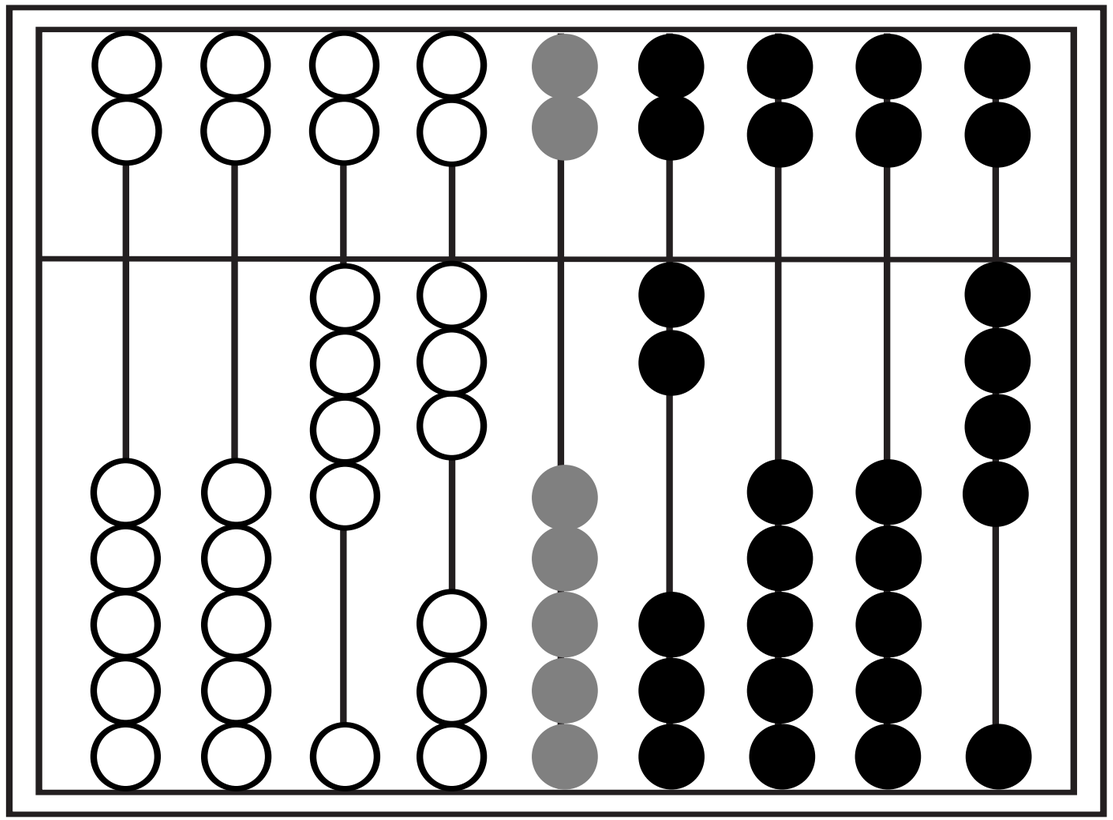
Abacus

Volume 43 2004



ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATORS

Volume 43 • Number 2 • December 2004 • A Publication of the OAME/AOEM

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From the Editor's Desk

…numbers have neither substance nor meaning, nor qualities. They are nothing but marks, and all that is in them we have put into them by the simple rule of straight succession.

- Herman Weyl

In this issue of *The Abacus* we shall be introducing our readers to some of the rich mathematical tasks and investigations which can be found at the NRICH website. For those of you who have visited this site, you already know that they have a range of delightful problems that span the grades. The site has kindly granted us permission to use their problems, investigations and articles in this journal. We express our deep appreciation to them for their willingness to share their resources with us! We know that you will find the tasks to be both meaningful and worthwhile. Please let us know how your students react to these tasks. We would like to know how empowering the tasks are and some of the ways the students solve the assigned tasks. Please send us some of the students' work and we shall publish as many of them as we can. As we know, a rich problem can be solved in many different ways and students have a way of surprising us as they construct knowledge and meaning for themselves.

We would also like to receive some problems or investigations from you or from your students. We know that every day many amazing and imaginative lessons are 'taught', and that teachers are continually creating for their students tasks that are seldom shared with others. We would like to have a "Share Your Ideas" section in which we could show case lesson tips, problems or insights gained from teaching a concept or lesson. It need not be a long article, just an idea or two will suffice.

We shall be starting some new columns in the next edition of *The Abacus*. Plans are under way to have articles written for each of the following sections: Mathematics and Literature, Concrete Materials Across the Grades and Technology Corner which will feature either *Geometer's Sketchpad*, Calculators or Tinkerplots. If there is another regular feature that you think we should include, please send your suggestions to us.

Looking forward to hearing from you!

Strange Numbers

printed from http://nrich.maths.org

We define "strange" numbers in the following way. All strange numbers are prime. Every one digit prime number is strange and a number of two or more digits is strange if and only if so are the two numbers obtained from it by omitting either its first or its last digit. Find all strange numbers.

This problem comes from the Hungarian magazine KöMaL.

Mathematics In The News: Mersenne Primes

Teacher: **Darlene, is 37 a prime number?**

Darlene: **Miss, why are prime numbers important? Why do I have to study prime numbers?**

Teacher: **Well, let's see…**

A scientist has used his computer to find the largest prime number found so far – written out, it would stretch for 25 km.

Primes are important to encryption and could lead to uncrackable codes.

Why do you believe that they could be used in this way?

The new figure, identified by Josh Findley, contains 7 235 733 digits, and would take someone the best part of six weeks to write out longhand.

One could ask the students, before giving them the answer, how long it would take a person to write the number.

He is a volunteer in the Mersenne.org research project called the Great Internet Prime Search (Gimps). Mr. Findley used his home computer and free software as part of an international grid of 240 000 networked computers.

The new number, expressed as 2 to the 24 036 583th power minus 1, has 7 235 733 decimal digits. It is nearly a million digits larger than the previous known prime number, and belongs to a special class of rare prime numbers called Mersenne primes.

Some Prime Facts!

- An integer greater than one is a prime if its only divisors are one and itself.
- A Mersenne prime is a prime of the form 2^p-1.
- The first Mersenne primes are 3, 7, 31, 127, etc.
- There are only 42 known Mersenne primes.
- The man whose name they now bear, the French monk Marin Mersenne (1588-1648), made a prediction about which values of "P" would yield a prime.
- Mersenne primes have been central to number theory since they were first discussed by Euclid in 350 BC.
- Historically, searching for Mersenne primes has been used as a test for computer software.
- The Fundamental Theory of Arithmetic states they are building blocks for numbers.
- Hamann's conjecture: Every even number is the difference between two primes.
- Arthur Hamann was a Grade 7 Illinois student in 1976.
- The prime factorization of any composite number is unique. This means, for example, that 24 can be factored in only one way, 2 x 2 x 3 (or 2 x 3 x 2), the order does not matter.
- As the numbers of the sequence increase, the intervals between primes, on the average, get longer, though in a somewhat haphazard way. In other words, primes gradually become more scarce as numbers get larger, and one can imagine that, at some point, they might run out.

(Continued on page 4)

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More than 2,000 years ago, however, the Greek mathematician Euclid proved that the sequence of primes continues forever. Suppose there is a finite number of primes, he argued. That means there's also a largest prime. Multiply all the primes together, then add 1. The new number is certainly bigger than the largest prime.

If the initial assumption is correct, the new number can't be a prime. Otherwise, it would be the largest. Hence, it must be a composite number and divisible by a smaller number. However, because of the way the number was constructed, all known primes, when divided into the new number, leave a remainder of 1. Therefore, the initial assumption can't be correct, and there can be no largest prime. ["Prime Theorem of the Century" by Ivars Peterson, Mathland, December 23, 1996] ▲

▲ 'Prime Magic'

printed from http://nrich.maths.org

Place the numbers 1, 2, 3,..., 9 one on each square of a 3 by 3 grid so that all the rows and columns add up to a prime number. Two solutions are considered to be the same if, as in the example shown, they contain the same six triples. How many different solutions can you find?

2	8	3
6	4	9
5	7	1

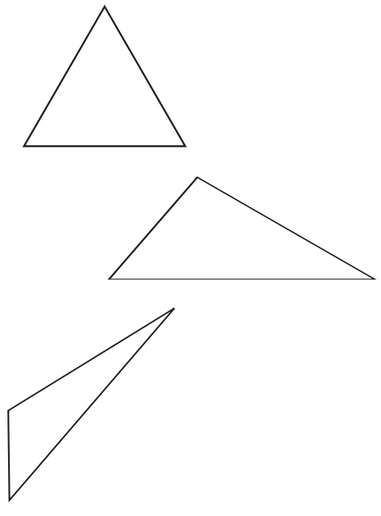
2	3	8
5	1	7
6	9	4

Show that it is impossible to place the numbers 1, 2, 3,..., 9 one on each square of a 3 by 3 grid so that the diagonals, as well as all the rows and columns, add up to prime numbers. ▲

▲ The Mathematical Eye

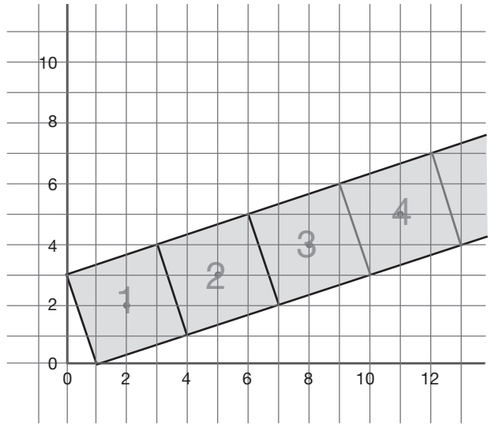
Find all the possible triangles that can be made with a 12 cm loop of string where all of the sides of each triangle are integers.

- How do you know when you have found all of them?
- Of the triangles drawn, which is the largest?
- Explain whether each triangle is scalene, isosceles, or equilateral, and also whether it is acute, right, or obtuse.
- What if the string were 13 cm? 14 cm? 15 cm?... Investigate to see whether there is a pattern.



Editor's Note: *If you assign this problem to your students, we would like to see the strategies they use in arriving at the answer.*

▲ Coordinate Patterns

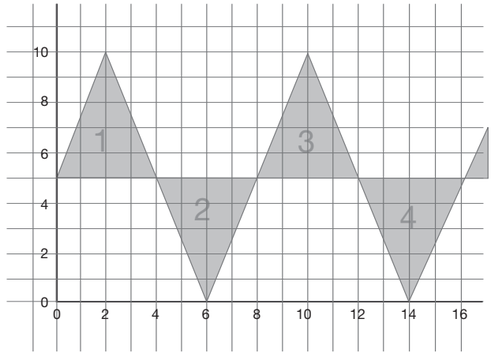


Squares

What are the coordinates of the centre of the 20th square?

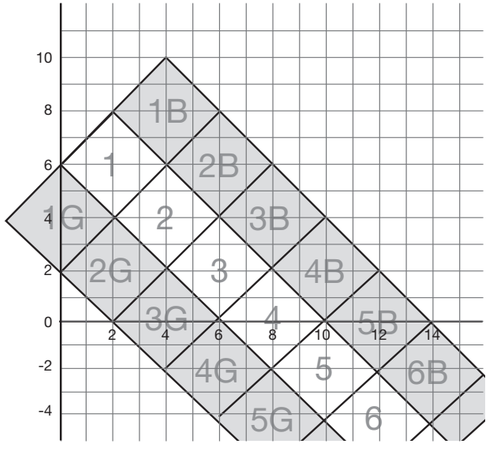
- What are the coordinates of the bottom left hand vertex of the 34th square?

  Imagine the sequence of squares extending to the left: ....-2, -1, 0, 1, 2, 3....
- What are the coordinates of the centre of the -15th square?
- What strategies are you using to answer these questions?



Triangles

- What are the coordinates of the top vertex of the 23rd triangle?
- What are the coordinates of the top left hand vertex of the 58th triangle?
- What strategies are you using to answer these questions?



And more squares

- What are the coordinates of the centre of 22B?
- What are the coordinates of the left hand vertex of 26G?
- What strategies are you using to answer these questions?

SOURCE:NRICH (http://nrich.maths.org)

▲ Inside Out

Materials Required:

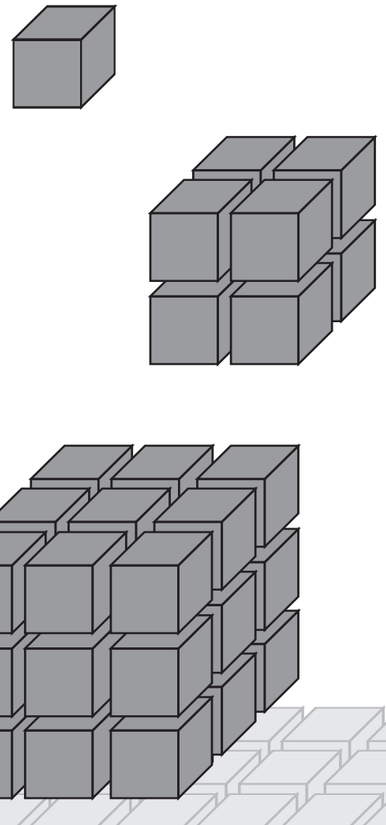
- Interlocking cubes

Imagine a (2x2x2) cube made from 8 smaller red cubes. Each small cube has 6 faces, making a total of 6 x 8 = 48 small faces.

On each side of the large cube you can see faces of 4 of the smaller cubes. By turning the large cube around you can see 24 small faces all together.

You dip the cube into a pot of yellow paint and wait for it to dry. Now you separate all the smaller cubes and you can see that 24 of the 48 small faces are yellow and 24 are still red.

This suggests that you can rearrange the cubes so that only red faces are on the outside and yellow on the inside. Then, by dipping the cube into a pot of green paint it seems reasonable to assume you will end up with 8 small cubes whose 48 faces are now all painted yellow or green (24 of each), none that are still red and none that have been painted twice. Is this true?



How about a 3 x 3 cube? This will have 27 smaller cubes, making a total of 162 faces, 6 x 9 = 54 being visible at any one time. This suggests you might need three pots of paint to colour all the faces of the smaller cubes without repetition and without having any red faces left.

Is this possible and how would you rearrange the cubes each time? Describe the system you are using.

How about a 4 x 4 x 4 cube and an n x n x n cube? ▲

Courtesy NRICH (http://nrich.maths.org)

▲ Dienes' Logiblocs

Materials Required:

- Attribute/Logic Blocks

A. These materials - a set of 48 "shapes" specifically designed to encourage logical thinking - have four different attributes:

Large or small (2)

Thick or thin (2)

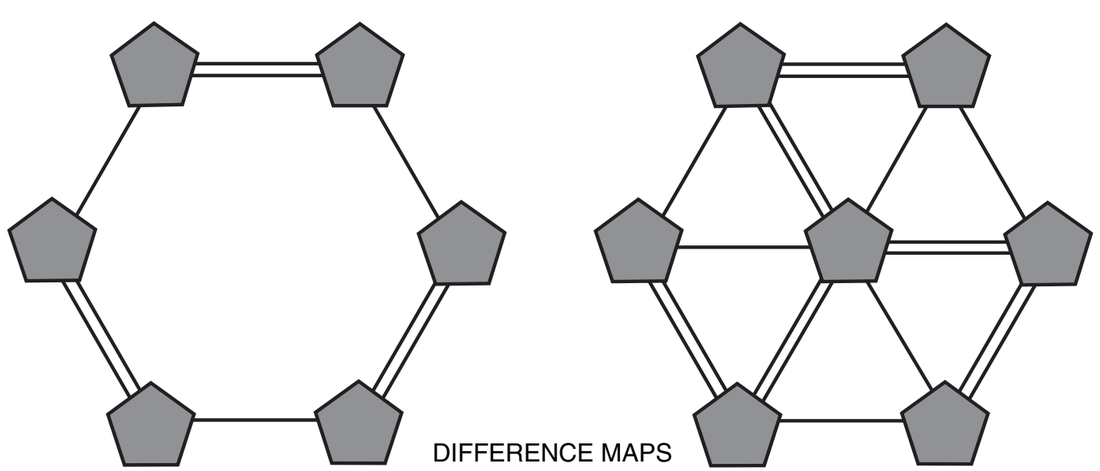
Colour - red, blue, yellow (3)

Square, circle, triangle and rectangle (4)

E.g. a large, thin, blue square is different in three ways from a small, thick, red square.

Difference maps can be pre-constructed and students invited to fill the spaces with appropriate shapes. One line means one difference, two lines two differences etc.

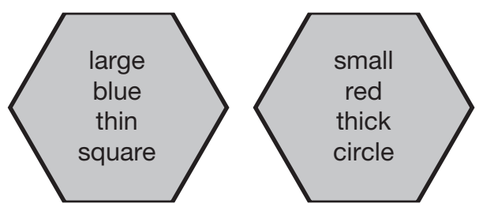
For example:



DIFFERENCE MAPS

B. Each of the 48 shapes in the set can be described on work cards containing those attributes which list their relevant size, colour, thin/thickness, shape.

The student picks a card, reads it and goes off to find the appropriate shape.▲



Work Cards

Courtesy NRICH (http://nrich.maths.org)

▲ A Long Division Algorithm

Many students struggle with the traditional long division algorithm. It is not uncommon to find students in some of the higher grades who cannot remember the traditional long division algorithm and who find the procedure somewhat mysterious and almost incomprehensible.

The following long division algorithm is worth considering. It is easy to execute and logical to follow. Students need to be informed that it is an algorithm and is in no way inferior to the traditional one.

As a preparation, review the multiplication of single digit by 20.

1 x 20 2x 20 3 x 20 4 x 20 5 x 20
6 x 20 7 x 20 8 x 20 9 x 20 10 x 20

Now pose the question.

How many teams of 6 students can we make from a total of 144 students?

4 24 R 0

```
   20
6)144
  120
   24
   24
    0
```

```
  24
 x6
 24
120
144
```

The first guess should be 20. Encourage students to find the largest multiple of 10.

We recommend the use of square grid paper to align the digits.

Some questions should be verified by multiplication.

If 144 ÷ 6 = 24, then 6 x 24 should be 24.

Continue practicing with questions in which the quotient is less than 30 including some with remainders.

1 21 R 0

```
   20
5)105
  100
    5
    5
    0
```

6 26 R 2

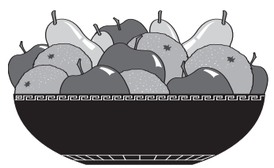
```
   20
3)80
  60
  20
  18
   2
```

For those students who are ready, extend the questions to include quotients less than 40. Precede this with practice of the multiplication of single digits by 30.

Include some questions where quotients are less than 20, as well as less than 30. If students are not guessing the highest multiple, return to lower limits for extra practice. With students who are ready continue this development to a maximum quotient of 99 using only single digit divisors. ▲

A Bowl of Fruit

Here is a bowl of fruit.



Half of the pieces of fruit in the bowl are apples. There are also 3 oranges, 2 pears and a banana.

How many apples are there in the bowl?

If, instead, one quarter were apples and one quarter were oranges and there were 4 bananas, 3 pears and 3 plums how many would be apples?

Courtesy NRICH (http://nrich.maths.org)