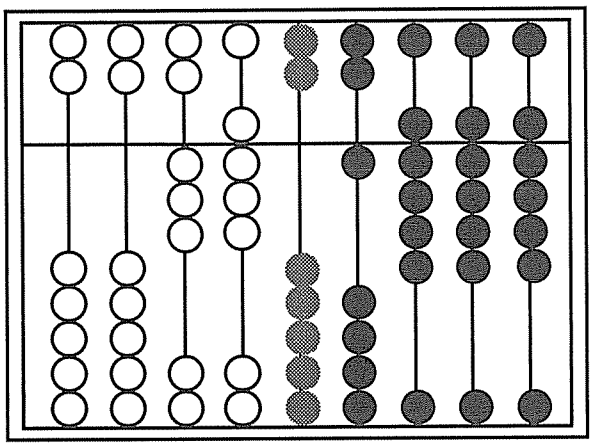
Abacus

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New CONTEST FORMAT NAMES

2000 AMC exam dates:

AMC⇨8 - TUESDAY, November 14, 2000
AMC⇨10 - TUESDAY, February 15, 2000
AMC⇨12 - TUESDAY, February 15, 2000
AIME - TUESDAY, March 28, 2000
USAMO - TUESDAY, May 2, 2000

www: http://www.unl.edu/amc

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NEW CONTEST, NEW FORMAT, NEW NAMES

The American Mathematics Competitions is pleased to announce a new contest as well as new names and a modified format for two of the current exams. The AJHSME is now the American Mathematics Contest ⇨8 (AMC ⇨8) and the AHSME is now the American Mathematics Contest ⇨12 (AMC ⇨12). The new contest is the American Mathematics Contest ⇨10 (AMC ⇨10), for students in grades 10 and below. This new contest will give more young students a chance to successfully participate in a significant mathematical problem solving experience.

Why should my school sign up? Because the AMC ⇨10 and AMC ⇨12 provide an excellent opportunity to challenge your students' mathematical abilities. It is but a means for furthering mathematical interest and development.

The AMC ⇨10 and AMC ⇨12 will each be 75 minutes long. They will both consist of 25 questions. Each correct answer is worth 6 points and a blank is worth 2 points. The AMC ⇨10 and AMC ⇨12 will have several common questions and will be given at the same time on the Tuesday befor the third Monday in February (the current AHSME date). The students should choose between the AMC ⇨10 and AMC ⇨12. The school team score will be determined from the AMC ⇨12. To qualify for the American Invitational Mathematics Examination a student must score at lease 100 points on the AMC ⇨12 or be in the top 1% of the AMC ⇨10 participants.

The registration fee for one or both contests is $30.00. One bundle of ten AMC ⇨12 is $12.00 and one bundle of ten AMC ⇨10 is $10.00. The first bundle of the AMC ⇨10 will be free for the year 2000 only.

A Program of the
Mathematical Association of America

▲ From the Editor's Desk

This issue is being written while my elementary teacher colleagues are finishing the school year with all the activities that June brings. I have to think ahead to September to assemble this collection of interesting mathematics.

Barry Onslow, Faculty of Education, UWO, brought *'A Teacher's Recipe for a Tasty Math Lesson'* by **Susan Tucker** to my attention. Susan, whose home is in Strathroy, wrote this for one of her projects in her B. Ed program this past year. Thank you Susan for an inspirational piece and thank you for bringing another teacher's work to my attention.

Susan wrote: "My bestest associate teacher, who is also my mentor and friend, teaches grade six at Wilfrid Jury Public School in London, Ontario. **Rhonda King** has 33 years experience (junior level and special education), is extremely artistic in a multitude of ways and is an award-winning associate teacher (Althouse 1997). Her poem is humour-filled, factual, and best of all, created to incorporate each student's name into it." Thank you Rhonda King for a cross-curricular piece of imagination and motivation.

In my daily teaching I have found students, especially grade 6-10, who are keenly interested in knowing how I can determine factors of numbers so quickly. For some students factor trees are very difficult. While I was thinking about this problem the *'Math Journal: Manitoba Association of Mathematics Teachers'* arrived. In this journal **Jerry Ameis**, Faculty of Education, University of Manitoba, approaches divisibility tests from a problem-solving situation by a search for a secret 'password' to protect your personal computer files.

The Ontario Curriculum Grades 1-8 (page 7) states:

> *The fundamental skills of calculation that students are expected to master are referred to as pencil-and-paper skills. For example, the pencil-and-paper expectations for Grades 4, 5, and 6 require the memorization of the multiplication tables and number facts. Students are expected to use calculators or computers to perform operations that are lengthier or more complex than those covered by the pencil-and-paper expectations.*

From the Provincial EQAO grade 3 results (1998) we know that when it comes to teaching basic facts of arithmetic there must be an understanding of numbers and counting which results in a number sense that will be the foundation of all future mathematics. Before I started school my parents gave me lots of opportunity to count before I could read or write the numbers. The tools were a wooden abacus with colourful beads and a drawer full of buttons in all shapes, sizes and colours. Not every child that comes to school has this background. I applaud the teachers who do try hard to put 'understanding' ahead of 'memorizing'. Can we find a way to put the historical 'abacus' into the curriculum for hands-on counting? Starting with this issue I am going to include information about the counter that we have featured on the title page for several issues.

The final math activity comes from the American Mathematics Competitions. Thank you to **Titu Andreescu** for providing some sample problems from the November 1998 competition with permission to print.

The annual OAME conference in Richmond Hill was splendid. I found so many of the workshops to be outstanding sources of classroom activities. I hope to highlight some of them in the December issue. Conversations with teachers indicated that they enjoy reading about activities that work in classrooms, and seeing students' projects. If you or your colleague have samples of a classroom activity, please contact me. I would be most pleased to consider it for a future issue.

Frances Schatz

▲ Answer Key

Divisibility for Secret Password

The answer is 324561.

Elementary Mathematical Challenges

Answers: 1. B 2. E 3. B 4. C 5. C
6. A 7. C 8. D 9. B 10. C

▲ A Teacher's Recipe For A Tasty Math Lesson

by Susan Tucker

Ingredients

2-3 juicy objectives

1 well done lesson plan

1 cup of enthusiasm

½ cup of content knowledge

⅓ cup of assessment measures

pinch of patience

dash of compassion

materials/equipment to taste

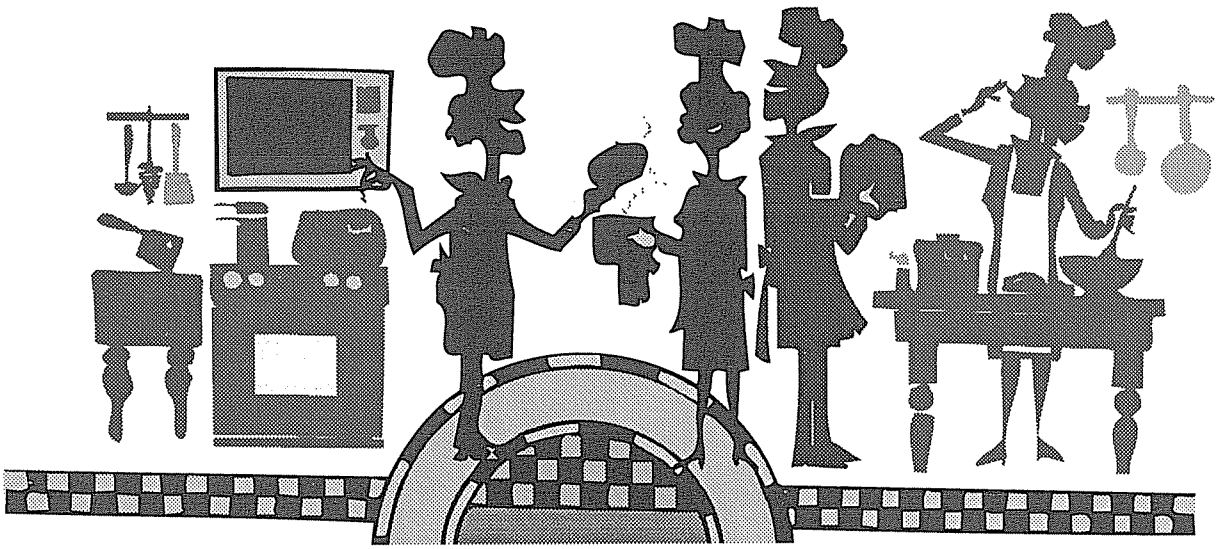
1 bunch experiential recommended!

Instructions

Begin preparing an awesome math lesson using the students' background knowledge as a springboard. Brush up your content knowledge by reading ahead and sampling a sprinkle of exercises. Combine 2 to 3 juicy objectives from the Ontario Curriculum and arrange complementary assessment measures. Mix activities that lend themselves to a blend of learning styles and methods. Taste-test your chosen activities and handouts *before* you assign them. Order materials *well* in advance, including spares in case items get lost or broken. Chill to prevent crumbling. Beat the clock and cut in an extra hour of sleep so that you can scatter the essential pinch of patience and dash of compassion as well as the energy to stir up a drizzle of fun. **Above all, get passionate about your topic!**

Equals 20 to 30 happy learned students and 1 content teacher.

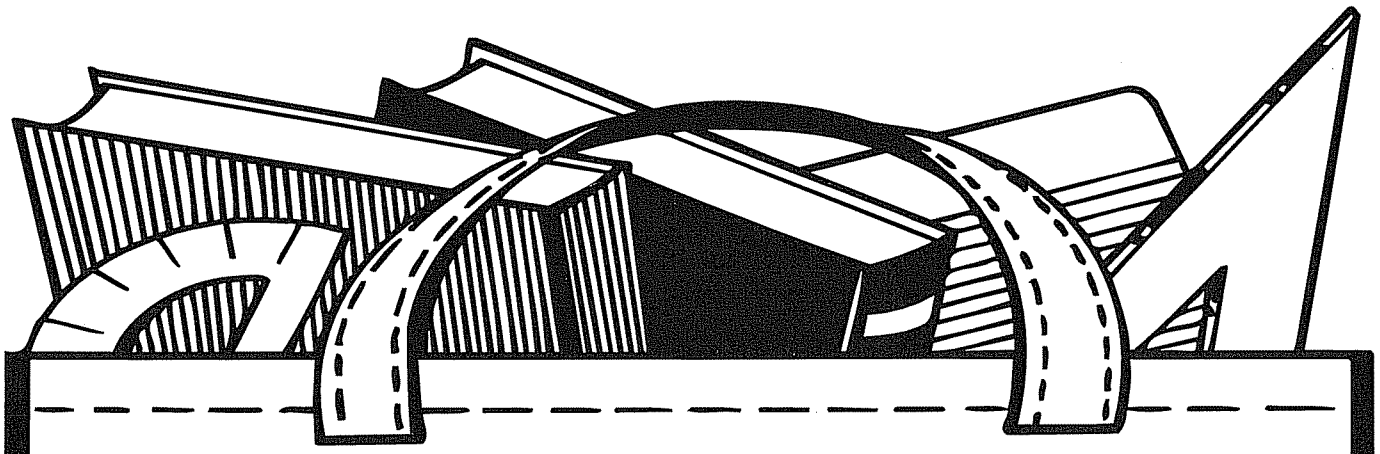
* *Radiate ½ cup of enthusiasm to initiate the lesson; add a splash every time you turn around.*



▲ FRACTIONS, FRACTIONS EVERYWHERE!

By Rhonda King, Wilfrid Jury Public School, London, Ontario

** Personalize, i.e., insert yours and your student's names!*
Dedicated to all parents, teachers and children who have suffered through fractions.



Fractions, Fractions Everywhere!
They really make **Chris** think.
They're oozing out her garden hose,
They're in **C.J's** kitchen sink!

Fractions, Fractions Everywhere!
Lindsay'd truly like to hide;
They're sneaking up her basement stairs,
'cause they're trying to get inside!

Fractions, Fractions Everywhere!
Kassie's coming quite untied.
They're ruining her life it seems,
No wonder they're feared far and wide!

Fractions, Fractions Everywhere!
Jason claims he's quite confused
James gets angry doing them,
They need to be defused!

Fractions, Fractions Everywhere!
Fiona stays awake at night
They're living in her closet,
And give her quite a fright!

Fractions, Fractions Everywhere!
They've grown on **Ryan's** lawn
Jeff found them in the fridge;
Oh how they wish they were gone!

Andrea's seen them at the mall,
Will I survive another day?
They've invaded **Jayden's** TV
How can they be kept at bay?

Fractions, Fractions Everywhere!
Has made **Darrel** start to worry
Andrew's eyes are rather glazed;
Dan wished they weren't in such a hurry.

Fractions, Fractions Everywhere!
My teacher makes it look simple.
Amanda gets her questions right,
And me – I get a pimple!

Willy seems to know the answers;
Jeremy and **Gabe** at least try.
Sometimes I put my hand up,
But then I forget why.

Fractions, Fractions Everywhere!
Craig can't take them any more.
One more common denominator,
And **Taylor** will run out the door!

Fractions, Fractions Everywhere!
Ms. King says, "You can do it!"
They're making **Aaron** crazy,
I'm not sure I can get through it.

Fractions, Fractions Everywhere!
Laura says, "they're just a number."
But what if **Michael** was a carpenter...
Would he need them to work with lumber?

Fractions, Fractions Everywhere!
Though they are just numbers in a book,
Because we'll use them all our lives,
They're worth a second look.

Maybe if I work each day
To get just one question right
The next day will be easier
When I don't put up a fight.

Each concept has a reason,
To be worked on hard at school.
But to really understand them,
I've got to keep my cool.

On that day in my next grade
When I hear we're doing fractions,
I'll sit up and listen carefully,
So I won't miss any action.

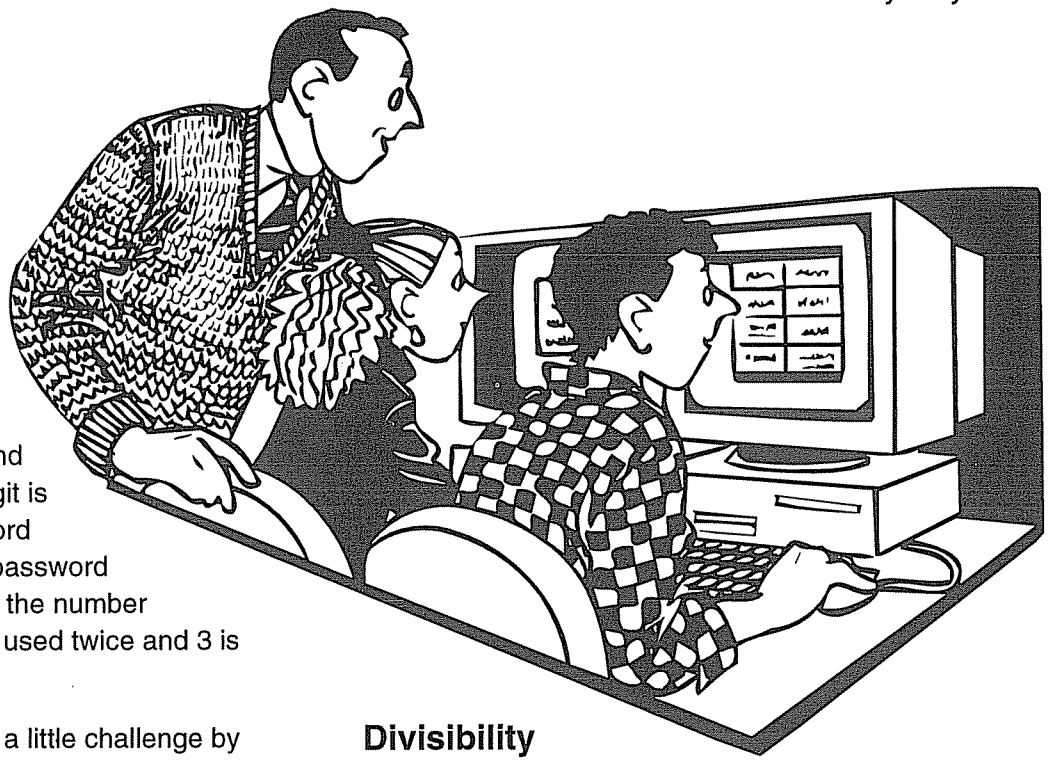
When I grow up, I think I'll be
A scientist for sure.
I'll work all day and through the night,
To find a fraction cure!

Children round the world will thank me
I know **Ms. King** will be surprised
For twenty years from now,
I'll get the Mathematics Nobel Prize!

(20 years later)
Fractions, Fractions, Everywhere!
The presenter is such a laugh...
He said that two of us won the award...
So it has to be cut in half!

▲ Divisibility for Secret Password

By Jerry Ameis



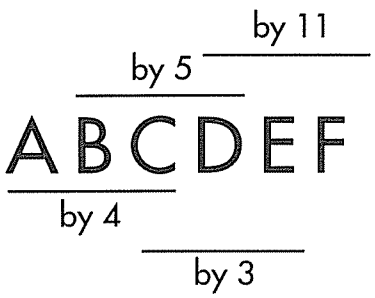
Problem

You want to protect your personal computer files from prying eyes. You can do that with a secret ìpasswordî number. Your computer only lets you create a 6-digit number that is made by using each of the digits: 1, 2, 3, 4, 5, and 6. This means that each digit is used only once in a password number. For example, the password number 651243 is valid but the number 125564 is not because 5 is used twice and 3 is not used at all.

You decide to give yourself a little challenge by creating a password number that has special mathematical features. Doing something special is always more interesting. You want your 6-digit number to have four divisibility features. Here is a description and diagram to illustrate what is wanted. The digits are labelled ABCDEF.

1. ABC must be divisible by 4.
2. BCD must be divisible by 5.
3. CDE must be divisible by 3.
4. DEF must be divisible by 11.

Diagram:



Divisibility Tests you need to know.

1. Divisibility by 4 requires that the number be even (divisible by 2). The 3-digit number must end in 0, 2, 4, 6, or 8. If the last two digits are divisible by 4, then the 3-digit number is divisible by four. Examples: 264 and 356.
2. Divisibility by 5 requires that the number end in 0 or 5.
3. Divisibility by 3 means that the sum of the digits is divisible by 3. Example: 135 is divisible by 3 because the sum (1 + 3 + 5 = 9) is divisible by 3.
4. Divisibility by 11 is a little tricky because the two outer digits of the 3-digit number must add to the middle digit. Example: 462 is divisible by 11 because 4 + 2 = 6. Note that 462 is 11 x 42.

Is more than one answer possible? Editor, Frances Schatz, would like to hear about your possible answers. The author has provided one possible answer hidden on another page.

▲ ABACUS: PART I: A LITTLE HISTORY

By Frances Schatz

If you like to trace your family roots or if you like to collect antiques then you will understand why this mathematician likes to collect and study old counting and calculating devices. Let me share what I have learned about the abacus.

Before modern calculators and computers, and before pencil and paper arithmetic, a counting board or abacus was used. The ancient arithmetician could do finger arithmetic on an abacus at lightning speed. This tool is most popular in China and Japan. Both the Japanese soroban and the Chinese suan-pan have a top and a bottom window where the top window beads count fives and the bottom window beads count ones. The major difference between the two styles is in the number of beads. The soroban has one five and four ones in each column while the suan-pan has two fives and five ones in each column (illustrated on title page). For the purposes of this article we will work with the suan-pan.



Figure 1

The word abacus probably is derived from a Semitic word abq meaning dust; thus, the device grew out of a sand tray used as a counting board. The Greek word abax means counting board. The formation of the Chinese symbol suan (figure 1) meaning 'to calculate' is interesting. The symbol is two hands holding a counting board made of bamboo (figure 2).



Figure 2

Why study the abacus?

It is a simple and user friendly device. It costs little and it could be homemade. It operates as reliably as one's fingers. It is a toy for children or adults. I guess that is why quite a few bankers and business CEO's have a fancy brass paperweight abacus sitting on their desks. So the abacus is decorative, meantenance-free, and a stress reliever. Have you noticed a busy executive talking on the phone and doodling with the brass ornament on the desk? In our multi-cultural society the abacus allows us to do arithmetic and problem solving without paper and pencil.

Design

My nine column abacus measures 14 cm by 10 cm. The frame and crossbar are wood. The pony beads are strung on fine wooden dowel. It uses 28 red beads, 7 blue beads and 28 gray beads. (Figure 3) The red beads (columns SRQP) are on the left, blue column X in centre and gray (columns ABCD) are to the right. The two colours are useful for subtraction and for work with integers. The three colours are great for multiplication or decimal work.

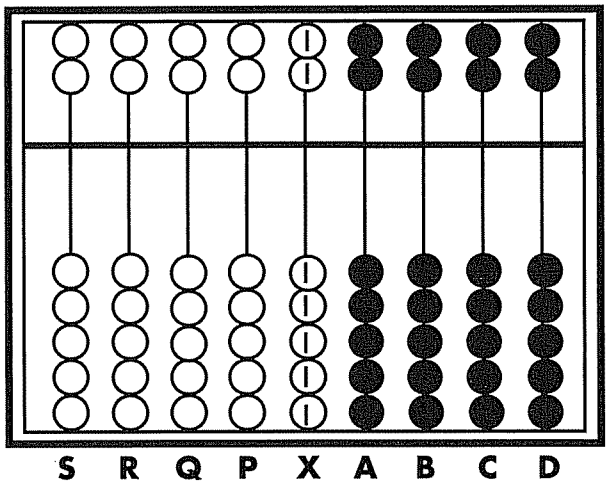


Figure 3

The crossbar divides the abacus into two windows 'heaven' and 'earth'. There are two beads in each column in heaven and five beads in each column in earth. Each column is an axis for counting. Heaven beads count five while earth beads count one. Starting at the right the place values of the nine columns for whole numbers are ones (D), tens (C), hundreds (B), thousands (A), ten thousands (X), hundred thousands (P), millions (Q), ten millions (R), hundred millions (S). The place values change if X is considered to be the decimal point.

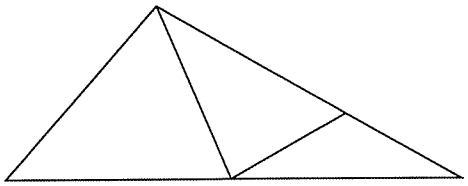
Part 2 will continue next issue with fingering and some addition.

▲ Elementary Mathematical Challenges

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American Junior High School Mathematics Examination 1998

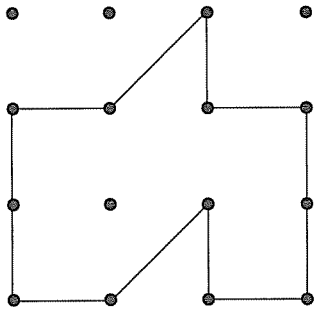
1. $\dfrac{\frac{3}{8}+\frac{7}{8}}{\frac{4}{5}} =$ (A) 1 (B) $\frac{25}{16}$ (C) 2 (D) $\frac{43}{20}$ (E) $\frac{47}{16}$

2. How many triangles are in this figure?
(Some triangles may overlap other triangles.)



(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

3. Dots are spaced one unit apart, horizontally and vertically. The number of square units enclosed by the polygon is



(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

4. For a sale, a store owner reduces the price of a $10 scarf by 20%.
Later the price is lowered again, this time by one-half the reduced price.
The price is now

(A) $2.00 (B) $3.75 (C) $4.00 (D) $4.90 (E) $6.40

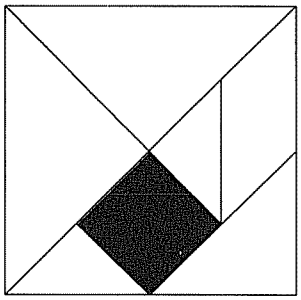
5. Harry has 3 sisters and 5 brothers. His sister Harriet has S sisters and B brothers.
What is the product of S and B?

(A) 8 (B) 10 (C) 12 (D) 15 (E) 18

6. $2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+4\left(1-\frac{1}{4}\right)+\ldots 10\left(1-\frac{1}{10}\right) =$ (A) 45 (B) 49 (C) 50 (D) 54 (E) 55

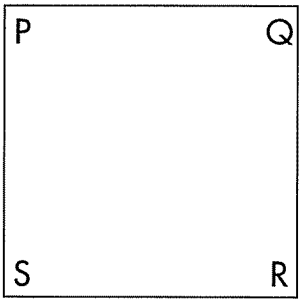
▲ Elementary Mathematical Challenges (cont'd)

7. What is the ratio of the area of the shaded square to the area of the large square (The figure is drawn to scale.)



(A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{12}$ (E) $\frac{1}{16}$

8. Let PQRS be a square piece of paper. P is folded onto R and then Q is folded onto S. The area of the resulting figure is 9 square centimetres. Find the perimeter of square PQRS.

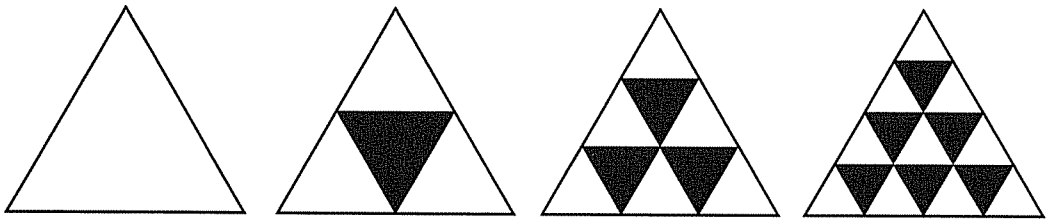


(A) 9 (B) 16 (C) 18 (D) 24 (E) 36

9. A 4 x 4 x 4 cubical box contains 64 indentical small cubes that exactly fit the box. How many of these small cubes touch a side or the bottom of the box?

(A) 48 (B) 52 (C) 60 (D) 64 (E) 80

10. If the pattern in the diagram continues, what fraction of the interior would be shaded in the eighth triangle?



(A) $\frac{3}{8}$ (B) $\frac{5}{27}$ (C) $\frac{7}{16}$ (D) $\frac{9}{16}$ (E) $\frac{11}{45}$