8 PAGE INSERT ELEMENTARY MATH

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FROM ARITHMETIC TO ALGEBRA – ALGEBRA AS GENERALIZED ARITHMETIC

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Every learner who starts school has already displayed powers to generalize and abstract from particular cases. Mason (2008) reminds us that such generalization and abstraction is the root of algebra. In our previous issue, Kaput (2008) described algebra as an expression of generalizations from arithmetic and quantitative reasoning. Such aspects of algebra include:

- looking at arithmetic expressions in a new way, in terms of their form rather than their numeric value when calculated.
- generalizing arithmetic operations, their properties, and their forms (e.g., commutativity, associativity, inverse relationships);
- building a syntactic aspect of algebra from the structure of arithmetic (e.g., an expression can be replaced with an equivalent expression)

So, the problems and commentary presented in this issue will focus on algebra as the study of structures and systems abstracted from computations and relationships that arise in arithmetic; that is, algebra as generalized arithmetic.

More specifically, in this issue, we will explore several concepts of generalized arithmetic: seeing the general in the particular; the potential of arithmetic for exposing methods of doing; arithmetic operations, properties, and forms; and a big idea of doing and undoing.

LINKS TO LITERATURE AND MANIPULATIVES



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Hello again! In this issue we will explore ways to focus on reasoning and generalizing arithmetic relationships as a strategy for making links between arithmetic and algebra.

Let's look at using the balance scale, and square tiles to investigate the commutative and distributive properties and the big idea of "doing and undoing."

RESEARCH SUMMARY – ALGEBRA AS GENERALIZED ARITHMETIC

The General in the Particular

Blanton (2008) describes a mathematical generalization as a statement that describes a general truth about a set of mathematical data. Mason, Graham, and Johnston-Wilder (2005) explain that algebra provides a manipulable symbol system and language for expressing and manipulating generality, especially in relation to a set of particular numeric cases or instances. An example of seeing the general in the particular is when learners notice from numerous multiplication examples that two numbers multiplied in any order will always result in the same product. So, generalized arithmetic refers to building generalizations about arithmetic operations, properties, and forms, as well as the properties of numbers.

Potential of Arithmetic for Exposing Methods of Doing

Arithmetic is not about memorizing hundreds of arithmetic facts. It is about learning generalizable methods of doing arithmetic calculations. Yet, arithmetic is often presented as a process of doing calculations to get a single number as an answer. The trouble with that approach is that the calculation processes are often obscured rather than illuminated. Try this problem.

If you had to multiply 32 by 15 in your head, how might you go about it? .

1. Partial Products

32 x 15 ->
5 x 2 = 10
5 x 30 = 150
10 x 2 = 20
10 x 30 = 300
15 x 32 = 480

2. 32 x 15 = (16 x 2) x 3 x 5 = (16 x 3) x (2 x 5) = 48 x 10 = 480

3. 32 x 15 = 15 x (4 x 8) = (15 x 4) x 8 = 60 x 8 = 480

4. 32 x 15 = (30 + 2) x 15 = (30 x 15) + (2 x 15) = 450 + 30 = 480

5. 32 x 15 = (30 + 2) x (10 + 5) = (30 x 10) + (2 x 10) + (30 x 5) + (2 x 5) = 480

Notice the different solutions; some include instances of the commutative, associative, and/or distributive properties. To develop notions of generality, the teacher might ask a question like, “Which calculation methods are specific to the problem? Which methods can be used for other multiplication problems? How do you know?” In this example, learners should not expected to memorize all possible calculations with numbers with a pattern like “(30 + 2) x something.” Rather, they need to be discerning generalized calculation methods using number properties and operation relationships.

Arithmetic Operations, Properties, and Forms

Students should know more than how to carry out arithmetic skills and procedures. Blanton (2008) reported that “generalizing arithmetic includes helping children see, describe, and justify patterns and regularities in operations on and properties of numbers. Although children might use this knowledge routinely, but implicitly when they’re doing arithmetic, algebraic thinking helps lift these properties out so that children are aware of them and can point to them as tangible objects of learning. In turn, this awareness reinforces their understanding of arithmetic concepts” (p. 12). Mason (2008) stated that generalized arithmetic is the result of learners generalizing their experience with numbers and expressing generality about properties of numbers arising from a variety of experiences in physical, mental, and symbolic worlds. Such mathematical work culminates in expressions of generality about rules of arithmetic operation and in the use of these rules for algebraic manipulation.

Commutative Property

Though generalizing is a natural thinking process, learners need time to notice that expressions have a sense of a generality, and they need ongoing experiences to express generality from numerous particular examples. Often a learner’s first attempt to express a generality turns out to be too wide-ranging, which is why initial expressions of generality should be treated as conjectures. Learners need to view conjectures as something that needs checking with particular examples or cases, followed by justifying at some later moment. Mason, Graham, and Johnston-Wilder (2005) described a child who noticed that 3 + 5 = 5 + 3, that 2 + 4 = 4 + 2, and that “anything plus anything = anything plus anything.” The child’s statement is an expression of generality that transcended from the particular cases he/she observed. Similarly, Blanton (2005) noted that children can learn to see and express generality, in terms of commutativity for addition

and multiplication initially using their natural language. As learners mature, they learn to express this arithmetic property in a more formal way that includes the use of symbols and literal variables. True/false and open number sentences are useful tasks for learners to solve in order for them to practise and consolidate their understanding of the commutative property of addition and multiplication.

Commutative property of addition	"You can add two number sin any order.	a + b = b + a	34 + 58 = 58 + 34 95 + 87 = □ + 95
Commutative property of multiplication	"You can multiply two numbers in any order."	a x b = b x a	34 x 58 = 58 x 34 95 x □ = 95 x 87

Associative Property

What ways can this problem be solved? 19 + 32 + 8 =? Can it be solved by adding 19 and 32 first? Could the problem can be made a bit easier to solve by grouping the 32 and 8 first: (19 + 32) + 8 = 19 + (32 + 8). Children may understand that they can group numbers in addition without changing the total sum, but they often don't think to use this strategy, even though the strategy can be very useful for simplifying equations. When analyzing the equation, 8 + 15 + 3 + 2 = 15 + 8 + 5, the use of a combination of the associative and commutative properties enables children to conclude that the statement is true without doing all the arithmetic to derive 28 = 28.

Associative property of addition	"If you're adding three numbers, you can add the first two, then the last, or you can add the last two, then the first"	(a + b) + c = a + (b + c)	(23 + 63) + 17 = 23 + (63 + 17) 69 + (21 + 82) = (69 + □) + 82

Further, Baek (2008) gathered evidence of children's intuitive knowledge of associative properties embedded in their multiplication strategies. Children were routinely encouraged to explain and discuss problem situations, solution strategies, and their reasoning about the strategies, and to learn from one another. This algebra-focused instruction employed three main strategies: representing children's strategies in more algebraic ways, providing problems that promote the partitioning of one or both factors and presenting true/false and open number sentences that explicitly examine the associative properties.

Associative property of multiplication	"If you're multiplying three numbers, you can multiply the first two, then the last or you can multiply the last two, then the first."	(a x b) x c = (a x b) x c	(34 x 58) x 2 = 34 x (58 x 2) (□ + 17) + 33 = 77 + (17 + 33)

Overgeneralizations of Commutative and Associative Properties

In the process of generating and discussing conjectures, children sometimes generate conjectures that look valid, yet they are not true. Carpenter, Franke, and Levi (2001) reported that such conjectures look very much like extensions of valid conjectures, but are based on form rather than on mathematical principles. So, children often over generalize from addition and multiplication which are commutative and associative to subtraction and division; but, subtraction and division are neither commutative nor associative.

Distributive Property

Baek's (2008) research about the associative property is similar for the distributive property, in that children's intuitive knowledge of the distributive property was often embedded in their multiplication strategies. Refer back to the problem on page 2, "If you had to multiply 32 by 15 in your head, how might you go about it?" Notice that some of the solutions had the distributive property embedded.

Also, Schifter, Monk, Russell, and Bastable (2008) reported that children naturally apply the distributive property in concrete situations. For example, where $65 was represented as money in the form of six $10 and one $5, students wrote the equation, 65 = 6 x 10 + 1 x 5. when students were asked how much money is needed for 11 people at $65 per person, students used the distributive property.

65 x 11 = (60 + 5) x 11
= (60 x 11) + (5 x 11)
= 660 + 55 = 715
65 x 11 = (60 x 10) + (60 x 1) + (5 x 10) + (5 x 1)
= 600 + 60 + 50 + 5
= 660 + 55 = 715 .

65 x 11

	10	10	10	10	10	10	5
10	100	100	100	100	100	100	50
1	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~5~~

Other Arithmetic Generalizations

Rather than investigating commutativity and associativity in itself, Schifter, Monk, Russell, and Bastable (2008, pp. 440-441) recommend that it is more fruitful for students to explore related ideas in number and operations, Some generalizations include:

- The order of addends does not affect the sum, but the order of the terms in a subtraction problem does affect the difference
- Subtraction undoes addition as in 22 + 8 – 8 = 22
- Any missing addend problem can be solved by subtraction and vice versa
- In an addition problem, if you subtract a certain amount from the 1st addend and add it to the other addend, the sum remains constant
- In a subtraction problem, if you subtract (add) the same amount from (to) both numbers, the difference remains constant
- The less you subtract, the larger the result
- Adding or subtraction 0 does not change the amount you start with
- Multiplying or dividing by 1 does not change the amount you start with
- If you add (or subtract) any number to an addend, the sum increases (or decreases) by that number
- If you double one factor and halve the other, the product remains constant
- In a multiplication problem, you can decompose one factor, multiply the parts by the other factor, and add the sub-products
- The more you divide by (the larger the divisor) the smaller the result (the smaller the quotient)

A Big Idea of Doing and Undoing

I'm Thinking of a Number

I am thinking of a number between one and ten; when I add two, I get six. What is my number? How did you find it?

The mathematical purpose of this type of problem, "I'm thinking of a number," is not about getting the answer. It is about explaining how the answer was determined. An undoing calculation process to the problem is "I started with 6, subtracted 2, and was left with the start number, four, which is between one and ten." This calculation process starts from an unknown number and gives a known result. Learners are provoked to reverse or undo the calculation sequence in order to determine the unknown number. Metacognitive questions like, "How is that operation undone?" provoke thinking. Undoing problems can include same operations or combinations of operations, such as the problems posed by Janine and Matthew.

Janine says, "I'm thinking of number that when I multiply it by 2 and add 3 the answer is 23."

Matthew says, "I'm thinking of a number that when I multiply it by 6, subtract 6, and multiply the result by 4, the answer is 4 less than 148."

Therefore, an "I am thinking of a number" problem sets learners with considerable choice of numbers, yet with each additional operation or constraint, learners must reason among fewer possibilities. Also, doing and undoing is one way of perceiving the relationship between addition and subtraction, multiplication, and division.

Think of a Number (THOAN)

Learners are often intrigued by a mathematician's skill in predicting answers to seemingly complicated sequences of calculations. Of course, it is all done with algebra. Mason, Graham, and Johnston-Wilder (2005) describe another problem design called THOANs or "Think of a number." In this example, a shape (□) is used to track the calculation process.

☐ -> 2 x ☐ -> (2 x ☐) + 4 -> ((2 x ☐) + 4) x 3 -> [((2 x ☐) + 4) x 3] ÷ 6 -> {[((2 x ☐) + 4) x 3] ÷ 6} – 2

Often learners solve THOANs by imagining movement on a number line or mentally keeping track of numerals. In fact, Mason (2008) explains that by tracking arithmetic; that is, tracking one or more numbers through a calculation, the role played by the number is illuminated; such is referred to as the algebraic use of numbers.

There is a natural link between "I'm thinking of a number" and "Think of a number" (THOANs) because they both involve mathematical doing, followed by undoing, except that the undoing in a THOAN is not simply done by reversing each operation in turn. In fact, while doing usually gives a single answer, the corresponding undoing often gives rise to a whole class of answers. Therefore, doing and undoing highlights the inverse relationship between addition and subtraction and between multiplication and division.

Think of a number, double it, add 4, multiply by 3, divide by 6 and subtract 2.

What is the original number.

How did you find it?

LET'S DO MATH: EXPLORING THE COMMUTATIVE PROPERTY

1a. Carmen had 21 flowers in her garden and planted 15 more. How many flowers did she have after the planting?



b. Doug's class collected 15 juice bottles on Monday to recycle. On Tuesday, 21 more recycle bottles are collected. How many did they have by the end of day on Tuesday?



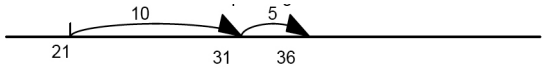
c. Explain how these two problems are alike and how they are different.

a. 

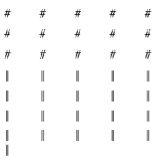
I added 15 new flowers (check). Then I wrote the numbers down, grouped them and added.

21 + 5 + 4 + 3 + 2 + 1 = 21 + 6 + 6 + 3
= 21 + 15 = 36

a. There were 21 flowers and Carmen added 15.
There are 36 flowers after the planting.



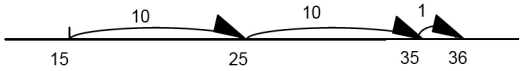
b.



I drew a 3 x 5 array that made 15.
I added 21 more in 5 rows of 4 plus 1.

The total was 15 + 5(4) + 1
= 15 + 20 + 1
= 36

b. Monday there were 15 bottles and 21 were gathered Tuesday.



- The numbers in these problems are the same, but because of the context, flowers and recycle bottles, the order of adding the addends was different.
- The same sum though the addends were combined in a different order.

LET'S DO MATH: EXPLORING COMMUTATIVE, ASSOCIATIVE, AND DISTRIBUTIVE PROPERTIES

True or False? Explain how you know.

1. (4 + 79) + 6 = (6 + 4) + 79
2. 16 + 9 = 16 + 4 + 5
3. 1 x 451 = 451 x 1
4. 23 x 5 = 5 x 23

Note: LS means the left side of the equal sign; RS means the right side of it.

1 -> **True**; LS is 83 + 6 = 89 and RS is 10 + 79 = 89; commutative property lets you reorder the addends
(4 + 79) + 6 = 6 + (4 + 69) = so whole expression is
(4 + 17) + 6 = 6 + (4 + 79) = (6 + 4) + 79= 10 + 79 = 89
(6 + 4) + 79 = 10 + 79 = 89 to make a 10, using (6 + 4) + 79

2 -> **True**; RS is 4 + 5 replacing 9 with 4 + 5 = 9. So, regrouping (16 + 4) + 5 to get 20 + 5 = 25

3 -> **True**; LS = 451 and RS = 451; can multiply numbers in any order and get the same product; any number multiplied by 1 is equal to the same number.

4 -> **True**; LS is 23 x 5 = 115 and RS is 5 x 23 = 115; equation is balanced; you can multiply numbers in any order and get the same product

5. $6 \div 3 = 3 \div 6$

6. $6 \times 7 = (6 \times 6) + (6 \times 1)$

7. $8 \times 13 = (8 \times 10) + (8 \times 2)$

5 -> **False**; LS of this equation is equal to 2; RS is equal to a fraction, 1/2

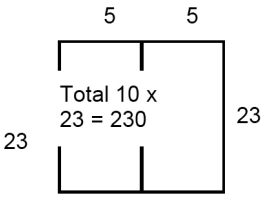
6 -> **True**; LS = 42 and RS = (36) + 6 = 42; if you forget 6 x 7 you can calculate (6 x 6) and add (6 x 1)

7 -> **False**; LS is 8 groups of 13 (104), RS is 8 groups of 10 (80) and 8 groups of 2 (16) which is the same as 8 groups of 12 (96) not 13 (104)

8. There were 5 classes of grade 4s from Alpha School going to the park.

   Each class had 23 people. The Principal wanted them to have 2 bottles of water for each student.

   How many water bottles do they need?



8 -> **Student 1:** This expression (5 x 23) x 2 represents the situation. I would rather calculate the numbers in a different order. I know I can do 2 x (5 x 23) because I can multiply numbers in any order and get the same product. Now I think I can go (2 x 5) x 23 and group the numbers differently. Now, I get 10 x 23 = 230. So, they need 230 water bottles.

Student 2: I see this open array as 23 x 5 doubled.
This is (23 x 5) x 2.
When I draw the picture I get 23 x 10 = 230.
So, it must be okay to break up the brackets and instead of writing (23 x 5) x 2, write 23 x (5 x 2) = 23 x 10 = 230
Multiplication is commutative – can multiply numbers any order; multiplication is associative – with 3 factors, you can multiply the factors in any order to make the calculations easier.
So (23 x 5) x 2 = 23 x (5 x 2) = 230
Total 10 x 23 = 230

9. The front of an apartment building with 12 floors has 15 windows on each floor.

   How many altogether?

9 -> **Student 1:**

$12 \times 15 = (3 \times 4) \times (5 \times 3)$
$= ((3 \times 4) \times 5) \times 3$ - associate the 5 with the (3 x 4)
$= 3 \times ((3 \times 4) \times 5))$ - commute the factors 3 and the whole big bracket
$= 3 \times (3 \times (4 \times 5))$ - associate factors inside big bracket
$= (3 \times 3) \times (4 \times 5)$ - associate factors keeping brackets
$= 9 \times 20 = 180$ - multiply 2 numbers

Student 2:

$12 \times 15 = (10 \times 15) + (2 \times 15)$
$= (150 + 30)$
$= 180$ - Distributive Property

LINKS TO LITERATURE AND MANIPULATIVES: EXPLORING THE COMMUTATIVE PROPERTY



Using the book, *12 Ways to Get to 11* by Eve Merriam, have children count the pictures from left to right and record an equation to represent the objects on the page (e.g., 9 + 2 = 11). Then, have the children count the pictures on the same page from right to left and record an equation to represent the objects (e.g., 2 + 9 = 11). Ask the children, "What do you notice about the equations?"

Student-Generated Examples of the Commutative Property for Addition

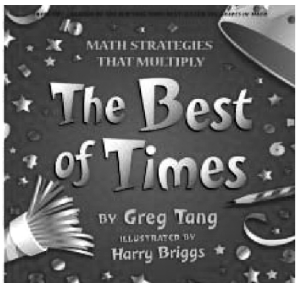


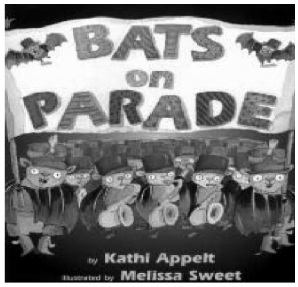
Ask children to model different addition equations for 7, to see if there are same pairs of addends with a different order (e.g., 5 + 2 = 7; 2 + 5 = 7). Compare and discuss their addition models. Children could explore the commutative property using a balance scale (e.g., Place 5 and 2 more blocks on one side and 2 and 5 more on the other side.). Ask children to explain and generalize what they notice (e.g., "When we put two amounts that are the same on each side, they will balance. The order the blocks down doesn't matter.")

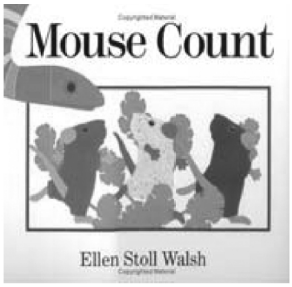
Student-Generated Examples of the Commutative Property for Multiplication

Similarly, students can investigate the commutative property in terms of factors of products, such as 2 x 3 = 3 x 2 = 6. They could use tiles and base ten blocks to create rectangular area models and round counters, beads, or paper clips to create array models. Larger whole numbers, decimal

numbers, fractions, and integers can be used to explore commutativity of multiplication. Have students discern similarities and differences between the model of the two expressions (e.g., same size (area), different orientation of the rectangle).







Exploring the Distributive Property

One of the strategies that Greg Tang uses to develop children's understanding of multiplication is representing the distributive property. A response to "What is 7 x 5?" includes "It's five 5s plus two 5s" or "7 x 5 = (5 x 5) + (5 x 2)." Children can model the array for this equation using square tiles.

Investigate other multiplication facts using the distributive property. In the picture book, "Bats on Parade," the bats in the parade line up by rows and columns of square numbers. A ruler can be used to investigate how many ways the children can compose numbers to show the distributive property of a composite number (e.g., 8 x 8 = 64 decomposed as 4 x 8 + 4 x 8 or 3 x 8 + 5 x 8).

Exploring the Concept of "Doing and Undoing"

The inverse relationship between addition and subtraction can be viewed from the perspective of "doing" and "undoing". In the book "Mouse Count", a slithery snake finds ten sleeping mice that he thinks will make a delicious meal. He pops them into a jar, counting "one, two, three." The captured mice outsmart him, "three, two, one . . . ," uncounting themselves as they scramble out of the jar and run home to be safe and sound.

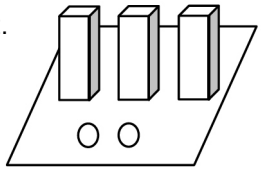
LET'S DO MATH: INVESTIGATING THE BIG IDEA OF "DOING AND UNDOING"

1. Draw the expressions.
   a. 1 unknown number plus 2
   b. 1 unknown number plus 2 doubled
2. Robyn starts with 26 counters. She puts an equal number of counters in each of 3 bags and has 2 left over. How many in each bag?
3. Think of a one-digit number.
   Add 4.
   Double it.
   Subtract 2.
   Divide by 2.
   Subtract 3.
   What is the answer? Why?
4. Choose any number.
   Add 4.
   Multiply by 2.
   Subtract your original number.
   Subtract 5.
   Subtract your original number.
   What do you get? Why?

Use a cube to represent an unknown number and counters to represent numbers.

1a. 

1b. 

2. 

There are 26 counters on the table.

26 – 2 = 24

Try 7: 7 + 7 + 7 = 21 too small

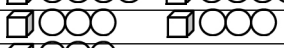
Try 8: 8 + 8 + 8 = 24

So, 8 x 3 + 2 = 24 + 2 = 26

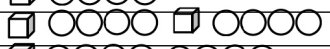
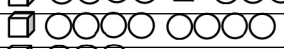
There are 8 counters in each bag.

Carry out these operations and explain what happens.

3. You always get what you started with. Like this…

7	9		*x*
11	13		*x* + 4
22	26		2*x* + 8
20	24		2*x* + 6
10	12		1*x* + 3
7	9		1*x*

4. You always get 3.

4	100		*n*
8	104		*n* + 4
16	208		2*n* + 8
12	108		1*n* + 8
7	103		1*n* + 3
3	*3*		*3*

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

Through the activities of analyzing, conjecturing, justifying, and symbolizing, children move from arithmetic to algebraic thinking. Look at the arithmetic you teach with "algebra eyes" so that you can structure arithmetic lessons around the broader goal to build algebraic thinking. Schifter, Monk, Russell, and Bastable (2008) recommend that learning experiences must provide opportunities for learners to puzzle over regularities of the number system.

Some strategies for prompting learners to become aware of possible generalities are as follows.

- Draw class' attention during discussion to how particular numeric solutions (or cases) work, in terms of the details of how learners draw, calculate, and/or count, so that the details of the generality could be construed.
- Prompt learners to construct their own examples that display similarities with or differences from a given example in order to extend their awareness of the range of possibilities to a problem, to develop their sense of generality; in fact, making choices and checking their conjectures supports learners in seeing themselves as active and competent mathematics problem solvers.
- Be explicit to students when varying particular aspects of a problem, so that they notice their shift of attention from particulars to those aspects that can be varied, changed (Mason, Graham, and Johnston-Wilder, 2005).
- Ask learners to step back and consider what is the same and what is different about various mathematical objects, such as mathematical diagrams, expressions, or even tasks or problems to draw learners away from calculations and focus their attention on important ideas or concepts (Schifter, Monk, Russell, and Bastable, 2008).

Suggested Readings

- Baek, J. M. (2008). Developing algebraic thinking through explorations in multiplication. In C. Greenes and R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook*, pp. 141-155. Reston, VA: NCTM.
- Blanton, M. (2008). *Algebra and the elementary classroom*. Portsmouth, NH: Heinemann.
- Carpenter, T., Franke, M. L., & Levi, L. (2001). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.
- Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. Carraher, and M. Blanton (Eds.), *Algebra in the Early Grades (5-18)*. Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.
- Mason, J., Graham, a., & Johnston-Wilder, S. (2005). *Developing thinking in algebra.* Thousand Oaks, CA: Sage Publishing.
- Mason, J. (2008). Making use of students' powers to produce algebraic thinking. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 57-94). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.
- Schifter, D., Monk, S., Russell, S., & Bastable, V. (2008). Early algebra: What does understanding the laws of arithmetic mean in the elementary grades? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 413-448.). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.

CLOSING

Our next issue will continue to focus on the relationship between arithmetic to algebra, in terms of understanding functional relationships with variables as single, unknown numbers.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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