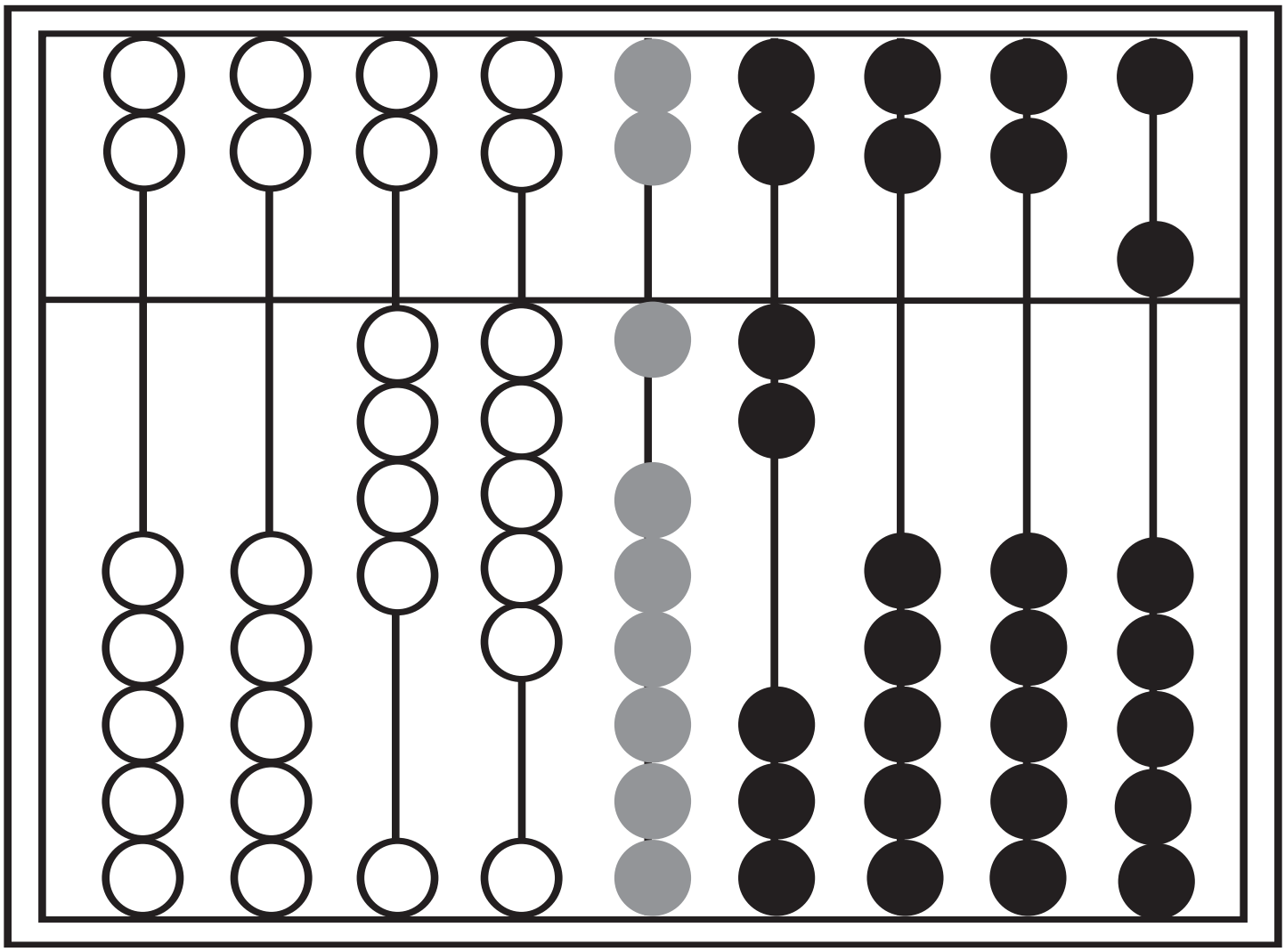
8 PAGE INSERT ELEMENTARY MATH

# Abacus

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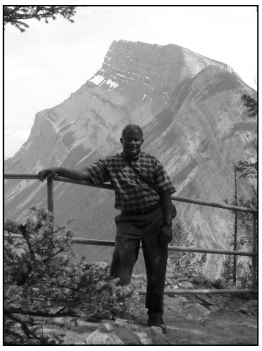
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Edited by Trevor Brown
1 Waterbury Drive, Weston, Ontario M9R 3X4
Tel: (416) 245-5780
E-mail: tbrown@edu.yorku.ca

# ▲ From the Editor's Desk

***Trevor Brown***



A new school year has just begun! We hope that you had a most enjoyable and relaxing break .

As we begin another school year, we realize that once again we shall face many challenges and surprises but as we have always done, we shall overcome them and continue doing what we are very good at doing, and that is teaching our students to develop a love and deep understanding of mathematics.

This summer I was present at a Mathematics Institute in Nova Scotia and heard the keynote address that was delivered by Dr. Marian Small, Dean of Education, University of New Brunswick. I would like to express our thanks to Marian and Nelson for granting us permission to present the main points of her presentation to a wider audience. You will notice that I have used her slides and tried to present them in such a way that you will see how she made the necessary connections and the insights she offered along the way. In future issues of *The Abacus* we shall present the main points of professional talks given by other prominent authors and educators.

## Professional Reading for Teachers

The book ***Developing Thinking in Algebra*** by John Mason with Alan Graham and Sue Johnston-Wilder is a must-read for teachers. To quote from the Preface:

*"This is one of a series of three books on developing mathematical thinking written by members of the influential Centre for Mathematics Education at the Open University in response to demand. The series is written for primary mathematics specialists, secondary and FE mathematics teachers and their support staff and others interested in their own mathematical learning and that of others.*

*These books integrate mathematics and pedagogy. They are practical books to work through, full of tasks and pedagogic ideas, and also books to refer to when looking for something fresh to offer and engage learners. No teacher will want to be without these books, both for their own stimulation and that of their learners."*

Some nuggets from *Developing Thinking in Algebra*:

**Task 1.1.2a Mental Multiplication**

If you had to multiply 32 by 18, in your head, how might you go about it?

**Task 1.4.1a Remainders and Multiples**

Write down a number that leaves a remainder of r when divided by m. Now write them 'all' down, as a general expression.

Generalize the observation that 2 and 7 have the same remainder when divided by 5.

**Task 2.1.3 Odd Sum**

Someone announced that the sum of consecutive odd numbers is always the difference of two squares. What do you think?

**Task 2.1.4 One Sum**

If two numbers sum to one, then which will be bigger – the square of the larger added to the smaller or the square of the smaller added to the larger?

## Reference:

John Mason with Alan Graham and Sue Johnston-Wilder (2005) *Developing Thinking in Algebra*. The Open University. ISBN 1-4129-1170-2.

# ▲ Focus Issue: Patterning and Algebra

### Task 1: Which is true?

The sum of three different positive integers is 30.

a) at least one is greater than 10,

b) all must be even,

c) at least one is even,

d) two of them could be multiples of 3,

e) the difference between one pair can be equal to the difference between another pair.

### Task 2: Which is true?

a) $x + y + z = 30$

b) If $x < 10$, $y < 10$, $z < 10$,
then $x + y + z < 30$

c) Use $x = 2k$ or $2k+1$;
$y = 2m$
or $2m+1$ and
$z = 2j$ or $2j+1$,…

d) Use $x = 3k$
or $3k+1$
or $3k+2$; $y = 3m$
or $3m+1$
or $3m+2$
and $z = 3j$
or $3j+1$
or $3j+2$

e) Use $x-y = z-y$ (so x, y, x are the numbers)
or $y-x = z-y$, so $2y = x + z$

### Commentary:

*What does the Patterning and Algebra Strand Look Like in the Curriculum*

~ Naturally, lots of algebra in high school and some in junior high, but what about before that?

Outcomes like:

**Grade 2:** demonstrate an understanding of the concept of equality by partitioning whole numbers to 18 in a variety of ways

**Grade 4:** determine, through investigation, the inverse relationship between multiplication and division.

*Reflection: And it's not just number..*

In what strands should you see these?

$A = s^2$

$P = 2l + 2w$

$V = lwh$

$C = pi \times r^2$

*Reflection: We teach our curriculum but our focus should be on big ideas*

Key Ideas

- Patterns represent identified regularities based on rules describing the patterns' elements.
- Any patterns can be represented in a variety of ways.
- Patterns underlie mathematical concepts and can also be found in the real world.
- Data can be arranged to highlight patterns and relationships.
- Relationships between quantities can be described using rules involving variables.
- Patterns represent identified regularities based on rules describing the patterns' elements.

  - 4. 6, 8, 10, …
  - $f(x) = \sin x$

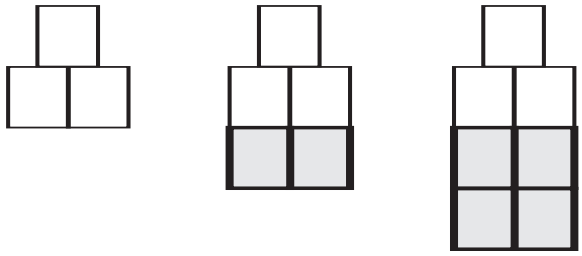
- Any pattern can be represented in a variety of ways.

  5, 10, 5, 10, 5, 10…

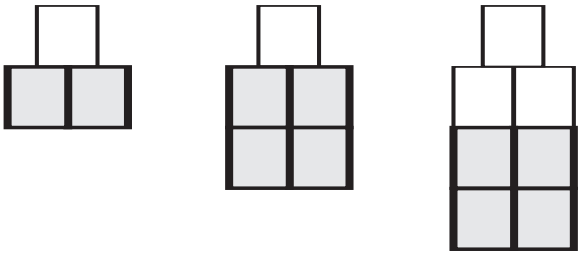
  as AB

  or as ○ □ ○ □ ○ □

- 3,5,7,9… as start with 3 and add 2



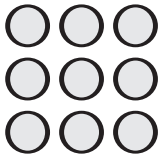
- or 3 + 2(n-1)



## Commentary:

Key Ideas

- Patterns underlie mathematical concepts and can also be found in the real world.
  - our place value system
  - measurement conversions
  - function transformations
  - arithmetic series
- data can be arranged to highlight patterns and relationships.
- arranging 6 dots like this to see it quickly



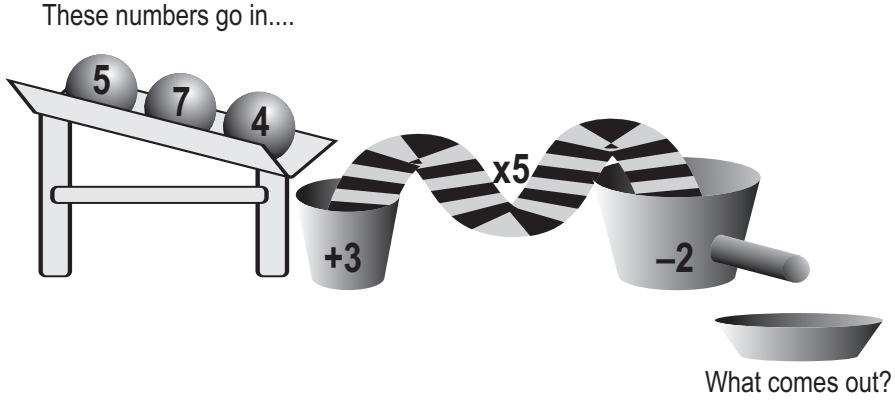
- arranging things in equal groups to see you can multiply
- using tables of values
- relationships between quantities can be described using rules involving variables.

  - 3x [] = [] + [] + []

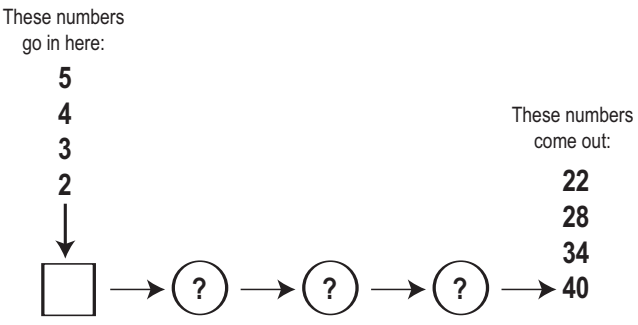
  ~ 1/n = n-1

  ~ (n2 = 2n + 1) = (n + 1)2

## Function Machines [NRICH]




This one is a bit more of a puzzle. Can you work out what happens in the three circles?



## Task 4: Patterns unveil Ideas

*Editor: How many different patterns do you notice as you scan the following?*

$1/3 = 0.\overline{3}\ldots$

$1/4 = 0.25$

$1/5 = 0.2$

$1/6 = 0.1\overline{6}$

$1/7 = 0.\overline{142\ 857}\ \ldots$

$1/8 = 0.125\ldots$

$1/9 = 0.\overline{1}$

$1/10 = 0.1$

$1/11 = 0.\overline{09}$

$1/12 = 0.083\ 83\ldots$

$1/13 = 0.\overline{076\ 923}$

$1/14 = 0.0\overline{71\ 428\ 5}$

$1/15 = 0.06$

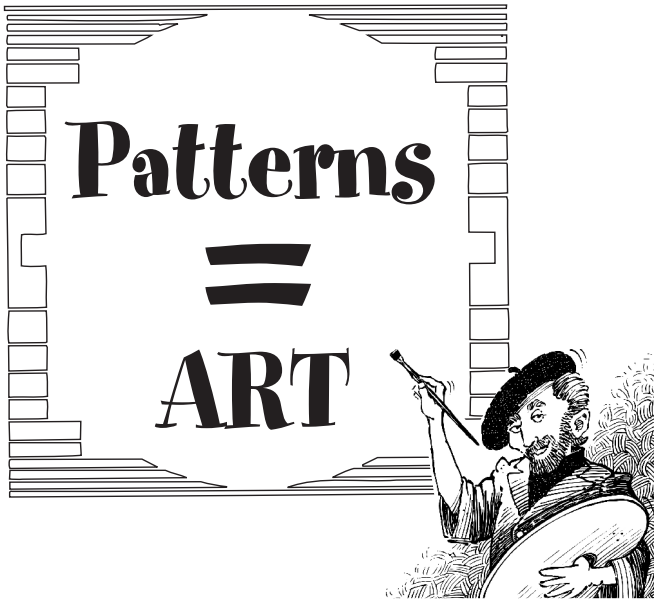
$1/16 = 0.062\ 5$

$1/17 = 0.\overline{058\ 823\ 529\ 411\ 764\ 7}$

$1/18 = 0.0\overline{5}$

$1/19 = 0.\overline{052\ 631\ 578\ 947\ 368\ 421}$

$1/20 = 0.05$




## Task 5: Patterns Unveil Ideas

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |
| 37 | 38 | 39 | 40 | 41 | 42 |

Every prime number greater than 3 is 1 greater or 1 less than a multiple of 6.

*Editor: Do you agree with this assertion? If you do, please give the reasons.*

## Task 6: Using Patterns Even in Higher Grades

To factor $x^2 - 3x - 4$

| **x** | **$x^2 - 3x - 4$** | |
|---|---|---|
| 1 | -6 | |
| 2 | -6 | |
| 3 | -4 | |
| 4 | 0 | |
| 5 | 6 | 6 x 1 |
| 6 | 14 | 7 x 2 |

$x^2 - 3x - 4 = (x + 1)(x - 4)$

## Commentary:

a) Why do Patterns and Algebra go Together?

- Patterns and statements of relationship describe regularities.
- Algebra provides the shorthand for those statements.

b) What is Symbol Sense?
How is it Like Number Sense?

~ Symbol sense describes the use of intuition and logic, rather than rules, to work with algebraic expressions.

Example 1

Selecting an appropriate form of a number for a calculation, e.g. 0.25 x 844

*Editor's note: Find a short way of performing this operation.*

Selecting an appropriate form for an algebraic expression to model a situation, e.g.sum of 3 consecutive numbers as (x-1) + x + (x + 1) compare this approach to the one that is normally used: x + (x +1) + (x + 2)

Example 2

Intuitively know when it is appropriate to use mental calculations, e.g. 144 x 5

*Editor's note: How would you solve this using mental calculation?*

Example 3

Intuitively knows when to use symbols to solve a problem or when not to, e.g. If my brother got $28 more, he would have 3 times as much as he has now. How much does he have now?

*Editor's note: In how many ways could you solve this problem?*

Example 4

Has an immediate sense of the similarities and differences between groups of numbers, e.g. multiples of 6 compared to powers of 6.

Discuss what is happening here.

Can scan an expression and have an immediate "estimate" of patterns that will emerge,

e.g. patterns for 10 +□ vs patterns for 10 – □

Discuss what is happening here.

## Commentary:

What can a variable mean?

- ~ It might represent the one (or some) value(s) to make a statement true: e.g. $5 + x = 20$ or $3x^2 < 50$
- ~ It might describe a relationship, e.g. $f(x) = 4x$ or $P = 4s$
- ~ It might represent an "identity" or statement that is always true: e.g. $5x + 5y = 5(x + y)$.
- ~ How can the student know which is which?

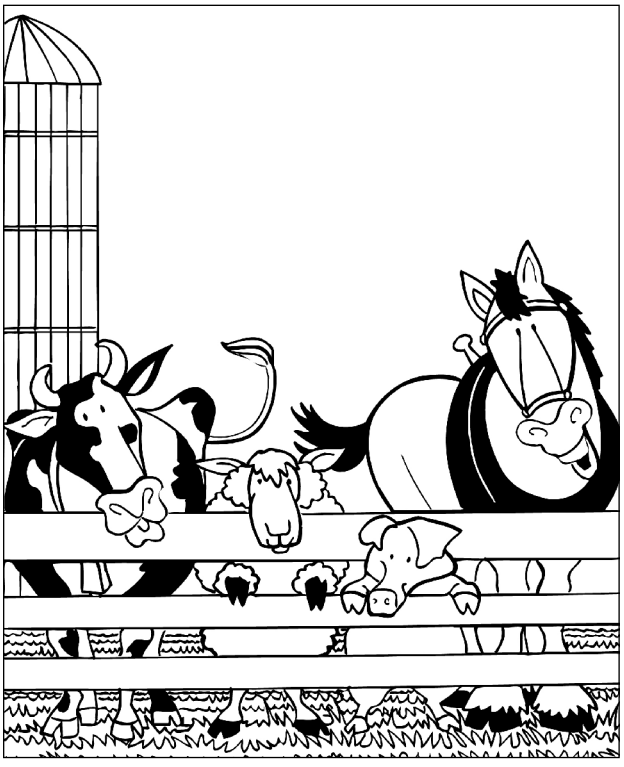
You try

- ~ Express some identities using symbols. Let's see how much variety we can get.

*Editor: Ask your students to perform this task by asking the question, "Express some identities using symbols".*

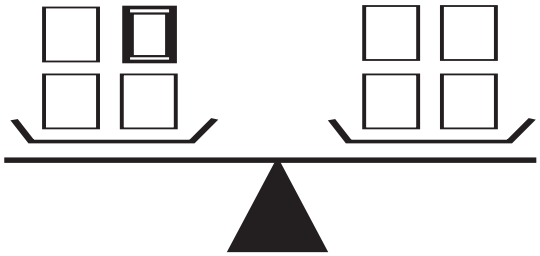
Some samples

- ~ $3x = x + x + x$
- ~ $5x – 2x = 3x$
- ~ $x^2 = (-x)^2$
- ~ $\sin x = \cos (90 – x)$

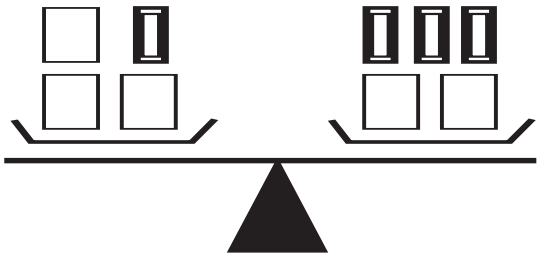


An equation as a balance

3 + 1 = 4 + 0



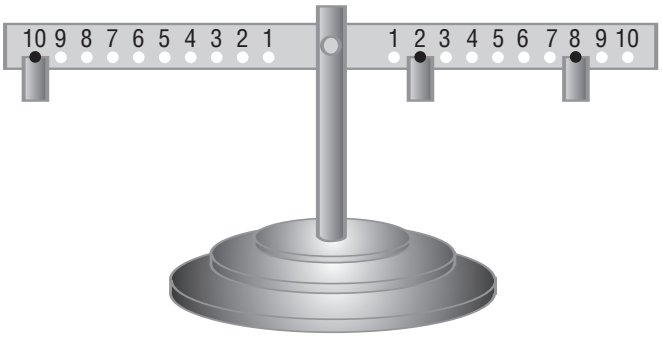
3x + 1 = 2x + 3



*Editor's note: May model this by also using a Mathematical balance.*

Example of a Mathematical Balance

These are hung below the numbers. It balances equal numbers, for example like this:




## Commentary: Solving equations

~ How do we make the solution meaningful to students?

3x – 2 = 4x – 8

# ▲ SOLVING PATTERNS PROBLEMS

### Task 7

~ A pattern begins with a 2-digit numbers and increases regularly by a 1-digit amount. What are the greatest and least possible values for the 10th term?

### Task 8

~ The greatest value comes from beginning with 99 and adding 9 each time.

The least value comes from beginning with 10 and adding 1 each time.

The values are 180 and 19

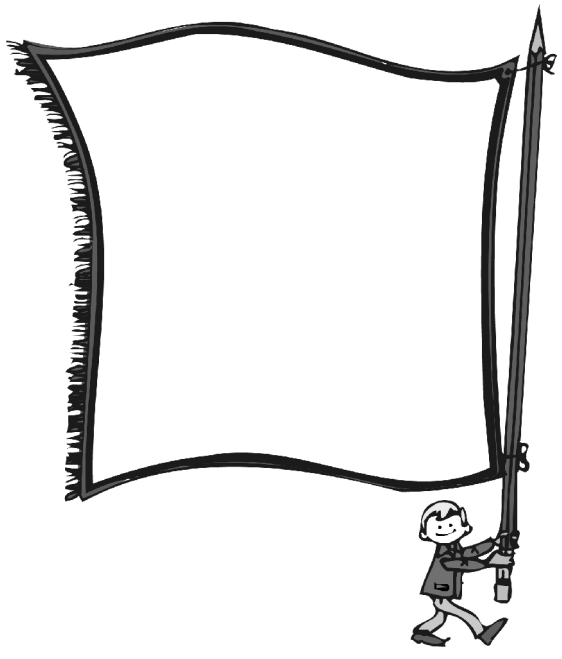
### Task 9

~ A pattern is formed like this:

If a number in the pattern is less than or equal to 20, you double it to get the next number.

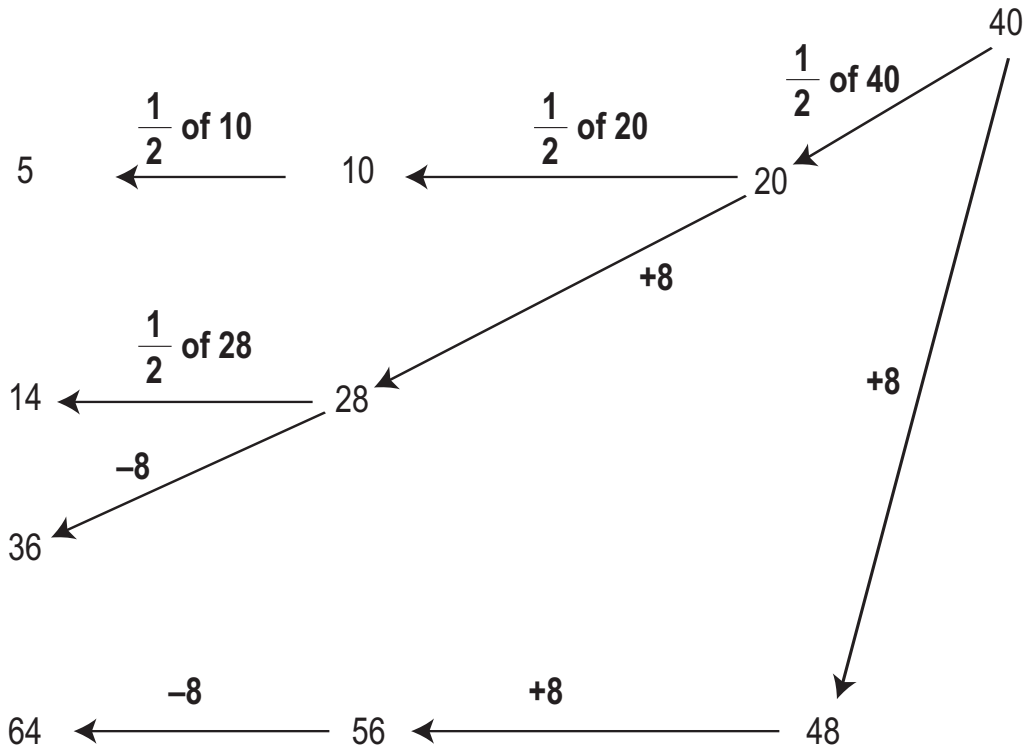
~ If a number is greater than 20, you subtract 8 to get the next number.

~ If the 4th number in the pattern is 40, what could the 1st number have been?



Possible Solution

To solve the problem, a student might use a diagram to work back from 40 to come up with one or more answers:




The first number in the pattern could have been 5, 14, 36 or 64.

## What our research told us

- Canadian students through grade 7 (and I think well beyond) are uncomfortable with variable use and, even when they use them, use them mechanically and not meaningfully.

## So what's our job?

- At the elementary level, our job is to help students understand the power of notation (e.g. 8 x 3 vs (3 + 3 + 3 + 3 + 3 + 3 + 3 + 3) and the value of and limitations of generalization.
- Our job at the secondary level is to make the use of variables not yet another set of Rules to learn but a way to translate our verbal expressions succinctly.
- It's actually possible to like it!