8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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REPRESENTING AND INTERPRETING PROBABILITY

TABLE OF CONTENTS

Abacus Editor Greetings 1
RESEARCH SUMMARY 2
Representing and Interpreting Probability
LINKS TO LITERATURE 3
- *Jumanji*
LET'S DO MATH 4-5
(Bansho (Board-Writing) Plan)
- One Die, Two Dice
LET'S DO MATH 6
- Two Choices plus Dishwashing
LINKS TO MANIPULATIVES . . . 7
- Colour Tiles and Spinners
LET'S DO MATH 7
- Predicting Likelihood
NEXT STEPS FOR YOUR PROFESSIONAL LEARNING. 8
- Suggested Readings

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Is there anyone who hasn't wished that they could win a provincial lottery? It is often said that we have the same chance of winning the lottery as the chance of being struck by lightening. If only a few people are hit by lightening, then only a few people win the multi-million dollar lottery prize. If that is the case, why do we buy lottery tickets? These two examples of probability are two examples of very unlikely events; that is, an event that probably won't happen. Weather forecasts are another way we use probability every day. What does it mean when the probability of precipitation (POP) is 50%? It may mean that there is 0% chance of rain in one part of the city and 100% chance of rain in another part of the city. A 50% chance of a rainstorm covering the entire city would also lead to a POP of 50%. Probability influences our lives every day. In fact, anyone who thinks and plans for their future uses probability in their planning process (e.g., deciding what the chances are of a specific event) In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for representing probability situations theoretically and through experiments and simulations. The problems we provide are designed for use within a three-part, problem-solving lesson structure.

LINKS TO LITERATURE AND MANIPULATIVES

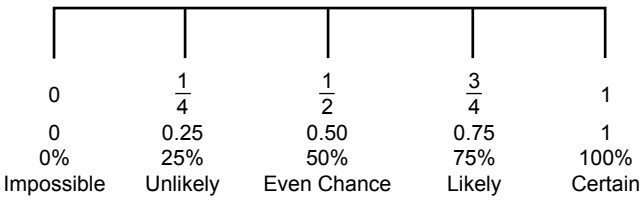


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As another school year comes to a close, it is likely that many of you are making plans for the summer based on the probability that the weather will be sunny and hot. Students also have experiences with probability. Watch a child opening presents and you might notice that they are making predictions about the contents. "This gift probably has new clothes in it." Also, probability provides an opportunity to explore the concept of fairness in a variety of social justice issues. In this issue I will take yet another look at colour square tiles in the context of probability, using a classic picture book as a context for solving a problem involving probability.

RESEARCH SUMMARY – REPRESENTING AND INTERPRETING PROBABILITY

Probabilities are associated with situations where the outcome is not known in advance or cannot be predicted. It is the study of uncertainty. Further, "probability is the study of measures of likelihood for various events or situations" (Small, 2009, p. 197). For younger students, probability language includes descriptions of situations, such as certain, impossible, and/or somewhere in between (like or unlikely). Also, young students could describe events using probability language like, always happens, likely to happen, unlikely to happen, never happens. Older students relate such probability descriptions to quantities (decimals, fractions, percentages) across a scale from 0 to 1, inclusive. An event that cannot occur has a probability (of happening) equal to 0 and the probability of an event that is certain to occur has a probability equal to 1.



Sample Space

In order to quantify probabilities, we need to define the sample space of an experiment and the events that may be associated with that experiment. The sample space is the set of all possible outcomes in an experiment. For example:

- If a die is rolled, the sample space S is given by $S = \{1,2,3,4,5,6\}$
- If two coins are tossed, the sample space S is given by $S = \{HH, HT, TH, TT\}$, where H = head and T = tail.
- If two dice are rolled, the sample space S is given by
  $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6);\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\ ;$
  $(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\ ;\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\ ;$
  $(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\ ;\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

An event is a subset of the sample space; a specific outcome of an experiment. For example:

- A die is rolled. Event, E is the set of possible outcomes where the number on the face of the die is even. $E = \{2,4,6\}$
- Two coins are tossed. Event, E is the set of possible outcomes where there is exactly one Head. $E = \{(HT),(TH)\}$
- Two dice are rolled. Event, E is the set of possible outcomes where the sum of the numbers on the faces of the two dice is equal to four. $E = \{(1,3),(2,2),(3,1)\}$

Theoretical Probability

Theoretical probability is based on the fact that all outcomes are equally likely. For example:

$$P(A)=\frac{n(A)}{n}$$

$P(A)$: probability of event A

$n(A)$: number of occurrences of A (number of favourable outcomes)

n: Total number of possible outcomes (sample space)

- What is the probability of getting a 3, when a die is rolled? The event is "getting a 3" is $E = \{3\}$. Sample space, $S = \{1,2,3,4,5,6\}$.
  The number of possible outcomes in E is 1 and the number of possible outcomes in S is 6. Probability of getting a 3 is $P(3) = 1/6$.
- What is the probability of getting an even number when a die is rolled? The event "getting an even number" is $E = \{2,4,6\}$. Sample space is $S = \{1,2,3,4,5,6\}$.
  The number of possible outcomes in E is 3 and the number of possible outcomes in S is 6.
  The probability of getting an even number, P(even number) = $\frac{3}{6}=\frac{1}{2}$.

Tree diagrams, area models, and tables are used to organize and determine theoretical probability.

Experimental Probability

Unlike theoretical probability, experimental probability uses real data from situations to determine how likely outcomes will occur in the future. For example, here is some data about people's colour preferences. If a person is selected randomly from the group of 20 people what is the probability that this person likes green? So, Event, *E* is "likes green". The probability of choosing a person who prefers the colour, green is represented as a fraction or ratio of the number of green outcomes and the total number of outcomes.

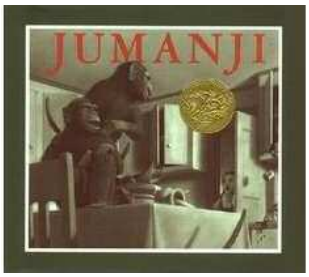
Color	Outcomes
red	6
blue	6
green	8
Total	20 (sample size)

P(*E*) or *P*(likes green) = number of green/total number of outcomes

P(*E*) or P(likes green) = $\frac{8}{20} = \frac{2}{5}$

According to Small (2009) "experimental probability is determined by the results of an experiment that has already occurred. In contrast, theoretical probability involves analyzing possible outcomes in advance and using logic and reason to predict what is likely to happen" (p. 203). The results of experimental probability approaches a theoretical probability when enough random samples are used; that is, with large enough sample spaces.

LINKS TO LITERATURE: *JUMANJI*



In the book, *Jumanji*, written by Chris Van Allsburg, in 1981, Peter and Judy find a game under a tree. They play the game and with each roll of the dice, the events described by the game board begin to materialize around them.

You may stop the story at various points to investigate the probability of rolling the number identified in the story. For example, read up to the point where Peter says, "If you roll a 12, you can get out of the jungle." Challenge the students to plan an investigation that determine the possible outcomes of rolling a sum of 12.

Strategy #1 – Cross Classification

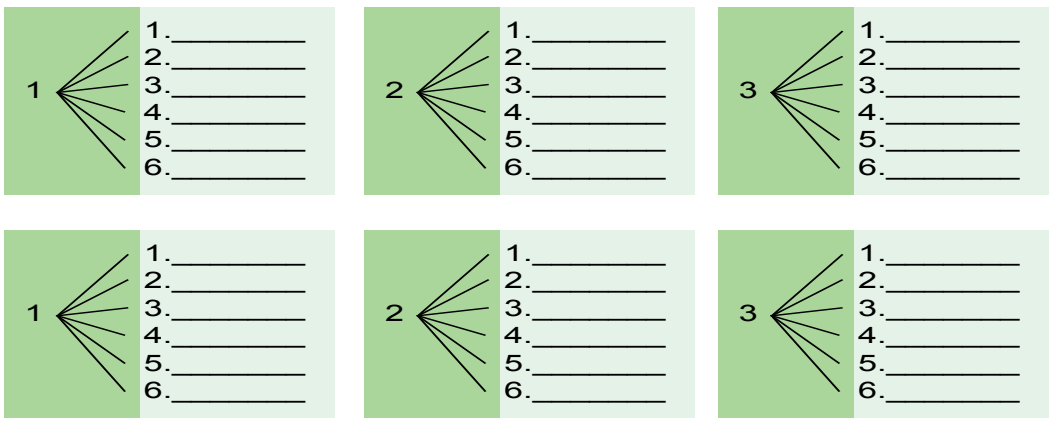
+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Strategy #2 – Concrete Graph

2	3	4	5	6	7	8	9	10	11	12
1+1	1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
	2+1	3+1	4+1	5+1	6+1	6+2	6+3	6+4	6+5	
		2+2	2+3	2+4	2+5	3+5	4+5	5+5		
			3+2	4+2	5+2	5+3	5+4			
				3+3	3+4	4+4				
					4+3					

Possible sums from two dice

Strategy #3 – a tree diagram



The book, *Jumanji*, was made into a movie in 1995. Watch the trailer on YouTube **http://www.youtube.com/watch?v=OJKHQLM8AbM** and investigate the probability in the following line from the trailer, "In the jungle you must wait until the dice say five or eight."

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

One Die, Two Dice Problem:

Consider the faces of a die. How many times should you expect a 5 to show, when you roll one die 30 times?



Solution 1

Rolled the one die and predict four 5s.

1	2	3	4	5	6
𝍸	𝍸	𝍸	𝍸	𝍸	𝍸

Solution 2

- *Equally likely chance of getting any number 1, 2, 3, 4, 5, or 6.*
- *30 rolls, get five 1s, five 2s, five 3s, five 4s, five 5s, and five 6s.*

***Probability Experiment of 30 Rolls** – close but not exactly the same*

***Equally Likely** - each face has equal chance of coming up on each die roll - 30 rolls ÷ 6 faces = 5 times for each face*

DURING (Working on It) – 15 to 20 minutes

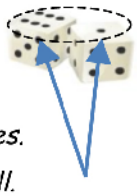
- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Predicting with Dice Problem:

If you roll two dice at least 10 times, how many times would you expect a 5 to appear?

a) Make a prediction. Explain.

b) Keep track and record your results.



What information are we going to use to solve the problem?

- *Roll 2 dice at the same time, at least 10 times.*
- *Keep track of dice combinations for each roll.*
- *Sum of dice (e.g., 1 die and 6 die -> sum of 2 dice is 7)*
- *How many times will a 5 will come up?*

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Experimental Probability

- ***Collect Data** - Record the number (frequency) of each dice sum for over 36 dice rolls (total number outcomes)*
- *Use data from experiment to determine "how likely a dice sum of 8" will come up after 36 dice rolls*
- ***Sample Space**- total possible outcomes in experiment - number of dice rolls is 36*
- ***Probability** of dice sum of 8 - using data collected*
- ***Event, E** is "dice sum of 8" (favourable event) using d*

P(E) or P(sum of 8) = number of sum 8/total number of outcomes = 5/36

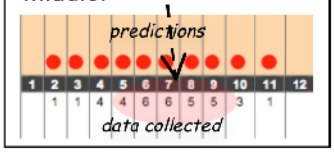
Predicting Outcomes

- *all numbers on the face of a die have equal chances*
- *different combinations 2 dice that sum to 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12*
- *more combinations for sums of 6, 7, 8 shown in both the data experiment OR by listing all possible sums*

Solution 1

Our prediction was really off. We expected all sums to be equally likely but it does not look like this is correct.

The outcomes are bunched up around the middle.



Solution 2

Sums	How	# times got it	Sums	How	# times got it
1	never	0	7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1	5
2	1+1	2	8	4+4, 3+5, 2+6, 6+2, 5+3	5
3	1+2 2+1	2	9	5+4, 4+5, 3+6, 6+3	4
4	2+2, 1+3, 3+1	3	10	5+5, 4+6, 6+4	1
5	1+4, 4+1, 2+3, 3+2	5	11	5+6, 6+5	1
6	3+3, 2+4, 4+2, 1+5, 5+1	7	12	6+6	1

Predicted All Sums Equally Likely Compared to Collected Data

Predicted All Sums Not Equally Likely - Listed All Possible Sum Combinations

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
- determine and represent all the possible outcomes in a simple probability experiment - represent using a common fraction, the probability that an event will occur in simple games and probability experiments	- express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely - predict frequency of outcome of simple probability experiment or game calculating and using theoretical probability of that outcome	- make predictions about a population given a probability - represent in a variety of ways all possible outcomes of a probability experiment involving 2 independent events

Theoretical Probability

- all outcomes for numbers (1 to 6) on dice are equally likely, BUT sums of dice rolls are not equally likely - some sums have more combinations of dice

- list all possible outcomes in a table or on a bar graph

- represent theoretical probability of an event (rolling dice sum 7) using fraction → 6/36

$$P(A) = \frac{n(A)}{n}$$

number of occurrences of A (number of favourable outcomes) → 6

probability of event A

total number of possible outcomes (sample space)

- all possible combinations of sums from 1 to 12, using 2 dice → 36

36 = 1+2+3+4+5+6+5+4+3+2+1

= 6+5+1+5+1+4+2+4+2+3+3

= 6+6+6+6+6+6 = 6x6 = 36

Probability of rolling dice sum of 7 = 6/36

Solution 3

Our prediction looked like group 1. But, we could not get 1 but we could get 12. So we started looking at what dice gave what sums. 5 comes up 4 times.

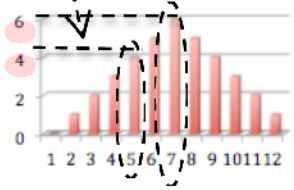
Sums	Total ways to get it	Probability
1	Impossible	0
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Comparing Predictions to All Possible Combinations of Sums

Solution 4

We tallied and graphed it. Getting 1 has a probability of zero as it can never occur with 2 dice.

The probabilities go up from impossible to most likely at sum of 7 and then back down to unlikely at 2, 3, 11, and 12. 5 comes up 4 times.



Graphing All Possible Combinations of Sums (Mountain)

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary:

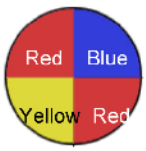
- ***Probability*** *describes how likely an event will occur.*
- ***Experimental probability*** *is data collected as the frequency of events occurring.*
  - *→ If rolling 2 dice 36 times (sample space), how often does the sum of 7 come up? Table shows it came up 6 out of 36 dice rolls → Experimental Probability (sum of 7)=6/36*
- ***Theoretical probability*** *is a ratio of the number of possible occurrences of a specific event in relation to total number of possible outcomes (sample space).*
  - *-> For a combination of sums using 2 dice, there are 36 different possibilities (sample space): for sum of 7 - there are 6 possibilities → Theoretical Probability(sum of 7)=6/36*
- ***Is the experimental probability always the same as the theoretical probability?***

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Coin Flipping Problem

What is the probability of spinning a red and flipping a head?





Solution 1

P(red) is $\frac{1}{4}$ and P(H) = $\frac{1}{2}$

So the probability of Red followed by heads is 1/8

Solution 2

Probability (RH) = $\frac{2}{8} = \frac{1}{4}$

Colour	Coin		Colour	Coin	
R	H	RH	B	H	BH
	T	RT		T	BT
R	H	RH	Y	H	YH
	T	RT		T	YT

Before (Getting Started)

Two Choices Problem:



Rosa just got married and is starting to think about having a baby. Her dream is to have one boy first and then a girl. What is the probability that she will have the children in the order she hopes for?

Solution 1

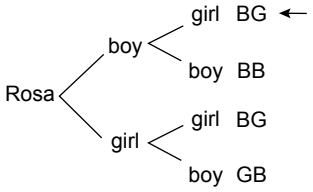
The likelihood that she will have a boy is half.

The likelihood that she will have a girl is half.

So the probability that Rosa will have a boy and a girl is one-half.

Solution 2

The probability that Rosa will have a boy followed by a girl is ¼ or one/fourth.



During (Working On It)

Dishwashing Problem:

Janine has three children two girls, Anise and Lada, and one boy, Ephrim. After breakfast, lunch and dinner, one child is chosen at random to do the dishes.

What is the probability that Anise will have to do the dishes in 2 of the 3 meals in a day?

After Consolidation

Solution 1

We could have AAA, AAL, AAE, ALA, AEA.

So the probability of Anise at two meals is 5 favourable outcomes.

Solution 2

Probability (A in 2 meals) = $\frac{7}{27}$.

B	L	D	DAY
		A	AAA
A	L	L	AAL
		E	AAE
		A	ALA
A	L	L	ALL
		E	ALE
		A	AEA
E	L	L	AEL
		E	AEE

B	L	D	DAY
		A	LAA
A	L	L	LAL
		E	LAE
		A	LLA
L	L	L	LLL
		E	LLE
		A	LEA
E	L	L	LEL
		E	LEE

B	L	D	DAY
		A	EAA
A	L	L	EAL
		E	EAE
		A	ELA
E	L	L	ELL
		E	ELE
		A	EEA
E	L	L	EEL
		E	EEE

Solution 3

We could have AAA, AAL, AAE, ALA, AEA, EAA, and LAA. So the probability of Anise at two meals is 7 favourable events out of 27 possibilities (3 x 3 x 3) or P(A in 2 meals) = $\frac{7}{27}$.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- *Solution 1* – Students identified some but not all of the favourable outcomes
- *Solution 2* – Students use the table to list all the possible outcomes – shows total favourable and total sample space.
- *Solution 3* – Students list favourable events and show how to calculate the total number of outcomes in the sample space.

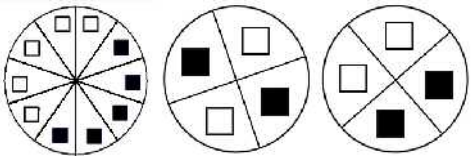
LINKS TO MANIPULATIVES: COLOUR TILES AND SPINNERS

"If you take a white tile out of the bag, you win a prize. Which bag will you choose? Why?"

Students who think additively may choose Bag A because it has twice as many white tiles as Bag B. Students who think multiplicatively may see that both bags are half white and half black.



Have students take and replace the tiles in each bag and keep track of the colours that they take. Then have students use the square tiles and fraction circles to create spinners.If the spinner points to a white square, you win a prize. Compare the results of spinning a spinner and taking tiles out of a bag. How do the results of using two of the following spinners compare?



LET'S DO MATH

Before (Getting Started)

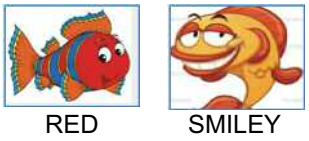
Predicting Likelihood Problem:

Which of the events in the chart are *impossible, unlikely, equally likely, likely* or *certain*? Identify at least one event for each category and then add two events of your own under the appropriate title.

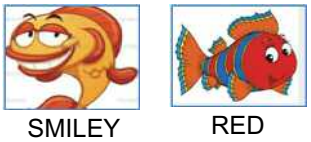
During (Working On It)

Predicting Fishy Events Problem:

The picture cards to the right represent the fish in a small aquarium. They are put in a bag.







1. What is the likelihood of each event?
   - picking ANGEL with all cards in the bag
   - picking SMILEY with all cards in the bag
   - picking RED with all cards in the bag
2. Which 3 cards are in a bag so that each probability statement is true?
   a) you are certain to pick RED
   b) it is likely that you will pick RED
   c) it is likely that you will pick SMILEY
   d) it is impossible to pick ANGEL

After (Consolidation)

Anticipating Student Response

Solution 1

Impossible	Unlikely	Equally likely	Likely	Certain
13	4	15	1,3	2

EVENTS

1 It will rain in April in Ontario.
2 My dog will sleep in my bed tonight.
3 I will grow to be taller than my mother.
4 I will play baseball this week.
5 I will play soccer this year.
6 I will read 10 books.
7 I will find an elephant in my closet tonight.
8 I will brush my teeth once today.
9 I will drink 4 full glasses of water.
10 I will have beans for dinner.
11 I will have some fruit today.
12 The sun will shine today.
13 The sun will shine tonight over my house.
14 When I roll a die, I will get a 4.
15 When I flip a coin, I will get heads.

Solution 1

1. a) Probability (Angel) = unlikely (Angel is 1 out of 6 cards)
b) P(SMILEY) = unlikely (2 out of 6 is still less than half)
c) P(RED) = equally likely – it is just as likely that you will pick RED as pick any other fish (3 RED and 3 other fish)

2. a) Put 3 RED cards in the bag only
b) Put 2 RED and 1 SMILEY in the bag
c) Put 2 SMILEY and 1 Angel (or RED) in the bag
d) put 3 other cards in bag – omit ANGEL

Solution 3

1 a) P(ANGEL) = $\frac{1}{6}$

b) P(SMILEY) = $\frac{2}{6} = \frac{1}{3}$

c) P(RED) = $\frac{1}{2}$

2 a) 3 RED cards

b) 2 RED 1 other

c) 2 SMILEY 1 other

d) 3 cards no ANGEL

Solution 2

Impossiblie	Unlikely	Equally Likely	Likely	Certain
P(ANGEL) = impossible with 0 ANGEL card out of 3 cards to pick	P(ANGEL) P(SMILEY)	P(RED)	P(RED) = likely with 2 RED out of 3 cards to pick P(SMILEY) = likely with 2 SMILEY out of 3 cards to pick	P(RED) = certain with 3 RED out of 3 RED to pick

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solutions 1 and 3?

- *Solution 2* – This solution shows the probabilities on the likelihood line from impossible to certain. So it is good to start with these understandings and using words to describe events.
- *Solution 1* – Students go through and support each argument with an explanation of why the likelihoods are where they are.
- *Solution 3* – Students here represent the probabilities with fractions.

$$\frac{\textit{number of favourable events}}{\textit{number of possible events}}$$

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these probability problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with probability concepts. Use different materials, such as counters, square tiles in bags, dice, spinners, and attribute blocks.
- Consider the purposes for representing and interpreting probability in local classroom and school community contexts. Some interesting contexts for probability include predictions of the weather and predicting outcomes of dice, card and board games and game playing strategies.
- Practise noticing the different strategies and the breadth of the mathematics your students use to describe, show and make sense of probability outcomes in relation to experimental and theoretical situations. Take note of the kinds of probability language students' use in the earlier years and the ways that students make sense of using fraction, decimal and percentage representations of probability. As well, provide ample opportunities for students to create theoretical models and compare them to experimental results, using tree diagrams, tables and cross-classification charts.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for reading data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Chapin, S., Koziol, A., MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades 3-5.* Reston, VA: NCTM.
- McMillen, S. (2009). Predictions and Probability. *Teaching Children Mathematics, 14*(8), 454.
- Nicolson, C. (2005). Is chance fair? One student's thoughts on probability. *Teaching Children Mathematics, 12*(2), 83.
- Quinn, R. (2001). Using attribute blocks to develop a conceptual understanding of probability. *Mathematics Teaching in the Middle School,* 6(5), 290.
- Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.
- Small, M. (2010). B*ig ideas from Dr. Small, Grades 5-8*. Toronto: Nelson Education.
- Young, E. (2010). Probability: A whale of a tale. *Teaching Children Mathematics, 17*(2), 106.

CLOSING

Our next issue will focus on proportional reasoning across the five strands (i.e., number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, data management and probability). As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca