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INSERT
ELEMENTARY
MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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ORGANIZING AND READING DATA

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ABACUS CO-EDITORS



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We collect data from the real world and analyze them in order to find out something about our world. In fact, data are often collected to answer questions and support or challenge hypotheses and arguments. However, because the amount of accumulated data can be overwhelming. It must be organized. The average (or mean), the median and the mode are three ways to organize data as measures of central tendency. Data can also be arranged and presented in charts, graphs or tables. Different charts are used for different purposes. From these charts, graphs or tables, a quick look can determine the answer to many questions. As you read data, ask yourself, What is the author's motivation for producing the diagram or chart? How does it support or refute an argument? What is its main message? Therefore, reading data is way to gather information in order to answer questions, to see relationships between sets of data and to raise further questions. So, when students read data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, topics in various subject areas, and so on. In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for reading quantitative and qualitative data. The problems we provide are designed for use within a three-part, problem-solving lesson.

LINKS TO LITERATURE AND MANIPULATIVES



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Venn diagrams, graphs, tables, and charts are tools that can be used in both mathematics and literacy contexts. Students can use these tools to write reports in subjects such as science, health, social studies, history, and geography. In addition to crossing subject areas, these tools also connect strands of the math curriculum. Students can use blocks to create patterns, then collect data about the number of blocks in the pattern, organize the data into a table of values, graph the data and then generalize the relationships algebraically. Students can collect data about the dimensions and areas of rectangles, organize the data in a table, graph the data, and generalize the relationships in measurement formula. Manipulatives such as found materials, fraction circles (for circle graphs) the Hundreds Carpet™, and pocket charts can be used to represent data graphically. In this issue we will look at the use of linking cubes. We will also look at using linking cubes to respond to some inquiries based on some ideas in two picture books.

RESEARCH SUMMARY

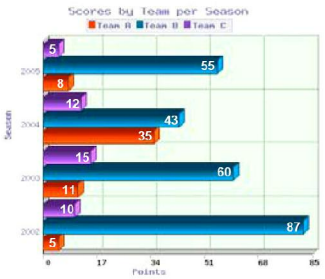
RESEARCH SUMMARY

Randall (2005) explains that “some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it. Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation” (pp. 20-21). An effective graph should communicate at a glance some overall information about the data. Graphs typically used in elementary classrooms, include:

Questions Completed

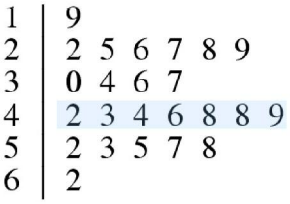


pictograph



bar graph

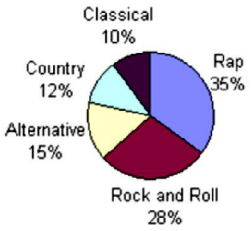
Number of Tags Collected



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3	0 4 6 7
4	2 3 4 6 8 8 9
5	2 3 5 7 8
6	2

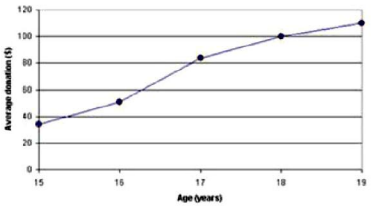
stem and leaf plot

Music Preference



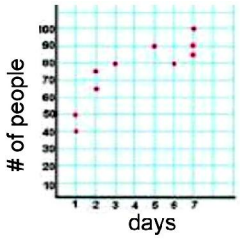
circle graph

Donations for Flood Relief



line graph

People Attending Show



scatter plot

When looking at the information contained within a graph, it is useful to consider: the title of the graph, the labels on the axes, the scale, or scales, used, the source of the data the size of the samples used (if appropriate). Ask students why this information is important to consider. More specifically, Small (2010) notes that students should be able to read individual specific details of the data from graphs, as well as work with multiple details. The kinds of conclusions that students might draw involve comparisons among data, such as which category has the greatest or least value, or which category has more data values than the other (i.e., pictographs, bar graphs, stem and leaf plot, circle graph). Line graphs and scatter plots are read to see trends and relationships between the variables on each axis.

Instructional Strategies

Russell (2006) explains that “at all grade levels, students need support to interpret their data representations in a way that helps them relate the data collected back to the context and to the actual event or information the data represent. Teachers can help students move away from reading graphs as if they are a collection of symbols towards using data to answer questions (pp. 27-28). Some teaching strategies include:

- having students develop their own representations from data they collected so that they experience connecting the symbols they use in familiar contexts
- developing with students, questions about comparing groups so that students have a reason to characterize each set of data as a whole
- encouraging students to notice where data is concentrated (e.g., About how much of the data set is part of that cluster? Is it more or less than half of the data? About what fraction of the data is it?)
- carrying out quick, fifteen minute data reading activities, asking questions like, “What do the data we collected and represented tell us about what we wanted to know?”

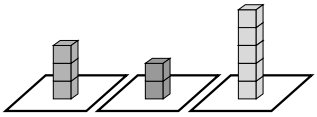
Whitin (2006) suggests these strategies for developing students’ as critical readers of data and measures of central tendency, such as:

- prompting students to question numerical information to consider and expose the intentions implicated in the data
- connecting data collection and analysis to the interests and background of the students, as they can be better critics if the data is tied to familiar contexts
- have students practice creating categories of the data to become more aware of the limits of their visual display
- asking questions like, “Who benefits from using the data in this way?” as it underscores the idea that people use data for a reason
- involving students in gathering data from different samples (e.g., adults, children; older children, younger children), so that they can compare the results. How are the data sets the same and different? What accounts for these similarities and differences?

LINKS TO MANIPULATIVES: LINKING CUBES

Linking cubes can be useful in making the transition from concrete graphs (K—1) to bar graphs, and for collecting discrete data (grade 2).

The class could consider an inquiry, such as, “What colour are your mittens?” Students can create a concrete graph with their mittens or respond to this survey by choosing a linking cube to represent the colour of their mittens. Posing questions such as which colour is most common can be extended to consider reasons for each frequency. (e.g., why don’t many parents buy white mittens for their children – too difficult to keep clean, they are harder to see in the snow if they come off). Students can extend this inquiry to investigate if there is a colour of mitten that is most common in the lost and found. Students can also collect data to investigate the changes in the number of mittens in the lost and found over time, compare the number of mittens to the number of gloves, collect data about where mittens are lost (inside the school and outside of the school) and compare the sizes of the mittens that are lost.



“How many pockets do you have?”, is a question that has appeared frequently in lesson plans. The teacher may ask students how they can keep track of the pockets as they count them. Students can put one linking cube in each of his/her pockets and then count the cubes. A variety of questions could be posed: are there differences in the number of pockets on clothes made for boys/girls or are there differences in the number of pockets on clothes in different cultures?

LET’S DO MATH

Before (Getting Started)

Data from Frequency Tables: Problem A

What conclusions can you draw from the data in this frequency table?



Solution 1

24 people answered the survey.

The highest number of people like snowboarding and that was about 20%.

20% of the people like sports with balls.

Solution 2

9 people like winter sports (snowboarding and hockey)

That is 9/24 about 38%.

Equal numbers of people picked skateboarding, soccer, and other.

The least popular were drama and badminton.

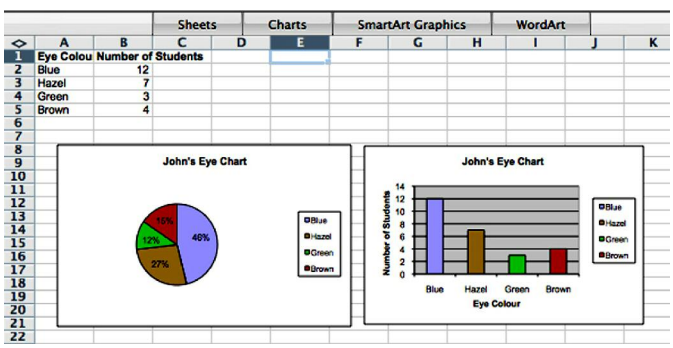
During (Working On It)

Frequency Tables and Graphs: Problem B

This survey was done in Australia. Data about people’s eye colour is presented in three forms:

- A frequency table (in EXCEL)
- A circle graph
- A bar graph

Which presentation of data is more useful?



LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

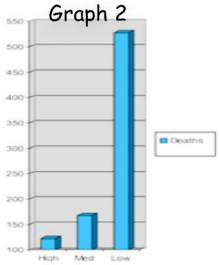
BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

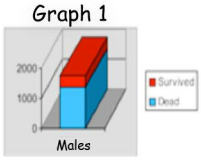
Perceptions of the Titanic Tragedy

Four Titanic survivors made graphs to represent their reactions to the event. Which comment matches which graph?
Person A: "Lots of people were saved."
Person B: "Most of the people who died were poor."
Person C: "The males were brave."
Person D: Working on ships is not safe."

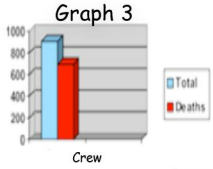
Graph 2



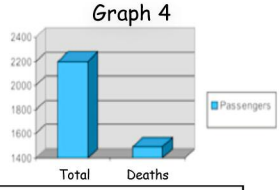
Graph 1



Graph 3



Graph 4



Solution 1

I think Graph 4 matches Person A comment. It shows the total passengers compared to the number who died.

Solution 2

Person A – Graph 4
Person B – Graph 2
Person C – Graph 1
Person D – Graph 3

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Oh! They Built the Ship Titanic Problem

How are the tables the same? Different? Analyze the data in the table and write statements about what you find. Support your statements with data.

Data Organized by Economic Status and Gender						
Economic Status	Number of people on the Titanic			Number of deaths resulting from the sinking of the Titanic		
	Male	Female	Total	Male	Female	Total
High Income	180	145	325	118	4	122
Middle Income	179	106	285	154	13	167
Low Income	510	196	706	422	106	528
Other	862	23	885	670	3	673
Total	1731	470	2201	1364	126	1490

Data Organized by Economic Status and Age						
Economic Status	Number of people on the Titanic			Number of deaths resulting from the sinking of the Titanic		
	Adult	Child	Total	Adult	Child	Total
High Income	319	6	325	122	0	122
Middle Income	261	24	285	167	0	167
Low Income	627	79	706	476	52	528
Other	885	0	885	673	0	673
Total	2092	109	2201	1438	52	1490

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

-the first two instances and in the third statement chose incorrect numbers to compare
- true that all the children who died were from low income families
-not all the children from low income families died (that would be 52/79=66%).

Solution 1

Same	*Different*
-People who died -Same totals -Rich, middle poor	*-One by gender -One by age*

We do not know who the "others" were – maybe workers.

Solution 2

-More men died than women 1731 – 470 = 1261
-More low income men died than high income men 422 – 118 = 304
-"All the low income children died 52/52."

Identifies Similarities and Differences

Absolute Differences

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
-read, interpret and draw conclusions from secondary data presented in charts, tables and graphs (including stem-and-leaf plots and double bar graphs)	-compare similarities and differences between 2 related sets of data using a variety of strategies	-demonstrate an understanding of how data from charts, tables and graphs can be used to make inferences and convincing arguments

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

1364/1490
ratio of number of women compared to total passengers

126/1490
ratio of number of men to total passengers

-easy to read chart that compares the correct numbers for two specific cases and draws a conclusion

Highlights/Summary:

Compare similarities and differences between 2 related sets of data by:

- *identifying similarities and differences*
- *calculating absolute differences in the data values of the sets of data*
- *calculating relative differences (using percents) in the data values of the 2 sets of data*
- *comparing specific cases to support argument*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Solution 3

1238 more men died than women: 1364 to 126

$\frac{1364}{1490} = 92\%$

$\frac{126}{1490} = 8\%$

All the children who died were from low income families died 52/52.
And 66% of the children from low income families died. $\frac{52}{79} = 66\%$
If you are poor. don't take your children on boats.

Solution 4

Both tables show the numbers who died on the Titanic; one shows the data by gender and the other by age.

Comparisons: number died / number on board

	Males	*Females*
High income	$\frac{118}{180} = 66\%$	$\frac{4}{145} = 3\%$
Middle income	$\frac{154}{179} = 86\%$	$\frac{13}{106} = 12\%$
Low income	$\frac{422}{510} = 83\%$	$\frac{106}{196} = 54\%$

"Middle and low income males should be careful when deciding to sail on a cruise ship. More than 9/10 of those on the Titanic died."

Relative Comparisons

Compares Cases and Draws Conclusions

Who Uses Computers at Home? Problem

Compare the data about computer use at home. What conclusions could you draw from the information in the charts?

Raw Data

	Sex		
UseComp	Female	Male	Total
Yes	45	45	90
No	2	8	10
Total	47	53	100

Percentage Data

	Sex	
UseComp	Female	Male
Yes	96	85
No	4	15
Total	100	100

Solution 1

Chart 1 shows the same number of males and females using a computer at home; Chart 2 says more women do - about 4 X as many men as women do not use computers at home.

Solution 2

Compare the number of women who use computers at home to the total number of women who responded to the survey →45/47 = 0.957 = 96%; For men 45/53 = 0.849 = 85%
Percent numbers (on the right) mean the whole population; about 10% more females use a computer at home than males.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

Anticipating Student Responses:

Solution 1

From the frequency table, I can see that 26 people answered the survey question.

The circle graph and the bar graph show a majority has blue eyes.

The circle graph suggests almost 50% of the people have blue eyes. I like the circle graph best. The bar graph just shows numbers like the frequency table.

Solution 2

All three representations show that blue eyes are most common. I do not think that would be true if we considered data from Ontario.

The circle graph shows the whole as 100% of the people and that really emphasizes that about 50% have blue eyes and the other 50% have one of 3 colours (hazel, brown or green).

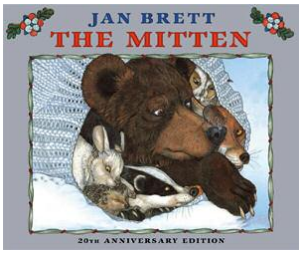
From least to most common: green, brown, hazel, blue.

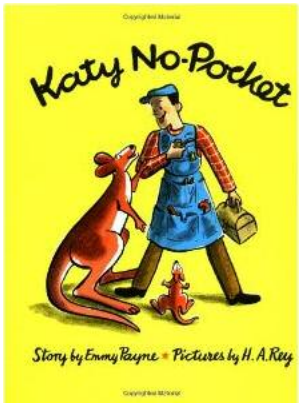
Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- Solution 1 reports on the graphs and the frequency table and the student says s/he likes the circle graph more but does not share reasoning
- Solution 2 talks about the most common occurrence of blue eyes and uses the circle graph to report on the relationships of blue being about half and the other three being the other half. The student refers to the circle being 100% or the whole. This student also comments on the fact that the data is from Australia and that it may be a different picture if it were collected in Ontario.

LINKS TO LITERATURE





The Mitten is a Ukrainian folktale that is available in a few versions as a picture book, one of which has been written by popular author Jan Brett. In this story, Nicki drops his white mitten in the snow, and goes on without realizing that it is missing. One by one, woodland animals find the mitten and crawl in; first a mole, then a rabbit, a badger and others, each one larger than the last. Finally, a big brown bear is followed in by a tiny brown mouse, which is, ironically, when the mitten explodes. Students can inquire into the question, “How big is your mitten?” by using linking cubes to measure the length, the width, and the capacity of their mittens. The cubes can then be organized graphically in order to respond to the original inquiry question (e.g., some materials can stretch to hold a lot of cubes and some cannot).

Katy No-Pocket by Emmy Payne is a classic book that can be used to investigate differences. Mother Katy Kangaroo doesn't have a pocket in which to carry her son Freddy like all the other mother kangaroos but other animals carry their babies in different ways. A wise owl sends Katy to the city to find a solution and a kind carpenter gives her an apron with many pockets. She uses her new pockets to not only carry her own baby but all the other babies too. Students can collect data to inquire about how many pockets are in their clothing (do all coats have pockets?), how do they use their pockets (what is in your pocket?) and how big is your pocket (how many cubes will your pocket hold?).

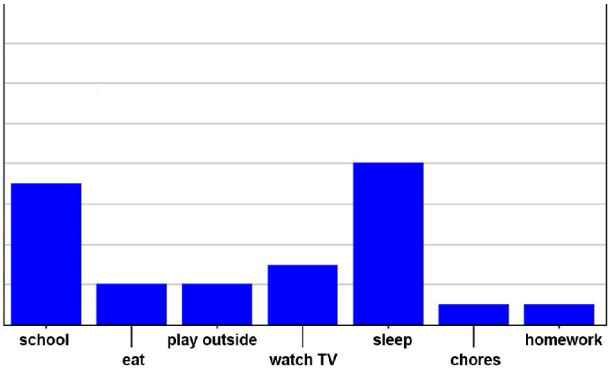
LET’S DO MATH – SCALES ON GRAPHS

Before (Getting Started)

Tracking Time Problem: Problem A

What does this graph tell you about the way students respond to the question, “On average, how many hours do you spend: sleeping, eating, studying, at school, playing outside, watching TV, doing chores, and working on homework?

How many hours do you spend on … ?



Solution 1

It says that students spend the most time sleeping and the next bundle of time at school. They spend the least time doing chores and on homework.

Solution 2

I speculate that each horizontal bar on the graph represents 2 h. Because when I use that idea to count I get 24 h and that is the total number of hours in a day

School	7	Sleep	8
Eat	2	Chores	1
Play outside	2	Homework	1
Watch TV	3	Total	24

During (Working On It)

Household Pets Problem: Problem B

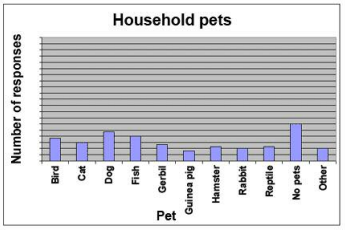
Jayden, Louis, and Gayle gathered the data randomly from 103 students in their school. What do their graphs suggest about the distribution of pets in the households of their peers?

The labels for the pets (on the horizontal axis) are in the same order as the names of the pets in the data table.

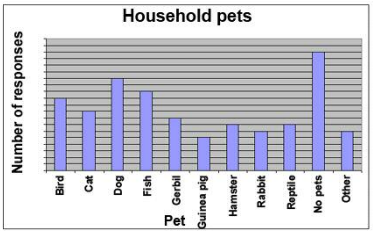
Pet	Number of responses	Pet	Number of responses
Bird	11	Hamster	7
Cat	9	Rabbit	6
Dog	14	Reptile	7
Fish	12	No pets	18
Gerbil	8	Other	6
Guinea pig	5		

After (Consolidation)

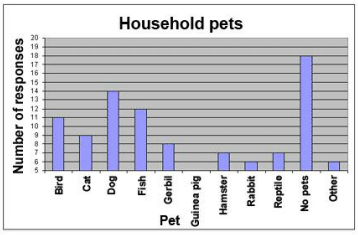
Gayle's graph



Louis's graph



Jayden's graph



Anticipating Student Responses:

Solution 1

Louis and Gayle's graphs look almost the same but Gayle's shows zero Guinea pigs. Her scale starts at 5 and counts by 1s. She should show that there is a break in her vertical scale.

Louis' must count by 1s too but starts at 0 because you can see 5 Guinea pigs counted.

Solution 2

Jayden and Louis did not show scales on their graphs. Louis seems to have counted by 1s. Jayden must have counted by 3s because "No pets" that supposed to show 18 is 6 marks high (18/6=3).

Louis' scale lets the differences show better because there is more differences in the heights when you count by 1s .

Solution 3

You have to have a scale on both axes in order to read information from a graph. However, all graphs show the relationships among the kinds of pets students reported. The most students reported no pets and then the most popular are dogs, fish, and birds.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- *Solution 1* reports on the problems in two of the graphs – highlighting the importance of recording a scale on both axes.
- *Solution 2* suggests scales for two of the graphs without report or compare the data.
- *Solution 3* reports on the relationships including the highest four categories without commenting on the scales.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these data problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with reading different sorts of data and different representations of data, such as data presented in advertisements that compare costs, news articles that show trends in growth or relationships between variables and data collected for local and school community events and issues>
- Think about how the context of the problem evokes the purpose and use of reading data (e.g., progress of litterless lunches in a school; comparing school absences just before the weekend and in the middle of the week; relationship between hours spent on homework and grades).
- Practise noticing the different strategies and the breadth of the mathematics your students use to read data. Take note of the number and kinds of solutions that show strategies for reading data (e.g., identifying greatest and least data value; compare data values among a set of variables; examining the effect of outliers or the addition or the omission of particular data values)
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for reading data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Chapin, s., Koziol, Al, MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades 3-5.* Reston, VA: NCTM.
- Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. J*ournal of Mathematics Education Leadership, 7*(3), 9-24.
- Graham, A. (2006). *Developing thinking in statistics.* Thousand Oaks, CA: Sage Publications.
- Russell, S. J. (2006). In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (pp. 17-29). Reston, VA: NCTM.
- Shaughnessy, J. M. (2006). Research on students' understanding of some big concepts in statistics. In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (pp. 77-98). Reston, VA: NCTM.
- Sheffield, L. et al. (2002). *Navigating through data analysis and probability in prekindergarten-grade 2*. Reston, VA: NCTM.
- Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.
- Small, M. (2010). *Big ideas from Dr. Small, Grades 5-8*. Toronto: Nelson Education.
- Whitin, D. (2006). Learning to talk back to a statistic. In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (31-39). Reston, VA: NCTM.

CLOSING

Our next issue will focus on key notions of reading and comparing graphs to make inferences (e.g., to tell a story, to provide evidence, to construct a message or argument), how teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca