TIPS4RM

Leading MATH Success

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Advanced Functions (MHF4U)

Final Evaluation

Introduction

A course performance task is provided to support teachers in assessing the overall expectations of this course through the lens of the mathematiccal processes. It allows for a balance of demonstration of the four categories of the Achievement Chart. Teachers could create a written examination to address the expectations that the performance task does not cover. Teachers should use their professional judgment to create additional questions, noting that not all specific expectations must be evaluated and that expectations addressed in the performance task do not need to be duplicated on a written examination.

Students should have had multiple opportunities to participate in similar instructional and assessment tasks/questions and to be assessed using rubrics based on the mathematical processes.

In creating the performance task the following criteria were considered:

- Focus on big ideas of the course:
  - identifying key features of polynomial, logarithmic, rational, exponential, and trigonometric functions;
  - making connections among numerical, graphical and algebraic representations of polynomial, logarithmic, rational, exponential, and trigonometric functions;
  - solving problems related to polynomial, logarithmic, rational, exponential, and functions or their combinations.
- Focus on critical thinking.
- Explicitly reference the mathematical processes.
- Cover several overall expectations and connections between and among them.
- Choose a context relevant to the student, e.g., environment, social justice.
- Provide students multiple entry points for solving the problem.
- Allow for four levels of performance.
- Assess with a rubric which clearly articulates distinguishing features.
- Involve modelling, synthesizing, analysing, and interpreting data.
- Allow access to a variety of resources and tools, e.g., open book, formula sheets, group brainstorm, technology.
- Engage students in actively doing and using mathematics.
- Include anticipation opportunities, e.g., prior reading, research, brainstorming.
- Ensure equity and balance – geographic, cultural, learning styles and resources.
- Include an opportunity to reflect on the processes they used.
- Include choice.

The task should be a SMART Problem:

- Specific
- Measurable
- Attainable
- Realistic
- Timely

MHF4U Performance Task

Lesson Outline

Big Picture

Students will be evaluated on:

- Creating appropriate algebraic models from a given set of data;
- Determining the key features of the models they derive and the models created from combinations of functions;
- Determining the rate of change for real-world applications;
- Determining and inferring the meaning of a composition of two functions for a real-world application;
- Solving a problem that is not easily accessible by algebraic techniques.

Day	Lesson Title	Math Learning Goals	Expectations
1	Sea the Earth Changing	• Make connections between algebraic, graphical and numeric representations. • Identify key features of functions and connect these to solve a problem. • Make inferences about the rate of change within a context. • Make inferences about the composition of two functions within a context. • Communicate their reasoning. • Determine approximate rates of change from a real world application.	A2, C1, D1, D2, D3, A2.1, 2.4 C1.2, 1.3 D1.4, 1.6, 1.9, 2.1, 2.2, 3.2, 3.3 CGE 2b, 3c, 4f
2	Changing into the Future	• Recognize real word applications of combinations of functions. • Determine the key features of the graphs of a function created by a combination of two functions. • Determine the approximate instantaneous rate of change from a real word application. • Solve a problem whose solution is not accessible by standard algebraic techniques.	A2, C1, C2, D1, D2, D3 A2.1, 2.4 C1.3, 2.1 D1.4, 1.6, 1.9, 2.1, 2.2, 2.5, 2.6, 3.2, 3.3 CGE 2d, 3c, 4f



75 min

Evaluation Goals

- Make connections between algebraic, graphical and numeric representations.
- Identify key features of functions and connect these to solve a problem.
- Make inferences about the rate of change within a context.
- Make inferences about the composition of two functions within a context.
- Communicate their reasoning.
- Determine approximate rates of change from a real world application.

Materials

- BLM 7.1.1–7.1.8
- graphing technology
- data projector
- computer

Assessment Opportunities

Minds On…

Whole Class → Discussion

To set the environmental context, students post words regarding global warming and the environment on a word wall. Possible words: natural disasters, oil production, oil consumption, urban population change over time (per capita over time), ice caps, Arctic sea ice extent, CO_2, global mean sea level, ozone depleting gas index, mean surface temperature, sea level change, thermal expansion.

Demonstrate how to identify an appropriate mathematical model for a given graph. Students use BLM 7.1.1 to identify algebraic models for the given graphical models. Discuss the causes and effects of global warming, highlighting that CO_2 is a major green house gas causing an increase in global temperature because of depletion of the ozone level. Students need to understand the concept of 'mean sea level.'

Continue setting the stage for the performance task, using BLM 7.1.2.

Clarify the nature and expectations for the task.

Explain the criteria used for assessing their performance (Rubric 7.1.8). Some questions are assessed analytically in the Application category for a total of 54 marks (BLM 7.1.3 A #4, B, C, D #1; BLM 7.2.2 #1, 2, 4, 5a, 6).

A discussion about critical literacy and the reliability of data presented would be appropriate.

The context could be set on the previous day by showing video clips displaying data, using "An Inconvenient Truth, a Global Warning" (documentary with Al Gore)

and

http://www.channel4.com/science/microsites/G/great_global_warming_swindle/index.html

("The Great Global Warming Swindle" – an alternate viewpoint).

Action!

Performance Task → Individual

Part A: Students pose and analyse three possible algebraic models for the Mean Sea Level vs. CO_2 Levels relationship (BLM 7.1.3). For each model they extrapolate the domain to zero and identify key graphical characteristics and appropriateness of fit. They determine which model is the most appropriate to the context.

Part B: Students use graphing technology to determine the sea level and rate of change at the earth's critical value of 450 ppm using their identified model from Part A.

Part C: Students determine when the critical CO_2 value will occur using the algebraic model identified in Part A for CO_2 concentrate as a function of time.

Part D: Students create a composite function for Mean Sea Level as a function of CO_2 levels based on time. They use this function to determine the year in which three cities across the world will begin flooding due to a rise in sea level.

BLM 7.1.5 provides a sample solution. BLM 7.1.7 contains additional data sets for use at another time. A sample solution for one model is provided.

For other function modelling resources go to http://www.statcan.ca/English/edu/mathmodel.htm

or

http://www.teacherweb.com/ON/Statistics/Math/
(in the E-STAT and Function Modelling folder).

Consolidate Debrief

Whole Class → Discussion

Ask: Can the CO_2 vs. Time graph be better represented by another model? Explain and create a new model and discuss how this change might affect the task.

Concept Practice
Skill Practice

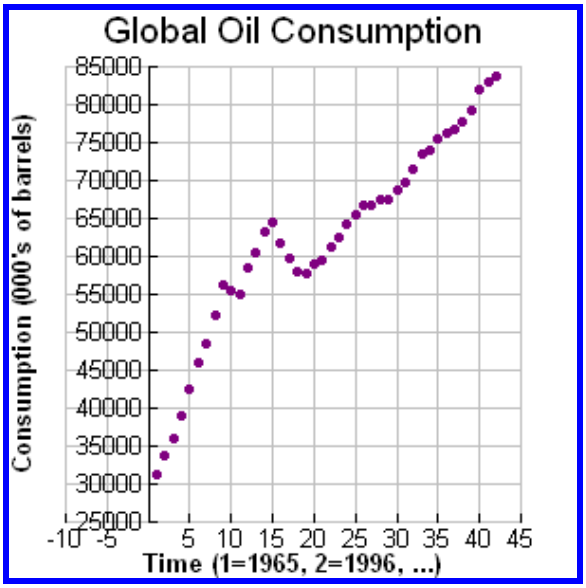
Home Activity or Further Classroom Consolidation

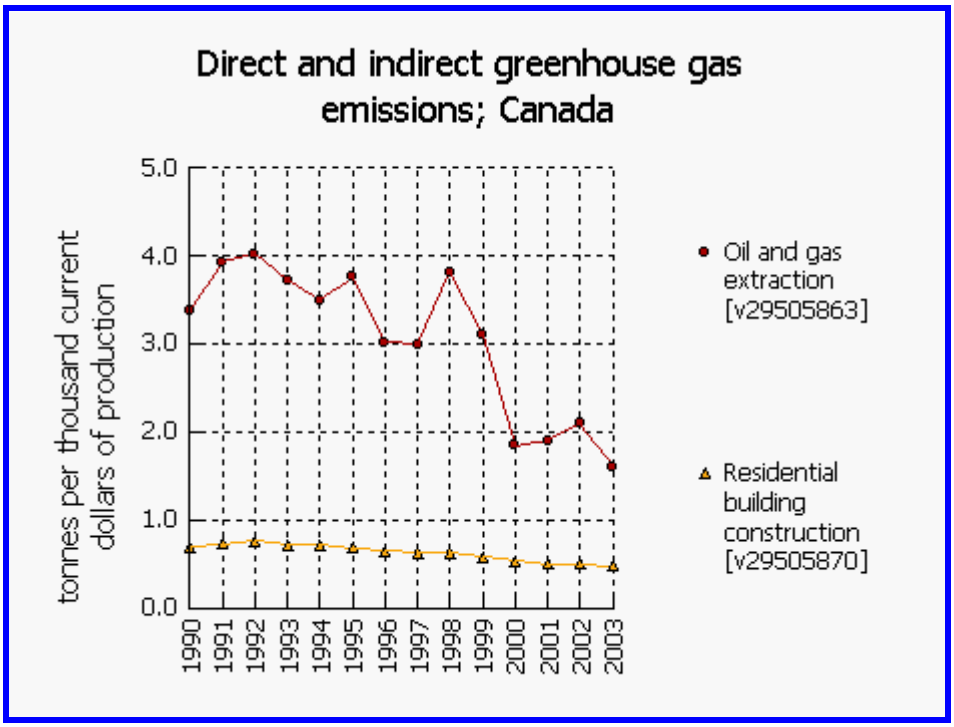
Complete question as preparation for the final exam.

BLM 7.1.4 and 7.1.6 provide a sample model and solution.

Provide review questions for the paper-and-pencil final exam.

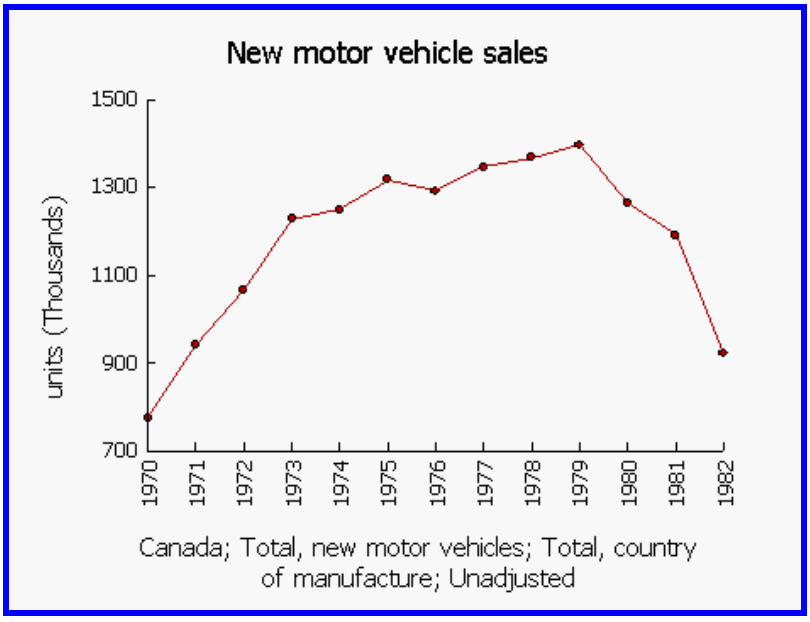
7.1.1: Our Changing Climate: Graphical Models



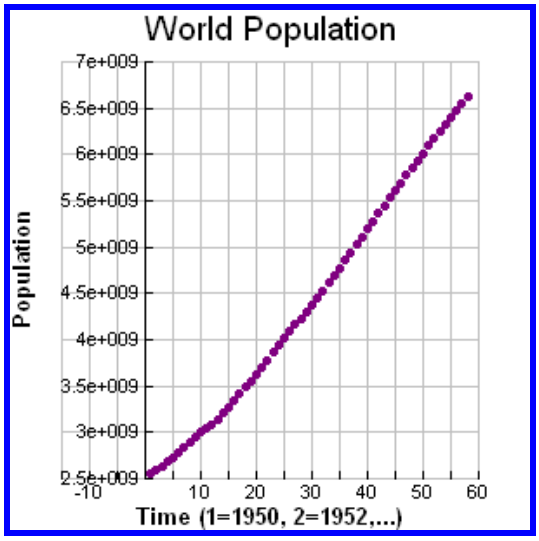


Source: Statistics Canada. Table 153-0033 - Direct and indirect greenhouse gas emissions (carbon dioxide equivalents), by industry, L-level aggregation, annual (tonnes per thousand current dollars of production) (graph), CANSIM (database), Using E-STAT (distributor).

7.1.1: Our Changing Climate: Graphical Models (continued)

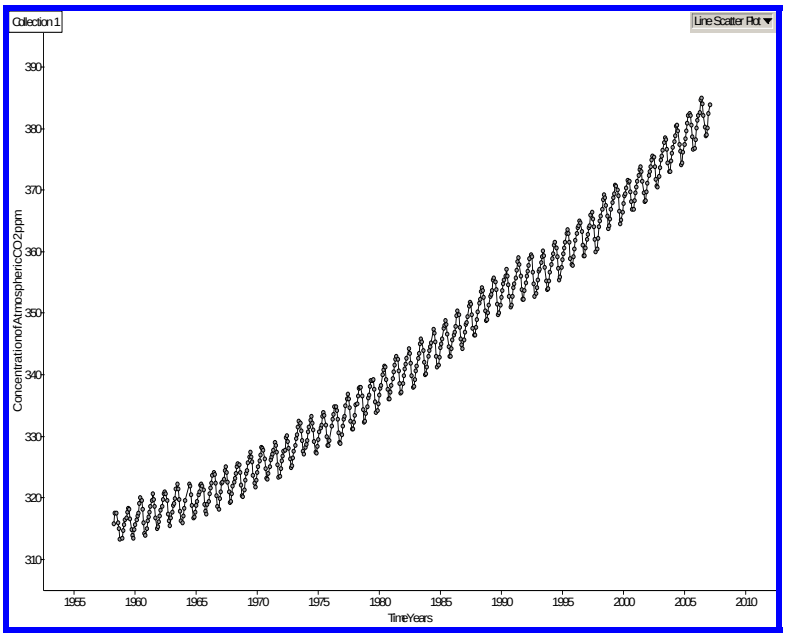


Source: Statistics Canada. Table 405-0002 - Road motor vehicles, fuel sales, annual (litres) (graph), CANSIM (database), Using E-STAT (distributor). http://estat.statcan.ca/cgi-win/cnsmcgi.exe?Lang=E&ESTATFile=EStat\English\CII_1_E.htm&RootDir=ESTAT/

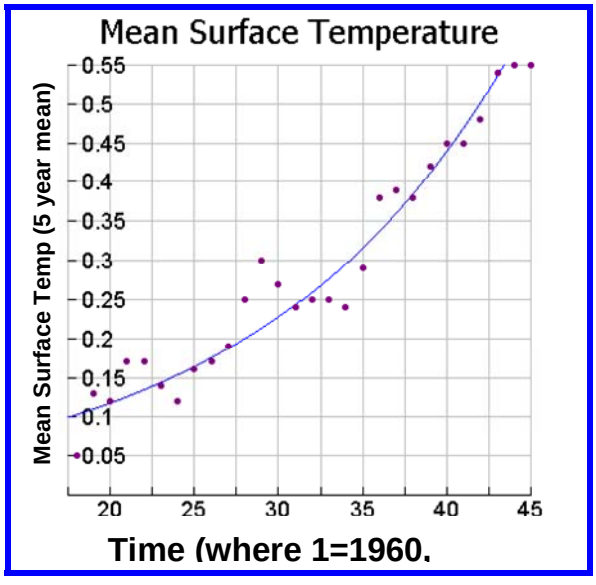


7.1.2: Setting the Stage

Carbon Dioxide (known as a greenhouse gas), contributes to the greenhouse effect. This gas absorbs heat released from the earth's surface and warms up the lower part of the atmosphere. Since James Watt invented the steam-powered engine in the mid-late 1700s, the levels of CO_2 have been increasing. At the same time, the earth's average/mean temperature has been on the rise.







There are numerous effects of global warming to the environment that include: Arctic ice cap retreat, raising sea levels and extreme weather events. All of these events have a great impact on human life.

How is atmospheric carbon dioxide affecting global sea levels? Can we predict the dangers of a rising sea level? Many experts agree that if the levels of CO_2 reach the critical value of 450 ppm, the damage to the environment will be irreversible. In what year will this occur? Can we do anything to stop global warming?

During the next two days you will be answering these key questions using mathematical concepts from this course.

7.1.3: Sea the World Change

Part A: Modelling the Earth's Global Sea Level

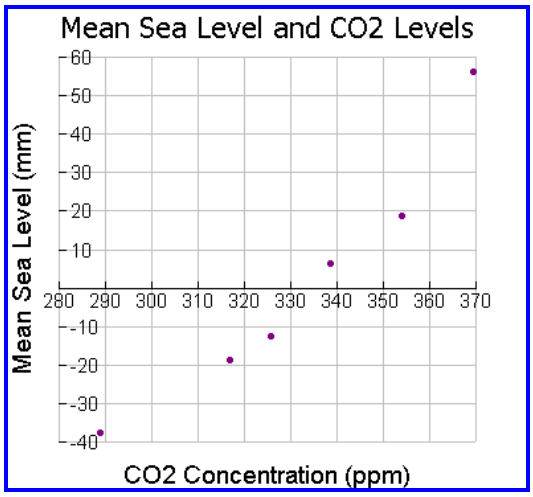
The global mean sea level has been increasing over the years. Is this due to global warming and greenhouse gases? What cities are in danger due to the rise in sea level?

The data below illustrates the relationship between the Global Mean Sea Level (mm) and the Global Atmospheric Carbon Dioxide Concentration (ppm).

Numerical Model

Atmospheric CO_2 Levels (ppm)	Global Mean Sea Level (mm)
288.98	–37.5
316.87	–18.75
325.65	–12.5
338.51	6.25
353.95	18.75
369.34	56.25

Graphical Model



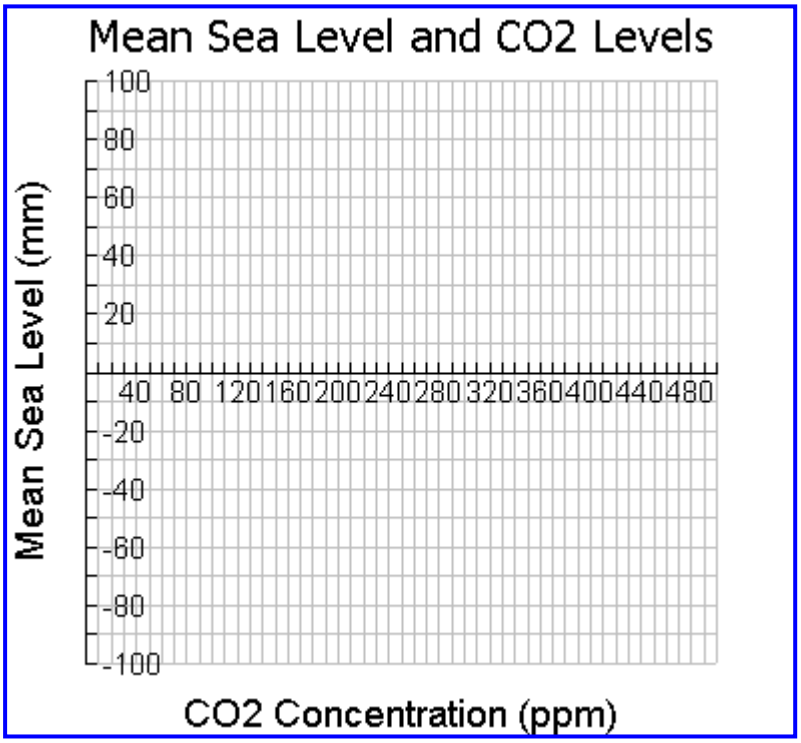
Task

1. Find three algebraic functions that can model the data. Each function should be from a different function family.
2. Extend the domain of the graph to start at zero.
3. Sketch the graph of each algebraic model using this domain, $0 \leq CO_2$ concentration ≤ 500.
4. Identify the key graphical characteristics such as range, end behaviours, zeros, intervals of increase/decrease, x- and y-intercepts.
5. Use the graph and the key characteristics of each function to describe how well the function represents the data and the context.
6. Identify which function best represents the data. Justify your choice based on the context and on the mathematics.
7. Use your identified function in Parts B-D.

7.1.3: Sea the World Change (continued)

Model 1: Quadratic Model

Graphical Model:
Plot the given points on the extended graph. Determine a quadratic model that fits the data. Sketch this model on the graph.



Algebraic Model:

Analysis:
Range:

Intervals of Increase/Decrease:

Zeros: *x*-/*y*-intercepts:

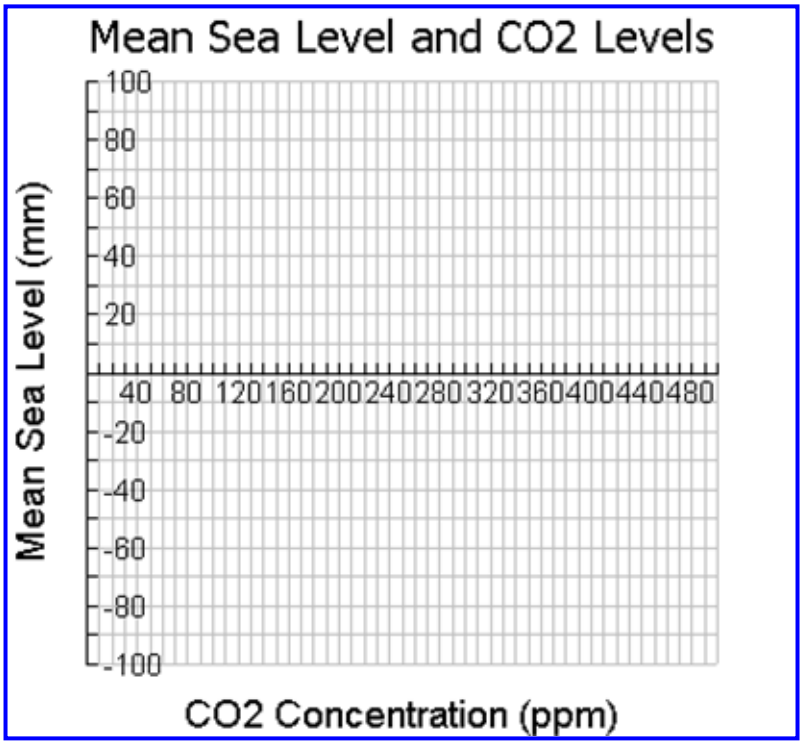
End Behaviours:

Appropriateness of model fit to the data and the context:

7.1.3: Sea the World Change (continued)

Model 2: Cubic Model

Graphical Model:
Plot the given points on the extended graph. Determine a quadratic model that fits the data. Sketch this model on the graph.



Algebraic Model:

Analysis:
Range:

Intervals of Increase/Decrease:

Zeros: *x*-/*y*-intercepts:

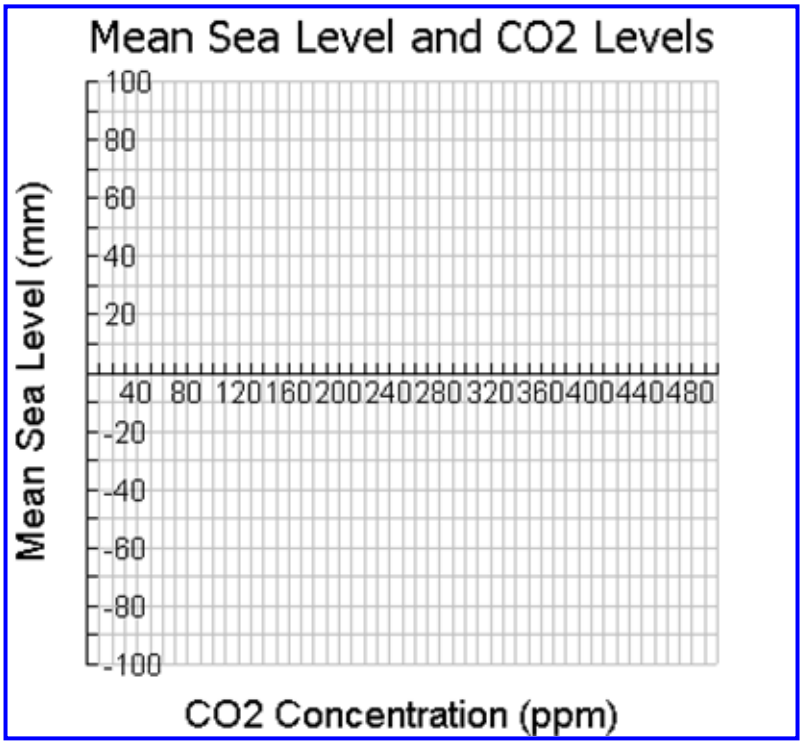
End Behaviours:

Appropriateness of model fit to the data and the context:

7.1.3: Sea the World Change (continued)

Model 3: Exponential Model

Graphical Model:
Plot the given points on the extended graph. Determine a quadratic model that fits the data. Sketch this model on the graph.



Algebraic Model:

Analysis:
Range:

Intervals of Increase/Decrease:

Zeros: *x*-/*y*-intercepts:

End Behaviours:

Appropriateness of model fit to the data and the context:

7.1.3: Sea the World Change (continued)

Part B: Sea Level and Rate of Change

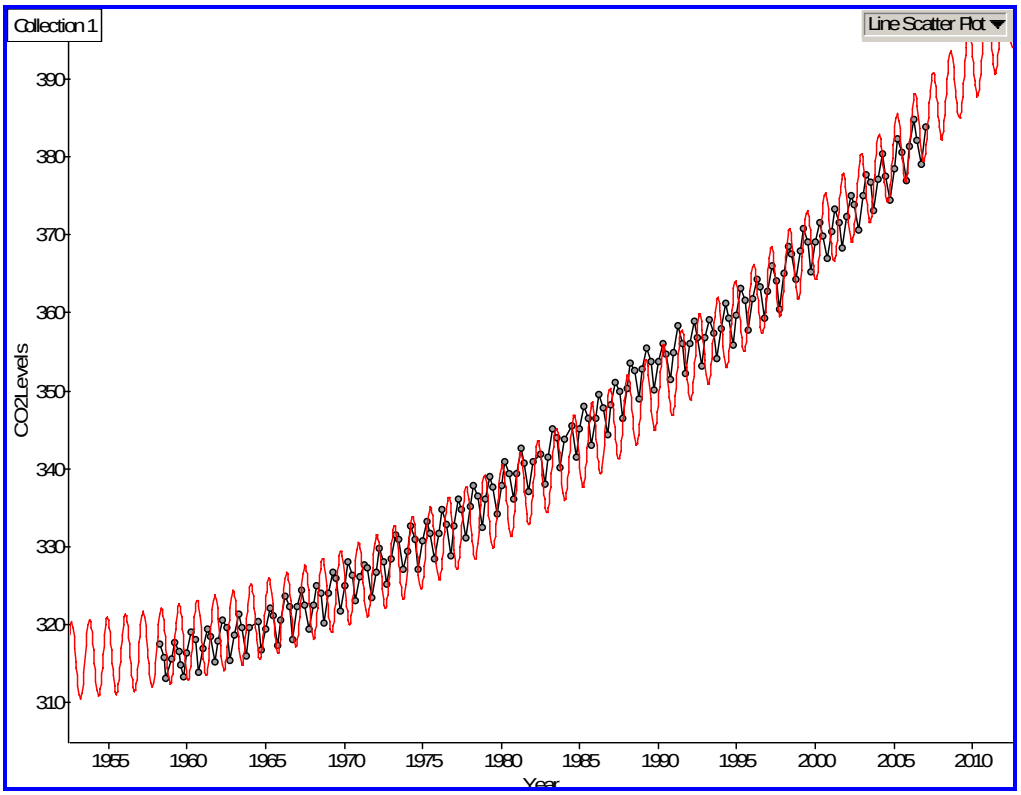
Using the most appropriate model you identified in **Part A**, what will the earth's sea level be if CO_2 concentration reaches the critical value of 450 ppm?

Today's CO_2 level is approximately 386 ppm. Determine the rate of change of the sea level based on today's CO_2 levels and the rate of change at the critical CO_2 level of 450 ppm. Compare the rates of change and describe the implications.

7.1.3: Sea the World Change (continued)

Part C: Time for Disaster

Below is a graph of CO_2 levels taken every 4 months since 1958.



The function $C(t) = 0.02(t-1948)^2 + 5\sin(5.5t) + 315$ models this data.

If we continue this pattern of CO_2 emissions, the earth will eventually reach the critical concentration of 450 ppm where the damage done globally becomes irreversible. Use the function above to determine the year the earth will reach the critical concentration.

7.1.3: Sea the World Change (continued)

Part D: A Global Problem

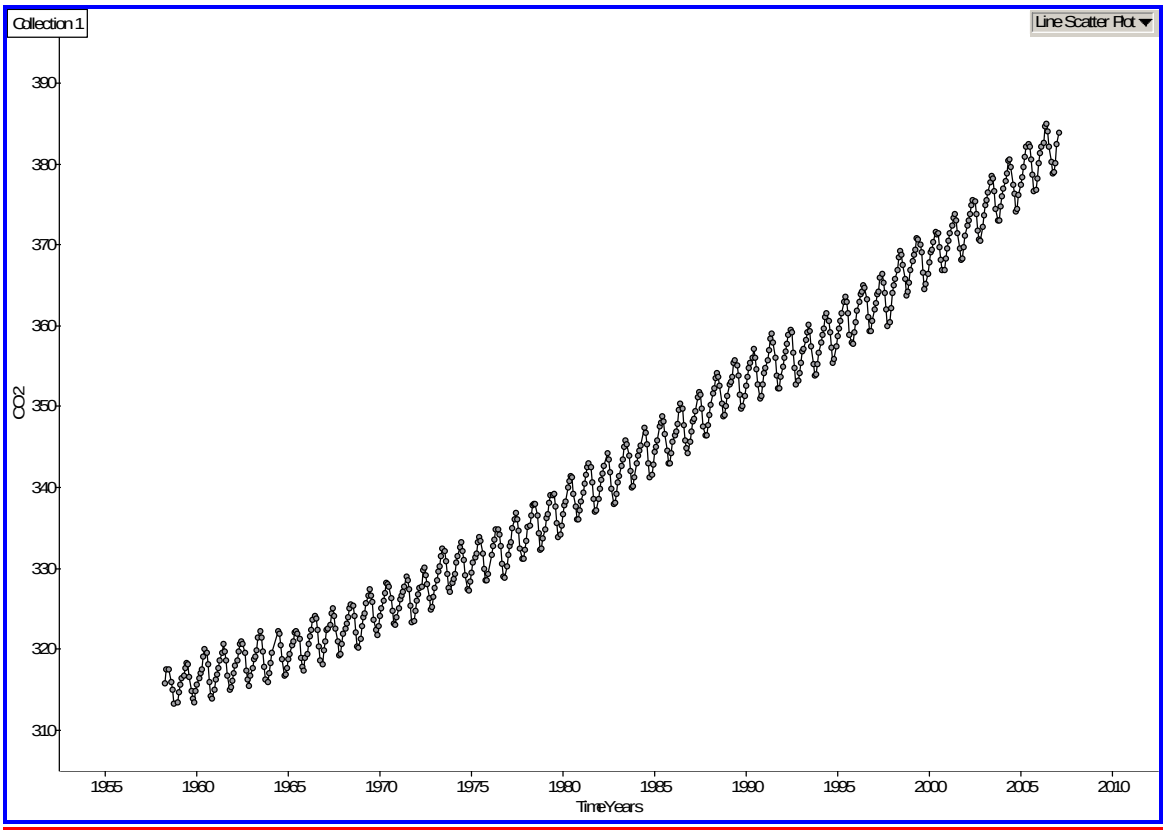
Since James Watt invented the steam-powered engine in the mid-late 1700s, the levels of CO_2 have been increasing. At the same time, the earth's average temperature has been on the rise. A result is the melting of the polar ice caps which translates to the rising sea levels. This would cause flooding of major world coastal regions putting millions of people in danger.

http://archives.cbc.ca/environment/climate_change/clip/14649/

Create a function that illustrates the Mean Sea Level as a function of time using your chosen model from **Part A**. Use this function to determine the year that these costal cities will be in danger of flooding.

City	Population	Elevation
Kozhikode,India	2.9 Million	1000 mm
Miami, USA	3.0 Million	2000 mm
Alexandria, Egypt	4.1 Million	5000 mm
Point Pelee, Ontario	30,000	177,000 mm

7.1.4: Changing Models



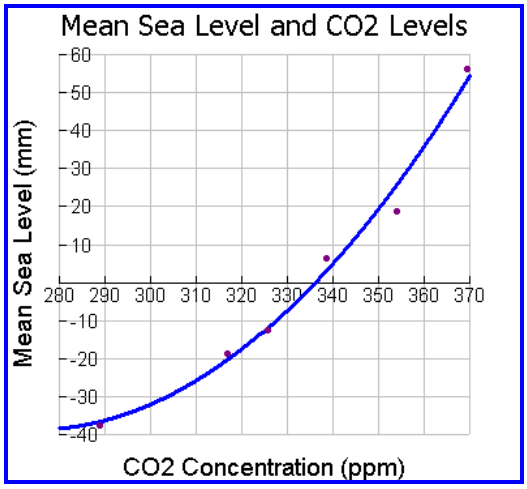
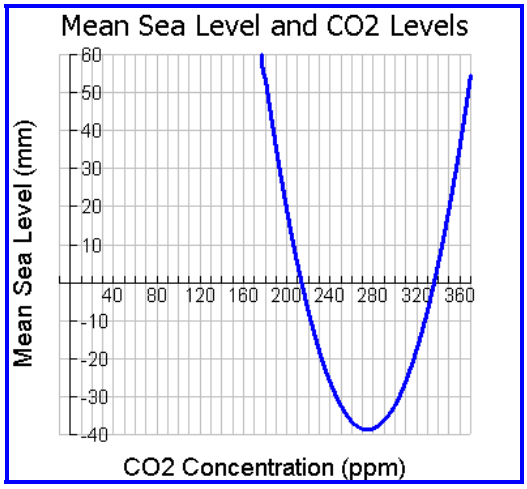
You used the function $C(t) = 0.02(t-1948)^2 + 5\sin(5.5t) + 315$ to model the data. This is the combination of two functions.

1. Identify the types of functions that were combined.

2. Could the graph be modelled by another combination of functions? If so, identify those functions and how this new combined function affects the results of this task.

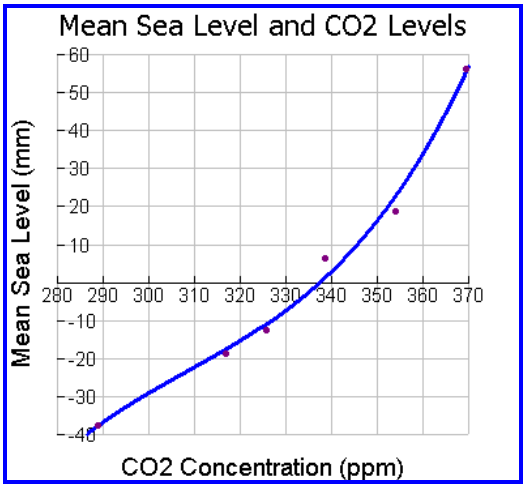
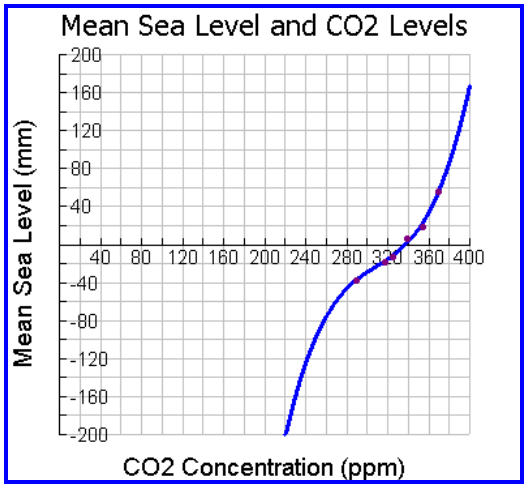
7.1.5: Sea the World Change Possible Solution (Teacher)

Part A: Modelling the Earth's Global Sea Level

Model	Student Solution
Model 1 – Quadratic Model  $M(c) = -0.007074c^2 + 0.977636c + 336.63$	Sketch Extending Domain  Range R : $\{y \mid y \geq -38.7278, y \in \Re\}$ Intervals and Increase/Decrease Decreases to x = 274.715, then increases End Behaviours As $x \to -\infty, f(x) \to \infty$ As $x \to \infty, f(x) \to \infty$ Justification of Model As $x \to -\infty, f(x) \to \infty$ implies that the sea level would be at a high level for a very low concentrations of CO_2. From the model, a minimum sea level is given at a CO_2 concentration of 274.715 ppm. However, it is possible to have lower levels of CO_2 which would result in the ice caps increases and much lower sea levels. Based on these characteristics this model does not suit the data well.

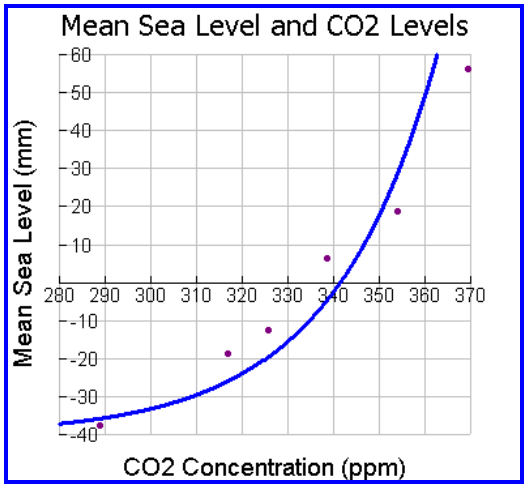
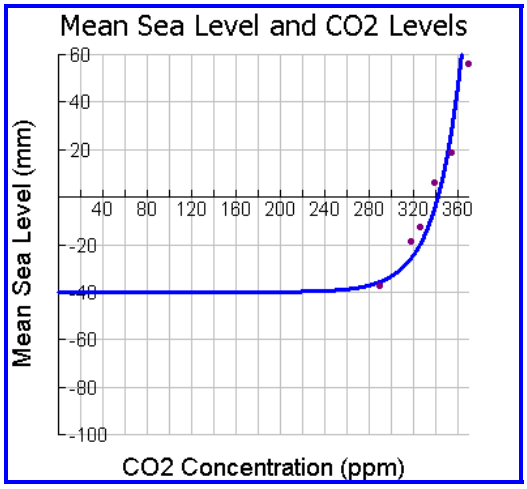
8 Marks

7.1.5: Sea the World Change Possible Solution (Teacher continued)

Model	Student Solution
Model 2 – Cubic Model  $f(x) = .000169x^3 - 0.15685x^2 + 49.08x - 5210.53$	Sketch Extending Domain  Range R : $\{y \mid y \in \Re\}$ Intervals and Increase/Decrease Always Increasing Zeros One zero at 337ppm End Behaviours As $x \to -\infty, f(x) \to -\infty$ As $x \to \infty, f(x) \to \infty$ The end behaviours are such that if, we have small amounts of CO_2 in the atmosphere, then the sea level would be at low levels. At a zero ppm of CO_2 the sea level would be –5205 mm or –5 m. This model fits the data better than the quadratic both algebraically and contextually. **This would be the model of choice!**

8 Marks

7.1.5: Sea the World Change Possible Solution (Teacher continued)

Model	Student Solution
Model 3 – Exponential Model	Sketch Extending Domain
	
$f(x) = 0.000016(1.0441)^x - 40$	Range R : $\{y \mid y > 40\}$ Intervals and Increase/Decrease Always Increasing Zeros One zero at 341.397 ppm End Behaviours As $x \to -\infty, f(x) \to 40$ As $x \to \infty, f(x) \to \infty$ Justification of Model This model is a better fit than the quadratic model but not as good as the cubic model. The end behaviours and range tell us that if we bring the CO_2 concentrations to low values, the mean sea level will never fall below 40 mm. If CO_2 is low, then the surface temperature is low resulting in a progressively larger ice cap. Hence, the sea level would not approach just one value.

8 Marks

7.1.5: Sea the World Change (Teacher Notes continued)

Part B: Sea Level and Rate of Change

Using the best model you chose from above, what will the earth's sea level be if it reaches the critical value of 450 ppm CO_2 levels?

Using the algebraic model that is better suited for this context, determine the Mean Sea Level at earth's threshold of 450 ppm CO_2.

$$f(C) = 0.000169C^3 - 0.156847C^2 + 49.0802C - 5210.53$$

The sea level at 450 will be:

$$f(450) = .000169(450)^3 - 0.156847(450)^2 + 49.0802(450) - 5210.53$$
$$= 549.603 \text{ is the best model.}$$

The mean sea level will be 549.6 mm or 0.5 m high when the earth reaches the critical CO_2 concentration of 450 ppm.

2 Marks

Today's CO_2 level is approximately 386 ppm. Compare the rate of change of the sea level based on today's CO_2 levels and compare it to the critical CO_2 level of 450 ppm.

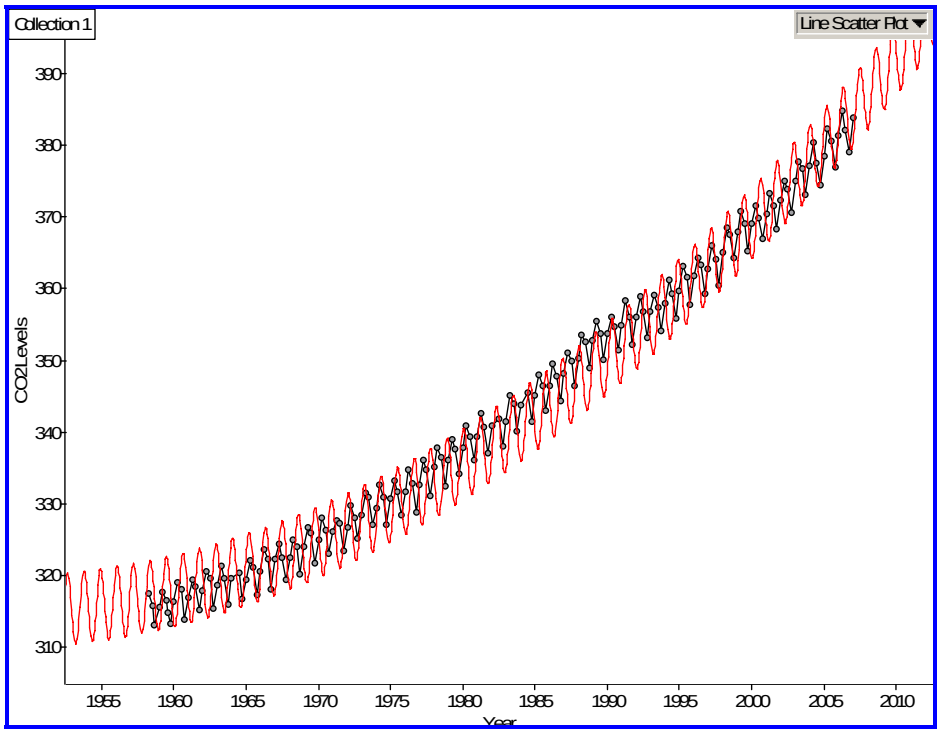
Present Rate of Change – Today	**Present Rate of Change at Critical CO_2 Level**
Students can use approximation of the instantaneous rate of change numerically and graphically 3.54 mm/ppm ***2 Marks***	Students can use approximation of the instantaneous rate of change numerically and graphically 10.59 mm/ppm ***2 Marks***

By approximating the rate of change, we can see that at today's CO_2 levels the sea are rising at 3.54 mm/ppm of CO_2. At the critical CO_2 concentration of 450 ppm, the sea would rise 10.59 mm/ppm of CO_2.

7.1.5: Sea the World Change (Teacher Notes continued)

Part C: Time for Disaster

Below is a graph of CO_2 levels taken every 4 months since 1958.



Using $C(T) = 0.02(t-1948)^2 + 5\sin(5.5t) + 315$,
we can determine when the critical concentration of 450 ppm by solving

$$450 = 0.02(t-1948)^2 + 5\sin(5.5t) + 315$$
$$135 = 0.02(t-1948)^2 + 5\sin(5.5t)$$

(Students must use technology since this equation cannot be solved using standard algebraic methods.)

t = 1865 or t = 2030

The critical CO_2 level value will occur in 2030.

We reject 1865 since this value has past. This also illustrates the flaw in this quadratic model. CO_2 levels have been gradually getting greater and greater over the years. These levels do not start at high levels then drop to a minimum as the model suggests. We may need another model to describe the data above.

7.1.5: Sea the World Change (Teacher Notes continued)

Part D: A Global Problem

Create a function that illustrates the Mean Sea Level as a function of time. Use this function to determine the year that these coastal cities will be in danger of flooding.

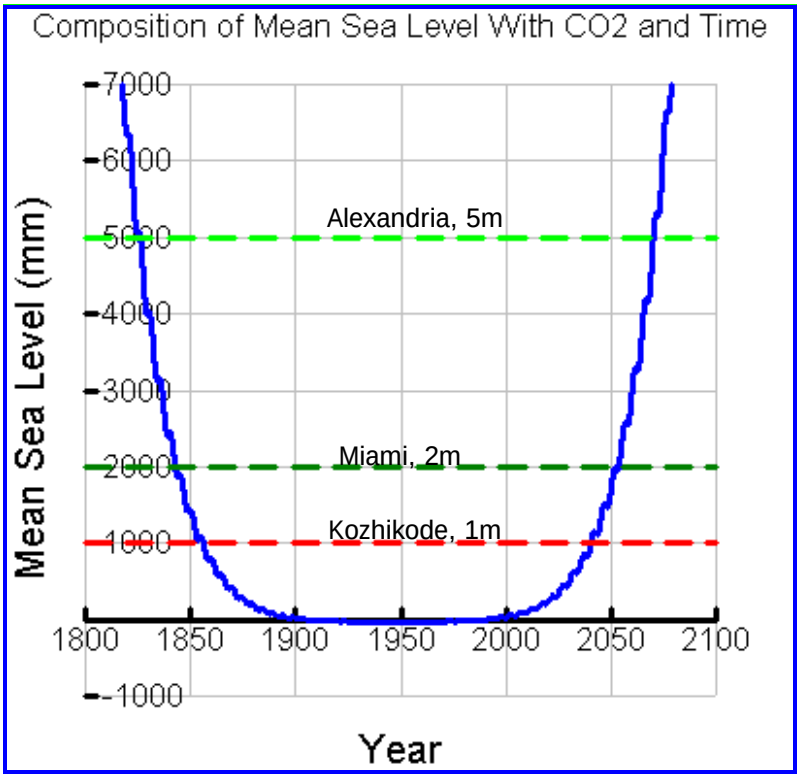
$$M\left(C\left(t\right)\right) = 0.000169\left[0.02\left(t-1948\right)^2 + 5\sin\left(5.5t\right) + 315\right]^3$$

$$-0.156847\left[0.02\left(t-1948\right)^2 + 5\sin\left(5.5t\right) + 315\right]^2$$

$$+49.0802\left[0.02\left(t-1948\right)^2 + 5\sin\left(5.5t\right) + 315\right] - 5210.53$$

2 Marks

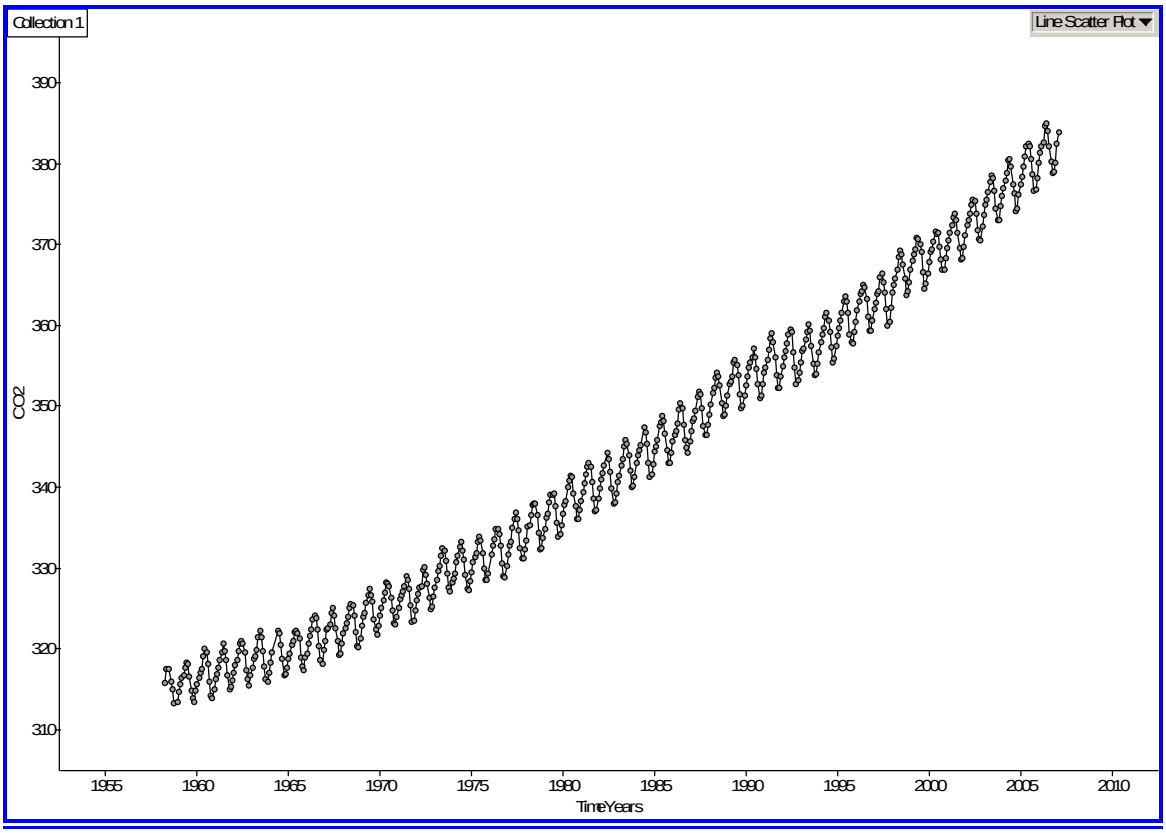
Students can use technology to create their graphs and solve using the intersect function of technology.



City	Population	Elevation	Year of Flooding
Kozhikode, India	2.9 million	1000 mm	2041
Miami, USA	3.0 million	2000 mm	2052
Alexandria, Egypt	4.1 million	5000 mm	2071
Point Pelee, Ontario	30,000	177,000 mm	2173

7.1.6: Changing Models (Teacher Notes and Possible Solution)

Students used a compound function for CO_2 levels with time that involved the combination of a trigonometric and quadratic function. Could the graph below be modelled by another function? What other compound functions could model this graph? How would this new function affect the results of this task?



The compound function of a sinusoidal and exponential could be a possible fit for this data. Since exponential model have a greater increasing rate of change than the quadratic, year the earth will reach 450 ppm CO_2 would happen earlier. The flooding of the cities and regions listed would also occur earlier.

7.1.7: More Data Sets

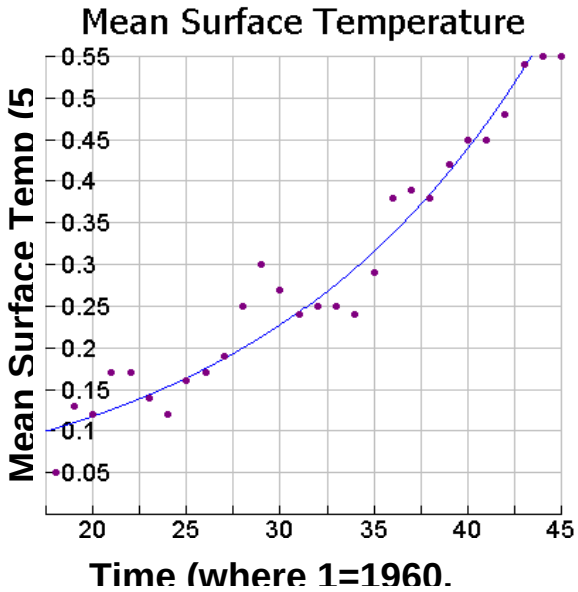
Mean Surface Temperature and Time

Numeric Model

Time (Year)	Time (where 1 = 1960, 2 = 1961, …)	Mean Surface Temperature (5 = year rolling mean)
1978	18	0.05
1979	19	0.13
1980	20	0.12
1981	21	0.17
1982	22	0.17
1983	23	0.14
1984	24	0.12
1985	25	0.16
1986	26	0.17
1987	27	0.19
1988	28	0.25
1989	29	0.3
1990	30	0.27
1991	31	0.24

Time (Year)	Time (where 1 = 1960, 2 = 1961, …)	Mean Surface Temperature (5 = year rolling mean)
1992	32	0.25
1993	33	0.25
1994	34	0.24
1995	35	0.29
1996	36	0.38
1997	37	0.39
1998	38	0.38
1999	39	0.42
2000	40	0.45
2001	41	0.45
2002	42	0.48
2003	43	0.54
2004	44	0.55
2005	45	0.55

Graphical Model



http://data.giss.nasa.gov/gistemp/graphs/

Algebraic Model

$h(t) = 0.031521(1.06809)^t$

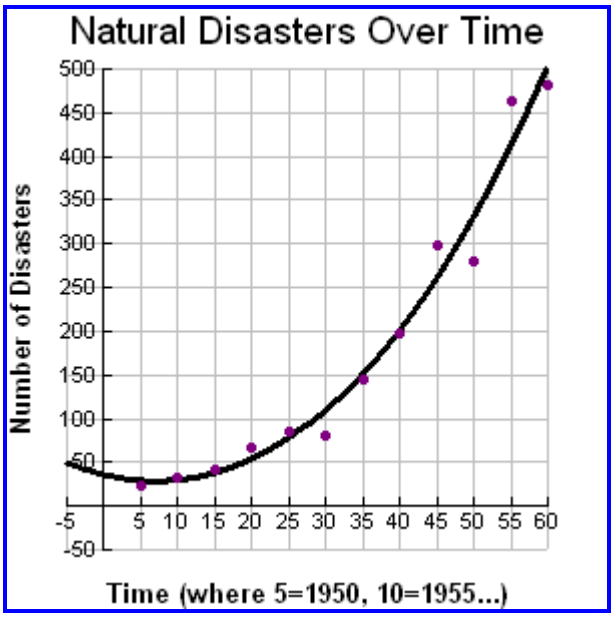
7.1.7: More Data Sets (continued)

Natural Disasters

Numeric Model

Year	Year (where 5 = 1950, 10 = 1955 …)	Disasters
1950	5	23
1955	10	32
1960	15	42
1965	20	66
1970	25	86
1975	30	81
1980	35	145
1985	40	197
1990	45	299
1995	50	279
2000	55	464
2005	60	481

Graphical Model



http://www.swivel.com/graphs/show/26885153

Algebraic Models

Exponential Model: $k(t) = 18.61(1.0556)^t$

Quadratic Model: $k(t) = 0.181t^2 - 3.271t + 44.114$

Cubic Model: $k(t) = 0.000476t^3 + 0.135t^2 - 2.017t + 36$ (graphed on scatter plot)

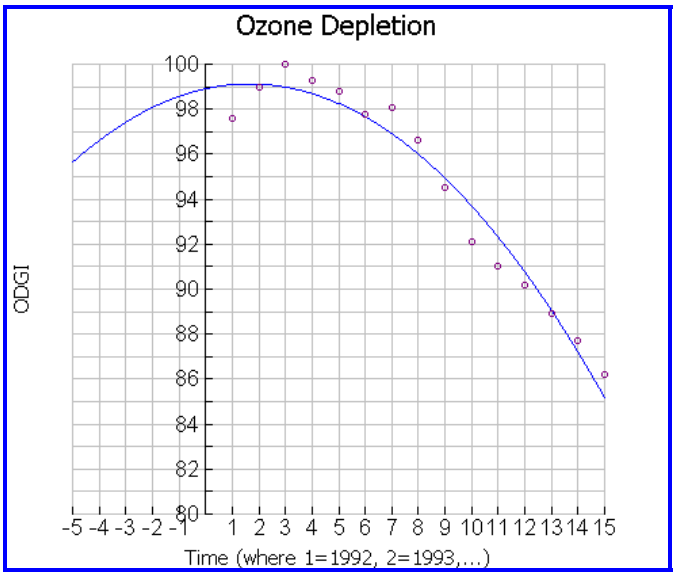
7.1.7: More Data Sets (continued)

Ozone Depletion

Numeric Model

Year	Year (where 1 = 1992, 2 = 1993 …)	Ozone Depletion Gases Index (ODGI)
1992	1	97.6
1993	2	99
1994	3	100
1995	4	99.3
1996	5	98.8
1997	6	97.8
1998	7	98.1
1999	8	96.6
2000	9	94.5
2001	10	92.1
2002	11	91
2003	12	90.2
2004	13	88.9
2005	14	87.7
2006	15	86.2

Graphical Model



http://www.esrl.noaa.gov/gmd/odgi/

Algebraic Model

$h(t) = -0.078369t^2 + 0.261411t + 98.9073$

7.1.7: More Data Sets (continued)

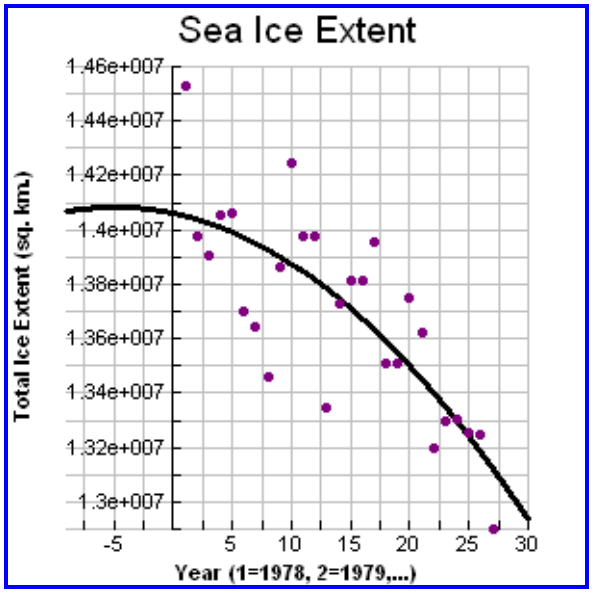
Sea Ice Extent

Numeric Model

Year	Year (where 1 = 1978, 2 = 1979, …)	Total Ice Extent (sq. km.)
1978	1	14529099
1979	2	13973253
1980	3	13906928
1981	4	14057762
1982	5	14061995
1983	6	13700599
1984	7	13640930
1985	8	13458771
1986	9	13860754
1987	10	14244787
1988	11	13975775
1989	12	13974090
1990	13	13346076

Year	Year (where 1 = 1978, 2 = 1979, …)	Total Ice Extent (sq. km.)
1991	14	13732243
1992	15	13815876
1993	16	13816173
1994	17	13953392
1995	18	13510143
1996	19	13508105
1997	20	13748543
1998	21	13624612
1999	22	13200600
2000	23	13296108
2001	24	13306383
2002	25	13256056
2003	26	13250588

Graphical Model



http://nsidc.org/data/smmr_ssmi_ancillary/area_extent.html

Algebraic Model

$k(t) = -947.382t^2 - 9103.32t + 14061500$

7.1.7: More Data Sets (continued)

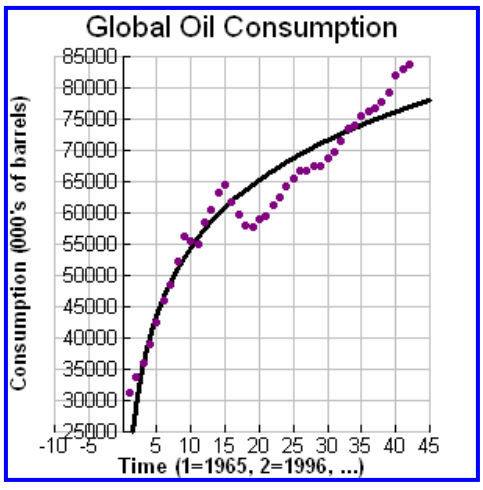
Oil Consumption

Numeric Model

Year	Year (where 1 = 1965, 2 = 1966 …)	Oil Consumption ('000s of barrels)
1965	1	31240
1966	2	33637
1967	3	36070
1968	4	39027
1969	5	42487
1970	6	46064
1971	7	48596
1972	8	52143
1973	9	56326
1974	10	55491
1975	11	54996
1976	12	58425
1977	13	60607
1978	14	63220
1979	15	64382
1980	16	61729
1981	17	59805
1982	18	58122
1983	19	57874
1984	20	59107
1985	21	59385

Year	Year (where 1 = 1965, 2 = 1966 …)	Oil Consumption ('000s of barrels)
1986	22	61167
1987	23	62454
1988	24	64277
1989	25	65615
1990	26	66828
1991	27	66809
1992	28	67521
1993	29	67376
1994	30	68666
1995	31	69833
1996	32	71493
1997	33	73595
1998	34	73929
1999	35	75547
2000	36	76281
2001	37	76829
2002	38	77739
2003	39	79158
2004	40	81900
2005	41	83077
2006	42	83719

Graphical Model



http://services.alphaworks.ibm.com/manyeyes/data/SS6NuKsOtha6c4E2rvWvK2

Algebraic Model

$g(t) = 36237.151\log(t) + 18073.806$

7.1.7: More Data Sets (continued)

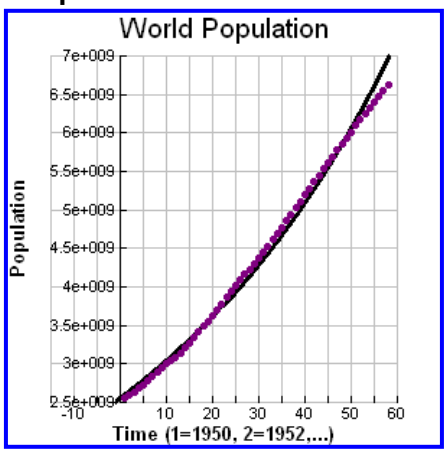
World Population

Numeric Model

Year	Year (where 1 = 1950, 2 = 1951, …)	Population
1950	1	2555898461
1951	2	2593043325
1952	3	2635100581
1953	4	2680437616
1954	5	2728297382
1955	6	2779658083
1956	7	2832536024
1957	8	2888278511
1958	9	2944698513
1959	10	2996946986
1960	11	3038930391
1961	12	3079552761
1962	13	3135560616
1963	14	3204953986
1964	15	3275941217
1965	16	3344855925
1966	17	3414981586
1967	18	3484617262
1968	19	3556354911
1969	20	3630875051
1970	21	3705987692
1971	22	3783442817
1972	23	3860605413
1973	24	3937265173
1974	25	4013371391
1975	26	4087382478
1976	27	4159863245
1977	28	4232579098
1978	29	4305305520

Year	Year (where 1 = 1950, 2 = 1951, …)	Population
1979	30	4380967472
1980	31	4457593483
1981	32	4534279227
1982	33	4614560605
1983	34	4694861502
1984	35	4774079669
1985	36	4854659097
1986	37	4937099145
1987	38	5022946463
1988	39	5109495298
1989	40	5194326719
1990	41	5281672973
1991	42	5365725046
1992	43	5448141769
1993	44	5529370029
1994	45	5609678819
1995	46	5691012889
1996	47	5771938438
1997	48	5852210712
1998	49	5932058339
1999	50	6011675931
2000	51	6090912914
2001	52	6169683257
2002	53	6248055839
2003	54	6325991838
2004	55	6403480654
2005	56	6480500311
2006	57	6557220076
2007	58	6633833596

Graphical Model



http://services.alphaworks.ibm.com/manyeyes/data/ScwMDMsOtha6Xjm5zgoRM2~

Algebraic Model

$$h(t) = 2547536009.48(1.017)^{t}$$

7.1.8: Summative Performance Task Rubric

Thinking					
Reasoning and Proving					
Criteria	**Below Level 1 Specific Feedback**	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Justifies an algebraic model chosen to represent the data appropriate to the context *(7.1.3 Part A #5, D #2)*		- justifies a model connecting few pertinent aspects of the problem	- justifies a model connecting some of the pertinent aspects of the problem	- justifies a model connecting the pertinent aspects of the problem	- justifies a model connecting the pertinent aspects with a broader view of the problem
Makes inferences, draws conclusions and gives justifications *(7.1.3 Part A#6, C 7.1.4, 7.2.2 #5b)*		- makes limited connections to the model and extrapolations when justifying answers	- makes some connections to the model and extrapolations when justifying answers	- makes direct connections to the model and extrapolations when justifying answers	- makes direct and insightful connections to the model and extrapolations when justifying answers
Communication					
Representing					
Identifies relevant key graphical features of the functions/scenarios *(7.1.3 #1)*		- identifies key features of the graphs/scenarios with major errors, omissions, or mis-sequencing	- identifies key features of the graphs/scenarios with minor errors, omissions, or mis-sequencing	- identifies key features of the graphs/scenarios with few or no errors, omissions, or mis-sequencing	- identifies key features of the graphs/scenarios accurately, clearly, succinctly and efficiently
Creates a graphical model to represent the problem. *(7.2.2 #3)*		- creates a graphical model to represent the problem with limited effectiveness; representing little of the range of the data	- creates a graphical model to represent the problem with some effectiveness; representing some of the range of the data	- creates an appropriate a graphical model to represent the problem with considerable effectiveness; representing most of the range of the data	- creates an appropriate and succinct graphical model to represent the problem with a high degree of effectiveness; representing the full range of the data
Communicating					
Uses mathematical symbols, labels, units and conventions correctly *(All Parts)*		- sometimes uses mathematical symbols, labels, and conventions correctly	- usually uses mathematical symbols, labels, and conventions correctly	- consistently uses mathematical symbols, labels, and conventions correctly	- consistently uses mathematical symbols, labels, and conventions, presenting novel or insightful opportunities for their use
Uses mathematical vocabulary appropriately *(All Parts)*		- uses common language in place of mathematical vocabulary or uses key mathematical terms with major errors	- uses mathematical vocabulary with minimal errors or uses some common language in place of vocabulary	- uses mathematical vocabulary appropriately	- consistently uses mathematical vocabulary appropriately, presenting novel or insightful opportunities for its use

The following questions are marked analytically in the category of Application for a total of 54 marks: BLM 7.1.3: A #4, B, C, D #1; BLM 7.2.2 #1, 2, 4, 5a, 6.



75 min

Math Learning Goals

- Recognize real world applications of combinations of functions.
- Determine the key features of the graphs of a function created by a combination of two functions.
- Determine the approximate instantaneous rate of change from a real world application.
- Solve a problem whose solution is not accessible by standard algebraic techniques.

Materials

- BLM 7.2.1–7.2.3
- graphing technology

Assessment Opportunities

Minds On…

Pairs → Activity

Students read each statement in the Anticipation Guide and select Agree or Disagree based on previous knowledge about energy sources (BLM 7.2.1). They briefly discuss their choices and reasoning with a partner.

Students will have an opportunity to revisit their choices in light of the work they complete during this day.

Whole Class → Activity Instructions

Clarify the expectations for Day 2 of the performance task (BLM 7.2.2). Explain that students will be investigating the impact of renewable energy on the level of CO_2 emissions in the atmosphere. The overall focus of this task is to investigate whether the threshold date introduced on Day 1 will be affected by renewable energy.

Students may need to be reminded of the CO_2 threshold of 450 ppm introduced on Day 1.

Action!

Individual → Summative Task

The first two pages of the task are the models related to wind energy, solar energy, and mean CO_2 emissions from the period 1997–2007.

Questions 1 and 2 require students to determine a function that is the sum (#1) and quotient (#2) of two functions from the three they were given.

The combinations they create in Question 2 forms the basis of Questions 3–5. Graphing technology is needed to graph the function in Question 2.

Students describe key graphical properties and determine a rate of change for this function.

Question 6 addresses the overall focus of this day. The answer to this question should show students that renewable energy resources are delaying the date by which the atmospheric concentration of CO_2 will reach 450 ppm.

Note: Question 2, requires students to find a combination of functions that represents a rate not a rate of change.

Consolidate Debrief

Individual → Reflection

Students revisit the Anticipation Guide (BLM 7.2.1) to complete the After column reflecting on what they have investigated, providing reasons for any responses that changed or stayed the same with those given at the start of class.

Home Activity or Further Classroom Consolidation

Complete questions to prepare for the exam.

Concept Practice
Skill Practice

Provide review questions to prepare students for the paper-and-pencil exam.

7.2.1: Anticipation Guide

Instructions:

- Check Agree or Disagree beside each statement below **before** you start the task.
- Compare your choice and explanation with a partner.
- Revisit your choices at the end of the task. Compare the choices that you would make **after** the task with the choices that you made before the task.

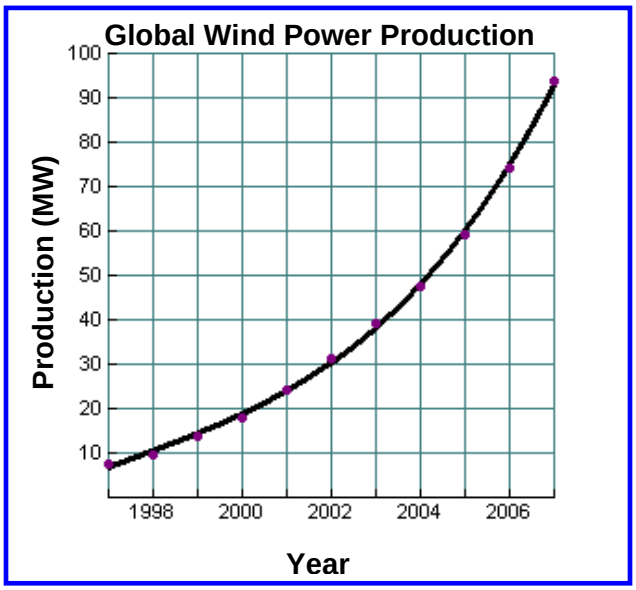
Before		Statement	After		
Agree	Disagree		Always	Sometimes	Never
		More energy is produced from solar energy than from wind energy.			
		The amount of power being produced from wind and solar energy is increasing exponentially over time			
		The amount of energy produced from renewable energy resources will have little or no impact on the date CO_2 emissions will reach 450 ppm.			

7.2.2: The Power of Nature

Over the past ten years the use of renewable sources of energy has become more popular. Both solar and wind energy are becoming more globally accepted as new sources of energy. Below is data on annual world wind energy production and world photovoltaic production (solar energy) from 1997 and data on atmospheric CO_2 emissions from 1997.

Wind Power

Year	Production (MW)
1997	7.475
1998	9.663
1999	13.696
2000	18.039
2001	24.32
2002	31.164
2003	39.29
2004	47.693
2005	59.033
2006	74.153
2007	93.849

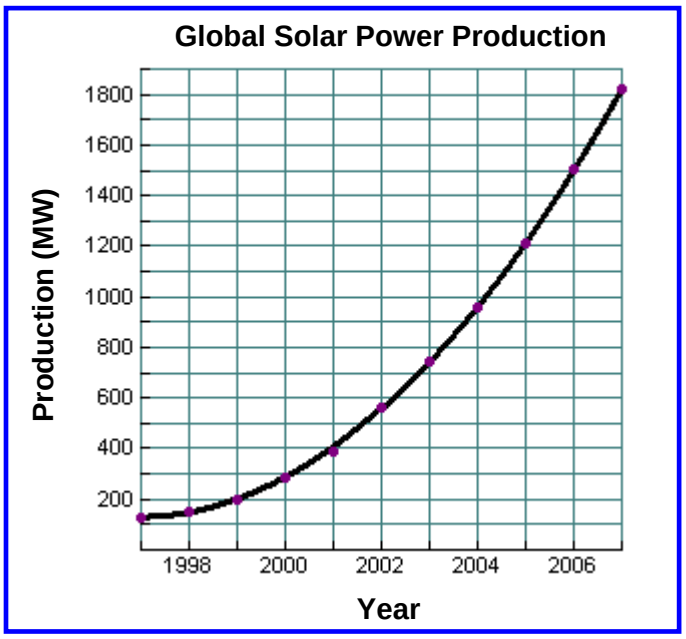


http://www.wwindea.org/home/index.php

$$W(t) = 0.061236t^3 - 367.002t^2 + 733178.3t - 488238151$$

Solar Power

Year	Production (MW)
1997	126
1998	153
1999	201
2000	288
2001	391
2002	560
2003	742
2004	961
2005	1211
2006	1504
2007	1818



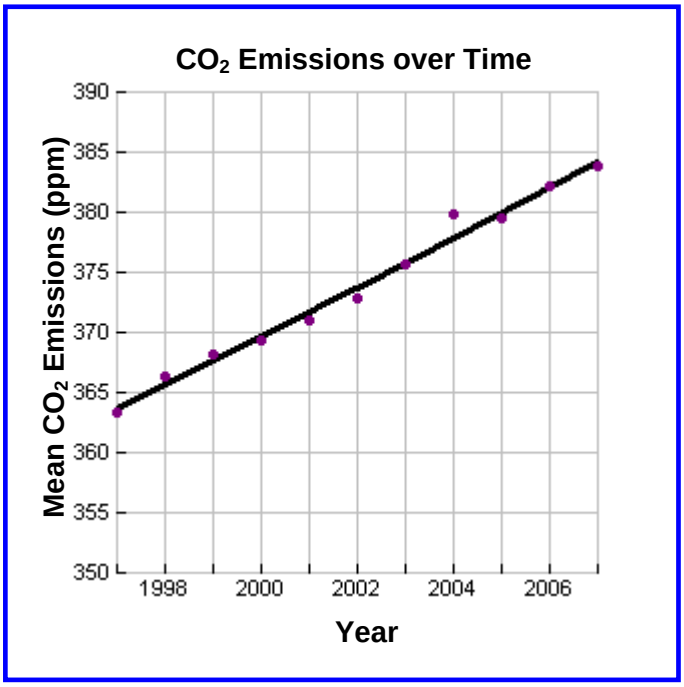
http://www.earth-policy.org/Indicators/2004/indicator12_data.htm

$$S(t) = 16.7821t^2 - 67026.3t + 66924727$$

7.2.2: The Power of Nature (continued)

CO_2 Emissions

Year	Mean CO_2 Emissions (ppm)
1997	363.35
1998	366.29
1999	368.24
2000	369.34
2001	370.92
2002	372.91
2003	375.61
2004	379.88
2005	379.51
2006	382.21
2007	383.84



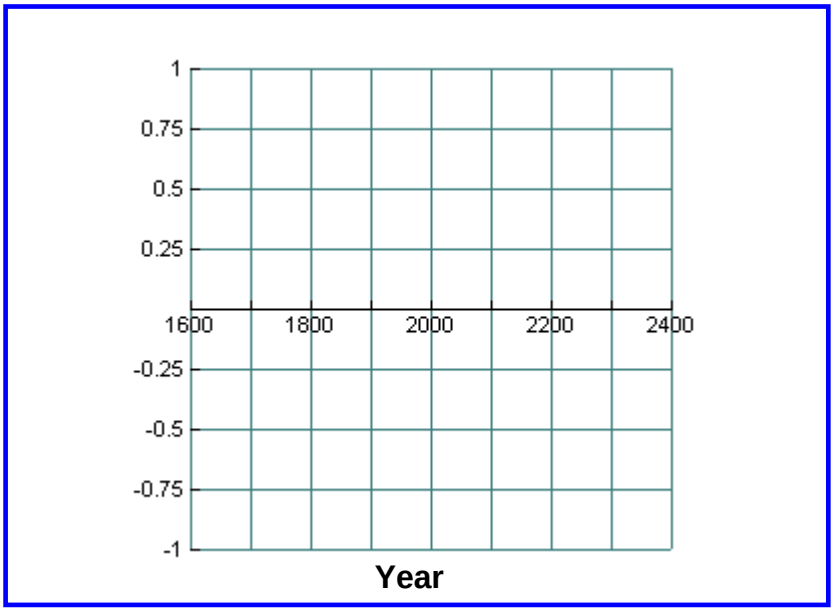
$$CO_2(t) = 0.0091465t^2 - 34.5707t + 32923$$

7.2.2: The Power of Nature (continued)

1. You determined the date 2030 is when the concentration of carbon dioxide will reach 450 ppm. How much **total energy** from renewable resources (solar and wind energy) will be produced at that time?

2. Using the three given functions for Wind Power, Solar Power and CO_2 Emissions, determine a combination of those functions that will model the rate at which CO_2 emissions are changing with respect to total energy produced from wind and solar power. (Do not simplify the expression.)

3. Use graphing technology to sketch the function from Question 2 on the grid below.



7.2.2: The Power of Nature (continued)

4. Using the graph and graphing technology to assist you, determine the **key features** of this function including:

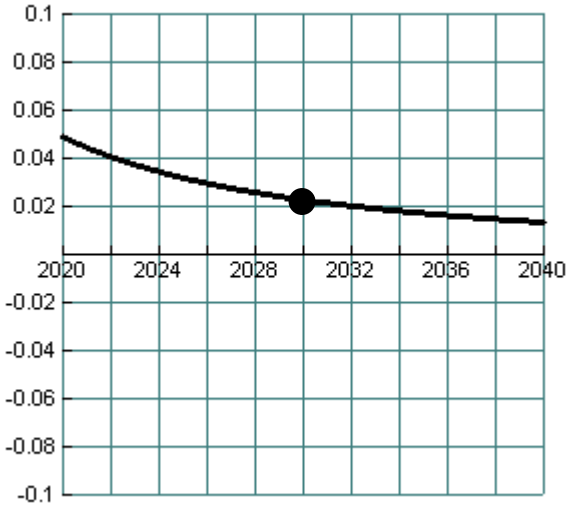
   End Behaviour:

   Domain and Range:

   Extremes:

   Intervals of Increase and Decrease:

5. a) Find the **rate of change** of the function you found in Question 2 numerically and graphically at the date by which the concentration of carbon dioxide will reach 450 ppm. A graph of the region around t = 2030 has been provided below.

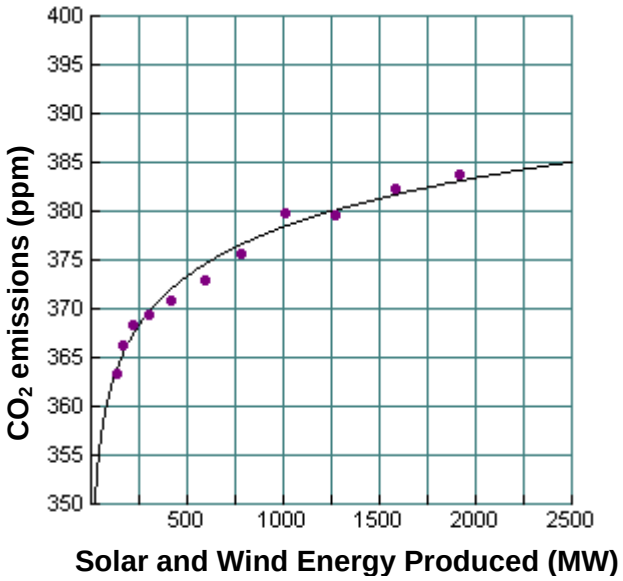


   b) Explain the meaning of the **rate of change** within the context of the problem.

7.2.2: The Power of Nature (continued)

6. The graphical, numeric, and algebraic models of carbon dioxide emissions vs. energy produced from wind and solar resources are provided below.

Solar and Wind Energy Produced (MW)	CO_2 Emissions (ppm)
133.475	363.3475
162.663	366.295
214.696	368.24
306.039	369.34
415.32	370.92
591.164	372.9125
781.29	375.6125
1008.693	379.8808
1270.033	379.512
1578.153	382.21
1911.849	383.84



$C(r) = 29.9329 \log r + 284.5044$ where C is the CO_2 emissions and r is the amount of solar and wind energy produced.

If we continue to produce wind and solar energy according to the models given, when will the Earth reach its sustainable CO_2 threshold?

7.2.3: The Power of Nature (Teacher Notes)

1. You determined the date 2030 is when the concentration of carbon dioxide will reach 450 ppm. How much **total energy** from renewable resources (solar and wind energy) will be produced at that time?

   Find the sum of the two functions representing solar and wind energy and define it as $T(t)$.

   Solar Energy: $S(t) = 16.7821t^2 - 67026.3t + 66924727$
   Wind Energy: $W(t) = 0.061236t^3 - 367.002t^2 + 733178.3t - 488238151$

   Total Energy:
   $T(t) = 16.7821t^2 - 67026.3t + 66924727 + 0.061236t^3 - 367.002t^2 + 733178.3t - 488238151$

   2 Marks

   $$\boldsymbol{T(t) = 0.061236t^3 - 350.2199t^2 + 666152t - 421313424}$$

   Evaluate: $T(t)$ at $t = 2030$:
   $T(2030) = .061236(2030)^3 - 350.2199(2030)^2 + 666152(2030) - 421313424 = 19237.7\ MW$

   There will be 19237.9 MW produced from renewable resources by 2030.

   1 Mark

2. Using the three given functions for Wind Power, Solar Power and CO_2 Emissions, determine a combination of those functions that will model the rate at which CO_2 emissions are changing with respect to total energy produced from wind and solar power. (Do not simplify the expression.)

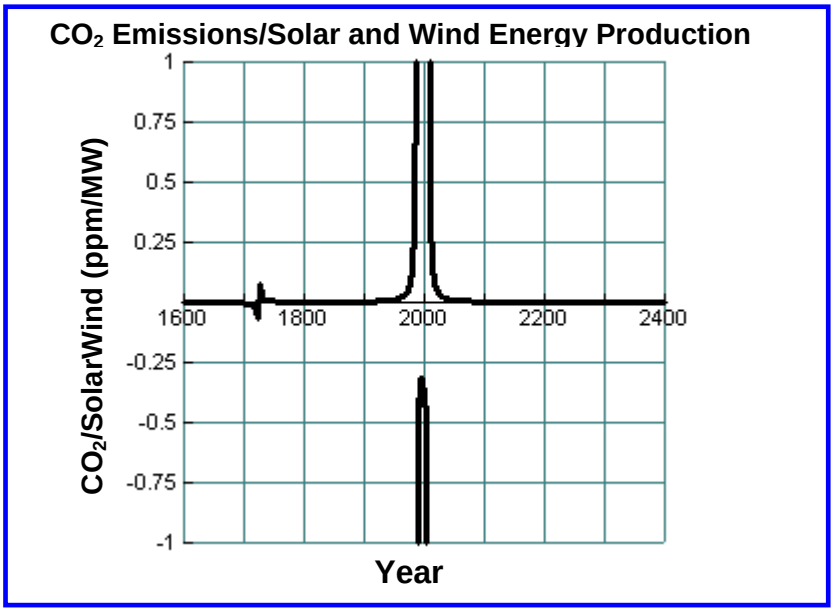
   Find the quotient function of the carbon emissions function ($CO_2(t)$) and the total energy produced from renewable energy resources ($T(t)$) and define it as $R(t)$.

   $$R(t) = \frac{CO_2(t)}{T(t)}$$
   $$= \frac{0.0091465t^2 - 34.5707t + 32923}{0.061236t^3 - 350.2199t^2 + 666152t - 421313424}$$

   1 Mark

7.2.3: The Power of Nature (Teacher Notes continued)

3. Using graphing technology, sketch the function from Question 2, on the grid below.



4. Determine the **key features** of this function including:

End Behaviour:

If we evaluate $R(t)$ at a very small value of t, the value of $R(t)$ is approaching zero.
Or as $t \to -\infty,\ R(t) \to 0$

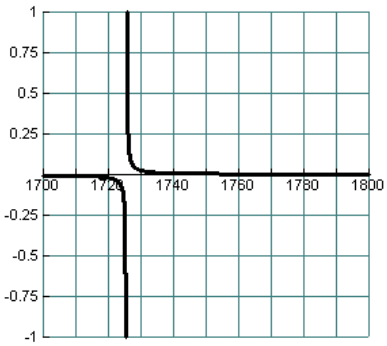
If we evaluate $R(t)$ at a very large value of t, the value of $R(t)$ is approaching zero.
Or as $t \to \infty,\ R(t) \to 0$.

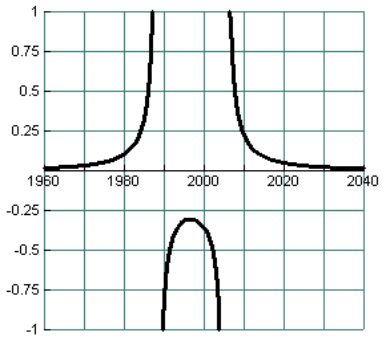
7 Marks

7.2.3: The Power of Nature (Teacher Notes continued)

Domain and Range

From the graph, we can see that there may be asymptotes around 1700 and 2000. If we zoom in on those areas we find the existence of vertical asymptotes at roughly 1725, 1987 and 2005 as seen on the screen shots below.





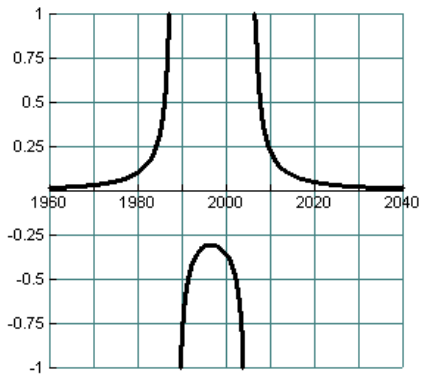
Therefore the domain is $\{t | t \in \Re,\ t \neq 1725, 1987, 2005\}$

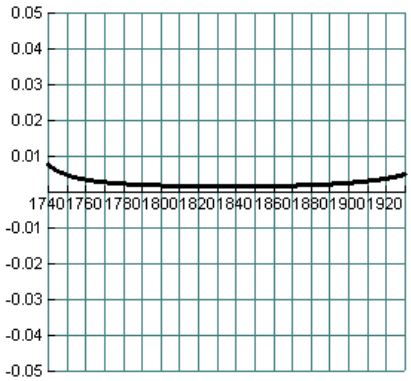
(**Note:** An argument can be made for not restricting the domain to negative values since we want to consider dates BC but we can also restrict it to the values on which the model is based.)

For the range, although in our graph it looks like the function stops decreasing around 1725, when we zoomed in above, we see that it does continue decreasing. If we investigate where there might be a y-intercept, we want to determine when $\mathbf{CO_2}(t) = 0$. When we graph $\mathbf{CO_2}(t)$, we see that it doesn't have a y-intercept. So the range is $\{R | R \in \Re,\ R \neq 0\}$.

Extremes

From the graph, we can see that there are no absolute maxima or minima. There does seem to be a local maximum at roughly 1997. The rate of CO_2 concentration per megawatt is roughly –0.3 ppm/MW. There must also be a local minimum occurring between 1800–1900. If we zoom in on this interval, we can see there is a minimum at roughly 1830 and the CO_2 concentration per megawatt is roughly 0.001 ppm/MW.





Intervals of Increase and Decrease

Intervals of Increase: From the graph, we get $1830 < x < 1987$ and $1987 < x < 1997$.

Intervals of Decrease: From the graph, we get $x < 1725$, $1725 < x < 1930$, $1997 < x < 2005$ and $x > 2005$.

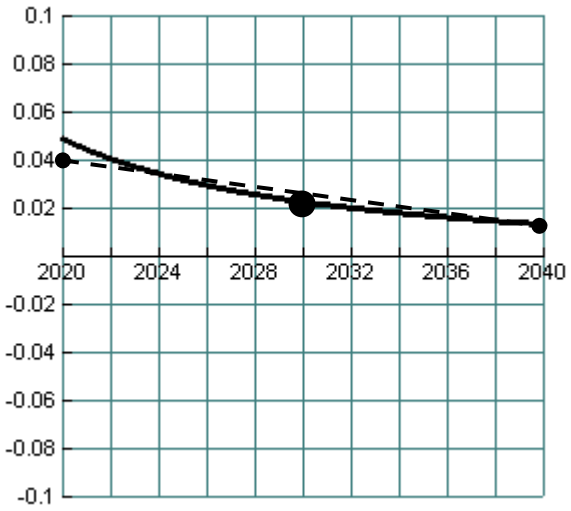
5. a) Find the **rate of change** of the function you found in Question 2 numerically and graphically at the date by which the concentration of carbon dioxide will reach 450 ppm. A graph of the region around $t = 2030$ has been provided below.

Numerically

Using values very close to $t = 2030$, we can determine the slope.

t	$R(t)$
2029.9	0.02282
2030.1	0.02254

3 Marks



Graphically

Draw a tangent to the curve at $t = 2030$. Using the endpoints of my tangent (2020, 0.039) and (2040, 0.01), determine the slope of the tangent to approximate the rate of change.

$$\text{Rate of change} = \frac{0.039 - 0.01}{2020 - 2040}$$
$$= \frac{0.029}{-20}$$
$$= -0.00145$$

3 Marks

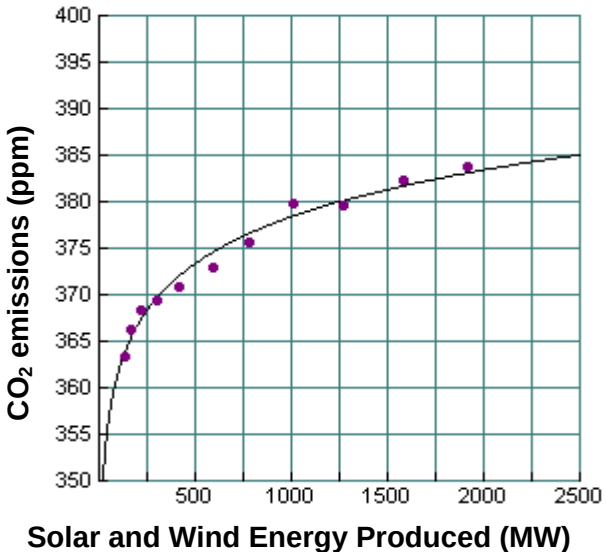
Note: The units for the rate change are ppm/MW/year

b) Explain the meaning of the **rate of change** within the context of the problem.

The rate of change is approximately –0.0014 ppm/MW/year. The means that the concentration of CO_2 per Megawatt produced by solar and wind energy is decreasing at this instant. In other words, if solar and wind energy continue to be produced at the levels we have seen, the concentration of CO_2 is slowed by the use of renewable energy resources.

6. The graphical, numeric and algebraic models of carbon dioxide emissions vs. energy produced from wind and solar resources are provided below.

Solar and Wind Energy Produced (MW)	CO_2 Emissions (ppm)
133.475	363.3475
162.663	366.295
214.696	368.24
306.039	369.34
415.32	370.92
591.164	372.9125
781.29	375.6125
1008.693	379.8808
1270.033	379.512
1578.153	382.21
1911.849	383.84



$C(r) = 29.9329 \log r + 284.5044$ where C is the CO_2 emissions and r is the amount of solar and wind energy produced.

If we continue to produce wind and solar energy according to the models given, when will the Earth reach its sustainable CO_2 threshold?

We first need to determine the amount of wind and solar energy being produced when we reach the sustainable CO_2 threshold, 450 ppm.

Let $C(r) = 450$: $450 = 29.9329 \log r + 284.5044$

$5.52889 = \log r$

$337976 = r$

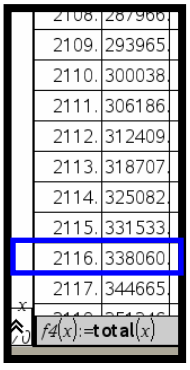
2 Marks

Therefore, there will be 337976 MW produced when the Earth reaches 450 ppm.

Now, using the $T(t)$ function determined earlier, find the time when we reach this level of energy production from solar and wind energy.

$$337976 = 0.061236t^3 - 350.2199t^2 + 666152t - 421313424$$

Using the Table feature of a graphing calculator, we can determine that the time when the level of renewable energy production reaches 337976 is approximately 2116.



3 Marks