Unit 3: Day 13: Summative – Understanding Annuities

Minds On: 2 Action: 70 Consolidate: 3 Total=75 min	**Learning Goal**: • To examine the effects of changing conditions on annuities • To demonstrate an understanding of mortgages and amortization tables	**Materials** • Graphing Calculators • BLM 3.S.1, 3.S.2

			Assessment Opportunities
	Minds On…	**Whole Class → Discussion** Lead a discussion about what important buying decisions involve annuities and the importance of understanding annuities.	
	Action!	**Individual → Completion of the Case Study on Annuities** Distribute BLM 3.S.1 and inform the students that this is a summative assessment. They will be using the graphing calculators to answer questions on annuities and interpret the results. Distribute the graphing calculators and remind students that they need to copy out the TVM solver screen for each question. **Mathematical Process Focus: Reflecting -** Students consider the implications of their answers by using them to make informed decisions.	
	Consolidate Debrief	**Whole Class→ Teacher Instruction** Collect the report and evaluate it using the scoring guide and communication rubric provided. **Expectations/Summative Assignment/Rubric:** Evaluate submitted work using the given scoring guide and communication rubric.	
Application		**Home Activity or Further Classroom Consolidation** Next day we will begin Unit 4 – Personal finance i.e. planning for your future. Your homework is to simply think about what 'you want to be', where you want to live, and how you think you'll get there.	

3.S.1 Understanding Annuities

KU	APP
___ 38	___ 12

Full solutions are required for each question, including a list of the values from the TVM-solver screen.

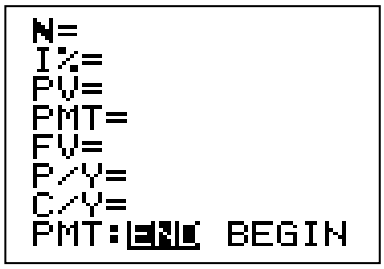
CASE STUDY

Akash has recently graduated from community college. He has just started his career and has established several financial goals. Two of his most important financial goals involve buying a car and buying a house. He would like to save enough money to buy a used car in 2 years and is in the research stage of buying a house.

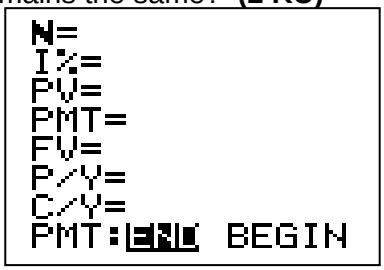
Part 1: Saving to Buy a Car in 2 Years → Effects of Changing the Savings Plan

Akash is saving money to buy a used car in 2 years. He will deposit the same amount into a savings account every month over the next 2 years. Then he will use the amount he has saved to buy a car. Help Akash by answering the following questions.

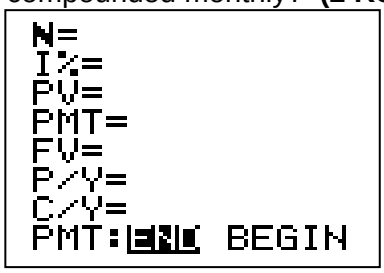
1. What price of car will he be able to afford in 2 years if he deposits $300 per month at 4.3% compounded monthly? **(2 KU)**



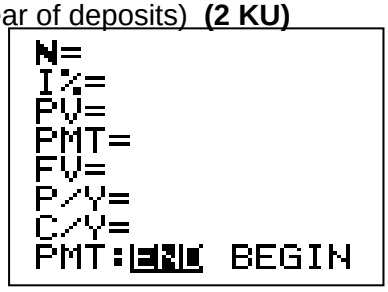
2. What price of car will he be able to afford if he increases his deposits to $350 per month but the interest rate remains the same? **(2 KU)**



3. What price of car will he be able to afford if his deposits remain at $300 per month, but he finds an investment that pays 5% compounded monthly? **(2 KU)**



4. What price of car will he be able to afford if he waits one year and then deposits $400 per month at 4.3% compounded monthly? (Thus only one year of deposits) **(2 KU)**



5. Using the information from above, describe how can Akash save the most money for his car? **(2 APP)**

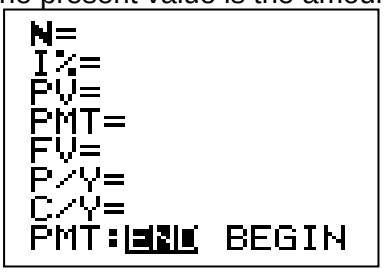
3.S.1 Understanding Annuities (continued)

Part 2: Buying the Car Now → Present Value Today or Payment Plan?

A local car dealer has a used car that would suit Akash. The dealer is offering monthly payments of $300 for 2 years. The payment plan already includes interest at 9% per annum, compounded monthly.

1. How much will Akash pay in total if he decides to use the payment plan of $300 per month for 2 years? **(2 KU)**

2. How much will he pay for the car if he pays the ***present value*** today? Remember that the present value is the amount *without* the interest. **(2 KU)**



3. How much <u>less</u> will he pay if he pays the present value today instead of accepting the payment plan? **(2 KU)**

4. Why might Akash prefer the payment plan even though it will cost him more overall? Make reference to the above calculations. **(2 APP)**

5. Why might it be better for Akash to save regularly for 4 years to buy a car, instead of buying this one now? Make reference to the above calculations. **(2 APP)**

3.S.1 Understanding Annuities (continued)

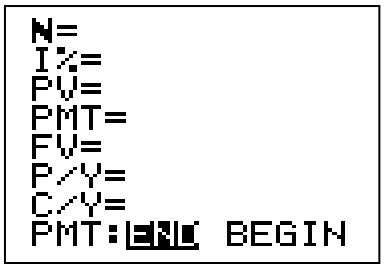
Part 3: Buying a House → Changing the Term of a Loan

Akash is starting to research buying a house. For a small two bedroom home in his area he can expect to pay $175 000. His bank offers mortgages for 6.5% interest compounded monthly and he must make a 5% down payment.

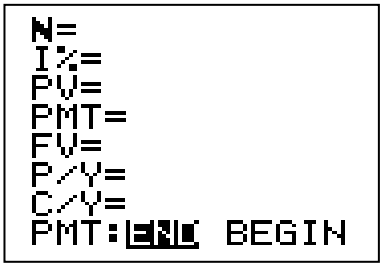
1. What amount does he need for the down payment? How much will the mortgage be after the down payment? **(2 KU)**

2. What will his monthly payments be if he uses a: **(4 KU)**

   a. 25 year mortgage



   b. 20 year mortgage



3. How much will he end up paying for the house for each term (including interest)? **(4 KU)**

   a. 25 year mortgage

   b. 20 year mortgage

4. What is the total interest paid for each term? **(2 KU)**

   a. 25 year mortgage

   b. 20 year mortgage

5. Why might Akash choose to pay off the mortgage in 25 years, even though it will cost him more in interest? **(2 APP)**

3.S.1 Understanding Annuities (continued)

Part 4: Reading an amortization table

Akash's bank has provided him with an amortization schedule for the first year of the 25-year mortgage that he is interested in.

Month	Principal	Interest	Payment	Principal Repaid	New Principal
1	166250.00	888.56	1122.53	233.97	166016.03
2	166016.03	887.31	1122.53	235.22	165780.82
3	165780.82	886.06	1122.53	236.47	165544.34
4	165544.34	884.79	1122.53	237.74	165306.60
5	165306.60	883.52	1122.53	239.01	165067.59
6	165067.59	882.24	1122.53	240.29	164827.31
7	164827.31	880.96	1122.53	241.57	164585.74
8	164585.74	879.67	1122.53	242.86	164342.87
9	164342.87	878.37	1122.53	244.16	164098.71
10	164098.71	877.06	1122.53	245.47	163853.25
11	163853.25	875.75	1122.53	246.78	163606.47
12	163606.47	874.43	1122.53	248.10	163358.38

Use the amortization schedule to answer the following questions.

1. What is the **total amount** paid after the first year? **(2 KU)**

2. What is the **total principal** paid after the first year? **(2 KU)**

3. What is the **total interest** paid after the first year? **(2 KU)**

3.S.1 Understanding Annuities (continued)

Akash's bank has also provided him with an amortization schedule for the **eleventh** year of the mortgage.

Month	Principal	Interest	Payment	Principal Repaid	New Principal
121	127034.53	678.97	1122.53	443.56	126590.97
122	126590.97	676.60	1122.53	445.93	126145.04
123	126145.04	674.21	1122.53	448.32	125696.72
124	125696.72	671.82	1122.53	450.71	125246.00
125	125246.00	669.41	1122.53	453.12	124792.88
126	124792.88	666.99	1122.53	455.54	124337.34
127	124337.34	664.55	1122.53	457.98	123879.36
128	123879.36	662.10	1122.53	460.43	123418.93
129	123418.93	659.64	1122.53	462.89	122956.04
130	122956.04	657.17	1122.53	465.36	122490.68
131	122490.68	654.68	1122.53	467.85	122022.83
132	122022.83	652.18	1122.53	470.35	121552.48

Use the amortization schedule to answer the following questions.

4. What is the **total amount** paid during the eleventh year? **(2 KU)**

5. What is the **total principal** paid during the eleventh year? **(2 KU)**

6. What is the **total interest** paid during the eleventh year? **(2 KU)**

7. Although the total amount paid is the same during the first year and the eleventh year, the interest paid and principal paid is different. What is the difference and why does it happen? **(2 APP)**

8. What will happen to the interest paid versus the principal paid in the final year of the mortgage? **(2 APP)**

3.S.1 Understanding Annuities (Teacher Notes)

Full solutions are required for each question, including a list of the values from the TVM solver screen.

CASE STUDY
Akash has recently graduated from community college. He has just started his career and has established several financial goals. Two of his most important financial goals involve buying a car and buying a house. He would like to save enough money to buy a used car in 2 years and is in the research stage of buying a house.

Part 1: Saving to Buy a Car in 2 Years → Effects of Changing the Savings Plan

Akash is saving money to buy a used car in 2 years. He will deposit the same amount into a savings account every month over the next 2 years. Then he will use the amount he has saved to buy a car. Help Akash by answering the following questions.

1. What price of car will he be able to afford in 2 years if he deposits $300 per month at 4.3% compounded monthly? **(2 KU)**

N _____24_____
I% _____4.3___
PV _____0_____
PMT___-300_____
FV___**7504.65**_____
P/Y____12_____
C/Y____12_____
PMT: END BEGIN

2. What price of car will he be able to afford if he increases his deposits to $350 per month but the interest rate remains the same? **(2 KU)**

N _____24_____
I% _____4.3_____
PV _____0_____
PMT____-350_____
FV____**8755.42**_____
P/Y_____12_____
C/Y_____12_____
PMT: END BEGIN

3. What price of car will he be able to afford if his deposits remain at $300 per month, but he finds an investment that pays 5% compounded monthly? **(2 KU)**

N ____24_____
I% _____5_____
PV _____0_____
PMT___-300_____
FV____**7555.78**_____
P/Y____12_____
C/Y____12_____
PMT: END BEGIN

3.S.1 Understanding Annuities (Teacher notes continued)

4. What price of car will he be able to afford if he waits one year and then deposits $400 per month at 4.3% compounded monthly? (Thus only one year of deposits) **(2 KU)**

N _____12_____
I% _____4.3_____
PV _____0_____
PMT___-400_____
FV____**4895.74**_____
P/Y____12_____
C/Y____12_____
PMT: END BEGIN

5. How can Akash save the most amount of money for his car? **(2 APP)**

Answers may vary.

Part 2: Buy the Car Now → Present Value Today or Payment Plan?

A local car dealer has a used car that would suit Akash. The dealer is offering $300 monthly payments for 2 years. The payment plan already includes interest at 9% per year, compounded monthly.

1. How much will Akash pay in total if he decides to use the payment plan of $300 per month for 2 years? **(2 KU)**

$300 x 24 = $7200

2. How much will he pay for the car if he pays the ***present value*** today? Remember that the present value is the amount *without* the interest. **(2 KU)**

N _____24_____
I% _____9_____
PV ___**6566.74**_____
PMT___-300_____
FV_____0_____
P/Y____12_____
C/Y____12_____
PMT: END BEGIN

3. How much <u>less</u> will he pay if he pays the present value today instead of accepting the payment plan? **(2 KU)**

$7200 – 6566.74 = $633.26

3.S.1 Understanding Annuities (Teacher notes continued)

4. Why might Akash prefer the payment plan even though it will cost him more overall? **(2 APP)**

   Answers may vary.

5. Why might it be better for Akash to save regularly for 3 years to buy a car, instead of buying this one now? **(2 APP)**

   He will be able to buy a better car and have no payments or interest to pay.

Part 3: Buying a House → Changing the Term of a Loan

Akash is starting to research buying a house. For a small two bedroom home in his area he can expect to pay $175 000. His bank offers mortgages for 6.5% interest compounded monthly and he must make a 5% down payment.

1. What amount does he need for the down payment? How much will the mortgage be after the down payment? **(2 KU)**

   $175 000 x 0.95 = 166 250

2. What will his monthly payments be if he uses a: **(4 KU)**

   a. 25 year mortgage

   N _____300_____
   I% _____6.5_____
   PV ___166250_____
   PMT___**-1122.53**____
   FV_____0_____
   P/Y____12_____
   C/Y____12_____
   PMT: END BEGIN

   b. 20 year mortgage

   N _____240_____
   I% _____6.5_____
   PV ___166250_____
   PMT___**-1239.52**____
   FV_____0_____
   P/Y_____12_____
   C/Y_____12_____
   PMT: END BEGIN

3. How much will he end up paying for the house for each term? **(4 KU)**

   a. 25 year mortgage

   25 x 12 x $1122.53 = $336 759

   b. 20 year mortgage

   20 x 12 x $1239.52 = $297 484.80

4. What is the total interest paid for each term? **(2 KU)**

   a. 25 year mortgage

   $336 759 – $166 250 = $170 509

   b. 20 year mortgage

   $297 484.80 – $166 250 = $131 234.80

3.S.1 Understanding Annuities (Teacher notes continued)

5. Why might Akash choose to pay off the mortgage in 25 years, even though it will cost him more in interest? **(2 APP)**

   The monthly payments are less.

Part 4: Reading an amortization table

Akash's bank has provided him with an amortization schedule for the first year of the mortgage that he is researching for a 25 year term.

Month	Principal	Interest	Payment	Principal Repaid	New Principal
1	166250.00	888.56	1122.53	233.97	166016.03
2	166016.03	887.31	1122.53	235.22	165780.82
3	165780.82	886.06	1122.53	236.47	165544.34
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5	165306.60	883.52	1122.53	239.01	165067.59
6	165067.59	882.24	1122.53	240.29	164827.31
7	164827.31	880.96	1122.53	241.57	164585.74
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9	164342.87	878.37	1122.53	244.16	164098.71
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11	163853.25	875.75	1122.53	246.78	163606.47
12	163606.47	874.43	1122.53	248.10	163358.38

Use the amortization schedule to answer the following questions.

1. What is the total amount paid after the first year? **(2 KU)**

   $1122.53 x 12 = $13 470.36

2. What is the total principal paid after the first year? **(2 KU)**

   Total Principal = $2891.64

3. What is the total interest paid after the first year? **(2 KU)**

   Total Interest = $10 578.72

3.S.1 Understanding Annuities (Teacher notes continued)

Akash's bank has also provided him with an amortization schedule for the **eleventh** year of the mortgage that he is researching.

Month	Principal	Interest	Payment	Principal Repaid	New Principal
121	127034.53	678.97	1122.53	443.56	126590.97
122	126590.97	676.60	1122.53	445.93	126145.04
123	126145.04	674.21	1122.53	448.32	125696.72
124	125696.72	671.82	1122.53	450.71	125246.00
125	125246.00	669.41	1122.53	453.12	124792.88
126	124792.88	666.99	1122.53	455.54	124337.34
127	124337.34	664.55	1122.53	457.98	123879.36
128	123879.36	662.10	1122.53	460.43	123418.93
129	123418.93	659.64	1122.53	462.89	122956.04
130	122956.04	657.17	1122.53	465.36	122490.68
131	122490.68	654.68	1122.53	467.85	122022.83
132	122022.83	652.18	1122.53	470.35	121552.48

Use the amortization schedule to answer the following questions.

4. What is the total amount paid during the **eleventh** year? **(2 KU)**

   $13 470.63

5. What is the total principal paid during the eleventh year? **(2 KU)**

   Total Principal = $5482.04

6. What is the total interest paid during the eleventh year? **(2 KU)**

   Total Interest = $7988.32

7. Although the total amount paid is the same during the first year and the eleventh year, the interest paid and principal paid is different. What is the difference and why does it happen? **(2 APP)**

   More interest is paid in the first year. Therefore more of the monthly payment is going towards the paying down the principal in the eleventh year.

8. What will happen to the interest paid versus the principal paid in the final year of the mortgage? **(2 APP)**

   Most of the monthly payment will be paying down the principal in the final year.

3.S.2: Understanding Annuities - Evaluation Rubric

Name: _____

	Category	Level 1	Level 2	Level 3	Level 4
Communication	Correct use of mathematical symbols, labels, units and conventions across a range of media.	Sometimes uses mathematical symbols, labels and conventions correctly.	Usually uses mathematical symbols, labels and conventions correctly	Consistently uses mathematical symbols, labels and conventions correctly.	Consistently and meticulously uses mathematical symbols, labels and conventions, recognizing novel opportunities for their use.
	Appropriate use of mathematical vocabulary.	Sometimes uses mathematical vocabulary correctly	Usually uses mathematical vocabulary correctly when expected.	Consistently uses mathematical vocabulary correctly when expected.	Consistently uses mathematical vocabulary correctly, recognizing novel opportunities for its use.
	Degree of clarity in explanations and justifications in reporting.	Explanations and justifications are partially understandable.	Explanations and justifications are understandable by me, but would likely be unclear to others.	Explanations and justifications are clear for a range of audiences.	Explanations and justifications are particularly clear and detailed.

Name: _____

	Category	Level 1	Level 2	Level 3	Level 4
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