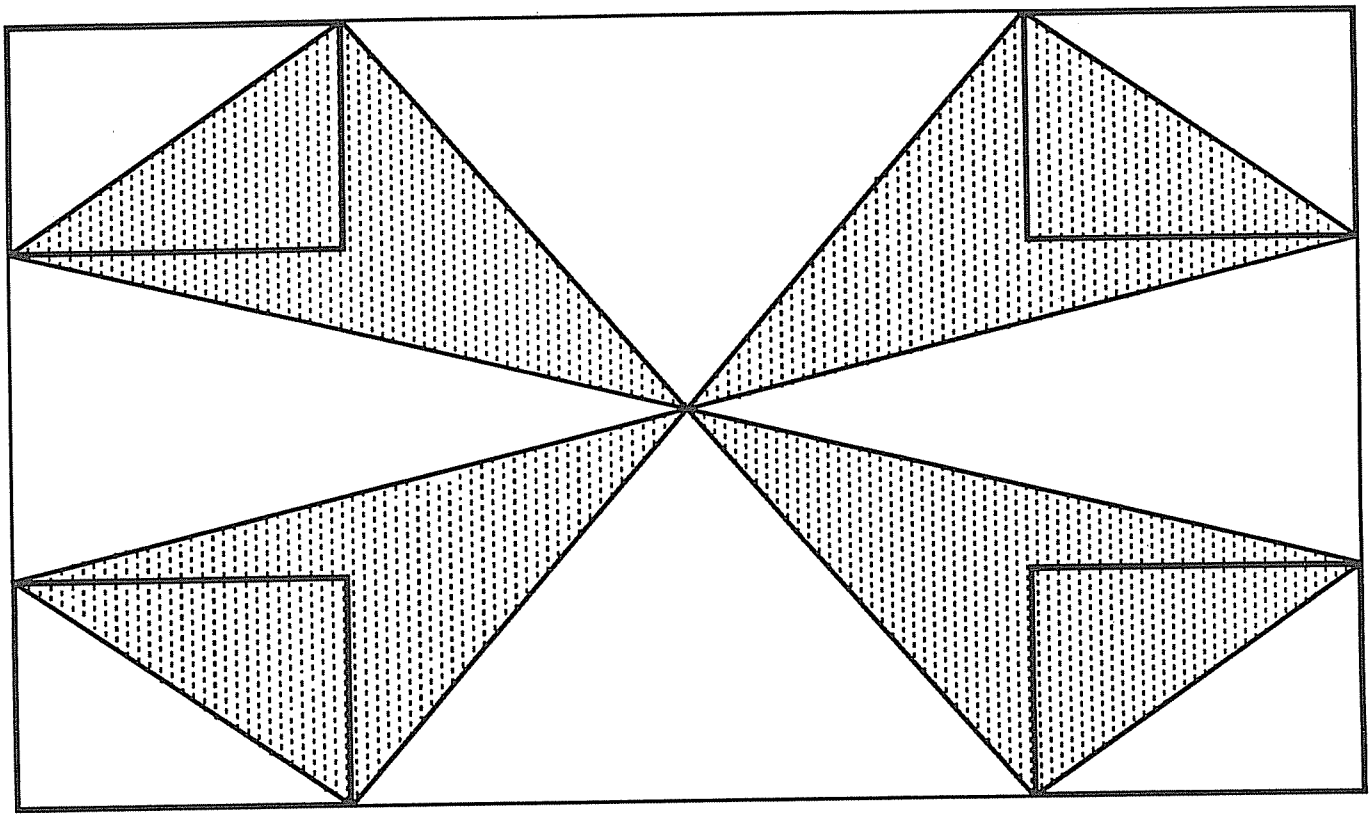
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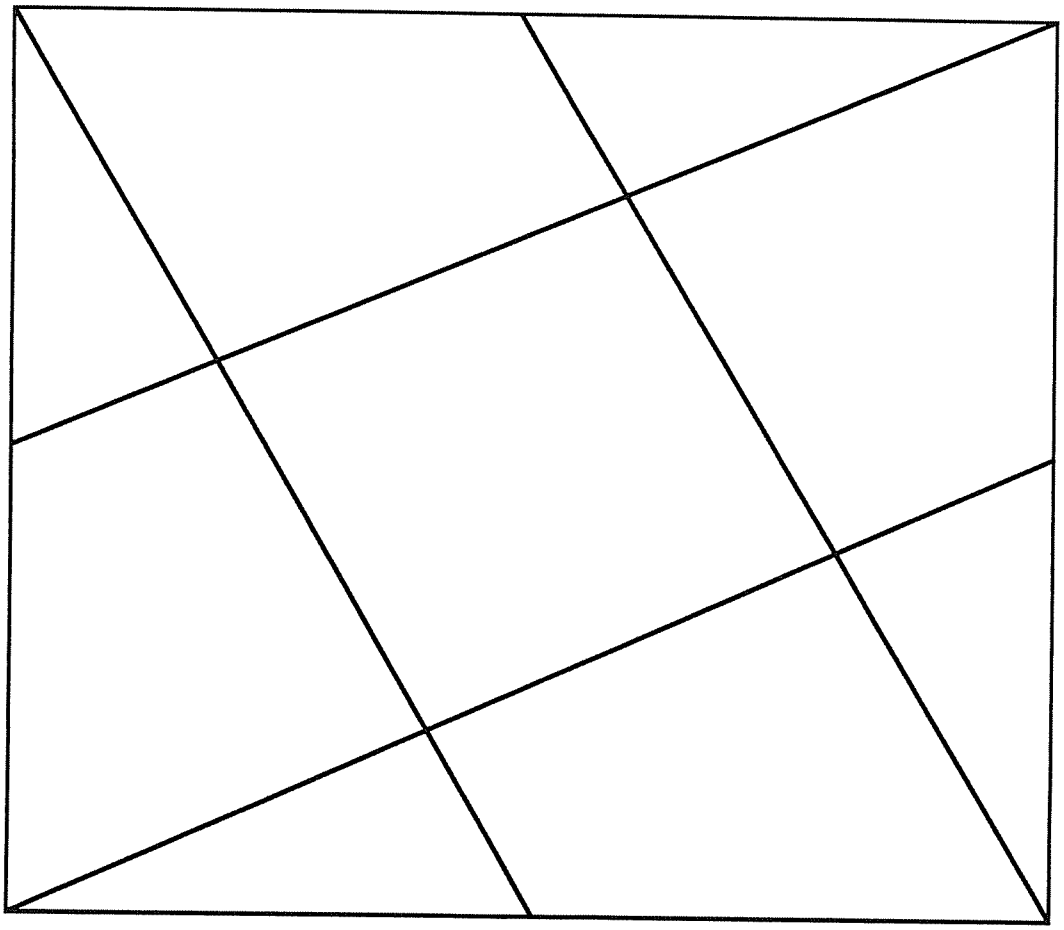
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Part of a Square

A large square is shown below. Within the square are a number of shapes.

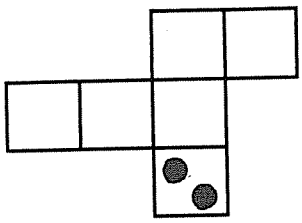
(a) Predict what fraction, and what percent, of the larger square the smaller square occupies.
(b) Cut out the square and each individual piece.
(c) Rearrange the pieces to find what fraction of the larger sqaure, the smaller square occupies.
(d) Use your results in (b). What percent of the larger square does the smaller square occupy?
(e) How accurate was your prediction in (a)?



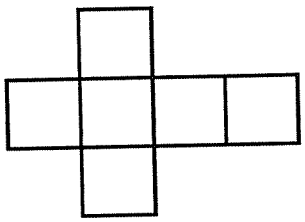
Forming a Die

1. Shown below is the net of a cube. The faces of a die are usually marked so that the opposite facess of the die sum to seven.

   (a) Predict what numbers will be placed on this net in order to form a die.
   (b) Cut out the net and form the die. How accurate was your prediction?



2. The net shown above is formed using exactly 6 squares. The squares are placed in such a way that they can be cut out and folded to form a cube. The following net can also be used to form a cube.

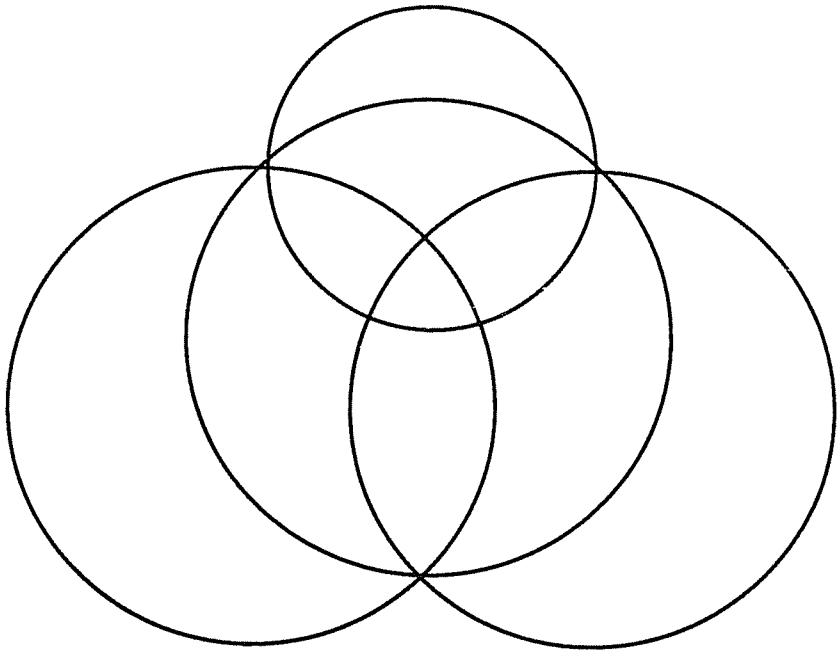


   (a) Cut out each square used to form the net above.
   (b) Place the squares in such a way that the resulting net cannot be used to form a cube.
   (c) Compare your net and the net shown above. How can you determine whether a net can be used to form a cube? Test your prediction using nets of your own.
   (d) Compare your net with others in the class. How are they alike? How are they different?

Thinking Visually

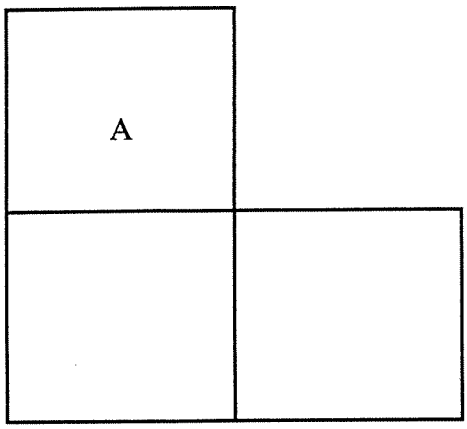
Four circles are shown below. The four circles are drawn in such a way that they overlap.

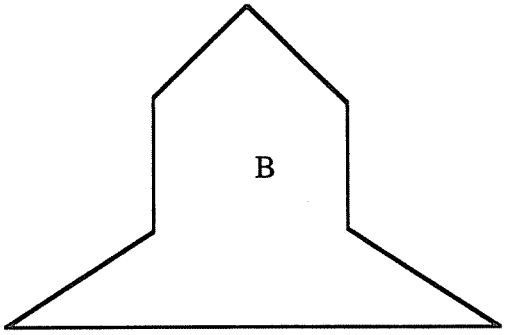
(a) How many regions are formed by the overlapping circles?
(b) Use the numbers 1, 2, 3, . . . , until you reach your number in (a). Place the numbers in each region, formed by the overlapping, circles so that the sum of the numbers in each circle is 28. Compare your solution to others in the class.
(c) Create a similar problem of your own. Solve the problem and then have someone in your class solve your problem.



Divide the Estate

Upon her death, P. A Scal left the following pieces of land to her four off spring. The offspring, being devoted "Mathly" people, decide to make copies of the land. The copies are shown below.





(a) Trace each of the above pieces of land. You will be using the copies to cut each of the pieces of land.

(b) The piece of land marked A can be divided using four cuts. Once divided, the land can then be put together to form a new piece of land that is square. The square piece of land has a hole in the middle of it that is also square. What cuts are needed in order to divide the property as described?

(c) The offspring decided that the land marked B should be divided equally amongst the four of them. How can the land be divided so that all four receive an equal share of land? What assumptions have you made when making the cuts to divide the estate amongst the offspring?

(d) Once the cuts in part (c) were made, the offspring found that the pieces of land can be put together to produce a trapezoid and a parallelogram. How can the pieces of land be put together to form the trapezoid and parallelogram? What assumptions have you made?

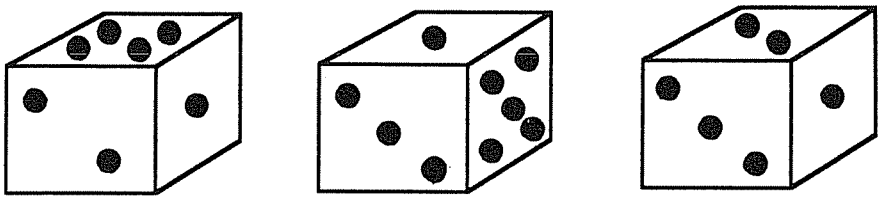
(e) Create a piece of land of your own that can be divided equally among the four offspring. Have someone in the class cut your piece of land accurately.

The Twilight Dice

"Submitted for your approval."

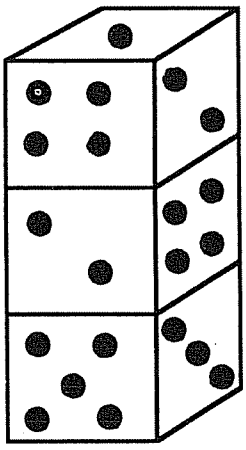
Three dice. Normally placid. However, on this day, they are going to stretch the limits of sight and of mind. They will confound, even amaze. They will set you up as a mystic in your own school. They will allow you to gather crowds, where once none stood. On this day, they will allow you to enter – the Twilight Zone.

1. Have a friend roll three dice – like those shown below.



(a) With your back turned, have your friend tell you the sum of the three faces showing.
(b) Have your friend select one of the die and add the numbers on any opposite faces to the sum given in (a). Do not allow your friend to give you this new sum.
(c) Write the sum your friend is thinking in (b) on a piece of paper. How can you correctly give the sum after your friend has completed the calculation.

2. Have a friend stack dice like the three dice shown to the right.

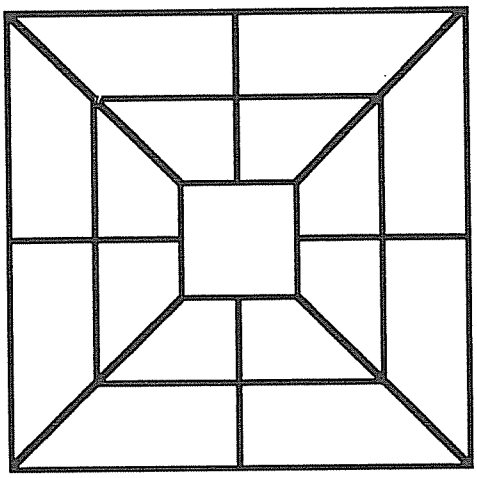


(a) Have your friend find the sum of the numbers on all hidden faces and write down the sum.

(b) Then ask your friend for the number on the top face of the top die. Using this number, and the number of dice used, you can write down the hidden sum. How can this be done? (Hint: Use dice of your own and arrange them in a stack. Find a pattern between the number of dice in the stack, the top number on the top die and what you know from the dice in Question 1.)

Tic - Tac - Toe – We Think

This is an ancient game, somewhat similar to tic-tac-toe. It was played by shepherds, who would scratch the following diagram in the sand. You may find it more convenient to draw your diagram on paper however.



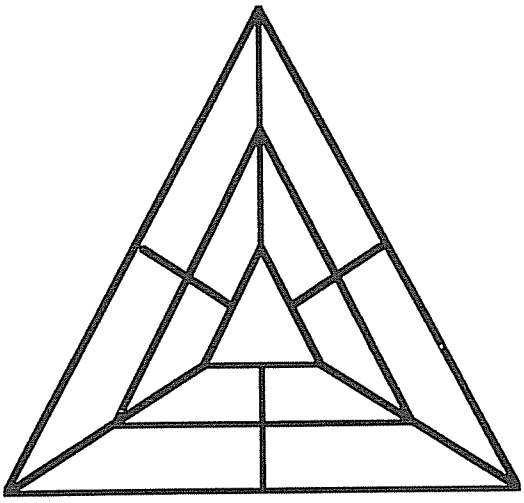
Two players take turns placing circles and crosses where the lines intersect. The first player to get "three-in-a-row" on any straight line wins.

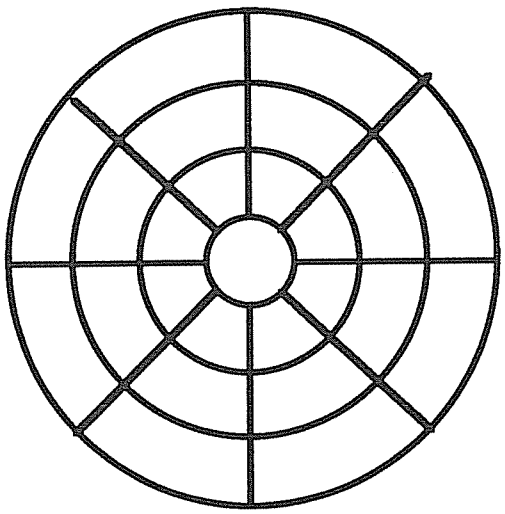
a. Play the game several times with a friend to get a feeling for it.

b. Find a strategy so that the person who starts can always win.

c. Extend your strategy so that the person who starts can always win regardless of where the first mark is placed.

d. Change the diagram and make the rules for a variation in the game (concentric circles, nested triangles, four-in-a-row). Is your variation an improvement?

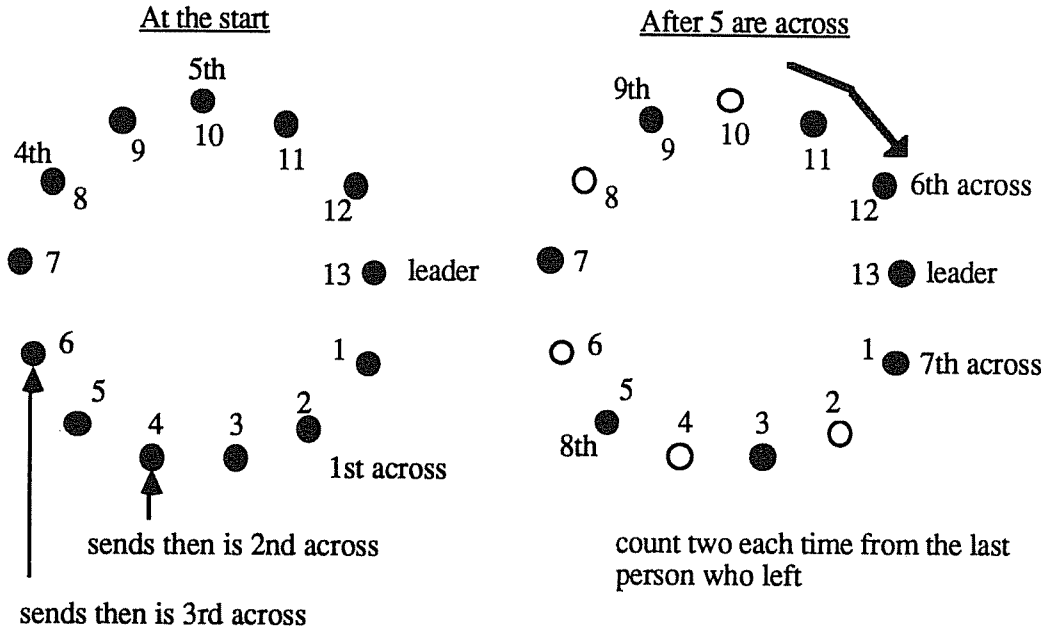




Last – But Definitely not Least

A small group of teachers are out hiking one day when they come to a deep ravine. Across the ravine is strung a pulley chair which requires one operator. All of the teachers lament loudly that one person will have to stay behind. Except for Lynn who does not like heights, but does not want to admit this to anyone. Being a mathematician Lynn thinks very hard and comes up with an idea...

Everyone is standing in a circle. Counting clockwise from the leader, the second person gets in the chair and the fourth person sends them across. Then the fourth person gets in the chair and the sixth person sends them across, etc. The last person will have to stay behind. Lynn plans to be that last person. Where in the circle does Lynn stand?



a) Fill in the table to show what place Lynn should take in each case.

Number of people in the original circle	Location of last person
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
⋮	

b) Where should Lynn stand if every 3rd person goes? Make a table and generalize it to N .

c) Where should Lynn stand if every 5th person goes? Generalize to N.

d) What if every 2nd person goes, then every 3rd person, then every 2nd person, then every 3rd person, etc?

e) Experiment with other patterns and generalize. Try counting the people off using the integers and starting at 2. The 2nd person goes, then the 3rd, then the 4th, then the 5th (the counting starts each time from the last person who left).

Solutions

Table for a) and b)

Number of people in the original circle	Location of last person for part a)	Location of last person for part b)
1	1	1
2	1	2
3	3	2
4	1	1
5	3	4
6	5	1
7	7	4
8	1	7
9	3	1
10	5	4
11	7	7
12	9	10
13	11	13
14	13	2
15	15	5
16	1	8
17	3	11
18	5	14
19	7	17
20	9	20

a) After one person is across we are back to the previous case (n - 1). So, add 2 to the location of the previous "last person". If that sum is greater than the number of people in the original circle start counting all over again.

Shortcut. Write the number of people in the circle in base 2. Then move the first digit of the base 2 number to the far right. That number is the location of the survivor.

Example: $12 = \underline{1}100_2 \Rightarrow 100\underline{1}_2 = 9$

b) As before, once the first person is across, we are back to the previous case. So, add 3 to the location of the previous "last person". If that sum is greater than the number of people in the original circle start counting all over again.

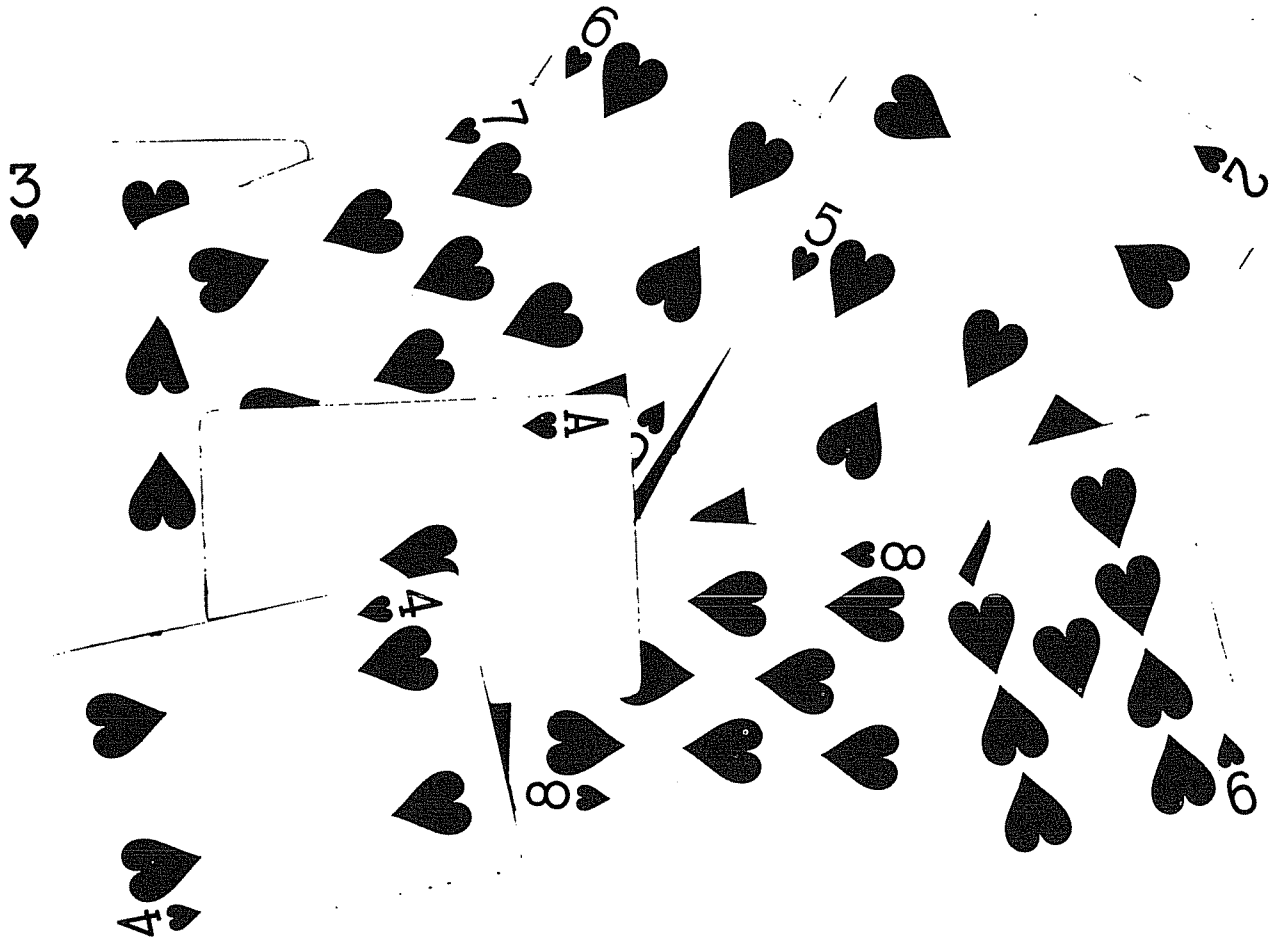
c) As before, but add 5.

d) This time alternately add 2, then 3.

Acey Ducey

There are 15 playing cards shown below. The ace has a value of either one or ten depending upon your needs. You are to play a game of fifteen with a friend. In this game, you pick up one card, your friend picks up the next, and so on until you have a total of fifteen. You may only select three cards.

Suppose you are to select first. What initial selection should you make so that you will win the game each time? Use the cards and play the game with a friend. Use the strategy that you have developed.



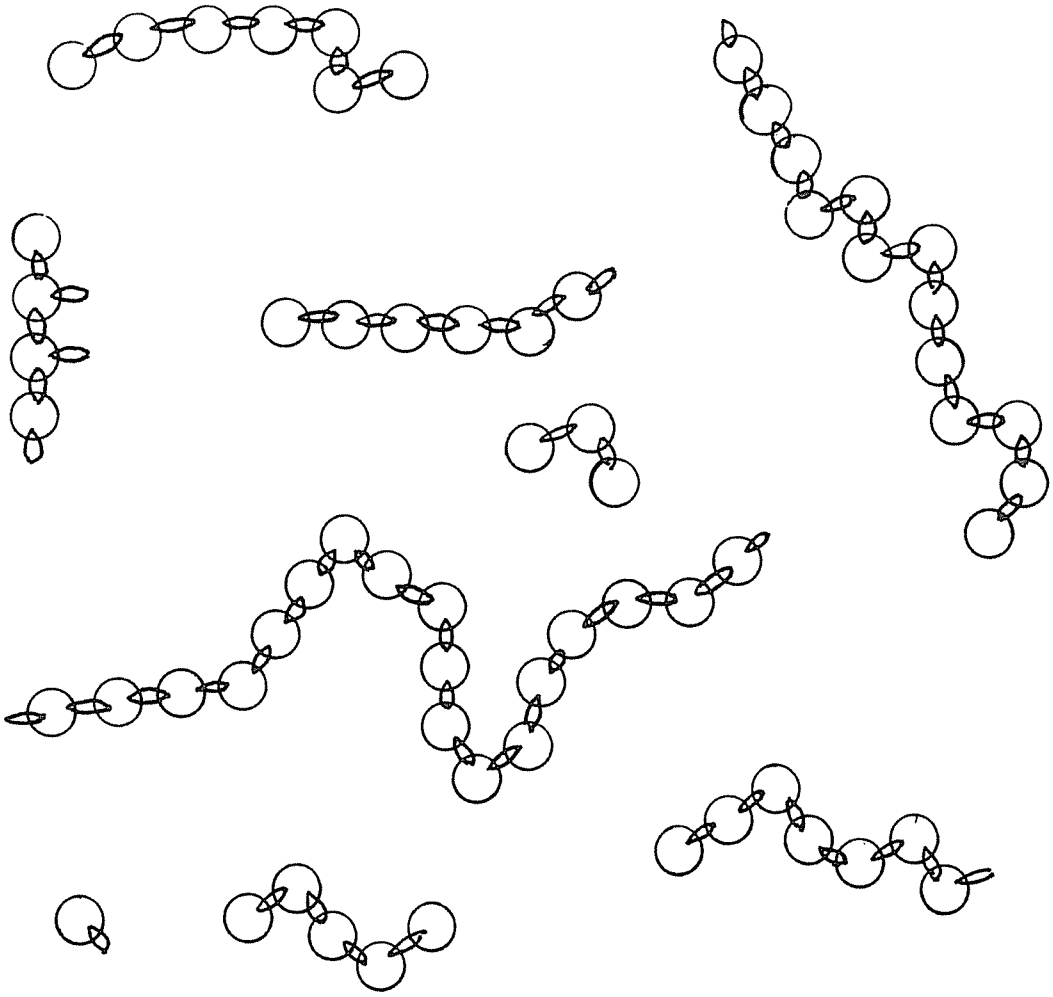
The Missing Link

Mr. Nee Juan Der Thall is the Missing Link Welder. An industrial chain has broken as shown below. An emergency call is placed to him by a company because this chain needs to be perfect in order to load and unload cargo. The chain itself is made of small rings and large rings. The rings are connected so that every large ring is connected to 2 smaller rings and 2 smaller rings are connected to 2 larger rings.

Because this is an emergency, the charge for fixing the chain is determined as follows.

- $2.75 to cut and connect a small link.
- $4.60 to cut and connect a large link.

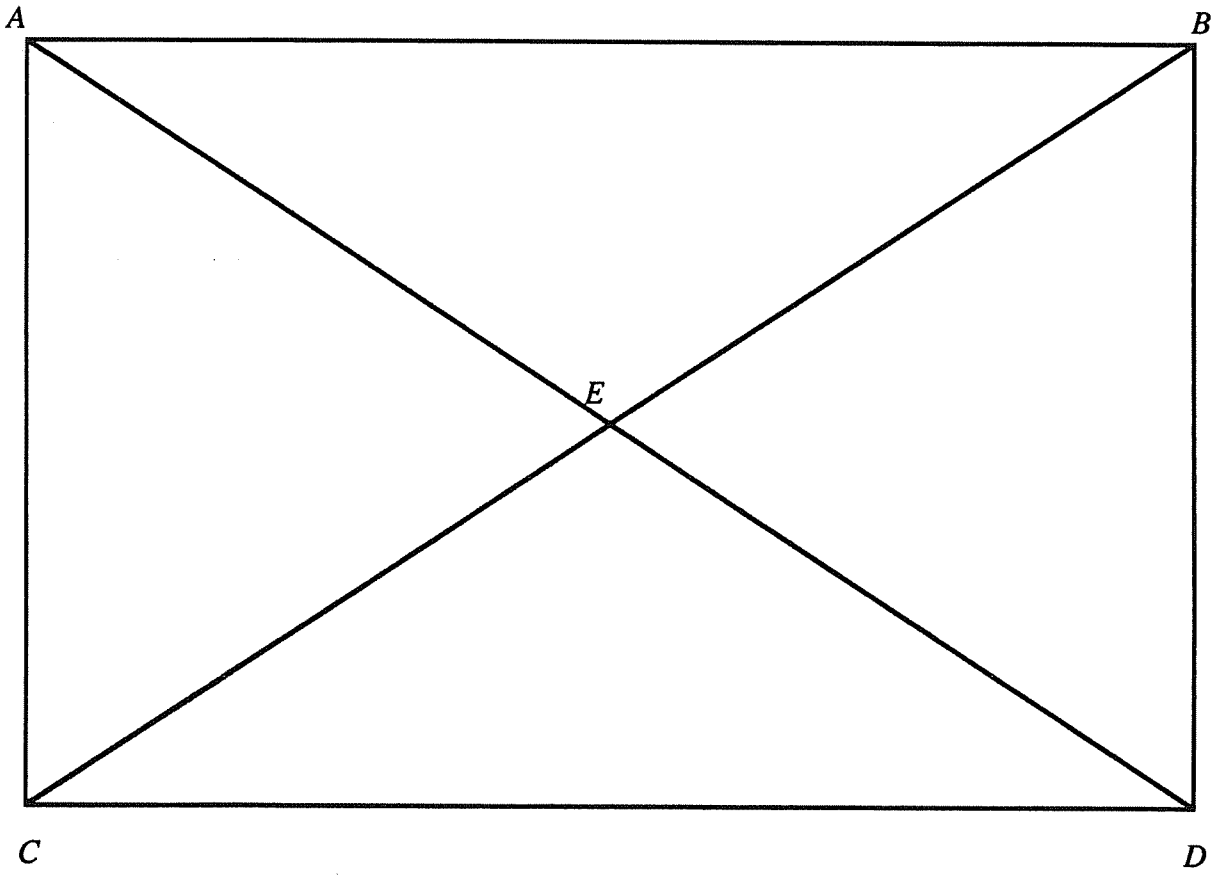
What is the least amount that should be paid to fix the chain?



Drawing a Circle

Shown below is a rectangle.
The diagonals are shown on the rectangle.
A pencil is placed at point C.
Make a hole at point C and place the pencil in the hole.

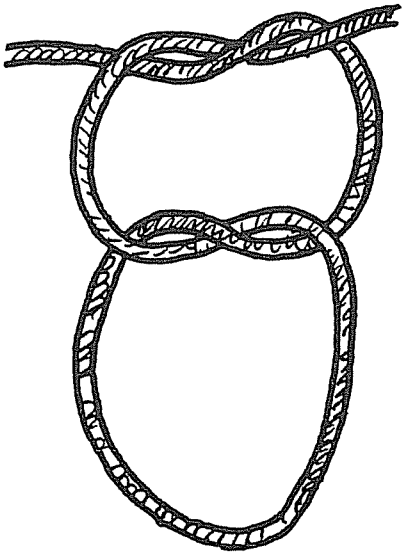
(a) Cut out the rectangle from this page and then trace it onto a piece of cardboard.
(b) Place a pin somewhere on the diagonal or sides of the rectangle. The pin is used to hold down the rectangle and becomes the centre of a circle. Spin the rectangle around the pin so that you draw a circle that has a radius of 1 cm.
(c) In how many different places can you put the pin so that you draw a cirlce with a radius of 1 cm?
(d) Where can you place the pin so that the circle drawn has the largest radius possible?

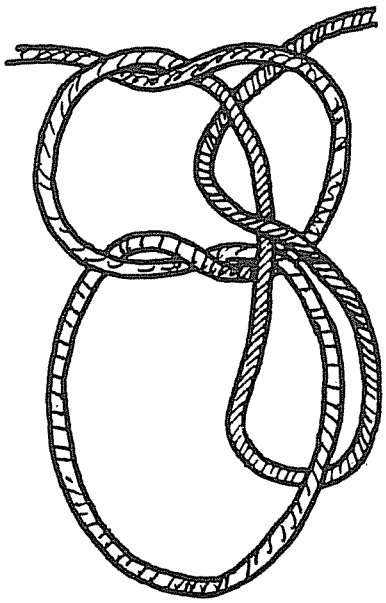


Why Knot??????????

You have probably tied many knots. From something simple, like a knot in your shoe laces, to something a bit more complex, like the knot in a bow tie, you have been working with knots for many years.

People who work around boats learn to tie many knots. I recently had the opportunity to watch a person working around the boats. She showed me the following two knots as she worked around the boat.





(a) Use string to form the knots exactly as shown above.
(b) Predict what will happen when you pull the two loose ends of the string.
(c) Pull the loose ends. What happened? Was your prediction correct?
(d) How can you explain what happened when you pulled the loose ends.

A Paradox

A paradox is often defined as two or more apparently legitimate arguments that lead to contradictory results. Two paradoxes are shown below. What went wrong?

Let	$a = 2$ and $b = 3$.
Thus,	$a + b = 5$.
Multiply both sides by $2a - 5$.	
	$2a^2 - 5a + 2ab - 5b = 10a - 25$
Add $5b - 5a$ to both sides.	
	$2a^2 - 10a + 2ab = 5a - 25 + 5b$
Factor	
	$2a(a - 5 + b) = 5(a - 5 + b)$
Divide both sides by $(a - 5 + b)$	
	$2a = 5$
	$a + a = 5$
But,	$a = 2$.
Therefore,	$2 + 2 = 5$.

A similar paradox is shown below. Use your results above to help you show what went wrong.

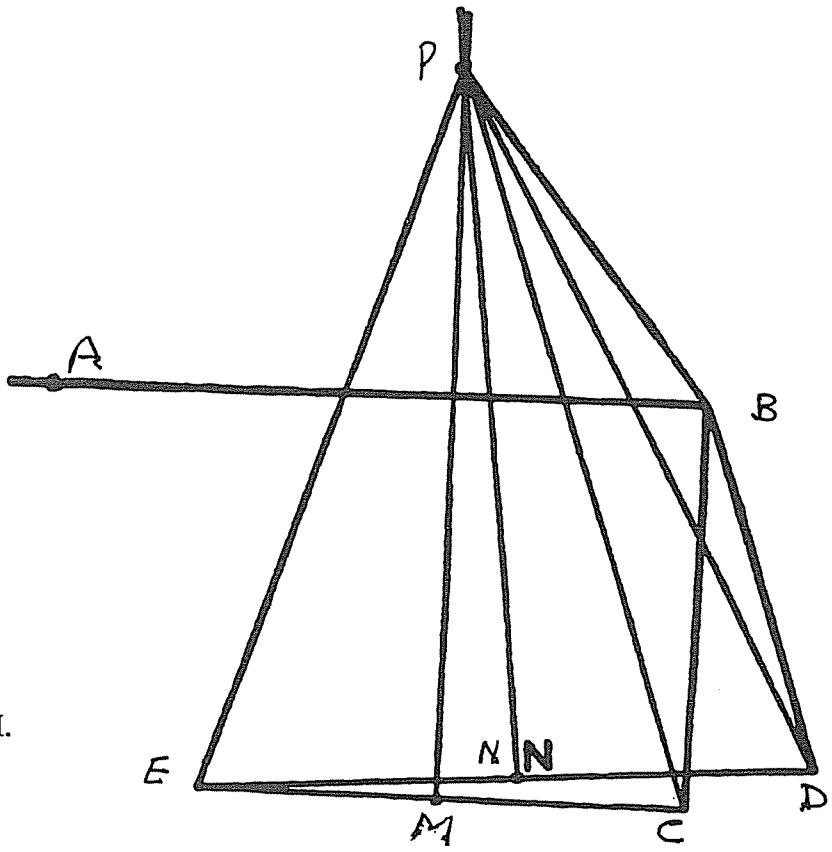
Let	$x = y$
Therfore,	$x^2 = y^2 = xy$
Now,	
	$x^2 - y^2 = x^2 - xy$
Factor.	
	$(x - y)(x + y) = x(x - y)$
Divide by $x - y$.	
	$x + y = x$
But, $x = y$.	
	$x + x = x$
	$2x = x$
	$2 = 1$

Now, try the proof on the next page. You should draw your own diagram to be sure all measurements and constructions have been done accurately. Then follow the proof. What went wrong?

Given
Let angle ABC be a right angle.
Let angle ABD be an angle greater than angle ABC.

Required
To prove that angle ABD $\cong$ angle ABC.

Proof
Construct BD $\cong$ BC.
Construct CE perpendicular to BC.
Construct MP as the perpendicular bisector of CE.
Join ED.
Construct NP as the perpendicular bisector of ED.
The two perpendicular bisectors meet at P.
Join PE, PC, PD, PB.



In ΔPEM and ΔPCM,
PM $\cong$ PM
EM $\cong$ CM
angle PME $\cong$ angle PMC.

Therefore, ΔPEM is congruent to ΔPCM.
Therefore, PE = PC.

In ΔPEN and ΔPDN,
PM = PM
EN = DN
angle PNE = angle PND.

Therfore, ΔPEN is congruent to ΔPDN.
Therefore, PE = PD.

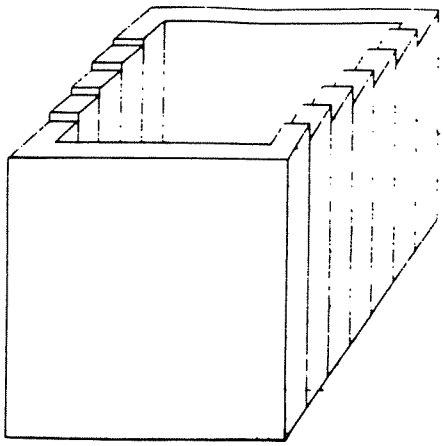
Similarly, PC = PD.

In ΔPCB and ΔPBD,
PB = PB
BC = BD
PC = PD.

Therefore, ΔPCB is congruent to ΔPBD.

By subtracting the common angle PBA, you see that angle ABC = angle ABD.

Is What You See What You Get?

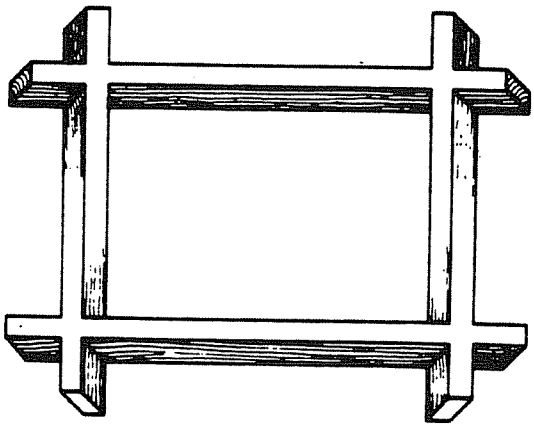


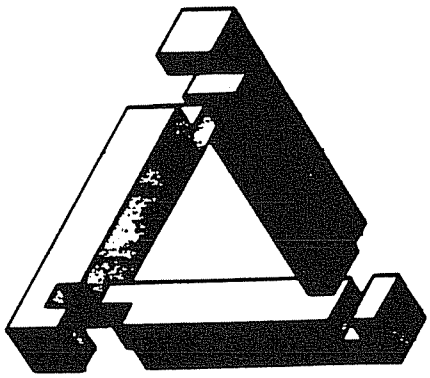
You have probably seen the works of M. C. Escher and the interesting use of perspective in his work. In fact, Escher is famous for his 'bending' of the rules of perspective to create unique effects.

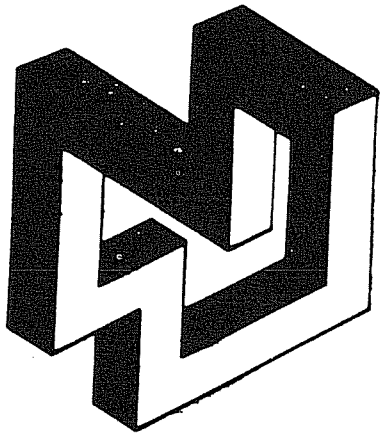
However, unless you are aware of what you are looking for, analyzing Escher's work can be difficult. Presented on this page are some simpler 'bendings' of perspective. Have your students analyze perspective in these simpler cases. Then, have them write a description of how perspective is 'bent' in each case.

The drawing shown above is called the endless stair and was first drawn by L. S. Penrose and his son, Roger, in 1958. The drawings shown below, from left to right, are

- Schapp's bookshelf.
- Something to Make.
- a Swedish Stamp.







Once your students have analyzed the drawings and summarized how perspective has been 'bent', have them try to draw a similar figure of their own. Have them compare their drawings with others in the class and summarize how they used the rules of perspective.