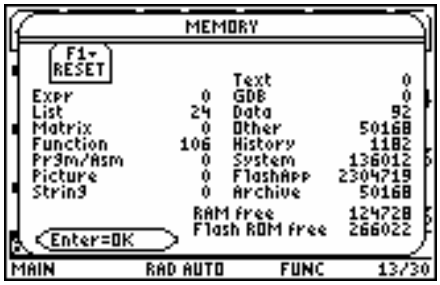
U4L4B – Box Problem – TI-89 CAS™ Teacher Notes

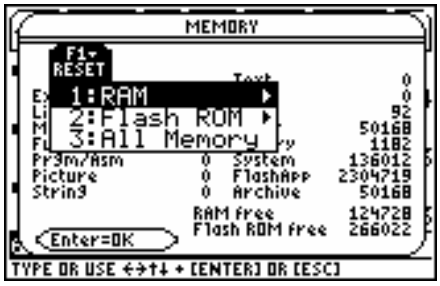
You are provided with a sheet of metal that measures 80 cm by 60 cm. If you cut squares out of the corners, you are left with a rectangle in the centre and four flaps that can fold up to form the sides of a box. Find the size of the square that you need to cut in order to maximize the volume of the box.

Resetting the Device

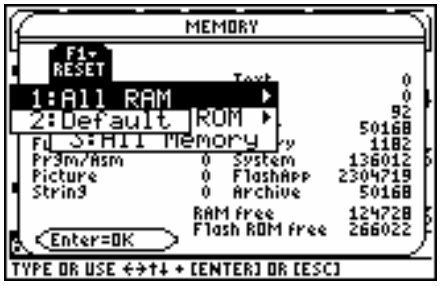
Before you do anything with the cubic function, reset the device so that you avoid any surprises left by the previous user. Press 2 followed by { in order to access the ↓ menu.



Press □ to begin the reset process. From the menu that pops up, choose ♦: Ram.



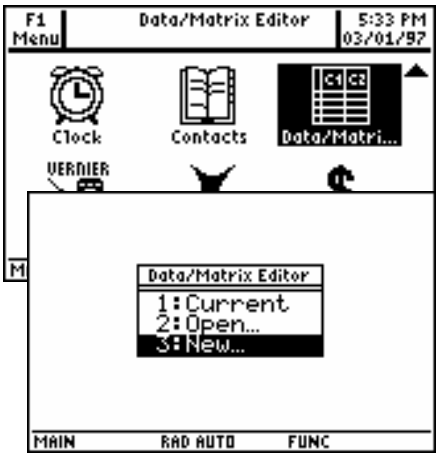
From the sub-menu that pops up, choose ♦: All Ram. You will have to press ÷ to confirm that you wish to proceed with clearing the memory and press ÷ a second time to proceed with the work that needs to be done.



Part 1 – Numerical Investigation

In the first section, you will look at the lists to investigate the relationship between the cut size and the length and width.

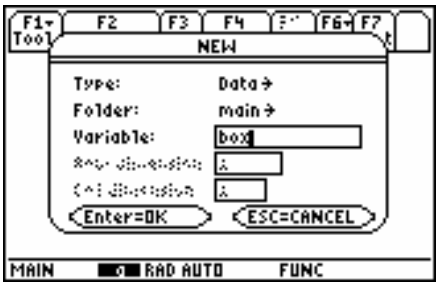
Press O and select the Data/Matrix application. Lists are handled in this application.



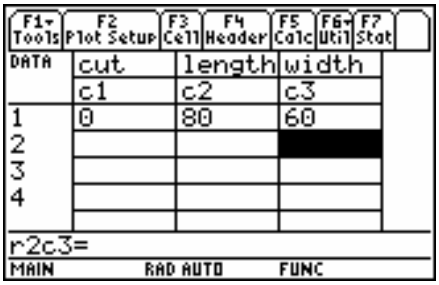
Select option ♠: New to begin a new set of lists.

U4L4B – Box Problem – TI-89 CAS™ Teacher Notes (cont.)

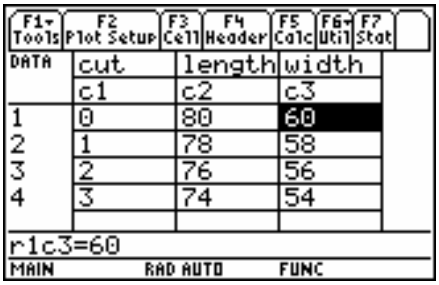
The TI-89 Titanium organizes files into folders for you. The default folder is “main”, but you could create your own if you choose. Use the down arrow key, ∆ to move to the “variable” field and enter the label “box”. The alpha lock will be on for you. Press ÷ to complete the window and move on.



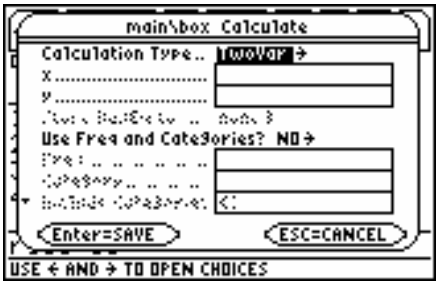
Label columns c1, c2 and c3 as “cut”, “length” and “width” respectively. Enter the initial values for the three variables.



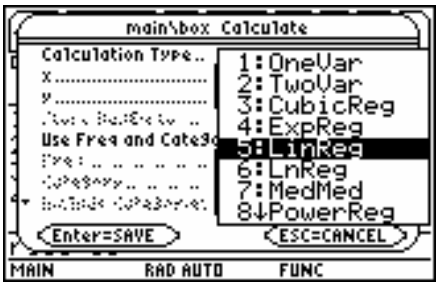
Let’s assume that the cut size increased by 1 cm in each line. As the size of the square increases by 1 cm, the length and width both decrease by 2 cm. Type the values into the columns as shown.



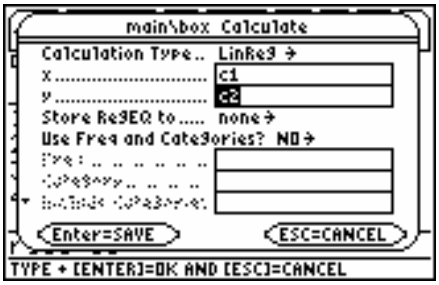
In order to find the relationship, you will perform a linear regression on length vs. cut and width vs. cut. Press □ to access the Math menu. The dialog box shown to the right will appear.



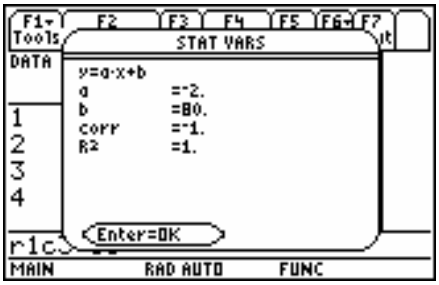
Press the cursor right key, B, to get the menu shown. Choose the LinReg option.



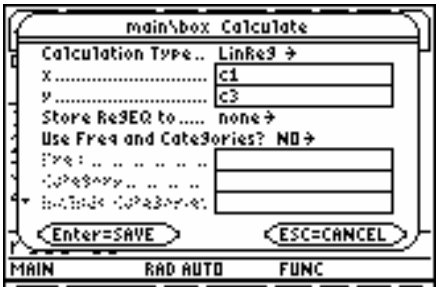
The dialog box will reflect the change. Move to the field for “x” and enter “c1” for column 1. For the “y” field, enter “c2”. You do not need to save the regression equation, so press ÷ to complete the calculation.



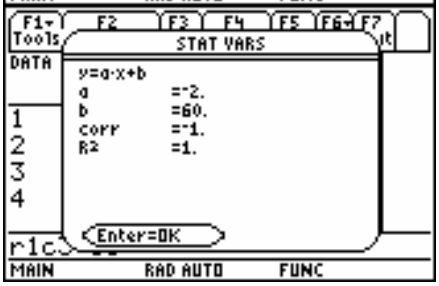
The results are displayed in columns D and E. From the slope and y-intercept fields, you can see that a formula for the length would be $l = 80 - 2x$.



Repeat the regression calculation for the second relationship. Choose “c1” for the X list again and “c3” for the Y list. Press ÷ to complete the calculation.



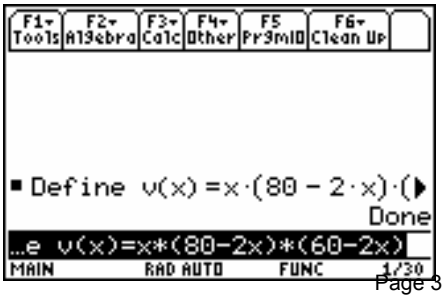
Looking at the results, you conclude that the width can be expressed as w = 60 – 2x.



Part 2 – The Algebraic Investigation

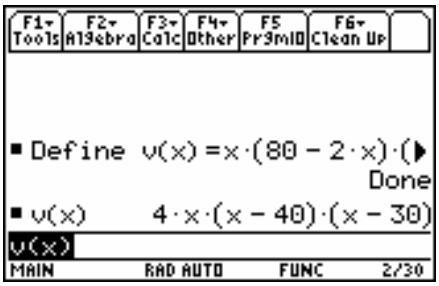
Now that you have a relationship between the length and cut size and also for width and cut size, you can use calculus to analyze the function and determine the cut size that will produce the maximum volume.

Press ∀ to enter the CAS. Define a new function, *v(x)*, as the product of the cut size, the length and the width.

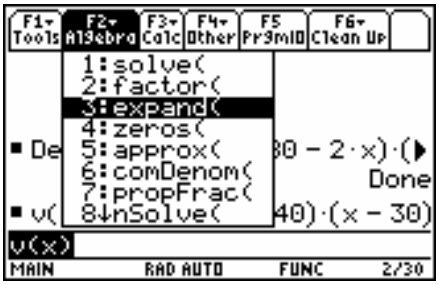


U4L4B – Box Problem – TI-89 CAS™ Teacher Notes (cont.)

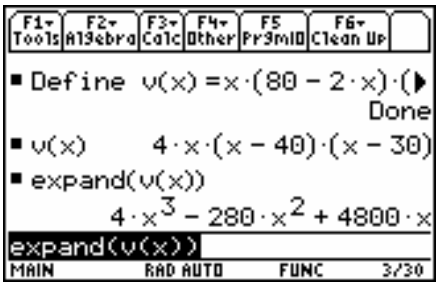
Display the function. Note that the CAS does some "simplification".



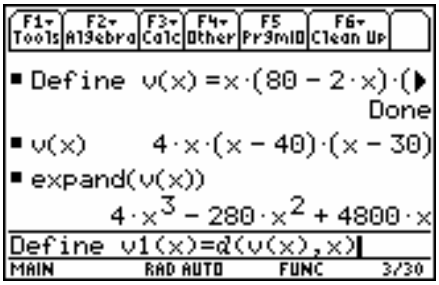
From the **Algebra** menu, choose Expand.



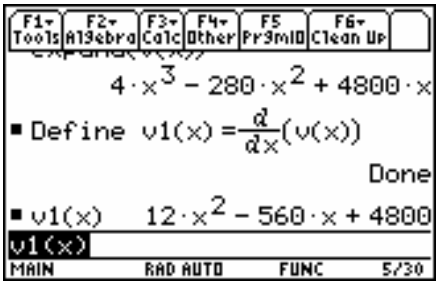
Complete the command as shown. The polynomial form of the function will be displayed.



Define a new function, *v1(x)*, as the derivative of *v(x)*.

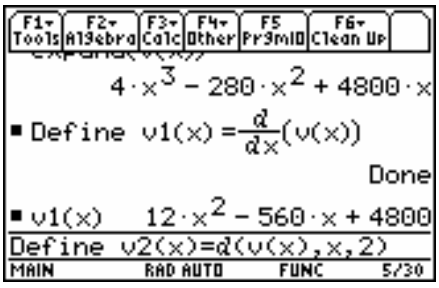


Display the new function.

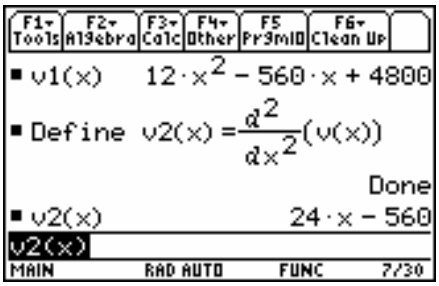


U4L4B – Box Problem – TI-89 CAS™ Teacher Notes (cont.)

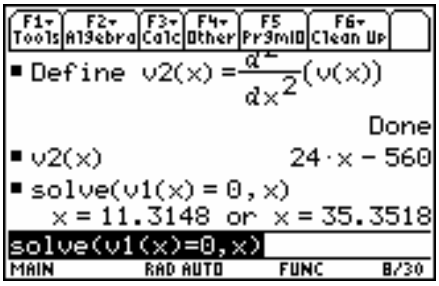
Before solving for the *x*-coordinates of the turning points, define another new function, *v2(x)*, to hold the second derivative.



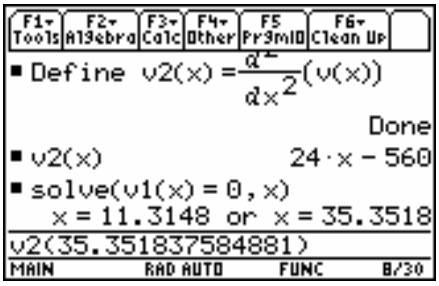
Display the new function.



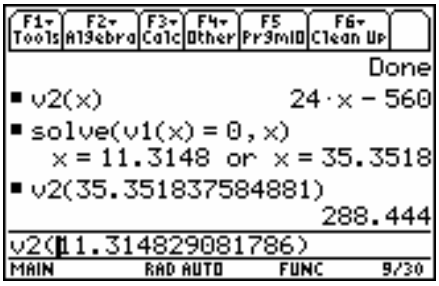
To find the *x*-coordinates of the turning points, use the Solve command to find the roots of the first derivative. In order to get decimal solutions, press ∞ followed by ÷.



Two solutions are shown which you will test in the second derivative to determine if the value(s) found are appropriate to the context of the problem. On a new line, enter v2(), move up to the solutions and press enter. The two solutions will be pasted into the entry line. Use the cursor keys and the 0 key to trim out the extra digits and characters.

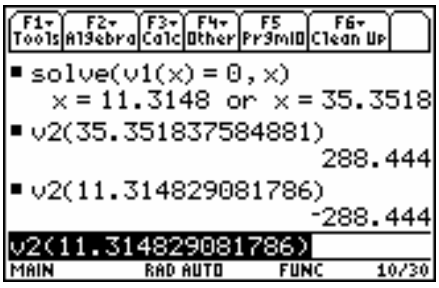


Since the value of the second derivative is positive, you conclude that this value represents the x-coordinate of a local minimum. In the context of the problem, you can also exclude this root since it produces a width that is negative. Substitute the smaller root into the second derivative in the same way.

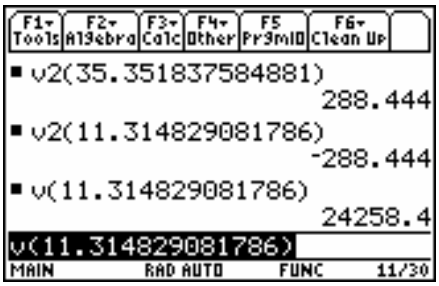


U4L4B – Box Problem – TI-89 CAS™ Teacher Notes (cont.)

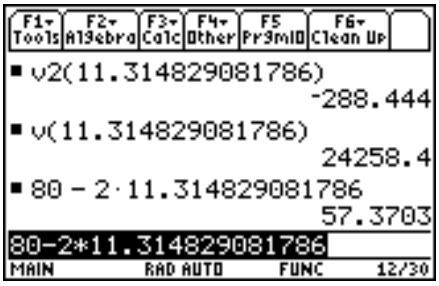
Since the value of the second derivative is negative, you conclude that the root represents the x-coordinate of a local maximum. Therefore, a cut size of 11.3 cm produces the maximum volume.



If you substitute this value into the function, you get a value that represents the maximum volume.



If you substitute this value into the expressions for length and width, you will get the other dimensions of the box of maximum volume.



So, a cut size of 11.3 cm produces a box with a length of 57.4 cm and a width of 37.4 cm.

