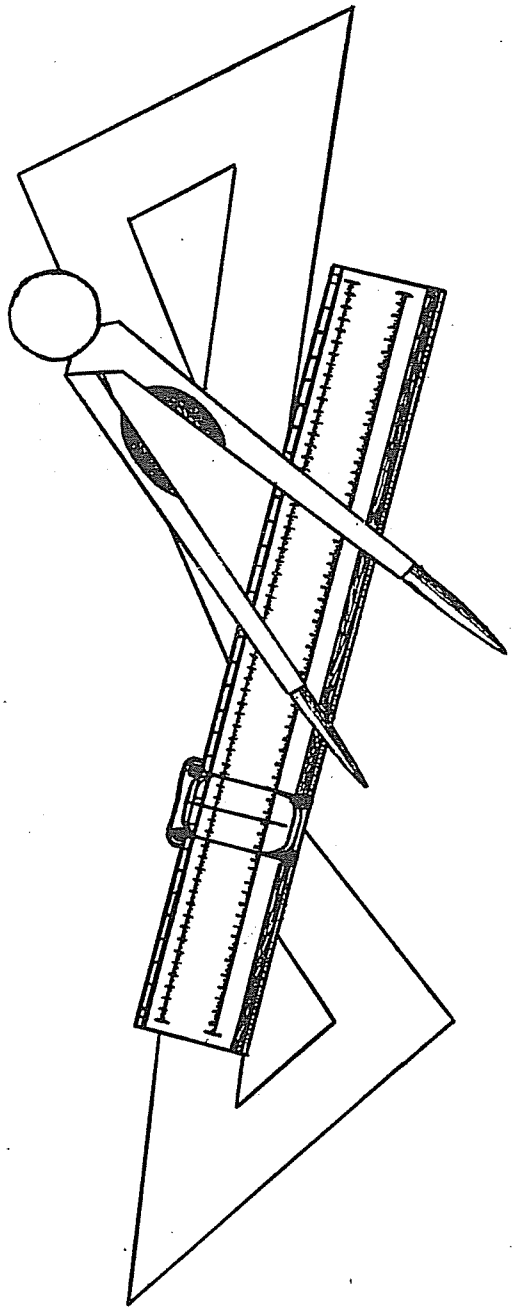
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ONTARIO MATHEMATICS GAZETTE

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THE ONTARIO ASSOCIATION FOR
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ONTARIO MATHEMATICS GAZETTE

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THIRTY

D. W. Alexander, President OAME

In February, 1962 the Ontario Mathematics Gazette's first issue was published under the editorial supervision of Dr. Ralph Stanton and the sponsorship of the Ontario Mathematics Commission and the Ontario Association of Teachers of Mathematics and Physics. The aims were to increase liaison between secondary school teachers and university professors while continuing the communication function of the Newsletter of the OATMP. In subsequent years liaison with the teachers of mathematics both in the elementary schools and the CAATS has been included in the goals of the journal.

A measure of the success in ataining the objectives is the subscription list which includes 627 Institutional Subscriptions and 117 Individual Subscriptions in addition to the subscriptions of OATM and OMC members.

The Ontario Mathematics Gazette has provided a means of the interchange of ideas among those concerned with mathematics education in Ontario. Under the leadership of the successive editors:

Dr. Ralph Stanton
Dr. J. E. Hogarth
Dr. D. H. Crawford
Dr. T.M.K. Davison
Dr. W. Eames and Dr. J.S. Griffith

and their editorial boards the journal has continually offered a wide range of stimulating, informative, thoughtful, thought provoking articles on mathematics and education.

Unfortunately the costs of such an ambitious journal and its distribution are high. While the cost was shared between the OMC and OATM with assistance from the Samuel Beatty Fund each could absorb its share without extreme difficulties. When the executive of the OAME examined the necessary budget for this journal, now the responsibility of the single organization, it was found that to continue would cost the organization the equivalent of $4 per member. This appeared indefensible when establishing priorities for the various projects demanding attention.

Reluctantly the executive has decided to cease publication.

The present editorial board may take pride in the fact that the final volume of the Gazette has maintained the high standars set since its creation. Hopefully in the future the OAME will gain the financial strength necessary to sponsor a publication of this stature. In the interim the OAME will send to members six times a year a less ambitious "newsletter". Initially there will be no separate subscriptions to this new publication its receipt being linked to either an individual or institutional membership in OAME. Present subscribers of the Gazette will receive appropriate refunds.

Thus we come to the end of the life of this journal and mark its conclusion with the journalist's traditional - THIRTY -.

MESSAGE FROM THE PRESIDENT OF THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

David Alexander, President OAME

On May 13, 1973 the Ontario Association for Mathematics Education was born. The creation of this organization signals the end of the separate activities of the Ontario Mathematics Commission and the Ontario Association of Teachers of Mathematics, each of which has made its own unique contribution to education in Ontario and beyond.

The intention of those who participated in the formation of the OAME is that it should continue the efforts of both of the organizations which have cooperated in the formation while avoiding duplication of efforts which have been evident in the work of the separate organizations.

The OAME in its structure is firmly committed to providing opportunities for individual participation. Provision is made for the formation of chapters which could reflect regional organization or special interest groups. An election will be held annually for positions on the executive and hence each executive will be annually accountable to the membership. As president in this initial year I feel that the strength of the organization lies in the executive acting as responsible representative of the membership. In the light of this assumption the executive has made, and will continue to make, decisions concerning the support of various projects. These decisions will be based on what the executive feels are reasonable priorities within the financial resources available. We would welcome any suggestions you might have as to areas to which the OAME should turn its attention.

Make the OAME <u>your</u> organization.

Membership applications may be obtained from:

Mr. M. B. MacGregor
Secretary Treasurer OAME
SARNIA, Ontario
N7T 4E9.

Committees

1. Projects Committee

   Functions
   (i) to stimulate and initiate suggestions for projects
   (ii) to recommend priorities on projects
   (iii) to recommend budget for specific projects
   (iv) to recommend sources of funds for specific projects
   (v) to recommend personnel, organization or chapter, etc. to act on specific project
   (vi) to receive reports of projects
   (vii) to recommend action to be taken on reports of projects.

2. Publications Committee

   Functions
   (i) to recommend policies concerning publications including types, frequency, distribution, financing, editors, etc.
   (ii) to organize assistance for editors such as editorial boards, regional representatives, etc.

3. Membership and Publicity Committee

   Functions
   (i) to initiate and stimulate ideas concerning publicity both for general public and potential members
   (ii) to initiate and stimulate ideas concerning membership promotion and records
   (iii) to establish any organization necessary for promoting memberhsip.

4. Meetings Committee

   Functions
   (i) to act as liaison with any groups organizing annual or regional meetings and workshops
   (ii) to stimulate regional meetings and workshops
   (iii) to set up guidelines to assist in organizing various types of meetings
   (iv) to set up organization of resource personnel to assist in organizing various types of meetings.

5. Chapter Relations Committee

   Functions
   (i) to act as liaison with chapters and prospective chapters

(ii) to stimulate organization of chapters
(iii) to set up guidelines for acceptance of application for chapter status
(iv) to set up procedure for application for chapter status
(v) to recommend to annula meeting those groups which should be granted chapter status.

6. External Affairs Committee

Functions
(i) to act as liaison with affiliated organizations
(ii) to act as "watch dog" of organizations, areas and situations where views of OAME should be expressed
(iii) to recommend procedures for study and action in situations mentioned above
(iv) to forward nominees as requested by NCTM or other organizations.

7. Manpower Committee

Functions
(i) to institute procedures necessary to create and maintain a "manpower" file
(ii) to make suggestions for appropriate personnel as requested by other committees and executive
(iii) to study and recommend changes in election procedures for OAME including possibility of mailed ballot for vice-president.

Committee Membership

OAME members are invited to communicate any thoughts on the activities of a particular committee to one of the appropriate committee members.

* indicates executive member

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N. Williamson	J. Mawdsley
*R. Leigh	B. Rachar
	*T. Griffiths

Membership & Publicity
J. Fencott
S. Pinkerton
*J. Routledge
D. McPhart

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Meetings
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O A M E

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SECOND MATHEMATICAL CONGRESS

A. G. Howson

The second International Congress on Mathematical Education held at the University of Exeter in 1972 attracted some 1,700 educators from 70 different countries. The formal programme of the congress was divided into three parts: plenary lectures given by six invited speakers, working groups on specialist topics organised on an international basis and the presentations mounted by various national committees for mathematics - presentations which included talks, films and demonstration lessons. There was an almost continuous showing of mathematical films and, in addition, certain 'fringe' exhibitions and demonstrations mounted by bodies such as the Open University, the School Mathematics Project and the Nuffield Mathematics Project, which - like their counterparts at the Edinburgh International Festival - generated as much interest as the more official activities. And there was a 'commercial' exhibition mounted by the EEA and the EPC.

What were the discernible trends? Perhaps the most significant was the fact that the honeymoon of experimentation now seems to be over. No longer is every experiment or project greeted with uncritical approval. As Professor Freudenthal remarked in his plenary lecture, 'There are sounder arguments for teaching subject matter than our ability to do so. More out spoken comments were to come from Rene Thom, perhaps the leading French mathematician of the day. Thom argued that those educators who placed emphasis on axiomatics, rigour and the early explicit introduction of the mathematical structures were guilty of a pedagogical and mathematical error. It is perhaps significant that, in general, British educators seemed in accord with Thom, whereas many of his compatriots disagreed violently with his thesis. Those of us who have watched developments in France, Belgium and certain other countries with great interest but with increasing concern will be curious to observe the effect of Professor Thom's criticisms.

That such criticism is now openly voiced is not, however, a matter for regret, for it serves to prove that mathematical education is now passing from infancy to adolescence and is learning something about self-discipline.

Other trends one noticed was a growing concern with the problems of the less able child. For too long mathematical educators have concentrated on the needs of the future mathematician and the future user of mathematics. Now more attention is being paid to the needs of the average child, to those of the socially disadvantaged and of the slow and reluctant learners. This problem is of world-wide concern for with the spread of educational opportunities, it is now arising in the developing countries.

Another whose needs have tended to be ignored in the past, is the student studying mathematics at a post secondary school level. Traditionally, mathematical education has concerned the under-eighteens. The university teacher was not concerned with pedagogical problems (at least, not among university students) and the problems within the colleges of education and other institutions of higher and further education were usually forgotten. Now more attention is being directed to ensuring that university and other students are not only given suitably designed mathematics courses, but that these are presented to them, and the students' work assessed, in a more stimulating and, as a result, more effective manner.

Many of the points I have made refer implicitly to the place of mathematics in the general education of the student. One cannot think of teaching mathematics in isolation, and for that reason one fears those who, when designing curricula, appear to look no further than the library shelves devoted to algebra, analysis, geometry and the like. It was encouraging, therefore, to see the empahsis laid at the congress on the applications of mathematics and on its origins. Mathematics is a subject which has evolved and is still enolving. It has grown in order to meet needs (indeed it is only in this century that certain sections have become divorced from applications), although those needs have varied greatly according to time and place. Just how much mathematics and those processes of thought which we term 'mathematical' depend upon the social setting was shown in a lecture by Dr. Edmund Leach, the social anthropologist.

Dr. Leach demonstrated how 'common-sense' notions of time do not always tally with those which we as teachers of mathematics have in so doing, he hinted at the way in which anthropology and the study of the growth of mathematics in primitive societies might help us to comprehend more fully the development of mathematical concepts in the childs mind.

The importance of taking the social background and even the language patterns of the child into consideration was further stressed by the Australian psychologist Hugh Philp, who told us of his work in New Guinea, and by speakers in several working groups.

Not only did mathematics grow to satisfy needs, but now more than ever is it used throughout the sciences - physical, biologica and social - and in technology, It is natural, then that several groups chose to discuss both these applications and the particular difficulties which arise when one teaches the subject to engineers, biologists, economists and others. One group specifically concerned itself with the problem of building links with other subjects at a school level. It is to be regretted that the recent advances in school mathematical and scientific curricula have proceeded almost independently of each other and that neither set of reformers appears to have taken advantage of the opportunities offered to them by the other. Clearly, the strengthening of these links is a matter which merits particular consideration in the coming years.

These, then, are some of the trends and problems one identified at the Congress. Of course, there were many others too detailed to be described here, for example, the exciting and novel computer-based work demonstrated by Seymour Papert and his colleagues from the Massachusetts Institute of Technology with the aid of a group of local 10-13 year olds. A much fuller account of the work of the congress, together with the scripts of the plenary lectures, the papers specially written by Professors Polya and Piaget, and a selection of other papers submitted to the various working groups, will appear in the Congress Proceedings to be published in 1973 by the Cambridge University Press under the title of *Developments in Mathematical Education*. It is to be hoped that this publication will enable a much wider audience to share in the benefits of a congress in which could be found (to quote from the publisher's blurb for a series of mathematical books) 'originality, creativity, enjoyment, deep thinking, inspiration and light'.

Not only was the congress successful mathematically, but the British Tourist Board will rejoice in the fact that delegates from all over the world left Exeter with a totally wrong impression of British weather.

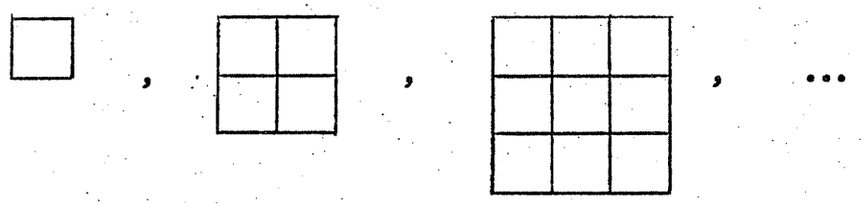
SEQUENCES

J. Norman C. Sharp
Co-ordinator of Mathematics
Etobicoke Board

Too often teachers begin a lesson on sequences by writing several terms of the blackboard to see if the students notice a pattern or to determine the next terms. For many this is an abstract approach no matter how simple the sequence. There are so many ways in which sequences can be determined by games, concrete materials or semi-concrete materials that teachers should take time to set up the materials in which the class can derive its own sequence. A sequence is an ordered set of numbers which can be set into a one-to-one correspondence with the natural numbers and hence is a function.

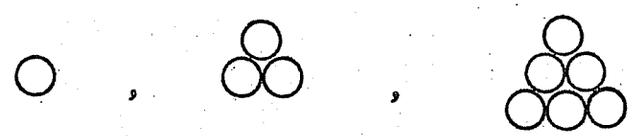
The following will give several situations from which sequences can be derived. What are the next terms? What is the nth term?

1. Square numbers



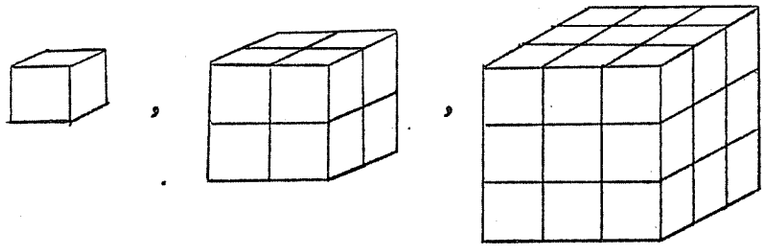
1 , 4 , 9 , ...

2. Triangular numbers (Billiard numbers)



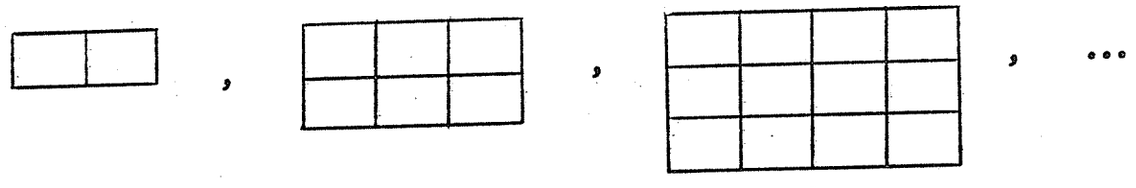
1 , 3 , 6 , ...

3. Cubic numbers



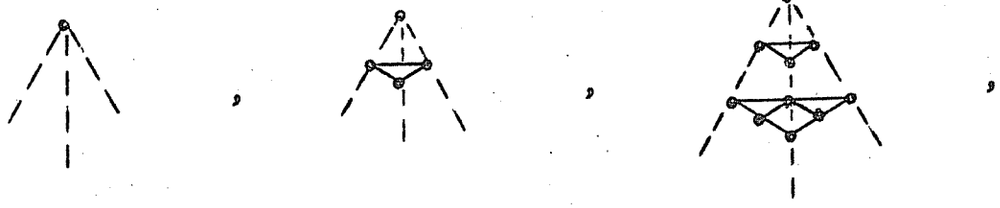
1 , 8 , 27 , ...

4. Rectangular numbers



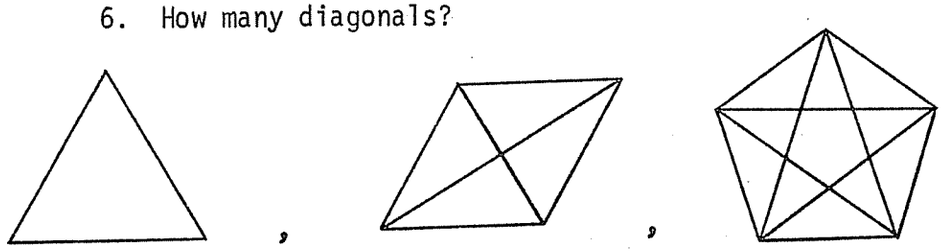
2 , 6 , 12 , ...

5. Pyramid numbers (triangular base)



1 , 4 , 10 , ...

6. How many diagonals?



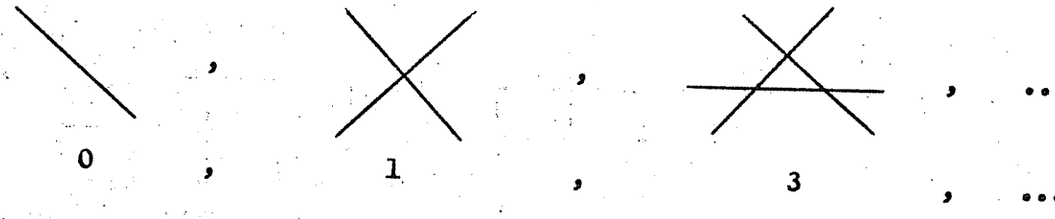
No. of sides	3	4	5	6	7	n
No. of diagonals	0	2	5			

7. How many line segments?

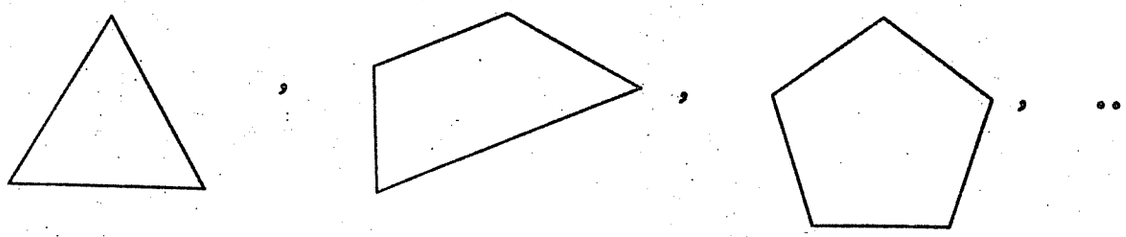


No. of points	1	2	3	4	5	6	n
No. of line segments	0	1	3				

8. How many intersections?



9. How many right angles in the sum of the interior angles?



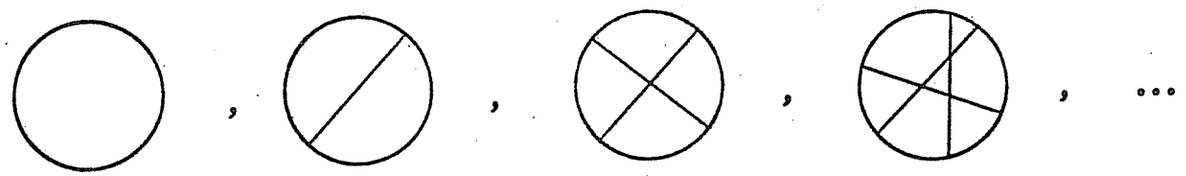
No. of sides	3	4	5	6	n
No. of right angles	2	4	6		

10. How many regions of the plane?



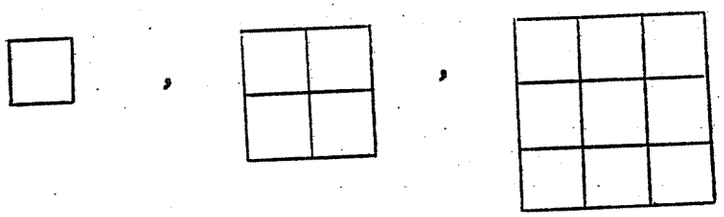
No. of lines	1	2	3	4	5	6	n
No. of regions	2	4	7				

11. How many regions in the circle?



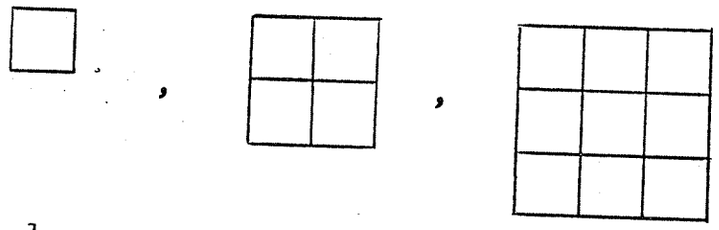
No. of chords	0	1	2	3
No. of Regions	1	2	4	7

12. The total number of squares.



1 , 5 , 14 , ...

13. The total number of rectangles in squares.



1 , 9 , 36 , ...

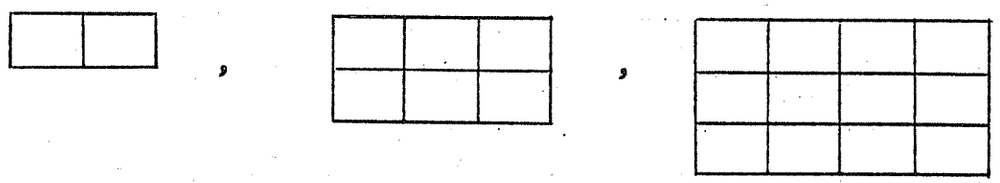
Compare your answers with (2).

14. The total number of triangles in triangles.



1 , 5 , 13 , ...

15. The total number of rectangles in rectangles.



3 , 18 , 60 , ...

(See Appendix for development of nth term.)

16. Climbing steps

A boy climbs a ladder 1 rung or 2 rungs at a time.

To climb a ladder of 1 rung there is 1 way
To climb a ladder of 2 rungs it can be done
(1 + 1) or (2) 2 ways
To climb a ladder of 3 rungs it can be done
(1 + 1 + 1) or (1 + 2) or (2 + 1) 3 ways
To climb a ladder of 4 rungs it can be done
(1 + 1 + 1 + 1) or (1 + 1 + 2) or
(1 + 2 + 1) or (2 + 1 + 1) or (2 + 2) 5 ways

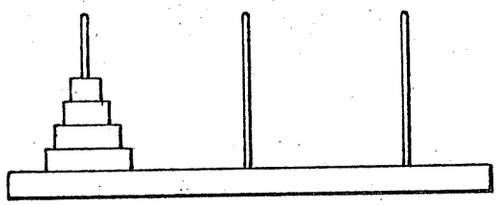
How many different ways can he climb a ladder of 6 rungs?

17. Wine and Water

Cask A contains 6 gallons of water, B contains 6 gallons of wine. Two gallons are removed from A, poured into B and mixed. Two gallons are removed from B, poured into A and mixed. These operations are continued. Make a tabke to show the amount of wine in A after each complete operation.

After operation	0	1	2	3	4	5	n
Gallons of Wine in A	0	$1\frac{1}{2}$	$2\frac{1}{4}$				

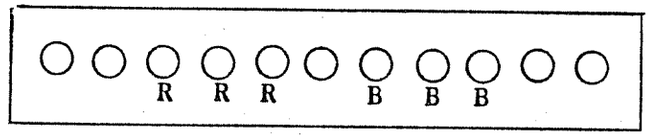
18. Tower of Hanoi



Move the discs to a second peg.
Move 1 at a time.
No disc can be placed on a smaller disc.

No. of discs	1	2	3	4	5	n
No. of moves	1	3	7			

19. Peg Moving



Interchange the pegs. Red pegs move right; blue pegs move left. Pegs can jump one of opposite colour.

No. of pegs on each side	1	2	3	4	5	n
No. of moves	2	8	15			

20. Number of Shapes

How many shapes can be made using congruent squares? All squares used must have an edge in common.

No. of squares	1	2	3	4	5	n
No. of shapes	1	1	2			

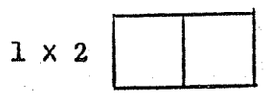
Students who work with these sequences will the proceed from the concrete to the abstract, particularly if they wish to find the nth term. An extension to this development would be to do further sequences from three dimensional figures, i.e., polyhedral numbers. An article describing these is in "The Mathematics Teacher", April 1973.

APPENDIX

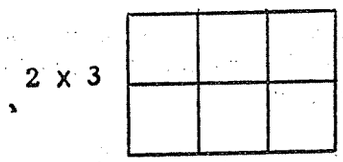
HOW MANY RECTANGLES?

Sequences and the inductive approach can often be used to solve mathematical problems. They can both be used to determine the number of rectangles with integral sides contained in a rectangle $n \times (n + 1)$. Using $n = 1,2,3,$

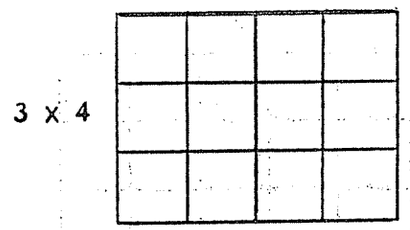
the number of rectangles can be counted.



3



18



60

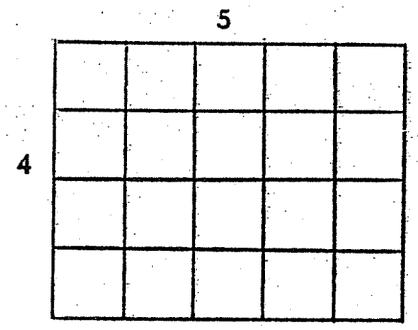
The numbers in the sequence become large very quickly and we realize that counting is not the best method nor is there any obvious law for determining the next terms in the sequence 3, 18, 60 ... . The following table helps to analyze the sequence.

Number of Rectangles

Type of Rectangle		Width 1	Width 2	Width 3	Width 4	Width 5	Total
1	2	3					3
2	3	15	3				18
3	4	42	15	3			60
4	5	90	42	15	3		150
n	(n + 1)	R_n	R_{n-1}	R_{n-2}	R_{n-3}	...	$\sum_{i=1}^{n} R_i$

A pattern is now forming which indicates that it is only necessary to determine the number of rectangles with width 1 unit and varying lengths.

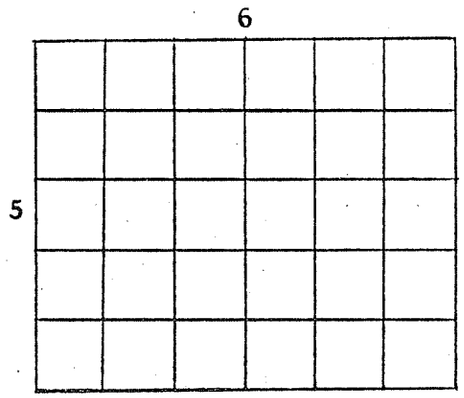
Consider a rectangle 4×5



Rectangles of width 1 unit are of the folloiwng types:
1 1, 1 2, 1 3, 1 4 and 1 5.

Type of Rectangle		Rows	Columns		
1 × 1	→		4 × 5	=	20
1 × 2	→	4 × 4	3 × 5	=	31
1 × 3	→	3 × 4	2 × 5	=	22
1 × 4	→	2 × 4	1 × 5	=	13
1 × 5	→	1 × 4		=	4
or $4(1+2+3+4)+5(1+2+3+4)$			Total		90

For a rectangle 5×6, rectangles of width 1 unit are of the following types: 1×1, 1×2, 1×3, 1×4 and 1×6.



Type of Rectangle		Rows		Columns	
1 × 1	→			5 × 6	= 30
1 × 2	→	5 × 5	+	4 × 6	= 49
1 × 3	→	4 × 5	+	3 × 6	= 38
1 × 4	→	3 × 5	+	2 × 6	= 27
1 × 5	→	2 × 5	+	1 × 6	= 16
1 × 6	→	1 × 5			= 5
or 5(1+2+3+4+5)+6(1+2+3+4+5)		Total			165

The number of rectangles of width 1 unit in an $n \times (n + 1)$ rectangle is then by induction $n\ (1 + 2 + 3 \ldots + n) + (n + 1)\ (1 + 2 + 3 + \ldots n) = (n + n + 1) \sum_{i=1}^{n} i$.

Simplifying, this becomes $R_n = (2n + 1)\ \frac{(n)(n + 1)}{2}$

$$= n^3 + \frac{3}{2} n^2 + \frac{1}{2} n$$

Examining $R_n = \frac{n(n + 1)(2n + 1)}{2}$ it will be notice that $R_n = 3 \sum_{i=1}^{n} i^2$. That is 3 times the sum of the squares of the natural numbers.

$$R_1 = 3 = 3(1^2)$$
$$R_2 = 15 = 3(1^2 + 2^2)$$
$$R_3 = 42 = 3(1^2 + 2^2 + 3^2)$$

The total number of rectangles is

$$S = R_1 + R_2 + R_3 + \ldots R_n$$

$$= \sum_{i=1}^{n} (i^3 + \frac{3}{2} i^2 + \frac{1}{2} i)$$

$$= \sum_{i=1}^{n} i^3 + \frac{3}{2} \sum_{i=1}^{n} i^2 + \frac{1}{2} \sum_{i=1}^{n} i$$

$$= \frac{n\ (n+1)}{4} + \frac{3}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{4} (n\ + n + 2n + 1 + 1)$$

$$= \frac{n(n+1)}{4} (n\ + 3n + 2)$$

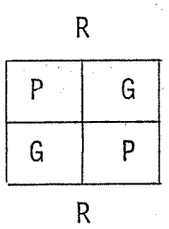
$$= \frac{n(n+1)^2(n+2)}{4}$$

Thus it can be seen that several sequences are involved in the determination of the number of rectangles and the inductive approach helps to find a formula for the number of rectangles which is the sum of several sequences.

INTEGER DRILL WITH DICE

Break class into units of 6 pupils:
Each unit consists of: Gold team - 2 players, 1 referee
Purple team - 2 players, 1 referee

Seating could be arranged as indicated.



Each unit received 1 green die and 1 red die.

Each player is allowed 10 throws - the object is to quickly find the sum of a positive number (green die) and a negative number (red die) but his teammate must call out the score correctly. The referee's decision is final. If there is an error team is penalized one point, if call is correct team is awarded one point.

When each player has had 10 throws the teams scores are totalled and the winning team becomes referees, while the previous referees form a team to play the losing team.

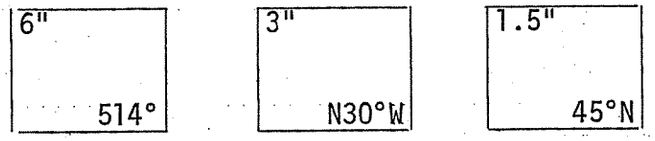
This game can be modified easily to handle drill in subtraction and multiplication.

SOC-A-VECTOR

P. Zolis

PURPOSE: This is a card game which may be used to introduce (and review) the concept of a vector, vector addition, subtraction and scalar multiplication.

METHOD: The deck consists of an arbitrary number of cards in the form shown below.



The game sheet can be a large sheet of graph paper 18" × 24" or regular size paper.

OBJECT OF THE GAME: The first player to draw a vector with its initial point inside the goal net, wins the game. Note that this game is essentially a soccer game but instead of using a ball, we use vectors.

NUMBER OF PLAYERS: Two or more players can play this game. You can also create teams and use this as a "discovery lesson" in introducing vectors.

CARDS: Blank cards may be obtained from the Arrco-Saxon Company, 3195 Airway Dr., Malton, Ontario.

NOTE: The preceeding article was designed and tested by: Paul Zolis, Department of Mathematics, West Hill Collegiate Institute, 350 Morningside Ave., West Hill, Ontario

SOC-A-VECTOR

P. Zolis

MATERIALS REQUIRED: (1) Game sheet - a soccer field with a net at each end.
(2) A deck of cards specially marked, indicating displacement at one corner and direction at the other.
(3) Protractor, ruler and two differend colour pens.

RULES:

Two or more persons can play this game, with one game sheet between them.

(1) Person reads card from a well shuffled deck.
- displacement at top left corner.
- direction at bot om right corner.

(2) After reading the card, person draws vector as indicated by card, (starting anywhere from his goal line and heading towards his opponent's net).

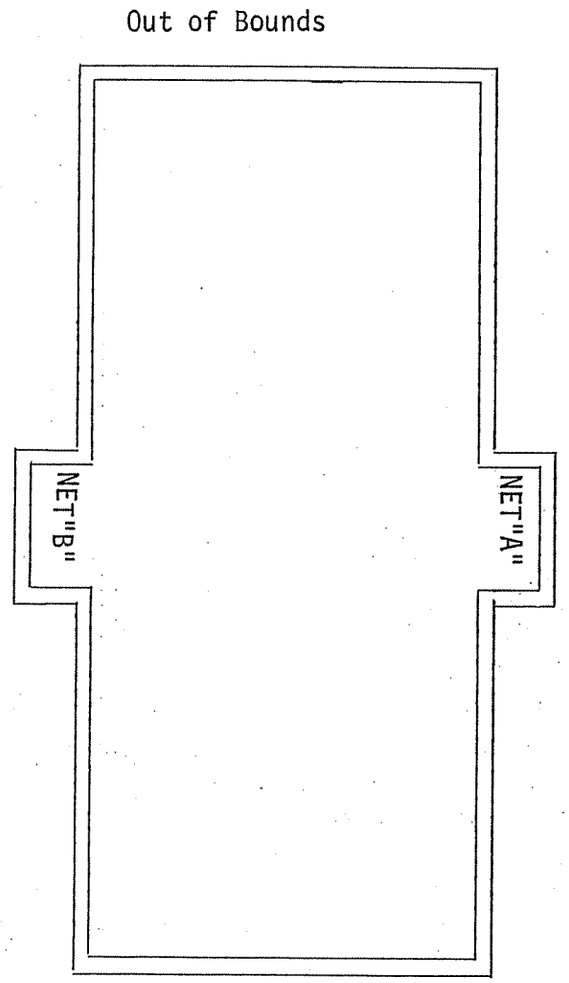
(3) The object of the game is to draw a series of vectors (following each terminal point) so as to reach his opponent's goal net.

(4) First person reaching the goal net wins the game.

(5) However, if the person goes over his own goal line, or out of bounds, then his opponent gets an extra turn, and draws another card.

(6) After having the extra turn, the person who initially went out of bounds or over his own goal line, takes the next card and draws this vector from the terminal point of the last vector before he went out of bounds.

(7) Remember, you must have your last vector inside the net. You do not win the game if you simply go over your opponents goal line (that is, at each side of the net). It must be within the net.


Out of Bounds
NET"B"
NET"A"
Out of bounds

THINK

Mrs. Sandra Bassett
Briar Hill Junior School

Purpose: Review of multiplication facts.

Level: Grades 4, 5 or 6

Materials: Teacher's Card and student cards.

RULES

- Students make up cards and selected answers from the 6X table to put under the T, the 7X for the H, the 8X for the I, the 9X for the N and the 3X for the K.

- Using the Teacher Card the teacher calls out various multiplication questions; student looks for the answer on his sheet and covers it as in Bingo.

- The first student to complete a row calls out think.

- The square in the centre of the card (72 in the sample) can be FREE.

Possible Adaptations: The game may be adapted to develop other operation skills by making a set of cards for development of each individual skill. The game could also be adapted for the learning or review of operations on fractions, decimals, etc.

T	H	I	N	K
6X	7X	8X	9X	3X
1×6 6	1×7 7	1×8 8	1×9 9	1×3 3
2×6 12	2×7 14	2×8 16	2×9 18	2×3 6
3×6 18	3×7 21	3×8 24	3×9 27	3×3 9
4×6 24	4×7 28	4×8 32	4×9 36	4×3 12
5×6 30	5×7 35	5×8 40	5×9 45	5×3 15
6×6 36	6×7 42	6×8 48	6×9 54	6×3 18
7×6 42	7×7 49	7×8 56	7×9 63	7×3 21
8×6 48	8×7 56	8×8 64	8×9 72	8×3 24
9×6 54	9×7 63	9×8 72	9×9 81	9×3 27

Sample Teacher's Card

T	H	I	N	K
12	21	24	81	9
6	42	40	27	18
54	56	72	45	27
36	35	16	36	6
48	63	32	18	15

Sample Student's Card

POPSICLE STICK GAME

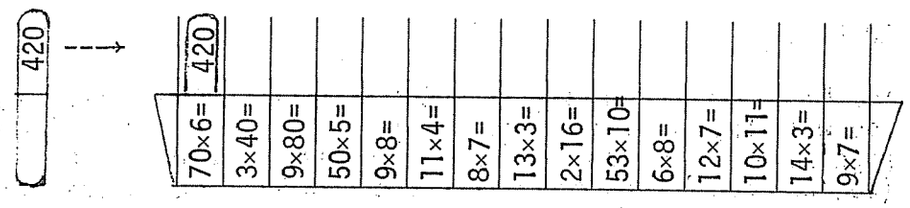
Miss Linda Thorn
Humewood Junior School

Purpose: Review of multiplication tables.

Level: Grades 5 or 6

Materials: Bristol board, popsicle sticks.

Popsicle Sticks Can Make You Smart



Sheet of bristol board with 15 pockets. Bundle of 15 popsicle sticks has answers as shown above.

Put the popsicle sticks in the right pocket.

Adaptations: Can be adapted for the development of all number facts with all four operations and with fractions.

A SPENDING SPREE

Miss H. L. Fox
C.E. Webster Junior Public School

Purpose: Experience with addition and subtractions of money

Level: Grades 3, 4

Materials: Mail Order Catalogues, such as Christmas catalogues

RULES

You have $100. You must buy 10 items. List the items that you will buy and write down the price of each item, and the page number. Person with ten items who is nearest $100, (but not over) wins the round if all prices and computations are correct.

Adaptations: Variation of money available and number of items.

THE EFFECT OF THE USE OF DESK CALCULATORS ON ACHIEVEMENT AND ATTITUDE OF CHILDREN WITH LEARNING AND BEHAVIOR PROBLEMS

Kan Advani
Educational Services Consultant
Frontenac County Board of Education

The report describes an experimental study conducted from January to June 1972 in a special class of 18 children (ages 12 to 15 years) with Learning and Behavior Problems for the purpose of determining the feasibility of using desk calculators in conjunction with classroom instruction in mathematics and their effect on the achievement, attitude and behavior of these children. Mathematics achievement was based on the results of the pre-test and post-test scores of the Stanford Achievement Test. Four calculators were placed in a corner of the classroom and students were encouraged to check their answers of math problems related to classroom work. The teacher also used the machines as a tool for the enrichment and reinforcement of a new unit. Comparison of the pre and post scores showed a significant difference (.01 level of confidence) and an analysis of questionnaire data indicated marked increases in student interest and positive attitudes toward mathematics. The author concludes that the use of calculators can facilitate mathematics instruction in a special class, provide a variety in the classroom, help release frustrations children have due to inaccessible numbers and help teachers in individualizing the math instruction.

INTRODUCTION

"Probably the greatest revolution in the teaching of arithmetic is the introduction of calculating machines." (6) Technology is gradually playing an important role in the present day education. We do find that some of these mechanical gadgets are able "to execute rather elaborate series of instructions (programs) and in many cases adequately serve the instructional needs of the students." (14)

STATEMENT OF THE PROBLEM

To determine whether or not Desk Calculators can be used in conjunction with classroom instruction in Mathematics with the children with Learning and Behavior Problems.

To determine whether or not the use of these machines will have any effect on the achievement, attitude and

behavior of the students.

HYPOTHESES

In an attempt to answer these problems the following hypotheses were formulated:

a. the use of the Desk Calculators in a class of children with Learning and Behavior Problems will result in their greater achievement on Standardized Tests,

b. the use of the Calculators will have a positive effect on the students' self-confidence and attitude toward learning thus improving their classroom behavior.

PROCEDURE

The criteria used to determine Mathematical Achievement of the 18 students were the results of Pre and Post tests on Stanford Achievement Test (Intermediate I - Partial Battery). The change, or difference, in the scores of each students was determined and the t - test of the differences between these mean differences was used, with the 5 percent level of confidence as the criterion for rejecting the null hypothesis.

EDUCATIONAL TREATMENT

Because these students were working at their individual levels in mathematics they were encouraged to complete their unit of work and then go to the calculators to check the answers.

When the teacher introduced a new unit with one student or a group of students which lent itself to reinforcement and enrichment by supplementary use of the calculators he brought that student or group to the Math Centre set in the far corner of the classroom and encouraged them to solve the problems of similar nature from the teacher prepared laboratory worksheets.

Quite often students brought the receipts of the grocery and other purchases made by their families and checked the totals on the machines.

SOURCES OF DATA

The study was conducted with eighteen boys and girls attending a special class at Victoria Public School, Kingston, Ontario. These youngsters, who were 12 - 15 years in age, were Educable Mentally Retarded and Neurologically Impaired.

They had I.Q. scores of about 68 to 116. The Calculators were received by the courtesy of local Olivetti Underwood dealer for a period of six months (January to June 1972). As the author himself was the teacher in-charge of the class, it helped keeping teacher variable relatively constant.

EARLIER EXPERIMENTS

After reviewing the past issues of "The Mathematics Teacher", and official journal of the American National Council of Teachers of Mathematics; "The Arithmetic Teacher" and ERIC (Educational Resources Information Center, U.S. Dept. of Health, Education and Welfare) Education Abstracts, I found a few articles and researches regarding the use of Calculating Machines in the regular classrooms but none referred to their use with the children with learning and behavior problems. Schlauch (11) in his study "The Use of Calculating Machines in Teaching Arithmetic" reported that the machines were a strong motivating force with no sacrifice of analytic skills.

Marsch (9) and Keough et. al (7) found that the use of calculators resulted in better achievement in mathematics.

Shoemaker (12) has developed materials employing the desk calculators in the High School Classroom, claiming

> "The calculator is a very important segment of the Mathematics laboratory. During the past few years, hundreds of instructors have found very favourable results in using the calculator as a means of reaching the slow achievers." (Page 24)

Broussard, Fields and Reusswig (3) stated that a program for low achievers from disadvantaged areas with emphasized real world applications and use of flow charts, calculators, and other materials resulted in significant achievement gain.

Beck (2), Stenzel (13), Gorenendyk (5), Longstaff, Stevens and King (8) found that motivation of the students was improved by the introduction of desk calculators into the classroom thus resulting in better attitude toward the subject. Reynolds (10) and Traverse and Knaupp (15) have urged the early introduction of computational aids in the elementary school curriculum. On the other hand, Cech (4) in his recent study conducted in a surburban High School of Illinois did not find improvement in the achievement or

attitude of the ninth graders. It, however, supported the hypothesis that the students could compete better with calculators than without them.

FINDINGS

Table 1
Comparision of the Pre - and Post - Test Scores (N:18)

	Mean Score	t
Pre-Test	40.5	3.09
Post-Test	43.0	
$p < .01$		

Table 1 shows that the post-test scores were higher than the pre-test scores and the difference was significant at the .01 level of confidence.

STUDENT INTEREST AND ATTITUDES

An analysis of questionnaire data indicated marked increases in student interest and positive attitude toward mathematics. When asked whether they would like to work on the calculators on the permanent basis, 17 out of 18 students answered affirmatively. One girl did not like the noise of the calculators while she was trying to concentrate on her work.

Students offered the types of responses categorized in Table II to an open-end item asking what was of most value doing math with the aid of the calculators.

Table II
Strength of Calculator Assisted Instruction
(N=18)

Responses	Frequency
It helped me understand better	6
It gives you the correct answer when you are wrong	5
It is fun	4
It is faster	3

Success breeds success, and failure does not provide motivation for learning nor does it release energy for learning. The sel-image of a child with learning and behavior problems, like any other child, is enhanced by experiencing success and diminished by frequent failure. Many of these children lack both hope of achievement and fear of not achieving. Machines helped in removing the fear of failure and provided structure through which child understands the computation process more clearly. Interest in math picked up and the children who were formally disinterested started finding problem solving a great fun.

In the beginning, students appeared rather overwhelmed with the complexities of the machines, but after this initial demonstration by the teacher, the enthusiasm and desire to master the "mechanical genius" rose. It didn't take them long to handle the 'tool' successfully.

Through these calculators it was easy to explain the why's of multiplications and divisions to some students. Most of the students gained an understanding of negative integers by using the calculators.

The presence of the calculating machines in the classroom brought a remarkable change in the behavior of the students. The calculators brought variety in the class. These children, who have comparatively short attention span, find it difficult to sit at one place for 40-45 minutes and concentrate on a task. A chance to go to the calculator did provide a change or a kind of rest and eased the tensions. Disruptive students became pleasant and the classroom atmosphere calmer and more controllable.

CONCLUSIONS

Based upon the data gathered from testing procedures and questionnaires, the following conclusions were drawn:

1. The student showed significant gains in Stanford Achievement Test scores from pre-test to post-test.
2. The attitudes of the students after working with the calculators for six months improved.
3. The calculators proved a help to the teacher in individualizing his instruction.
4. Calculators provided a variety in the classroom.
5. Calculators helped releasing some of the frustrations children had due to inaccessible numbers which resulted in better behavior.

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It may be suggested that further studies be conducted to determine the effectiveness of the calculator at different levels with the Exceptional Children and under varying conditions.

PRELIMINARY NOTICE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
TORONTO CONFERENCE

On November 2-4, 1973, the Name-of-the-site meeting of the National Council of Teachers of Mathematics will be held in Toronto. The Conference is devoted to provide sessions of interest to teachers of mathematics at all levels. Speakers on the program include:

Lola May
Frederick Mostellller
Robert Rourke
Max Sobel

Sessions will be held at the various levels K-6, 7-9, 10-13, and sections are also included for particular interest to the Colleges of Applied Arts and Technology.

In addition to the many sessions devoted to aspects of mathematics teaching both at the elementary and secondary level, a large number of workshops are provided to accommodate the needs of all participants at the conference.

A full complement of other features related to mathematics teaching is also provided for those attending the conference

- school exhibits display
- current free and inexpensive materials display
- films and other media can be previewed
- publishers display

Sightseeing and tours are being arranged to help fill in those "off" hours: visit the various unique features of Toronto while you are here. A program of hospitality will be available for wives who attend the conference.

Plan to attend. Mark the date

NOVEMBER 2-4, 1973

For information, contact:

Frank Ebos (NCTM)
Publicity Chairman
Faculty of Education
University of Toronto
TORONTO, Ontario M5S 2R7

Jack LeSage (NCTM)
Publicity Co-chairman
Ministry of Education
40 Eglinton Avenue, East
TORONTO, Ontario M4P 1A6

NEW COURSES IN MATHEMATICS

Have you written a new course in mathematics? We have heard of some. For example: *"Mathematics For the Social Sciences"* by Peter Weygang, Levack High School.

If you have other provisional course outlines we would be glad to receive a copy and help you publicize it.

Send an outline to PROJECTS COMMITTEE, O.A.M.E, 247 Bright Street, Sarnia, Ontario.

CANADIAN MATHEMATICS OLYMPIAD

ADVANCE ANNOUNCEMENT

Prior to 1973, candidates for the Canadian Mathematics Olympiad were chosen, on a provincial quota basis, by provincial committees. Ontario's allottment is 50 students and the Ontario committee consists of A. J. Coleman, G.F.D. Duff and K. D. Fryer (chairman). In the past, the top 50 students in the M.A.A. Contest have been selected.

In addition to the candidates chosen above, it is now possible to have some other candidates write the Olympiad. These candidates must be nominated on a formal application form by a school principal and a token fee is required. At this time, final details for the 1974 Olympiad are not known. However, the 1973 fee was $2.00. The contest was written on May 10, 1973 and nominations from principals were required by April 22, 1973.

It is assumed that nomination forms for the 1974 Olympiad (for those students not in the top 50 of the M.A.A. Contest) will be available from the Ontario chairman, K. D. Fryer, Faculty of Mathematics, University of Waterloo.

A CASE FOR INDIVIDUAL PROGRESS IN MATHEMATICS

D. Attridge
Thornlea Secondary School

We recognize the fact that every student in every classroom is not always ready at every time that every teacher wants to teach every topics. After each lesson some students understand thoroughly, some have a general understanding and others have tuned right out, or do not understand the topic at all. Thus, the next day, the teacher has to meet his students in various degrees of understanding. Now what does he do?

As teachers, we attempt to keep the better students happy by giving them enriched material, or we let them become bored while we attempt to bring the average students up to par. Similarly the weaker students will need a great deal of attention. How do we as teachers treat all of these problems (and there are many degrees of these problems)? How do we preserve some sanity as well as keep the attention of the majority of our students?

General Mathematics Classes

It is not my purpose to suggest that the following idea is the answer to all our woes, but I do suggest it as an alternative for teaching students who are generally not strong in mathematics. At our school we call them phase 3 students who follow the general mathematics programme (as compared to the advanced programme). Let me suggest the type of student who might be in these classes. He could be a very hard working student who lacks good background and/or sound logical thinking. He could be a very frustrated student who never seems to succeed for all manner of reasons, and thus has come to almost hate the subject of mathematics. He might be in a school where mathematics is compulsory, and he may not want to take the subject. Finally, he may have been prevented from taking other courses at the school due to a lack of prerequisites or because he was a discipline problem. Whatever the reasons, our general mathematics classes are quite a hodge-podge of students. The teacher has quite a job on his hands to keep control, let alone attempt to teach any mathematics.

At Thornlea S.S. we have tried a number of ways to stimulate these students to do some mathematics. Some have

worked and others have been less than successful. However, one of the ideas that seems to have caught on is that designed by a former teacher John Falk. John believed that every student has his good days and his poor ones, and that every student has his own rate of learning some material. Therefore, rather than keeping all students at the same place at all times, he wanted to let the students work at their own individual rates. If some want to, or can, work faster than others, then let them do so. Her is our method of operation.

Organization

First the teachers discuss what core material should be taught in each of our 12 courses in the four years of general mathematics (since we have a trimester system, we offer three courses per year with each term lasting about 13 weeks). The core material is located in the various texts in the school. Since mathematics textbooks are generally not written for students but for the teachers using them, we felt that the texts were too difficult for the students to read. This is a pet peeve of mine about textbooks in mathematics. While they are supposedly written for students to use, so often the amount of information that must go into the texts must cover all the known restrictions. With all of this added material, the general student cannot sift the wheat out from the chaff, and thus he finds the text incomprehensible.

Since the textbooks were not entirely satisfactory, John sat down and over the last three years rewrote the core material in the hopes that it would be easily readable and understandable to the students themselves. John's material gives the student the opportunity to try the problems himself and to generate some self-motivation through his own success.

Operation

When a student begins each term it is determined, in consultation with the teacher, where he is, to date, mathematically. He is placed in one of our courses and he starts reading and working with our prepared material for that course. Since a class may consist of students working on different courses, depending on their previous background, the teacher is constantly changing from one subject area to another as the questions arise. Each course is divided into from 3 to 5 different ideas. It is thought that the average student would be able to cover all of these ideas in the 13

week term alloted. However he may not be that alert and it may take him more than the 13 weeks to complete the required material. If this is the case then he will receive an incomplete rating at the end of that term. He reregisters for the course in the next term and attempts to complete the material then. If he does so, then he may begin the material for the next successive course.

When the student gets the material for the first idea of a course, he is expected to read it and attempt to do all of the problems involved. He may seek assistance from his colleagues and/or his teacher for as long as he wishes. When he feels that he understands the material sufficiently well, he asks to write a test on that material. The teacher, in order to determine if he is actually prepared, gives him a work sheet on which there are the very basic concepts involved in the packaged material. If the student seems to comprehend these very basic points, then he will be allowed to write the test. If not, the teacher will specifically point out some points for him to study further.

Testing

Each test is made up of from 4 to 6 questions, and the student must be able to do all of them satisfactorily (at our school "satisfactorily" means success at an 80% level or better). If he gets all of the questions correct then he obtains the next idea of work and applies himself accordingly. However, if he does not do all of them correctly, then the teacher and the student sit down and quietly, and in some detail, discuss the reasons why the student was unable to do the work. It is possible that he just made some minor mechanical errors. Then again he might have made some very serious errors and the teacher would have to explain these to him. However it is hoped that the worksheet would have indicated this ahead of time.

After the discussion the student goes back to his notes etc. and works on the unfinished areas. Then he writes another test on only the questions that he missed. This procedure continues until the teacher is satisfied that the student has proficiency in ALL of the area of the core material. When he finishes all of the units or ideas in one course then he gets his credit of either a pass or honours. If a student covers at least one-third of the material in one course in any term then he will not fail. He will receive an incomplete report at the end of that term.

Recipe Card Tests

The tests that John designed are also rather unique. Suppose there are to be six questions on a test. Each of these questions is printed on different coloured recipe cards. To be prepared for numerous tests, four similar questions are placed on the same coloured recipe cards. Thus, when a student wishes to write a test, he picks one card from each of the different coloured sets. He takes them to his desk and writes his own individual test. At the same time other students can pick six other different coloured cards for their individualized test. These cards are brought back to the teacher with the answers and the results are discussed in conference the very next day. Because of the different individualized tests, there seems to be little or not cheating.

Perhaps an example of a test on analytic geometry might help. Suppose one is testing slopes, intercepts, and equations of straight lines. The pink cards might all have the coordinates of two different points, and the question is to find the slope of the line segment joining these two points. The green cards might all have the equation of a line on it (different of course for each green card) and the student is to find the slope and the y-intercept of that line. The yellow cards might all have the coordinates of two points on them and the student is to derive the equation of the line passing through these two particular points, etc.

It should be noted that this material would never have been available if John had not been a part-time teacher. He is a former mathematics department head at a nearby school but left the classroom to go into the publishing business. However he still wanted to teach and try some experimental material. Thus he came to Thornlea S.S. to teach for one period per day. He himself admits that he would never have been able to do the research and writing if he had not been a part-time teacher. We are most fortunate to have his material to duplicate and now we have a ready-made programme for our phase three students, particularly at the grades 10 and 11 stage. John readily admits that the material is far from perfect and it is our intention to revise it as we see fit and if time allows.

Initially the programme requires a great deal of organization. Not only does the material have to be written but it must then be duplicated. Each idea generally covers about 10-15 pages of notes, and each course usually

has 5-6 ideas involved. Then each idea is tested with 5-6 questions, each duplicated four times, with different numerical values. Then there is a great deal of marking and the recording of these marks. Often a student will write the initial test and have to repeat two or three questions quite a few times before he has done them to our satisfaction (80%).

Most teachers are "hams" and like to be at the centre of attention. However this programme does not allow for much of the socratic approach at the front of the class. At the same time, no matter how good that "ham" might be, he does not always reach all of his students. This programme does allow for a great deal of very individualized help for a student when he needs it and where he needs it. Every day we are quietly talking to students with their own indivudual problems, giving them hints, and building up their background.

Reaction

How does the student feel about the whole affair? Remember the type of student that I described in these classes. We have found that the majority of student sare quite happy with this procedure. They like to have the material to work on at their own rate of progress. They do not have to be pressured into a test when not prepared or when others, including the teacher, happen to be ready. They like it because they will not fail provided that they show a good effort and attempt at least one-third of the ideas in each term. They may not receive a pass mark, but at least they will not have a failing report on their record. They will receive an incomplete result for that term's work.

Problems

Of course there are problems. Initially we concede that some students cannot settle down to working through material and finding the essential core material involved. Here is where the teacher at the front of the class would be of some assistance. Some students need to know exactly what they are to do each day. However we do constantly direct students with their particular problems when they ask. Then again we know that some students will not ask for help. It should be noted that we do NOT use this method for our advanced or phase 4 students, but only for the general mathematics classes in grades 10 and 11. In the odd case we offer this method to students who have already tried the grade 9 course and did not achieve success. Since we

are most interested in getting basic core facts into their heads, we think we have been quite successful to date.

We have few discipline problems because the students know exactly what they are to do. We admit that we do not always get them to classes but some students enjoy working on their own in the library or at home, or some other quiet place. We keep a close check on attendance but do not demand it every day. If a history essay is due one day or an English test is coming up the next period, why shouldn't the student use his time as he feels is best for him? We feel that 85% of our students are achieving at their own individual rate of progress, and their attendance indicates the same. Of course some students skip constantly and they will receive an unsatisfactory report. But they have failed because of their decision not to turn up in class, not because they could not do the work. Herein lies the benefit of our trimester system. After 13 weeks a student can start afresh and begin again. He can recognize that he made a mistake in the last term and he can begin anew. Too often, in the past, this student could not get a fresh start until a whole year had passed, and he may have become despondent and frustrated. At Thornlea he can get back on the right track after only 13 weeks.

Summary

In summary I would like to restate what I feel are the benefits of this system:

- the teachers like it because the students must do the material at an 80% proficiency level.
- the students like it because they can work at their own rate with no pressure from anyone but themselves. They will not fail if they keep consistently trying.
- there is little or no discipline problem and even the toughest kid feels that the system is fair.
- a student can work ahead and, if successful, do more than one term's work in any one term, thus completing courses ahead of schedule.

The problems lie in the fact that it takes a long time for preparation of suitable material. The students cannot always work under this system (but what system works for all students?). There is a great deal of marking and record keeping but this is compensated with little daily lesson prepartion. All in all the system seems to be running very nicely. When most people involved like the programme, perhaps we are doing the right thing at the moment.

A COMPUTER PROGRAM FOR TEST MARK NORMALIZATION

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For many years it has been customary to represent students marks as a percentage of the maximum mark; e.g. 17/25 = 68%. This procedure has two major failings:

(a) It assumes that a teacher can always design a 'fair' test. This is one in which the average hard working student obtains a comfortable pass.

(b) The teacher consciously or subconsciously adjusts the marking scheme so that the maximum is a 'nice' number, such as 10, 20 or 25. This process usually weights the test items in a somewhat arbitrary manner.

As a result of these two weaknesses it is not uncommon for an entire class to pass a test with high marks, or alternatively for every student to 'flunk'; which in turn raises the question of test score adjustment. I maintain that even an experienced teacher has difficulty in producing class tests which are 'fair', and especially now that heavy work loads and numerous meetings have eroded the time previously spent on the careful preparation of tests. It is apparent that a convenient and rational method for adjusting marks is badly needed.

The mark adjustment systems commonly used do not bear thinking about; such as 'adding a convenient mark to all scores', 'multiplying by a suitable factor', or 'mapping the raw scores onto a set of scores obtained from a normal distribution'; and so on. Any mark adjustment procedure should satisfy the following criteria:

(c) The normalized score (i.e. the score obtained by adjusting the raw score) should be easy to interpret by students, teachers, parents and administrators.

(d) The ranking of the students must be preserved.

(e) The relative spacing of the test scores must be maintained. By this I mean that if there is a large difference in raw scores between two

students, then a correspondingly large difference must be observed between their normalized scores. (Note: This is a major failing of most adjustment procedures.)

Eighteen months ago my colleague, Mr. Jim Burns, and I developed a computer program which would handle raw scores subject to the above criteria. We have been using this procedure in the mathematics department, where it has been accepted by students and staff as being fair, efficient and helpful. The development of the mark adjustment procedure hinges upon two basic assumptions:

(f) That in a reasonable sized sample - say 50 or more - the average performance of the class is the best available measure of what the students actually know. Unfortunately this seldom agrees with what the teacher thinks they should know!

(g) That the best available measure of the difference in performance between two students is the difference in their "Z" scores.

$$Z \text{ score} = \frac{\text{Students Raw Score - Mean Score for Sample}}{\text{Standard Deviation for sample}}$$

This assumption may be argued; but it does have the merit that it has some basis in statistics, and that it avoids the absurd results such as those produced by considering percentage differences or mark ratios.

Fundamentally our program ascribes a new value to the mean of the raw scores; (which can be thought of as a change of origin) and then changes the scale. It is apparent that the raw score distribution curve and the normalized score distribution curve are strictly similar. To obtain this end the computer program follows these steps:

(h) Reads the raw scores for each students. (S)

(i) Calculates the mean of the raw scores. (A)

(j) Calculates the standard deviation for the sample of raw scores (D).

(k) Calculates a Z score for each student.
$Z = (S - A)/D$

(ℓ) Calculates a Normalized Score (NS) for each

student NS = M + ZT.
Where 'M' is the false mean of the normalized scores - the new 'origin'.
Where "T" is the false standard deviation of the normalized scores - the new 'scale'.

The values of M and T are set by the teacher, usually in compliance to socially acceptable norms. For example a mean of M = 60 and standard deviation of T = 15 will produce, from any set of raw scores, a set of normalized scores which range from about 35 to 85, with, of course, a mean of 60. A very bright class might have M = 63 - 65; a very homogeneous class T = 12; and a heterogeneous class a T value of about 17. A teacher needs to reflect for some time on these parameters - experience and standardized tests will help in setting realistic values. However it is most important to realize that even when these parameters are chosen badly that relatively little error can result, only 2 or 3 marks. (I assume that no teacher with any sense of professional responsibility will choose M = 80 and T = 5!). This possible error is far less than the uncertainty produced by assuming that the test is 'fair', and then simply using the raw scores. In this program the M and T parameters may be changed at will; this is often necessary as some classes go to pieces and others grow together as the year progresses. The removal of a few "bad eggs", or the addition of a bright student can dramatically change the output of a class, and would warrant changes in M and T.

This program has a capacity of 12 tests for students; in addition to printing the raw scores for each student for each test, (so that students can check for any recording errors on the part of the teacher), it prints out the normalized scores for each test, and the term average for each student. Students who are absent for a test are not included in that particular calculation, and although this changes the sample from test to test, the change is neglibly small if the sample itself is a reasonable size. Absentees are denoted by -.1.

Advantages: The main advantages of this system are:

1. That it permits all test marks to be presented in the same manner, and subject to the same interpretation.

2. There is a considerable saving in teacher time. The marks are recorded directly onto optical scanning data

cards, (OMR) and since the term average is immediately available, that end of term computation session is avoided. The program was originally produced in punched cards; however, most teachers find the OMR more convenient.

3. Students have ready access to their marks, and are aware at all times of their progress, or lack of it. The students may take remedial action before it is too late.

STUDENT RAW SCORES

STUDENT NAME	1	2	3	4	5	6	7	8	9	10	11	12
CODE ABLE VERI	2.0	1.6	3.3	0.8	4.9	2.4	1.1	2.3	3.5	0.0	0.0	0.0
CODE BOTTOM JOHN	0.5	0.3	0.8	0.0	1.2	0.7	0.3	0.5	1.1	0.0	0.0	0.0
CODE CHANCER LIL	1.8	0.7	1.9	0.7	4.2	1.2	1.1	2.1	2.0	0.0	0.0	0.0
CODE DODGER JIM	1.3	-0.1	2.3	-0.1	-0.1	1.7	0.9	2.0	3.2	0.0	0.0	0.0
CODE EXIT BOB	0.6	0.5	0.5	0.1	0.3	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0
CODE FAY E JANE	1.1	0.9	2.0	0.6	3.2	1.7	1.0	1.8	3.1	0.0	0.0	0.0
CODE GUIDANCE NEED	1.2	0.7	1.5	0.6	4.3	2.1	0.8	1.4	3.2	0.0	0.0	0.0
CODE HOPE BOB	1.1	0.6	1.7	0.5	3.2	1.5	0.7	1.2	2.5	0.0	0.0	0.0
CODE INERT SAM	0.7	0.6	1.1	0.3	1.5	0.9	0.4	0.8	1.2	0.0	0.0	0.0
CODE JAMES JESSE	1.3	-0.1	2.6	0.5	3.7	-0.1	0.6	1.3	1.7	0.0	0.0	0.0
CODE KONSTANT AM	1.3	1.0	2.2	0.6	3.9	1.8	0.9	1.7	3.1	0.0	0.0	0.0
CODE LOSER JACK	0.5	0.3	1.0	0.5	1.4	1.0	0.4	0.7	1.8	0.0	0.0	0.0

STANDARDIZED NORMAL SCORE

STUDENT NAME	1	2	3	4	5	6	7	8	9	10	11	12	A
CODE ABLE VERI	8.8	9.6	9.0	8.0	8.0	8.6	7.9	8.2	7.9	0.0	0.0	0.0	8.4
CODE BOTTOM JOHN	3.9	4.2	4.1	2.9	4.2	3.6	3.5	3.5	3.6	0.0	0.0	0.0	3.7
CODE CHANGER LIL	8.2	5.9	6.3	7.4	7.3	5.1	7.9	7.7	5.2	0.0	0.0	0.0	6.8
CODE DODGER JIM	6.5	-0.1	7.0	-0.1	-0.1	6.5	6.8	7.4	7.4	0.0	0.0	0.0	7.0
CODE EXIT BOB	4.3	5.0	3.6	3.6	3.3	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	3.9
CODE FAYRE JANE	5.9	6.7	6.4	6.8	6.3	6.5	7.3	6.9	7.2	0.0	0.0	0.0	6.7
CODE GUIDANCE NEED	6.2	5.9	5.5	6.8	7.4	7.7	6.2	5.9	7.4	0.0	0.0	0.0	6.5
CODE HOPE BOB	5.9	5.5	5.9	6.1	6.3	6.0	5.7	5.3	6.1	0.0	0.0	0.0	5.9
CODE INERT SAM	4.6	5.5	4.7	4.8	4.5	4.2	4.1	4.3	3.8	0.0	0.0	0.0	4.5
CODE JANES JESSE	6.5	-0.1	7.6	6.1	6.8	-0.1	5.2	5.6	4.7	0.0	0.0	0.0	6.1
CODE KONSTANT AM	6.5	7.1	6.8	6.8	7.0	6.8	6.8	6.6	7.2	0.0	0.0	0.0	6.9
CODE LOSER JACK	3.9	4.2	4.5	6.1	4.4	4.5	4.1	4.0	4.9	0.0	0.0	0.0	4.5

// XEQ

Teachers, guidance counsellors, students and parents can interpret this information easily:

Examples: (1) 'Need Guidance' found that his marks were dropping, and after a visit to the counsellor organized his work habits more effectively, with good results.

(2) 'Bob Exit' soon noticed the writing on the wall - and transferred to a lower level; much more efficient than waiting a full term.

(3) 'Jim Dodger' is often absent on test days, but he is obviously a gifted student.

(4) 'Lil Changer' is one of those 'variable stars'; good one day, poor the next. This seems a good case for guidance and parental involvement.

Note: 'CODE' is a four element space for an identification number e.g. M232.

```
                    REAL ITEST(80,12)
      DIMENSION     NAME(80,12)
      READ          (8,1) AVMK,DEV
 1    FORMAT        (2F4.2)
      WRITE         (3,2)AVMK,DEV
 2    FORMAT        (4X,'THE AVERAGE '
     X'MARK IS',7X,F4.2,
     X/,4X,' THE ',
     X'STANDARD ',
     X'DEVIATION IS',2X,
     XF4.2)
                    J=0
                    ISNO=0
                    RCNT=0.0
34                  J=J+1
      READ          (8,3)
     X(NAME(J,I),I=1,9),
     X(ITEST(J,K),K=1,
     X12)
 3    FORMAT        (9A2/6F3.1/6F3.1)
      IF            (ITEST(J,1)+.9)34
     X,43,34
43                  J=J-1
      DO            1121=1+J
*     IF            (I-1)111,110,111
110   WRITE         (3,10)
 10   FORMAT        (1H1,35X,7HSTUDENT
     X,11H RAW SCORES)
      WRITE         (3,11)
 11   FORMAT        (1HO,10H  STUDENT
     X,5H NAME, 6X,2H1 ,
     X15H   2    3    4
     X15H   5    6    7
     X15H   8    9    10
     X14H   11   12    )
111   WRITE         (3,12)
     X(NAME(I,L),L=1,9),
     X(LTEST(I,N),N=1,
     X12)
 12   FORMAT        (1HO,9A2,12F5.1)
112   CONTINUE
      DO            133 K=1,12
                    ISNO=J
                    TOT=0.0
                    DO 121 I = 1,J
                    IF(ITEST(I,K)+.1)
     X120,113,120
```

```
113                   ISNO=ISNO-1
                      GO TO 121
120                   TOT=TOT+ITEST(I,K)
121    CONTINUE
                      IF(TOT-0.0)122,140
     X,122
122                   RMEAN=TOT/
     XFLOAT(ISNO)
                      SDTOT=0.0
       DO             130I=1,J
                      IF(ITEST(I,K)+.1)
     X123,130,123
123                   RCNT=(RMEAN-ITEST(
     XI,K))**2
                      SDTOT=SDTOT+RCNT
130    CONTINUE
                      RSTDV=SQRT(SDTOT/F
     XLOAT(ISNO))
                      DO 132I=1,J
                      IF(ITEST(1,K)+.1)
     X131,132,131
131                   TSCR=AVMK+(((ITEST
     X(I,K)-RMEAN)*DEV)/
     XRSTDV)
                      ITEST(I,K)=TSCR
132    CONTINUE
133    CONTINUE
140                   K=K-1
       DO             213I=1,J
       IF             (I-1)210,143,210
143    WRITE          (3,13)
 13    FORMAT         (1H1,25X,7HSTANDAR
     X,13HDISED NORMAL ,
     X5HSCORE)
       WRITE          (3,20)
 20    FORMAT         (1HO,10H   STUDENT
     X,5H NAME, 6X,2H1 ,
     X15H   2    3    4
     X15H   5    6    7
     X15H   8    9    10
     X14H   11   12   A)
210                   TOT=0.0
                      ISNO=0
                      AVRG=0.0
       DO             212L=1,K
       IF             (ITEST(I,L)+.1)
     X211,212,211
211                   TOT=TOT+ITEST(I,L)
```

```
                        ISNO=ISNO+1
                        AVRG=TOT/
     XFLOAT(ISNO)
212   CONTINUE
      WRITE             (3,21)
     X(NAME(I,L),L=1,9),
     X(LTEST(I,N),N=1,
     X12)
     X,AVRG
 21   FORMAT            (1HO,9A2,13F5.1)
213   CONTINUE
      CALL EXIT
      END
```

The program is designed for 80 students and 12 tests. The dimensions can easily be changed; however the relevant format statements would also need to be changed.

* By adding further instructions here the heading "Student Raw Scores" etc. can be made to appear at the top of each sheet, or for blocks of students, say every 20.

BIRTHDAYS, CARDS, AND MATCHES: AN EXPERIENCE IN PROBABILITY

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In contemporary mathematics curricula, probability is playing an increasingly important role and when introducing probability concepts many teachers make use of problems which have surprising solutions. These non-routine problems are often interest-generators which produce student involvement. "Matching" problems are sometimes used for this purpose.

For instance, a well known probability query is known as *the birthday problem*. This problem is: In a room of n people, what is the probability that at least 2 people share the same birthday anniversary? A surprising outcome of this problem is that in a room containing 23 or more randomly selected people, the probability of at least one "match" is greater than $\frac{1}{2}$. For an extensive solution of this problem, the reader may consult any of the references listed at the conclusion of the article.

A similar type of experience may be performed with ordinary (52 card) decks of playing cards: Suppose that each of a group of n students ($n \leq 52$) has his own deck of well-shuffled playing cards. If each draws out a random card from his deck, what are the chances that at least two will draw exactly the same card (same suit, same numerical value)? This experience may be considered preferable as an illustration to the birthday problem since it may be repeated easily; for obvious reasons the birthday problem is not as readily repeatable with a single group of people!

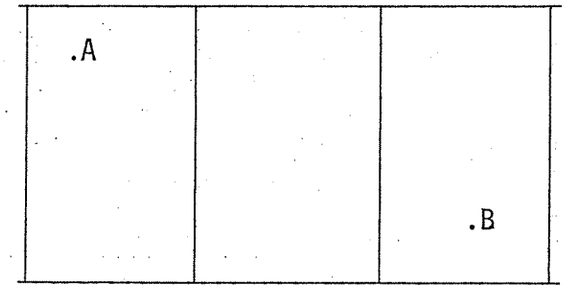
Some students will "solve" this problem by plausible but incorrect techniques. For instance, some will speculate that if there are 13 people drawing and 52 different cards from which to choose, a match will occur about $\frac{13}{52}$ or $\frac{1}{4}$ of the time. However, experiments suggest a much greater likelihood. A consideration of the probabilities involved indicates that, in fact, in the experiment using 13 people, a match will occur, on the average, about 80 percent of the time. Why is this figure so high?

To answer this question, let us list some pertinent probability principles:

1. If an experiment has n equally likely outcomes and if m of them are considered favourable, then the <u>probability</u> of a favourable outcome is m/n.

2. If the probability that a certain outcome occurs is m/n, the probability it does <u>not</u> occur is $1 - m/n$.

3. (Fundamental Principle of Counting) Suppose task A can be performed in m ways and, after task A is completed, task B can be performed in n ways. Tasks A and B can then be consecutively performed in mn ways. This principle can be similarly extended to any number of consecutive tasks -- for instance in the drawing below there are 120 ways $(5 \times 4 \times 6)$ of going from point A to point B by going through each "wall" exactly once. One such path is depicted on the drawing.



Returning to the card problem, suppose two people, June and Cindy, draw cards, each from their own decks. In how many ways can this be done? Since June can perform her task in 52 ways and Cindy can then perform her task in 52 ways, by principle 3 the two tasks can then be performed in 52×52 ways.

How many of these possible outcomes yield <u>no</u> match, that is, the cards drawn are different? To get <u>no</u> match visualize June first drawing her card - she can do this in 52 possible ways. When she has completed this, Cindy then draws her card in one of <u>51</u> ways (remember that she is not to match June). Thus by principle 3, there are 52×51 possible outcomes for which no matches occur. Principle 1 then indicates that the probability of <u>no</u> matches for the June-Cindy experiment is

$$\frac{52 \times 51}{52 \times 52}.$$

Principle 2 then implies that the probability of at least one match is

$$1 - \frac{52 \times 51}{52 \times 52} .$$

If three people, June, Cindy and Ginger are involved, the reasoning is similar. The three people can draw three cards in $52 \times 52 \times 52$ ways. To get no matches, let June draw her card (52 choices), let Cindy draw a card different from June's (51 choices) and let Ginger draw a card different from both June's and Cindy's (50 choices). Three different cards can be therefore drawn in $52 \times 51 \times 50$ ways. The probability of no matches is then

$$\frac{52 \times 51 \times 50}{52 \times 52 \times 52} .$$

The probability of at least one match is

$$1 - \frac{52 \times 51 \times 50}{52 \times 52 \times 52} .$$

Continuing this pattern, if n people draw cards, each from his own deck, $(52)^n$ different outcomes result. Since $(52)(51)(50) \ldots (52-n+1)$ of these outcomes yield no matches, the probability that no matches will result is

$$\frac{(52)(51) \ldots (52-n+1)}{(52)^n} .$$

The probability of at least one match resulting is then

$$1 - \frac{(52)(51) \ldots (52-n+1)}{(52)^n}$$

(Formula 1).

Table 1 includes the results of performing the calculations indicated in Formula 1.

TABLE 1

No. of people drawing cards	Probability of at least one match
2	.019 or 1.9%
3	5.7%
4	11.1%
5	18.0%
6	25.9%
7	34.4%

No. of people drawing cards	Probability of at least one match
8	43.2%
9	52.0%
10	60.3%
11	67.9%
12	74.7%
13	80.5%
14	85.4%
15	89.3%
16	92.4%
17	94.7%
18	96.5%
19	97.7%
20	98.5%
⋮	
38	99.9999%

For example, note that the probability of at least one match when 13 people draw cards is 80.5%; most beginning probability students would predict a somewhat smaller probability.

These probability experiences are but two examples of problems which have surprising but interesting solutions. The creative mathematics teacher can build his own collection of challenging and motivating problems if he only asks the question: "What happens if ...?"

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BOOK REVIEWS

History of Mathematics, by D. E. Smith, General Publishing Co. Limited (Dover) ($9.50 Vol. I & II) pp. 1254.

It has long been our contention that students of the pedagogical art in Education Faculties could find more use for the history of their disciplines, than for some of the methodology so often taught, and which seems to change with the vagaries of fad and circumstance. ("open-concept school" = "big, new, city, little, red, country, school house"; "spiral approach" = "we know that drill and repetition are necessary, but don't like the sound of the words"; "help underachievers attain their intellectual aspirations" = "how can we get those slow kids to learn <u>something</u> which will be of use to them in life".)

This book, in two volumes, confirms our long-standing opinion in two ways:
(1) it affords a good, connected insight into the development of much of the elementary mathematics used today
(2) it reveals the way "civilized" and "intellectuals" viewed others fifty years ago, thus casting appreciable light on some very modern social ills.

Of point (1) we should perhaps say that much of the material treating western Europe is probably still valid, but the sections in China, Japan and the Middle East will bear very extensive revision in light of modern research by such people as Price, *Science Since Babylon* and Price, *The History of Chinese Science and Technology* and discoveries made in wrecks off Turkey and Greece by the National Geographic Society.

We view the entire first Chapter on "Prehistoric Mathematics" as a waste of time, based almost entirely on unprovably conjecture, and replete with offensive phrases

such as "when the primitive savage began to develop number names, the process was a tedious one", "... use of small numbers ... all his meager life demanded of mathematics ..." "... judging by other races of low intelligence ..." "... number sense ... inferred from the speech of certain lower types of savages ...". We firmly believe that a history book should be just that, and unbased conjecture has no place in it. It is sad, too, how accurately the phraseology quoted above reflects the persistence of the idea of Manifest Destiny and Herrenvolk well into the present century. (If, indeed, they be dead even now.) It goes without saying that the Australian aboriginal lack of vocabulary is featured on page 7 of Vol. I. More modern work has, of course, largely laid this one to rest, but we feel that students studying these books should be so advised by their teachers.

As to good points, there is an immense volume of useful knowledge compressed into the 1254 pages of text, and taken as a whole, it is well thought out and balanced. For $9.50 it seems a good buy, and a student could always check out the objectionable Chapter One with a more modern work such as Menninger, *Number Words and Number Symbols*.

We should be inclined to recommend the latter in place of Smith except that the emphasis is so very different. In fact, we do not know personally of any other book in English, at anything near the price, with such a wide scope, extending as it does, from country to the elements of Calculus.

To sum up, this book is qualifiedly recommended as a text, and should certainly be in libraries, perhaps with addenda warning against the more objectionable portions. It is perhaps our duty, in this connection, to point out that the Chronological Table at the end of Vol. I is reasonably accurate only from about 1650 B.C. on in light of more modern sources.

L. D. Black, Lakehead University

Letters on Probability, by Alfred Renyi, Wayne State University Press, 1973 ($2.95) pp. 86.

A clever collection of expository gems that belongs in every mathematics library, this little book has enough depth for a mathematics major, yet can be easily followed

by a grade school pupil.

It purports to be a correspondence between Pascal and Fermat in the 1600's in the form of a collection of letters. This genre is very old and respected, although we found it a little forced, and in the hands of a lesser master than Renyi could easily have been grotesque. We had the privilege twelve years ago of hearing Renyi and his lovely daughter doing a Socratic Dialogue at Manitoba, and have been a fan every since. We recommend anything that bears his name, and this little volume is no exception.

L. D. Black, Lakehead University

The Exact Sciences in Antiquity, by O. Neugebauer, General Publishing Co. Limited (Dover) ($2.95) pp. 240.

This reviewer had many short hours of delight out of reading this book. Perhaps its most unique feature is its goegraphical and chronological chart facing the title page. This enable one to get a world perspective of history almost at a glance. The actual photographs of primary soucres near the end of the book were also most edifying, and inspire great respect for the men who have been able to reconstruct, however partially, the glories of past intellectual achievements from the pitifully inadequate traces that have remained with us.

The first chapter is particularly fascinating for the insight it gives us into the methods of scientific historian. Chapter III goes further into the matter of sources, and the evaluation thereof. The tragedy of indiscriminate excavation without subsequent publication is exposed, and we are forced to the conclusion that in this, as in other of men's affairs, scientists and their backers have not proven to be more wise than their illiterate predecessors.

The rest of the text per se is a fine exposition of Egyptian, Babylonian and Hellenistic Astronomy and Science. This reviewer was very happy to see in Appendix I, the old lie dispelled that "Copernican astronomy was accepted in preference to Ptolemaic because it was simpler" which has been foisted off by generations of well-meaning teachers and semi-competent school texts on innocent, unsuspecting students. Instead, the author expounds ... "the very elegant methods ... of ... one of the greatest masterpieces of

scientific analysis every written" ... . It is also gratifying that the author allows that practitioners of this art must have been aware of the discrepancy between observed and predicted values of lunar diameter, although retaining the overall theory because it predicted longitudes correctly.

It was also interesting to find that it is now known that ibn ash-Shatu corrected Ptolemy's lunar model almost 200 years before Copernicus, although whether the latter was aware of this, we do not know.

This reviewer was gratified, because we have long felt it an unfair criticism of Ptolemy the fact that he based everything on circles, when that, in effect, is what one does with Fourier Series anyway.

For a minor criticism: we feel the title demonstrates a certain provincialism of outlook, in view of the lack of chapters on Hindu, Middle American, and African (other than Egyptian) antiquity. This is truly a minor complaint, and we do not hesitate to recommend the book accepting the author's choice of subject matter limitation and living with it.

L. D. Black, Lakehead University

Numerical Computing, by L.F. Shampine and R.C. Allen, Jr., W. B. Saunders Co., 1973, pp. 258.

This introductory text in Numerical Anslysis is designed as a one semester course for students with a background knowledge in Calculus and FORTRAN. As such it does not, of course, pretend to give a complete account of methods available.

It covered interpolation, approximation, integration and linear, non-linear and differential equations, and contains also a discussion on floating point arithmetic. FORTRAN programs for some of the methods discussed are included.

The presentation is mostly clear and straight forward and there are plenty of examples (with solutions!). There are also adequate references for additional reading.

Minor and major criticisms: why is the reader advised in the introduction to peruse the book in a different order to that in which it is printed, and the absence

of an index is to be regretted.

K. H. V. Booth
Lakehead University

Desi Decimal Series: (1) The boy who didn't like division, (2) Metric activities, (3) Desi learns short cuts, (4) Temperature, (5) History, (6) Liquid volume and (7) Desi goes metric, by Calogeridis, Mutton and Richard, Holt, Rinehart and Winston of Canada, Limited, Toronto, 85¢ each.

This new series, consisting of seven soft cover booklets which range from 25 to 45 pages each, is part of the current outpouring of metrication materials. As such it is not worse than much of the rest that is presently flooding the market, and perhaps better than some.

Its best feature is the happy integration of reading and arithmetic, so that many children will be likely to think of it as a story book rather than an arithmetic text. The black and white cartoons on each page are quite effective. The text itself deals sympathetically with the student who is likely to find himself caught and confused during the transition period from pound-inches to metrics.

However the text is long on rules but short on instilling metric sense or even common sense, and mathematical thinking. For instance the boy-in-the-story is asked by the grocer to divide a bag of rice containing 8350 ounces into one pound bags. That is over 500 pounds. Some bag! Similarly he is asked to pour a large, hat-box shaped medicine jar which contains 653,821 millilitres into litre jars. A medicine jar containing some 150 gallons? The recommended procedure for finding the answer is to move the decimal point 3 places to the left. Little explanation! The mechanical approach is further emphasized by tables which state that in order to change miles to kilometer, one multiplies by 1.6, while to convert from kilometres to miles one multiplies with 5/8.

If conversion tables were deemed necessary, a statement that 1 mile = 1.6 km would have been more desirable. That way it would have been left to the student to decide whether he ought to divide or multiply. As it is the tables may confuse the memory of the students as it apparently confused the authors or whoever was responsible who wrote

that in order to change "quarts to litres -- divide by 1.14". It should of course have read: multiply by 1.14. Similarly the table converting liters to quarts should read divide by 1.14 instead of multiply.

The few activities are of the type where the student is presented with a completely worked out example before being asked to do an almost identical one, save for the numbers.

The overall assessment is that while this series is better than some of the material already on the market, it still is not good enough to recommend for classroom use.

G. Vervoort, Lakehead University

College Mathematics for General Education by Nichols, Holt, Rinehart and Winston, Inc., 1970, pp. 300; $10.95.

This book was written in response to the demand for a course in the elements of mathematics for the non-majors in mathematics or science. These students, hopefully, will increase their appreciation of mathematics through structural study and, at the same time, concurrently with this study, will become competent in the fundamental skills. Fundamental notions are presented in a simple and clear manner.

The book starts out with the study of logic and sets, the basis for structural study. A study is then made of the field (well-defined), of finite systems (various bases) and of the notion of groups (concisely, and to the point). The chapter names and section headings are set off in attractive colouring.

The number systems, the natural numbers, the whole numbers, the integers and the rationals, are looked at structually. The real numbers are introduced. There is a chapter on graphs, equations and inequalities. There is an appropriate section on using equations to solve problems.

These academic considerations number six chapters and this is the basic course. There are five further chapters on numer theory, exponents and radicals, probability, trigonometric ratios and functions and, in conclusion, an introduction to matrices.

There are 162 pages in the basic six-chapter course, plus 133 pages more for the optional five chapters. The study of the while book would require about four lecture hours weekly for a semester.

In the chapter on number theory, are the usual considerations regarding primes. There are problems on the greatest common divisor and least common multiple. The chapter closes with a word on the fundamental theory of arithmetic.

The chapter on exponents and radicals is the standard treatment of powers and roots. A word about scientific notation closes the chapter.

The chapter on probability takes the student with set notation through the probability of the occurrence of two events.

Trigonometric Ratios and Functions consideres first similar triangles. The trigonometric ratios are defined for the right triangle. The solution of right triangles and the computation of inaccessible distances follow. The chapter closes with the analytic definitions of trigonometric functions on the unit circle.

The concluding chapter is on matrices. 2×2 matrices are added and subtracted. We have the theorems on commutativity and associativity. 2×2 matrices are multiplied. This leads to the multiplicative identity and inverse. Mention is made of higher order matrices and of their applications.

Each chapter closes with a glossary of significant terms in the chapter, followed by a set of review exercises. There are answers to selected problems in the back of the book.

A manual for the instructor accompanies the textbook. This manual contains answers for all problems found in the text, and there is an alternate set of review exercises for each chapter.

All in all, the treatment of topics is standard and orderly, and the topics chosen are those with some appeal for the average liberal arts, non-mathematics majors.

A student who mastered the whole text would be quite knowledgeable in the mathematics field.

Marc A. Laframboise
University of Detroit

Relations, Transformations and Statistics, by Elliot, Fryer, Gardner and Hill, Holt, Rinehart and Winston, pp. 467 (1972).

This book is in large measure a response to the need for three books to fit the three Grade XIII mathematics courses fo the Ontario curriculum. This one is intended for what we call the Math A course. The authors have taken their previous book *Functions, Relations and Transformations*, and combined it with their texts *Algebraic Structures and Probability* and *Vectors and Matrices* to produce this new edition. Besides, there are new additions, such as the chapters on mathematics of investment and statistics (presentation of data).

The first three chapters are idential to chapters two, three and four of the *Functions, Relations and Transformations* text. Chapter four on translations was the seventh chapter of the Functions text. Continuing, chapter five (reflections and dilatations) is the former chapter nine; chapters six (trigonometric functions), seven (compound angles) and eight were chapters five, six and eight respectively. Chapter nine and ten (on matrices) were the former chapters seven and eight respectively of *Vectors and Matrices*. As previously mentioned, these are identical, word for word. The examples and exercises are the same. The only apparent difference is that the authors have put important points to be remembered in blue print for emphasis - a useful idea, really. Chapters thriteen, fourteen and fifteen (arrangements and subsets, the binomial theorem, and probability) were chapters two, four and five of the *Algebraic Structures and Probability* text. I must say I am sorry to see that the authors still refuse to call permutations and combinations as such, and still use this "arrangements and subsets" title.

In the new section, the mathematics of investment protion is well done and we see the use of the $a_{n\rceil}$ and $s_{n\rceil}$ formulas rather than that ugly geometric series method. Tables greatly facilitate what used to be a tedious calculation. Emphasis is on the motivation and usage of these

tables. The statistics section is clear and presents many worked out examples. This is like a breath of fresh air in comparison to Del Grande's text, *Relations*, which gives a treatment on statistics that is just awful. There are graphs that explain the frequency distributions well.

All in all, if you enjoyed the older texts of this series, this new one will not disappoint you. The language is clear and not too much confusion can follow. There is a good set of exercises in each chapter, and the teacher will not find himself lacking for material to teach. In terms of depth, the book by Del Grande is more comprehensive. Unfortunately its presentation at times is too insular. What other books talk of "winding functions", for example? A good text ought to prevent students from finding other standard texts in later courses to be of "unfamiliar notation". The one by Scroggie et al is simply unsuitable for the average Grade XIII student, in that it is just too wrapped up in symbology and the set theory approach. Hence this one seems to be a pleasant alternative.

Myer Kalin
South Carleton High School

Numeration Systems, No. 6 in a series of seven optional topics supplemental to Mathematics 410, by L. Levine, published by Copp Clark.

This booklet (54 pages) contains optional material for Grade 10 students.

The first 7 pages concern the historical use of number systems other than our decimal (base 10) system. I suggest it should be eliminated or thoroughly revised. The statement that no older number system combined the use of a base with place value and a "zero" symbol is incorrect. Indeed, the Maya had such a system. The information on a Greek system of numeration is misleading as no indication is given that at different times and places the Greeks used different systems. The treatment of the Babylonian system and in particular the examples of large numbers written in this system are incorrect.

The remainder of the booklet consists of techniques used in changing from a number system in a given base to a number system in a different base. Many examples are given and particular attention is paid to binary notation

because of its role in the operation of computers. Also briefly considered are binary codes in which binary notation is used to represent the digits of a decimal numeral. The latter part of the booklet is written in an easy to read and clear manner. A puzzling aspect of this portion is the appearance of diagrams with no explanation of their significance in relation to the text.

Finally the presence of the Americanism concerning Abraham Lincoln and his Gettysburg address is not likely to be appreciated by Ontario students.

M. P. Closs
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Mathematical Pursuits One, by R. Wigle, P. Dowling, P. Jennings, Macmillan of Canada (356 pages, $6.95).

Mathematical Pursuits One is a refreshingly different text which is useful for every teacher of Mathematics. It is the first in a new series of Canadian texts designed to help and encourage secondary school students to experience mathematics, and the reviewer believes that Departments of Mathematics should give this series serious consideration when deciding on a new text.

The series was developed when Robert Wigle and his associates at Aldershot High School (Burlington) became dissatisfied with their methods of teaching mathematics. They searched for materials of greater relevance and interest to students, and became convinced that the imaginative ideas presented by the School Mathematics Project in England constituted a better approach to teaching Mathematics. Wigle and his group experimented with this material and then began work on a Canadian program which would retain the flavour of the School Mathematics Project while offering topics more in line with Canadian curriculum needs. Their experimental programs have gone through two revisions, have been class tested and the final form is being published.

Mathematics Pursuits One is now off the press (Macmillan, $6.95) and is intended for Grade IX, and may be used for both average and above average students.

Chapters in *Mathematical Pursuits* can stand as independent units and hence it is possible for a teacher to

choose the topics and order best suited to his situation. Chapter headings of *Mathematical Pursuits One* are: (1) Co-ordinate Systems and Relations, (2) Basic ideas in Arithmetic, (3) Angles and Polygons, (4) Symmetry, Reflection and Rotation, (5) Patterns in Algebra, (6) Equations and Inequalities, (7) Ratio and Proportion, (8) Areas and Volumes, (9) Similar Figures, (10) Introduction to Vectors, (11) Patterns in Numbers, (12) Pythagorean Theorem and Irrational Numbers, (13) Presenting Data, (14) Introduction to Probability, (15) Review Exercise.

In each Chapter, the authors have addressed themselves directly to the student, with a minimum of explanation and clear simple illustrations. There are no "model solutions" but the exercises are largely developmental with built in drill. Thus the material is ideal for individual progress but could be used effectively in the more traditional way. The reviewer can see four ways of using this text. Firstly, it could be used very effectively as a class text in the usual way. Secondly, the teacher could have a class set, and have the class do some chapters on their own. Thirdly since this series grew out of a desire of the authors to develop a program that would allow their students to pursue the study of mathematics at their own rate, the text is ideal for those also who prefer individual progress and/or individualized programming. Lastly, the text could be used as a source book for excellent teaching ideas, particularily ways of introducing topics.

Although the many topics included are all presented at a suitable level for Grade IX, and are all interesting and worthwhile, the reviewer believes that the text would have been even more valuable if fewer topics had been attempted and the pages thus saved devoted to more work on some basic topics. For example, Chapter 4 (Symmetry, Reflection and Rotation), Chapter 9 (Similar Figures) and Chapter 10 (Introduction to Vectors) could have been left for Grade X, and more material could have been added on Arithmetic (particularly Business Arithmetic and applications of percentage) Equations (especially word problems) and Mensuration (particularly volume which gets a very skimply treatment).

Although a first look at the text might lead one to think there is insufficient drill and review, the authors have done a good job of "building in" the review in the development of new topics. They have been especially

successful in using co-ordinates in several chapters, in showing how order, patterns and relationships pervade all aspects of mathematics and in showing algebra-geometry relationships in various topics.

The text emphasizes the use of concrete materials, models and the process of trial and error to solve problems and build hypotheses. The material is presented to allow for inductive teaching and learning because the authors talk directly to the students and lead him in discovery. Alternative solutions are accepted. Skills are reinforced by numerous exercises and by using the skill and concept in a different context in a later chapter.

The authors intend to prepare a teachers' guide and are willing to conduct a workshop in any area.

In conclusion, the reviewer wishes to congratulate R. Wigle and associates, and Macmillan Company for producing a text that speaks to the student and allows him to progress in Mathematics more on his own terms.

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