### Big Picture

Students will:

- develop and apply the formula: Volume = area of the base × height to calculate volume of right prisms;
- understand the relationship between metric units of volume and capacity;
- understand that various prisms have the same volume.

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title</th>
<th>Math Learning Goals</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Exploring the Volume of a Prism</td>
<td>• Develop and apply the formula for volume of a prism, i.e., area of base × height.</td>
<td>7m17, 7m34, 7m36, 7m40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Relate exponential notation to volume, e.g., explain why volume is measured in cubic units.</td>
<td>CGE 5d, 5e</td>
</tr>
<tr>
<td>2</td>
<td>Metric measures of Volume</td>
<td>• Determine the number of cubic centimetres that entirely fill a cubic decimetre, e.g., Use centimetre cubes to determine the number of cm³ that cover the base. Many layers are needed to fill the whole dm³?</td>
<td>7m35, 7m42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Determine how many dm³ fill a m³ and use this to determine how many cm³ are in a m³.</td>
<td>CGE 3b, 4a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems that require conversion between metric units of volume.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Metric Measures of Capacity and Mass</td>
<td>• Explore the relationship between cm³ and litres, e.g., cut a 2-litre milk carton horizontally in half to make a 1-litre container that measures 10 cm × 10 cm × 10 cm. This container holds 1 litre or 1000 cm³.</td>
<td>7m35, 7m42</td>
</tr>
<tr>
<td></td>
<td>(See Metric Capacity and Mass – My Professional Practice)</td>
<td>• Determine that 1 cm³ holds 1 millilitre.</td>
<td>CGE 3b, 4a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems that require conversion between metric units of volume and capacity.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Volume of a Rectangular Prism</td>
<td>• Determine the volume of a rectangular prism, using the formula Volume = area of the base × height.</td>
<td>7m34, 7m40, 7m42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems involving volume of a rectangular prism.</td>
<td>CGE 4b, 4c</td>
</tr>
<tr>
<td>5</td>
<td>Volume of a Triangular Prism</td>
<td>• Determine the volume of a triangular prism, using the formula Volume = area of the base × height.</td>
<td>7m34, 7m40, 7m42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems involving volume of a triangular prism that require conversion between metric measures of volume.</td>
<td>CGE 3c, 5d</td>
</tr>
<tr>
<td>6</td>
<td>Volume of a Right Prism with a Parallelogram Base</td>
<td>• Determine the volume of a parallelogram-based prism, using two methods.</td>
<td>7m35, 7m40, 7m42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Determine that the volume of the parallelogram-based prism can be calculated, using the formula: Volume = area of the base × height.</td>
<td>CGE 5f</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems involving volume of a parallelogram-based prism.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Volume of a Trapezoid-Based Prism</td>
<td>• Determine the volume of a trapezoidal-based prism.</td>
<td>7m23, 7m34, 7m38, 7m40, 7m42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Solve problems involving volume of trapezoidal-based prisms.</td>
<td>CGE 5f</td>
</tr>
<tr>
<td>Day</td>
<td>Lesson Title</td>
<td>Math Learning Goals</td>
<td>Expectations</td>
</tr>
<tr>
<td>-----</td>
<td>-------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>8</td>
<td>Volume of Other Right Prisms</td>
<td>• Determine the volume of right prisms (with bases that are pentagons, hexagons, quadrilaterals, composite figures), using several methods.</td>
<td>7m23, 7m34, 7m40, 7m42, CGE 3b</td>
</tr>
</tbody>
</table>
| 9   | Linking Surface Area and Volume     | • Apply volume and area formulas to explore the relationship between triangular prisms with the same surface area but different volumes.  
• Estimate volumes.                                                              | 7m23, 7m42, CGE 4c, 5a |
| 10  | Surface Area and Volume of Right Prisms | • Investigate the relationship between surface area and volume of rectangular prisms.                           | 7m23, 7m42, CGE 4c, 5a |
|     | *GSP*®4 file: PaperPrism.gsp        |                                                                                      |              |
| 11  | Summative Performance Tasks         | • Assess students’ knowledge and understanding of volume of prisms with polygon bases.                       | CGE 3a, 3c   |
|     | *lesson not included*               |                                                                                      |              |
| 12  | Summative Performance Task          | • Skills test                                                                         | CGE 3a, 3c   |
|     | *lesson not included*               |                                                                                      |              |
Unit 10: Day 1: Exploring the Volume of a Prism

Math Learning Goals
• Develop and apply the formula for volume of a prism, i.e., area of base × height.
• Relate exponential notation to volume, e.g., explain why volume is measured in cubic units.

Minds On… Whole Class → Guided Instruction
Show a cube and ask: If the length of one side is 1 unit:
• What is the surface area of one face? (1 unit²)
• What is the volume? (1 unit³)
• Why is area measured in square units?
• Why is volume measured in cubic units?

Using a “building tower” constructed from linking cubes, lead students through a discussion based on the model:
• Why is this a right prism?
• What is the surface area of the base?
• What is the height of the building?

Count the cubes to determine the volume of the building.

Action! Pairs → Investigation
Invite students to ask clarifying questions about the investigation (BLM 10.1.1). Students create several more irregular prisms of various sizes, using BLM 10.1.1, Building Towers. Students display their findings in the table.

After investigating the problem with several samples, state a general formula for the volume of a prism:

\[ \text{Volume} = \text{area of the base} \times \text{height} \]

Students test their formula for accuracy by constructing two other towers.

Curriculum Expectations/Oral Questioning/Anecdotal Note: Assess students’ understanding of the general formula Volume = area of the base × height.

Consolidate Debrief Whole Class → Student Presentation
As students present their findings, summarize the results of the investigation on a class chart.

Orally complete a few examples, calculating the volume of prisms given a diagram.

Reinforce the concept of cubic units.

Home Activity or Further Classroom Consolidation
A prism has a volume of 24 cm³. Draw prisms with this volume. How many possible prisms are there with a volume of 24 cm³ with sides whose measurements are whole numbers?

Concept Practice Application Skill Drill
10.1.1: Building Towers

Name: 
Date: 

Each tower pictured here is a prism. Build each prism and determine the volume of each building by counting cubes.

Tower A  
Tower B  
Tower C  

1. Complete the table of measures for each tower:

<table>
<thead>
<tr>
<th>Tower</th>
<th>Area of Base</th>
<th>Height of Tower</th>
<th>Volume (by counting cubes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What relationship do you notice between volume, area of the base, and height?

3. State a formula that might be true for calculating volume of a prism when you know the area of the base and the height of the prism.

4. Test your formula for accuracy by building two other prism towers and determining the volume. Sketch your towers. Show calculations on this table.

<table>
<thead>
<tr>
<th>Tower</th>
<th>Area of Base</th>
<th>Height</th>
<th>Volume (by counting cubes)</th>
<th>Volume (using your formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Explain why your formula is accurate.
# Unit 10: Day 2: Metric Measures of Volume

## Math Learning Goals
- Students will determine the number of cubic centimetres that entirely fill a cubic decimetre, e.g. determining the number of centimetre cubes that will cover the base of an object. How many layers are needed to fill the whole dm³?
- Students will determine how many dm³ fill a m³ and use this to determine how many cm³ are in a m³.
- Students will solve problems that require conversion between metric units of volume.

## Materials
- Centimetre cubes
- BLM 10.2.1
- BLM 10.2.2
- BLM 10.2.3

## Minds On…
**Small Groups ➔ Review**
Students work in co-operative groups to review metric conversions using BLM 10.2.1

Some students may need assistance to complete the personal benchmark part.

Here are a few examples:
- mm = thickness of a dime; cm = width of a pinkie finger; dm = width of their palm

## Action!
**Pairs ➔ Investigation**
Students complete BLM 10.2.2 in pairs to discover that:
- \(1\text{dm}^2 = 100\text{cm}^2\) and \(1\text{dm}^3 = 1000\ \text{cm}^3\)

Distribute centimetre cubes for them to use as a manipulative to verify their predictions and solutions to the questions.

## Consolidate
**Whole Class ➔ Discussion**
Take up BLM 10.2.2. Reinforce the process for converting metric units of volume. When converting between dm and cm, use 10 as the ‘conversion number’.

When converting between:
- dm³ to cm³ ➔ multiply by 10 x 10 x 10 or \(10^3\).
- dm³ to cm³ ➔ multiply by 1000 (\(10^3\))

\[\text{e.g. } 3\ \text{dm}^3 = \underline{_______} \ \text{cm}^3 \ (\text{Answer: } 3000)\]

## Concept Practice Application
**Home Activity or Further Classroom Consolidation**
Complete BLM 10.2.3.
10.2.1: Metric Conversions

Complete the following in your co-operative grouping. Make certain all members of the group understand the work.

1. $1 \text{ cm} = \underline{\phantom{00}} \text{ mm}$  
   $1 \text{ dm} = \underline{\phantom{0}} \text{ cm}$  
   $1 \text{ m} = \underline{\phantom{0}} \text{ cm}$  
   $1 \text{ m} = \underline{\phantom{0}} \text{ dm}$

2. Complete the following conversions. For each one, show what you are thinking.

   e.g. $3 \text{ m} = \underline{\phantom{0}} \text{ cm}$
   
   $3 \text{ m} = 300 \text{ cm}$

   $10 \text{ mm} = \underline{\phantom{0}} \text{ cm}$
   $450 \text{ cm} = \underline{\phantom{0}} \text{ dm}$

   $4 \text{ m} = \underline{\phantom{0}} \text{ dm}$
   $50 \text{ dm} = \underline{\phantom{0}} \text{ m}$

3. For the following measurements, think about something in real life that would help you remember and visualize that measurement (e.g. $1\text{ mm} = \text{ thickness of a dime}$)

   $1 \text{ cm} =$

   $1 \text{ dm} =$
10.2.1: Metric Conversions Answers

Complete the following in your co-operative grouping. Make certain all members of the group understand the work.

1.  
   1 cm = 10 mm 
   1 m = 100 cm 
   1 dm = 10 cm 
   1 m = 10 dm 

2. Complete the following conversions. For each one, show what you are thinking. 
   
   e.g. 3 m = ______ cm  
        3 m = 300 cm 

   10 mm = 100 cm 
   4 m = 40 dm 
   450 cm = 45 dm 
   50 dm = 5 m 

3. For the following measurements, think about something in real life that would help you remember and visualize that measurement. (e.g. 1mm = thickness of a dime) 
   
   1 cm = width of your pinkie finger 
   1 dm = width of your palm
10.2.2: Converting Metric Units of Volume

Grade 7

Part A - Estimations
1. Estimate the area of each of the following in cm$^2$:
   This page ___________________ Your desk ___________________
   An item of your choice ___________ Item name ________________

2. Estimate the volume of each of the following in cm$^3$:
   The inside of your desk _______________ The classroom _______________
   An item of your choice _______________ Item name ________________

Part B - How many cm$^2$ are there in a dm$^2$?

Complete the measurements on each side of the square to show what its dimensions are in decimetres and in centimetres.

The square is called a decimetre square (dm$^2$), why do you think it is called that?

---

Fill the inside of the square with centimetre cubes.

How many cm cubes fit inside of the decimetre square?

Area = _____________cm$^2$

This means that

1dm$^2$ = ___________ cm$^2$

Approximate the area of each item from Part A in dm$^2$ using the decimetre square you created.

This page _______________ Your desk ___________________

An item of your choice ________________

Use each of your answers in dm$^2$ to determine the area of each item in cm$^2$. How close were your estimations?
10.2.2: Converting Metric Units of Volume (cont.)    Grade 7

Part C - How many cm\(^3\) are there in a dm\(^3\)?

Create a decimetre cube using your centimetre cubes.

How many cm cubes did you use?____________________________

Volume of the decimetre cube = ______________cm\(^3\)

This means that

\[
1\text{dm}^3 = \underline{\text{_______}} \text{cm}^3
\]

Approximate the volume of the items from Part A in dm\(^3\) using the decimetre cube you created.

The inside of your desk ___________________

The classroom ______________________

An item of your choice ____________________

Use each of your answers in dm\(^3\) to determine the volume of each item in cm\(^3\). How close were your estimations?

In summary

\[
1\text{dm} = \underline{\text{____}} \text{cm} \quad 1\text{dm}^2 = \underline{\text{_______}} \text{cm}^2 \quad 1\text{dm}^3 = \underline{\text{_______}} \text{cm}^3
\]

How can you use your new knowledge to help you make better estimations for areas and volumes?
10.2.3: Converting Units of Volume

1. \(3 \text{ dm}^3 = \underline{\text{__________}} \text{ cm}^3\) \(5 \text{ dm}^2 = \underline{\text{__________}} \text{ cm}^2\)

\[
53500 \text{ cm}^3 = \underline{\text{_______}} \text{ dm}^3\] \[457 \text{ dm}^2 = \underline{\text{________}} \text{ cm}^2\]

2. The area of the base of a storage container is 1500 dm\(^2\). The height is 30 dm.
   a) What is the volume of the container in cm\(^3\)?

3. a) Can you picture a 1 metre cube created out of centimetre cubes? How many cubes would it hold?

b) The volume of container A is 0.25 m\(^3\). The volume of container B is 45 000 cm\(^3\). Which container is larger? By how many cm\(^3\) is it larger?
10.2.3: Converting Units of Volume Answers Grade 7

1. 3 dm$^3$ = 3000 cm$^3$   

   3 dm$^3$ = 3000 cm$^3$  

   5 dm$^2$ = 500 cm$^2$   

   5 dm$^2$ = 500 cm$^2$  

   53500 cm$^3$ = 53.5 dm$^3$   

   457 dm$^2$ = 45700 cm$^2$   

2. The area of the base of a storage container is 1500 dm$^2$. The height is 30 dm.  
   a) What is the volume of the container in cm$^3$?  

   \[ V = 1500 \times 30 = 45000 \text{dm}^3 \]  

   \[ \text{multiply by 1000 to convert to cm}^3 \]  

   \[ V = 45000000 \text{cm}^3 \]  

3. a) Can you picture a 1 metre cube created out of centimetre cubes? How many cubes would it hold?  

   \[ 1\text{m} = 100\text{cm by 100cm by 100cm} \]  

   \[ = 1,000,000 \text{cm}^3 \]  

b) The volume of container A is 0.25 m$^3$. The volume of container B is 45 000 cm$^3$. Which container is larger? By how many cm$^3$ is it larger?  

   \[ \text{Container A} : \ 0.25 \text{m}^3 = 250 \text{000 cm}^3 \]  

   \[ \therefore \text{Container A is larger by 205 000 cm}^3. \]
### Math Learning Goals

- Students will explore the relationship between cm$^3$ and litres, e.g. cut a 2-litre milk carton horizontally in half to make a 1-litre container that measures 10 cm x 10 cm x 10 cm. This container holds 1 litre or 1000 cm$^3$ of liquid.
- Students will discover that a container with a volume of 1 cm$^3$ can hold 1 millilitre of liquid.
- Students will solve problems that require conversion between metric units of volume and capacity.

### Whole Class → Guided Instruction

Show the milk carton to the class, pointing out the measurement on the carton.

- What is the measurement? (Answer: 2L)
- What is this a measure of? (Answer: capacity)

Cut a 2-litre milk carton in half. Introduce the term *capacity* as being the amount a container can hold. The capacity of the original container is 2 litres (2L)

- What is the capacity after the carton is cut in half? (Answer: 1L)

Ask students for suggestions for determining the volume of the carton in cm$^3$. (Possibilities: students could use unit cubes and fill it, and then count – could use cubes from last day and compare, or they could measure the dimensions of the carton). Use methods suggested to determine the volume of the half container. (Answer: 1000 cm$^3$)

### Small Groups → Discussion

Have students work in co-operative groupings to answer the questions below. Remind them that volume is the amount of space an object takes up.

- What is the relationship between capacity and volume? (Students should discuss that the measurements have to be related in some way as they are giving a quantity to the same container).
- What is the volume of 1 cm$^3$? (Answer: 1 mL = 1 cm$^3$)

### Whole Class → Note Taking

Summarize the results of the above questions with the entire class and then do a few conversions, similar to the following examples:

- 500 mL = ____ cm$^3$ (Answer: 500); 5 cm$^3$ = ____ mL (Answer: 5);
- 450 cm$^3$ = ____ L (Answer: 0.45); 3.5 L = ____ cm$^3$ (Answer: 3500)

### Whole Class → Discussion

Reinforce that:

1 litre = 1000 cm$^3$ = 1 dm$^3$ = 1000 mL
1 mL = 1 cm$^3$
1 000 000 cm$^3$ = 1000 dm$^3$ = 1 m$^3$ = 1 kL

Point out objects in the classroom that will help students recognize the volumes that are equivalent.

### Home Activity or Further Classroom Consolidation

Complete BLM 10.3.1
10.3.1: Metric Measures of Capacity and Mass

1. Fill in the following chart.

<table>
<thead>
<tr>
<th>Volume</th>
<th>10 cm³</th>
<th>6 m³</th>
<th>___ m³</th>
<th>___ cm³</th>
<th>250 mm³</th>
<th>___ m³</th>
<th>3 m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>____ mL</td>
<td>____ L</td>
<td>6 kL</td>
<td>3 L</td>
<td>____ mL</td>
<td>500 L</td>
<td>____ mL</td>
</tr>
</tbody>
</table>

2. a) A prism has a base with an area of 20 cm², and a height of 2 cm. Calculate the volume of the prism in cm³.

b) Calculate the capacity of the prism in millilitres (mL).

3. The cargo hold of a truck has a base with an area of 25 m², and a height of 4 m. How many 5-litre containers can the truck carry?

4. Will the milk in a 2 L container fit into 5 glasses if each glass has a volume of 350 cm³? Explain your answer.
1. Fill in the following chart.

<table>
<thead>
<tr>
<th>Volume</th>
<th>10 cm³</th>
<th>6 m³</th>
<th>6 m³</th>
<th>3000 cm³</th>
<th>250 cm³</th>
<th>0.5 m³</th>
<th>3 m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>10 mL</td>
<td>6000 L</td>
<td>6 kL</td>
<td>3 L</td>
<td>250 mL</td>
<td>500 L</td>
<td>30000 mL</td>
</tr>
</tbody>
</table>

2. a) A prism has a base with an area of 20 cm², and a height of 2 cm. Calculate the **volume** of the prism.

   \[
   \text{Volume} = \text{area of base} \times \text{height} \\
   = 20 \text{ cm}^2 \times 2 \text{ cm} \\
   = 40 \text{ cm}^3 \\
   \]

   b) Calculate the **capacity** of the prism in millilitres (mL).

   We know \(1000 \text{ cm}^3 = 1000 \text{ mL}\).
   So \(40 \text{ cm}^3 = 40 \text{ mL}\).
   The capacity of the prism is 40 mL.

3. The cargo hold of a truck has a base with an area of 25 m², and a height of 4 m. How many 5-litre containers can the truck carry?

   \[
   \text{Volume} = \text{area of base} \times \text{height} \\
   = 25 \text{ m}^2 \times 4 \text{ m} \\
   = 100 \text{ m}^3 \\
   100 \text{ m}^3 = 100 000 000 \text{ cm}^3 = 100 000 \text{ L} \\
   100 000 \text{ L} / 5 \text{ L} = 20 000 \text{ containers}. \\
   \]

4. Will the milk in a 2 L container fit into 5 glasses if each glass has a volume of 350 cm³? Explain your answer.

   **Glass**
   \(V = 350 \text{ cm}^3\)
   Capacity of each glass is 350 mL.
   5 glasses would hold \(5 \times 350 \text{ mL} = 1750 \text{ mL}\)

   So, 2 L container (2000 mL) would not fit into the 5 glasses.
   250 mL will be left over.

   **OR**

   **Container**
   \(V = 2L\)
   \(= 2000 \text{ mL}\)
   \(= 2000 \text{ cm}^3\)

   Number of glasses needed = \(2000 \text{ cm}^3 / 350 \text{ cm}^3\)
   \(= \text{approx. 5.7 glasses needed. 0.7 glasses short}\)
   Therefore, you would need one more glass to hold all of the milk in the container.
Unit 10: Day 4: Volume of a Rectangular Prism

Math Learning Goals
• Determine the volume of a rectangular prism using the formula: Volume = area of the base × height.
• Solve problems involving volume of a rectangular prism.

Minds On… Whole Class ➔ Sharing/Discussion
Students share their diagrams and solutions for prisms with a volume of 24 cm³ (Day 1). Students build these with linking cubes (assume the prisms are using integer dimensions). Relate the dimensions to the factors of 24.
Using concrete samples of a rectangular prism, ask students:
• Will the volume be the same or different when the prisms are oriented vertically or horizontally?
• Is the base of a rectangular prism clearly defined or can it change?
• What do we mean by “dimensions of a prism?”

Action! Pairs ➔ Investigation
Students use a rectangular prism to show that the “base” is interchangeable but the volume remains the same (based on the general formula of Volume = area of the base × height). They investigate how to use the formula to calculate volumes of several examples of horizontally and vertically oriented prisms, and show their calculations to justify their conclusions.

Curriculum Expectations/Oral Questioning/Anecdotal Note: Assess students’ understanding of the general formula Volume = area of the base × height.

Consolidate Debrief Whole Class ➔ Reflection
Students share their investigation and justify their explanations, using diagrams and calculations.

Home Activity or Further Classroom Consolidation
• Make two or three sketches of rectangular prisms with whole number dimensions with volume:
  a) 27 cm³?
  b) 48 cm³?
• Why are there many more prisms of volume 48 cm³ than 27 cm³?
• Choose a volume for a rectangular prism that can be generated by several different sets of measurements with whole number dimensions. Explain.
• Complete the practice questions.

For any prism: 
V = area of base × height
For rectangular prisms: 
V = (l × w) h
When calculating volume of a rectangular prism, any of its faces can be thought of as the base.

Materials
• models of rectangular prisms
• linking cubes

Assessment Opportunities
- For any prism: 
  V = area of base × height
- For rectangular prisms: 
  V = (l × w) h
- When calculating volume of a rectangular prism, any of its faces can be thought of as the base.
Unit 10: Day 5: Volume of a Triangular Prism

Math Learning Goals

• Determine the volume of a triangular prism using the formula
  \[ \text{Volume} = \text{area of the base} \times \text{height} \]

• Solve problems involving volume of a triangular prism that require conversion
  between metric measures of volume.

Whole Class ➔ Sharing

Students share their sketches of prisms with volumes 27 cm³ and 48 cm³ and the responses to the questions. Students should use the term factors when explaining the relationship of the measures. Make a list of rectangular prisms that can be generated by several different sets of measurements. Discuss the relationship of these measures to the factors of a number.

Whole Class ➔ Discussion

Using concrete samples of a triangular prism, ask students:

• What can be altered in the volume of a prism formula to make the formula specific for a triangular prism?
• Will the volume be the same or different when the prism is oriented vertically or horizontally?
• What do we need to think about when applying the volume formula to a triangular prism?

Pairs ➔ Investigation

Students use a triangular prism to develop a formula specific to their prism (based on the general formula of Volume = area of the base × height.) They investigate how to use this formula to calculate volume of several horizontally and vertically oriented prisms, and show their calculations to justify their conclusions.

Curriculum Expectations/Oral Questioning/Anecdotal Note: Assess students’ understanding of the general formula Volume = area of the base × height.

Whole Class ➔ Reflection

Students share their investigation findings. Focus discussion on the need to identify the triangular face as the “base” when using the formula \( V = \text{area of base} \times \text{height} \) for a triangular prism. Connect this discussion to the idea of stacking triangles either vertically or horizontally to generate the triangular prism.

Discuss the need for \( h \) and \( H \) in the formula for volume: \( h \) is perpendicular to \( b \) and refers to the triangle’s height, \( H \) is the perpendicular distance between the triangular bases. Discuss each of these in relationship to rectangular prisms. If students understand that all right prisms have a Volume = (area of base) (height) they should not get confused by multiple formulas.

Students complete BLM 10.5.1.

Home Activity or Further Classroom Consolidation

Concept Practice

Sketch and label the dimensions of a triangular prism whose whole number dimensions will produce a volume that is:

a) an even number  
b) an odd number  
c) a decimal value

Explain your thinking in each case.
10.5.1: Volume of Triangular Prisms

Show your work using good form and be prepared to tell how you solved the problem.

1. Determine the volume of the piece of cheese.
   Create a problem based on the volume.

2. Determine the volume of the nutrition bar.
   Create a problem based on the volume.
10.5.1: Volume of Triangular Prisms (continued)

3. Determine the volume of air space in the tent.
   The front of the tent has the shape of an isosceles triangle.
   Create a problem based on the volume.

   ![Diagram of a triangular prism]

   1 m
   60 cm
   2 m
   1.6 m

4. a) If you could only have 1 person per 15 m³ to meet fire safety standards, how many people could stay in this ski chalet?

   ![Diagram of a cone-shaped chalet]

   Height of chalet = 7.1 m
   4.0 m
   7.5 m
   5.0 m

   **Hint:**
   Think about whether the height of the chalet is the same as the height of the prism. Which measurements are unnecessary for this question?

   b) How much longer would the chalet need to be to meet the safety requirements to accommodate 16 people?
Math Learning Goals

• Determine the volume of a parallelogram-based prism using two methods.
• Determine that the volume of the parallelogram-based prism can be calculated using the formula: Volume = area of the base x height.
• Solve problems involving volume of a parallelogram-based prism.

Minds On…

Whole Class ➔ Demonstration
Display two triangular prisms with congruent bases, e.g., use polydron materials or two triangular prism chocolate bars, or two triangular prisms cut from the net on BLM 10.6.1.

Students measure and calculate the volume of one of the prisms. Demonstrate how the two triangular prisms can be fitted together to make a parallelogram-based prism.

Action!

Pairs ➔ Investigation
Students respond to the question: How can the volume of the parallelogram-based prism be determined, knowing the volume of one triangular prism?

They find a second method for calculating the volume of a parallelogram-based prism and compare the two methods.

They verify that their findings are always true by creating several other parallelogram-based prism measurements.

The volume of a parallelogram-based prism can always be determined by decomposing it into two triangular prisms. (The formula Volume = area of base x height will determine the volume for any right prism.)

Consolidate Debrief

Whole Class ➔ Discussion
Debrief the students’ findings to help them understand that the volume of a parallelogram-based prism can be determined by determining the area of the parallelogram base, which is composed of two congruent triangles and is (b x h) multiplied by the height (H) of the prism. The volume of the parallelogram-based prism can also be determined using the formula:

\[ \text{Volume} = \text{area of the base} \times \text{height of the prism.} \]

Model the solution to an everyday problem that requires finding the volume and capacity of a parallelogram-based prism.

Home Activity or Further Classroom Consolidation

• Write a paragraph in your journal: There is one formula for all right prisms. It is… Here are some examples of how it is used….

OR

• Complete the practice questions.

Curriculum Expectations/Demonstration/Marking Scheme:

Assess students’ understanding of the general formula for right prisms.

Provide students with appropriate practice questions.
10.6.1: Triangular Prism Net
**Unit 10: Day 7: Volume of Trapezoidal-Based Prism**

**Math Learning Goals**
- Determine the volume of a trapezoidal-based prism using several methods for using the formula \( V = \text{area of the base} \times \text{height} \) to determine if there is a relationship.
- Solve problems involving volume of trapezoidal-based prisms.

**Minds On… Whole Class → Review**
Review the definition and characteristics of a trapezoid. Recall methods for calculating the area of a trapezoid.

**Action! Pairs → Investigation**
Students complete the investigation:
Can the formula \( V = \text{area of the base} \times \text{height} \) of the prism be used to determine the volume of trapezoid-based prisms instead of decomposing the trapezoid?
Investigate to determine the volume of a trapezoid-based right prism by decomposing the trapezoid into triangles and rectangles, using different decompositions.
Compare the solutions from the decomposition method to the volume calculated using the standard formula.
Write your findings in a report. Include diagrams and calculations.
Prompt students who are having difficulty decomposing the trapezoid by suggesting some of these possibilities:

**Problem Solving/Application/Checkbrick:** Assess students’ problem solving techniques, as well as their communication in the report.

**Consolidate Debrief Whole Class → Discussion**
Discuss the need for \( h \) and \( H \) in the formula and the importance of the order of operations.
Focus the discussion on the fact that the standard formula \( V = \text{area of the base} \times \text{height} \) of the prism always works for right prisms. Volume can also be calculated by decomposing into composite prisms.

\[
V = \left( \frac{(a + b)h}{2} \right) (H)
\]

**Concept Practice Home Activity or Further Classroom Consolidation**
Complete worksheet 10.7.1.
10.7.1: Designing a Box

A local pet food company wishes to package their product in a box. The preliminary box design is shown on the left.

Box A

Box B

1. Determine the volume of the box on the left. Verify your calculation using an alternate method.

2. Box B has the same volume as Box A. What is the height of Box B? Explain how you know.

3. Design a new box, Box C, with the same volume as the two boxes above.

Alternate
Build Box A and B. Be sure B has the same volume as A. Fill them up to check for equal volume.
Math Learning Goals

- Determine the volume of right prisms (with bases that are pentagons, hexagons, quadrilaterals, composite figures), using several methods.

Materials

- BLM 10.8.1

Assessment Opportunities

- Have some of the previously constructed figures available for student reference.

Minds On…

Whole Class ➔ Presentations

Students discuss the solution to the homework problem. Some students share their design for Box C. The class checks the dimensions for correctness.

If students built Boxes A and B, have them explain their method and prove that their volumes were the same.

Action!

Whole Class ➔ Brainstorming

Use a mind map to brainstorm a list of other possible shapes that could form the base of a right prism.

Students sketch the 2-D shapes on the board—pentagons, hexagons, quadrilaterals, and composite figures.

Learning Skills (Class Participation)/Observation/Mental Note: Assess students’ participation during the brainstorm.

Pairs ➔ Practice

Students decompose the shapes displayed into triangles and rectangles. They discuss how they would determine the area of the shape of the base in order to calculate the volume of that prism, e.g., \( V = \text{area of base} \times \text{height} \); decompose the prism into other prism shapes with triangular and rectangular bases.

Pairs ➔ Problem Solving

Students complete BLM 10.8.1.

Consolidate Debrief

Whole Class ➔ Presentation

Students present and explain their solutions.

Concept Practice

Home Activity or Further Classroom Consolidation

Design two right prisms with bases that are polygons. The prisms must have an approximate capacity of 1000 mL.
10.8.1: Designing a Gift Box

Determine the volume of the gift box designed by the students from Trillium School.

Shape of the base of the box:  

Side view of the box:

Volume of the box:

Capacity of the box:
**Math Learning Goals**
- Apply volume and area formulas to explore the relationship between triangular prisms with the same surface area but different volumes.
- Estimate volumes.

**Materials**
- rectangular tarp or sheet
- connecting cubes
- BLM 10.9.1

**Assessment Opportunities**
This activity might be done outside or in a gymnasium. Consider using a rope to hold the peak of the tent in place.

Students might investigate changes when the fold is moved from lengthwise to widthwise.

**Minds On...**

**Small Groups ➔ Discussion/Presentation**
Students share solutions for homework questions assigned on Day 8 for volume of right prisms with polygon bases. Each small group presents one solution to the whole class.

**Whole Class ➔ Investigation**
Place a large tarp on the floor/ground. Invite six students to become vertices of a triangular prism tent. Four of the students are to keep their vertices on the ground. They stand on the corners of the tarp. The remaining two students stand on opposite sides of the tarp, equidistant from the ends, to become the fifth and sixth vertices. These two vertices gradually raise the tarp until a tent is formed. Note that the “ground” vertices have to move. Invite two or three other students to be campers.

Students verbalize observations about the tent’s capacity as the tent’s height is increased and decreased. Ask: Does it feel like there is more or less room?

**Action!**

**Pairs ➔ Model Making**
Students simulate the tent experiment using a sheet of paper and connecting cubes. Data may be collected in a two-column chart – height of the tent vs. number of connecting cubes that will fit inside the tent without bulging the sides.

**Consolidate Debrief**

**Think/Pair/Share ➔ Discussion**
In pairs, students respond to the question: Is the following statement sometimes, always, or never true?

Two triangular prisms with the same surface area also have the same volume.

Ask probing questions to ensure that students realize that investigation of this statement differs from the tent investigation since the floor and the triangular sides were ignored in the tent scenario, but cannot be ignored in this question.

Ask students if their conclusion would be the same for closed and open-ended prisms.

**Whole Class ➔ Discussion**
Discuss how an experiment might be designed to confirm or deny hypotheses about the relationship between surface area and volume.

**Home Activity or Further Classroom Consolidation**
On worksheet 10.9.1, make two folds using the two solid lines. Form a triangular prism. Imagine that it also has paper on the two triangular ends. Sketch the prism and its net. Take the measurements needed to calculate the surface area (including the two triangular ends) and volume. Label the diagrams with the measurements. Calculate the surface area and volume. Repeat the process for the prism formed using the two broken lines.

Make a statement regarding your findings that relates surface area and volume.

**Curriculum Expectations/ Application/Marking Scheme:** Assess students’ ability to calculate the area and volume of triangular prisms.
10.9.1: Triangular Prisms
**Math Learning Goals**
- Investigate the relationship between surface area and volume of rectangular prisms.

**Materials**
- BLM 10.10.1
- interlocking cubes

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**
Use GSP®4 file Paper Folding To Investigate Triangular Prisms to check student responses and investigate additional scenarios (Day 9 Home Activity).

**Action! Pairs → Investigation**
Pose the question:
If two rectangular prisms have the same volume, do they have the same surface area?

Students investigate, using BLM 10.10.1:
- a) For prisms with the same volume, is the surface area also the same? (no)
- b) What shape of rectangular prism has the largest surface area for a given volume?

**Individual → Written Report**
Students individually prepare a written report of their findings.

**Communicating/Presentation/Rating Scale:**
Assess students’ ability to communicate in writing and visually their understanding of surface area and volume as a result of their investigation.

**Consolidate Debrief Whole Class → Student Presentations**
Students present their findings and apply the mathematics learned in the investigation to answer this question:
Why would a Husky dog curl up in the winter to protect himself from the cold winds when he is sleeping outdoors? (If the dog remains “long and skinny” he has greater surface area exposed to the cold. If he curls up, he has less surface area exposed to the cold, and thus he would lose much less body heat. Although his volume stays the same, his surface area decreases as he becomes more “cube-ish,” or spherical.)

**Home Activity or Further Classroom Consolidation**
Complete the practice questions.
10.10.1: Wrapping Packages

Three different rectangular prism-shaped boxes each have a volume of 8 cubic units. Does each box require the same amount of paper to wrap? Let’s investigate!

1. a) Verify that each rectangular prism illustrated above has a volume of 8 cubic units.
   b) Draw the net for each rectangular prism box.
   c) Determine the amount of paper required by calculating the surface area.
      (Ignore the overlapping pieces of paper you would need.)
   d) Describe your findings.

2. a) How many different rectangular prism boxes can be designed to have a volume of 24 cubic units?
   b) Draw several of the boxes, labelling the dimensions.
   c) How much paper is required to wrap each box?
   d) Describe your findings.

3. Investigate wrapping rectangular prism boxes with a volume of 36 cubic units.
   Determine the dimensions of the rectangular prism with the greatest surface area.

4. Write a report of your findings. Include the following information, justifying your statements.
   • Describe how surface area and volume are related, when the volume remains the same.
   • Describe the shape of a rectangular prism box that uses the most paper for a given volume.
   • Describe the shape of a rectangular prism box that uses the least paper for a given volume.
Paper Folding to Investigate Triangular Prisms

If two triangular prisms have the same surface area will they have the same volume?

The dynamic model below shows a piece of paper with two fold lines. If possible, the folded sections are joined so that the paper becomes an open-ended triangular prism.

Change the prism by changing the location of the folds.

Any red point in the dynamic model can be dragged.

Paper Triangular Prisms

Imagine you are looking at the edge of the paper.
The paper is folded at point D and point E.

Paper Folding to Investigate Triangular Prisms

1) What paper size are you using?
2) Where do you want to place your folds?
3) If two triangular prisms have the same surface area do they have the same volume?

Volume of Prism = 9.8 cm³
Surface Area of Prism (without triangles) = 38.8 cm²
Surface Area of Prism (with triangles) = 41.3 cm²

Area of Triangular Face = 1.4 cm²