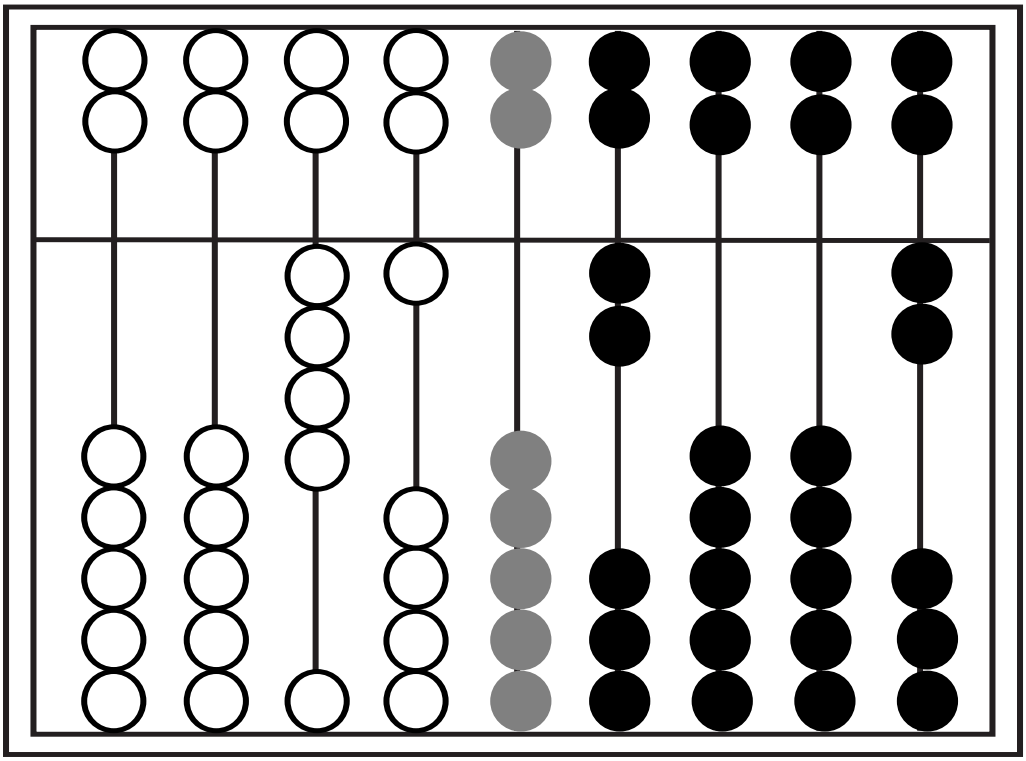
Abacus

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Volume 41 • Number 1 • SEPTEMBER 2002 • A Publication of the OAME/AOEM

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Volume 41 2002



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▲ From the Editor's Desk

When I returned from the NCTM Annual Conference 'Realizing the Vision of School Mathematics' held in April, I was energized by the many interesting activities that can happen in elementary mathematics classrooms. This issue will bring several to your attention.

Tara Tasker, a teacher at Waverley Public School in Bowmanville and a member of Pine Ridge, a chapter of OAME, wrote her own math resource entitled 'Math Wizards' for grade three patterning. She identified a student submission she would like to share with readers. With parental permission and school permission the *Abacus* is pleased to share **Tyler Zammit's** story 'The Hungry Gupey'. It contains a growing pattern in the illustrations, an AB pattern with WHAT the gupey eats and a +1,+2,+3 pattern with HOW much the gupey eats. When you read this story you'll have to pretend that it was done in a roll and slide book format reproduced at a 35% reduction from original size.

P. Gill Dunn, the 'Mathman' and I had an informal breakfast meeting at the B&B where we were staying on the Saturday of the OAME annual conference at Barrie. Gill, a retired mathematics teacher, loves to share his Mathman projects. His presentations motivate students to excellence, raise self-esteem, and encourage creative thinking as well as reveal the 'fun' in math. The game, SIXTRIX, is part of his Mathman's manipulatives kit workshop. You may be interested in visiting his website for other nifty classroom ideas: http://mathmanhere.tripod.com/

Stewart Craven, chairperson for the Ontario Mathematics Olympiad 2002 held in Toronto this past June, kindly shared competition material with me for *Abacus* readers.

The National Council Teachers of Mathematics (NCTM) has recently released some new publications for elementary teachers. As your Ontario NCTM representative I would be pleased to arrange for you to get copies of any of the following. You can contact me regarding availability and Canadian pricing:

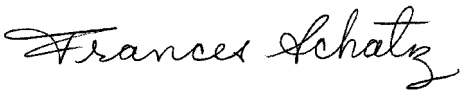
Navigating Through Algebra has PreK-2, 3-5, and 6-8 available as books with CD-ROM. This series presents activities to stimulate students thinking about patterns, equality, and variables leading to senior elementary algebra and functions.

Navigating Through Geometry offers the same as the algebra series. Fundamental ideas of geometry are introduced, developed, and extended with student activities.

The Young Child and Mathematics focuses on children from ages 3 to 8 and offers classroom vignettes and teacher-tested activities for this age group.

If you have an activity that you enjoy using in your K-8 class, be sure to forward it to *Abacus*. If you have a request for an activity to meet a particular curriculum expectation, don't hesitate to contact the editor who will do the research.

Have a great startup with your new classes.



Frances Schatz

▲ SOLUTIONS/SCORING RUBRICS FOR OMO 2002

1. 4 points

1	2	3-4
Little evidence that the student can approach the problem.	Organized approach. Partially correct answers.	Organized correct answer(s).

ANSWERS: Student should show some evidence of organization through sketches or drawings.

a) D b) F

2. 4 points

1	2	3-4
Little evidence of an organized approach.	Some organization. Partially correct.	Well organized. Correct answer(s).

ANSWERS:

a) Since 4 divides 8 twice there are 2X2X2 cubes. i.e., 8 cubes will fit

b) Since we can fit 3 layers of 4 cubes - exactly 12 cubes will fit.

3. 4 points

1	2	3-4
Shows little understanding of probability.	Shows some understanding of probability. Partially correct method.	Understands probability. Correct answer.

ANSWER: There are 12 jellybeans in total and 5 are green. Hence the probability of Nikita picking a green bean is 5 out of 12 or 5/12.

4. 4 points

1	2	3-4
Partial analysis. Computational errors.	Some attempt at analysis. Mostly accurate computation. Attempt at systematic elimination.	Well organized systematic elimination. Correct solution.

ANSWER: Consider where to place the smaller digits

(X X X)	(X X)
3	3
or	or
4	4

Place larger digits in other places; complete computations and compare products.

SOLUTION: 467 x 35 = 16345

5. 6 points

1	2-3	4-5	6
Unsure of how to approach the problem or definition of 'prime'. Unsure of order of operations	Organized approach. About 50% of solutions correct. Some ability to use order of operations	Well organized. Almost all answers are correct.	Well set out. Complete accuracy.

ANSWERS: (N.B. There are several possible correct solutions)

2	(3/3 + 3/3) or 3 - 3/3
3	3 or 3/3 + 3/3 + 3/3 etc
5	(3 + 3/3 +3/3)
7	given
11	3 X 3 + 3/3 + 3/3
13	3 X 3 + 3 + 3/3
17	3 X 3 + 3 X 3 - 3/3
19	3 X 3 + 3 X 3 + 3/3
23	3 X 3 + 3 X 3 + 3 + 3 - 3/3
29	3 X 3 X 3 + 3/3 + 3/3
31	3 X 3 X 3 + 3 + 3/3

6. 8 points (2 points for a, 3 for b, 3 for c)

1-2	3-4	5-6	7-8
Shows little evidence of analysis of the patterns	Analyses the patterns with some success. Partial accuracy.	Analyses patterns in organized ways with considerable success. Considerable accuracy.	Thorough analysis. Accurate solutions.

ANSWERS:

a) Algebraically this is based on 3 x + 2 where x is 0,1,2 etc.
So the sequence is 2,3,8,11,14,17,20.

b) This is a Fibonacci sequence where each term is obtained by adding the two previous ones.
i.e., (-2) + (-4) gives -6 and (-4) +(-6) gives -10
and (-6) + (-10) gives -16

c) The pattern here is rotational symmetry - the centre of the 1,8, midway between 11, 69, 96 and the centre of the 0 and middle 1 of 111.

So we need another two digit number with rotational symmetry - there are only four 11 and 88, 69 and 96. So the missing number is 88.

7. 8 points

1-2	3-4	5-6	7-8
Shows little understanding of ratio. Finds it difficult to approach the problem.	Shows partial understanding of ratio	Organized approach. Explains analysis. Mostly successful.	Thorough organization. Concise solution/ explanation. Correct answers.

ANSWER: If the number of cherries is increased by 20% then there are 3 + (.2) 3 = 3(1.2) = 3.6 cherries.

If the number of grapes is decreased by 20%, then there are 5 - (.2)(5) = 5(0.8) = 4.0 grapes.

So the new ratio of cherries to grapes is 3.6 to 4 or 36 to 40 which reduces 9 to 10 or 9:10 or 9/10.

8. 10 points a) 3 points b) 7 points

0-2	3-5	6-8	9-10
Little evidence of a systematic organization. Partially correct lists of numbers	Correct list and total for a). Some organization and a partial list for b).	Organized approach obtains a total for a) and has considerable success in working for a partial solution for b).	Well organized. Correct answers for a and b including shortcuts by grouping numbers

SOLUTION:

a) We consider all these numbers 345, 354, 435, 453, 534, 543.
3(888) = 2664.
Note: that the numbers can paired (3445 and 543) for quick calculation.

b) We now need to consider all multiple digit cases:
333, 343, 353, 334, 335, 344, 355,
433, 434, 454, 443, 445, 433, 455,
555, 535, 545, 553, 554, 533, 544
These can be added more easily by grouping in 3s, such as (333 + 444 + 555) or (343 + 454 + 535).
Both give 1332 and there are seven such groups.
So the total is 7(1332) - 9324.
Added to the total from a) this gives 9324 + 2664 = 11988.

▲ Patterning Picture Books: Teacher Notes

by Tara Tasker

The first week we read a variety of books that have patterns in them and we discuss all of the patterns we see. These patterns include text patterns (repetitions in text), number patterns (growing or shrinking patterns), patterns in the illustrations and patterns within the actual story.

We discuss the differences and similarities of the books. We discuss the components of a picture book: do the illustrations match the text?, why is there a lot of text on a page?, what is important about the cover and title page?, and why is there only a few words with a picture?

The students complete *The Story Planner* and *Editor's Checklist*. In this issue you will also find my *'Math Wizards©: Patterning Booklet Rubric'*. Your students will need the following:

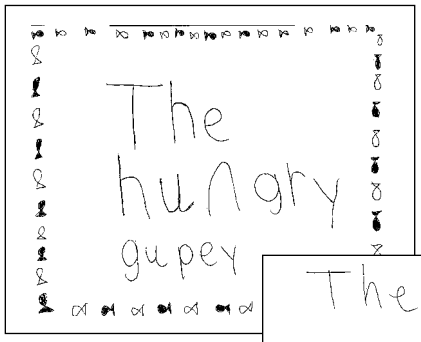
Directions for a Roll and Slide Book

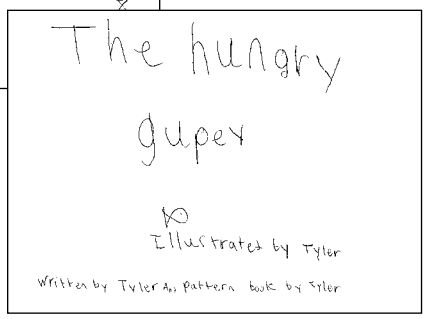
1. Fold an 8 1/2" x 13" paper in half (hamburger style).
2. Along the fold, measure 5 cm in from either side.
3. Cut in between the 5 cm markings, removing a sliver of the fold and creating a slit in the middle of the original paper.
4. Repeat steps 1-2 for several more pieces.
5. On the pieces from step 4, cut slits up to the 5 cm markings creating two slits (one on top and one on the bottom).
6. Roll the paper with the two slits and slide it through the paper with the slit in the middle.
7. Unroll the two slit paper when the middles meet.

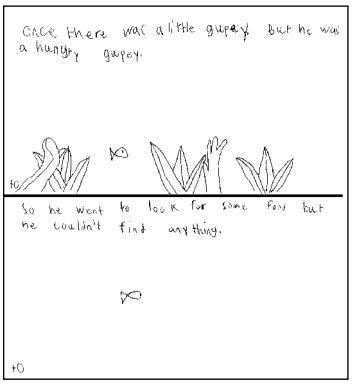
Book List

TITLE	AUTHOR	ISBN #
The Mitten	Jan Brett	0-399-23109-9
The Very Hungry Caterpillar	Eric Carle	0-399-22690-7
There was an old lady who swallowed a fly	Illustrated by Pam Adams	0-85953-018-3
The Greedy Triangle	Marilyn Burns	0-590-48991-7
A Cloak For The Dreamer	Aileen Friedman	0-590-48987-9
*Joseph had a little overcoat**	Part of *Our Wonderful World Guided Reading Series* by Prentice Hall Ginn, Canada provided by the Ontario Government	0-7702-2851-2

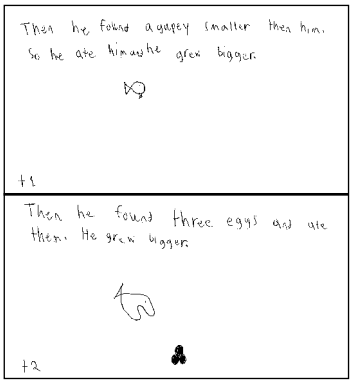
*The book *Something from Nothing* by Phoebe Gilman could be substituted ISBN# 0-590-74557-3



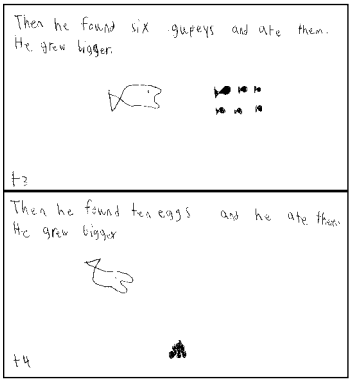




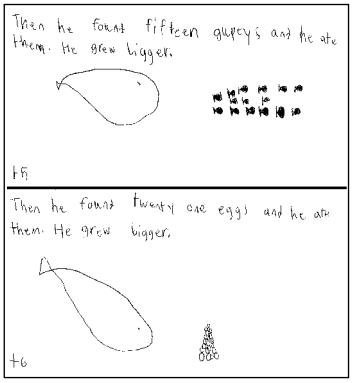
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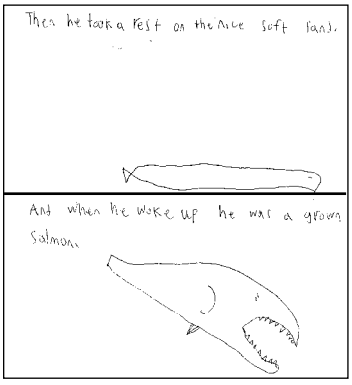
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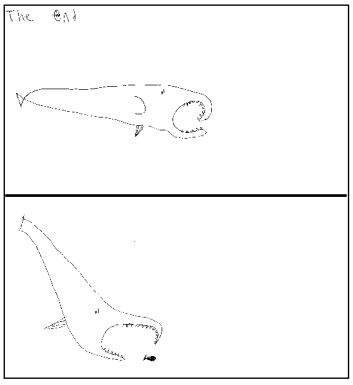
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▲ SIX TRIX Teacher Notes

by P. Gill Dunn

SIX TRIX is a very simple game to play as a one-player game. For the two-player game, I have not been able to exhaust all the strategies. (This game has remained with me over 32 years that I've been a teacher.)

One Player Game

You need a 3 by 3 square grid consisting of 9 squares and the numbers 1, 1, 1, 2, 2, 2, 3, 3, 3. The object of this game is to find as many ways to place the numbers on the 3 by 3 square grid so that the sum of the three numbers is six.

It is quite interesting how different students approach this puzzle. When I have seen a student set up the problem this way (figure 1), I'll ask that student how many different ways does (s)he see that totals six? Often the response is four ways and this is where I interject that I see two other ways. Now the student realizes that the diagonal is another possibility. Another interesting observation is when a student can only get seven ways of doing this puzzle and knows that there is an 8th way. See figure 2 and 3. For figure 3, suggest changing the centre number for figure 2. Only when (s)he has found all eight ways will I then explain the rules for the two player game. A student who only knows seven ways will be at a disadvantage when (s)he plays another student who know the maximum number of ways of obtaining six.

1	2	3
1	2	3
1	2	3

figure 1

Two Player Game

Before I explain this 2 player game, let me digress for a moment. This puzzle began with the language arts curriculum. In my grade 5 class there was a very bright Italian boy (Montreal 1964) who caught onto this game very quickly when he played me. He had to write the instructions for the single game. Next he wrote 5 or 6 rules for the two-player game so that any other student from the class of 34 boys could read his instructions, solve the puzzle and then be able to play him in the 2 player game. This proved to be quite a challenge for "Mario" to improve his writing skills in his second language.

This is how the two player game is played: Each player receives the numbers 1,2,3. The remaining numbers 1,2,3 area turned face down and shuffled. Each player now selects one of the numbers and the remaining face down number is turned over and placed in the centre of the 3 by 3 grid. The player who has just picked the largest number that was face down has the option of going first or second. The game begins by the first player playing one of his/her numbers on the 3 by 3 grid. A point is scored as soon as 3 of these numbers add up to six. Each player takes a turn in placing his/her numbers in one of the empty squares remaining. Round 1 concludes when all the numbers have been played. Remove the numbers from the board and proceed with round 2. It is the same procedure as round 1 except that second player starts. Continue the game for several rounds until one of the players scores 15 points. Originally, the game ended at 10 points but I extended it to 15 in order to allow for better strategies to work within the game. (I'm still not sure if 15 is the best number to decide the winner.)

It should be noted here that when you score points in this game you can sometimes allow your opponent to score 1 point while you score 2 points. For example, (see figure 4) if player A puts his 1 in row 2, column 3 and scores a point, player B places his 3 in the upper right corner and scores 2 points.

2	2	1
3	1	2
1	2	3

figure 2

Some students may not have any success winning at this game because they need to learn a defensive strategy, especially when they have to go first. Here is an opportunity for these students to be exposed to some fun "mathematical research". I would ask them to revisit the single player game and investigate the minimum number of ways you can score six. For example in figure 5 there is only one way in which the sum of the three numbers adds up to six. Now have them go back and challenge their opponent using this strategy and see if the results are any different. (I never had a chance to use this technique with a whole class to see if it really helps the weaker student – I would appreciate some feedback here.)

2	1	3
1	3	2
3	2	1

figure 3

1	2	
2	3	
3	1	2

figure 4

2	3	3
1	3	2
1	1	2

figure 5

It would be very easy to set up a class tournament where an eventual class champion would be declared the overall winner. All students could be invited to write down the strategies used and to share their information in a collaborative setting. With the new emphasis on communication in Mathematics, is a four player game possible? I will await your feedback on this idea.

▲ Math Wizards© Activity

Math Wizards© Story Planner for Patterning Picture Book

The pattern in my story will be _____

_____

_____

WHO are the characters? _____

_____

_____

WHAT are they doing? _____

_____

_____

WHERE are they? _____

_____

_____

WHEN is it? _____

_____

_____

The Problem: _____

_____

_____

The Solution: _____

_____

_____

The Beginning: _____

_____

_____

The Middle: _____

_____

_____

The End: _____

_____

_____

Math Wizards© Editors' Checklist

Does your story have

- ☐ A beginning
- ☐ A middle
- ☐ An ending
- ☐ A pattern

What is your pattern rule?

_____

_____

Explain the choice of your pattern rule:

_____

_____

Are your ideas

- ☐ In order
- ☐ Clear
- ☐ Interesting

I checked my capitals

- ☐ At the beginning of sentences
- ☐ In names
- ☐ In dates

I checked my punctuation for

- ☐ Periods
- ☐ Question marks
- ☐ Exclamation marks

☐ I checked my story for spelling

☐ I reread my WHOLE story after editing it

☐ My pictures give clues to the words and the story.

▲ Math Wizards© - Patterning Booklet Rubric

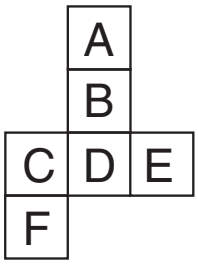
	LEVEL1	LEVEL 2	LEVEL 3	LEVEL 4
Understanding of patterning concepts in written story	limited understanding of concepts through partially complete and unclear explanations in the story	understanding of concepts is incomplete based on appropriate but inaccurate explanations in the story	solid understanding of concepts through complete and appropriate explanations and procedures in the story	complex and/or thorough understanding of concepts explained throughout the story
Communication of patterning knowledge through the written story	• mathematical language is imprecise or inappropriate • no closure or reference to the original pattern is provided	• Some mathematical language and symbols are used appropriately, but are compromised by errors or vagueness • closure to the problem is provided, but may be ambiguous or incomplete	• mathematical language and symbols are used appropriately, no errors of terms • closure to the pattern is communicated clearly	• mathematical language and symbols are used purposefully and with elaboration to enhance the story • strong descriptions of patterns provided, with extensions and/or alternatives
Communication of patterns in illustrations	• illustrations are not used to communicate pattern in story	• Illustrations are communicating some patterns but are incomplete	• Illustrations are clear, precise and represent the text.	• Illustrations are used purposefully to communicate pattern and enhance the written text
Applications of patterns in illustrations	• patterns include major errors and omissions	• Patterns are often correct but include several minor flaws or errors	• Patterns are accurate with only a few minor errors • some additional patterns are provided	• patterns are accurate in illustrations • additional patterns are provided in illustrations and are connected to the main pattern in the story
During the production of the story the student	Required assistance	Required some assistance	Was independent	Was independent
The overall presentation of the story	Needs improvement	Is satisfactory	Is good	Is excellent.

▲ Ontario Mathematics Olympiad 2002 -

1. If the figure shown is folded to form a cube,

   a) what symbol would be opposite to "A"?

   b) what symbol would be opposite to "B"?



2. a) How many 4cm X 4cm X 4cm cubes will fit in 8cm X 8cm X 8cm box?

   b) How many 4cm X 4cm X 3cm cubes will fit in 8cm X 8cm X 9cm box?

3. A jar contains 3 red jellybeans, 4 blue ones, and 5 green ones. Nikita takes one jellybean from the jar. What is the probability that she will pick a green jellybean?

4. What is the smallest possible product when multiplying a three-digit number by a two-digit using each of the digits 3, 4, 5, 6, and 7 only once?

5. Use only the number 3 and the basic operations (+,–,x, and ÷) to write each prime number up to 32. For example, 7 could be written as 3 + 3 + 3 ÷ 3.

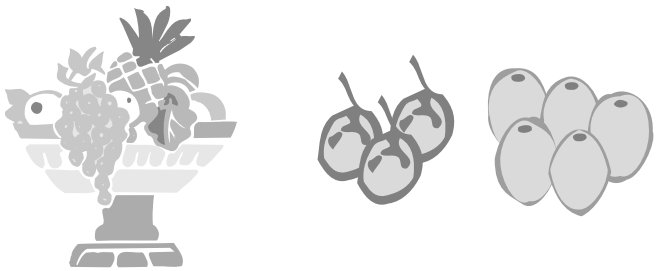
6. Here are some number sequences. Find the missing numbers in each sequence and explain the pattern.

   a) 2, 5, 10, 17, 26, ___, ___, ___

   b) 2, -2, 0, -2, -2, -4, ___, ___, ___

   c) 1, 8, 11, 69, ___, 96, 101, 111

7. The ratio of cherries to grapes in a fruit salad is 3 to 5. If the number of cherries is increased by 20% and number of grapes is decreased by 20%, what is the new ratio of cherries to grapes?



8. a) What is the sum of all three-digit numbers made of the digits 3, 4, **and** 5 (e.g., 435, where no repeat digits are allowed)?

   b) What is the sum of all three-digit numbers made up of the digits 3, 4 **or** 5 (e.g., 343, where repeat digits are allowed)?

▲ Six Trix Game

Object of the game

Cut out the number grid. Find as many ways to place the numbers on the 3 by 3 square so that the sum of the three numbers in any direction is 6.

1	1	1	2	2	2	3	3	3