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SECOND ANNUAL CONFERENCE

Ontario Association for Mathematics Education

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WRITE TO THE SECRETARY TREASURER

For other upcoming items of interest see inside back cover.

1973 - 74 EXECUTIVE

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FROM THE EXECUTIVE OF OAME

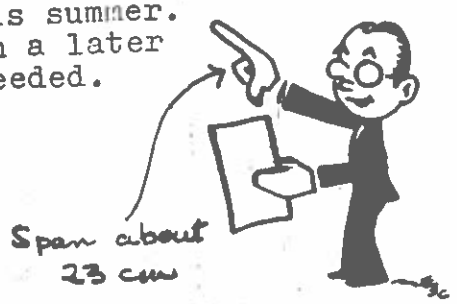
We trust that you had an enjoyable summer and that all is going well with the work of this year. O A M E looks forward serving you and having your support as we work together for Mathematics Education in Ontario.

FROM THE EDITOR'S DESK

Many thanks for the items sent in this summer. If not included here, yours will be in a later issue. More PRIMARY MATH items are needed.

SEND YOUR MATH GAME, puzzle, activity, book review, ideas to:

ABACUS EDITOR
c/o CITY MATH CENTRE
OAK PARK SCHOOL
40 HUNT CLUB DRIVE
LONDON N6H 3Y3



ITEM CREDITS: This Issue

MORE TO IT- M.Loring, Tangent, Alberta _ -K Newsl.

HAVE WE GONE FAR ENOUGH Dr. Usiskin (Philadelphia Math Teachers' Newsl.)

PAYMENTS PARADOX- D.Duncan B.Litwiller, Assoc.Profs.Math University of Iowa

GAME OF TWOS-submitted by A.W.Harris, A.C.E. London

EXPANDED NOTATION-D.H.Skypek Emory University

NEXT ISSUE: BEGINS THE SAGA of BILL & JOE-Laurier S.S.



Letters To The Editor

AS SOON AS WE GET SOME LETTERS WE WILL FILL THIS SPACE.

ABACUS ITEMS MAY BE REPRINTED WITHOUT FORMALITY EXCEPT CUSTOMARY ACKNOWLEDGEMENTS

TRY IT YOURSELF CORNER

Originality not claimed. Solutions elsewhere in this issue. Submit appropriate problems to ABACUS Editor

(1) In a box there are 64 slips of paper, each inscribed with a counting number from 1 to 64 inclusive. There are no duplications and no omissions. The slips are identical (except for the written number) and we are well stirred; five slips are then drawn at random, without replacement. What is the probability that the slips will have been drawn in ascending order (not necessarily consecutive)? (Contributed)

(2) Without using the North Pole as starting point, can you locate a point (on the surface of the earth) from which you can travel 5 miles south, thence 5 miles west, thence 5 miles north, at which time you will be back at the starting point? Locate at least 2 such points, in different latitudes. (Contributed)

• • •

INDIVIDUALIZED FLEXIBILITY?

Individualized instruction is sweet in theory and sour in practice. Only a very few so-called individualized mathematics programs truly provide flexibility for students in all three desired aspects, content to be learned, mode of learning, and rate of progress. So often, only the third aspect, rate, is made flexible.

Research has shown that mastery of mathematical skills and retention of mathematical knowledge depend primarily upon two things: (1) understanding of concepts, and (2) periodic, spaced - review of learned concepts and skills. Most individualized programs provide neither. Thus, it is no surprise that test results are showing that students are not retaining the concepts and skills they are exposed to through self-study.

The most promising of the individualized mathematics programs are those that utilize a variety of learning modes (presentations, discussions, small group work, variety of learning materials, etc.) and also have built in a carefully planned program of continual, periodic spaced-review.

WHAT DO YOU THINK?

More to it than You Think

Consider the sentence, "Five is greater than four." To master this, the child must learn five different concepts:

1. *Vocabulary* The word "greater" is not in the average Grade I child's vocabulary. Even the word "less" is not very often used, as these children tend to say, "He's got more than me," rather than, "I've got less than him." So the meaning of the words "greater" and "less" have to be taught.

2. *Word Recognition* Grade I children at this stage are only just starting to read, so they will not know any of the words in the sentence by sight.

3. *Grammar* The word "greater" applies to the first word in the sentence which is the subject. We take this for granted, but it is not so easy for children. I have found some Grade II children who didn't have this clear, as was shown by the type of mistakes they made. Whenever they had a sentence of the type, "___ is one greater than 7", they would write the correct numeral; but if the sentences had the missing number at the end, as in "7 is one greater than ___", they made mistakes. It appears that their thinking was something like, "Here is 7, and here is 'greater', so I must make 7 greater", rather than thinking, "The first number must be greater." This is even more difficult for Grade I students.

4. *Reading from left to right.* Children at this stage have not all grasped firmly the concept of expressing the order of spoken words by a left-to-right progression, but this is necessary if they are to put in writing the grammatical concept described above.

5. *Number* Finally we come to the mathematical concept that the number five is greater than the number four.

When I finish analyzing in this fashion, I am filled with admiration for the children who do succeed in mastering all this!

I have found it easier to use the symbols $>$ and $<$ first, because whichever way round you look at them, the point is always toward the lesser number, and therefore the "left to right" concept is not needed, so at first you just teach concepts (1) and (5). Then the children read what they have written, and so start to learn concepts (3) and (4). When they can read better the words may be substituted for the symbols.

Another source of confusion is in the type of picture used to teach the concepts of "greater" and "less". This reinforces an error common among young children - that of thinking that the child who has four small candies has more than the child who has three large ones. Three bears are greater than two bears, but they are not greater than two elephants, which is what some of the pictures teach. To teach this is, in effect, to teach that three dimes are greater than two quarters. To teach the correct concepts the pictures should be of objects that are the same, or at least of the same size.

Success-oriented instruction, where the success of every student is a major objective, is a must in mathematics education. Too often, mathematics teachers seem guilty of failure - oriented instruction where the main objective seems to be to guarantee failure on the part of a significant proportion of the students.

CREATIVITY

Creativity is an attitude in the teacher's mind and the students'. It should be a spirit which surfaces often in the classroom and in a variety of ways.

The classroom will never be dull or commonplace where creativity exists. So try it; you'll like it. It is a challenge, but students remember these experiences when you see them in later years; they are not forgotten. And considering all the mathematics taught, isn't that a worthwhile achievement?

* *

(Adapted from a now out-of-print Bell Telephone "Think Creative" booklet.)

* Could this be the beginning of the restoration of The Lost Art ?

Many younger teachers may wonder what we oldtimers did in the pre-television days when a trip to Niagara Falls was a four hour drive from Toronto and a one hundred dollar increment was a big raise. Keen teachers used to spend hours on end doing old Problems Papers. These problems constituted a special grade 13 exam for students trying for scholarships. In 1957, the Department of Education published solutions and comments each year. As I recall, Stanton and Fryer got into the act by publishing solutions in the Waterloo Bulletin. But -- 30 years ago, us oldtimers had to work them out for ourselves. Doing old Problems Papers is the Lost Art.

All this reminiscing was brought about by a request from Fred Pulfer of Peterborough for Question 4 on the 1947 paper. Yes, I had a solution to the question in my old files! I used to spend many pleasant evenings with people like Harold Minieley of Kitchener just doing problems. I understand the younger breed of teacher spends his (oops, her) time travelling the globe, having boisterous parties or just complaining about the P.T.R. I enjoyed doing old Problems Papers and I submit the oldest paper that I worked on as a sample. ➟

UPPER SCHOOL PROBLEMS, 1923

NOTE:—*Ten questions constitute a full paper.*

1. (*a*) Find the nth term of the series
$$5+6+9+18+45+126+\ldots\ldots$$
(*b*) Find the coefficient of x^n in the expansion of
$$(1+x)^n:(1-x).$$

2. Given $x^2+y^2+z^2=1$,
$a^2+b^2+c^2=1$,
$ax+by+cz=1$,
prove $x=a$, $y=b$, and $z=c$.

3. Given two circles and a straight line AB, construct a straight line parallel and equal to AB and with its extremities one on each of the two given circles.

State completely the conditions determining the possibility of a solution, and the number of solutions for this problem.

4. If P is any point on AB, one side of a given triangle ABC, and Q is the intersection of AC and the polar of P with respect to the circumscribing circle, show that
$$PQ^2=PA.PB+QA.QC.$$

5. Find the square root of $(1-\sin A)(1-\sin B)$ in the form $\sin x-\sin y$.

6. Solve $\tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36}$.

7. The square on the tangent from any point P of one circle to another circle is proportional to the distance of P from the radical axis of the two circles.

8. If Q is any point on the circumference of $x^2+y^2=1$, and P is the point on OQ produced, such that QP is equal to the ordinate of Q, find the equation of the locus of P.

9. Find the number of different ways in which 8 different books may be distributed to 4 people, it being required that each person shall receive at least one book.

10. Prove that the circle through the feet of the perpendiculars from the vertices of any triangle upon the opposite sides, bisects these sides of the triangle and also bisects the lines joining the vertices of the triangle to the intersection of the perpendiculars.

11. If the perpendiculars from A, B, and C, the vertices of a triangle upon any straight line, be denoted by p, q, and r, then for a certain angle θ we have $p-q=c\sin(B+\theta)$, $r-q=a\sin\theta$. Eliminate θ from these two equations and reduce the result to the form—
$$a^2(r-p)(q-p)+b^2(p-q)(r-q)+c^2(p-r)(q-r)=4T^2.$$
What is the meaning of T in this equation?

12. The four sides of a square circumscribing a circle cut any other tangent to the circle harmonically.

13. Find a simplified expression for the value at the beginning of the first year of twenty annual payments of a tax of \$1 upon each person in the country on Jan. 1st of each year, payable on Dec. 31st of that year, given that the initial population is 10,000,000, that its increase owing to the excess of births over all losses is at the rate of one per cent. per year on the population on Jan 1st, and that there is a steady immigration of 200,000 per year. The rate of interest is 4%.

14. Prove that the distance between the centres of the circumscribed and the inscribed circles of a triangle is a mean proportional between R and $R-2r$, where R and r are the radii of these circles.

15. If the feet of the perpendiculars from the vertices of any triangle upon the opposite sides are L, M, and N, prove that LMN is $\frac{1}{2}R^2\sin 2A\sin 2B\sin 2C$.



LOST ART cont'd.

My notes show that I got all the questions except #12. The problems are probably too hard for most young teachers - but they get easier in the 1950's. If enough of you are tired of globe hopping, boisterous parties and complaining about P.T.R., perhaps the Lost Art will help you fill your idle hours.

If the response to this article is favourable, I will suggest to the editors of the Abacus that a Lost Art column be established in which more problems can be submitted and some of your replies may appear. Maybe teachers do other things than travel, party and complain. Let's hear from you.

Send responses to: THE LOST ART
c/o John Del Grande, Math.Coord.
Bd. of Ed. for Borough of North York
5050 Yonge St. Willowdale

A further response to the thought "What Mathematics Means to Me"

First of all, mathematics is fundamental in giving a method of understanding and making effective use of our physical world.
Next it is a tantalizing and compelling challenge in the pursuit of mathematical problems, giving vistas of opportunities for further investigation
Our problem as teachers is to convey to our pupils a feeling for the above and also equip him to deal with everyday things.

An ORME member



A man is one fourth of the way across a railroad trestle when he sees a train that is one trestle length behind him approaching the trestle. Which way should he run?

The combined age of Martha and Ann is 60 years, and Ann is as old as Martha was when Ann was 1/4 the age that Martha will be when Ann is as old as Martha is now. How old is Ann?

Have We Gone Far Enough With the New Mathematics?

1. There have been 3 types of changes in the school math curriculum in the past 15 yrs.
   a. Attempts to make school math more like the field of math.
   b. Attempts to change methodology.
   c. Attempts to relate to life or things non-mathematical.

It is the first of these that has normally been called the "new math."

2. Society has not understood what math is about.
   a. Research indicates that even 11th grade math students feel most math has been here since ancient times
   b. Most adults can't do their "old math"
   c. Early reform efforts seemed to show that new math must be hard. This is not so — math does not have to be hard to be good.
   d. **Skills are down everywhere** — not just in math. It's society as much as our curriculum.

3. An example of "newer" mathematics — the general treatment of congruence through transformations
   a. It's easy and the old definitions of congruent segments, angles, triangles come out as what they should be — theorems.
   b. Applications — e.g., mass-produced objects, right and left hands.
4. Second example — treatment of division
   a. Students think division of fractions is different from division of decimals which is different from division of complex numbers. Symbols don't help; witness:
5. Have We Gone Far Enough?
   a. Would we think a biology teacher was good if he or she did not keep up with the changing field?
   b. Other disciplines have changed more than we have in the past 15 years.
   c. Math is constantly changing — witness computers, probability, statistics, linear algebra — all burgeoning since World War II.
   d. **We have never gone far enough.** We must keep up with our discipline and its applications. Students learn no math from TV, movies, or outside school. If we do not teach them good mathematics, they will not learn it.

ANSWERS TO.

TRY IT YOURSELF CORNER

(1) The probability is 1 over 5! or 1/120. The original number of slips need not have been 64; the result is the same if the number of slips is 5 or any larger number.

(2) Imagine a circle of latitude, near the South Pole, whose circumference is 5 miles. Any point 5 miles. Any point 5 miles north of the circle will satisfy the requirements. To obtain a point of different latitude, imagine a circle of latitude, again near the South Pole, whose circumference is 2-1/2 miles; and any point 5 miles north of that circle will also serve.

A PAYMENTS PARADOX

Mathematics teachers and students are often interested in problems which appear to be straightforward but have surprising solutions. An interesting example is illustrated by this problem. Which of the following jobs would provide you with the better salary:

Job A—$8000 the first year and a raise of $800 for each succeeding year.

Job B—$4000 the first six months and a raise of $200 for each six months period

It is evident that in the first year Job A provides the better salary. Routine arithmetic calculations yield these results:

Job A will pay $8000 for the first year while Job B will pay $4000 the first six months and $4200 the second six months for a total salary of $8200 for the first year.

A plausible conjecture, however, is that beginning in the second year, the salary for Job A will surpass that of Job B. After all, one would think that $800 increase once a year is twice as good as two increases of $200 for a year! Routine calculations, however, yield a surprising result. Salaries earned during the second year are:

Job A—$8800

Job B—$4400 + $4600 (Recall a salary increase of $200 for every six months period) or $9000

Similarily the salaries during the third year are:

Job A—$9600

Job B—$4800 + $5000 or $9800

Note that for each of the first three years Job B paid $200 more per year than Job A; thus, a $200 raise each six month period produces a better total increase than an annual raise of $800. You will find that this is also true for all succeeding years.

ANOTHER ONE-LINER

Take one letter away from a six-letter word and leave twelve.

THE GAME OF TWO'S

Those of you who own the Encyclopedia Britannica may have read the article on Mathematics in the 1972 Yearbook. In recent years the annual review of mathematical developments has been written by Dr. Irving Kaplanski of the University of Chicago, a Toronto graduate who, along with John Coleman was on the team which won the Putman (or was it Putnam?) prize in the 1930's.

Kaplanski reported that Fermat numbers ($F_n=2^{2^n}+1$) are in the news again. John Billhart and Michael Morrison have factored $F_7=2^{128}+1$. As I have not room to write the factors in the margin, I suggest you look up the 1972 Yearbook if you are interested.

The purpose of this note is to describe a mathematical recreation having an interesting new unproved theorem connected with it.

"An entertaining new puzzle that passed around in mathematical circles in 1971 illustrated how a simply stated problem in number theory might be surprisingly difficult. Start with any odd number. Multiply by 3, add 1, and then divide by the appropriate power of 2 so as to get an odd number again. Then repeat the procedure. For instance, starting with 29 we get in succession $(3x29+1)\div8=11$, then 17, 13, 5, 1. The question is: do we eventually reach 1, starting from any odd number? There appeared to be no way of attacking this problem by known methods."[1]

This puzzle could provide an interesting game for young students. Competitors might choose a starting number, and then the object could be to achieve a minimum number of steps, (eg. $(5X3+1)\div16 = 1$) or the greatest number of steps (try 73). Speed in mental calculations might also be stressed. Memorization of the powers of 2 would give an advantage. TRY IT ON YOUR MINI-CALCULATOR !

[1] The 1972 Britannica Book of the Year, page 447.

DATES

(PLEASE SEND ANNOUNCEMENT)

Leadership Math Seminar-Niagara Falls Oct.23-25.
School Science & Math Assoc.-Cincinnati Nov.7-9.
Prog.Book. Dr.Rouse, Miami Univ.Oxford,Ohio45056)

YORK UNIVERSITY - MATH '75
MAY 28-30 1975
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Other Canadian Math Conferences- Quebec Assoc. Prof. 28-30
- Nova Scotia, Manitoba & Alberta Assoc. mid October.

A C T I V I T I E S on E X P A N D E D N O T A T I O N

To provide drill on expanded notation (and perhaps some insight at the symbolic level):

1. Give each student enough 5 by 8 inch cards to make for himself the following sets of numeral cards:

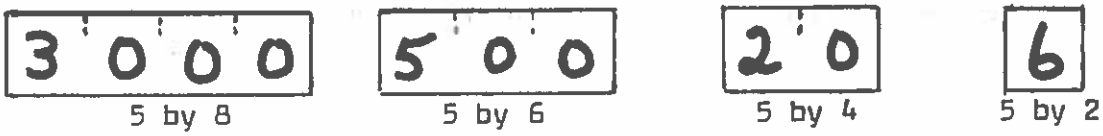
(a) Nine 5 by 8 cards on which the student writes 1000, 2000, 3000, 4000, ..., 9000;

(b) Nine 5 by 6 cards (cut 5 by 8 cards into two cards -- a 5 by 6 and a 5 by 2) on which the student writes 100, 200, 300, ..., 900;

(c) Nine 5 by 4 cards (cut 5 by 8 cards in half) on which he writes 10, 20, 30, ..., 90;

(d) Nine 5 by 2 cards on which the student writes 1, 2, 3, ..., 9.

In writing the numerals on the cards, the students should be careful to write each digit in a 5 by 2 space, as pictured here:



2. Now, if the four cards shown in the figure above are clipped together, by joining the right-hand edges, one has

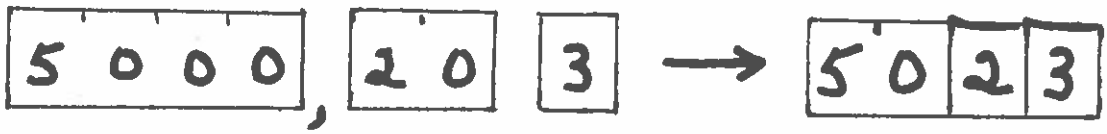


, the sum
3000 + 500 + 20 + 6.

3. As a class activity, the teacher names a number less than 10000 and each student must select the appropriate cards to put together to form the corresponding numerals. For instance:

Teacher: "Five Thousand Twenty Three"

Student:



4. Alternate activities might include:

(a) students working in small groups, with one of them calling out the numbers to be represented.

(b) students using the cards to reproduce numerals written on a work-sheet or those in textbook exercises.

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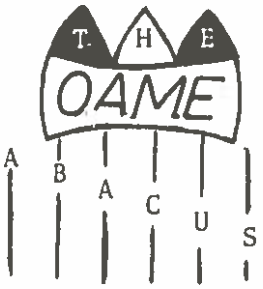


SOME TARGET DATES FOR INTRODUCTION OF METRIC MEASURE

1. Metric weather reporting to the public to begin in April April, 1975.
Phase 1 — Reporting temperatures in Celsius degrees
Phase 2 — Reporting precipitation in centimeters
Phase 3 — Reporting windspeeds in kilometers per hour
Reporting visibility in kilometers.

2. Metric units for the grain handling trade is planned for the 1976-77 crop year. (Use of the metric tonne.)

3. Provincial highways departments plan conversion of all factors involved in highways during the period 1975-79. Metric highway signs will be introduced in September 1977.



"MATHEMATICS EDUCATORS
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