8 PAGE INSERT ELEMENTARY MATH

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FROM ARITHMETIC TO ALGEBRA – EQUALITY AND RELATIONAL THINKING

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Welcome back to a new year of the *Abacus*. What do you think elementary algebra is about and how does understanding it develop? Algebra includes many aspects, such as generalizing beyond specific numeric examples, describing and representing patterns and functions, building equations and expressions using symbolic representations with integers and variables, and manipulating symbols to simplify equations, to prove relationships, and to solve for unknowns (single or a range of numeric values). In fact, Kaput (2008, p.10) described algebra as a merging of two identities – a cultural artifact (expression of generalizations in increasingly systematic, conventional symbols systems) and as human activity (algebraic reasoning which is "syntactically guided action on symbols expressed in conventional symbol systems.") More specifically, Kaput (2008) said that these two core aspects are embodied in three strands:

- Strand 1 – "Algebra as the study of structures and systems abstracted from computations and relations, including those arising in arithmetic (algebra as generalized arithmetic) and in quantitative reasoning"
- Strand 2 – "Algebra as the study of functions, relations, and joint variation
- Strand 3 – "Algebra as the application of a cluster of modeling languages both inside and outside of mathematics.

Yet, for algebra, equations are much more than the representation and manipulation of arithmetic. Number equations or number sentences are based on an understanding of the meaning of the equal sign and equivalence. In fact, equivalence is about knowing that two expressions may be equivalent even though they don't look alike; such is a big idea in early algebra. So, for this issue, we will examine the big idea of equality and equivalence using true/false and open number sentences.

LINKS TO LITERATURE AND MANIPULATIVES



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Greetings! In this issue, we will be exploring the idea that algebraic thinking is a fundamental aspect of your mathematics curriculum. So, let's revisit the use of bead strings and the rekenrek to see what role they play in the development of algebraic thinking. As well, let's continue to examine closely some picture books that focus on numbers that can be composed and decomposed in more than one way. Such contexts can be springboards for designing true/false sentences. If you know of other picture books that show numbers being composed and decomposed in more than one way, please share them so that they can be featured in future issues of the *Abacus*.

RESEARCH SUMMARY – ABOUT EQUALITY AND RELATIONAL THINKING

Building Generalization from Arithmetic and Quantitative Reasoning

Student Misconceptions of Equivalence:

Blanton (2008) explained that one of the first symbols that students encounter in school is likely the equal sign. But what meaning do they attach to this symbol? What number would you put in the box to make this number sentence true? 8 + 4 = ☐ + 5. According to Carpenter, Franke, and Levi (2003), the answer 7 is not common; two common responses have been found to be 12 and 17. Try it with the students in your school to compare your school results to their research (p. 9).

Number to Put in Box to Make Number Sentence True (% of Solutions)				
Grade	**7**	**12**	**17**	**12 and 17**
1 and 2	5	58	13	8
3 and 4	9	49	25	10
5 and 6	2	76	21	2

From such data, Carpenter, Franke, and Levi (2003) inferred that most elementary students and many older students do not understand that the equal sign denotes a relationship between two equal quantities. As a result, students have significant misconceptions about the meaning of the equal sign. Similarly, Knuth, Alibali, Hattikudur, McNeil, and Stephens (2008, p. 516) showed that a large number of grades 6 to 8 students defined the equal sign from an operational rather than relational view.

Percent of Students Providing Their Best Definition of an Equal Sign (n = 375)			
Best Definition	**Grade 6**	**Grade 7**	**Grade 8**
Relational	29	36	46
Operational	58	52	45

In both research cases, students are interpreting the equal sign as a command to carry out a calculation. Blanton (2008), Carpenter, Franke, and Levi, 2003, and Small (2005) explained that this longstanding problem is exacerbated by the fact that many calculators require the equal sign key to be pressed to get the answer, thus reinforcing their interpretation of the equal sign as synonymous with “here comes the answer.” Further, Carpenter, Franke, and Levi (2003) and Fosnot and Lent (2007) reported that the repetitive use of arithmetic tasks that have students compute an expression and then write their answer immediately after the = symbol also contributes to their misconceptions about the equal sign. Researchers (Blanton, 2008; Carpenter, Franke, and Levi, 2003; Fosnot and Lent, 2007; Small, 2005) agreed that this misconception limits students’ capacity to learn and flexibly represent and use fundamental arithmetic ideas with understanding which is key to developing algebraic reasoning.

Developing Students’ Understanding of Equivalence:

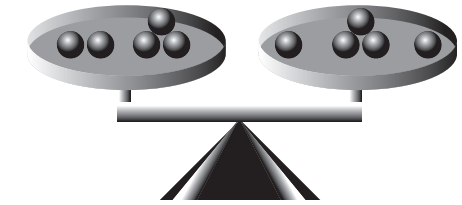
According to Fosnot and Lent (2007), primary students struggle to understand statements like, 5 + 3 = 4 + 4. They wonder how it is possible that the two expressions can be equal when the numbers are different. So, at first, students add the numbers on each side to determine both expressions are equivalent, but when using compensation (e.g., 1 is taken from 5 and added onto 3), their need to do the arithmetic reduces. For students in grades two and three, equivalence can be expanded to include true/false number sentences like 8 + 2 + 10 = 12 + 4 + 4. For such contexts, students generalize that equivalence is maintained no matter what operation is used, as long as the number operation is applied to both sides of the equation. Thus, the generalization ☐ + 8 + 2 + 10 = 12 + 4 + 4 + ☐ is true for any number, ☐.

Carpenter, Levi, Franke, and Zeringue (2005) and Molina, Castro, and Mason (2008) explained that when students solve open number sentences using relational thinking, they are looking at expressions and equations in their entirety. That means, that students are noticing the fundamental number properties of the number sentences to transform them, rather than calculating the answer for each numeric expression. Students who solve number sentences by using relational thinking (RT) are considering arithmetic expressions from a structural perspective rather than from a computational one. They say that students’ choice of solutions represents the mathematical structures to which they

are attending. When students consider sentences and expression as a whole, they look across the equal sign, as well as within each numerical expression to make distinctions and establish a relationship between the elements. Other students proceed to do computations paying attention only to the numbers involved and operations to perform on them. Therefore, to develop relational thinking, have students look at the sentence before they calculate and ask questions which draw their attention to the sameness or difference of some elements in the sentence.

Further, Small (2005) suggested that it is important for students to recognize that the equal sign is a way to say that the same number has two different names, one on either side of the equal sign. Carpenter, Franke, and Levi (2003) and Fosnot and Lent (2007) note the significance of students understanding the algebraic role of the equal sign as signaling a relationship between quantities, such as 6 + 8 "is equivalent to" or "is the same as", □ + 4.

According to Greenes, Cavanagh, Dacey, Findell, and Small (2002), the concept of variable is fundamental to algebra; in the early years, students explore the use of various symbols, such as geometric shapes (square, triangle) and letters to represent unknown quantities that do and do not vary. Variables are used to generalize arithmetic properties. Quantities that are not given and do not vary are often referred to as unknowns or missing elements when they appear in equations. So, for example, □ + 4 = 10, the square stands for the unknown or missing addend. In the early years, the pan balance is used as a model for representing equations (pans in balance) and inequalities (pans out of balance). As students explore ways that balance can be maintained, they learn that adding the same mass from both pans, and/or substituting objects of equal mass will not cause the pans to go out of balance.



Pan balance

Furthermore, Fostnot and Lent (2007) explained that students need numerous experiences in composing and decomposing numbers before they realize that numbers can be grouped and turned around in a variety ways to produce sums that stay the same (e.g., commutativity and associativity for addition). Carpenter, Franke, and Levi (2003) suggested enabling student discussion about their perspectives about how the equal sign is used in order to understand which ideas support particular conceptions and to resolve differences in perspectives. In fact, discussing the equal sign in the context of specific mathematical tasks can provide a focus for students to articulate their ideas and challenge their concepts. Such mathematics tasks include justifying "true/false number sentences" and solving "open number sentences."

Considerations for Designing True/False and Open Number Sentences:

Different mathematical contexts can provoke students to examine and "reconsider the conceptual perspectives they and others have taken. Such contexts including the manipulation of number sentences in particular ways that provoke students' understanding of the equal sign. Carpenter, Franke, and Levi (2003, pp. 16-17) describe the following sequences:

Varying the Structure of True/False Number Sentence	Including Zero in True/False Number Sentence	Open Number Sentences
3 + 5 = 8	9 + 5 = 14	3 + 5 = □ or 3 + □ = 8
8 = 3 + 5	9 + 5 = 14 + 0	8 = 3 + □ or □ = 3 + 5
8 = 8	9 + 5 = 0 + 14	8 = □
3 + 5 = 3 + 5	9 + 5 = 13 + 1	3 + 5 = □ + 5
3 + 5 = 5 + 3		3 + 5 = □ + 3
3 + 5 = 4 + 4		3 + 5 = □ + 4

First, in varying the structure of the number sentence, students are experiencing different placement of the equal sign and different number of addends on either side of the equal sign within a number sentence. Second, by including zero, students can see that more than one number appears after the equal sign. In fact, the number after the equal sign no longer appears as the answer. Third, asking the question, "What number can you put in the box to make this number sentence true?" is another way to structure number sentences that challenge students' conceptions of the equal sign. As well, students will realize that changing the order of numbers does not change the value of the quantity. Note: Such variations of number sentences can be written for the other three operations (e.g., subtraction, multiplication, and division).

Finally, according to Mason (2008), elementary students possess mathematical powers to think and articulate abstractions and generalities of numbers and number operations. In fact, he says when students examine number sentences, they teach themselves to focus on some features as foreground and ignoring other features as background. Such power enables students to distinguish between different elements of number sentences, as well as to notice relationships between elements of number sentences and between the features of the elements. So, let's consider that the design of true/false and open number sentences should build on students' natural powers to detect relationships between details of the number sentences. The design of true/false and open number sentences should be designed to provoke students to shift their attention from specific elements (e.g., placement of the equal sign, number of addends) to relationships between numbers within and across equivalent expressions (e.g., 1 more than, 1 less than). Mason (2008) reminds us that "if learners' attention is drawn to different dimensions of possible variation, then they are more likely to form a rich web of meaning for a concept." (p. 88)

LET'S DO MATH: DETERMINING EQUIVALENCE USING TRUE/FALSE SENTENCES

Some samples of true/false equations within a three-part problem solving-based framework are provided below.

BEFORE

Activating Knowledge and Experience

True or False?
Explain how you know.

1. 7 = 7
2. 3 + 4 = 7
3. 7 = 4 + 3
4. 9 + 5 = 14
5. 9 + 5 = 14 + 0
6. 13 = 14 + 1
7. 6 + 3 = 7 + 2

- First three **True** equations re-establish the idea that the equal sign indicates that the two expressions or quantities are equivalent.
- 2nd and 3rd equations nudge students towards making a conjecture about the order of addends (e.g., commutative property); knowing that addition is commutative, reduces the number facts that students need to learn by almost half.
- 4 -> **True** - compare the sum of two addends to a single number; that is, 9 + 5 = 9 + 1 + 4 = 10 + 4 = 14
- 5 -> **True** - zero as one addend supports students to accept equations with more than one number after the equal sign. As well, they can examine the result when zero is added to a number
- 6 -> **False** - compare 13 and 14, noticing that 14 is 1 more than 13 and +1 makes it 2 more than 13
- 7 -> **True** - 6 is 1 less than 7 and 3 is 1 more than 2, so the two expressions are equivalent

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DURING

Working On It

True or False?
Explain how you know.

1. 43 = 40 + 3
2. 80 + 2 = 92
3. 76 = 7 + 60
4. 56 = 30 + 26
5. 7 + 18 = 6 + 18
6. 23 + 8 = 23 + 7 + 1
7. 5 + 14 + 22 = 15 + 22 + 4

- 1 -> **True**; 43 is on the left and it is made up of 4 tens and 3 ones which is the expression on the right 40 + 3 = 4 tens and 3 ones.
- 2 -> **False**; 80 + 2 is 8 tens and 2 ones and on the right there are 9 tens and 2 ones or 8 tens and 12 ones.
- 3 -> **False**; 76 is 7 tens and 6 ones and on the right we have 6 tens and 7 ones. The 7 and 6 have different place values on the left and right.
- 4 -> **True**; on the left we have 56 which is 5 tens and 6 ones and on the right we have 3 tens plus 2 tens and 6 ones which equals 5 tens and 6 ones.
- 5 -> **False**; 18 is on both sides of the equal sign but the numbers added to 18 on the left, 7, and right, 6, are not equal
- 6 -> **True**; on the right side, 23 + 7 + 1 is the same as 23 + 8 which is the same as the left side
- 7 -> **True**; 22 is on both sides, so there is equivalence there; 14 on the left is 1 less than 15 and 5 on the left is 1 more than 4

AFTER

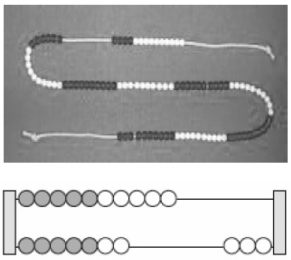
Practice

True or False?
Explain how you know.

1. 18 = 1 + 80
2. 18 = 8 + 10
3. 30 + 4 = 43
4. 94 = 4 + 90
5. 63 = 30 + 33
6. 8 + 48 = 48 + 2 + 6
7. 267 + 18 = 266 + 18
8. 22 + 3 + 14 = 15 + 4 + 22

- 1 -> **False**; 18 decomposes to 10 + 8 not equivalent to 1 + 80
- 2 -> **True**; 18 decomposes to 10 + 8 = 8 + 10 (commutativity)
- 3 -> **False**; 43 on the right is 4 tens plus 3 ones but the left side shows 3 tens and 4 ones
- 4 -> **True**; the left shows 90 + 4 and the right shows the same in the other order but when adding, you can add the addends in any order
- 5 -> **True**; 63 can be decomposed to 30 + 30 + 3 = 30 + 33
- 6 ->**True**; 48 is on both sides and I see 8 on the left which is equal to 2 + 6
- 7 -> **False**; 18 is on both sides so there is equivalence there; 267 on the left side is 1 more than 266
- 8 -> **False**; 22 is on both sides, so there is equivalence there; 14 on the left which is 1 less than 15, and 3 on the left is 1 less than 4

LINKS TO MANIPULATIVES: DEVELOPING EQUIVALENCE WITH A REKENREK



One strategy for introducing a math tool is to provide students with some time to explore the tool and then ask them to make statements about what they notice about the tool. Ask students to make observations about bead strings. Record their observations. For example, ½ of the beads are white and ½ of the beads are red. These observations can be used to develop true/false statements such as the following:

"You noticed that 50 beads are red and 50 beads are white. You also noticed that there are 10 beads that are the same colour together and 10 sets of beads. Is this true or false? 50 + 50 = 10 x 10." Students who are thinking relationally may say, "I know that 50 is the same as 5 x 10 so 50 + 50 = 5 x 10 + 5 x 10 or 50 + 50 = 2 x 5 x 10 so 50 + 50 = 10 x 10 is a true statement."

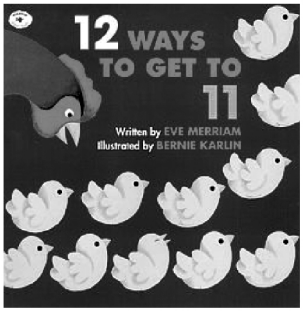
Similarly, students might make the following observations about the quantity shown on this rekenrek. "I see 10 red and 7 white beads." "I see 5 red beads, 5 white beads, 5 red beads, 2 white beads." " I see 20 beads minus 3 beads." " I see double 7 beads and 3 more."

These observations can be used to develop true/false statements, like: 10 + 7 = 5 + 5 + 5 + 2 or 10 + 7 = 20 – 3 or 10 + 7 = 2 x 7 + 3

What relationships might students notice? How might students explain their reasoning?

LINKS TO LITERATURE

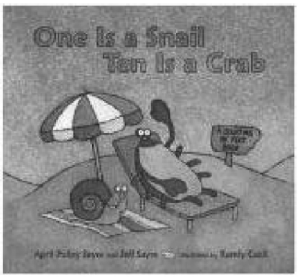




You can design true/false and open number sentences in relation to the situations described in picture books that illustrate the composition and decomposition of numbers in different ways. Examples of such books include, *One Hundred Ways to Get to 100* and *12 Ways to Get to 11*. Read the following two pages from *12 Ways to Get to 11*. '"Pick up nine pine cones from the forest floor and two acorns. At the circus, six peanut shells and five pieces of popcorn."

Build students' understanding of equality by using numbers to record number relationships from the picture book text. Here is another sample approach … "We are finding ways to get to 11 in this book. Here I see 11 = 9 + 2. On the next page I see that 6 + 5 = 11. Is this true? 9 + 2 = 6 + 5." Students who think relationally will notice that when the 9 decreases by 3 to get 6, the 2 increases by 3 to get 5.

Ask students to make other true number relationship statements using other pages of the book. As well, challenge students to describe and record false number relationship statements for the number 11.



You can use similar approaches for the book, *One is Snail Ten is a Crab*, since it depicts the composition of numbers in different ways. For example, on page 50, it shows five crabs or ten dogs and a crab can be expressed with true/false number sentences, such as: 50 = 4 x 10 + 10 or 10 x 5 = 4 x 10 + 10 x 1." Or, "50 legs is the same as 10 dogs each with 4 legs plus 1 crab with 10 legs or 5 crabs with 10 legs each is the same as 10 dogs with 4 legs each plus 1 crab with 10 legs."

Further, ask students to use the counting feet in the story to compose and decompose other numbers. Ask them to record representations symbolically as true/false number sentences.

LET'S DO MATH: ESTABLISHING EQUIVALENCE USING OPEN NUMBER SENTENCES

Some samples of open number sentences within a three-part problem solving-based framework are outlined below.

BEFORE

Activating Knowledge and Experience

What does the = sign mean? What number will make the open number sentences true?

1. 3 + 4 = ☐ + 4
2. 7 = 7 + ☐
3. 7 = 6 + ☐
4. 8 + 4 = 9 + ☐
5. 6 + ☐ = 7 + 5

- Many students think the equal sign means, "Here comes the answer." So, they interpret 3 + 4 = ☐ + 4 as having the answers 7 and 11. Also, they think 3 + 4 = 7 and then think they need to calculate 7 + 4 = 11. It is important to establish the idea that the equal (=) sign describes a relationship of equality between the expression on the left and the expression on the right.
- 1 -> 3 + 4 = **3** + 4.
- Use the 2nd and 3rd activating equations to make explicit the idea that a number can sit by itself on the left of an equal sign. Many have come to believe that equations that look like this are wrong. In addition, these two equations should activate a conversation about the relationships between the numbers on the left and right. In the 2nd equation 7 = 7 + **0**.
- 3 -> left side has 7, the right has 6 which is 1 less than 7 so the blank must be a number 1 more than 0; 7 = 6+**1**.
- 4 -> examine both sides of the equation and compare, saying, "9, on the right, is one more than 8, on the left, so to balance the right side with the left I need a number on the right that is one less than 4 from the left, 8 + 4 = 9 + **3**."
- 5 -> 6 is one less than 7 so need a number, 1 more than 5 to balance the equation. So, 6 + **6** = 7 + 5

DURING

Working On It

What number will make the open number sentence true?

1. ☐ + 9 = 9 + 3
2. 4 + 8 = ☐ + 8
3. 27 + 1 = 28 + ☐
4. 25 + ☐ = 2 + 26
5. ☐ + 33 = 34 + 5
6. 73 + ☐ = 75 + 12
7. 111 + ☐ = 5 + 109
8. ☐ + 523 = 281 + 525
9. 354 + 773 = ☐ + 350
10. 204 + 867 = 204 + ☐

- The first two equations again establish the idea that equations can have a variable (unknown number) represented by ☐ on the left or the right and still be made to represent equivalence. **3** + 9 = 9 + 3 and 4 + 8 = **4** + 8.
- While discussing solutions to the 3rd and 4th equations, ask students questions to make explicit the comparison of numbers that are 1 more or less than each other. In number 4, the position of the numbers to be compared (25 is first but 26 is the second number) has changed so students need to look within the expressions on both sides for numbers that are close in value to each other when they compare.
- 3 -> 27 + 1 = 28 + **0** because 27 and 28 are close in value; 27 on the left is 1 less than 28, so add 0
- 4 -> on the right is 26 which is 1 more than 25 on the left, so need to add a number that is 1 more than 2 to make equivalence
- 5 -> starting on the side where ☐ is located, the comparison is 1 less, so add 1 more than 5; **6** + 33 = 34 + 5

- 6 -> comparison is 2 less than – add 2 more than to 12; 73 + **14** = 75 + 12.
- 7 -> 111 is 2 more than 109 – number must be 2 less than 5; 111 + **3** = 5 + 109;
- 8 -> 523 is 2 less than 525 – number is 2 more than 281; **283** + 523 = 281 + 525
- 9 -> 350 is 4 less than 354 – number is 4 more than 773; 354 + 773 = **777** + 350
- 10 -> 204 is equal to 204, so need a number is equal to 867; 204 + 867 = 204 + **867**;

AFTER

Practice

What number will make the open number sentences true?

1. 84 + 62 = ☐ + 62
2. ☐ + 62 = 84 + 62
3. 61 + ☐ = 84 + 62
4. 91 + ☐ = 118 + 90
5. 92 + 118 = ☐ + 91
6. 92 + ☐ = 92 + 163
7. 92 + 163 = ☐ + 168
8. 159 + ☐ = 92 +162
9. 837 + 108 = ☐ +110
10. 824 + ☐ = 117 + 834

- 1 -> 84 + 62 = **84** + 62
- 2 -> **84** + 62 = 84 + 62
- 3 -> 61 + **85** = 84 + 62; 61 is 1 less than 62 so number is 1 more than 84
- 4 -> 91 + **117** = 118 + 90; 91 is 1 more than 90 so number is 1 less than 118
- 4 -> 92 + 118 = **119** + 91; 91 is 1 less than 92 so number is 1 more than 118
- 6 -> 92 + **163** = 92 + 163
- 7 -> 92 + 163 = **87** + 168; 168 is 5 more than 163 so number is 5 less than 92
- 8 -> 159 + **95** = 92 + 162; 159 is 3 less than 162 so number is 3 more than 92
- 9 -> 837 + 108 = **835** + 110; 110 is 2 more than 108 so number is 2 less than 108
- 10 -> 824 + **107** = 117 + 834; 834 is 10 more than 824 so number is 10 less than 117

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

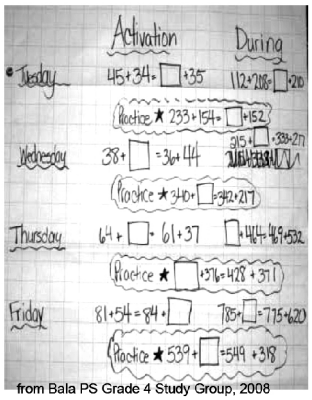
Application to Your Classroom

Daily 10-15 Minute Mini Lessons:

Have students experience these types of problems during fifteen minute daily mini-lessons, possibly beginning at the start of the morning. Use a problemsolving framework: Before (Activation), During (Teaching), and After (Practice). Have your students practise solving true/false or open number sentences for 5 minutes (e.g., 2 or 3 times a day before and/or after recesses). For practice:

- post one or two equivalent expressions for students to solve before and/or after recesses in a designated area (e.g., by the classroom door, outside class wall, mental math bulletin board)
- have the students explain their solutions in pairs
- choose two students to explain their thinking so the class can compare solutions.

Listen to your students' mathematical thinking to discern their understanding of the meaning of the "equal sign." Ask yourself, how "more than" and "less than" relationships between the numbers and expressions are evident in the students' explanations.



from Bala PS Grade 4 Study Group, 2008

Designing Sequences of Mini-Lessons - True/False and Open Number Sentences:

To suit your students' learning needs, prepare a developmentally appropriate sequence of mini-lessons, by systematically structuring true/false and open number sentences.

- Start with true/false sentences, then open number sentences.
- Start with addition, then subtraction; then multiplication, then division.
- Start with single digit, then two-digit, then three-digit numbers. See grade-specific curriculum expectations for size of numbers, number of addends, and mental math strategies.
- Vary the position of the equal sign (e.g., a + b = c; c = a + b; a = a; a + b = c + d).
- Vary the position of the box (variable – unknown)
  (e.g., ☐ + 3 = 8 + 2; 6 + ☐ = 10 + 2; 34 + 36 = ☐ + 37; 235 + 36 = 238 + ☐).
- Vary the number relationship structure (e.g., 1 more than, 1 less than, 2 more than, 2 less than, 3 more than, 3 less than).
- Vary the number of addends on either side of the equal sign
  (e.g., two addends -> 2 + ☐ = 5 + 3; three addends -> 12 + ☐ + 34 = 10 + 36 + 22; four addends -> 34 + 25 + 10 + 70 = ☐ + 50 + 20 + 35).

Such variations of number sentences can be written for the other three operations (e.g., subtraction, multiplication, and division).

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the equivalence problems and solutions are listed below:

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- Molina, M., Castro, E., and Mason, J. (2008). Elementary school students' approaches to solve true/false number sentences. PNA 2(2), pp. 75-86.
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CLOSING

Our next issue will continue to focus on the relationship between arithmetic to algebra, in terms of understanding arithmetic structure and properties and variables by solving problems that focus on the big idea of doing and undoing operations.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@edu.yorku.ca or mkestell@sympatico.ca