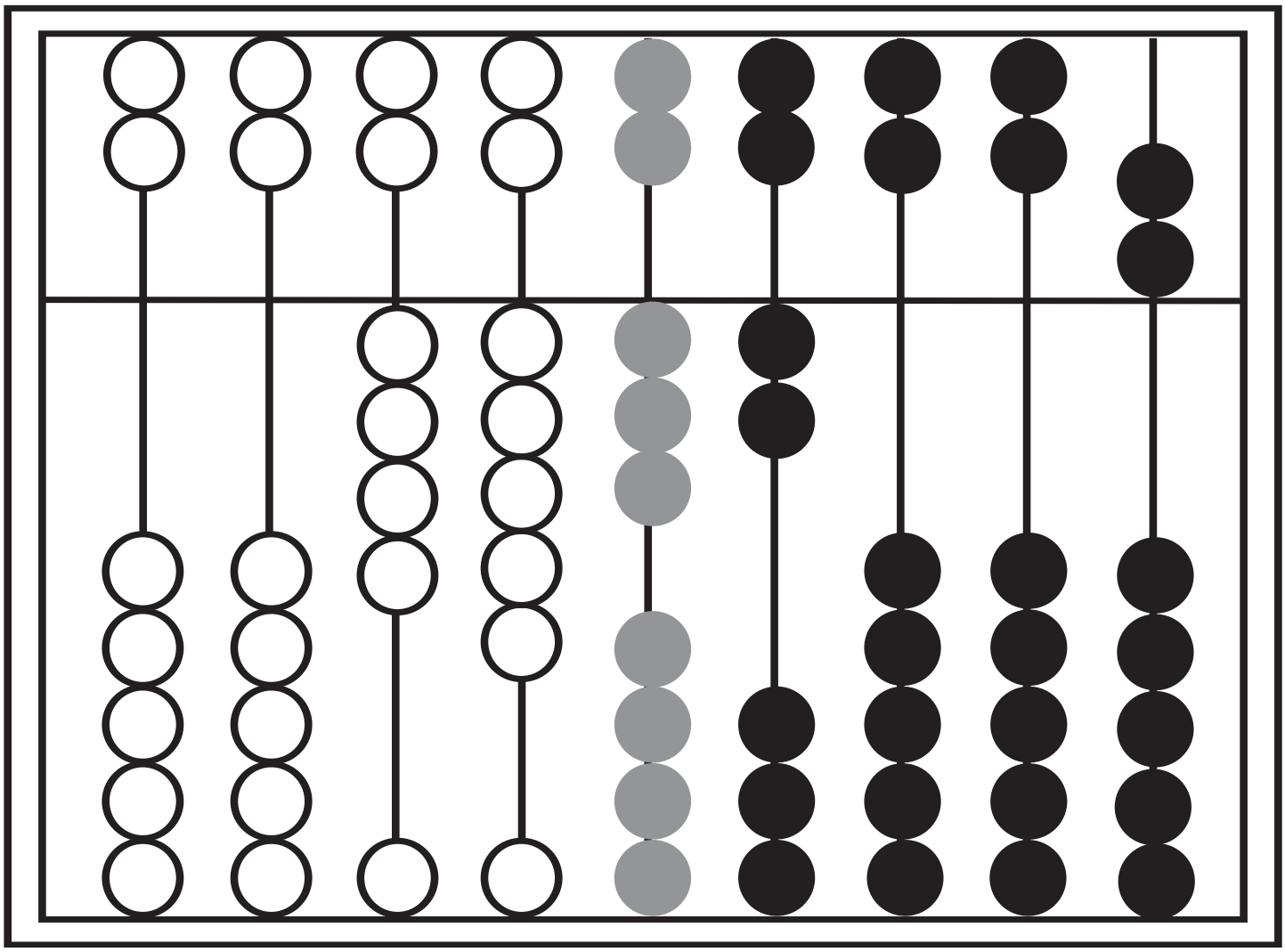
8 PAGE INSERT ELEMENTARY MATH

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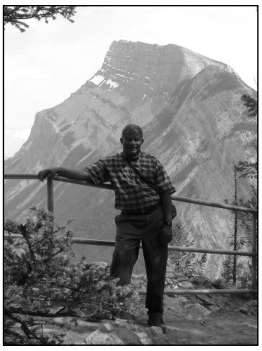
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▲ From the Editor's Desk

Trevor Brown

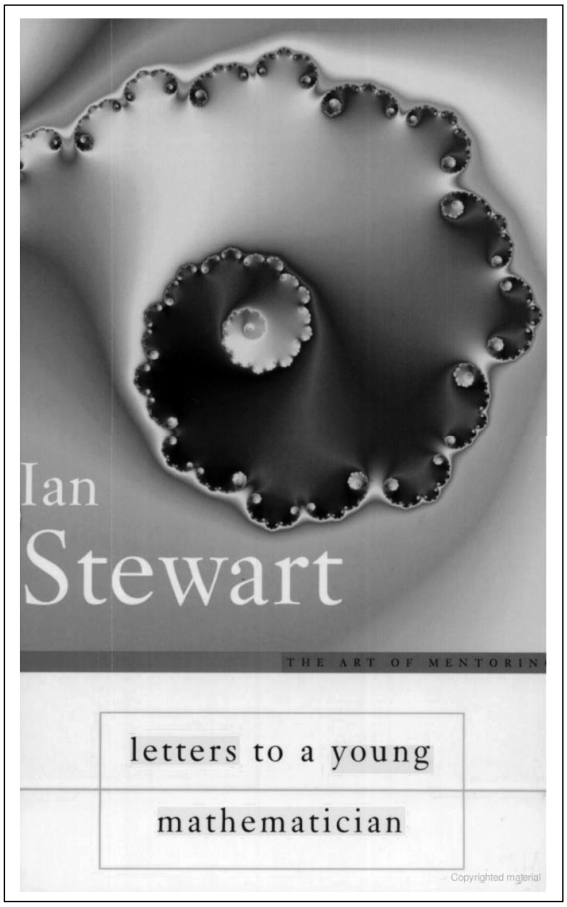


Many teachers have read and appreciated the many books and articles written by the British mathematician Ian Stewart. I recently came across the following review of his most recent book, a book that is vintage Ian Stewart.

Letters to a young mathematician by Ian Stewart

Reviewed by Marianne Freiberger

With this collection of letters Ian Stewart, accomplished mathematician, science writer, and even science fiction writer, accompanies a young and imaginary student on her path to becoming a professional mathematician. The letters address the questions that arise naturally at the crucial points in "Meg's" career, from leaving school and pondering whether to take a maths degree, through to becoming a fully established mathematician wondering how to juggle teaching and research.



Stewart's answers to Meg's questions give a fascinating insight into maths and mathematicians, combining down-to-earth advice on practical matters, like how to become unstuck on a maths problem, with some serious maths and philosophy. In a light and conversational tone Stewart explains how to go about learning maths and how to teach it, exposes the maths that lies hidden (or not so hidden) in the world around us, explains some big open maths questions and even asks if God is a mathematician.

The result is a collection of 21 essays that contains something of interest for everyone, from the young mathematician of the title to anyone who is curious about the subject. In fact, it's probably those with a maths aversion that, could they be tricked into reading the book, would get the most out of it. Stewart does a great job at differentiating the tedious and seemingly irrelevant equations and formulae we encounter at school from the beautiful and fascinating world of ideas that constitutes "real" maths and motivates mathematicians. With uncontrived examples of the maths inherent in the world around us — in pigeons, dogs and vegetables for example — Stewart convincingly makes the point that our "entire life bobs like a small boat on a vast ocean of mathematics". His descriptions of what inspires mathematicians show that their aim is not to drown all the world's beauty in this ocean (as you might think when you first encounter a maths book full of symbols and numbers) but to take a greater share in it. In fact, some of this book's chapters would make an invaluable addition to the school maths syllabus.

For those that don't need convincing of the beauty and uses of maths, Stewart

offers some deeper insights. He explores some of the big problems, like Fermat's last theorem, the four colour theorem and the Poincaré conjecture. His chapter about mathematical proofs explains why mathematicians are so obsessed with them and gives some non-technical, almost non-mathematical examples. And for those who are actively doing maths, the young mathematicians of the title, he has some valuable advice on anything from how to write down a proof to how to deal with the maths community.

This book is accessible to pretty much everyone. The maths is presented in a way that is easy to understand, equations appear very rarely and mostly only in parentheses and Stewart successfully avoids all mathematical jargon. The tone is gentle and light and the self-contained nature of the individual letters allows you to skip those that don't seem too appealing.

I'd recommend this book to anyone with an interest in maths, whether they are young or old, a budding mathematician or an amateur. If you are a teacher, you will find some great anecdotes and examples in this book. And if you are a mathematician, this book might finally get your friends to understand what it is you do all day and why you're doing it.

Book details:

Letters to a young mathematician (Art of mentoring) Ian Stewart
Hardcover - 224 pages (2006)
Basic Books ISBN:0465082319

"In Mathematics the art of posing problems is easier than that of solving them."

Georg Cantor

▲ Blackcurrantiest

nrich

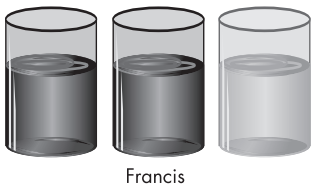
Lizzie's Dad is making some blackcurrant squash for Lizzie and her friends.

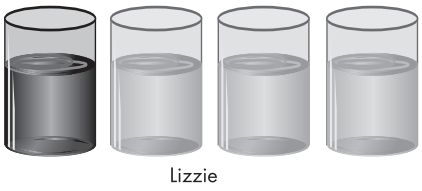
A dark shaded glass stands for a measure of blackcurrant and a light shaded glass for a measure of water:

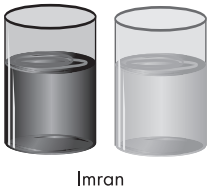


Can you decide which of these drinks, when mixed, will be the blackcurrantiest? (In other words which one will have the strongest blackcurrant taste.)









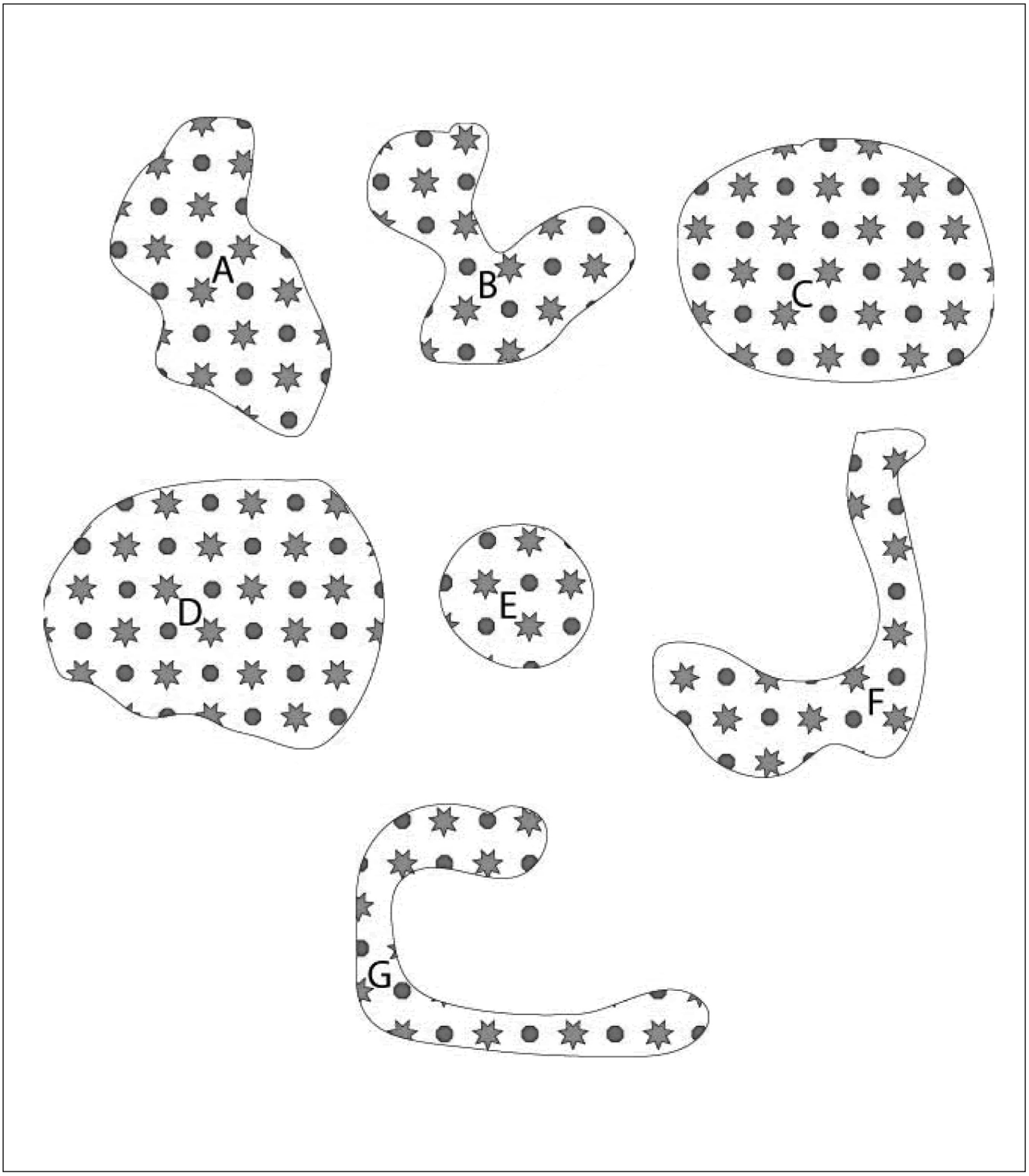
Order the drinks from the blackcurrantiest to the one which would have the least blackcurrant flavour.

Can you explain why you have put them in that order?

▲ Wallpaper

from nrich

Arrange these pieces of wallpaper in order of size. Put the smallest first.



▲ Some Mathematical Nuggets

Problem 1



Perimeter of 6 units Perimeter of 8 units

A unit square has perimeter 4 units. When two such squares are placed, non-overlapping, in a plane, the minimum perimeter of the shape they form is 6 units.

For three squares the minimum perimeter is 8 units.

What is the minimum perimeter when (i) four unit squares are placed, non-overlapping, in a plane? (ii) how about six squares? (iii) eight squares? (iv) one hundred squares?

Problem 2



I have three discs.

They have a number on each side.

Three of these numbers are shown.

I toss the discs up into the air and add the three numbers that I see.

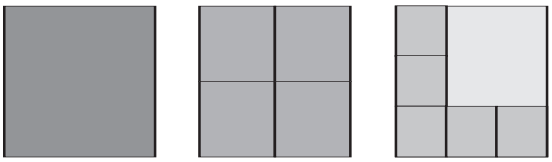
After a while I notice that all the totals I get are consecutive numbers.

What numbers are on the other sides of these discs?

Problem 3

Imagine a square of paper. Can you cut it up into smaller squares?

For example,



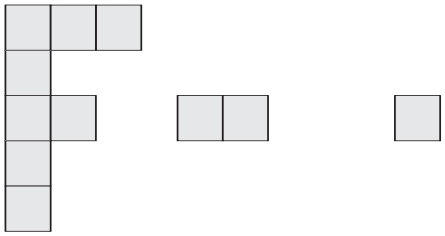
Is it possible to cut our square into any and every number of squares?

That means, of course, with no paper left over.

Problem 4

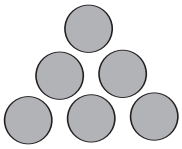
There are many ways the following three shapes can be placed so that all touch (no overlapping) and the resulting shape has line symmetry.

There are seven ways in which this can be done.

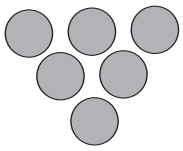


Problem 5

In the diagram there are 6 discs arranged in the shape of a triangle.



It is not too difficult to show that, by moving three discs only the 6 discs can be made to point in the opposite direction, like the discs shown to the right.



What happens with 15 coins?

If you put 15 coins in a triangular shape with 5 rows, what is the smallest number of coins that you can move that will point the triangle in the opposite direction?

Problem 6

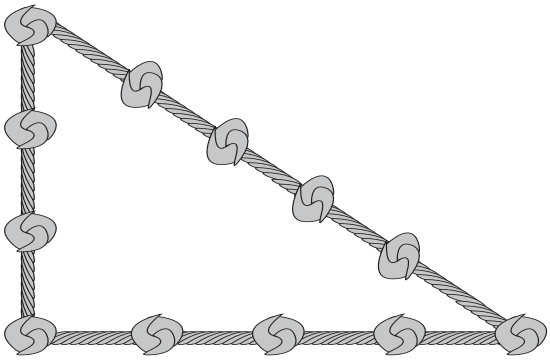
Using each of the ten digits once, find the two five-digit numbers which have the largest possible product.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

□ □ □ □ □ × □ □ □ □ □

Problem 7

The ancient Egyptians were said to make right-angled triangles using a rope with 12 equal sections divided by knots.



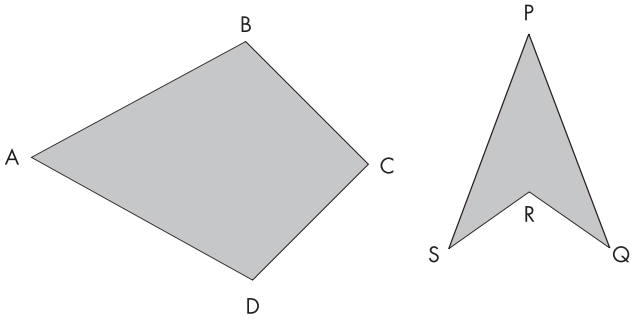
If you have a rope knotted in such a way what other triangles can you make? They must have a knot at each corner.

Of the triangles you have, which has the largest area?

What regular shapes can you make ?

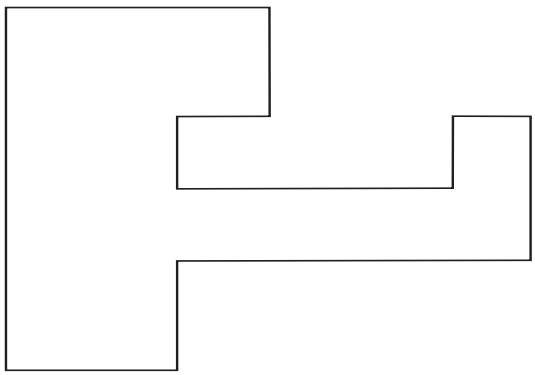
Problem 8

Derive a formula for finding the area of any kite.



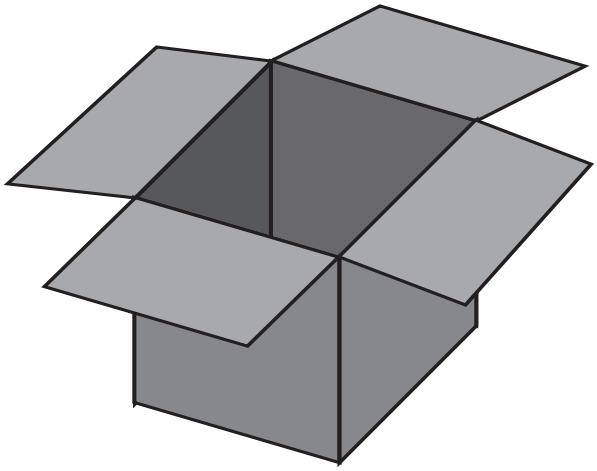
Problem 9

There are 12 segments parallel or perpendicular to each other on the diagram below. The lengths of at least how many segments do you have to measure if you want to know the perimeter of the shape?



Problem 10 NRICH

Chris and Jo decide to play a game.



They put two red and four blue ribbons in a box.

They each pick a ribbon from the box without looking (and without replacing them).

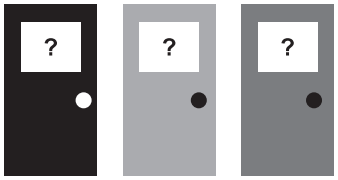
Jo wins if the two ribbons are the same colour and Chris wins if the two ribbons are a different colour.

Is the game fair?

If not, how many ribbons of each colour would you need in the box to make it a fair game?

Is there more than one way to make a fair game?

Problem 11



A game show host hides a prize behind one of three doors. The contestant has to guess which door hides the prize. The rules of the game are as follows.

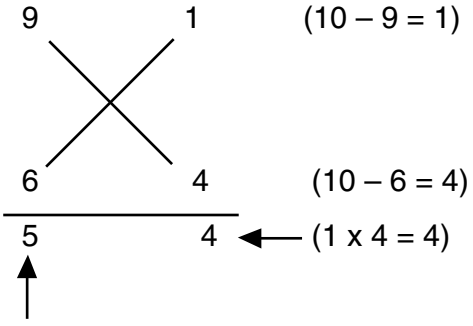
Firstly, the contestant chooses a door and tells the host this is the one she thinks the prize is behind. The host must then open one of the other doors. Of course, the host does not want to reveal the whereabouts of the prize so he always opens a losing door.

The host then asks the contestant if she would like to stick with the door she originally chose or switch to the other unopened one.

Should she switch doors?

Problem 12

According to Robert Ecorde (1510? – 1558), an Oxford University mathematician and teacher, one only needed to learn the multiplication tables as far as five by five. To multiply 9 by 6, for example, he suggested the following method:



So, 9 x 6 = 54. Try a few more examples and see that the method always works.

▲ Mathematics: A Human Endeavour

Heron's Formula

Heron's formula for the area of a triangle with sides of length a, b, c is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where} \quad s = \frac{a+b+c}{2}$$

Use Heron's formula to show that a triangle with sides of length 5, 5 and 6 has the same area as a triangle with sides of length 5, 5 and 8.

The Hailstone Sequence

The hailstone sequence is not the problem's only name. You may also see it referred to as the Syracuse problem, the Collatz problem or simply as the 3n+1 conjecture. A conjecture is simply a mathematical statement which has not yet been proved.

To our knowledge the principal conjecture "the sequence terminates at the value 1 for all starting values greater than 0" is still unproved, despite rumours to the contrary!

Start with any positive integer (an **initial seed**) and obtain a sequence of numbers by following these rules.

1. If the current number is even, divide it by two; else if it is odd, multiply it by three and add one.
2. Repeat.

Let's try a few numbers to see what happens:
n=3; 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ...
n=5; 16, 8, 4, 2, 1, 4, 2, 1, ...
n=7; 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ...

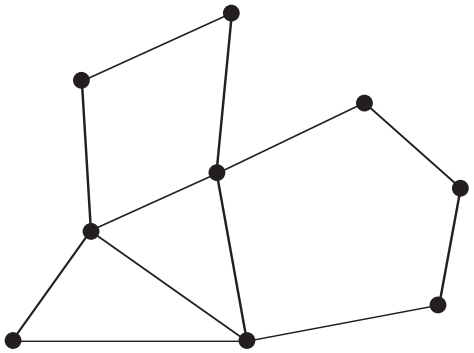
Question: Does every initial seed yield a sequence that eventually hits 1 (and then repeats: 4,2,1,4,2,1,...)?

The Collatz conjecture says yes, but this has never been proved. However, it has been shown true for every number ever tried!

▲ The Euler Characteristic of a Surface

from: Mudd Math Facts]

Draw any number of dots on your page. Now connect the dots with lines, subject to the following rules: lines may not cross each other as they move from dot to dot, and every dot on your page must be connected to every other dot through a sequence of lines.



Now count the number dots (D), lines (L), and regions separated by lines (R). (Don't forget to count the outside as a region too.) Compute D – L + R. What do you get?

No matter how you started, the number you will always get is 2!

In Figure 1, D = 9, L = 12, R = 5, and indeed, D – L + R = 2.

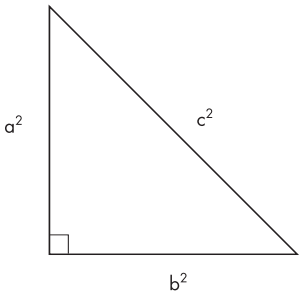
Pythagorean Triples

Which triples of whole numbers {a, b, c} satisfy

a2 + b2 = c2 ?

Such triples are called Pythagorean triples.

You probably know {3, 4, 5} and {5, 12, 13}. But can you classify all possible Pythagorean triples?



ANSWER: It can be proven that all Pythagorean triples are of the form

{ $M^2 – N^2$, 2MN, $M^2 + N^2$ }

for some integers M and N, or they are multiples of this form.

Thus setting M = 2,N = 1 gives {3,4,5} and M = 3,N = 2 gives {5,12,13}.

Colour Tiles

John and Brenda were making designs with square tiles.

They each had eight tiles.

John made the design in figure A and Brenda made the design in figure B.

When they counted the number of sides of each design, they were surprised to find that the two shapes had different numbers of sides.

They wondered how many shapes with different number of sides they could make with eight tiles.

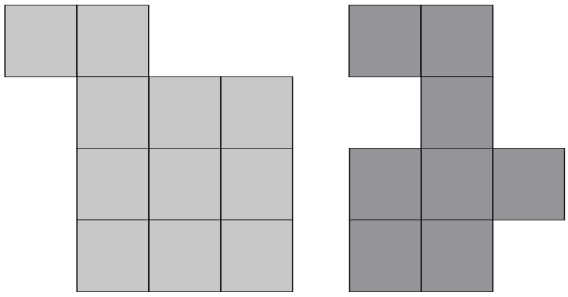


Figure A (8 sides) Figure B (12 sides)

They agreed that in making their designs, they would always put the edge of one tile exactly along an edge of another tile and that no shape would have holes.

Under these conditions, how many shapes with different numbers of sides can be made with eight tiles?

How would the situation change if Jeff and Carol used nine tiles? Seven tiles? Twenty-two tiles?