: Day 7: Interesting Intersections Part 1 — Grade 12

75 min

Math Learning Goals
- Determine the intersections of two lines, and a line and a plane in three space given equations in various forms and understand the connections between the geometric and algebraic representations.

Materials
- BLM 6.7.1
- BLM 6.7.2

Assessment Opportunities

Minds On…

Whole Class → Discussion
Lead a discussion where students identify the four possible representations of a system of two lines in 3-space (1 point, coincident, parallel, skew) and the three possible representations of a system of a line and a plane in 3-space (one point, coincident & parallel). Refer to Smart Ideas file Overview.ipr for details of representations.

Make use of the pictures students collected for home extension Day 6 to demonstrate relevant scenarios.

Action!

Groups → JigSaw
Learning Skills Teamwork/Observation/Rubric/Written note: Circulate and make note of students' teamwork performance.

Form heterogeneous groups of at least four students per home group. Assign four experts per home group using numbered heads. Using samples on teacher BLM 6.7.1 for the four expert groups, students solve systems of two equations in 3-space. Use an assortment of parametric and vector forms.

- Expert group 1 solves a system of 2 lines with one point of intersection and a system of a line parallel to a plane.
- Expert group 2 solves a system of 2 parallel distinct lines and a system of a line that intersects the plane.
- Expert group 3 solves a system of 2 lines coincident lines and a system of a line parallel to a plane.
- Expert group 4 solves a system of 2 skew and a system of a line in the plane.

Students return to home groups and share and summarize findings using graphic organizer in BLM 6.7.2.

Mathematical Process Focus: Communicating: Students communicate their understanding of the various permutations of systems of equations of 2 lines and a line and a plane in 3-space.

If home groups contain more than 4 students, ensure that the expert groups are equally balanced.

Reference the eLearning Ontario "toolkit" for graphing lines and planes in 3D

Consolidate Debrief

Whole Class → Discussion
Lead a discussion to ensure students understand all possible scenarios of systems of 2 lines in 3-space and of systems of a line and a plane in 3-space.

Practice

Home Activity or Further Classroom Consolidation
Complete assigned practice questions.

Describe the pictures gathered in the previous lesson according to the types of systems encountered in this lesson.

Choose consolidation questions based on observations of need

6.7.1 Sample Systems of Equations (teacher)

✂

	Systems of a Line and a Plane
Group 1: Solve the following system $(x,y,z)=(-5,2,-7)+t(3,2,6)$ $x=s \quad y=-6-5s \quad z=-3-s$ $s,t\in\Re$	Group 1: Solve the following system $x=5+t \quad y=4+2t \quad z=7+2t$ $2x+3y-4z+7=0$ $t\in\Re$
Group 2: Solve the following system $x=1+t \quad y=2+t \quad z=-t$ $x=3-2s \quad y=4-2s \quad z=-1+2s$ $s,t\in\Re$	Group 2: Solve the following system $(x,y,z)=(4,6,-2)+t(-1,2,1)$ $2x-y+6z+10=0$ $t\in\Re$
Group 3: Solve the following system $(x,y,z)=(1,1,1)+t(1,2,-3)$ $(x,y,z)=(3,5,-5)+s(-2,-4,6)$ $s,t\in\Re$	Group 3: Solve the following system $(x,y,z)=(2,1,4)+t(1,0,1)$ $3x-4y-3z-9=0$ $t\in\Re$
Group 4: Solve the following system $x=-2+s \quad y=1+3s \quad z=7s$ $(x,y,z)=(3,-3,4)+t(5,-4,-2)$ $s,t\in\Re$	Group 4: Solve the following system $x=2-t \quad y=4-t \quad z=1+t$ $3x-y+2z+6=0$ $t\in\Re$

6.7.2: Systems of Two Lines and Systems of a Line and a Plane

After each expert has shared in your home group, summarize the findings by completing the following table. The description can be either words or a sketch.

System of 2 Lines in 3-Space		System of a Line and a Plane In 3-Space	
Description	Number of Intersection Points	Description	Number of Intersection Points

Unit 6: Day 8: Interesting Intersections Part 2

Grade 12

75 min

Math Learning Goals

- Determine the intersections in three-space of 2 planes and 3 planes intersecting in a unique point given equations in various forms and understand the connections between the graphical and algebraic representations of the intersection.

Materials

- BLM 6.8.1, 6.8.2, 6.8.3, 6.8.4
- card stock and straws
- scissors

Assessment Opportunities

Minds On…

Pairs → Exploration

Curriculum Expectation (C3.3)/Observation/Oral feedback: Observe student success in completed BLM 6.8.1 and provide oral feedback where required.

Using card stock as models for planes, students predict the three possible solutions for a system of two planes in 3-space.

Students use BLM 6.8.1 to summarize the information the normal vectors and constants provide for each possible solution type.

This is a consolidation of concepts developed in previous lessons in this unit

Assessment for Learning to ensure student readiness to proceed

Whole Class → Teacher Led Instruction

Using the systems from BLM 6.8.1, demonstrate elimination and substitution as methods for solving the systems algebraically. Ensure students make the connection between the geometric and algebraic representations by asking:

Describe how the algebraic solution indicates whether the planes intersect or not?

How do you differentiate algebraically between coincident planes and planes intersecting in a line?

Print sufficient quantities of BLM 6.8.3 on coloured card stock. If laminated the sets can be reused

Action!

Groups → Investigation

In heterogeneous groups of 3 or 4, students complete BLMs 6.8.2 and 6.8.3 by building the model of the system and solving it algebraically using elimination or substitution.

Mathematical Process Focus: Representing and Connecting

Students represent intersection of three planes geometrically and connect the algebraic solution to the geometric model.

Refer to Teacher BLM 6.8.4 for solutions

Reference the eLearning Ontario "toolkit" for graphing lines and planes in 3-D

The connection to the scalar triple, $\vec{a}\cdot\vec{b}\times\vec{c}$ will be made in the next lesson

Consolidate Debrief

Whole Class → Discussion

Ensure students make the connection between the geometric and algebraic representation by asking:

What is the significance of the algebraic representation as it relates to the geometric model?

What observations can be made about the normal vectors to the planes in BLM 6.8.3? (Answer: Normal vectors are not scalar multiples or coplanar.)

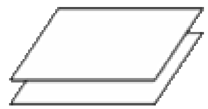
Will these properties be true for all systems of 3 planes with a unique solution?

Journal Entry

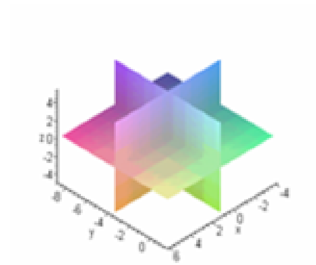
Home Activity or Further Classroom Consolidation

Predict how the relationship among normal vectors to 3 planes will change for the other geometric scenarios summarized in BLM 6.6.3?

6.8.1 Characteristics of Normal Vectors for Intersecting Planes

System of Equations	Description	Sketch	# intersection points	Analysis
$2x+3y-2z=5$ $6x+9y-6z=12$	Two distinct parallel lines		0	Normal vectors are scalar multiples of each other. Constants are not the same multiple of each other.
$2x-13y-6z=7$ $(x,y,z)=(0,-1,1)+$ $s(3,0,1)+t(4,2,-3)$				
$x-3y+6z=13$ $x=1+2s+5t$ $y=-4s-t$ $z=2+s+2t$				

6.8.2: Intersection of Three Planes Investigation



Problem:
In this investigation over the next two days you will consider the different ways 3 planes can intersect and answer the following questions:

- How is the concrete representation related to the algebraic solution?
- How are normal vectors used to verify the model?

Procedure:

Part A, Geometric Model:

1) Cut out and assemble the set of coloured cards representing planes by matching like letters. Observe and describe the intersection of this geometric model. Make a sketch of your model

2) Predict how the algebraic solution will indicate this intersection. Hint: Consider the possible geometric models and corresponding algebraic solutions of 2 lines in 2-space.

3) Using straws to represent the normal vector to each plane, describe the relationship among the normal vectors using terms such as parallel (collinear), coplanar, non-coplanar.

Part B, Algebraic model:

1) Solve the system algebraically using the equations of the planes.

2) Does your solution match your prediction from above? How do the normal vectors confirm your prediction model?

3) Summarize the connection between the algebraic solution and the geometric model.

6.8.3 Intersection of Three Planes – Set 1

B A C

$$x + 2y + 3z + 4 = 0$$

$$x - y - 3z - 8 = 0$$

D A E

B E

$$2x + y + 6z + 14 = 0$$

D C

6.8.4 Intersection of Three Planes: Solutions & Conclusions

(Teacher)

Set 1:

$\prod_1$: $x + 2y + 3z + 4 = 0$ (1)
$\prod_2$: $x - y - 3z - 8 = 0$ (2)
$\prod_3$: $2x + y + 6z + 14 = 0$ (3)

(1) – (2) $3y + 6z + 12 = 0$ (4)
2 × (1) – (3) $3y - 6 = 0$ (5)

$y = 2$
substitute into (4)
$z = -3$
substitute into (1)
$x = 1$

Conclusions:

The three planes intersect in the point in space $(1, 2, -3)$
The normal vectors are non coplanar (i.e., form a basis for $\Re^3$)
The scalar triple of the normal vectors, $\vec{a} \cdot \vec{b} \times \vec{c}$ as will be demonstrated in the next lesson.

75 min

Math Learning Goals

- Determine the intersections of 3 planes in three space given equations in various forms and understand the connections between the graphical and algebraic representation of the intersection
- Recognize that if $\vec{a}\cdot\vec{b}\times\vec{c}\neq 0$ is true then the three planes intersect at a point
- Solve problems involving the intersection of lines and planes in three-space represented in a variety of ways

Materials

- card stock
- chart paper and markers
- BLMs 6.9.1, 6.9.2, 6.9.3, 6.9.4, 6.9.5
- data projector

Assessment Opportunities

Minds On…

Groups → Placemat

Using chart paper and working in heterogeneous groups of 3 or 4, students individually in their section of the placemat list/sketch all the possible intersections, or non-intersections, of 3 planes in 3-space. As a group, students consolidate and classify their findings and then write in the centre of the placemat.

Whole Class → Discussion

Consolidate group findings using a graphic organizer.

Provide groups with 3 pieces of paper to represent plane intersections.

For information on placemats see p. 66 of *Think Literacy: Cross-Curricular Approaches , Grades 7 12*

Action!

Groups → Investigation

Curriculum Expectation (C4.4)/Observation/Mental note: Circulate, listen, and observe student proficiency at determining the intersection of 3 planes by solving a system of 3 equations.

In heterogeneous groups of 3 or 4, students complete sets 2 through 4 on BLM 6.9.1, using BLM 6.8.2 as guide, by building the model of the system and solving it algebraically using elimination or substitution.
Students use BLM 6.9.2 record their findings.

Solutions to sets 2 through 4 are on BLM 6.9.4

Assessment for Learning to ensure student readiness to proceed

Consolidate Debrief

Whole Class → Discussion

Lead a discussion to consolidate student understanding of the information tabulated on BLM 6.9.2 using the following guiding questions.

What does the algebraic solution tell you about the uniqueness of the solution?

How can you use normal vectors to distinguish between 2 different models with similar algebraic solutions?

Observe the values of the scalar triple product, $\vec{a}\cdot\vec{b}\times\vec{c}$. What geometric conclusions about the normal vectors, and subsequently the planes, can be deduced from this calculation?

Mathematical Process Focus:
Reasoning and Proving: Students use normal vectors to classify the geometric solutions to the various intersections of three planes.

Pairs → Extension

Students complete BLM 6.9.3.

Curriculum Expectation (C4.3, C4.7)/Worksheet/Checkbric: Collect BLM 6.9.3 and assesses student work using a checkbric.

Use power point file Intersection of 3 planes.ppt (BLM 6.9.5) to consolidate student understanding of the possible solution types for the intersection of three planes. (BLM 6.9.2)

Reference the eLearning Ontario “toolkit” for graphing lines and planes in 3-

Assessment as learning to allow students to check their understanding

Home Activity or Further Classroom Consolidation

Application

Extend your collections of visual examples of combinations of points, lines and planes that have a finite distance between them

6.9.1 Intersection of Three Planes – Set 2

A $x + 2y + 3z + 4 = 0$ B

B $x - y - 3z - 8 = 0$ C

C $x + 5y + 9z + 10 = 0$ A

6.9.1 Intersection of Three Planes – Set 3

A B

$$x + 2y + 3z + 4 = 0$$

A

$$x + 3y + 2z + 3 = 0$$

B

$$2x + 6y + 4z - 7 = 0$$

6.9.1 Intersection of Three Planes – Set 4

A

$x + 2y + 3z + 4 = 0$

A

B

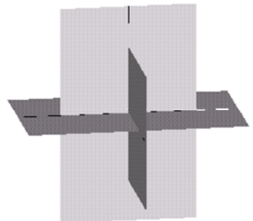
$x - y - 3z - 8 = 0$

B

$x + 5y + 9z + 16 = 0$

6.9.2 Characteristics of Normal Vectors for Intersecting Planes

Record the information found from the investigation on BLM 6.8.2.

Equations	Description	Sketch	Solution	Additional Information
Set 1: $x+2y+3z+4=0$ $x-y-3z-8=0$ $2x+y+6z+14=0$	Three planes intersection in a point.		$(1, 2, -3)$	Normal vectors: $\vec{a}=(1,2,3)$ $\vec{b}=(1,-1,-3)$ $\vec{c}=(2,1,6)$ $\vec{a}\cdot\vec{b}\times\vec{c}=-18$
Set 2:				
Set 3:				
Set 4:				

Graphic source: www.rwgrayprojects.com/Lynn/iop/iop.html

6.9.3: Connecting Algebraic and Geometric Models of the Intersection of Three Planes

1) List and sketch any other configuration of 3 planes which you have not modeled in the investigation 6.9.2.

$\vec{a} \cdot \vec{b} \times \vec{c} =$	$\vec{a} \cdot \vec{b} \times \vec{c} =$
$\vec{a} \cdot \vec{b} \times \vec{c} =$	$\vec{a} \cdot \vec{b} \times \vec{c} =$

2) If you solved the system of equations that are represented by these planes, what is the nature of solution would you expect?

3) How would you use normal vectors to identify the configuration?

4) a) Explain how, and in which models, the scalar triple product $\vec{a} \cdot \vec{b} \times \vec{c}$ of normal vectors is helpful?

b) Record the value of the scalar triple product, $\vec{a} \cdot \vec{b} \times \vec{c}$, for each case above.

6.9.4: Intersection of Three Planes: Solutions and Conclusions (Teacher)

Set 2

$\prod_1$: $x + 2y + 3z + 4 = 0$ (1)
$\prod_2$: $x - y - 3z - 8 = 0$ (2)
$\prod_3$: $x + 5y + 9z + 10 = 0$ (3)

(1) – (2) $3y + 6z + 12 = 0$ (4)
(1) – (3) $-3y - 6z - 6 = 0$ (5)
(4) + (5) $3y - 6 = 0$ (5)
$6 = 0$ contradiction
$\therefore$ no solution no intersection

Conclusions

a) normal vectors are not scalar multiples
$\therefore$ no parallel planes
b) scalar triple product is zero
$\therefore$ normal vectors are coplanar (& no solution)
$\therefore$ planes form a triangular prism in space

Set 3

$\prod_1$: $x + 2y + 3z + 4 = 0$ (1)
$\prod_2$: $x + 3y + 2z + 3 = 0$ (2)
$\prod_3$: $2x + 6y + 4z - 7 = 0$ (3)

$2 \times$ (2) – (3) results in: $13 = 0$ a contradiction
$\therefore$ no solution no intersection

Conclusions

a) $\prod_2$ and $\prod_3$ have parallel normal vectors BUT are not merely multiples of the same plane
$\therefore$ parallel planes

6.9.4: Intersection of Three Planes: Solutions and Conclusions (teacher - continued)

Set 4

$\prod_1$: $x + 2y + 3z + 4 = 0$ (1)
$\prod_2$: $x - y - 3z - 8 = 0$ (2)
$\prod_3$: $x + 5y + 9z + 16 = 0$ (3)

(1) – (2) $3y + 6z + 12 = 0$ (4)
(1) – (3) $-3y - 6z - 12 = 0$ (5)
(4) + (5) $0 = 0$ always true, (always a solution)
$\therefore$ solution exists but not unique

Consider (4): $y = -2z - 4$
If $z = t$ then $y = -2t - 4$
And $x + 2(-2t - 4) + 3t + 4 = 0$
$\therefore x = t + 4$

i.e., $(x, y, z) = (t + 4, -2t - 4, t)$, a line in space

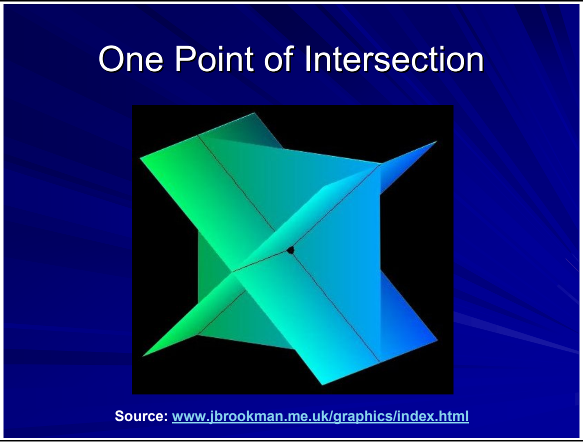
Conclusions

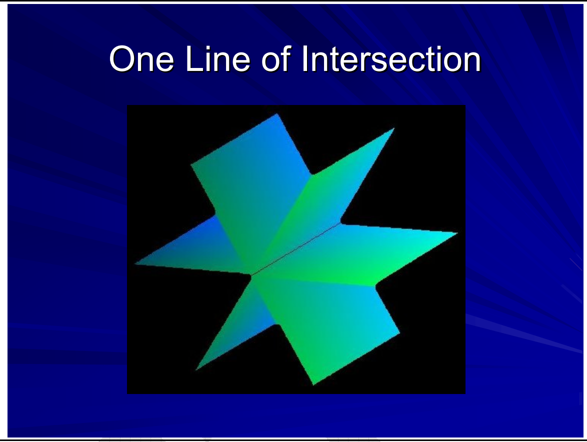
a) and b) as in **Set 2**

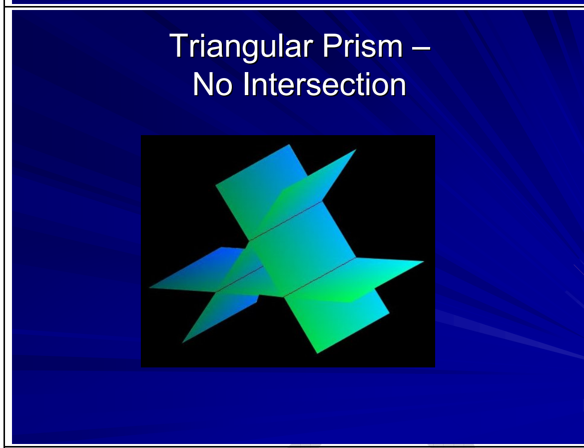
c) Comparing the equations to **Set 1**, $\prod_3$ has been translated so that the lines of intersection of the three planes, taken in pairs, are now coincident.

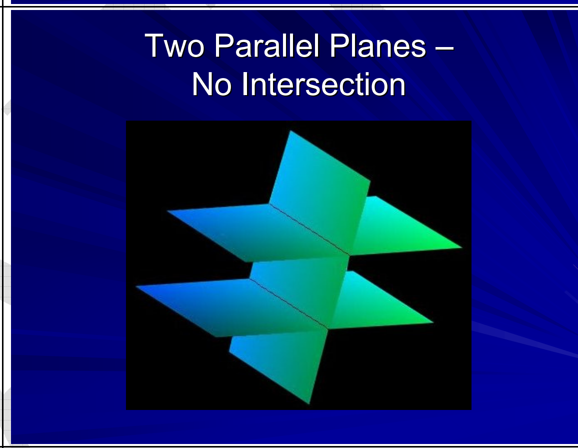
6.9.5 Intersections of Three Planes Slides

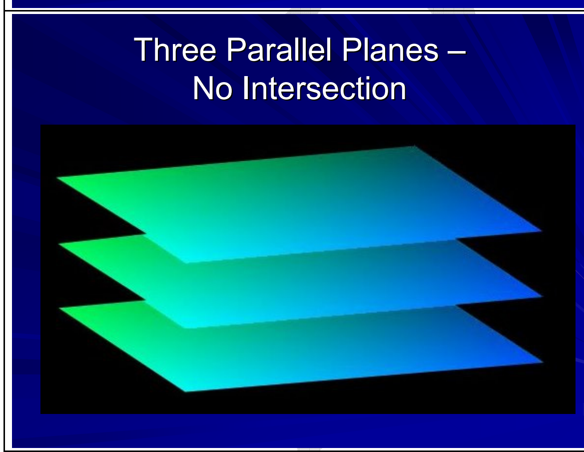
(Presentation software file: Intersection of 3 planes.ppt)











Source: www.jbrookman.me.uk/graphics/index.html

75 min

Math Learning Goals

- Calculate the distance between lines and planes in three space with no intersection
- Solve problems related to lines and planes in three-space that are represented in a variety of ways involving distances

Materials

- BLM6.10.1
- coloured pipe cleaners OR straws OR wooden skewers
- card stock

Assessment Opportunities

Minds On…

Whole Class → Think, Pair, Share

Students classify the cases in which there is no common intersection making use of the models developed using BLM 6.6.2.

Whole Class → Discussion

Lead a discussion to summarize the cases using a chart or graphic organizer. (Note: the models that do not intersect can be divided into two categories: those that have no intersections (two parallel lines, two skew lines, a line parallel to a plane, two parallel planes) and those that have partial intersections which were considered on Day 8). Establish "look fors" (normals) and procedures to identify each model in the first category.

Make use of the pictures students collected for home extension Day 6 to demonstrate relevant scenarios.

Significant emphasis will be placed on normal vectors to classify and determine the distances required.

Action!

Expert Groups → Guided Exploration

In heterogeneous groups of 3 or 4, students build models: two parallel planes and a line parallel to a plane, using BLM 6.10.1.

Whole Class → Discussion

Curriculum Expectation/Observation/Mental note: Observe to identify students who have quickly grasped the concepts for the purpose of them completing a D.I. extension activity during Consolidate time.

Summarize the group findings from BLM 6.10.1. (i.e., The distance between a line and a plane and between parallel planes can be determined by projecting any vector connecting a point on the line to a point on the plane or connecting two points, one on each plane, onto the common normal.)

Mathematical Process Focus:
Connecting: Students connect prior concepts and procedures**.**

Straws can be connected end to end or cut to a more appropriate length.

It will be necessary to "suspend" lines and planes in space using an appropriate tool.

Reference the eLearning Ontario "toolkit" for graphing lines and planes in 3-D

Students must recognize that it is impossible to determine the common normal between two parallel lines in 3-space.

Consolidate Debrief

Whole Class → Think, Pair, Share → Discussion

Pose the question: "Can the distance between parallel lines in 3-space be determined using a similar approach to the exploration above? Why or why not?" Establish the understanding for determining the distance between planes and lines in 3-space.

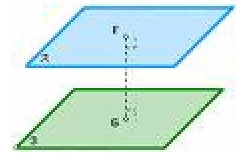
Small Group D.I. Extension: Provide a large model of two skew lines. (Suggest using a large cardboard box and two metre sticks placed appropriately through its interior) As in the exploration above, students note the common normal and a vector between the two lines, and demonstrate geometrically and algebraically the distance between the two lines.

Practice

Home Activity or Further Classroom Consolidation

Select appropriate questions for practice.

6.10.1: BLM How Far Can It Be? Investigation



1. Using card paper and straws, construct a model of two parallel planes.

2. Construct a vector whose magnitude represents the distance between the planes.

3. Construct a common $\overrightarrow{\textbf{normal}}$ to the planes. How would you determine this $\overrightarrow{\textbf{normal}}$ algebraically?

4. Make a conjecture about a relationship between the $\overrightarrow{\textbf{normal}}$ in step 3 and the vector in step (2).

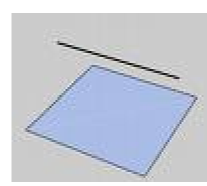
5. Construct a vector with one end on each of the planes. How would you determine the points and the vector algebraically?

6. What is the relationship among the vectors constructed in steps 2, 3 and 5 in terms of a projection?

7. Explain why the projection of the vector in step 5 onto the $\overrightarrow{\textbf{normal}}$ is independent of the choice of that vector.

8. Describe what you would do *algebraically* to find the distance between two parallel planes.

9. **Repeat** steps 1 through 7 for a line parallel to a plane.
   Describe what you would do algebraically to find the distance from a line to a parallel plane.



10. **Reflect:** What does it mean if the distance calculated in either of the above cases is zero?

Images: intermath.coe.uga.edu/dictnary/descript.asp?t...,
mathworld.wolfram.com/ParallelLineandPlane.html