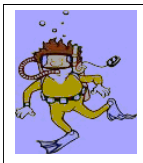
MHF 4U Unit 2 –Rational Functions– Outline

Day	Lesson Title	Specific Expectations
1 (Lesson Included)	Rational Functions and Their Essential Characteristics	C 2.1,2.2, 2.3
2 (Lesson Included)	Rational Functions and Their Essential Characteristics	C 2.1,2.2, 2.3
3 (Lesson Included)	Rational Functions and Their Essential Characteristics	C 2.1,2.2, 2.3
4	Rationale Behind Rational Functions	C3.5, 3.6, 3.7
5 (Lesson Included)	Time for Rational Change	D1.1- 1.9
6-7	JAZZ DAY	
8	SUMMATIVE ASSESSMENT	
TOTAL DAYS:		8

Unit 2: Day 1: Rational Functions and Their Essential Characteristics			MHF 4U
Minds On: 5 Action: 55 Consolidate:10 Total=75 min	**Learning Goal:** Investigate and summarize the characteristics (e.g. zeroes, end behaviour, horizontal and vertical asymptotes, domain and range, increasing/decreasing behaviour) of rational functions through numeric, graphical and algebraic representations.		**Materials** Graphing calculators BLM 2.1.1 - 2.1.2
			Assessment Opportunities
Minds On…	**Whole Class → Discussion** What is a rational function? Compare and contrast rational and polynomial functions What are the restrictions on rational functions?		
Action!	**Pairs → Investigation** Complete Investigation: Scuba Diving (BLM 2.1.1) Lesson – Rational Functions (BLM 2.1.2)		
Consolidate Debrief	**Whole Class → Discussion** Summarize the characteristics of a rational functions Discuss how to find zeroes, end behaviour, asymptotes etc of a rational function		
	Home Activity or Further Classroom Consolidation to be developed by the classroom teacher		



2.1.1 Investigation: Scuba Diving

Scuba divers must not hold their breath as they rise through water or their lungs may burst. This is because the air, which they have breathed to fill their lungs underwater, will expand as the scuba diver rises and the pressure on the body reduces. At every depth, the diver wants 4 litres of air in her lungs for breathing.

If a diver holds her breath, the volume of the air in her lungs varies with the pressure in the following manner:

Volume (at new pressure) $= \dfrac{original\ volume \times original\ pressure}{new\ pressure}$

The pressure is 1 atmosphere at the surface and increases by 1 atmosphere for every 10 metres below the surface.

1. A diver takes a 4-litre breath of air at the surface and descents without breathing. Using the formula above, complete the following table.

Depth (D) in metres	0	10	20	30	40	50	60
Pressure (P) in atmospheres	1	2	3				
Volume (V) of air in lungs in litres	4	2					

2. (a) A diver takes a 4-litre breath of air from her tank at 60 metres. Imagine that she can ascent without breathing. Complete the following table.

Depth (D) in metres	0	10	20	30	40	50	60
Pressure (P) in atmospheres	1	2					
Volume (V) of air in lungs in litres							4

(b) Using the answers from the above table, find the relationship between P and D
Check the rule for D = 20, 30 and 40

(c) Using the answers from the above table, find the relationship between V and P
Check this rule for P = 3, 4 and 5.

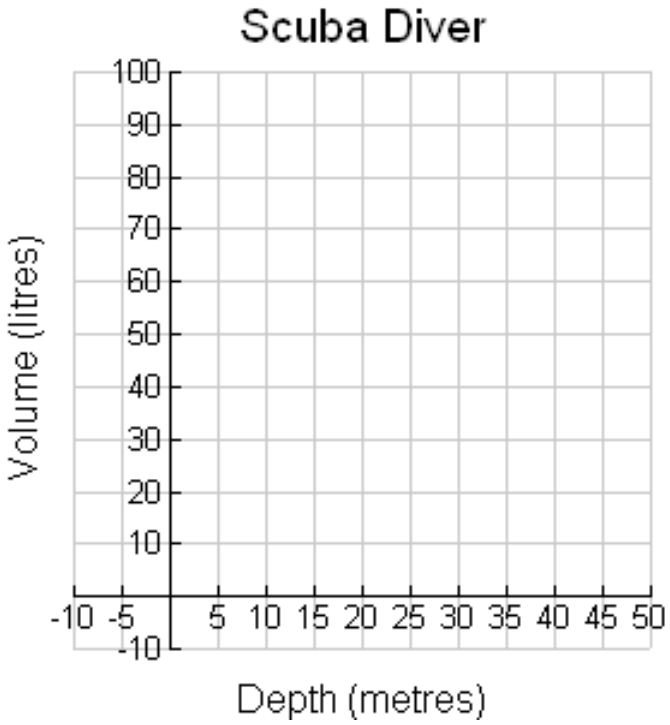
(d) Use algebra to show that V = $\dfrac{280}{D+10}$



2.1.1 Investigation: Scuba Diving (Continued)

(e) Use your graphing calculator to graph the volume of air against depth of the diver in metres.

(f) Sketch your graph on the grid provided.



(g) Look at your table and your graph and decide through what depths the diver should be most careful about breathing to avoid bursting her lungs.

(h) What happens to the graph as the depth increases?

(i) What happens to the graph as the depth moves into the negative values?

2.1.2 Rational Functions and their Essential Characteristics: Day 1 (Teacher Notes)

Definition of a Rational Function

A **rational function** has the form h(x) = $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials

The domain of a rational function consists of all real number except the zeroes of the polynomial in the denominator. g(x) $\neq 0$

The zeros of h(x) are the zeroes of f(x) if h(x) is in simplified form.

1. Finding the Domain and the Intercepts

   What is the domain of each rational function? Determine the x- and y-intercepts. Then graph y = f(x) with graphing technology and estimate the range.

   (a) $f(x) = \frac{7}{x+2}$ (b) $f(x) = \frac{x}{x^2 - 3x - 4}$

   (c) $f(x) = \frac{2x^2 + x - 3}{x^2 - 4}$ (d) $f(x) = \frac{x-2}{x^2 - 2x}$

Vertical and Horizontal Asymptotes

The graph of a rational function has at least one asymptote, which may be vertical, horizontal, or oblique. An oblique asymptote is neither vertical nor horizontal.

Vertical Asymptote can be found where the function is undefined. You'll find the vertical asymptotes algebraically by setting the denominator equal to zero and solving. The solutions will be the vertical asymptotes.

Since the vertical asymptote is opposite to the domain. The solutions will be the values that are not allowed in the domain.

Horizontal Asymptote indicates the general behaviour far off to both sides of the graph. It can be found by setting up a chart for large negative and positive values of x.

The graph of a rational function never crosses a vertical asymptote but it may or may not cross a horizontal asymptote.

2.1.2 Rational Functions and their Essential Characteristics: Day 1 (Teacher Notes continued)

2. Find the vertical and horizontal asymptotes of g(x) = $\frac{x+3}{x^2-4}$.

E.g. For x 64

x	g(x)
100	
500	
1000	
2000	
5000	
10000	
100000	
700000	

x	*g(x)*
-100	
-500	
-1000	
-2000	
-5000	
-10000	
-100000	
-700000	

General Rules on Finding the Horizontal Asymptotes
Other than using the table of large positive and negative values, the horizontal asymptote can also be found by comparing the degrees of the numerator $f(x)$ and the denominator $g(x)$ of a rational function.

Given that the numerator and denominator are polynomials in x of degree n and m, respectively.

1. If $n < m$, then the horizontal asymptote is $y = 0$.
2. If $n = m$, then the horizontal asymptote is $y = \frac{coefficient\ \ of\ \ x^n\ \ term}{coefficient\ \ of\ \ x^m\ \ term}$.
3. If $n > m$, there is no horizontal asymptote.

3. Use the general rules above, determine the horizontal asymptote for $g(x) = \frac{x+3}{x^2-4}$. Do you have the same answer as in Question #2?

4. Let $f(x) = \frac{x^2}{x^3-2x^2-5x+6}$. Find the domain, intercepts, and vertical and horizontal asymptotes. Then use this information to sketch an approximate graph.

Unit 2: Day 2: Rational Functions and Their Essential Characteristics				MHF 4U
Minds On: 10 Action: 55 Consolidate:10 Total=75 min		**Learning Goal**: Demonstrate an understanding of the relationship between the degrees of the numerator and the denominator and horizontal asymptotes Sketch the graph of rational functions expressed in factored form, using the characteristics of polynomial functions.		**Materials** Overhead projector BLM 2.2.1- 2.2.2
				Assessment Opportunities
	Minds On…	**Whole Class → Discussion** On the overhead projector play Name That Asymptote (BLM 2.2.1)		
	Action!	**Whole Class → Lesson** Lesson on horizontal and oblique asymptotes (BLM 2.2.2)		
	Consolidate Debrief	**Whole class →Discussion** Discuss how to identify if a rational function will have an oblique asymptote Describe the type of oblique asymptote		
		Home Activity or Further Classroom Consolidation to be developed by the classroom teacher		

2.2.1 Name That Asymptote (Overhead)



For the following functions name the vertical and horizontal asymptotes

1. $f(x) = \dfrac{x}{2x - 8}$

2. $g(x) = \dfrac{x - 2}{x^2 + 5x + 6}$

3. $h(x) = \dfrac{2x^2 - 5x}{x^2 - 1}$

2.2.2 Rational Functions and their Essential Characteristics: Day 2 (Teacher Notes)

1. Discuss what happens to the value of the function $f(x) = \frac{1}{x}$ as x→+4 and x→-4

   (Concept of a limit could be introduced at this time)

2. On a graphing calculator, adjust the window to (**Xmin** = -5, **Xmax** = 5, **Ymin** = -5, and **Ymax** = 5). Graph the functions and its reciprocal on the same axes.

   i) Compare the two graphs and make connections between the rational function and its reciprocals or linear/quadratic functions.
   ii) Make connections between the equations and key features of the graphs.

   (a) $y = x - 3$ $\qquad$ $y = \frac{1}{x-3}$

   (b) $y = 2x - 3$ $\qquad$ $y = \frac{1}{2x-3}$

   (c) $y = x^2 - 3$ $\qquad$ $y = \frac{1}{x^2-3}$

   (d) $y = (x-1)(x+3)$ $\qquad$ $y = \frac{1}{(x-1)(x+3)}$

3. A photocopying store charges a flat rate of $2 plus $0.05/copy.

   (a) Write a function f(x) to represent the average cost per copy.

   (b) Determine what happens to the function as x becomes very large.

   (c) What is the significance of this value?

4. Find the horizontal asymptote for $g(x) = \frac{10x^2 - 2x - 1}{5x^2 - 4x + 1}$ and determine where the function values are positive or negative.

2.2.2 Rational Functions and their Essential Characteristics: Day 2 (Teacher Notes continued)

5. Oil spills in the ocean are contained by various sizes of rectangular booms. The formula $S(w) = \frac{200\ 000}{w} + 3w$ relates the total length of boom required for a particular rectangular design of area $100\ 000\ m^2$ and width w metres. For which intervals of width, is the length of the boom **increasing or decreasing?**

6. **Oblique Asymptotes**

For rational functions, linear oblique asymptotes occur when the degree of the numerator is exactly one more than the degree of the denominator. The equation of the linear oblique asymptote can be found by dividing the numerator by the denominator.

Determine the oblique asymptote for y = $\frac{2x^3 - x^2 + 3}{x^2}$

Use the oblique asymptote and the vertical asymptote to sketch the graph.

Unit 2: Day 3: Rational Functions and Their Essential Characteristics				MHF 4U
Minds On: 5 Action: 55 Consolidate:10 Total=75 min	**Learning Goal**: Investigate and summarize the characteristics (e.g. zeroes, end behaviour, horizontal and vertical asymptotes, domain and range, increasing/decreasing behaviour) of rational functions through numeric, graphical and algebraic representations. Solve rational inequalities, where the numerator and denominator are factorable Approximate the graphs of rational functions and use this information to solve inequalities			**Materials** BLM 2.3.1
				Assessment Opportunities
Minds On…	**Whole Class → Discussion** What is the difference between an equation and an inequality? How would we solve a rational inequality? e.g. $\frac{x}{x+7} < \frac{-x}{x-2}$			
Action!	**Whole Class → Lesson** Lesson – Rational Functions and Solving Rational Inequalities (BLM 2.3.1)			
Consolidate Debrief	**Whole Class → Discussion** Summarize the characteristics of a rational functions Discuss how to find zeroes, end behaviour, asymptotes etc of a rational function Discuss how to solve a rational inequality			
	Home Activity or Further Classroom Consolidation . to be developed by the classroom teacher			

2.3.1 Rational Functions and their Essential Characteristics: Day 3 (Teacher Notes)

Re-cap

Definition of a Rational Function

A **rational function** has the form h(x) = $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials

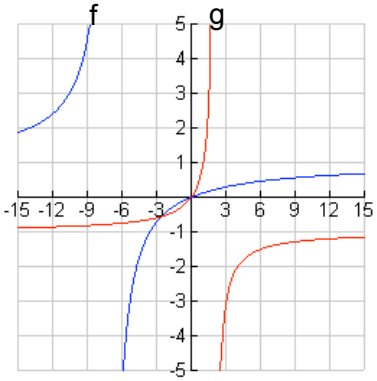
The domain of a rational function consists of all real numbers except the zeroes of the polynomial in the denominator. g(x) $\neq$ 0
The zeroes of h(x) are the zeroes of f(x) if h(x) is in simplified form.

1. Let f(x) = $\frac{(x+5)(x-4)^2}{x^4(x-2)}$. Find the domain, intercepts, and vertical and horizontal asymptotes. Then use this information to sketch an approximate graph.

2. Solve $\frac{x}{x+7} < \frac{-x}{x-2}$ by graphing the functions $f(x) = \frac{x}{x+7}$ and $g(x) = \frac{-x}{x-2}$

   (a) Find the intersection points of the two graphs. Determine when f(x) is less than g(x) from the graph.

   (b) Find the intersection point(s) algebraically.



Unit 2: Day 5: Time for Rational Change				MHF4U
Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Learning Goal**: Solve problems involving average and instantaneous rates of change at a point using numerical and graphical methods Investigate average rates of change near horizontal and vertical asymptotes			**Materials** BLM 2.5.1
				Assessment Opportunities
	Minds On…	**Small Groups → Investigations** Work in groups to solve one problem listed on BLM 2.5.1 Discuss major points of their investigation		
	Action!	**Small Groups → Presentation** Present solutions to their problem to the class		
	Consolidate Debrief	**Whole Class → Discussion** Discuss major points of investigations and any conclusions that can be drawn. Ask questions for clarification		
		Home Activity or Further Classroom Consolidation .Complete remaining problems on BLM 2.5.1		

2.5.1 Rate of Change Problems

1. The electrical current in a circuit varies with time according to $c = \frac{3s^3 - s^2 + 5s}{s^3 + 10}$, where the current, ***c***, is in amperes, and time ***s*** is in seconds. Find the average rate of change from 0.75 seconds to 1.5 seconds, and find the instantaneous rate of change at 1.5 seconds. Identify any vertical asymptotes.

2. As you get farther from Earth's surface, gravity has less effect on you. For this reason, you actually weigh less at higher altitudes. A person who weighs 55kg can use the function's $W(h) = \frac{6400(55)}{h + 64000}$ to find their weight, ***W*** in kgs, at a specific height, ***h*** in feet above sea level, above the Earth's surface. Find the average rate of change from heights of 750 ft to 1200 ft above sea level, and find the instantaneous rate of change at 1200 ft above sea level.

3. A child who weighs 34 kg is seated on a seesaw, while a child who weighs 40kg is situated on the opposite end of the seesaw. The function $D(x) = \frac{34x}{40}$ gives the distance that the 40 kg child must sit from the center of the seesaw when the 34 kg child sits *x* meters from the center. The seesaw is 9m long. Find the average rate of change in distance as the lighter child's distance changes from 1.5m to 2.5m, and find the instantaneous rate of change at 2.5m.

4. The pitch ***p*** of a tone and its wavelength ***w*** in meters are related to the velocity ***v*** in m/s of sound through air and can be modeled by the function $p = \frac{v}{w}$. For a sound wave with velocity of 985m/s, find the average rate of change in pitch as the wavelength changes from 35m to 45m, and find the instantaneous rate of change at a wavelength of 45m.

5. The average speed of a certain particle in meters per second is given by the equation $S(t) = \frac{2t^2 + 8t + 5}{t + 3}$. Find the average rate of change as time changes from 0.5 seconds to 3.5 seconds, and find the instantaneous rate of change at 3.5 seconds.

6. After you eat something that contains sugar, the pH or acid level in your mouth changes. This can be modeled by the function $L(m) = \frac{-20.4m}{m^2 + 36} + 6.5$, where ***L*** is the pH level and ***m*** is the number of minutes that have elapsed since eating. Find the average rate of change from 1.5 minutes to 3 minutes, and find the instantaneous rate of change at 3 minutes.

2.5.1 Rate of Change Problems (Continued)

7. Airbags are a standard safety feature of most cars. Without airbags, a person or object would move forward at the velocity that the car is moving. A person inside a car traveling at 8m/s and weighing 8000kg would follow the function $v(m) = \frac{8000 - m}{8000 + m} \times 5$, where ***m*** represents the mass of the person. Find the average rate of change as the mass of the person changes from 57 kg to 75 kg, and the instantaneous rate of change at 75 kg.

8. The ratio of surface area to volume of a cylinder of radius 7cm is given by the function $f(h) = \frac{14\pi h + 98\pi}{49\pi h}$, where ***h*** represents the height of the cylinder in cms. Find the average rate of change as height changes from 4cm to 9cm, and find the instantaneous rate of change at 9cm.

9. Explain why the line ***y = x*** is an asymptote for the graph of $y = \frac{x^3}{x^2 + 1}$.

10. Explain why the line ***y = -x*** is an asymptote for the graph of $y = \frac{1 - x^2}{x}$.

ANSWERS:

Average Rate of Change	Instantaneous Rate of Change
1. 0.9629	1.1
2. -0.0065	-0.0069
3. -1.275	0.85
4. -0.6254	-0.4864
5. 2.3932	2
6. -0.3733333	-0.2720
7. -0.0012296	-0.0012
8. -0.055554	-0.02
9 & 10. Has an oblique asymptote because the degree of the numerator is exactly one greater than that of the denominator. The equation of that asymptote is found through synthetic division (the quotient).	