8 PAGE INSERT ELEMENTARY MATH

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PROPORTIONAL REASONING IN NUMBER SENSE AND NUMERATION

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ABACUS CO-EDITORS



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Many topics within the school mathematics curriculum require proportional reasoning, such as problem solving with fractions, decimals, percentages, scaling factors, unitizing, place value, and multiplication and division operations, to name a few. Proportional reasoning is fundamental to real-world and every-day situations, such as: determining which soccer team has the best ratio of experienced to new players; sharing submarine sandwiches among three people; determining if 5 cents less per bagel is better than a 5% discount on a dozen bagels; calculating the sale price when everything is 40% off.

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for representing proportional reasoning situations within the number sense and numeration strand only. The problems we provide are designed for use within a three-part, problem-solving lesson structure.

LINKS TO LITERATURE AND MANIPULATIVES



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Although proportional reasoning first appears as a subheading in grade 4 in the Number Sense and Numeration strand of the Ontario Mathematics Curriculum (2005), some experiences with counting contribute to the development of proportional reasoning. Unitizing is one of the principles of counting. Unitizing occurs when a student is able to understand that a single object may stand for more than one thing. For example, children come to realize that one train of ten cubes has the same value as ten single cubes; one nickel is equal to five pennies; one dot pattern of four can also represented as four single dots. In this issue, let's look at the use of dot plates, ten frames, number cubes and a picture book to develop the concept of unitizing.

RESEARCH SUMMARY – PROPORTIONAL REASONING (RATIO AND PROPORTION)

About Proportional Reasoning

Proportional reasoning is about being able to make comparisons between quantities in a multiplicative, rather than additive way. For example, multiplicative comparisons can be described using terms, such as double, half, three-times greater; rather than comparing objects as being smaller or bigger. Small (2010, 2012) explains that proportional reasoning involves the deliberate use of multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another. Consequently, proportional reasoning is commonly interchanged with the term, multiplicative thinking, as an implicit multiplicative comparison is used. For example, there are 6 marbles in 1 bag. How many marbles are there in 4 bags? You are compelled to compare the number of marbles in 1 bag to the number of marbles in 4 bags in a multiplicative way. Therefore, "proportional reasoning involves a sense of co-variation, multiple comparisons and the ability to mentally store and process several pieces of information" (Post, Behr and Lesh, 1988, p. 79).

From Additive to Multiplicative Thinking

Though this Caterpillar Leaves Problem is intended to provoke students' multiplicative thinking, Kenny, Lindquist and Heffernan (2002) report that students use a range of strategies that include additive and multiplicative thinking:

Caterpillar Leaves Problem:

A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars.

- 15 leaves → think the amount to be added must remain constant rather than the multiplier remaining constant; since there were 3 more leaves than caterpillars, then there should be 3 more leaves for the 12 caterpillars (15 leaves)
- 15 leaves → focused on 10 more caterpillars rather than 6 times as many caterpillars, so added 10 more leaves to original 5 leaves, keeping the amount added constant
- 30 leaves à since 5 is for 2 caterpillars, adds 2-1/2 leaves for each caterpillar, repeatedly adds 2-1/2 leaves 12 times; multiplies 2-1/2 leaves x 12 caterpillars (size of group is 2-1/2; number of groups is 12)
- 30 leaves → 5 leaves for 2 caterpillars, so groups the 12 caterpillars into 6 groups and multiplies 5 leaves (for 2 caterpillars) x 6 groups; uses a table of values, showing # of caterpillars as 2, 4, 6, 8, 10, 12 in relation to leaves, 5, 10, 15, 20, 25, 30

Siemon and Breed (2006, p. 4) say that "multiplicative thinking develops as each element (equal groups, number of groups, total) is abstracted … dealt with as a mental object."

Modelled	Abstracted
Equal Groups Makes all, counts all	Trusts the count, sees equal group as a composite unit
Number of Groups Sees in terms of each group, counts all groups, 1 group, 2 groups, 3 groups	Can deal with number of groups in terms of part-part-whole understanding, e.g. 6 groups is 3 groups and 3 groups or 5 groups and 1 group
Total Arrived at by counting all, skip counting	Total seen as a composite of composites, e.g. 18 is 2 nines, 9 twos, 6 thress, 3 sixes

Ratio and Proportion

Solve this ratio problem in 2 different ways. How do your solutions compare to the solutions below?

• part to part – clear : green → 5 : 10 → 15/10 = 3/2
(3/2 times (x) more clear than green marbles)

• part to whole – clear : total marbles → 15 : 45 → 15/45 = 1/3
(1/3 of total marbles are clear)

Marbles Problem 1:

In a bag of 45 marbles, 15 are clear, 10 are green, and 20 are solid black. What comparisons can be made?

Ratio is a multiplicative comparison of two or more quantities, which can be expressed as a fraction of two quantities of the same kind (part to part, part to whole). Also, a ratio is the division of two quantities, having the same units. The ratio has no unit. It is a numerical quantity, which indicates how many times one quantity is greater or less than the other. (Lobato, Ellis, Charles, & Zbiek, 2010; MOE, 2012; Small, 2010, 2012).

How does the solution to this problem 2 change multiplicatively from problem 1? Why?

• part to part – clear : green → 2 x 15 : 2x10
→ (2)15/(2)10 = 3:2 (meaning that 3/2 times (x) more clear than green marbles)

• part to whole – clear : total marbles → 2 x 15 : 2 x 45
→ (2)15/(2)45 = 1/3 (meaning that 1/3 of the total marbles are clear)

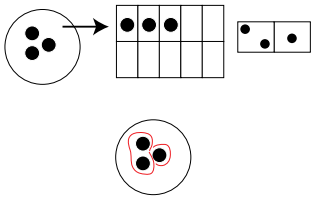
Marbles Problem 2:

What is the ratio of clear marbles to green marbles and total marbles if there are double the bags of marbles?

A ***proportion*** is a relationship of equality between two ratios. In a proportion, the ratio of 2 quantities remains constant as the corresponding values of the quantities change. A ***continued ratio*** is a comparison of more than 2 quantities having the same units.

LINKS TO MANIPULATIVES: DOT PLATES, TEN FRAMES AND NUMBER CUBES

Unitizing can be developed by flashing familiar dot configurations, from number cubes /dice, dominoes, dot plates, and five and ten frames. Ask students, “What number did you see?” Students can respond by holding up fingers to show the number seen. Providing each student with materials such as a set of number cards, ten frame cards or ten frames and counters provides students with experiences to represent the number in a variety of ways.



- Flash this dot plate. Students might choose this ten frame or a domino to respond to what they saw.
- Ask students, “How did you see the number?” They may be seeing a dot pattern as one unit as well as a collection of single units. (i.e., thinking of the dot pattern as a single unit of three as well as a collection of three ones).
- A child who cannot unitize will begin at one and count by ones to determine sums of basic combinations; count all the objects in a set by ones instead of seeing smaller sets of twos, threes, and fours imbedded in the set.

These actions support students in beginning to think of a quantity.

Unitizing is a precursor to counting on, using ten as a unit, part/part/whole and place and value.

LESSON LEARNING GOALS

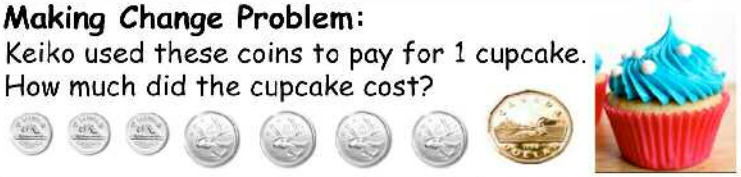
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

Making Change Problem:

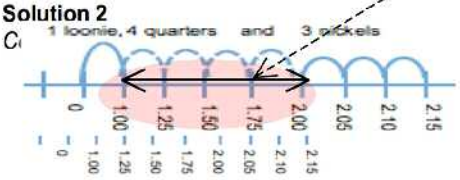
Keiko used these coins to pay for 1 cupcake.
How much did the cupcake cost?



Solution 1

3 nickels: 5, 10, 15 ..... 15 cents
4 quarters: 25, 50, 75, 100 ... 1 dollar
1 loonie: 100 cents ..... 1 dollar
Keiko paid $2.15 for the muffin.

Solution 2



4 quarters for 100¢
25¢ x 4 = 100¢

4 quarters for 1 dollar ($1.00)
$0.25 x 4 = $1.00

100¢ for 1 dollar
$0.01 x 100 = $1.00

size of group *- 25¢, $0.25, or $0.01*
# of groups *- 4 or 100*

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Coin Box Problem 1:

A grade 4 class store started with a coin box with $30.00. After 1 week, they had 24 nickels, 22 quarters and 35 loonies.
How much money did they make in their first week?

What information are we going to use to solve to solve the problem?
- *$30.00 at the start*
- *24 nickels, 22 quarters, 35 loonies after 1 week*
- *How much more money than $30 is in the coin box now?*

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria for the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas,

multiplicand *- size of group (e.g., nickel, 5 cents, or $0.05)*
multiplier *- number of groups (e.g., 24 or 4 groups of 6 nickels)*
product *- result of multiplication ($1.20 = 4x6x$0.05 = 24 x $0.05)*

Proportional Relationships are multiplicative
1 x 4 x $0.25 = 1 x $1.00 (1 group of 4 quarters)
5 x 4 x $0.25 = 5 x $1.00 (5 groups of 4 quarters)
1 group -> $1.00; 5 groups -> $5.00 has a multiplicative relationship of X 5

Solution 1
(a) 35 loonies is $35.00

(b) 22 quarters
-4 quarters make $1.00
-5 groups of 4 quarters
(20) makes $5.00
-2 more quarters (22) makes $5.50

(c) 24 nickels
-6 nickels make $0.30
- 4 groups of 6 nickels
(24) makes $1.20

$41.70 -$30.00=$11.70
(money made)

Solution 2
1 nickel →$0.05
10 nickels →$0.50 (x 10)
20 nickels →$1.00 (x 10, x 20)
24 nickels →$1.20 (x 20)

1 quarter →$0.25
10 quarters →$2.50
20 quarters → $5.00 (x 22)
22 quarters →$5.50 (x 22)

$1.20 + $5.50 + $35.00
= $41.70
Subtract start of $30.00
$41.70 - $30.00= $11.70
They made $11.70.

Making Groups of Same Value Coins Near to $1.00 Unit

Scaling Up (Multiplying by Larger # of Groups) Starting at Unit of 1

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
-explain concept of equivalent fractions using concrete materials -demonstrate … simple multiplicative relationships involving whole number rates	-represent ratios found in real-life contexts, using concrete materials, drawings and standard fractional notation	-determine through investigation the relationships among fractions, decimals, percents and ratios -demonstrate an understanding of rate as a comparison or ratio of two measurements with different units

Ratio

- multiplicative comparison of two or more quantities of the same kind (part to whole - 1 nickel to total 24 nickels)
- has no unit
- is a numerical quantity - how many times (multiplier) one quantity is greater than the other quantity.

Equivalent Ratios *- are equivalent fractions*

$\frac{35 \text{ loonies}}{1 \text{ loonie}} - \frac{3500 \text{ cents}}{100 \text{ cents}}$

24 nickels = 120 cents
1 nickel 5 cents
X 25
22 quarters = 550 cents ÷ 22
1 quarter 25 cents
X 25
35 loonies = 3500 cents
1 loonie 100 cents

- ratio as fraction
- doesn't matter which quantity is numerator or denominator (X or ÷)

Values in ratio table added together (2+20=22) quarters corresponds with $0.50+$5.00 = $5.50 (value of quarters)

2 values changing together
x 4 top and bottom
x 20 top and bottom

A proportion *is a relationship of equality between two ratios. The ratio of 2 quantities remains constant as the corresponding values of the quantities change.*

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary:

Ratios are multiplicative (proportional) comparisons (part to whole) of 2 or more quantities of the same unit.
Represent (equivalent) ratios as:
- a numerical quantity (not unit, a constant) stays the same as corresponding values change

X 25
X 22 (1 quarter = 25 cents
22 quarters 550 cents) X 22
X 25

- showing how many times (multiplier) one quantity is greater than the other quantity
- equivalent fractions (1/22 = 25/550 or 22/1=550/25)

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Solution 3

Nickels	1	4	20	24
Value	0.05	0.20	1.00	1.20

Quarters	1	2	20	22
Value	0.25	0.50	5.00	5.50

Loonies	1	35
Value	1.00	35.00

$1 + $5 + $35 + $0.20 + $0.50 - $30 = $41.70 - $30 = $11.70 (1 week earned)

Solution 4

X 24 (1 nickel = 5 cents
24 nickels 120 cents.) X 24
X 25
X 22 (1 quarter = 25 cents
22 quarters 550 cents) X 22
X 25
X 35 (1 loonie = 100 cents
35 loonies 3500 cents.) X 35

(120 + 550 + 3500) cents
4170 cents = $41.70
less $30 → $11.70
earned in 1 week

Ratio Table
Multiplicative Relationships

Ratio (Fraction)
# of Coins to Coin Values

Coin Box Problem 2:



1 roll of coins has either 25 loonies, 40 quarters or 40 nickels. These coins were deposited at the bank:
15 rolls of loonies; 5 rolls of quarters; 84 nickels. How much money was deposited?

Solution 1

25 loonies →$25
1 roll → $25
15 rolls → 15 x $25 = $375

40 quarters -> $10
1 roll → $10
5 rolls → $50

40 nickels → $2
80 nickels → $4
84 nickels → $4.20

Total:
$375.00
50.00
+ 4.20
$429.20

Solution 2

loonies
$25 x 15 = $375.00

quarters
$0.25 x 40 x 5 = $50.00

nickels
$0.05 x 84 = $4.20

$375 + $25 + $25 + $4.20
= $400 + $25 + $4.20
= $429.20

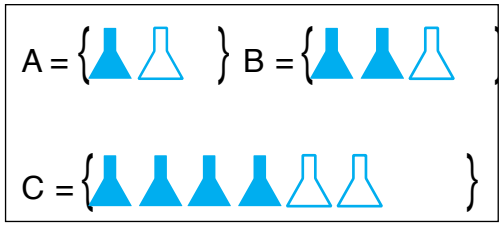
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH

Before (Getting Started)

Blueberry and Water Mixtures Problem:

How do the mixtures compare to one another?



Key

represents 1 part blue

represents 1 part clear

Solution 1

Mixture A

part to whole → blue to clear
→ 1 to 1 → 1 : 1
→ 1/1 blue to clear
1 part blue to 1 part clear water

Mixture B

part to part → blue to clear
→ 2 to 1 → 2/1 blue to clear
2 parts blue to 1 part clear water

2 >1 → Mixture B is bluer than Mixture A.

Solution 2

Mixture A

part to whole → blue to total → 1 to 2 → 1 : 2
1/2 of the mixture is blue

Mixture C

part to whole → blue to total → 4 to 6 → 4 : 6
4/6 of the mixture is blue

Mixture B

part to whole → blue to total → 2 to 3 → 2/3 = 4/6
2/3 or 4/6 of the mixture is blue

Mixtures B and C are 2/3 blue; Mixture A is 1/2 blue
1/2 < 2/3 → Mixture A is lighter blue than mixtures B and C



During (Working On It

Ladybugs and Grasshoppers Problem

The ratio of ladybugs to grasshoppers is 12:11 in a garden. If there are 1955 insects, then how many ladybugs and grasshoppers are there?

After (Consolidation)

Solution 1

Part + part = 12 + 11 = 23 insects in one group

$\frac{1955}{23}$ = 85 size of each group

85 groups of 23 insects = 1955 insects
12 x 85 = 1020
11 x 85 = 935
12 to 11 = 1020 to 935 = ladybugs to grasshoppers

Solution 2

12 to 11 means part to whole is 12 to 23.

23 x 50 = 1150
23 x 30 = 690 (1150 + 690 = 1840)
23 x 5 = 115
23 x 85 = 1955

$\frac{\textit{lady bugs}}{\textit{insects}} = \frac{12}{23}$

$\frac{\textit{lady bugs}}{1955} = \frac{12 \times 85}{23 \times 85} = \frac{1020}{1955}$

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2?

- *Solution 1* – shows comparison of 23 insects in one group to 1955 altogether; determines 85 groups and multiplies once by 12 and once by 11.
- *Solution 2* – proportions are used to solve the problem, using fractions and equivalent fractions.

LINKS TO LITERATURE: *ONLY ONE*



Is it math or is it language? The picture book, *Only One* is a counting book that includes collective nouns.

"There can be 11 cows. But there is only one herd."	"There may be 12 eggs, But there is only one dozen."

- Read the book to the students and ask, "What did you learn about the number one?"
- Sample response: "One can mean one like one sun in the sky, but sometimes is means more than one."
- Review the book and look for examples of when one refers to a specific number e.g., one dozen is 12 and when one refers to a quantity that can change like, the number of cows in one herd.
- Collect and record examples of one as a unit and the specific quantity referenced, e.g., one hundred, one ten frame, one nickel, one hand, one baseball team, one hour.

LETS DO MATH – BUTTERFLY PROBLEMS

Before (Getting Started)

Butterfly Problem 1:

Each butterfly has 4 wings.

How many wings are on 26 butterflies?

Solution 1

BF	1	2	3	5	10	20	25	26
Wings	4	8	12	20	40	80	100	104

25 butterflies have 100 wings

26 butterflies have 104 wings

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

During (Working On It)

Butterfly Building Problem 2:

Students are building butterfly models at a science fair. Each butterfly needs 4 wing pieces, 1 body and 2 antennae. If Jan has 29 wings, 8 bodies and 13 antennae, how many complete butterfly models can be built?

After (Consolidation)

Anticipating Student Responses:

Solution 1

Wings (4)	4	8	12	16	20	24	28	Only 29
Body (1)	1	2	3	4	5	6	7	8
Antennae (2)	2	4	6	8	10	12	Only 13	
Number of models	1	2	3	4	5	6		

We skip counted by the size of each group until we reached the total number of each part available. Then we knew they could make 6 complete butterflies.

Solution 2

Wings: 4 x 5 = 20
4 x 2 = 8
28 wings → 7 butterfly models.
29 wings not enough for 1 more model.

Antennae: 2 x 7 = 14 but we only have 13 → so only 6 butterfly models can be made. With 8 bodies, 6 models can be made.

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solutions 1?

- *Solution 2* – Students used guess and test with multiples of 4 to determine 28 wings would make 7 butterflies. They went on to reason the same way with antennae.
- *Solution 1* – Students skip counted, using a table to record until they got to the total number of parts available at the science fair. When they got to the maximum number for that part they stopped and saw the answer in the bottom row. They would use 24 wings, 6 bodies, and 12 antennae to make the 6 butterfly models.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these proportional reasoning problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with noticing and using multiplicative (proportional) relationships inherent in multiplication and division operations, in comparing part-whole relationships and equivalence between fractions, decimals and percentages, and in representing situations using ratios. Use different models of representation, such as open number lines, polygon grids, and sets of objects, visible in concrete materials and visuals, such as counters, base ten blocks, square tiles, pattern blocks, ten frame, dot plates and fraction bars to model proportionalities.
- Practise noticing the different strategies and the breadth of the mathematics your students use to describe, show and make sense of multiplicative thinking (proportional reasoning) in relation to solving multiplication, division, fraction and ratio problems. Take note of the kinds of proportional reasoning language students' use from primary to junior years (e.g., equal size groups, size of group, number of groups, fractions as part- to-part and part-to-whole ratios).
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for transitioning from additive to multiplicative thinking (e.g., partitioning a whole into equal size groups, doubling using an array grid, multiplicand (size of group), multiplier (number of groups), multiples, halving,) as the students discuss and analyze each other's solutions to co-construct success criteria for the lesson learning goal.

Suggested Readings

Readings used in the Research Summary and design of the problems and solutions include:

- Karplus, R., & Peterson, R. (1970). Intellectual development beyond elementary school II: Ratio, a survey. *School Science and Mathematics, 70*(9), 813-820.
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CLOSING

Our next issue will focus on proportional reasoning within the measurement strand. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca