# ▲ Editorial

*by Gary Flewelling*

The Ontario Ministry of Education has set out to restructure education from K to OAC. Programs from Grades 1-9 are now to be based on the learning outcomes contained in the working paper *The Common Curriculum, Grades 1-9,* February 1993. This document supersedes *The Formative Years,* 1975, and the subject guidelines for Grades 7 to 9, including those developed under *Ontario Schools, Intermediate and Senior Divisions* (OSIS), 1989. Before the mathematics component of the program envisaged in *The Common Curriculum* can become a classroom reality, the learning outcomes will need to be interpreted, expanded upon, and clarified. There also needs to be a clear understanding of the new emphasis in mathematics education and how the new mathematics program should differ from the present one. This inaugural "focus issue" of *The Ontario Mathematics Gazette* has been designed to provide for some of this needed clarification for mathematics in grades 7, 8 and 9, the Transition Years.


## Acknowledgements

As editors, we are pleased to acknowledge the contribution of several people in the preparation of this issue of the Gazette.

Of course, we are grateful for the tremendous work of the two teams that prepared ***The Focus on Renewal in Mathematics*** and ***The Transition Years: A Working Document.*** These two documents form the foundation of the entire issue.

Many teachers from across the province contributed articles and activities. Each is identified with his/her article. Amy Parker gathered and edited Abacus material, which we have blended into the activity package. Mary Codrington, of the Wellington County Board of Education and Lynn Hamilton, of the University of Guelph, created the computer files for our desktop publishers, who in turn, did their usual superlative work. Jennifer Kells, a mathematics student at the University of Guelph, provided many hours of invaluable (and free!) help proofing the document.

Finally, we thank OAME for its sponsorship of this very special Focus edition of the Ontario Mathematics Gazette.

**Gary Flewelling,** *Guest Editor*
Wellington County Board of Education

**Jack Weiner,** *Editor*
University of Guelph


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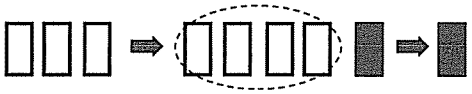
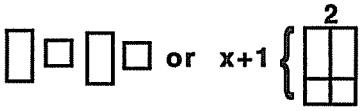
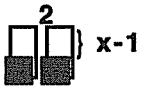
# MANIPULATIVES: USING INTEGER CHIPS

*Physical models allow students to use concrete things to help them think about and come to an understanding of abstract things. Shown below is a developmental sequence, using coloured chips, to teach students how to carry out operations on integers.*

| Numerical Expression | Physical Representation | Comments |
|---|---|---|
| 1 | ○ | a green chip |
| 3 | ○○○ | 3 green chips |
| -1 | ● | a red chip |
| -4 | ●●●● | 4 red chips |
| (any pair of opposites) | ○ and ●, or ○○ and ●● etc. | equal numbers of each chip |
| 0 | ○● (or any pair of opposites) | the important "zero property" |
| 3+(-4) |  | using the zero property to combine integers |
| (-2) + (+4) + (-5) |  | Of which chip are there more? *(gives sign of sum).* How many more? *(gives magnitude of sum)* |
| 3-5 |  | add zeros when necessary |
| -3 – (-4) |  | student eventually concludes, "to subtract, add opposite" |
| 3-4+5-6 *[think 3 + (-4) + 5 + (-6)]* | without chips | Of which are there more? (ans. -ve) How many more? (ans. 2) |
| 2x(-3) |  | use integer addition 2 groups of -3 (=-6) |
| 3 x (-3) = ___ 2 x 6 = ___<br>2 x (-3) = ___ 1 x 6 = ___<br>1 x (-3) = ___ 0 x 6 = ___<br>0 x (-3) = ___ -1 x 6 = ___<br>-1 x (-3) = ___ -2 x 6 = ___ | without chips | use patterns to give meaning to (-ve) x (-ve) and (-ve) x (+ve) |
| a) 12÷(-2) = ?<br>b) (-12)÷(-3) = ? | without chips | use multiplication knowledge<br>a) ? x (-2) = 12<br>b) ? x (-3) = -12 |

# MANIPULATIVES: USING ALGEBRA TILES

*Shown below is a developmental sequence, using coloured tiles, to teach (Grade 9) students how to work with algebraic expressions. The emphasis here is to develop algebraic skills necessary for simplifying and solving linear equations. Note how the developmental sequence parallels that of the integer sequence on the previous page.*

| Algebraic Expression | Physical Representation | Comments |
|---|---|---|
| 1 |  | a green square with area 1 sq. unit  |
| x |  | a green rectangle with area x sq. units  |
| 3x |  | 3 green rectangles |
| –x |  | a red rectangle |
| Any pair of opposites |  | |
| 0 |  or any pair of opposites | "zero property" |
| 2x +3 |  | unlike tiles |
| 3x–4x |  | add zeros if necessary |
| 2x –x – 3x *[think: 2x+(–x)+(-3x)]* |  | Of which tile are there more? (ans. -ve) How many more? (ans. 2). Therefore answer -2x |
| 2x+1+x–4 |  | combine like terms |
| (3x-1)–(2x–2) |  | adding zeros when necessary. student concludes "to subtract, add opposites" |
| 2 (x+1) |  | multiplication as addition; multiplication as area |
| 2 (x–1) |  | multiplication as area using zero property |

# ▲ Student Math Journals

*In this article M. Elaine Harvey from Kingston shares her experiences with student math journals.*

Tell your class that they are going to begin keeping a math journal. Students will have five or ten minutes at the end of math class every Friday to make their journal entries. They will write about something to do with the class, their understanding of the work, or their goals or achievements in the course. Tell the students that you will read what they write and that you will frequently respond in writing, but that you will honour the confidentiality of their journal entries. You will not show their journals or discuss their comments with anyone else. However, give them permission to show comments to anyone they wish. The students who wrote the comments that are quoted in this article have given me written permission to publish them anonymously. Keep the journals locked in the classroom so that there is no security risk or danger of losing them. I have changed any names that appear in their comments. It is important to emphasize to your students that the journals are for communication only and not for evaluation.

I am usually pretty casual about my instructions as to what I want the student to write. Usually at the beginning of the course I ask them to say whether they like math and what kind of achievement they are aiming for in the course.

**Student:** *Math courses seem to be the stable factor in my mark record.*

When I read those entries, I usually keep a tally and afterwards tell the students that, say, 60% of them like math, 20% hate it and the rest can tolerate it.

After the first week or two of the course I ask them to indicate whether they have any positive or negative comments to make about how the class is run. As I read their journal entries I write my own responses and make notes on general trends that I feel are worth sharing with the class. When I adjust my teaching as a result of their comments, I want them to know that it was their comments that made the difference. I also make a list of those comments that I want to follow up in person such as the following:

**Student:** *I'm a little bit confused on some of the long division questions but after practising I think I'm getting along a bit better.*

Sometimes I will ask them how they feel about the course so far or how they feel about the test that is coming up or the one that they just got back.

**Teacher:** *Your unit 1 mark is 86%. I was very pleased with that.*

**Student:** *Hey not Bad. But Unit 2 is gettin wierd I don't understand.*

Note that I never mark spelling or grammar errors in the students' journals because I use them as a communication tool. As such the reader has just as much obligation to understand as the writer has to express the ideas clearly. Acting in an authoritative manner by marking errors limits the freedom of expression of the student.

Sometimes I don't give any suggestions at all:

**Student:** *We lost our first football game and I was quite upset. Math is going well.*

**Teacher:** *Sorry about the loss. I hope the rest of the season goes better. Your Unit 1 math mark is 84%. I'm pleased with that. Are you?*

**Student:** *Yes, I am pleased! Hopefully it will continue.*

**Another student:** *Mary (not her real name) and I help each other with our math but I will try not to interrupt the class anymore. Sorry! I'm not really pleased with that mark but I know it's my fault and I will try and bring it up.*

I never write instructions or questions for journal entries on the board because I am trying to break down the barriers to communication that traditionally exist between teacher and student. I want to make it as casual as possible.

I obtained class sets of duotangs for the journals in colour codes—one colour for each class. I have reused these several times. I find the colour coding useful, but if your school can't afford duotangs, surely you or your students can be innovative and create some kind of cover. Just remember that uniformity is an advantage if you are carrying them home to read or handing them out in the classroom.

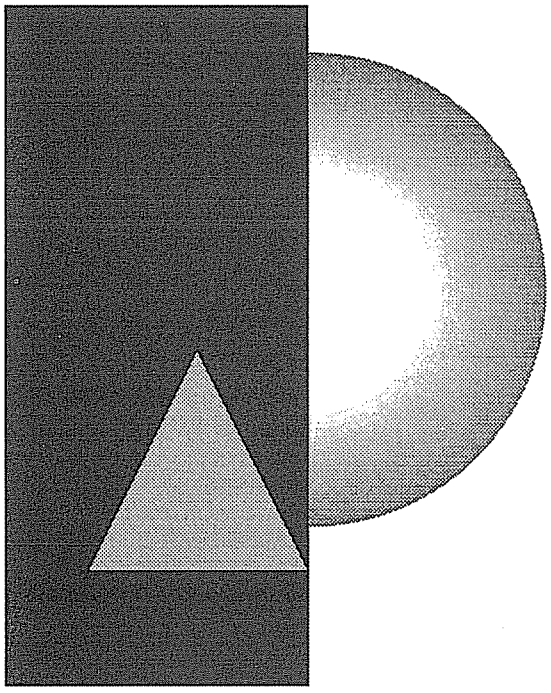
With encouragement students will begin to write notes within their math questions. Perhaps they will explain what method they should use even though they are unable to get started with a question. Or they may note that they recognize that an answer doesn't make sense and give the reason they feel that way. Sometimes one might say, "I'm not sure how to do this but I'll try anyway". This is one way of giving themselves permission to make mistakes without erasing their whole answer.

Soon they will be ready to attempt a written explanation of a particular mathematical technique. It is best to start with a technique that you are sure the students understand. When they become accustomed to doing this, you might be able to ask them to learn a new concept on their own and write a description of it. It would be considered effective if other students could follow it. This may open the door to students instructing each other, or to conducting interviews with other students or adults to research a variety of topics.

# ▲ Learning Activities

*The kind of activities in which students are involved and the way they are involved are two crucial factors in creating meaningful learning experiences. The following pages contain sample activities, each of which exhibits a number of the following characteristics:*

- **relevant to students**
- **challenging and engaging**
- **involves the use of manipulatives**
- **integrates problem solving**
- **has an investigative flavour**
- **encourages the student to make generalizations**
- **encourages discussion**
- **involves students working in small groups**
- **open-ended**
- **makes connections to various areas in mathematics, to other subjects and to interesting contexts**
- **encourages reflection**
- **encourages the meaningful involvement of a wide range of students**
- **encourages the use of a variety of assessment techniques.**



"Strategies for using learning materials should include activities that enable students to work co-operatively. In working with others, students encounter different perspectives and approaches that can help them improve their own information-gathering skills and enhance their ability to understand and apply the information they have collected."

(Teaching Destreamed Grade 9 Classes, pg. 5)

# ▲ The Buffalo Problem

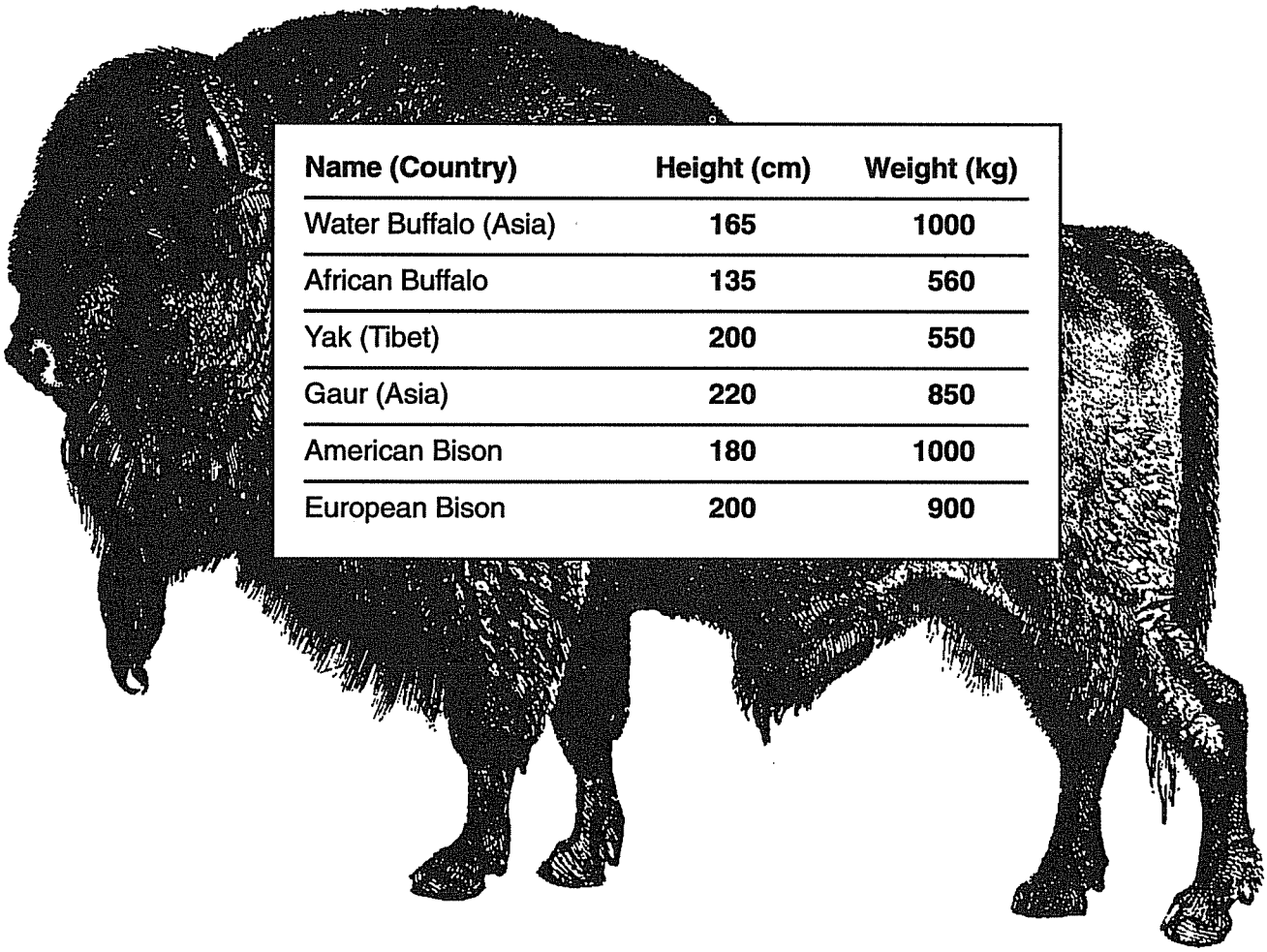
*The following question appeared in an insert in a '93 NCTM News Bulletin.*
*Such open-ended problems are a wonderful source of mathematically rich activity and discussion.*

This problem should be given to small groups to work on. Students can be asked to individually write up their responses to the question. The teacher will have to employ a holistic marking strategy here.

## A Sample Holistic Scoring Strategy

1. Place student responses in four or five piles from high to low levels of performance.
2. Identify the characteristics that place a response on a given pile (holistic scoring criteria).
3. Write a level or grade on each student's work along with comments justifying the rating. Comments concerning how to improve the rating could also be added.

### *Which is the Largest Buffalo?*



| Name (Country) | Height (cm) | Weight (kg) |
|---|---|---|
| Water Buffalo (Asia) | 165 | 1000 |
| African Buffalo | 135 | 560 |
| Yak (Tibet) | 200 | 550 |
| Gaur (Asia) | 220 | 850 |
| American Bison | 180 | 1000 |
| European Bison | 200 | 900 |

# ▲ Some Number Problems

*Instead of giving students a page of computation questions to do, try one of the following problems, from Lorna Morrow of North York, as a group activity and listen to the group discussions. Then have the groups present (and attempt to justify) their solutions.*

**Write fifteen numbers between 3.4 and 3.5**

The mathematics: the knowledge that no matter how close two numbers are, there is always another one between them. This is known as the density of numbers. Students will need to understand both tenths and hundredths to complete this problem. Notice that there is no period at the end of the sentence. This is to avoid confusing children who are really not certain what to do with decimal points or who might have difficulty distinguishing between decimal points and periods.

**A number rounded to the nearest tenth is 9.8**
*Describe all such numbers.*

The mathematics: density (as above); place value; rounding. Students may be able to identify several numbers that fit the description given above; it is a more difficult task to describe the set of such numbers.

**Write three fractions between 1/4 and 1/3.**

The mathematics: common denominators; equivalent fractions; density. It is interesting to observe students as they change the given fractions to twelfths (3/12 and 4/12) only to find that this does not give them the "answer". As a hint, you may wish to ask them why they chose 12 as a denominator or if they could use any other denominator.

**Suppose the '8' key on your calculator is broken.**
*Describe how you could still use your calculator to calculate 357 x 888*

The mathematics: the meaning of multiplication; the multiplication algorithm; distributive property. This problem demands number sense rather than the rote application of a memorized procedure. There are several ways to complete the problem: 357 x 1000 - 357 x 112 or 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 3570 + 3570 + ... (8 times) ... + 357 + 357 + .. (8 times)

**Suppose the decimal point on your calculator is broken.**
*Describe how you could still use your calculator to calculate 34.25 + 17 + 8.1*

The mathematics: place value; addition of decimals. One of the simplest ways to solve the problem is to write each number using hundredths, then add on the calculator without the decimal points (i.e. 3425 + 1700 + 810) and then place the decimal point correctly.

**The answer to a division question is '4R2'**
*Write four different questions that could have this answer.*

The mathematics: meaning of division; the division algorithm; division as the inverse of multiplication. This problem points out the meaninglessness of such an answer since 4R2 could mean four and two-sevenths (from 30 ÷ 7) or four and two-fifths (from 22 ÷ 5) or four and two-thirtieths (from 122 ÷ 30). That is, the "R2" has no clear meaning unless we know what the original division question was. Notice that as soon as we suggest a meaning for "R2" we have solved the problem – i.e. four and two sevenths as an improper fraction is thirty sevenths or 30 ÷ 7.

**The answer to an addition question is '3 1/2'.**
*Write four different questions that could have this answer. How many could you write? Explain.*

The mathematics: addition of fractions; meaning of fractions; common denominators. The simplest answer is probably 3 + 1/2 which is also the meaning of 3 1/2. Some students may be uncomfortable calling "3" a fraction because of previous experience, but since "3" can be written in a/b form (eg. 6/2, 9/3) it is a fraction. Some students may recognize simple patterns and write 3 + 1/2; 2 1/2 + 1; 2 + 1 1/2; etc. Such students are exhibiting number sense and should be commended.

**Why can't 12 x 13 = 135?**
**Why can't 162 ÷ 9 = 15?**
**Why can't 37 + 58 + 19 = 97?**

The mathematics: number sense; divisibility rules; estimation. For the first problem, students should recognize that the units digits, 2 and 3 cannot give a units digit of 5 in the answer. For the second problem, use similar reasoning. For the third problem, the sum of the units digits must give a units digit of 4, not 7; estimating suggests that the answer must be about 120.

**Consider 6394 ÷ 12. Invent a story where the answer is a) 532 b) 533 c) 532 R10 d) 532 5/6 e) 532.8333...**

The mathematics: number sense; an understanding of the meaning of division; recognition that some situations demand 'rounding down', some 'rounding up', some a more exact answer than others. In other words, students must recognize the importance of context in assigning meaning to mathematics.

# ▲ Finding Out About Yourself

*Math activities, such as this one from Jan Scully of York Region, can teach students about the world in which they live and about themselves.*

## Work with a partner:

- My height is _____.
- The distance around my forehead is _____.
- The distance between my eyes is _____.



- The distance around my waist is _____.
- The distance around my fist is _____.
- The distance from my knee to the floor is _____.
- The length of my foot is _____.
- The width of my foot is _____.
- The distance around my neck is _____.



- The distance across my eyes is _____.
- The distance from the base of my nose to the base of my chin is _____.
- The distance from the end of one outstretched hand to the end of the other outstretched hand is _____.
- The distance across my four fingers is _____.
- The distance from my knee to the top of my head is _____.
- The distance from my elbow to my wrist is _____.

## Fill in the blanks with the name of the body lengths.

| | | *Estimated ratio* |
|---|---|---|
| _____ | to _____ | = 1 to 1 |
| _____ | to _____ | = 1 to 2 |
| _____ | to _____ | = 3 to 1 |
| _____ | to _____ | = 4 to 1 |
| _____ | to _____ | = 2 to 3 |

1. Compare your results in the chart with another pair.
2. How are your results alike? different?
3. Write about what you found out about yourself.

# ▲ Field of Vision

*In this activity from Jan Scully, York Region, angle measurement is not carried out in isolation but it is connected to an interesting topic of investigation, field of vision in animals.*

## Work in a group of four:

In this activity you will perform measurements to determine the greatest angle you can see in a vertical direction. WORK SAFELY!

- To have an accurate angle measure, you must keep your head still while you are being tested. However, you can move your eyes as much as you have to in order to see the object.
- Invite a member of your group to stand close to the chalkboard and look straight ahead.

1. Draw a line on the blackboard outward from their eye parallel to the floor. *(See figures)*
2. Have them look upwards. Hold a pencil above their head.
   - Move the pencil outward until they tell you they can see it.
   - Draw a segment from their eye to that part of the pencil they first saw. (see figure 2)
3. Have them look downwards. Put the pencil beside one of their shoes.
   - Move the pencil outward along the floor until they tell you they can see it.
   - Draw a segment from their eye to that part of the pencil they first saw. (see igure 3)
4. Measure and record the 3 angles made between the segments.
5. Repeat steps 1-4 for each member of your group.

Figure 1

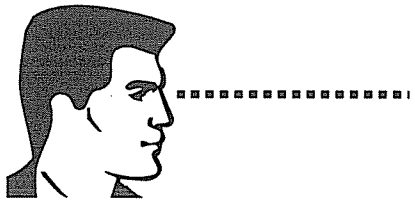


Figure 2

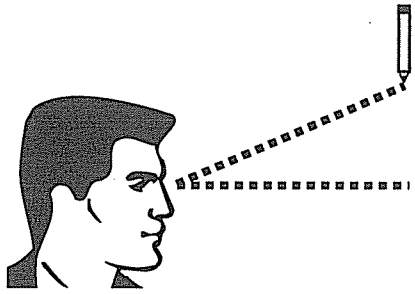
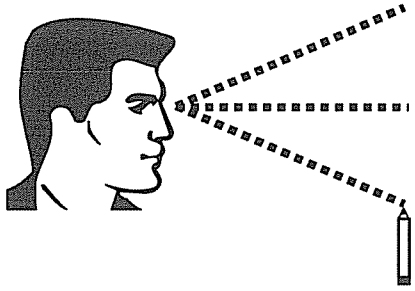


Figure 3



6a. Compare the size of the angles.

- What is the size of the 2 smaller angles?
- the greater angle?

The greater angle is the vertical field of vision.

- What is your vertical field of vision?
- Do boys and girls have a different field of vision?
- Write a report about the above angle. Share your report with other groups.

6b. Create an activity that can measure the greatest angle you can see in a horizontal direction. This angle is called the horizontal field of vision.

Research information about the horizontal field ot vision for animals that prey upon other animals like wolves, foxes, hawks, and owls. How does the field of vision for the above animals compare to animals that do not prey upon others like budgies and canaries?

# ▲ Filling an Area: No Gaps and No Overlaps

*The following geometry activity, submitted by Barry Scully of York Region, has groups of students investigating which simple 2-dimensional shapes will tile a surface.*

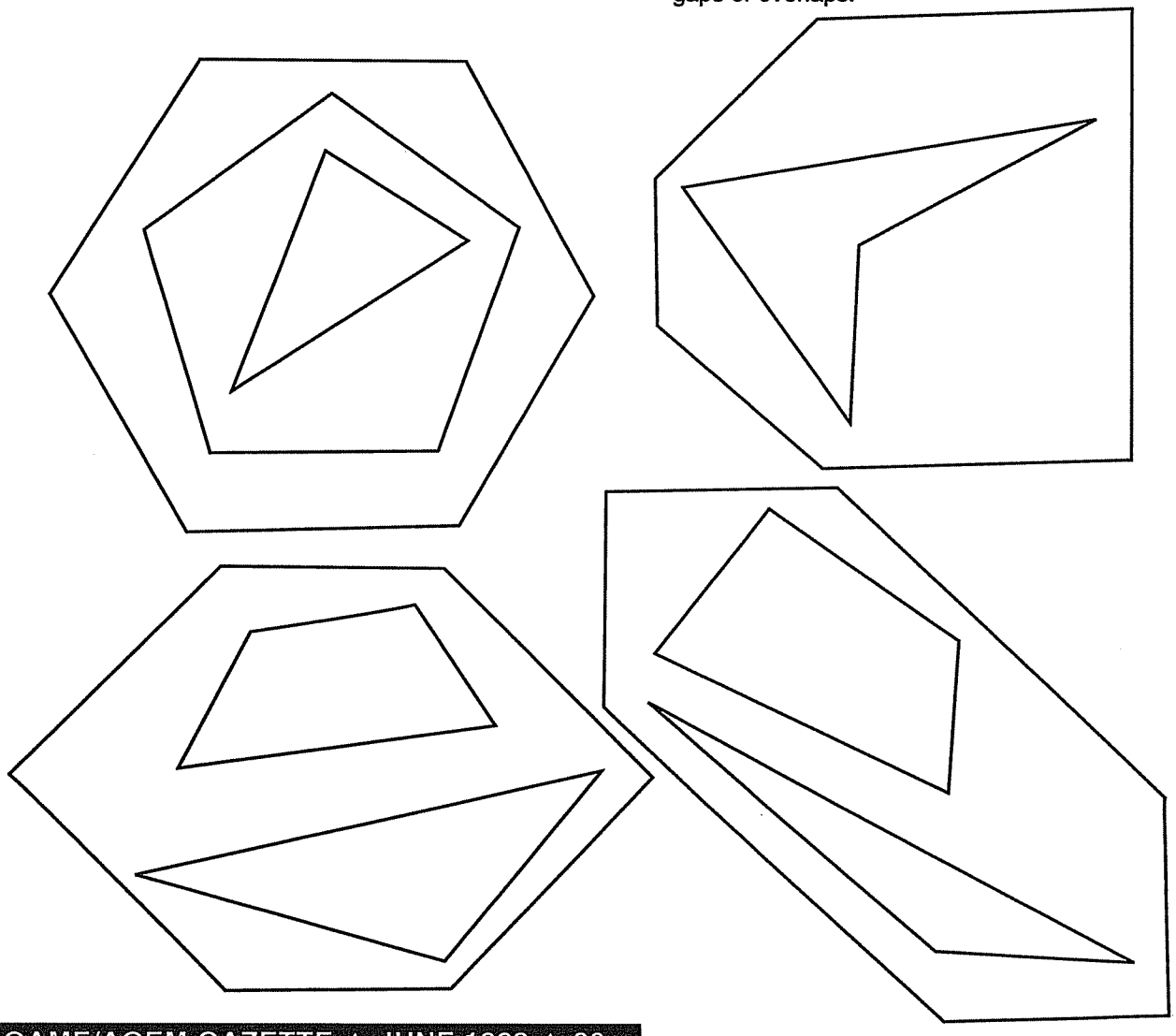
- Work with a partner.
- You will need scissors and construction paper.

## Instructions:

- Choose one of the figures below to use as a template.
- Cut about 12 copies of the figure from construction paper.

## The problem:

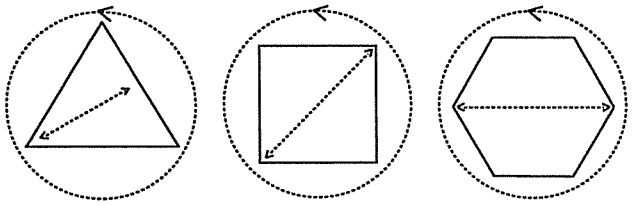
- Arrange the 12 copies to cover an area with no gaps and no overlaps.
- Repeat for other figures.
- Make a list of the figures that can cover an area with no gaps or overlaps.
- Compare your results with other groups.
- Why do you think some figures work and others do not?
- Find or create figures that can cover an area with no gaps or overlaps.



# ▲ Circumference of a Circle: an Investigative Approach

*A change in approach to a familiar topic can often lead to students having a better understanding of the associated concepts and procedures. This is illustrated in the following activity developed by Gary Flewelling.*

*Have students set up and conduct an experiment that will allow them to answer this question.*

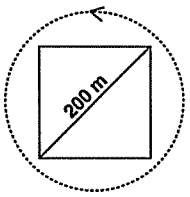


## How much farther is it around a regular polygon compared to the distance across it?

Break students into groups of 2 to 5. Each group should select a different family of regular polygons, for example, one could investigate equilateral triangles. Other groups could investigate squares, pentagons, hexagons, octagons, decagons, etc. Each group should find or construct ten different-sized members of their polygon family, share the measurement and calculation workload and average their results.

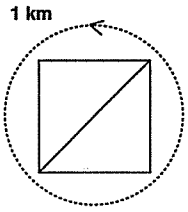
Have the groups pool their results in a table on the blackboard or overhead, and copy it in their notes. Through questioning have the class note that each regular polygon has associated with it its own particular constant. For example, the distance around any square is about 2.8 times as long as the distance across the square. This answer assumes that the distance across a square is defined as the distance from one vertex to its opposite vertex. (The answer is 4 if the distance across is defined as the distance between midpoints of opposite sides.)

At this point, you can pose two types of questions involving the indirect measurement of the perimeter of a regular polygon given its width and the indirect measurement of the width of a polygon given its perimeter.




### Example 1

Calculate the perimeter of a square that is 200 metres wide. (answer: about 2.8 x 200 = 560m)




### Example 2

Calculate the distance across a square that has a one kilometre perimeter. (answer: about 1000 ÷ 2.8 = 375m)

Have each group make up two problems like the examples above, and place them on the board for the others to solve. Have students write in their journals, with examples, what they learned.

Follow this up the next day with a similar investigation involving circles, with perimeter being referred to as circumference and distance across referred to as diameter. Include a typical set of application questions involving either the indirect measurement of circumference or diameter given the circle's diameter (or radius) or circumference, respectively.

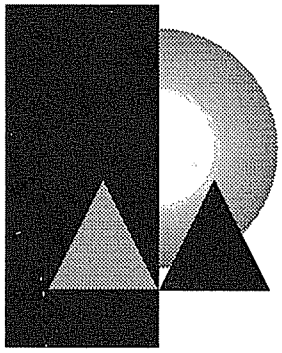
# ▲ Estimation Problems

*The following series of estimation problems, submitted by Lorna Morrow of North York, are taken from, adapted from, or triggered by "The I Hate Mathematics! Book" and "Math for Smarty Pants" by Marilyn Burns from Little, Brown and Co.*

Students can work with the problems one at a time or they can work at the whole set of problems in pairs or small groups. In the latter case, give students these instructions:

1) Select a problem
2) Estimate an answer
3) Describe a method you could use to check your estimate
4) Carry out your method, if possible.

The value here is in the discussion of the methods and the realization that some quantities will always be estimates since there is no practical way to measure them. Other quantities may be approached through sampling or surveys.



"Schools shall provide for heterogeneous and flexible groupings of students within classes and shall ensure that all students have an opportunity to achieve the learning outcomes specified in The Common Curriculum, Grades 1 to 9."

(Transition Years, Grade 7, 8 and 9, pg. 4)

## Problems

1. If you had a piece of string as tall as you are, how many times do you think you could wrap the string around your head?
2. Which is greater—the length of your foot or the distance around your wrist? Your neck?
3. Is there enough paper in a roll of toilet paper to go all the way around this room?
4. Is a large box of cereal taller or farther around?
5. If you were to do a survey on the street in front of the school, do you think you would see more people wearing shoes with shoelaces or more wearing shoes without shoelaces?
6. How many pieces of popped popcorn do you think would fit into a wastebasket?
7. Which do you think would be worth more—a stack of nickels as tall as you or a row of dimes laid edge to edge from your toes to your head as you are lying down?
8. Is a soft drink can farther around than it is tall?
9. Try to draw a rectangle the size of a dollar bill, a circle the size of a quarter, a rectangle the size of a chiclet, a circle the size of a telephone ear piece. Can people do this accurately?
10. About how long a line could you draw with one new pencil before it was worn out?
11. How many cars could be lined up bumper to bumper between here and your city hall?
12. If you asked people to choose their favourite one-digit number, what do you think would be the most commonly chosen number?
13. If you were to try spending $1 000 000.00 by spending $100.00 every hour, day and night, how long would it take you to spend it all?
14. Which would you rather have — your weight in nickels or your weight in quarters?
15. Could you carry $1 000 000.00 in $2.00 bills? in $10 bills? in $1 coins?

# ▲ Investigating the Surface Area of a Sphere

*Jan Scully, from York Region, brings nutrition* into the math classroom with this hands on measurement activity. (*At the end of the activity the students eat their oranges.)*

## Work in Pairs:

- Measure the circumference of an orange.
- One partner completes the first 3 columns of the chart.
- The second partner cuts the orange peel and finds the area it covers by arranging the peel on centimetre grid paper.
- Partners work together to complete the last three columns of the chart.
- Pairs share their findings by recording them on a common chart.
- What could be a formula we could use to find the surface area of a sphere?

| Circumference | Radius | Area of circle | Surface Area of sphere | $\frac{\text{SA sphere}}{\text{A circle}}$ | Surface area of sphere |
|---|---|---|---|---|---|
| | $r = \frac{c}{2\pi}$ | | | | $= \square \times$ Area of circle |

# ▲ A Pythagorean Pyramid

*In this `hands on' construction activity submitted by Lorna Morrow from North York, students make three congruent pyramids. The pyramids are specially selected to fit together to form a cube.*

You will need: scissors, tape or glue, paper, geometry construction instruments

- Using one decimetre as a unit, draw an enlargement of Diagram 1.
- Add tabs as shown in Diagram 2.
- Make two more copies of this figure so that you have three altogether.
- Cut out the figures and fold on the line segments to make pyramids. Use glue on the tabs to help hold the figure together.
- Fit the three pyramids together to construct a cube.

Compare the volume of the cube with the volume of one pyramid. Compare the base area of the cube with the base area of a pyramid. Compare the height of the cube with the height of a pyramid.

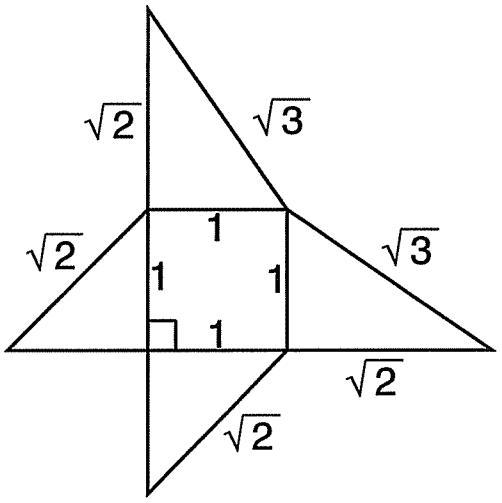



Diagram 1

## Teacher Notes:

This activity gives students practice in using construction instruments (compasses, protractors, transparent mirrors, set squares, rulers, etc.)

In order to enlarge the figure, students need to solve the problem of constructing a segment that is √2 units or √3 units long. Students also need some familiarity with scale drawings or dilatations.

Fitting the three pyramids together to construct a cube is a problem in itself demanding spatial abilities.

Students may conclude that the area of a pyramid on the same base and having the same height as a given cube has one-third the volume of the cube. Generalizing to all rectangular prisms is not likely at this stage. However, see the "Prisms and Pyramids" activity for an extension that leads to a more general conclusion.

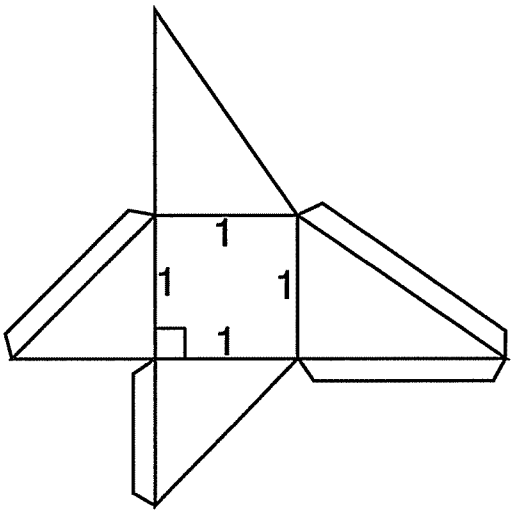
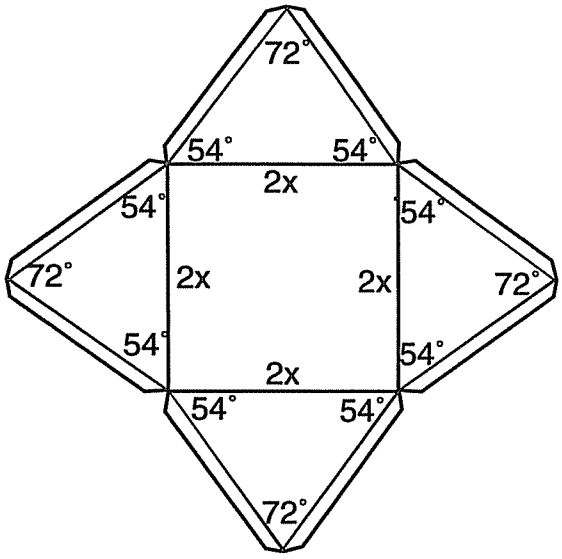


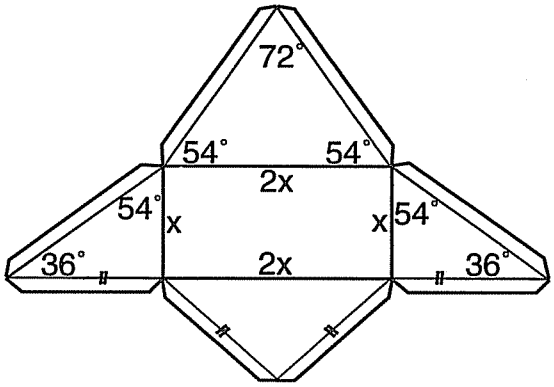

Diagram 2

# ▲ Prisms and Pyramids

*This activity, also submitted by Lorna Morrow, is an extension of the previous activity "A Pythagorean Pyramid". The end product is a set of hinged pyramids that fold into a rectangular prism and demonstrate the fact that the volume of a pyramid is 1/3 the volume of its related prism.*




Pyramid 1 (Make 1)




Pyramid 2 (Make 4)

You will need: scissors, tape or glue, paper, geometry construction instruments.

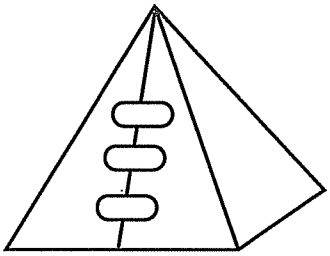
Copy the diagrams above. The side length of the square (2x) should be between 7 and 10 cm to get a model of reasonable size.

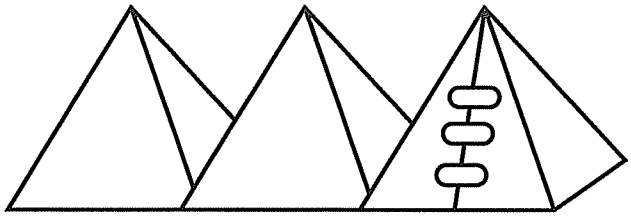
- Cut out the figures, fold, and glue or tape to make 5 pyramids.
- Tape two of Pyramid 2 together to make a hinged copy of Pyramid 1.
- Repeat with the other 2 copies of Pyramid 2.

You should now have 3 copies of Pyramid 1, two of which are hinged on one face.

- Line up the pyramids as shown.
- The hinged pyramids are on the ends with the hinged faces on opposite sides.
- Tape the bases of the pyramids together.
- Fold the model to build a rectangular prism.

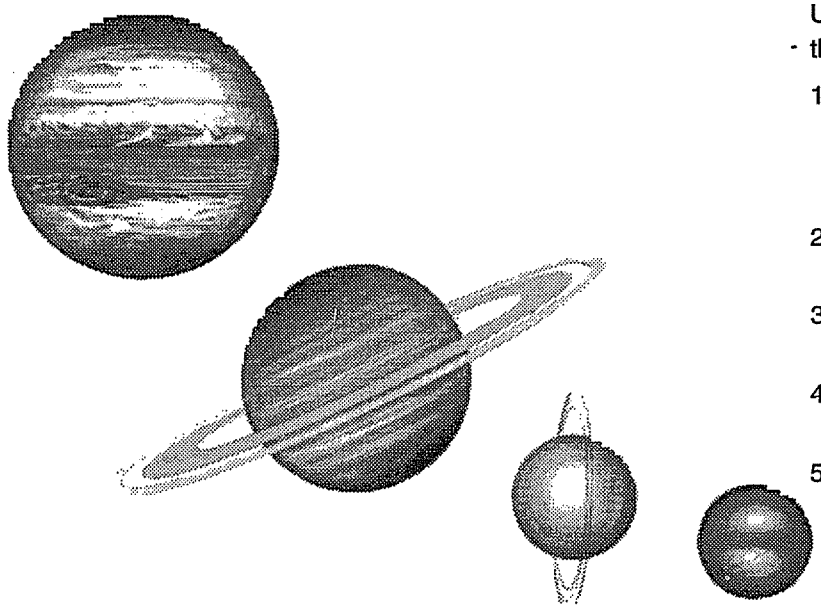
Compare the volume of the prism with the volume of Pyramid 1. Compare the base area of the prism with the base area of Pyramid 1. Compare the height of the prism with the height of Pyramid 1.





# ▲ The Solar System: Algebra in Context

*Modelling situations by linear equations or inequations and connecting mathematics to the world outside of the mathematics classroom are two of the emphases of algebra in the Transition Years. This activity from Gary Flewelling of Wellington County can be done by students working independently or in small groups.*



Use the information in the following table to help you answer the following questions:

1. If the sun had a mass of one tonne (about the mass of a car) then the earth would have a mass of about 3 grams. Explain how a person could come up with such a result.
2. If the sun had your mass, calculate the mass of the earth.
3. If the sun had a diameter that allowed it to just fit in your classroom, how big would the earth be?
4. Make a scale model of our solar system that would fit into your classroom.
5. Make up and solve three more problems about our solar system. Share these problems with another group of students.

**OUR SOLAR SYSTEM**

| | Diameter (km) | Distance to sun ($10^6$km) | Mass ("earths') |
|---|---|---|---|
| **SUN** | 1 392 000 | | 332 830 |
| **MERCURY** | 4878 | 57.9 | 0.38 |
| **VENUS** | 12 104 | 108.2 | 0.8 |
| **EARTH** | 12 756 | 149.6 | 1.0 |
| **MARS** | 6787 | 227.9 | 0.1 |
| **JUPITER** | 142 800 | 778.4 | 317.8 |
| **SATURN** | 120 000 | 1423.8 | 95.2 |
| **URANUS** | 51 200 | 2868.7 | 14.5 |
| **NEPTUNE** | 48 860 | 4492.1 | 17.2 |
| **PLUTO** | 2300 | 5926.5 | 0.002 |

# ▲ Discovery with Cubes

*Don't include too much detail in your instructions, only enough to give students a starting point to develop their own approaches and strategies. In this activity, a few students will find it helpful to have a set of interlocking cubes available so that they can make and study physical models of the cubes involved in this activity.*

Some cubes have been stacked and glued together to form these larger cubes.

If each large cube is dropped into a bucket of paint and completely submerged how many of the smaller cubes will end up:

a) painted on more than three sides?

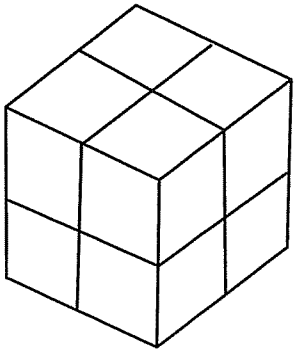
b) painted on three sides?

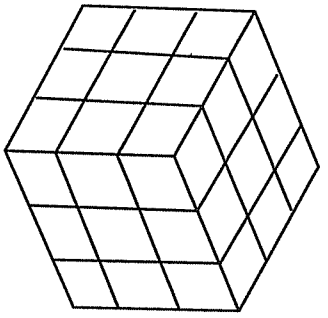
c) painted on two sides?

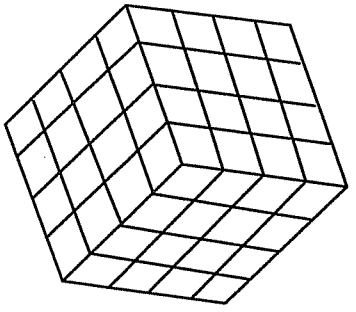
d) painted on one side?

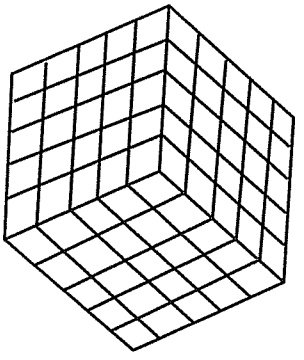
e) not painted?

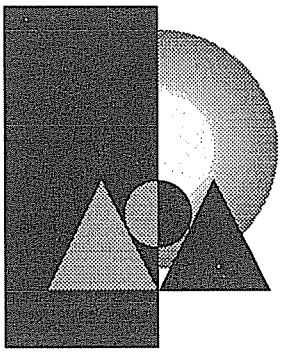
1. Record your results in a table.
2. Add to your table the results for a 6 x 6 x 6 cube.
3. Predict table entries for three other larger cubes. Explain the patterns you used to make these predictions.
4. What would your table entries be for an n x n x n cube?











"A good rule of thumb is to try to include in every learning situation opportunities for direct experience, reflection, instruction and application."

(Teaching Destreamed Grade 9 Classes, pg. 2)

# ▲ STUDENT BODY GRAPHING

*This is a simple yet effective activity developed by Stu Telfer, Waterloo County, that gives concrete meaning to the notion of function and ordered pairs. It offers an exciting change of pace by allowing students to do mathematics outside the classroom.*

### *MATERIALS NEEDED*

- length of rope (optional), line markings for grid

### *OVERVIEW OF ACTIVITY*

- A large set of X - Y axes is drawn on an outside playing field. Lines can be drawn very easily using lime line markings on a grass field or using sidewalk chalk in a parking lot. Just using large markers such as orange pylons will also work. If available, a grid can be drawn on a field using Super Stripe (white athletic paint used to line fields).
- The unit markings on the X-axis are approximately 1 metre or 1 student pace apart. The units on the X-axis should be marked, but it is not essential to mark units on the Y-axis.
- Group your students into teams. Teams of 11 are ideal (5 positive, 5 negative and zero) but are not essential.
- Have one group take their position on the X-axis with a student at each unit marking, all facing the first and second quadrant. Students are given a command and each is asked to take that many paces.

**Example:** *Teacher –*
"Take as many paces as twice the number you are standing on".

The student standing at +3 would take 6 paces forward, the student standing at -2 would take 4 paces backward, etc. Of course, the result will be the students lined up in a "reasonably" straight line of slope 2. If appropriate the teacher could ask the students to state the equation of the straight line (y = 2x) that is the result of the command.

Periodically switch groups. It is possible to show the intersection of two lines by giving one command to a group, telling them to remain in place, then giving a different command to the second group. Places where two students are together are points of intersection (or a discipline problem).

If a length of rope is available, have students hold on to a rope to give a better picture of a straight line.

### *HINTS AND SUGGESTIONS*

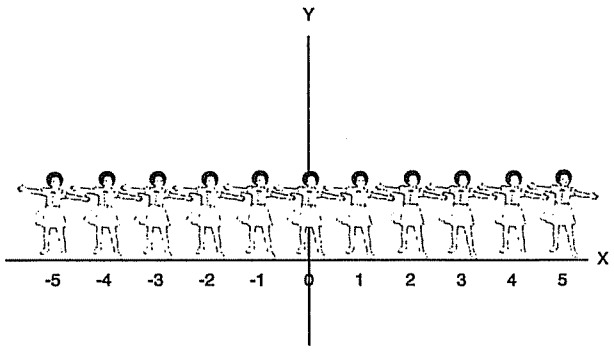
- Have the students "practise pacing" before your initial commands so that they understand what is expected and to allow paces to be fairly equivalent in length. An explanation for students on the negative X-axis is important.
- Keep the first few examples simple. For some you may wish to restrict activity to the first quadrant only (positive X values only).

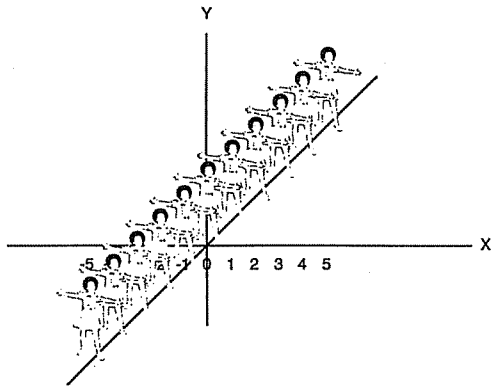
### *EXAMPLES OF COMMANDS*

- "Take as many paces as double the number you are standing on. (Allow students to move.) Now take one more step forward." (y = 2x + 1)
- "Turn around and take as many paces as the value you are standing on!" (y = -x)

### *EXTENSION*

- If students have had previous experience with graphing straight lines, first tell them an equation (eg., y = 2x - 1) and then ask what instruction(s) would generate this line.
- Activities can be created to illustrate properties of parallel lines (eg., one group paces y = 2x and the other y = 2x + 4) and the properties of slope (eg., one group paces y = 2x and the other paces y = 3x or y = -2x).



# ▲ The Mathematics of the Pendulum

*The math classroom should frequently have the look and feel of a science lab with students making conjectures and verifying them through experimentation. The pendulum investigation, submitted by Barry Scully from York Region, is a good example of such an activity.*

- Work in a group of three. Answer the following questions before you begin the activity. Answer them individually, then share your thoughts with the group. Try to come to some agreement before beginning the activity.
- A grandfather clock uses a pendulum, gears, and masses to keep proper time. Not all clocks are the same height but they all "tell" the same time. How do you think clock-makers determine the length of the pendulum to make an accurate clock?
- At the end of each pendulum is a mass. How do you think clock-makers determine what size of mass is needed to make an accurate clock?

Now collect the materials you need: string, washers, stopwatch, masking tape, ruler, graph paper

***Problem A:***

- How long is the pendulum that has a time of swing (a period) of 1 s and a mass of one washer?
- Write about how your group solved this problem. Can you think of any other ways you might have solved the problem? Write about them.

***Problem B:***

- Change the length of the pendulum and determine the corresponding time of swing for each new length. Make a table:

| Length of Pendulum (cm) | Time of Swing (s) |
|---|---|
| | |

- Record this information on a graph. What type of graph did you choose? Why?
- Make 3 observations from your graph. What effect does the length of the pendulum have on the time of swing?

***Problem C:***

- Change the mass at the end of the pendulum and determine the corresponding time of swing for each new mass. Make a table:

| Number of Washers | Time of Swing |
|---|---|
| | |

- Record this information on a graph. What type of graph did you choose? Why?
- Compare this graph with the graph from Problem B. Record your observations.
- What effect does the mass at the end of the pendulum have on the time of swing?

***Wrap-up:***

- Compare the results of Problems A, B, and C with your original theory. Adjust your theory if necessary.

*(Note 1. The teacher might wish to add an activity in which students explore the effect of amplitude of swing on the period of the pendulum.*

*Note 2. Pendulums in grandfather clocks are not simple pendulums. As an extension, students could research the nature of clock pendulums.)*

# ▲ Activities to Stimulate Statistical Concept Development

*In this article on Data Management and Probability, Eric Muller, Department of Mathematics, Brock University, illustrates with three sample activities the importance of exposing students in the Transition Years to the concepts of variability and quantity of information.*

Data Management and Probability are exciting and important parts of the Transition Years mathematics curriculum. They are exciting because they can absorb and stimulate students at all levels. They are important because of their widespread use to analyze various types of information. This area of applied mathematics is called Statistics. It is worth noting that statistical experiments are controlled by the scientific process, which requires that repetitions of an experiment lead to the same conclusion, within a specified level of error. These conditions will appear, on the surface, contradictory. My advice is not to emphasize the making of decisions but rather to provide experiences which explore alternatives.

Which statistical concepts should our students grasp by the time they complete their transition years? The concept of **variability** is at the top of my list. Students start to explore variability by comparing and pooling their results with those of the other students in the class. Pooling exposes students to a second important statistical concept, namely that of the **quantity of information.** In most situations, one improves one's understanding of the situation by increasing the amount of information. When dealing with large sets of data, avoid repetitive and boring massaging of data; use spreadsheets to do that. I recommend that our activities in data management aim at generating **relative frequency distributions,** as these suggest a bridge to probabilities. In fact they are experimental probability distributions. Our students should leave their transition years with some experience of bias and a desire to question data statements in news reports and advertising. Plan on developing a project with a teaching colleague who has some expertise in communications.

We now outline the types of activities which would enrich the transition years experience. Data gathering activities such as, the number of sports articles in a newspaper, the number of cars passing the school per minute, the weight of candy eaten per student per day, etc., can be good for involving students, raising their awareness of certain issues and developing their data management skills. However, these activities fall short because the underlying mathematical models are complex. In other words, there is a difficult gap between the experiments and the probability models which statisticians would use in these situations. My suggestion is to expose students in their transition years to activities involving manipulatives such as dice and spinners. There the underlying probability model is accessible to the mathematically talented student. The emphasis in these activities should be on the type of information data to record and on what is the most suitable way to display the data in order to meet one's objectives. The purpose of the activities should not be paper and pencil exercises in probability.

In the December 1987 issue of the Gazette I described a game which uses two dice. The advantage of this game is that students can grow with it; it has various levels of play. The game is typical of the activities I would recommend for the transition years. Here are three more activities to expose students to the concepts of variability and quantity of information. (In the first two the probability distributions are known, in the third a comparison is made with published results).

## Activity 1: Exploring the distribution of coloured chips in a bag.

A large number of transparent chips, of four of more colours, is counted and placed in a opaque bag. Taking turns each student chooses, without looking, a fixed number of chips (say 10), records the distribution and replaces them in the bag before the next student makes a choice. Based on their information, and given the total number of chips in the bag, each student is asked to make a prediction of the distribution in the bag.

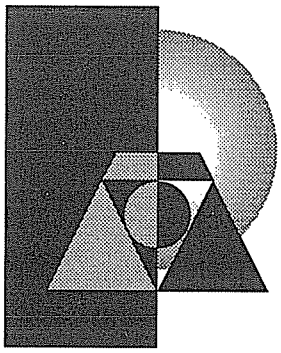
The samples and predictions are compared and their variability discussed. Each student is asked to repeat the sampling this time choosing twice as many chips as before. The change in variability of the samples and predictions compared to the first sampling is discussed. The activity concludes with a comparison of the pooled information with the actual distribution.

## Activity 2: Predicting the minimum number of rolls of a die required to reach or exceed a given total.

Before starting the experimentation students are asked to give a worst case (maximum number of rolls) to get 50 (say) and a better case (minimum number of rolls) to get at least 50. Each student then performs the experiment a number of times. The information is now pooled and a suitable display of all the data is produced. What was the worst case - the most rolls? What was the best case - the least rolls? Given this information what would the class use as its best estimate of the number of rolls to get at least 50? Is this information useful to estimate the total we would obtain by rolling the die a fixed number of times (20 say)? The experiment should be repeated with spinners, for example, a spinner with 5 equally probable numbers and a spinner where the angles associated with each number are not equal, i.e., not all numbers are equally likely.

## Activity 3: What do we mean when we say that our town had 5.3 cm of rain in September?

This activity involves generating data and comparing class results with those published in the local newspaper. Each student will need a flat bottomed clear plastic container with vertical sides, for example, a flat bottomed plastic bottle could be cut. Using a permanent marker, a vertical centimetre scale is drawn with 0 at the bottom. Each student is asked to place his or her container outside in an open space and to record the depth of water after each rainfall. At the end of the month the individual totals are entered in a suitable table and the total rainfall is estimated and compared with a newspaper report. Discussion should centre on possible causes of the variability of individual totals. Also discuss the location, within the class distribution, of the quantity reported in the newspaper.



"Basing the curriculum on problems seen in context improves learning for all types of learners, including high achievers and students who have difficulty with traditional methods of learning."

(Teaching Destreamed Grade 9 Classes, pg. 2)

# ▲ PROBLEM-SOLVING BINGO

*This problem-solving bingo game is an idea from the TOPS Problem-solving Decks put out by Dale Seymour publications several years ago. Lorna Morrow of North York has used it from grade 2 to grade 8, "always with success".*

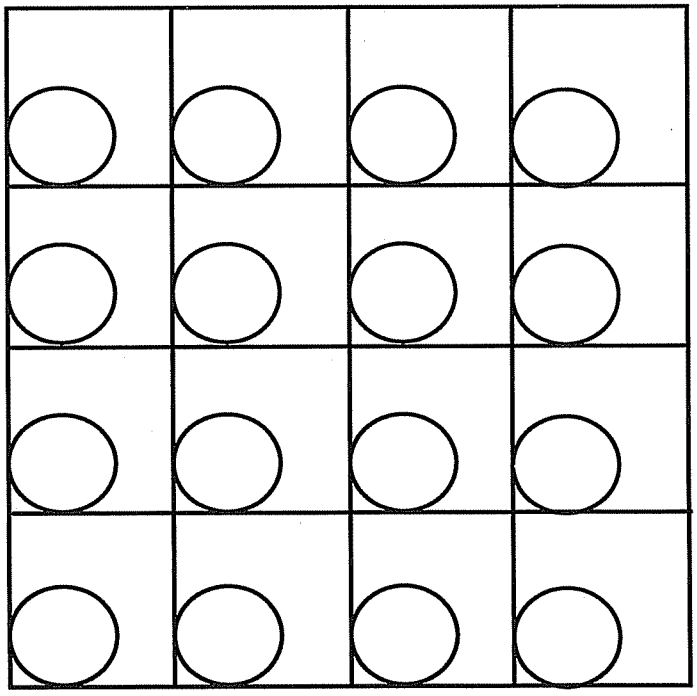
## Rules:

1. In each square of the board write one of the numbers from the choices given. Do not look at the problems.
2. Select a problem and solve it. Look for the answer on your bingo board. If the answer is on the board indicate this by writing the number of the problem in the small circle in that square on the bingo board.

   If the answer is not on your bingo board, or if you have already used that answer for another problem, you "lose a turn". That is, you have solved a problem but it does not help you towards bingo.
3. First team to get four in a row marked on the board is the winner.

***Choose from***

| | | | |
|---|---|---|---|
| 1 | 6 | 7 | 8 |
| 10 | 12 | 13 | 18 |
| 20 | 24 | 30 | 36 |
| 45 | 54 | 62 | 64 |
| 96 | 162 | 200 | 450 |
| 26 500 | | | |



## Teacher Notes:

- Students may work individually, but pairs or small groups are very effective.
- Each team of players needs one bingo board and one collection of problems. A convenient way to package the problems is to reproduce a set for each team, cut the problems apart and put them into an envelope.
- Scrap paper, calculators, and manipulatives should be available for students to use as they wish.
- Problems can be taken from any source — old text books, supplementary materials, NCTM publications, etc.

  Two good NCTM sources are:

"Mathematical Challenges for the Middle Grades: From the Arithmetic Teacher" William D. Jamski (Ed.), NCTM, and the companion volume "Mathematical Challenges for the Middle Grades: From the Mathematics Teacher Calendar Problems".

This particular set of problems is taken from Gauss contest papers over the last few years. A good source of such problems is "Problems, Problems, Problems" Volume 4 (and the forthcoming Book 6) from Canadian Mathematics Competition, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1.

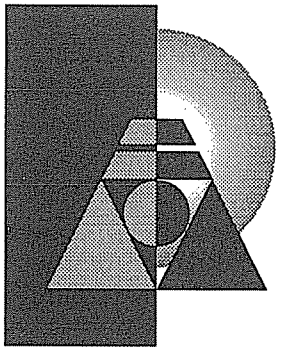
Answers are given here for easy checking of "Bingo". However, if students interpret problems differently and can justify alternative answers, these should be accepted.

***The problem number is given first, then the answer.***

| | | | |
|---|---|---|---|
| 1-12 | 2-7 | 3-12 | 4-26 500 |
| 5-1 | 6-96 | 7-200 | 8-8 |
| 9-12 | 10-24 | 11-62 | 12-450 |
| 13-13 | 14-10 | 15-6 | 16-18 |
| 17-64 | 18-162 | 19-54 | 20-10 |
| 21-20 | 22-45 | 23-36 | 24-30 |

## Problems:

1. One mouse said to another, "If you give me one piece of cheese, then we will have an equal number of pieces." The other mouse replied, "If you give me one piece, then I will have double the number that you have." How many pieces did they have together?
2. Each number in a sequence is obtained by adding the two previous numbers. The 6th, 7th and 8th numbers of the sequence are 29, 47, and 76. What is the third number in the sequence?
3. A railway engine is 12 m long. A model is build to 1/100 scale. What must be the length of the model in centimetres?
4. If Bob earns $25,000 a year and Frank earns 6% more than Bob, then how many dollars does Frank earn in a year?
5. If the average of the five numbers 4.9, 3.1, 7.2, k, and 9.3 is 5.1, then what is the value of k?
6. When 16 litres are added to a gas tank that is half-full, then the tank is two-thirds full. What is the capacity of the tank in litres?
7. To make concrete, mix 4 shovels of stone, 2 shovels of sand, and 1 shovel of cement. How many shovels of stone would be needed to make 350 shovels of concrete?
8. When 62 is divided by a certain number, the quotient is 7 and the remainder is 6. What is that certain number?
9. What is the greatest possible product of two whole numbers that have a sum of 7?
10. If 3/4 of a number is 12, then what is 3/2 of the same number?
11. What is the sum of all the even divisors of 32?
12. What is the sum of the digits in the tens places of all two-digit whole numbers?
13. In a set of five numbers, the average of the first two numbers is 10, and the average of the last three is 15. What is the average of all five numbers?
14. A certain number is multiplied by 8. This result is then increased by 8. The final answer is 88. What was the original number?
15. Of 33 students in a class, 18 belong to the mathematics club, 17 belong to the science club, and 4 belong to neither club. How many students belong to both clubs?
16. On a test consisting of 30 questions, Sue had 50% more right answers than she had wrong answers. Each answer was either right or wrong. How many answers did Sue have right?
17. If you travel 48 km in 45 minutes, what is your speed in kilometres per hour?
18. What is the sum of all whole numbers, less than 100, that are multiples of 6 and end in a 4?
19. If 12 345 679 x 9 = 111 111 111, and 12 345 679 x n = 666 666 666, then what is the value of n?
20. Two whole numbers are in the ratio of 2 : 5. The product of the two numbers is 40. What is the greater number?
21. Arin and Jim together have $1.35. Arin has only nickels while Jim has only dimes. Arin has six more coins than Jim. How many coins do the two of them have together?
22. A ball that bounces straight up and down falls, initially, from a height of 16 m. Each time, it bounces up to half the height from which it just fell. When the top of its bounce is 1 m from the floor, what is the total distance, in metres, that it has travelled?
23. If one-third of a number is 6, then what is twice the number?
24. A 70 cm by 84 cm rectangle is entirely covered with congruent square tiles. What is the least number of tiles required?



"The program must be organized in such a way that the interrelationships among subjects and topics are evident and meaningful to the student".

(Transition Years, Grade 7, 8 and 9, pg. 3)