8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

Volume 48 • Number 2 • December 2009

MAKING SENSE OF FRACTIONS – COMPARING AND ORDERING FRACTIONS

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A Publication of the OAME/AOEM

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Quantities represented by natural numbers are more easily understood than fractions. We can count and say how many apples are in a bag. But fractions cause difficulty to most people because they involve relations between quantities. What is 1/2? One half of what? If Aki and Sandie both spent 1/2 of their pocket money on snacks, they may not have spent the same amount of money each. Why is that?

We've noticed that students need to realize that the same fraction may refer to different quantities (1/2 of 8 and 1/2 of 12 are different) and that different fractions may be equivalent because they refer to the same quantity (e.g., 1/3 and 3/9). Also, it is possible that the errors that students make when working with fractions can be seen as a consequence of their difficulty in understanding that natural and rational numbers involve different ideas. For example, one common error that students make with fractions is to think that 1/3 of a sandwich is smaller than 1/5 of the same-size sandwich because 3 is less than 5. Yet most children readily recognize that a sandwich shared among three children gives bigger portions than the same sandwich shared among five children.

In this issue, the Research Summary and problems (with multiple solutions) will focus on the underlying notions of comparing and ordering common, proper, and improper fractions and mixed numbers, conceptually and procedurally. As suggested in our previous Abacus issues, consider solving these problems yourself first before examining the solutions we offer you. Also, consider how these problems can be used in a three-part problem solving lesson framework. Finally, enjoy doing math!!!

LINKS TO LITERATURE AND MANIPULATIVES



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When creating a problem based on a manipulative, mathematical connections between different strands of the mathematics curriculum can be made more explicit. Let's revisit a familiar fraction problem that provokes the use of mathematical ideas and strategies, such as composing and decomposing shapes, exploring the properties of symmetry, comparing areas of shapes, using proportional relationships, collecting and organizing data in a table, and generalizing number relationships. Also, I'm featuring children's picture books, in which you will meet two characters who learn more about who they really are when things are done half way or when a person is one out of three. What are the different ways that fractions are represented and used in the story? Enjoy!

RESEARCH SUMMARY – COMPARING AND ORDERING FRACTIONS

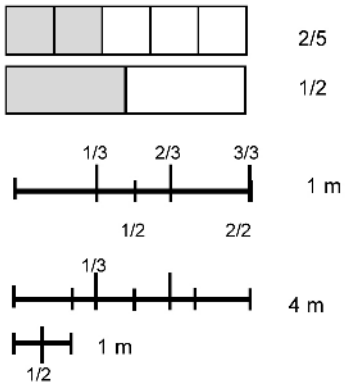
In order to develop fraction sense (e.g., representing fractions, comparing and ordering fractions, fraction equivalency), most students need extended periods of time examining different physical models depicting fractions. Concrete materials are necessary tools for students to use to develop a conceptual understanding of comparing and ordering fractions, in terms of different models of representation:

- region - Base Ten blocks, Cuisenaire rods, folded paper (in rectangles), fraction rectangles and circles, and pattern blocks
- set of objects - pattern blocks and tangrams (as objects, not by congruency), coloured counters, and balance scales
- linear measurement - analog clocks, centicubes aligned to a ruler, Cuisenaire Rods (focus on relative lengths), folded paper strips, and graduated cylinders (measuring volume or capacity).

However, some educators believe that using rules or algorithms for comparing fractions are effective for getting correct answers. But, without conceptual understanding, when students implement these rules they are not thinking about the relative sizes of fractions. Van de Walle and Lovin (2006) remind us that if students are taught the common denominator rule before they have had the opportunity to think about the relative size of various fractions, there is little chance that they will develop any number sense about fractions. Therefore, a conceptual understanding needs to be built on comparisons of relative sizes. So, according to Randall (2005), Small (2009), Smith (2002), and Van de Walle and Lovin (2006), there are several important principles underlying the concept of relative fraction sizes.

Same Whole, Different Whole

Fractions can be compared only if the whole is known. Two fractions can be compared easily when they are parts of the same whole such as 2/5 and 1/2. It is common for students to choose 2/5 to be larger, since the digit 5 is more than 2 and 2 is more than 1. Yet, using direct comparison of a concrete model, students can see that fifths are smaller in size than halves, and even 2/5 is smaller in size than 1/2.



However, when two fractions are from two different wholes, it is difficult to make a direct comparison. If you compare 1/2 and 1/3, 1/2 > 1/3 when 1/2 and 1/3 are of the same whole. But, if 1/2 and 1/3 are of two different wholes, it may not be true that 1/2 > 1/3. In this example, 1/2 and 1/3 are not of the same whole (i.e., 1 m and 4 m).

Same Denominator, Compare Numerator

If two fractions have the same denominator, the one with the greater numerator is greater.

3/5<4/5, 12/3<6 2/3, and 1 6/9>1 2/9

Same Numerator, Compare Denominator

If two fractions have the same numerator, the one with the greater denominator is less.

3/5<3/4, 3/5>3/6, and 12/5<12/3

Benchmark Numbers

Often, fractions can be compared in relation to benchmark numbers, such as 0, 1, and 1/2. When one fraction is greater than 1/2 or 1 and the other fraction is less than 1/2 or 1, a comparison of fractions can be made through reasoning.

Using the Benchmark, 1/2

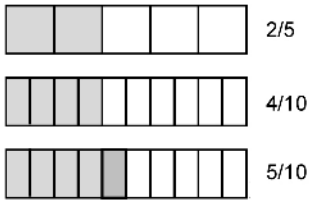
2/6<3/4, since 2/6<3/6 or 1/2 and 3/4>1/2 so, 2/6<1/2< 3/4

Using the Benchmark, 1

8/5>2/3, since 8/5>1 and 2/3<1

Common Denominators, Common Numerators

Fractions can be compared by renaming them with same or common denominators, For example, focusing on same or common denominators (or wholes), 2/5 can be renamed as 4/10 and 1/2 can be renamed as 5/10. Then 2/5 < 1/2 because 4/10 < 5/10. Also, fractions can be compared by renaming them with same or common numerators. For example: 1/2 can be renamed as 2/4, so 2/5 < 2/4 because fifths are smaller than fourths and there are the same number of fourths and fifths (same numerator), so 2/5 must be smaller.



Fractions are Dense

No matter what two different fractions are selected, there is always a fraction in between two fractions. For example, between 1/4 and 2/4, there are an infinite number of fractions. 1/4 and 2/4 can be renamed with greater denominators to determine fractions between them.

1/4 = 2/8 and 2/4 = 4/8, so 3/8, is between 1/4 and 1/2.

Or, 1/4 = 5/20 and 1/2 = 10/20, so 6/20, 7/20, 8/20, and 9/20 are also between 1/4 and 1/2.

Developmental Learning Sequence

Smith (2002) reports that there is a developmental sequence of strategies for comparing and ordering fractions.

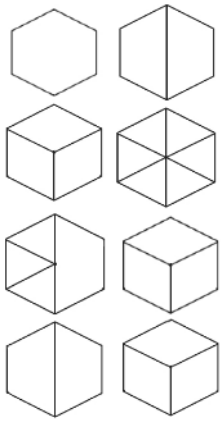
- First, students commonly construct and examine the corresponding divided quantities by directly comparing or sketching them on paper.
- When students are not satisfied with visual judgments from their diagrams or concrete models or become frustrated with representing fractions like elevenths and thirteenths, they begin to reason more analytically about the size and number of the parts in each fraction. With lots of experience in reasoning about fractions using divided quantity diagrams and models, they come to understand that thirteenths are smaller than elevenths (i.e., 7/11 > 7/13). Shifting towards reasoning directly about relative fraction sizes by comparing numerators and denominators (i.e., same denominator, compare numerator; same numerator, compare denominator) is a key strategy. Reasoning directly about the numerical component is more efficient and enables students to apply their knowledge of whole numbers.
- For the third milestone, students think about fractions in terms of where they lie relative to important numerical reference (or benchmark) points, 0, 1/2, and 1.
- Finally in the late junior grades, students use numerical transformations for fractions, such as conversions to common denominator, common numerator, decimal numbers, and/or percentages.

Therefore, using particular manipulatives for comparing and ordering fractions has a significant role in helping students develop concepts of relative fraction sizes. With sufficient concrete experiences, students will be prepared to seek out more efficient, precise, and strategic methods, that in turn, will lead towards generalizations that can be stated as rules or algorithms for comparing and ordering fractions.

LINKS TO MANIPULATIVES: PATTERN BLOCKS

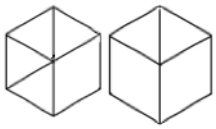
Have students use combinations of other pattern blocks to make composite shapes congruent to the yellow hexagon. In order to build language and the habit of comparing fractions one to another, prompt students to talk about the relationships they see, by asking questions like, "What do you notice?

- *the yellow hexagon is 2 times bigger than the red trapezoid because it takes two red trapezoids to cover the same area as the yellow hexagon*
- *3 blue rhombuses or 6 green triangles cover the yellow hexagon*
- *1 red trapezoid is half as much as 1 yellow hexagon because 1 red trapezoid covers half of 1 yellow hexagon*
- *half of 1 yellow hexagon can be covered by 1 red trapezoid and the other half by 3 green triangles or 1/2 = 3/6 (grade 4 and up)*
- *2 trapezoids cover 1 hexagon and 3 rhombuses, so 1 trapezoid is a larger fraction of the hexagon than 1 rhombus (record as 1/2 > 1/3 (grade 4 and up)*



As students continue to explore, they will combine different pattern blocks. Pose questions like, "What is the same and different about these two hexagons?"

- *2/6 of both hexagons is green triangles*
- *2/6 of one of the hexagons is green triangles and 2/3 blue rhombus'; 4/6 of the other hexagon is green triangles and 1/3 blue rhombus*



Listen for students who call each part of these hexagons one fourth because they used four pattern blocks to cover the area. Try a similar problem with other fraction materials such as fraction circles.

LESSON LEARNING GOALS

- Curriculum Expectations (MOE, 2005) (not to put on the board with students, for board (bansho) plan only)
- Show relevant curriculum expectations the grade before and after the grade-specific learning goal, to see development of mathematical thinking across grade levels; Use the curriculum expectations to determine the appropriateness of the problems chosen for Before (activation), During (lesson), and After (practice)

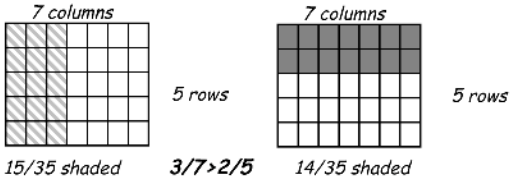
BEFORE (Getting Started)

- 5 to 10 minutes
- Activating students' mathematical knowledge and experience that is directly related to the mathematics in the lesson problem
- Use a smaller problem similar to previous lesson problem
- Use student work responses for class analysis and discussion to highlight key ideas and/or strategies

Paper Problem:

Which is larger -- 2/5 of a sheet of paper or 3/7 of a same sized sheet? How do you know?

- *Half of 5 is 2.5; Half of 7 is 3.5; The first offer, 2/5, is 0.5 out of 5 units from the middle. The second offer, 3/7, is 0.5 units out of 7 units from the middle. 0.5 out of 7 is smaller than 0.5 out of 5 as the pieces are smaller with 7 cuts than with 5 cuts. So, 3/7 is closer to half than 2/5.*
-



DURING (Working On It)

- 20 minutes
- Understand the problem, Make a Plan, Carry Out the Plan
- Understanding the problem – Ask, "What information from the problem will we use to make a plan to solve it? Explain."
- Teacher records in a list, below the problem, the information that students identify (We don't want different solutions to different problems; we want different solutions to the same problem. Students are learning to discern important information (i.e., problem definition))
- Students solve the problem on chart paper with markers (visible for later discussion) in pairs or in small groups
- Teacher circulates to record different student solutions, in addition to the ones the teacher has anticipated prior to the lesson

Chocolate and Lemon Cake Problem:

Jarred had two cakes of the same size that he took to a party. He cut the chocolate cake into 12 pieces and the lemon cake into 8 pieces. His guests ate 5 slices of chocolate and 3 slices of lemon cake. Did they eat more chocolate or more lemon cake? Explain.

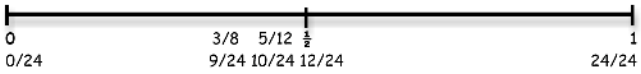
What information will we use to solve this problem?

- *chocolate cake cut into 12 equal parts, lemon cake into 6 equal parts*
- *chocolate and lemon cakes – same size (area and volume)*
- *Which is more cake - 5 chocolate slices, 3 lemon slices?*

AFTER (Consolidation)

- 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion
- Teacher decides which solution to share 1st, 2nd, 3rd based on mathematical relationships between the solutions
- Students (authors) explain and discuss their solutions with their friends
- During discussion, solutions are organized to show mathematical elaboration from one solution to the next and towards lesson learning goal
- Teacher mathematically annotates on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students, using students' ideas, relating students' informal vocabulary with formal vocabulary

proper fractions *-> 5/12, 3/8, 9/24, 10/24;* ***benchmarks*** *-> 0, ½, 1*
"Order these fractions from least to greatest on a ***number line****."*



numerator*-> # of parts of cake eaten;* ***denominator****->total # of parts of cake*

Counted and compared number *of chocolate slices (5 out of 12) and number of lemon slices (3 out of 8) eaten*

What about the size of the cake slices? *… each of lemon slices are larger than each of the chocolate slices*

- drew 2 same size ▭ (lem, choc)
-divided lem into 8 parts (equal size) and choc into 12 parts (equal size)
- made a 3rd same size rectangle and divided it into 24 parts (equal size)
→12 x 2 = 24 (2 X parts of choc)
→ 8 x 3 = 24 (3 X parts of lem)
- placed lem rectangle over 24 parts rectangle→ 9 parts lem (3 x 3)
- placed chocolate rectangle over 24 parts → 10 parts choc (5 x 2)

Solution 1


chocolate 12 pieces


lemon 8 pieces


5 out of 12 eaten


3 out of 8 eaten

There is more chocolate cake eaten than lemon cake → 5/12 > 3/8

24ths or 24 parts in one whole
common whole *– both cakes are divided into 24 equal parts*
-chocolate cake eaten -> 5/12 or 10/24
-lemon cake eaten -> 3/8 or 9/24

Solution 2
If both cakes were cut into 24 pieces, then 5/12 would be like 10/24 and 3/8 would be like 9/24

So because 10/24 > 9/24 then 5/12 > 3/8 by a little bitty-bit.

Greater Number of Chocolate Slices than Lemon Slices

Number of Parts of the Whole are the Same for Both Cakes

Note: Mathematical annotations includes mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing approach/strategy, and questions to further thinking and practice. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
- compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts - compare fractions to the benchmarks of 0, ½, 1	… compare and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers using a variety of tools and using standard fractional notation	… compare, and order fractional amounts with unlike denominators, including proper and improper and mixed numbers using a variety of tools and using standard fractional notation

AFTER (Highlights/Summary)

- 5 minutes
- Teacher revisits in the different student solutions the key ideas, strategies, and models of representation that are related to the lesson learning goal
- Teacher records key ideas, strategies, and models of representation separately, so the students can see the explicit focus of learning from their solutions, as it relates to the lesson learning goal

Highlights/Summary:

1. When comparing fractions,
- *same numerators – lesser fraction has greater denominator*
- *same denominators – greater fraction has greater numerator*
- *compare to benchmark fractions like 0, 1/2 or 1*

2. To get common (same) denominator (or common whole):
- *multiply denominators together – like folding paper two ways*
- *find a multiple of both denominators and make equivalent fractions with like denominators – compare numerators*

2. Common Multiple Strategy for Common Denominator
- What number is a common multiple for both denominators?
- What number can both denominators be divided into equally?

1. Multiplication Strategy for Common Denominator
-> 5/12 and 3/8 -> 12 x 8 = 96 (same total # of parts)
-> 40/96 > 36/96 because (5x8)/(12x8) > (3x12)/(8x12)

1 can be different fractions -> 8/8, 12/12, 1/1, 84/84

***benchmarks** for comparing fractions, 0, ½, 1*

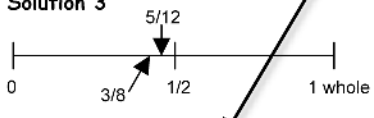
AFTER (Practice)

- 10 minutes
- Teacher chooses 2 or 3 problems, similar to the lesson problem for students to solve individually or in pairs as a scaffold)
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to use a strategy different from the one they used for the lesson problem to solve practice problems

Pie Problem:

Jess baked two (same size) pies for a party. She cut the cherry pie into 6 equal slices and the pumpkin into 12 slices. At the party, people ate 3 slices of the cherry pie and 6 slices of the pumpkin pie. Did people eat more cherry or pumpkin pie?

Solution 3



5/12 is 1/12 away from 6/12 or 1/2
3/8 is 1/8 away from 1/2
1/12 < 1/8 (numerators same)
so 5/12 > 3/8

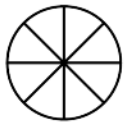
Using Benchmarks, 0, ½, 1 to Compare Proper Fracitons

Solution 4
If we think of 12 x 8 we get 96, a common denominator.
Then write each fraction with that denominator…

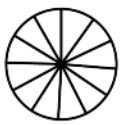
5/12 x 8/8 = 40/96
3/8 x 12/12 = 36/96 ***-> 40/96 > 36/96***
3/8 < 5/12
So, the family ate more chocolate cake than lemon.

Multiplying Denominators to Get Common Denominator

Solution 1



Cherry 6 slices
3 gone
3/6 pie gone or 1/2



Pumpkin 12 slices
6 gone
6/12 gone or ½

People ate same amount, ½ of each pie.

Solution 2

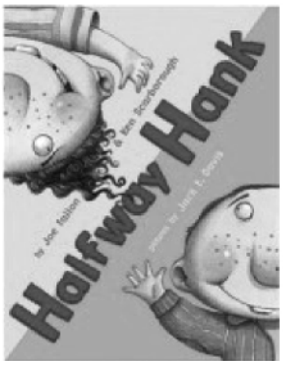
- 3/6 is the same as ½

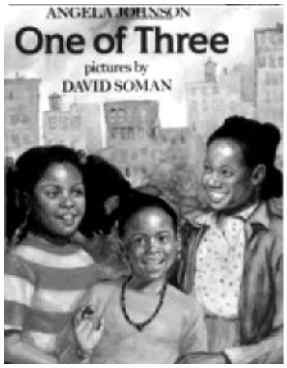
- 6/12 is the same as 1/2

They ate equal amounts of the whole pies.

Pumpkin pie people had more smaller slices.

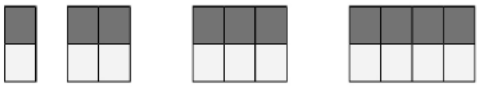
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.





LINKS TO LITERATURE

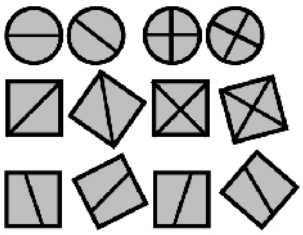
- Prompt students to find examples showing ways that the illustrators communicate mathematical ideas in the books, *Halfway Hank* and *One of Three*
- Ask students to identify examples of halves in the drawings in *Halfway Hank* or explain 1 of 3 in *One of Three*. Keep track of the examples on the board in a chart."
- Challenge the students to be illustrators by drawing a picture that could be in either of the books. Prompt students to explain how their pictures show fractions.
- Provide students with materials like square tiles, geoboards, and pattern blocks, so that they can illustrate halves or 1 out of 3 in different ways. See example of square tiles showing 1/2.



LET'S DO MATH – BEDROOM PROBLEM

Before (Getting Started)

Provide a sheet of 4 circles and 4 squares and have students show half of 1 circle and one-fourth of 1 square, in different ways.



- Rotate the circle and square and ask students if 1 part is still 1/2 or 1/4.
- Rotate the dividing lines on the square slightly and ask students if 1 part is still 1/2 or 1/4.

During (Working On It)

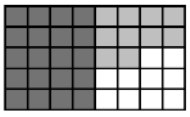
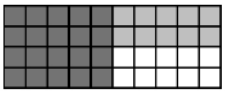
Jong draws his room on grid paper using 40 square units. He shows half of the bedroom area for his bed and half of the rest of the area for his desk and books and the rest for his clothes.

(a) What could his bedroom drawing look like? How much space is left for his clothes?

(b) Show your work using a labeled diagram.

After (Consolidation)

Anticipating Student Responses:

Solution 1	**Solution 2**	**Solution 3**
 - *Dark shading is half. Light shading is half of the space that I was like* - *It looks like light shading is half of dark shading.* - *He needed one-fourth for desk and things and there was one-fourth left for clothes and shoes.*	 - *Dark shading is half – 20 squares* - *Light shading is half of the space that was left. (half of 20 is 10)* - *It looks like one-fourth of the whole.* - *So there is one-fourth left for clothes and shoes.*	- *Half of 40 is 20 for bed 40 – 20 is 20 squares left* - *Half of 20 is 10. 10 out of 40 is one-fourth. 40 – 20 – 10 is 10 and 10 out of 40 is one-fourth.*

Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 –> made a room 5 by 8 to show 40 squares – then shaded 20 for the bed – saw that there were 20 squares left and said 10 would be for the desk and stuff and 10 would be for clothes. If you break 40 into four parts you get 10 + 10 + 10 + 10 so 10 must be one-fourth of the whole.
- solution 2 –> the room is 4 x 10 rather than 5 x 8 but still shows half as 20 and half of half as 10.
- solution 3 –> considers half of 40 as 20 then half of the 20 left is 10 and 10 is left for clothes – then reasons that 10 is one-fourth of 40 because 10 + 10 + 10 + 10 = 40.

LET'S DO MATH – CHESTNUTS PROBLEM

Before (Getting Started)

The chestnuts are falling from a tree next door. Abe, Barb, and Connor collected 24 chestnuts. Connor collected 1/6, Abe 1/4 and Barb 1/2 of the chestnuts. Who collected the most? How do you know?

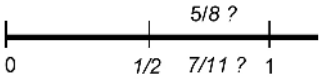
- Provide students with counters, square grid paper, or other concrete materials that can be used to represent the chestnuts as a set of objects in a group or array
- Record students' responses using same size groups or an array and numerical and word expressions such as: fractions, division, 1 group out of 6 groups has 4 chestnuts.

During (Working On It)

Emilio has finished 7/11 of their classroom homework. Jenny has completed 5/8 of their classroom homework. Who has completed more homework?

After (Consolidation)

Anticipating Student Responses:

Solution 1	**Solution 2**	**Solution 3**
 *Half of 7 is 3.5, so 7/11 is greater than 1/2.* *Half of 5 is 2.5, so 5/8 is greater than 1/2.* *Jenny has done more.* *3/4 of 11 is 8.25* *3/4 of 8 is 6* *But this does not help me. I do not know.*	*If I draw a grid and show 11 rows and 8 columns, I will have 88 spaces.* *7/11 x 8/8 = 56/88 and 5/8 x 11/11 = 55/88* *So 7/11 is the greater number because 56 > 55 and the denominators are the same.* *Emilio has finished a little bit more than Jenny.*	*Emilio has 3 of 11 parts to go and Jenny has 3 of 8 parts to go.* *3/11 is less that 3/8 because if you divide the same number into more parts, each part is smaller.* *So if 3/11 < 3/8 then 8/11 must be greater than 5/8. 7/11 > 5/8* *Emilio has finished more than Jenny.*

Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 –> shows student trying to compare to benchmark fractions focusing on the numerator only.
- solution 2 –> shows student comparing 7/11 and 5/8 by folding paper the make 88 spaces. They then determine that 7/11 is equivalent to 56/88 and 5/8 is equivalent to 55/88 so 8/11 > 5/8 by making a grid with 88 spaces involves the same mathematics as getting a common denominator of 88 and writing equivalent fractions with that denominator.
- solution 3 –> has dealt with the remaining fraction of homework each one has to complete his or her set. Emilio has 3/11 left to finish and Jenny has 3/8 left to complete. Because numerators are the same, can say 3/11 < 3/8 so the other part of the fraction, 8/11 must be less than 5/8.

For Professional Discussion:

What if the Chestnut Problem was written this way:

"Emilio has finished 7 out of 11 homework assignments. Jenny has completed 5 out of 8 homework assignments. Who has completed more homework?"

Would the anticipated solutions shown above still be relevant for this version of the problem? How do you know?

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these fraction comparison problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., common fractions, same denominator/different numerator; same numerator/different denominator, improper fractions, and mixed numbers) and/or the context of the problem.
- Think about how the context of the problem evokes particular ways to compare and order fractions (e.g., same denominator/different numerator, different denominator/same numerator, benchmarks of 0, 1/2, 1)
- Provide students with non-examples of comparing and ordering fractions and have students explain why the comparison is inaccurate. For example, Jon says that 3/5 is smaller than 4/6. Kim disagrees with John. What do you think? Explain using a concrete model.
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number of solutions that show strategies for comparing and ordering fractions. Which model of fractions (i.e., region, set, linear measurement) do students use more often?
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third. Refer back to the developmental learning sequence for comparing and ordering fractions.
- Think ahead of what mathematical annotations you will record on and around the student solutions to render explicit key mathematical ideas, strategies, and terms of comparing and ordering fractions as the students discuss and analyze each other's solutions.
- Choose 4 student work samples to analyse further in terms of their mathematical details.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Baroody, A. (1999). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction.* NJ: Lawrence Erlbaum Associates.
- Cramer, K., & Henry, A. (2002). Using manipulative models to build number sense of additon of fractions. In B. Litwiller & G. Bright (Eds.) *Making sense of fractions, ratios, and proportions* (pp. 41-48). Reston, VA: National Council for Teachers of Mathematics, Inc.
- Randall, C. (2005). Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics. *Journal of Mathematics Education Leadership,* (7)3, 9-24.
- Small, M. (2009). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics, Grades 4 - 8.* Toronto, ON: Nelson Educational Ltd.
- Smith, J. P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller and G. Bright (Eds.) *Making sense of fractions, ratios, and proportions* (pp. 3-17). Reston, VA: National Council for Teachers of Mathematics, Inc.
- Van de Walle, J., & Lovin, L. (2006). *Teaching student-centered mathematics, Grades 3-5.* New York: Pearson Education, Inc.

CLOSING

Our next issue will focus on key notions of fractional equivalence, how teachers need to know these notions for teaching, as well as, ways that teachers can use these mathematical ideas and pedagogical strategies through problem solving-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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