A B A C U S

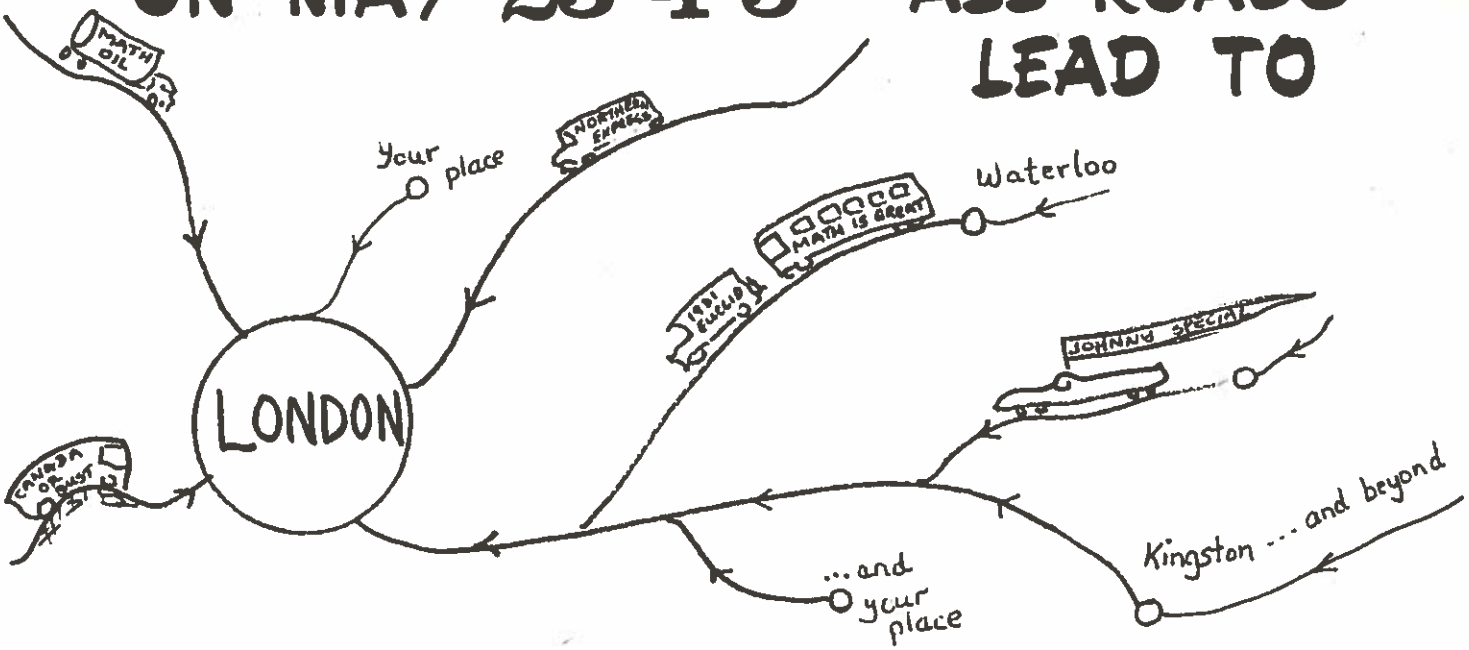
LONDON · THE FOREST CITY · THE NEW WITH THE OLD
OAME 1975

OAME

A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

VOLUME TWO NUMBER FOUR MAY 1975

ON MAY 23-4-5 ALL ROADS LEAD TO



THE SECOND ANNUAL CONFERENCE

Ontario Association for Mathematics Education

SOCIAL SCIENCES BUILDING
U.W.O.
LONDON

YOUR MEMBERSHIP IN OAME INCLUDES
THE A B A C U S YOU CAN COUNT ON IT!

WRITE TO THE SECRETARY TREASURER

ST. CATHARINES IN '76

1974 - 75 EXECUTIVE

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GREETINGS FROM THE EXECUTIVE:

We look forward to meeting, in London, with all members of OAME both "old" and "new." We anticipate an excellent conference and know that the members and friends of our association will join in making it a success.

The first two years of the Abacus are coming to a close. Under the capable editorship of Brock Rachar, the Abacus is doing a great job in helping the classroom teacher in her or his daily work. The executive and council of OAME wish to thank Brock for a job well done.

With the September issue of the Abacus, Andy Czempinski of Iroquois Falls Secondary School will take over as editor for a two year term. We wish him well!

John Egsgard,
President, OAME.

FROM THE EDITOR

I have appreciated the opportunity of serving OAME as editor of ABACUS since its beginning.

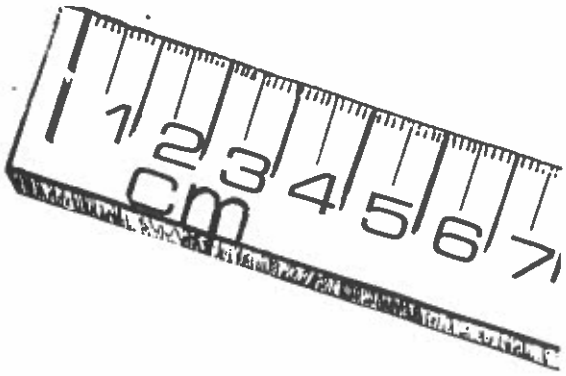
The contributions of members and friends have made possible what we have hoped has been an interesting, useful and varied publication.

We know that Andy will continue to receive this support and wish him the very best as he takes over.

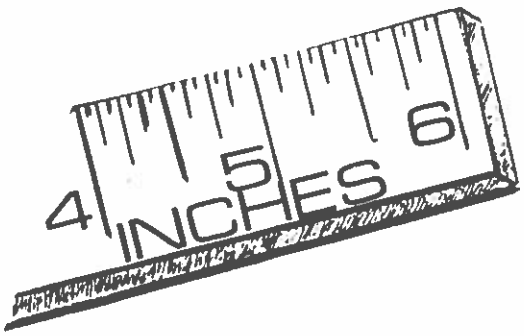
Let me take this opportunity to wish my many friends in mathematics an enjoyable summer and express my hope to see you at the conference in London.

Sincerely,

Brock Rachar.







AT U.W.O. MAY 23, 24, 25

Do you remember when there were HIGH SCHOOL ENTRANCE EXAMINATIONS?

Compare some example questions to those from C. T. B. S. (1967)

THE OLD	THE NEW.
What is the difference between these two types of exams? They reflect different eras and different purposes. Let us take a moment to analyse the questions given in the February issue.	
A. 5/8 = 0.625 8)5.000 understanding of division sign division facts required	2/3 = .66, 5/10 = .5, .7, 1/6 = 0.16 equivalence required order required (number line)
B. 2758 x6308 mechanical skills in multiplication needed	389 understanding of place x27 value needed mechanical skill 7780
C. a two step problem 750 pupils bought 27,000 total value 27,000 x 25¢ average value $\frac{27000 \times 25}{750}$ reading a worded problem understanding of average relationship $1 = 100¢	a one step problem average egg production $\frac{11,616}{66}$ reading a worded problem understanding of term average
D. $3\frac{5}{9} + 4\frac{2}{3} - 2\frac{5}{6}$ $= 3\frac{10}{18} + 4\frac{12}{18} - 2\frac{15}{18}$ $= (3+4-2) + (\frac{22}{18} - \frac{15}{18})$ $= 5\frac{7}{18}$ 2 step mechanical common denominator required	$1/4 + 3/8 = 5/8$ interpreting a picture diagram to see a fraction number sentence.

Purposes:

The High School Entrance Exam

This examination given at the end of Grade 8, was a measuring stick. It measured subject matter known. It set a basic standard of work required of all students before they were allowed to continue their further mathematical studies.

Thus the elementary school was expected to meet a set of uniform requirements. These requirements were in large measure the following: ability to compute with whole numbers, fractions and decimals; to solve word problems and do simple geometric contructions.

The entrance exam was a measuring instrument at the end of a pupil's Elementary School life.

C. T. B. S

"A good test tells two things about you. First it tells how much you know about the subject - i.e. understanding the number system, mathematics terms and operations, problem solving. Second, it shows how you stand among other pupils in your grade."

Accompanying any test is a detailed skills classification list so that the teacher may aid pupils in the redevelopment of that area. Skills which are basic to living as well as basic to later work in the Secondary School.

Some of the items in the February issue were taken from the London Board of Education Mathematics Diagnostic - Achievement Survey, 1973.

This survey was set up in part because it was felt that C.T.B.S. did not really fit the aims and objectives in mathematics in that city. It was felt that there was a greater need to investigate the understanding of concepts, and perhaps less requirement for measuring the needlessly long mechanical arithmetic skill.

Concern for the future. item from C.T.B.S.

If set K is a subset of set Q, and set Q is a subset of set L, which statement below is always true?

1) Set L is a subset of set Q.
2) Set L is a subset of set K.
3) Set Q is a subset of set K.
4) Set K is a subset of set L.

The majority of the Mathematics taught in the Grade 8 program seems to be that which is required in higher level maths. Perhaps we should give consideration to the inclusion of topics which will be of vital concern to our young people as they leave school and enter the work force. Even more, we might well be concerned at the attitudes developed, that each person might feel confident to use the mathematics learned in the years in school.

R. S. Smith Faculty of Education University of Western Ontario

(1946)

A dealer bought a radio listed at $65.00. He received a discount of 29 % for paying the account promptly.

a) What was the net cost of the radio to the dealer?

b) If the dealer allowed 40% of the net cost for overhead and profit, what was the selling price of the radio?

Mr. Dort's gas and electric bill for Nov. was $9.60. If he paid it by Dec. 10, he would receive a 7½ % discount. How much would Mr. Dort save by paying the bill by Dec. 10 ?

1) $.71 3) $1.15

2) $.72 4) not given

(1944)

Write the correct formula to be used to work each of the following:

a) Find the volume of a tank 2 yd. long, 4 ft. wide, 1 yd. deep.

b) What is the interest on $730 for 50 days at an interest rate of 4 % per annum?

c) Find the area of a triangle which is 4 in. in height and has a base of 7 in. in length.

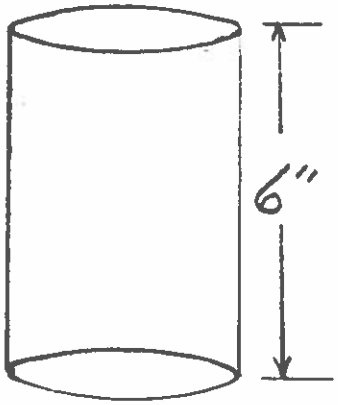
The area of the base of the cylinder below is 12 square inches. What is the volume of the cylinder?

1) 18 cu. in.

2) 30 cu in.

3) 60 cu. in.

4) 72 cu. in.



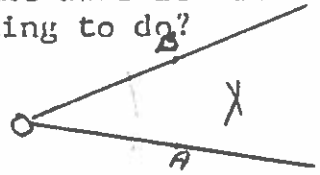
(1940)

Using a scale of 1/8 in. to the rod, draw a triangle to represent a piece of property whose sides measure 20 rd., 15rd., & 25 rd.

a) By using ruler and compasses, bisect the largest angle.

c) Using a protractor, find the measure remaining angles.

A student performed the construction below with a straight edge and a compass. What does it seem the student was attempting to do?

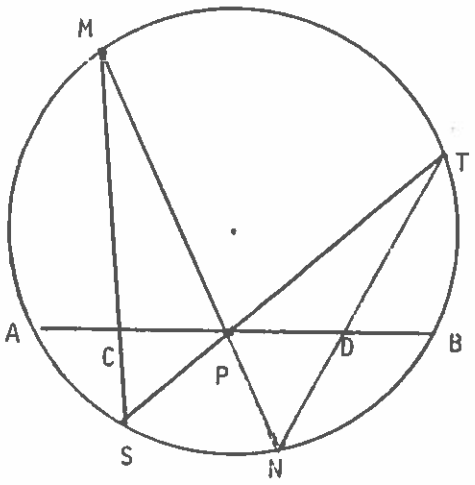


1) bisect angle AOB
2) construct an equilateral triangle
3) construct a pair of parallel lines
4) construct a rectangle.

The Lost Art

Last issue's challenge didn't have specific problems to solve. Here are a few that may be of interest.

1. This problem was issued as a challenge by Ann Harris of Althouse College several years ago. The problem is an old chestnut and those of you who remember P. A. Petrie of O.C.E. might recall it as his famous "butterfly" problem. Given a circle and any point P in its interior. Chord AB is drawn through P such that AP = PB. $\overline{MN}$ and $\overline{ST}$ are two chords through P. If $\overline{MS}$ intersects $\overline{AB}$ at C and $\overline{TN}$ intersects $\overline{AB}$ at D, prove CP = PD.



2. This is a good problem suggested by Frank Asbury. A cube 10 cm to the edge is held on a horizontal surface so that one of its longest diagonals is vertical. What is the distance of each corner of the cube from the horizontal surface?

3. Here's a question from the 1948 Problems paper that uses the idea of symmetry.

Factor: $a^4(b - c) + b^4(c - a) + c^4(a - b)$

4. Here's a problem from the 1940 paper. I remember writing the paper. I missed the problem on the exam but solved it when I got home.

On the Russian Peasant Multiplication Algorithm

- *William J. Bruce* (ALTA. MATH.TEACH.) DELTA-K

This simple algorithm for multiplication of natural numbers involves addition, multiplication only by two, and division only by two. It is commonly known as the Russian peasant method because of the limited arithmetical skills required. We consider an example in detail and then prove by mathematical induction that the algorithm holds for any two natural numbers.

Our problem is to find the product of 226 and 35 and the format given below is chosen so as to fully explain each step:

	A	B		Product
a	226	35	b	ab
2a	452	17	b_1	$ab = 2ab_1 + a$
4a	904	8	b_2	$ab = 4ab_2 + 2a + a$
8a	1808	4	b_3	$ab = 8ab_3 + 2a + a$
16a	3616	2	b_4	$ab = 16ab_4 + 2a + a$
32a	$\frac{7232}{7910}$	1	b_5	$ab = 32ab_5 + 2a + a$

The procedure illustrated above is as follows:

1. Double a = 226 in column A to obtain 2a = 452. Reduce b = 35 to an even number, namely 34, and divide 34 by 2 to obtain $b_1 = 17$ in column B. Our new product, $2ab_1$, is short by a = 226, since we reduced b by 1. We now have

$$ab = 2ab_1 + a.$$

2. Double 2a = 452 in column A to get 4a = 904. Reduce $b_1 = 17$ to an even number again, namely 16, and divide 16 by 2 to obtain $b_2 = 8$ in column B. Our new product, $4ab_2$, is short by an <u>additional</u> amount 2a = 452, since we reduced b_1 by 1. We now have

$$ab = 4ab_2 + 2a + a.$$

3. Double 4a = 904 in column A. Since $b_2 = 8$ is even, it is divisible by 2 and we obtain $b_3 = 4$. Our new product, $8ab_3$, is not short by any <u>additional</u> amount,

since b_2 is even. We now have

$$ab = 8ab_3 + 2a + a.$$

4. Continue this procedure to obtain 16a and 32a in column A, and b_4 and b_5 in column B. In these steps no additional omissions have been made in obtaining $16ab_4$ and $32ab_5$. Thus

$$ab = 16ab_4 + 2a + a$$

and

$$ab = 32ab_5 + 2a + a.$$

Now $32ab_5$ is trivially obtained by multiplying by 1. Hence we have

$$P = ab = 32ab_5 + 2a + a$$
$$= 7232 + 452 + 226 = 7910.$$

We can verify that this is correct since

$$b_5 = \frac{b_2}{8} = \frac{b_1 - 1}{16} = \frac{\frac{b-1}{2} - 1}{16} = \frac{b-3}{32}$$

so that

$$P = 32a \frac{b-3}{32} + 2a + a$$
$$= ab - 3a + 2a + a = ab.$$

This final step in the algorithm can now be stated briefly. Every time the number in column B becomes even, there is no remainder upon division by 2, hence for even numbers in column B we omit the corresponding numbers of column A. The sum of the remaining numbers in column A gives the product ab.

The simplicity of the steps in the example indicate that, intuitively at least, we can see that this algorithm generalizes for products of any natural numbers. Our proof by induction proceeds in the following manner -

The algorithm is true for b =1.

We assume it is true for $b \leq k$, $k \leq b-1$.

Case I: b even. We must prove true for b = k+1.

Now $a(k+1) = 2a \frac{k+1}{2} = 2ab_1 = 2a \frac{b}{2} = ab$, since b is even.

Case II: b odd. We must prove true for b = k+1.

Now $a(k+1) = ak + a = 2a \frac{k}{2} + a = 2ab_1 + a$, since b is odd.

But $2ab_1 + a = 2a \frac{b-1}{2} + a = ab$.

It follows by the principle of mathematical induction that the algorithm is true for every natural number b. Note that the algorithm is commutative and that the number of steps will be fewer when we choose the smaller number for division by 2.

POLAR BEARS AND THE FLAT EARTH

by Tom O'Shea,
7 - 30 Baylor Avenue,
Winnipeg

It is ironic that while many writings on mathematics are available which stress the need to link creative thought and mathematics the average mathematics classroom today (the reader's excepted of course) indicates little attempt to bridge the gap between formal mathematics and the free play of imagination.

My experiences in teaching Geometry, particularly introductory Geometry at the Grade Ten level, have led me to believe that there is tremendous scope for the imagination at this point. In particular, the introduction of the undefined terms of point, line, and plane may be approached in imaginative ways. For example, one can ask the class to define a straight line or to give examples of a straight line and then successively demolish the examples presented even to the point of arguing that the path of a ray of light through space is not what we conceive of as "straight" since light is affected by gravity. (And what better point to bring in a discussion of "black holes" - surely one of the wildest creations of the imagination.)

Such a discussion points out to the students that many of their cherished assumptions of our existence are based on what is, in their experience, flimsy evidence or simply hearsay evidence. One particular development that I have used successfully with a number of classes is the forceful, sincere (apparently) presentation of the theory that the earth is flat. This initially is met with guffaws and jeers of derision. But, as in the matter of the straight line, student arguments may be quite handily "answered" and a good case made for the flat, stationary earth. Red herrings such as "Do you really think that the Australians are hanging from the earth by their feet?" or "Is it conceivable to you that on the surface of the earth you are travelling at 1000 mph?" in many cases bring long discussed brain-power to bear on the topic.

The mathematical spinoffs of a persistent argument of this type are almost unlimited. In this example students develop an interest in: knowing how we measure the distance to the sun and the stars, the difference between circular and elliptical orbits (and at this point it is most instructive to point out the almost circular orbit as opposed to the usually exaggerated presentation of elliptical planetary paths), and the curvature of the earth as it affects our measurement of distance and area.

As an example of this technique of provocation I would like to present an article which I prepared for our student newspaper using the local situation when I was teaching in Churchill, Manitoba. The article appeared in the section of the paper set aside for guest articles called the "The Madman Speaks". Perhaps the title was appropriate.

"The recent excitement about the polar bear threat to the Churchill area has produced results that have gone unnoticed by the public but have produced a profound new spirit of inquiry among mathematicians in several locations.

"Dr. R. S. Perlman of the University of Fensterwurfer, West Germany, has propounded the following problem -- 'If a polar bear is transported 150 miles from a garbage dump, how long will it take him to return to the dump if he travels at an average speed of 7 1/2 miles per day?' A quick calculation gives an answer of 20 days. But, surprisingly, the evidence indicates that the bears around Churchill do not conform to traditional mathematics as their average time to return is 15 days.'

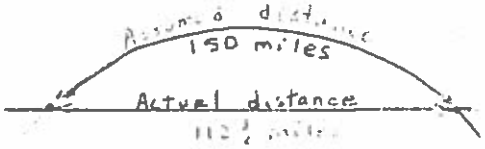
"This leads to two possible conclusions -- either (i) the bears are hungrier than 7 1/2 miles per day, or (ii) 150 divided by 7 1/2 does not equal 20. The first possibility has been discarded on the word of the Department of Game and Fisheries as their examinations of the bears established a maximum hunger factor of 8 miles per day. The second possibility, proposed by Dr. Chee Yin Ghee at the Institute for Mathematical Advancement, does not seem to hold much attraction for most mathematicians. For to concede that 150 divided by 7 1/2 is, in fact, 15 would indicate that there are complications involving calculations with fractions which exceed those encountered by most high school students.

"Mr. James Quigley, F. R. S., of the University of Shangri-la, has suggested that the whole question has no meaning since he has never seen a polar bear and doubts that such an animal exists.

"Eenowtooak of the Canadian High Arctic Research Board agrees with Quigley's statement that the question has no meaning, but on different grounds. He argues that although he knows that polar bears exist, he is certain that numbers larger than 6 do not exist and, therefore, such a calculation would be nonsensical. Furthermore, he maintains that the practical problems of dividing 6 seals among 3 people giving 2 seals to each person has meaning, but dividing 150 (if such a number exists) miles by 7 1/2 miles per day (if such a number or thing exists) is a typical Kabloona exercise in futility.

"It is ironic that the so-called experts (especially Mr. Chee Yin Ghee) have failed to recognize the basic fact that would explain the entire situation. The simple fact is that the earth is <u>flat</u> rather than spherical.

"Consider the diagram at the right. The assumed distance of 150 miles along a circular arc leads to a distance of 112 1/2 miles along the chord which would conform to the distance along a flat surface.



"Now 112 1/2 divided by 7 1/2 is exactly 15 and is in perfect agreement with the experimental figures!

"I have communicated the above calculations to the gentlemen mentioned previously. Messrs. Quigley and Eenowtooak still are of the opinion that the question has no meaning. Dr. Chee continues to maintain that our methods for dealing with fractions are all all wrong. Dr. Perlman has agreed to submit the idea to his graduate students. He feels that in it there may lie implications for the military strategy of West Germany since the distance from Munich to Ottawa may not be as large as previously thought."

* * * * *

WHAT'S YOUR NAME WORTH?

<u>OBJECTIVE:</u> to review or introduce column addition and additon by endings.

<u>MATERIALS:</u> a list of the last names of the students in your class.

<u>PROCEDURE:</u> Assign the letters of the alphabet values in this way:

- assign the letters A to J inclusive the numerals
  0 to 9
- assign the letters K to S inclusively the values
  10 to 90 (using tens only)
- assign the letters T through Z the values
  100 to 700 (using hundreds only)

Using the above information find the value of your name?

Example: Your last name is Blow the value of your name is found in this way:

B = 1
L = 20
O = 50
W = 400

Your name is worth 1 + 20 + 50 + 400 = 471

Use the class list to find the values of the names of the other students in your room. Who has the greatest valued name?

* * *

March 4–6, 1976 – NCTM Name-of-Site, Detroit



Metric Research

Dr. Howard James Brooks of Platteville, Wisconsin recently completed a doctoral dissertation in an attempt to determine whether the metric system is best presented on its own or in relation to the customary system of measurement. The dissertation was entitled "A Comparison of Instructional Approaches In Teaching The Metric System To Seventh Grade Students With Varying Mental Ability."

The conclusions of the study were:

1. Both of the instructional approaches used to present the metric system were significantly more effective in terms of achievement for students with high mental ability as compared to students with low mental ability. It was concluded therefore that both treatments were more effective for high ability students than for low ability students. The same was true when the retention was checked two weeks later.

2. It was also ascertained that immediately after the treatments were administered, the mean scores of those students receiving exclusively metric instruction were not significantly different from those students who received metric instruction in relationship to the customary system. It was concluded therefore that teaching the metric system exclusively on its own is no more effective than teaching it in relationship to the customary system.

3. In terms of retention, the high mental ability students receiving metric instruction exclusively scored significantly higher than those students receiving metric instruction in relationship to the customary system. It was concluded therefore that in terms of retention, instruction presented exclusively metric was significantly more effective for students with high mental ability than instruction presented metric in relationship to the customary system.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Canada's nickel 25 cents won't buy you much, but the coin substitutes as a good 5 gram mass. A half-gallon milk carton closely approximates a square decimetre in cross section: a height of 10 cm, accordingly, marks off 1 litre. What other common objects do you know that have useful metric measures? (Canada's "big cent" of grandfather's day, significantly, had diameter 'exactly' one inch.)

METRIC UPDATE by H. D. Allen

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

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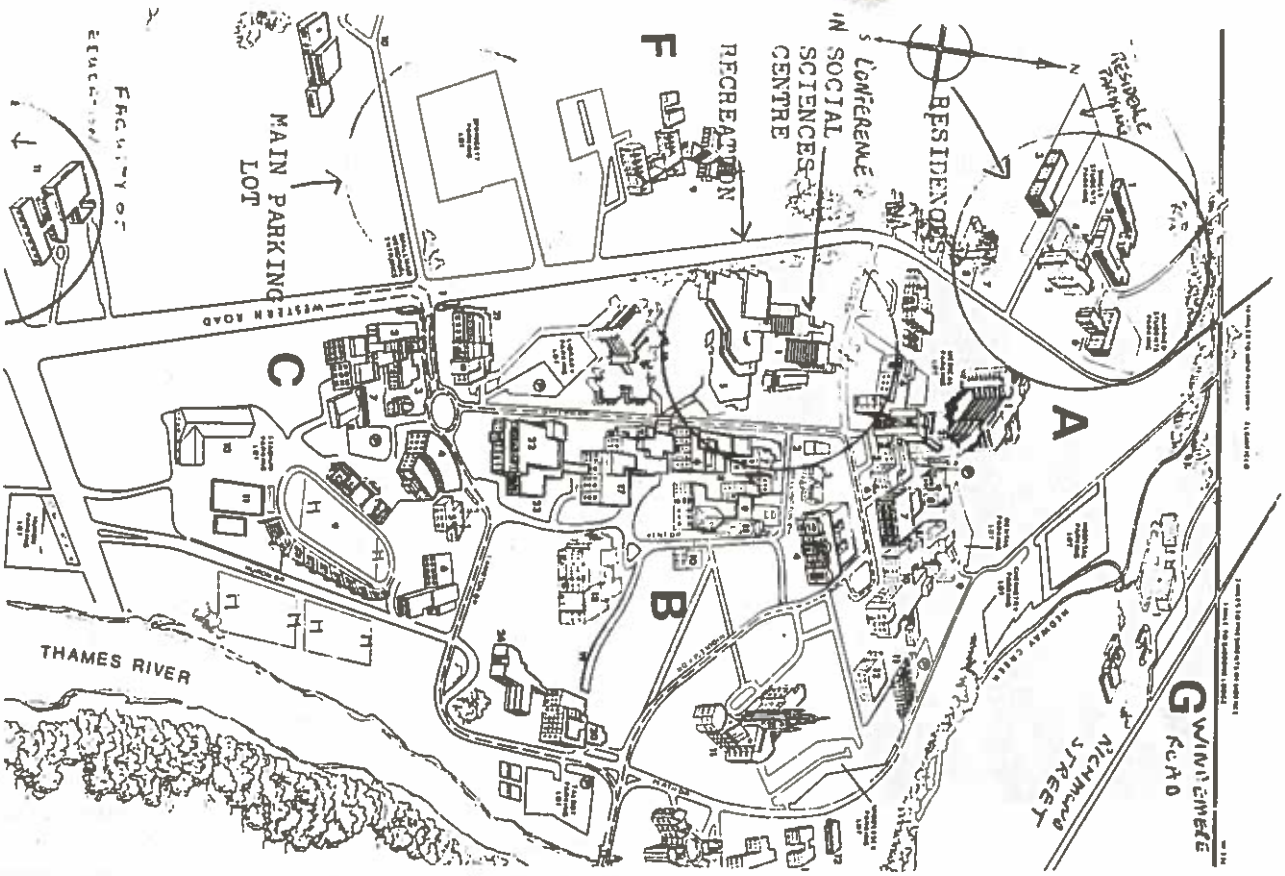
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