Unit 4: Day 16: Summative Assessment: Ferris Wheel Day 1

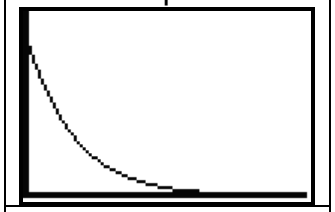
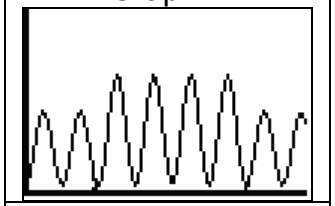
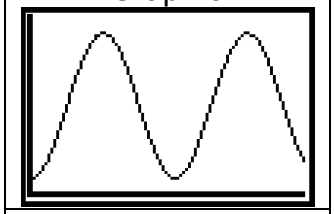
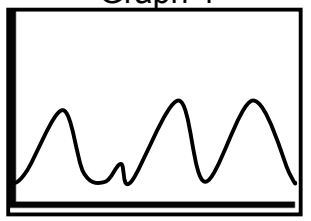
MCT 4C

Minds On: 10 Action: 60 Consolidate: 5 Total=75 min	**Math Learning Goals:** • Identify sinusoidal functions arising from a real-world application and explain restrictions that the context places on the domain and range. • Solve problems involving a sinusoidal function by using a given graph, table of values or equation. • Represent sinusoidal data, descriptions, or graphs with an equation. • Determine the amplitude, period, and phase shift of sinusoidal functions. • Sketch graphs of sinusoidal functions. • Reasoning about and connecting different models	**Materials** • BLM 4.16.1-4.16.4 • Scissors • Glue sticks

			Assessment Opportunities
	Minds On…	**Pairs → Activity** Student pairs work on one copy of BLM 4.16.1. Students match graphs to picture or description of amusement park ride. Students write their reasons on reverse of the sheet. Reasoning for matches can be shared between pairs, as a class. Students should ignore the relative heights of one ride compared to another but focus on how the height changes over the time of the ride. Solution: Ferris Wheel – Graph 3; Drop Ride – Graph 1; Roller Coaster – Graph 4; π-rate Ride – Graph 2.	Listen for students with misconceptions. They may need scaffolding for he individual portion on BLM 4.16.2-4.16.4
	Action!	**Whole Class → Discussion** Discuss overall design of activity on BLM 4.16.2-4.16.4 and the assessment process. **Individual → Investigation** Each student should be given copies of BLM 4.16.2, BLM 4.16.3 and four copies of BLM 4.16.4. Students should also be given scissors and glue to paste parts once they have sorted them. Encourage students to use time efficiently. Observe and offer prompts as needed. **Mathematical Process Focus: Connecting** – Students make **connections** between mathematical concepts and real-word applications. **Mathematical Process Focus: Representing** – Students create different **representations** of mathematical ideas.	Day 1 of this assessment should be marked analytically. Four copies of BLM 4.16.4 are needed so they can sort the pieces they have and complete the missing pieces.
	Consolidate Debrief	**Whole Class → Discussion** Students submit their completed copies of BLM 4.16.4. Discuss issues and strategies used to complete the activity.	Feedback from work completed on Day 1 can be provided to students that did not come up with correct models.
	Reflection	**Home Activity or Further Classroom Consolidation** Journal: Reflect on the strategies you used to match the pieces and fill in the missing information.	Self-assessment can be used to determine how well students completed the activity.

4.16.1 Whose Ride is it Anyway?

Match the pictures or descriptions of rides at an amusement park in the left column with the graphs of height versus time in the right column. Provide reasons for your matches on the back.

Ride	Graph
Ferris Wheel 	Graph 1 
Drop Ride – You are seated in a chair at a height of 35 m and fall to the ground. The ride slows down as you reach the ground.	Graph 2 
Roller Coaster 	Graph 3 
π-rate Ride – You board the ride at ground level. The ride starts swinging back and forth slowly and continues swinging back and forth but getting faster and higher. The ride then slows down until it stops.	Graph 4 

4.16.2: A Ride in The Park

Background:

You are an engineer and you have been asked to design a Ferris wheel to be built as an addition to an existing amusement park that is already inside a mall atrium. The loading dock to the Ferris wheel is always on the second floor of the mall.

Your employer has asked you to design **four** different Ferris wheels to be presented to a panel of investors. Your presentation is almost complete but your computer crashed! Luckily parts of your presentation have been recovered. Your deadline is tomorrow and you need to piece together all the recovered material and re-create all the missing parts.

Day 1:

In your presentation there were five components to each design:

- A numerical model (table of values)
- An algebraic model
- A graphical model
- A descriptive model (description of the ride)
- Model specifications (amplitude, period, radius, shifts and translations)

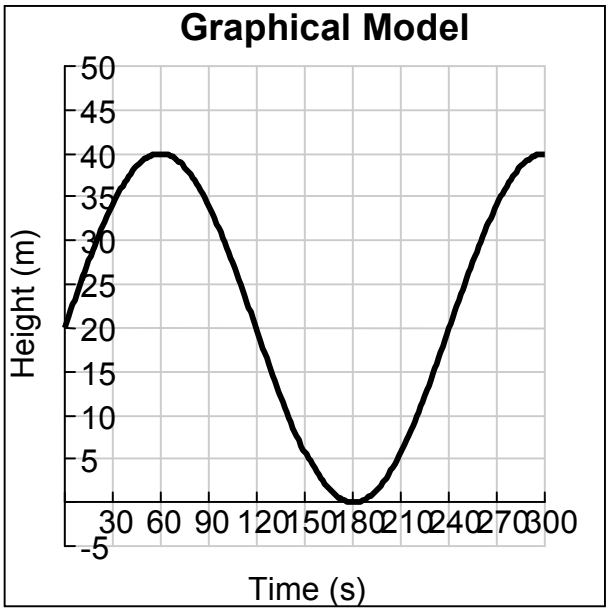
Your task:

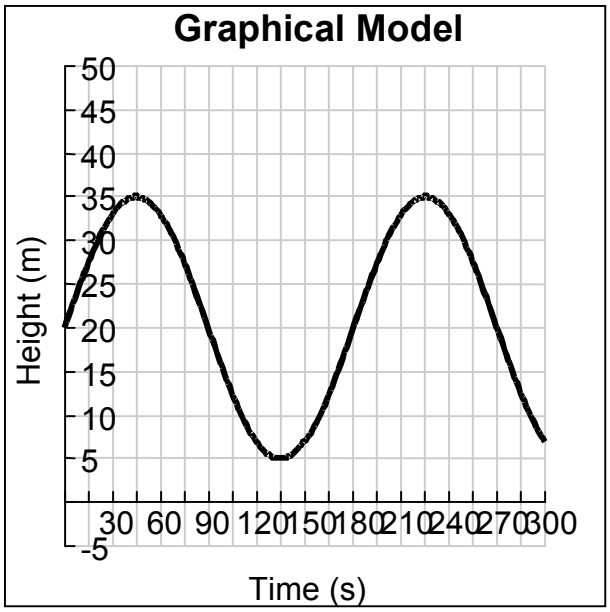
The pieces of the four models that were recovered are found on BLM 4.16.3. Your job will be to match any that go together and then provide the models that are missing (i.e. the numerical model; algebraic model; grapical model; descriptive model; and model specifications).
Use scissors to cut out the models that were recovered. Four copies of BLM 4.16.4 have been given to you so you can paste the recovered pieces and add the missing pieces for each Ferris wheel.



4.16.3: The Pieces of the Puzzle

Here are all the recovered pieces of information:





Algebraic Model:

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
12	6	6	80

Descriptive Model:

You load the Ferris wheel on the second floor. It completes one revolution in 120 seconds.

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
30	15	15	180

4.16.3: The Pieces of the Puzzle (continued)

Numerical Model:

x	y
0	20
10	24.24
20	26
30	24.24
40	20
50	15.76
60	14
70	15.76
80	20
90	24.24
100	26
110	24.24
120	20
130	15.76
140	14
150	15.76
160	20
170	24.24
180	26
190	24.24
200	20
210	15.76
220	14
230	15.76

Numerical Model:

x	y
0	20
10	25.18
20	30
30	34.14
40	37.32
50	39.32
60	40
70	39.32
80	37.32
90	34.14
100	30
110	25.18
120	20
130	14.82
140	10
150	5.86
160	2.68
170	0.68
180	0
190	0.68
200	2.68
210	5.86
220	10
230	14.82

4.16.4: Putting the Pieces Together

Use this template to sort the pieces you have been given and fill in the missing ones.

Ferris Wheel Model #: _____

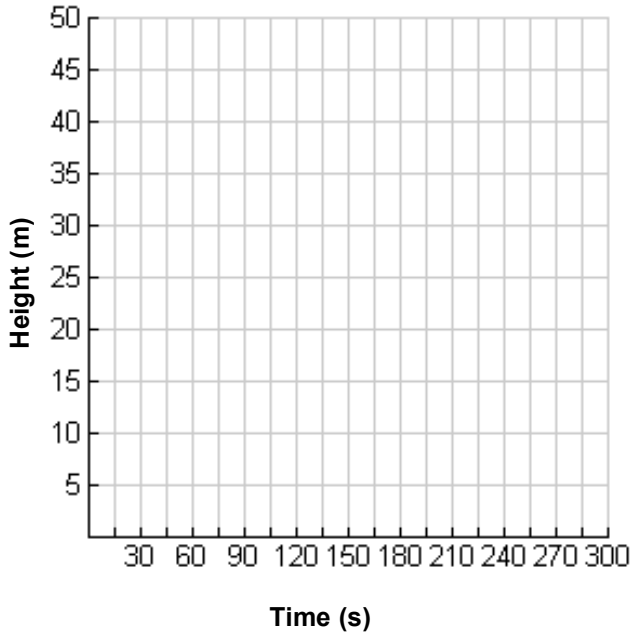
x	y
0	
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	
130	
140	
150	
160	
170	
180	
190	
200	
210	
220	

Descriptive Model:

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)

Algebraic Model:



4.16.4: Putting the Pieces Together – Teacher Solutions

Ferris Wheel 1

x	y
0	20
10	25
20	28.66
30	30
40	28.66
50	25
60	20
70	15
80	11.34
90	10
100	11.34
110	15
120	20
130	25
140	28.66
150	30
160	28.66
170	25
180	20
190	15
200	11.34
210	10
220	11.34
230	15
240	20

Descriptive Model:

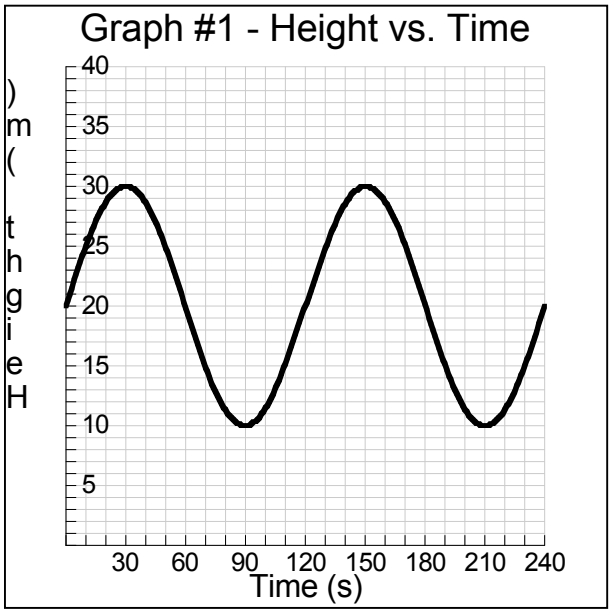
You load the Ferris wheel on the second floor. It completes one revolution in 120 seconds.

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
20	10	10	120

Algebraic Model:

$y = 10\sin 3x + 20$



4.16.4: Putting the Pieces Together – Teacher Solutions
(continued)

Ferris Wheel 2

x	y
0	20
10	24.24
20	26
30	24.24
40	20
50	15.76
60	14
70	15.76
80	20
90	24.24
100	26
110	24.24
120	20
130	15.76
140	14
150	15.76
160	20
170	24.24
180	26
190	24.24
200	20
210	15.76
220	14
230	15.76
240	20

Descriptive Model:

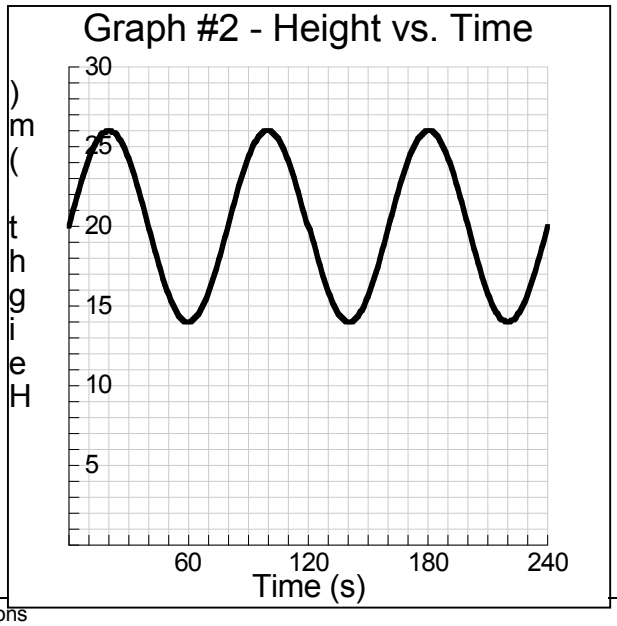
You load the Ferris wheel on the second floor. It completes one revolution in 80 seconds.

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
12	6	6	80

Algebraic Model:

$y = 6\sin 4.5x + 20$



4.16.4: Putting the Pieces Together – Teacher Solutions
(continued)

Ferris Wheel 3

x	y
0	20
10	25.18
20	30
30	34.14
40	37.32
50	39.32
60	40
70	39.32
80	37.32
90	34.14
100	30
110	25.18
120	20
130	14.82
140	10
150	5.86
160	2.68
170	0.68
180	0
190	0.68
200	2.68
210	5.86
220	10
230	14.82
240	20

Descriptive Model:

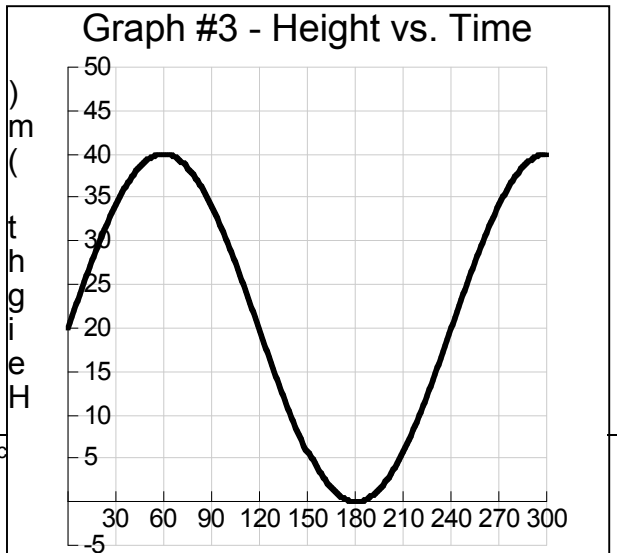
You load the Ferris wheel on the second floor. It completes one revolution in 240 seconds.

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
40	20	20	240

Algebraic Model:

$y = 20\sin 1.5x + 20$



4.16.4: Putting the Pieces Together – Teacher Solutions
(continued)

Ferris Wheel 4

x	y
0	20
10	25.13
20	29.64
30	32.99
40	34.77
50	34.77
60	32.99
70	29.64
80	25.13
90	20
100	14.87
110	10.36
120	7.01
130	5.23
140	5.23
150	7.01
160	10.36
170	14.87
180	20
190	25.13
200	29.64
210	32.99
220	34.77
230	34.77
240	32.99

Descriptive Model:

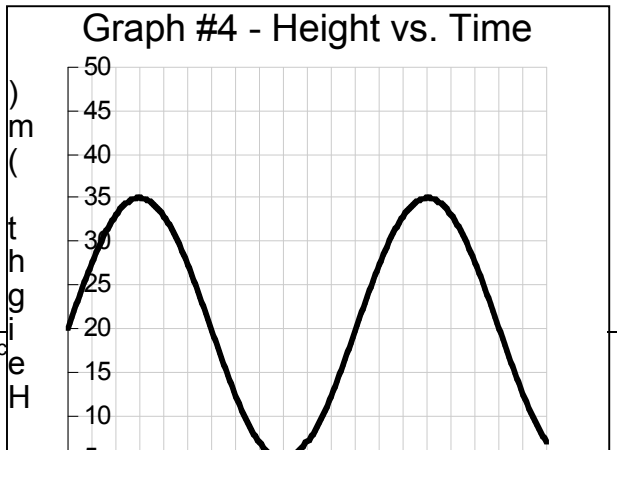
You load the Ferris wheel on the second floor. It completes one revolution in 180 seconds.

Model Specifications:

Height of the wheel (m)	Radius (m)	Amplitude (m)	Period (s)
30	15	15	180

Algebraic Model:

$y = 15\sin 2x + 20$



Unit 4: Day 17: Summative Assessment: Ferris Wheel Day 2

MCT 4C

Minds On: 10

Action: 60

Consolidate: 5

Total=75 min

Math Learning Goals:

- Identify sinusoidal functions arising from a real-world application and explain restrictions that the context places on the domain and range.
- Solve problems involving a sinusoidal function by using a given graph, table of values or equation.
- Represent sinusoidal data, descriptions, or graphs with an equation.
- Determine the amplitude, period, and phase shift of sinusoidal functions.
- Sketch graphs of sinusoidal functions.
- Reasoning about and connecting different models

Materials

- BLM 4.17.1-4.17.2

			Assessment Opportunities
	Minds On…	**Whole Class → Discussion** Return students' submitted work on BLM 4.16.4 with feedback so that they can proceed with today's task.	
	Action!	**Individual → Investigation** Distribute copies of BLM 4.17.1 and 4.17.2. Briefly discuss the overall expectations of the task and how they relate to the assessment rubric (BLM 4.17.2). Students should be encouraged to look at criteria and determine their performance level. Observe and offer prompts as needed. **Curriculum Expectations/BLM 4.17.1/Mental Note:** Observe students misconceptions as they complete the task. Student requiring additional support should access the CLIPS activites on trig.	Grade 11 Trig CLIPS will be posted on the OAME website at http://www.oame.on.ca/main/index1.php
	Consolidate Debrief	**Whole Class → Discussion** Collect completed work from BLM 4.17.1 and rubric. Discuss which Ferris wheel design meets the criteria set out by the task.	Self-assessment can be used to determine how well students completed activity.
Concept Practice Application		**Home Activity or Further Classroom Consolidation** Prepare for pencil and paper test that addresses the expectations that were not covered in the final summative task.	

4.17.1: The Right Fit

Day 2: You've put together your presentation and your employer and panel of investors are very interested in your designs. However, only one design will considered. Before you select your design, you need to consider the following:

- The Ferris wheel will be placed in a shopping mall in the atrium.
- The loading platform will be on the second floor, which is 20 m above the ground level.
- There should be three full revolutions for each ride.
- Each ride must last between 7-12 minutes.

Your Task:
Choose one design that will meet all the above criteria. Justify your choice. Include the reasons why you rejected each of the other models.

Ferris Wheel	Accept or Reject	Justification
1		
2		
3		
4		

4.17.1: The Right Fit (continued)

After your design is accepted, the investors would like you to analyze the ride further.

1. Recall that the ride must last between 7 and 12 minutes. If the Ferris wheel completes exactly three revolutions, what is the:

a. maximum period for a 12 minute ride?

b. minimum period for a 7 minute ride?

2. Describe any restrictions on the height of the wheel.

3. If the Ferris wheel loaded on the third floor which is 40 m above the ground and was open to the floors below, describe the change(s) to the:

a. equation.

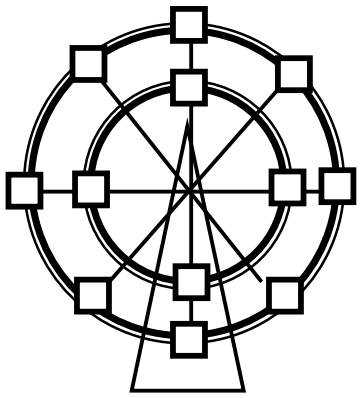
b. height of the wheel.

c. Is there another design that meets the previous criteria if the ride is on the third floor? Explain.

4.17.1: The Right Fit (continued)

4. If the loading platform were changed to the first floor, how would that affect the graph and equation for the design that was accepted?

5. The accepted design will be changed slightly. The Ferris wheel will have an inner wheel, which will board on the first floor (see diagram). The inner wheel will rotate in the opposite direction and rotate at the same speed as the outer wheel (i.e. 1 revolution in 180 seconds). The radius of the inner wheel will be 13 m. Mary boards the outer wheel at the exact same time as Frieda boards the inner wheel. At what time(s) will they be at the same height during the first revolution? Support your answer with reference to graphs, equations or tables of values.



4.17.1: The Right Fit – Teacher Solutions

Day 2: You've put together your presentation and your employer and panel of investors are very interested in your designs. However, only one design will considered. Before you select your design, you need to consider the following:

- The Ferris wheel will be placed in a shopping mall in the atrium.
- The loading platform will be on the second floor, which is 20 m above the ground level.
- There should be three full revolutions for each ride.
- Each ride must last between 7-12 minutes.

Your Task:
Choose one design that will meet all the above criteria. Justify your choice. Include reasons why you rejected each of the other models.

Ferris Wheel	Accept or Reject	Justification
1	Reject	*3 revolutions = 3(120) / 60* *= 6 min* *Since the ride must last between 7 and 12 minutes, this design is not long enough.*
2	Reject	*3 revolutions = 3(80) / 60* *= 4 min* *Since the ride must last between 7 and 12 minutes, this design is not long enough.*
3	Reject	*Ferris wheel hits the ground.*
4	Accept	*3 revolutions = 3(180) / 60* *= 9 min* *Since the ride must last between 7 and 12 minutes, this design is long enough and the wheel is always above the ground.*

4.17.1: The Right Fit – Teacher Solutions (continued)

After your design is accepted, the investors would like you to analyze the ride further.

1. Recall that the ride must last between 7 and 12 minutes. If the Ferris wheel completes exactly three revolutions, what is the:

a. maximum period for a 12 minute ride?

Maximum time = 12 minutes
Maximum period = 12 / 3
= 4 minutes
= 240 s
Therefore, the maximum period of the wheel is 240 s.

b. minimum period for a 7 minute ride?

Minimum time = 7 minutes
Minimum period = 7 / 3
= 420 / 3
= 140
Therefore, the minimum period of the wheel is 140 s.

2. Describe any restrictions on the height of the wheel.

Since the wheel can not touch the ground and the wheel's centre is 20 m above the ground, the maximum height of the Ferris wheel must be less than 40 m because a height of exactly 40 m would result in the wheel dragging on the ground and a height of more than 40 m would result in the wheel travelling underground.

3. If the Ferris wheel loaded on the third floor which is 40 m above the ground and was open to the floors below, describe the change(s) to the:

a. equation of the accepted design.

The vertical shift would now be 40 and the equation would become y = 6 sin 4.5x + 40

b. maximum possible height of the wheel.

Since the centre of the wheel is now 40 m above the ground, the maximum height of the Ferris wheel must be less than 80 m because a height of exactly 80 m would result in the wheel dragging on the ground and a height of more than 80 m would result in the wheel travelling underground.

c. Is there another design that meets the previous criteria if the ride is on the third floor? Explain.

Since all of the rejected designs did not meet the time requirement of lasting between 7 and 12 minutes, they still cannot be accepted.

4.17.1: The Right Fit (continued)

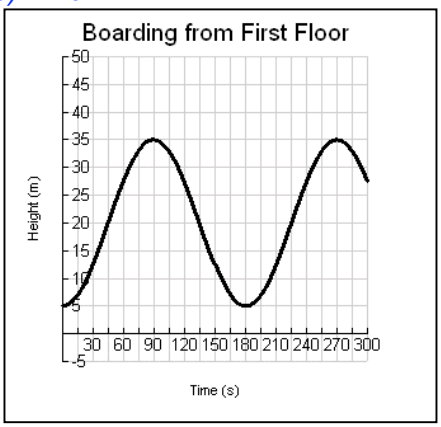
4. If the loading platform were changed to the first floor, how would that affect the graph and equation for the design that was accepted?

The height of the rider for one revolution would be:

Time (s)	*0*	*45*	*90*	*135*	*180*
Height	*5*	*20*	*35*	*20*	*5*

This would affect the algebraic model. The amplitude, period and vertical shift would be the same. But there would now be a phase shift of 45 seconds to the right (or left). The new equation would be: y = 15 sin2(x – 45) + 20

The resulting graph would look like:



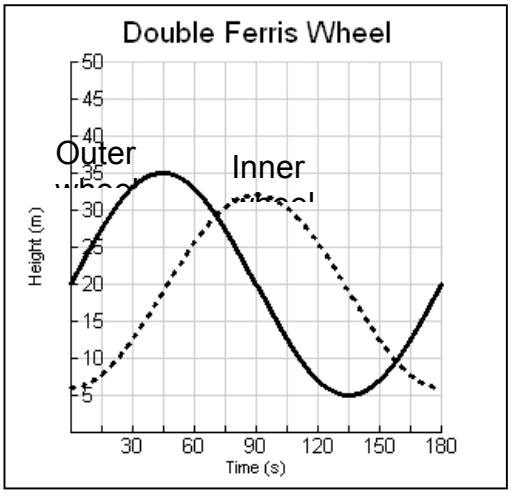
5. The accepted design will be changed slightly. The Ferris wheel will have an inner wheel, which will board on the first floor (see diagram). The inner wheel will rotate in the opposite direction and rotate at the same speed as the outer wheel (i.e. 1 revolution in 180 seconds). The radius of the inner wheel will be 13 m. Mary boards the outer wheel at the exact same time as Frieda boards the inner wheel. At what time(s) will they be at the same height during the first revolution? Support your answer with reference to graphs, equations or tables of values.

The height of the rider on the inner wheel for one revolution would be:

Time (s)	*0*	*45*	*90*	*135*	*180*
Height (m)	*6*	*19*	*32*	*19*	*6*

The algebraic model of a rider on the inner wheel would be y = 13 sin2(x – 45) + 19.

If we plot the equation or data from the table on the graph for the outer wheel we could read the points of intersection, which is when the riders are at the same height. Looking at the graph, the points of intersection are about (70, 30) and (160, 10).
The riders will therefore be at the same height at 70 seconds and 160 seconds.



4.17.2: Summative Assessment Rubric

Connecting				
Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Making connections among mathematical concepts and procedures **All parts of day 2.**	Makes weak connections	Makes simple connections	Makes appropriate connections	Makes strong connections
Reasoning and Proving				
Formulation and defence of a hypothesis or conjecture **All parts of Day 2.**	Forms a hypothesis or conjecture that connects a few aspects of the problem	Forms a hypothesis or conjecture that connects some aspects of the problem	Forms a hypothesis or conjecture that connects sufficient aspects of the problem	Forms a hypothesis or conjecture that connects aspects of the problem with a broader view of the problem
Making inferences, conclusions and justifications **All parts of Day 2.**	Justification of the answer presented has a limited connection to the problem solving process and models presented	Justification of the answer presented has some connection to the problem solving process and models presented	Justification of the answer presented has a direct connection to the problem solving process and models presented	Justification of the answer has a direct connection to the problem solving process and models presented, with evidence of reflection
Communicating				
Correct use of mathematical symbols, labels, units and conventions across a range of media (media production) **All parts of days 2.**	Sometimes uses mathematical symbols, labels and conventions correctly	Usually uses mathematical symbols, labels and conventions correctly	Consistently uses mathematical symbols, labels and conventions correctly	Consistently and meticulously uses mathematical symbols, labels and conventions, recognizing novel opportunities for their use
Appropriate use of mathematical vocabulary (preparation for media consumption and production) **All parts of day 2.**	Sometimes uses mathematical vocabulary correctly when expected	Usually uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly, recognizing novel opportunities for its use
Degree of clarity in explanations and justifications in reporting **All parts of day 2.**	Explanations and justifications are partially understandable	Explanations and justifications are understandable by me, but would likely be unclear to others	Explanations and justifications are clear for a range of audiences	Explanations and justifications are particularly clear and detailed