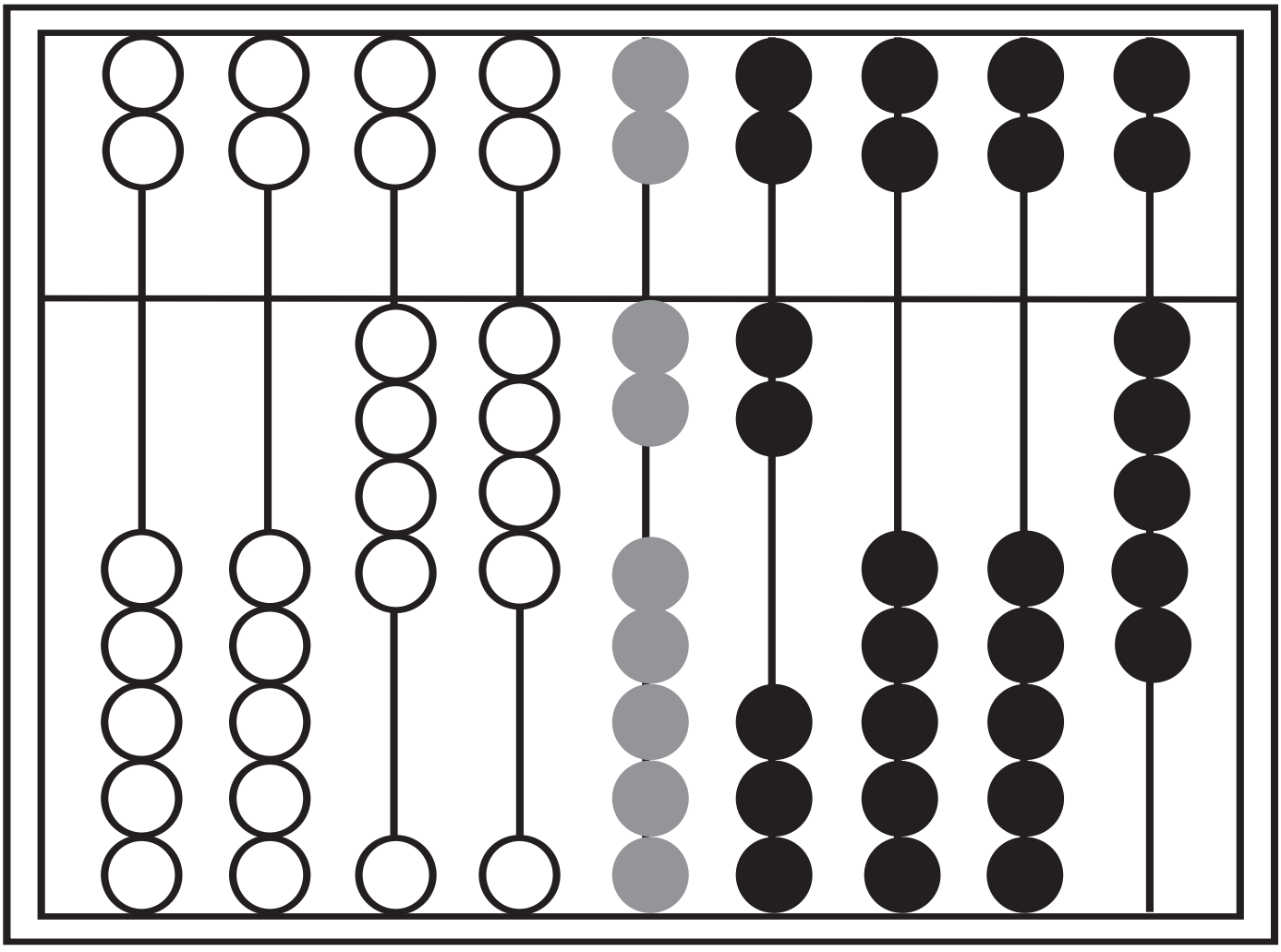
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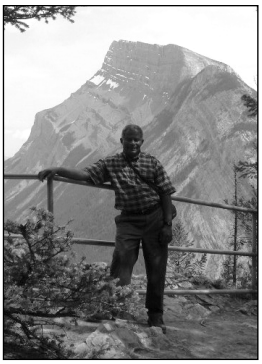
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From the Editor's Desk

Trevor Brown



By now, most of the readers of this magazine have tried to solve one of the Su Doku puzzles. This puzzle has recently become a regular feature of many of the national newspapers. The puzzle can certainly be very challenging and addictive. All the puzzles have only one solution. Hints and strategies are available on the web site www.sudoku.com on which it states that there is no mathematics involved.

I recently had a friendly discussion with a colleague as to whether Su Doku qualifies as mathematics. In the discussion he mentioned some of the strategies he has used to solve the puzzles he has attempted and how one of his colleagues, an ex-math teacher uses a different strategy when some them. I am not sure whether we resolved whether or not

Su Doku would qualify as mathematics. The discussion does. However, cause one to ask the question, "What constitutes mathematics?"

There are many interesting mathematical aspects of Su Doku worth pursuing, and here we offer some.

Let's look at a simpler Su Doku puzzle – a 4 by 4 version. The objective is similar to that of the standard puzzle. In this case, one is required to complete the grid so that every row, and every column and every one of the 2 x 2 grids contains the digits 1 to 4.

Is the solution to this puzzle unique?

1			
			2
3			
			4

Here are some additional ones to try.

In each case, determine whether the solution is unique.

1			2
3			4

1	2	3	4

1	2		
		3	4

Mathematician have calculated that there are 6 670 903 752 021 072 936 960 nine-by-nine number patterns that can be turned into Su Doku puzzles. If you were to do one every second of every day, how long do you think it would take you to solve all of them?

How long does it take you to solve a Su Doku puzzle?

Here is the original Su-Doku puzzle. Have a go!

	6		1		4			
		8	3			6		
2								1
8			4		7			6
		6				3		
			9		1			4
5								
		7	2		6	9		
	4				8		7	

▲ Some Points of Departure ~ Ratio

Problems involving **ratio** have exercised the minds of mathematicians for a long time, and ratio has been regarded for sometime as a core area in mathematics. Indeed, not too long ago, most of the arithmetic that students experienced in school involved ratio.

The activities included in this section are not intended to be used in any particular order. You could try these for yourself and discuss with your colleagues what ideas of ratio are involved, or you could assign them to your students. These tasks show some of the many mathematical situations that require a knowledge and understanding of ratio

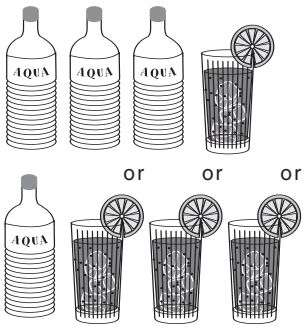
Some research, which has offered a good deal of insight into the strategies and techniques involved in the solving of ratio problems, was undertaken by Noelting and his associates. The test that he gave to a number of his students involved the comparison of the relative '**oranginess**' of two drinks made by combining a certain number of glasses of orange juice with a certain number of bottles of water.

The following are some of the tasks that they suggested could/should be given to students as some starting points to ascertain their understanding of ratio.

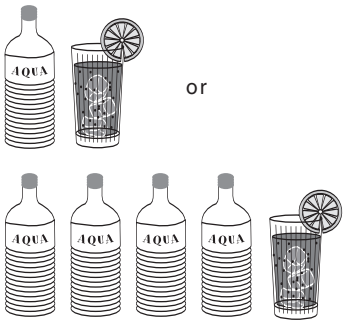
Why not try the following with your students?

Which Is Orangier?

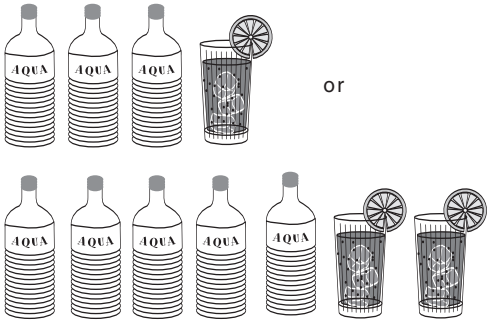
a)



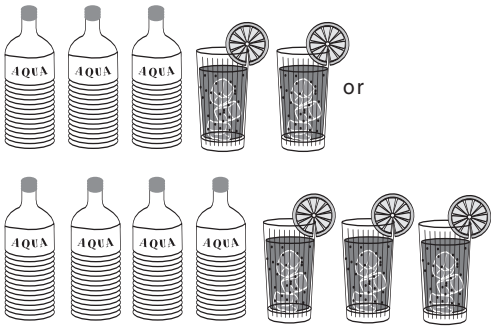
b)



c)



d)



Make up your own 'orange" tasks.

Task 1: Mixing It Up

To make a jug of orange squash use two glasses of orange juice and five bottles of water.



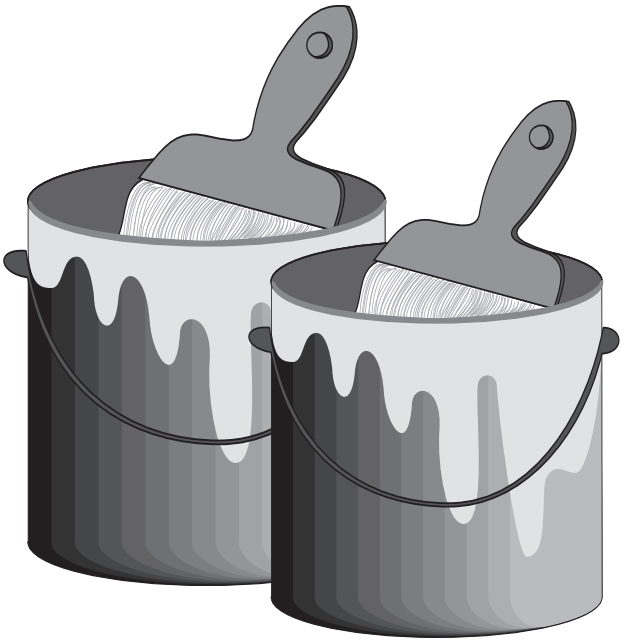
What happens to the strength of the mixture if I add an extra glass of orange juice?

What is I add an extra bottle of water?

What happens if I add one of each, two of each, etc.?

Task 2: Ratio of Red Paint to White Paint

Paint is to be mixed so that the ratio of red to white is 3 to 2.



If a mixture is to have 6 cans of white,

How many cans of red paint are needed?

If you want to make 30 cans of the mixture, how many cans of red paint and of white paint will you need?

Task 3: One Day I'll Catch Up

My dad is four times older than I.

When will he be three times as old?

When will I catch up?



Task 4: Quick Percentages

If you want to increase something by 10%, it's easy, you just multiply it by a magic number!

Does this work?

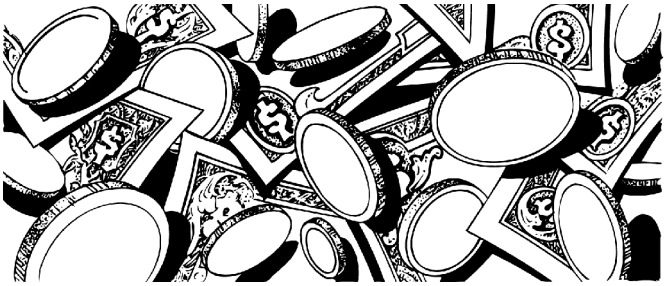
What is the 'magic number'?

Does this work for other percentages?

What about decreasing?

Task 5: Discount

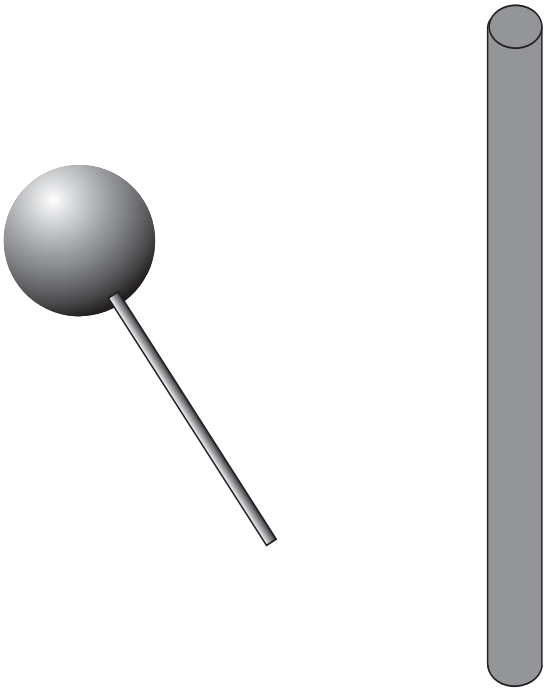
In a warehouse you are given a 20% discount on some goods but you have to pay an additional 15%.



Which do you want calculated first? Why?

Task 6: Better Estimate

Nadia estimates the length of two objects – a lollipop stick and a wooden pole.



	Actual Length	Estimate
Lollipop stick	12 cm	10 cm
Wooden pole	2 m	1.80 m

Which is the better estimate? Why?

Teacher's Note: Could this form the basis of a practical, measurement activity which could bring out the idea of ratio?

Task 7: Fractions Change

Is each of the following statements true or false?

Give reasons for selecting the answer you do.

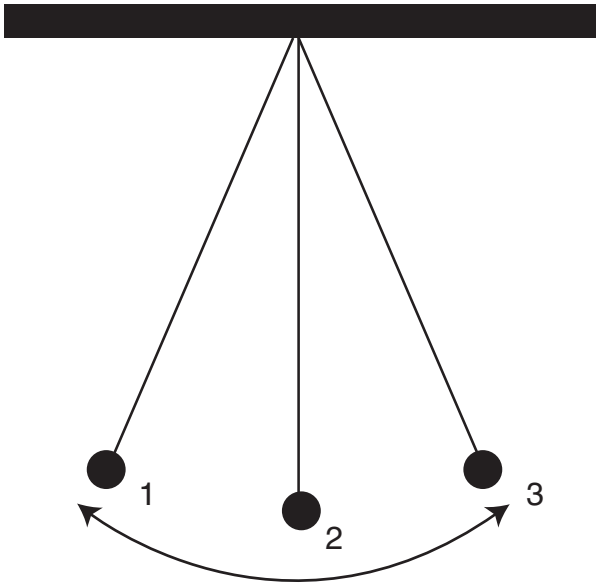
a) Fractions get bigger when you make the numerator bigger.

b) Fractions get smaller when you make the denominator bigger.

c) Fractions stay the same when you make both the numerator and the denominator the same.

Fractions 8: Pendulum

Make a pendulum which will swing back to its starting position in one second



Now arrange it so that it swings back in two seconds.

What did you change to do this?

Task 9: Crew and Passengers

The Mayflower is reported to have had 20 crew and 102 passengers.

The Titanic had 889 crew and 1324 passengers.

What is the ratio of crew to passengers on each ship?

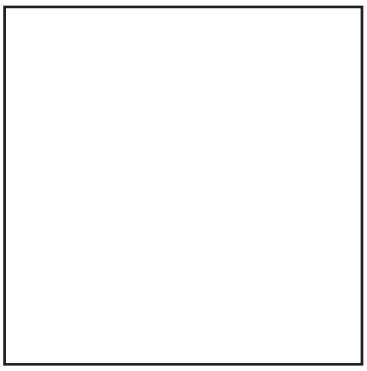
Why do you think there is a difference?

Task 10: Perimeter of Square B

The perimeter of square A is 24 cm.

The area of square B is $\frac{1}{4}$ the area of square A.

What is the perimeter of square B?



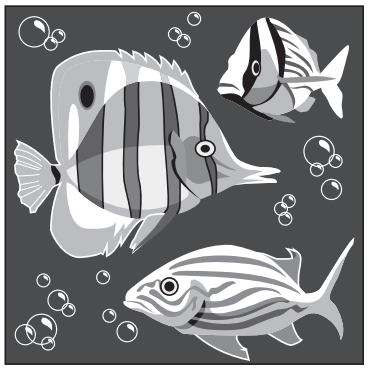
Task 11: Jane's Aquarium

Jane's aquarium contains goldfish, turtles, and snails.

There are 16 legs, 10 shells, and 36 eyes in the aquarium.

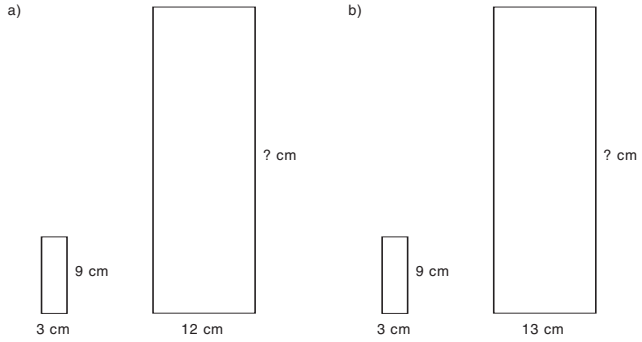
How many creatures of each type are there?

(Hint: Snails have two eyes, one shell, and one leg.)



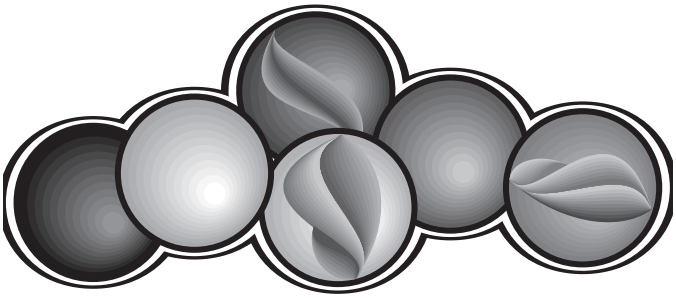
Task 12: Rectangles

If the two rectangles are the same shape, what is the height of the larger one?



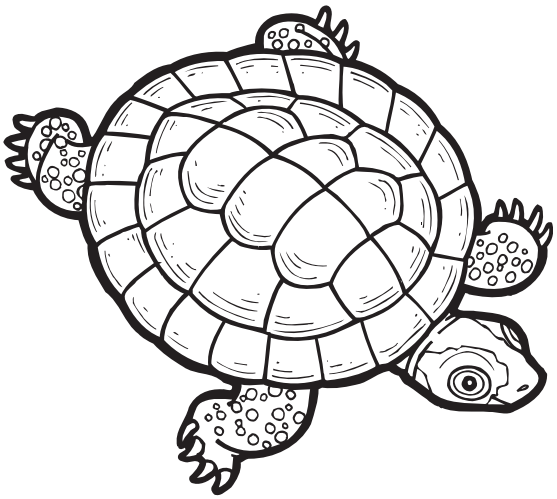
Red Marbles to Blue Marbles

The ratio of red marbles to blue marbles in a package is 3 to 5.



25 blue marbles are in the package.

What is the smallest number of marbles, and of which colour, would you have to add to the package of marbles so that the ratio of red marbles to blue marbles becomes 5 to 8?



Boys: Girls

In Ms. Simms' classroom, there are 12 boys and 15 girls.

In Mr. Craven's classroom, there are 8 boys and 6 girls.

In Ms. Jones' classroom, there are 4 boys and 5 girls.

Which two classrooms have the same boy-to-girl ratio?

On one occasion Ms. Simms' class joined Mr. Craven's class.

What was the resulting boy-to-girl ratio?

On another occasion, Ms. Jones' class joined Mr. Craven's class.

What was the resulting boy-to-girl ratio?

Find the Pattern - Part A

Consider the geometric sequence: 1, 1/2, 1/4, 1/8. 1/16, 1/32,…

What is the ratio between successive terms?

Find the sum of the first two numbers in the sequence.

Now find the sum of the first three numbers.

Continue finding the sums in this manner.

What happens to the sum of these numbers?

Part B

Now consider the geometric sequence: 1, 3/2, 9/4, 27/8, 81/16,…

What is the ratio between successive terms?

Find the sums as you did above.

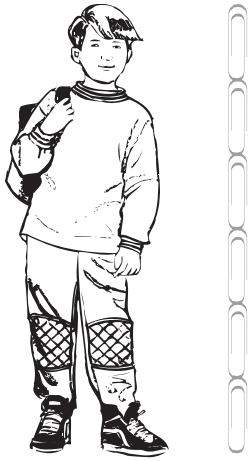
What happens to the sum of these numbers?

Form a geometric sequence.

What do you predict will happen to its sum?

Mr. Short and Mr. Tall

You can see the height of Mr. Short measured with paper clips.



Mr. Short has a friend, Mr. Tall.

When we measure their heights with match sticks.

Mr. Short's height is four match sticks.

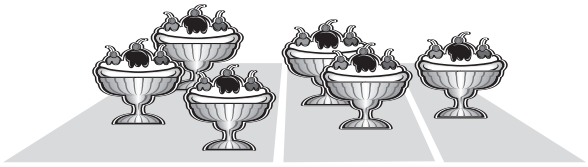
Mr. Tall's height is six match sticks.

How many paper clips are needed for Mr. Tall's height?

Three Tables

In a room, there are three tables on which there are bars of chocolate as shown below.

Ten students are lined up outside the room.



One at a time they enter the room and sit at any table. They all like chocolate, so at the end the chocolate at each table is shared out between them.

Once the students have sat down, how is the chocolate shared out?

Is there a fair arrangement so that all students will receive an equal amount of chocolate?

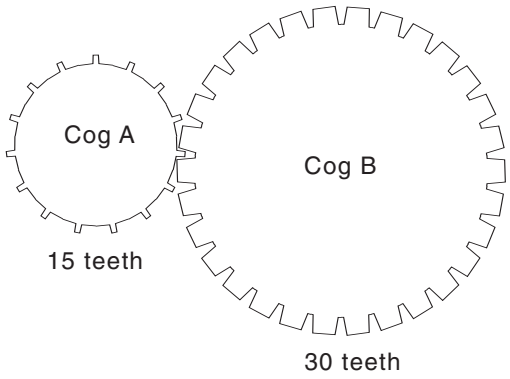
If not, why not and what could you change so that a fair arrangement is possible?

Gears

Gears are used to transmit motion from one rotating shaft to another.

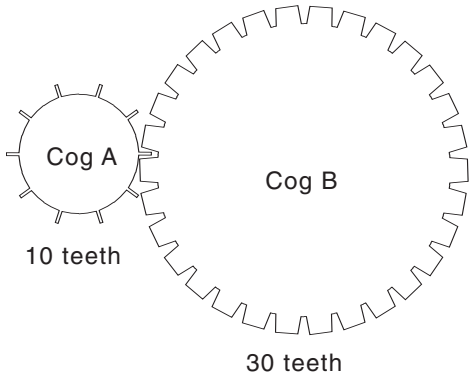
They are a vital element of many modes of transport (example, bicycles, cars), but are of equal importance in such things as clock mechanisms, drills or toys.

A very simple example is shown below



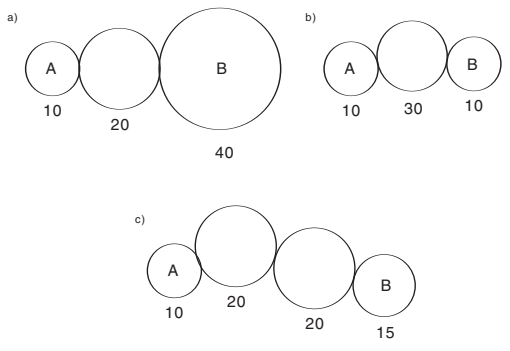
Task 1

If A makes a) 2 turns b) 10 turns, how many turns will B makes?

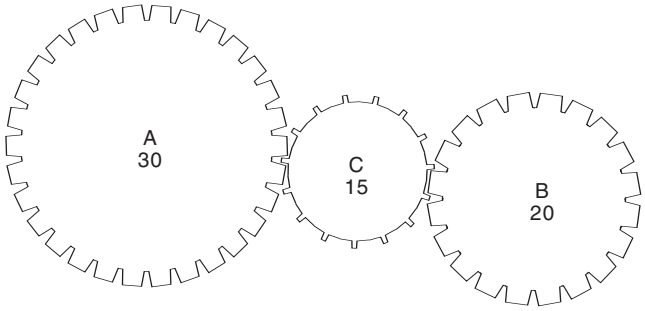


Task 2

For each of the examples below, decide through what angle B turns for one complete turn of A.



Task 3



Most gearing systems involve a number of cogs.

If A makes 6 complete turns, how many will B make?

Can you deduce the number of turns of B without using the information about C?

EXTENSION

If you had available cogs with 60, 36, 12 and 24 teeth, design a system such that the output shaft turns a) 5 and b) 1/3 times as fast as the input shaft.

APPLICATION

Ratios and Growth

Ratios play an important role in growth patterns involving plants, animals, and geometric shapes. In the following activities you will investigate patterns of aging and of rectangles.

Ratios and Aging

In 1990 Nancy was twice as old as her brother Peter. She was 10 years old and he was 5 years old. Will she always be twice as old? Investigate what happens to the ratio of their ages over a lifetime.

What if Nancy were three times as old as Peter in 1990?

Investigate the ratio of their ages over a lifetime.