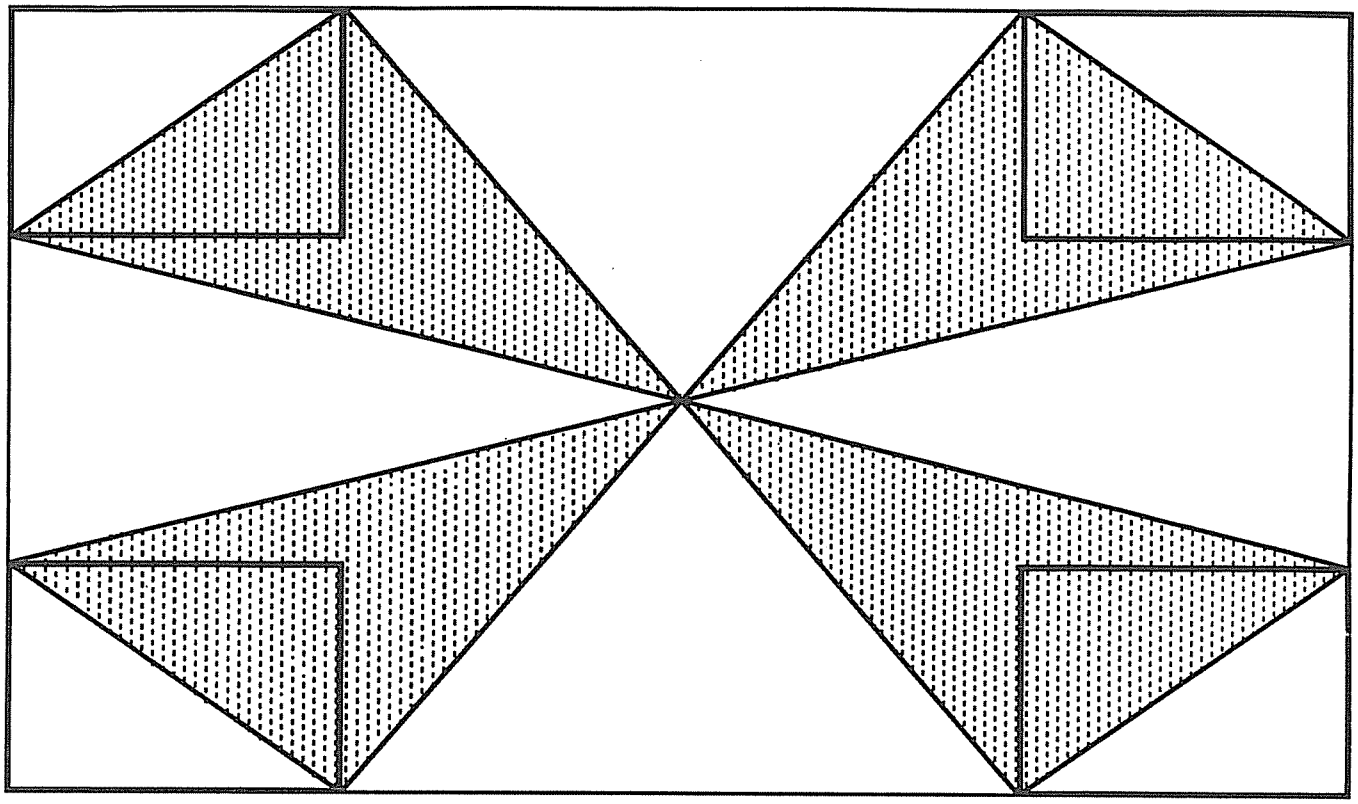
Volume | | |
Number 2

abacus

publication of the O. A. M. E.

April 1990



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Mr. Moe B. Us and His Daughter

As most young women do, Mr. Us's daughter, Knott, wished to marry. Moe wished to have a son-in-law who was not only a good husband, but also a good problem solver. Thus, the following problem was posed for all of her boyfriends. The one who was successful in its solution, was allowed to marry the "fair maiden" Knott.

The problem was to join the corresponsing letters on the strip of paper, using only straight lines, without ever crossing the lines. Cut out a strip of paper, like the one shown below, and find the correct solution.



Once you have found the solution that will allow Knott to be married, cut along the lines that you have drawn.

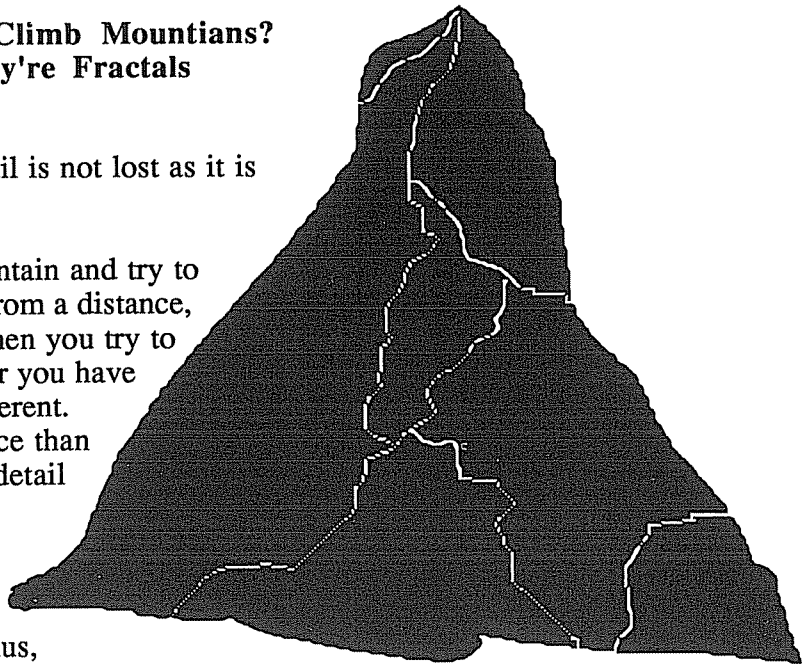
- What do you think will happen when the cuts are made?
- Make the cuts. Was your prediction correct?

The solution to the problem is done using the Moebius strip. What you, and Moebius, have now discovered is that it is possible to make a strip with only one surface and only one edge.

Now that you have seen how a Moebius strip is formed, try the following investigation. It will use the ideas discovered by August Ferdinand Moebius in 1858.

(a) Cut out a strip of paper similar to the one above.
(b) Give the strip of paper two half twists and attach the ends.
(c) Draw a line through the middle of the strip in (b) and then cut along the line.
(d) What do you notice after the cut has been made?
(e) Repeat parts (a) and (b) for a new strip.
(f) Draw two lines along the strip in (e).
(g) Cut along both lines in part (f). What do you notice after the cuts have been made?

Why Do People Climb Mountians? Because They're Fractals



Simply put, a fractal is any object whose detail is not lost as it is magnified.

For example, suppose that you look at a mountain and try to draw it to scale. If you look at the mountain from a distance, the sides will look very smooth. However, when you try to climb the mountain and then measure how far you have traveled, then the drawing will look much different. You will have traveled a much greater distance than appears on the scale drawing. However, the detail of the mountain has not been lost.

Compare this to a circle. A circle looks round until you magnify an arc of the circle. As it is magnified, the arc tends to straighten out. Thus, you have lost the detail of the circle.

The following exploration is a representation of a fractal. It is called the Koch curve and can be used to help you draw a snowflake. In fact, this curve has been called the snowflake curve. Before you try the exploration below, research more about snowflakes. Compare your research to the curve that you draw. What other mathematical concepts do you see?

Exploration 1

(a) Construct an equilateral triangle. Each side should be at least 20 cm in length. What is the perimeter of your triangle?

(b) Divide each side of your equilateral triangle into three equal parts. Use the middle part of each side and draw new equilateral triangles. You will now have a 12-sided polygon. What is the perimeter of your polygon?

(c) Once again, take each side of the polygon in (b) and divide the sides into three equal parts. Draw an equilateral triangle using the middle part of each side.

How many sides does your polygon have?
What is the perimeter of your polygon?

(d) Repeat the process at least twice more.

What pattern do you see?
How many sides will a triangle divided n times have?

Exploration 2

Repeat the process above using a shape of your choice. Compare your shape to others in your class.

How are the shapes alike?
How are the shapes different?
What patterns do you see?

Where else can you see the idea of a fractal in nature?

Poker Expressions

Many of your students will already know how to play card games. Poker is but one of the games that can be played using combinations of cards. The object of the game is to get the best possible five card hand. Depending upon the type of game chosen, there may or may not be wild cards. Cards that are deamed to be wild, are so for the entire game.

This one game, by itself, can produce much mathematics - especially in the field of probability. Players are trying to win by having the best hand of all the players involved. Since cards are often being replaced as the game proceeds, the probability of getting the card(s) you want is continuously being calculated.

This game can be used to help your students practise their "mental math" skills as well as their skills with probability. On the next three pages, you will encounter 52 different expressions. Some of them are verbal while others are expressions that must be calculated. The 52 expressions count for a total of only 13 different numbers. In other words, if one expression is evaluated to be 42, then there are 4 expressions that will have a value of 42. To have your students play poker, follow these directions.

1. Cut out the expressions and tape them on a deck of 52 playing cards.

2. Deal out the cards to a group of students. Each student will get 5 cards. Briefly explain how the game of poker is played and then allow the students to invent their own poker game. However, ensure that each group of students has no more than five players and that no more than five cards are given out at the same time.

3. Tell your students that they will have to evaluate the expressions on the card in order to make their hand the best that it can be. For example, if the student has two cards that both have a value of 42 and two cards that both have a value of 0.1, then they have two pair.

You can make variations of this game depending upon the level and the ability of your class. For instance, if you have a Grade 12 class, then you may wish to place trigonomertric expressions on your cards since many trigonometric expressions have the same value.

0.1	one tenth	0.3 x 0.5 - 0.05
$\frac{1}{10}$	4^2	12.3 - 9.2 + 0.9
smallest prime number mulitplied by 8	thirty two divided by four plus 8	(17 x 10 - 2) ÷ 4
one half of eighty four	$6^2 + 6$	LCM of 7 and 6
square root of 100	10 x 10 x 10 ÷ 10	GCF of 10 and 70
0.25×20^2	0.25	$\frac{1}{4}$

twenty five hundredths	$\frac{1}{2}$ of $\frac{1}{2}$	18 x 5 ÷ 10 - 4
The sum of the least two prime numbers.	square root of twenty five	The value of x in $3x = 15$.
The number of days of Christmas.	LCM of 4 and 3.	77 - 13 x 10 ÷ 2
GCF of 24 and 60.	Product of the least three prime numbers.	7 x 5 - (33 ÷ 11 x 2 - 1)
Six times the number of cards in your hand.	0.5 x 200 x 0.3	one third
0.33	$\frac{1}{3}$	three ninths

$25 + 35 - 15 - 10 - 7$	The number of days in the shortest month of the year.	$1.4 \times 100 \div 5$
The sum of the first seven grade levels.	The only prime number between 24 and 30.	$72.5 \div 2.5$
1.45×20	The number of days in February during a leap year.	$2 \times 3 \times 3 \times 10$
One half of the number of degrees in a circle.	Sum of the angles in a triangle.	$0.25 \times 10 \times 0.45 \times 10$
The number on which the decimal system is based.	$10^5 \div 10^4$	0.1×100
Sum of the first three prime numbers.		

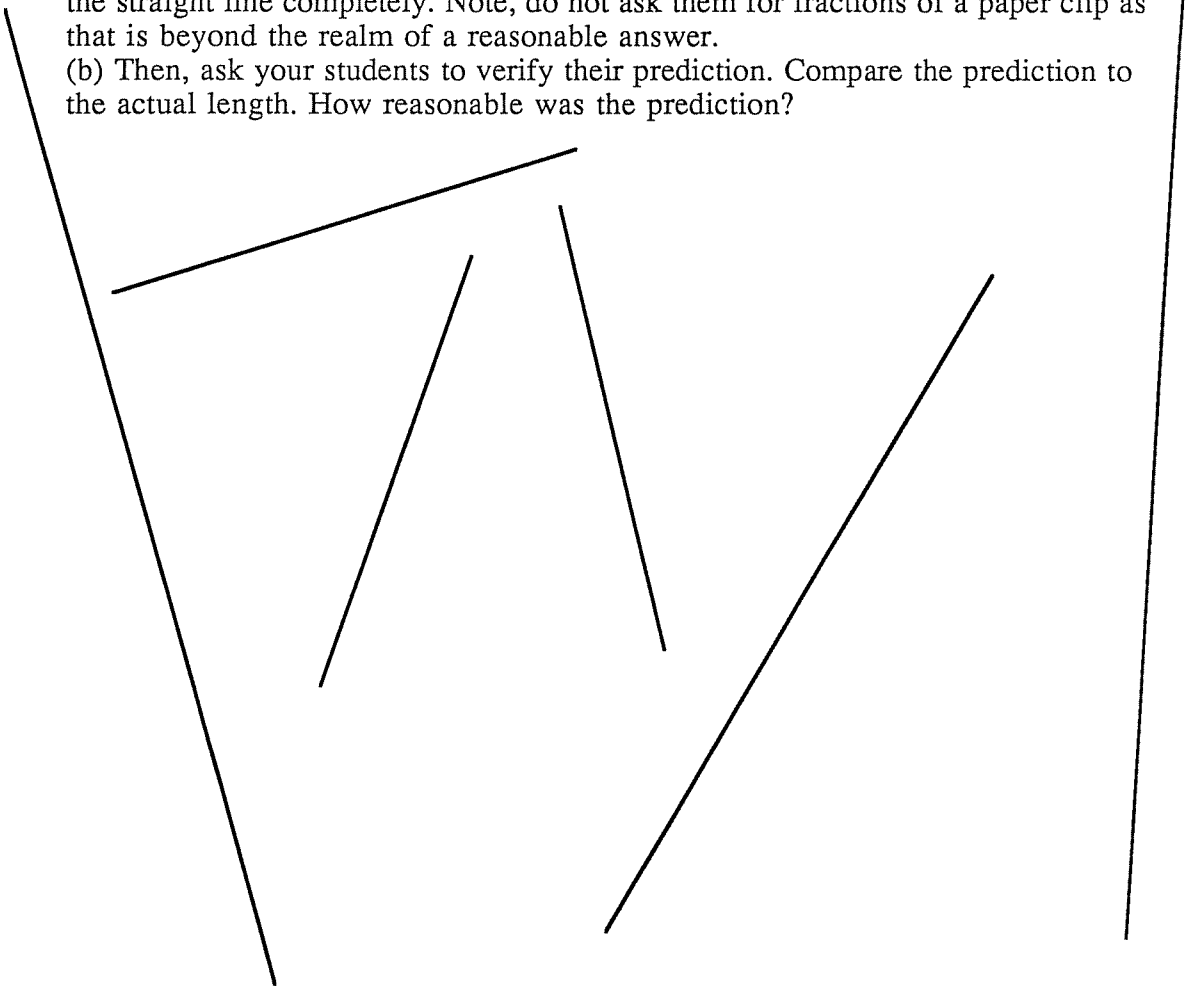
Math and Paper Clips

No! We haven't lost our minds. There is mathematics that can be done with paper clips. To do the mathematics presented in this section, the kids will use their "Intuitive" skills with ratios and develop their skills with estimation and approximation.

Before you begin, talk about the paper clip. Since there are all different sizes of paper clips, make sure that all students have the same size paper clips. Similarly, because there may be a shortage of paper clips in your class, have your students work in groups to perform the activities. As an extension, you could repeat each of the following activities using paper clips that have been straightened out.

Activity 1: Measuring Linear Distances

Give the following straight lines to your students.
(a) Have your students predict how many paper clips will be needed to fit along the straight line completely. Note, do not ask them for fractions of a paper clip as that is beyond the realm of a reasonable answer.
(b) Then, ask your students to verify their prediction. Compare the prediction to the actual length. How reasonable was the prediction?



Activity 2: Measuring the Distance Around

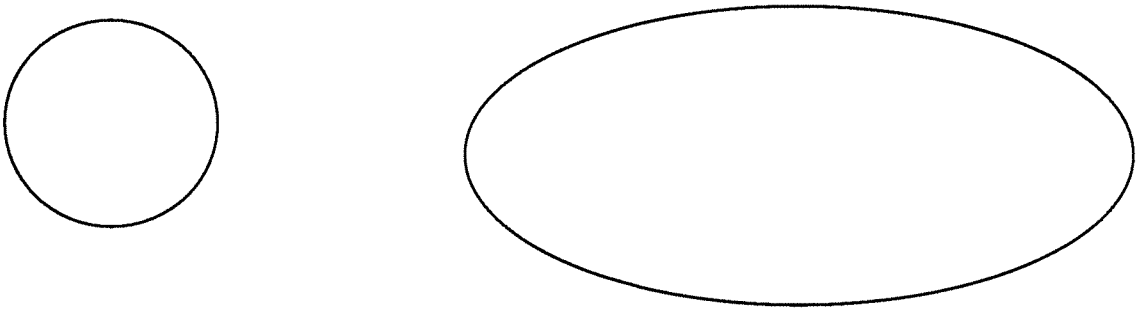
There are many objects in the class where the students can use their paper clips to find the distance around. Some of them are shown below. Remember, before they do the actual measuring, have them predict what they think the measurement will be. Also, it may be preferable to have them make a "chain" of paper clips so that the measurement around is made much more simply and accurately than otherwise would have been with only one paper clip.

(a) The distance around a text book.
(b) The distance around the desk.
(c) The distance around a window.
(d) The distance around the teacher's desk.
(e) The distance around a door.
(f) The distance around the class room.

Activity 3: Measuring Circular Distances

So far, we have had the student measuring only linear distances. Now, let's use their skills with circumference in order to predict and measure objects that are circular.

To begin , have them measure the distance around the following shapes.



Now, have them measure the distance around the following circular shapes. As always, have them make a prediction first and then measure using the "Paper Clip Chain" in order to determine the accuracy of their prediction.

(a) The distance around a fist.
(b) The distance around a head.
(c) The distance around a waist.
(d) The distance around a tree trunk.

Voyageur II

One of the most exciting voyages has just taken place. The meeting of Voyageur II with the outer regions of our solar system. As a result, in the middle of this issue, we are now going to look at the voyage of Voyageur (if you'll pardon the pun) and some of the discoveries made during its journey. The following is a brief summary of the Voyageur expedition. Then, you will find some questions based on the information found in this summary. You may wish to have your students research and write about the discoveries in more detail than is presented in these pages.

The Journey

The Voyageur's journey began in 1977 and took the satellite a distance of about 4.48 billion kilometres. It took pictures of whatever it saw and relayed all the photographs back to the Jet Propulsion Laboratory in Pasadena, California. Eager scientists, "surviving mainly on pizza, coffee and the strength of their own curiosity", awaited the encounter with Neptune.

And then it happened. Voyageur discovered the "Pink and Blue" moon of Neptune, called Triton. The discovery that this was not a gas of different colours, but indeed a planet was very exciting. But, this was not all that was to be discovered. Neptune was also discovered to be, like Saturn, a planet with rings! Not one or two rings but, yes, five distinct rings surround Neptune.

Throughout Voyageur's 12 year journey, it has sent back to Earth candid shots of Jupiter, Neptune, Saturn and Uranus. However, the photographs taken of the Neptune are the clearest and most detailed. To take these photographs, Voyageur directed itself within 5000 km of the surface of Neptune with a speed of 100 000 km/h. The planet, other than Neptune, that it got closest to was Saturn at a distance of 41 600 km. In fact, after 12 years of travelling, Voyageur was within 26 km of where scientists predicted it would be. They have compared this to sinking a putt, on a golf course, that is 3600 km in length.

The signals that were sent back to Earth from Neptune took about 4 h and 6 min. These signals were sent at the speed of light and were so faint that it required 38 giant radio antennae on four different continents to catch each signal. Even more amazing is that these signals reached us from Neptune where storms can create winds of up to 640 km/h. These winds are capable of blowing many of the rings that surround the planet.

Just where Neptune finds the energy to create such winds is still a mystery. The earth gains much of its strength from the sun. However, Neptune gains only one thousandth as much energy from the sun as that of the earth. It is assumed that the centre of Neptune must contain some sort of "Heat Engine" similar to those of Saturn and Jupiter, from which the planet can draw an endless supply of energy.

By the end of October, 1989, Voyageur had sent over 5 trillion bits of information. This is enough information to fill 26 encyclopedias. From the Voyaguer space probe, and the Pioneer space probes, we have learned much about our solar system. On the next page, you will see many of the facts that are now known about our solar system.

Sun

Diameter: 1 384 000 km
Age: 4.5 billion years
Temperature of surface:
48.6 million °C

Mercury

Diameter: 4850 km
Distance from the sun: 58 000 000 km
Time to orbit sun: 88 d

Venus

Diameter: 12 032 km
Distance from the sun: 107 000 000 km
Time to orbit sun: 225 d

Earth

Diameter: 12 739.2 km
Distance from the sun: 57 600 000 km
Time to orbit sun: 365d

Mars

Diameter: 6755 km
Distance from the sun: 225 000 000 km
Time to orbit sun: 687 d

Jupiter

Diameter: 141 968 km
Distance from the sun: 773 000 000 km
Time to orbit sun: 11.9 years

Saturn

Diameter: 119 296 km
Distance from the sun: 1 417 000 000 km
Time to orbit sun: 29.5 years

Uranus

Diameter: 52 096 km
Distance from the sun: 2 853 000 000 km
Time to orbit sun: 84 years

Neptune

Although Voyageur II has found that there are five rings surrounding the planet, other information is still being examined and the calculations and conclusions still being made.

Pluto

Diameter: 3040 km
Distance from the sun: 5 866 000 000 km
Time to orbit sun: 248 years

Some facts about voyageur.

Total cost of the voyageur to date: $865 000 000
Launched on August 20, 1977
Has a mass of 800 kg.
The voyageur has enough energy to travel for another 296 000 years.

Problems

1. (a) What is the speed of light?
   (b) How long did it take for the signals to reach the earth from Neptune?
   (c) Use your results in (a) and (b). How far is Neptune from the earth?

2. (a) What was the average speed of the Voyageur?
   (b) Suppose the Voyageur does in fact travel at this speed for the next 296 000 years, will the satellite ever reach the nearest star other than the sun? Give reasons for your answer.

3. (a) What speed of wind will account for a hurricane-like wind here on Earth?
   (b) How many times greater is the speed of the wind on Neptune than that of a hurricane on Earth?

Use the facts about our solar system presented on the previous page to solve the remaining problems. Remember to read each problem carefully before solving and use your library to research as needed.

4. Draw a diagram of our solar system. Remember that the sun is the centre of our solar system and use an appropriate scale to draw your diagram.

5. One year on Earth is about 365 d. This is the time that it takes the earth to travel around the sun exactly once.
   (a) Suppose you lived on Mercury. How long do you think that one year would last? Give reasons for your answers.
   (b) What would your age be on Mercury?
   (c) What would be the ages of the members of your family on Mercury?
   (d) What assumptions have you made in solving parts (b) and (c)?

6. (a) Suppose you live one year. What is the age of a dog in "Dog Years" after you have lived for one year?
   (b) A dog is 42 years old. What is the age of this dog on Jupiter? What assumptions have you made in solving this problem?

7. (a) How much did it cost to build the SkyDome in Toronto?
   (b) How much did the entire Voyageur program (which includes the launching and monitoring of 2 satellites) cost?
   (c) How many times greater is the cost of the Voyageur program than the cost of the SkyDome?

8. Draw a scale drawing to show the diameter of all the planets in our solar system. Use an appropriate scale in your drawing.

9. List the planets in ascending order according to the length of time they take to travel around the sun.Why do you think that these differences arise? Write a short paragraph to justify your answer.

Magic Squares

Has some of the magic been lost in the magic square because you teach them to all of your students each year? Have your students lost interest in the magic square because they have worked with them every year since Grade 2? Well, now we are going to put some of the magic back into the magic square.

Look at the two magic squares shown below. How are the squares alike? How are they different?

5	22	18
28	15	2
12	8	25

five	twenty two	eighteen
twenty eight	fifteen	two
twelve	eight	twenty five

In both magic squares, each row, column and diagonal sum to the same number. Now, prepare a third magic square using the number of letters in each written number of the magic square above to the right. What do you notice?

The same process can be used with the magic square shown to the right. Once you have found your new magic square, repeat the process with the magic square you have created.

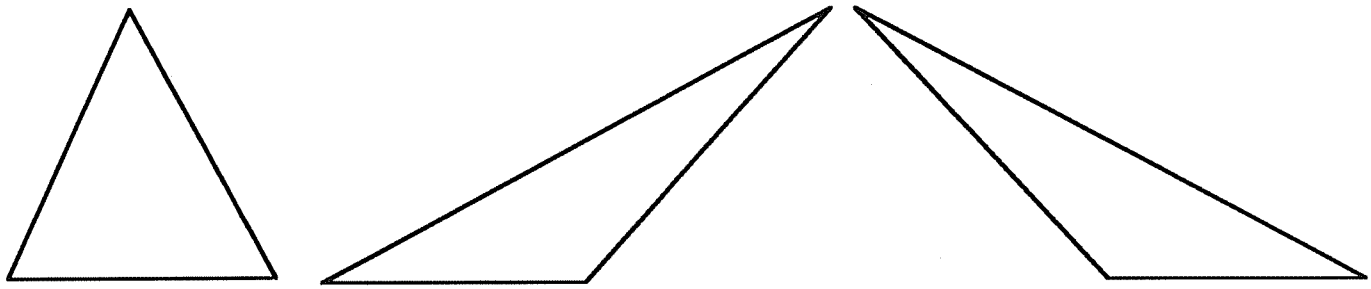
44	61	57
67	54	41
51	47	64

Finally, invent your own magic square that has these properties. You have now put some of the

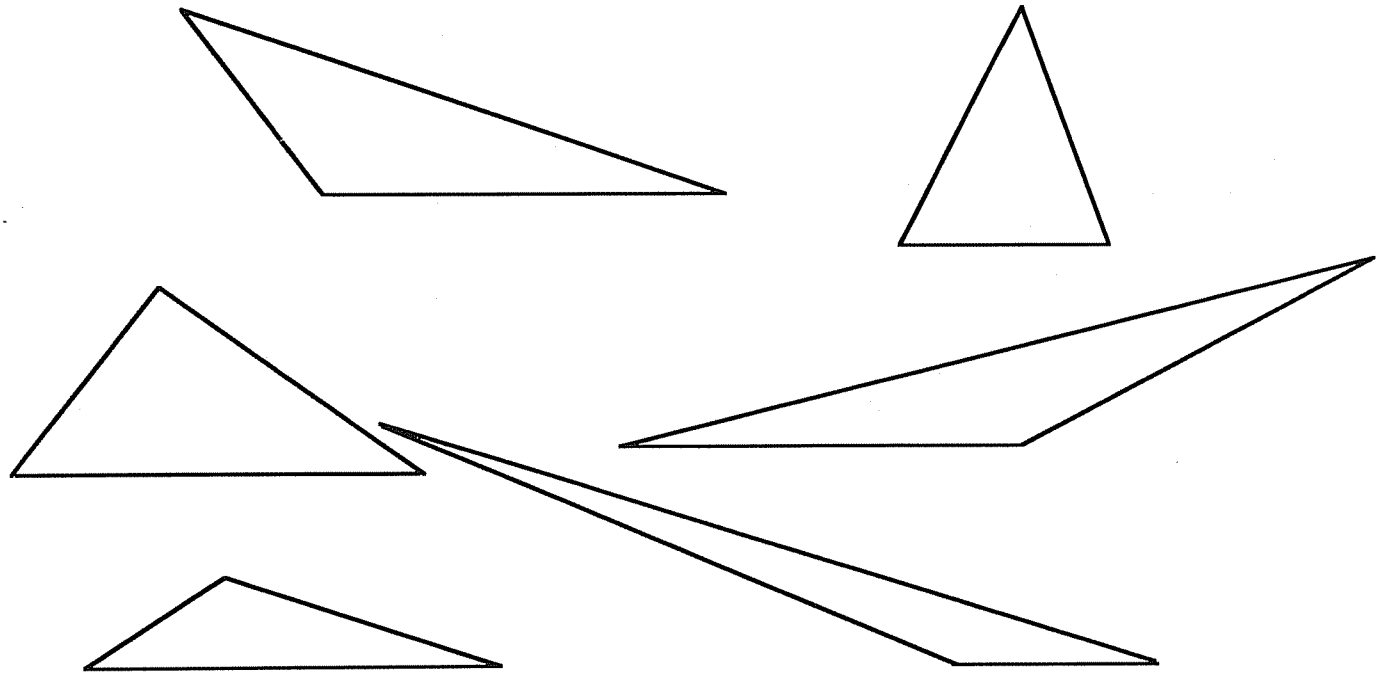
Thinking Visually

Sometimes, what appears to be obvious, is in fact not. Developing a student's ability to think visually, and to think using patterns, can be started by using your student's skills with area.

After completing your lesson finding the area of a triangle, present your students with the following three triangles. Ask them to estimate which triangle covers the greatest amount of area.



Because the base length and the height of each of the above triangles is the same, they will each cover the same area. If your students did not arrive at that conclusion, have them measure the base length and the height of each triangle to prove it for themselves. Then, ask them to determine which of the following triangles have equal areas. Be careful. So as to not make the solution too obvious, not all triangles have the same area.



Working Upside Down

You have all heard, and used, the problem solving process of working backwards to solve a problem. However, there is an extension of this topic that will enable you to solve problems by working upside down. For example, look at the following pattern.

$0! = 1$
$1^2 = 1$
$2^2 = 1$
$3^2 = 9$
$4^2 = 16$

Notice that the numbers in the left hand column all sum to 10. The square of 10 is 100. Now turn the page upside down and find the sum of the numbers that appear in the new left hand column. You will see that the numbers 91 + 6 + 1 + 1 + 1 also has a sum of 100.

Some other problems that can be solved, and patterns that can be seen, are shown in the following exercises. Remember, to work with these problems, think of the strategy of "working upside down."

1. (a) Use the following sums.
   (i) $68 + 89 + 96$ (ii) $68^2 + 89^2 + 69^2$
   (b) Find the sum of each expression in (a).
   (c) Now, turn the page upside down and write each new expression.
   (d) Find the sum of the each new expression. Compare your answers to your answers in (b). What do you notice?
   (e) Create an expression of your own using the properties explored in this section. Compare your to others in your class.

2. The magic square shown to the right is called the IXOHOXI magic square. It has the same magic sum if your read it as it is presented or if your read it upside down. In fact, even the name is the same whether it is read normally or upside down or even backwards.

8818	1111	8188	1881
8181	1888	8811	1118
1811	8118	1181	8888
1188	8881	1818	8111

There are other 4 x 4 magic squares that are magic squares whether you read it the way it is presented or whether you read it upside down.
(a) Try to find one such magic square.
(b) What other properties does IXOHOXI have?

Math In Our Songs

As your local Disk Jockey might say

"Whoaaaaaaaa!!! We are going to get down!" (Hopefully, we are going to get back up again.)

Well, we are going to get down. Then, we are going to get right back up again and try some more problems. The problems that you see on this page are all based on some songs that you have probably heard at some point. Have fun remembering the song and then try the problem(s) that follow.

Rock Around the Clock Tonight

One, two, three o'clock, four o'clock rock
Five, six, seven o'clock, eight o'clock rock
Nine, ten, eleven o'clock, twelve o'clock rock
We're gonna rock - around - the clock tonight.

1. (a) At what time will the dance start?
   (b) At what time will the dance end?

2. For how many hours will the dance last?

3. If the dance ended at a time that divided the clock into two parts, where the part on each side of the clock had the same sum, at what time will the dance end?

4. Who wrote the song?

The Gambler

You've got to know when to hold them. Know when to fold them,
Know when to walk away. Know when to run.
You never count your money. When you're sitting at the table.
There is time enough for counting.
When the dealing's done.

1. Kenny Rogers was singing about a card game.
   (a) Kenny could make hands with five cards. How many five card hands are possible?
   (b) In how many ways can you get a 10, Jack, Queen, King and Ace of Hearts?
   (c) What is the probability of getting the "Royal Flush" in (b)?

2. If the five card hand in Question 1, part (b) is the best hand in the game, where does the hand of four aces fall? Give reasons for your answer.

3. Suppose that four of the 52 cards in the deck are wild. What is the probability of obtaining a "Royal Flush" with the wild cards?