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▲ Editorial



LOUIS LIM, EDITOR
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Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning (Principles and Standards for School Mathematics, *2000, p. 24).*

Graphing calculators, Calculator-Based Rangers (CBRs), *Geometer's Sketchpad* software, *Fathom* statistical software, and the Internet are some tools that have revolutionized the way mathematics is now taught and learned. Technology has played a significant role in shifting the focus of traditional skill and procedural acquisition to conceptual understanding, modelling, reasoning, and problem solving: "technological tools extends the range of problems accessible to students and also enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modelling" (p. 25). These tools also provide a visual picture or graph to help students make the transition to the (abstract) algebraic model.

There may be concern that technology weakens students' computational skills. But is sole acquisition of computational skills and procedures the only focus of mathematics learning? The NCTM's *Principles and Standards for School Mathematics* (2000) reminds us, "as with any teaching tool, it (technology) can be used well or poorly" (p. 25). Take for instance the class set of TI-92 Plus graphing calculators that are in my mathematics department at Unionville HS. Is technology being used to replace computations that were traditionally completed by paper-and pencil? The TI-92 Plus, like the Ministry licensed *Maple* and *Theorist* softwares, have built-in computer algebra systems. Much of the computations that involve algebra and calculus can now be done by pushing buttons without any understanding. For instance, the derivative of $y = (2x + 3)^{15}$ can be calculated using the TI-92 Plus by pressing, F3, selecting 1, and then inputting (2x + 3) ∧15, x). A teacher could use the TI-92 Plus more appropriately, I think, by having students **develop** a rule for calculating derivatives using the Chain Rule. I have used the following worksheet in my calculus classes so students see that:

$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ and that $\frac{d}{dx}f(g(x)) \neq f'(g(x))$:

Function	Guess the Derivative	Actual Derivative Using TI-92 Plus
$y = (2x + 3)^{15}$		
$y = (4x + 7)^{20}$		
$y = (2x - 7)^3$		
$y = (2x^5 - 7)^3$		
$y = (x^2 - 5x - 1)^3$		
$y = (x^2 + 3x - 8)^5$		
$y = (5x^2 + 3x - 7)^4$		
$y = (3x+5)^{\frac{1}{2}}$		
$y = (x^4+1)^{\frac{1}{2}}$		
$y = (3x^2-5x+5)^{\frac{1}{2}}$		

Technology is not a luxury item for select classrooms since the Ontario mathematics curriculum documents mandate their use by saying: "collect data, using appropriate equipment and/or technology..."; "...through investigations facilitated by graphing technology"; "...through investigation, using dynamic geometry software"; "...using spreadsheets or other appropriate software"; "use graphing technology effectively..."; "demonstrate, using technology ..."; "use technology effecting in making and testing conjectures..."; "use the Internet effectively..."; "create database or spreadsheet templates..." Although I know that the majority of schools do not have TI-92 Plus graphing calculators, Ministry funding for TI-83+ graphing calculators, CBRs, and the Ministry licensed *Geometer's Sketchpad* and *Fathom* softwares make meeting these expectations possible. Most significant is that **every** mathematics course in the new four-year high school curriculum requires technology beyond that of a scientific calculator.

Our elementary school teaching colleagues need not remain out of the loop. With the provincial license to have the *Geometer's Sketchpad* available in every publicly funded school and faculty of education, grade 7 students can now explore the triangle congruence postulates using this software (see Fred Ferneyhough's *Technology Corner* in the June, 2001 issue of the *Ontario Mathematics Gazette*). In fact, with the Ministry license, every school also has a CD-Rom version of Key Curriculum Press' *Geometry Activities for Middle School With the Geometer's Sketchpad* for grades 5 – 8. Access to the computer lab may be a challenge for some (many?) schools, but that is a "fight" I don't think we can give up.

To end, I spent some time during the past summer "messing with" the *Geometer's Sketchpad*, Version 4. The updated version has expanded capabilities that should change the way we teach algebra and calculus. I think it is now misleading to call the software "Geometer's Sketchpad" since it is no longer just for exploring geometry. Surprisingly, none of the new textbooks for the *Advanced Functions and Introductory Calculus* course make use of the *Geometer's Sketchpad.* Yet, the graphing capabilities and the drag feature allow students to visualize and move points, which should develop greater understanding of concepts. Nick Jackiw, the designer of the *Geometer's Sketchpad*, called the software "dynamic manipulation", at his keynote address at OAME'2000 in Ottawa. Jackiw stated that the *Geometer's Sketchpad* creates a learning environment where manipulation is direct, motion is continuous, and the environment is immersive. Included are two worksheets that compare the way I taught and will now teach the limit of the secant line and the tangent line:

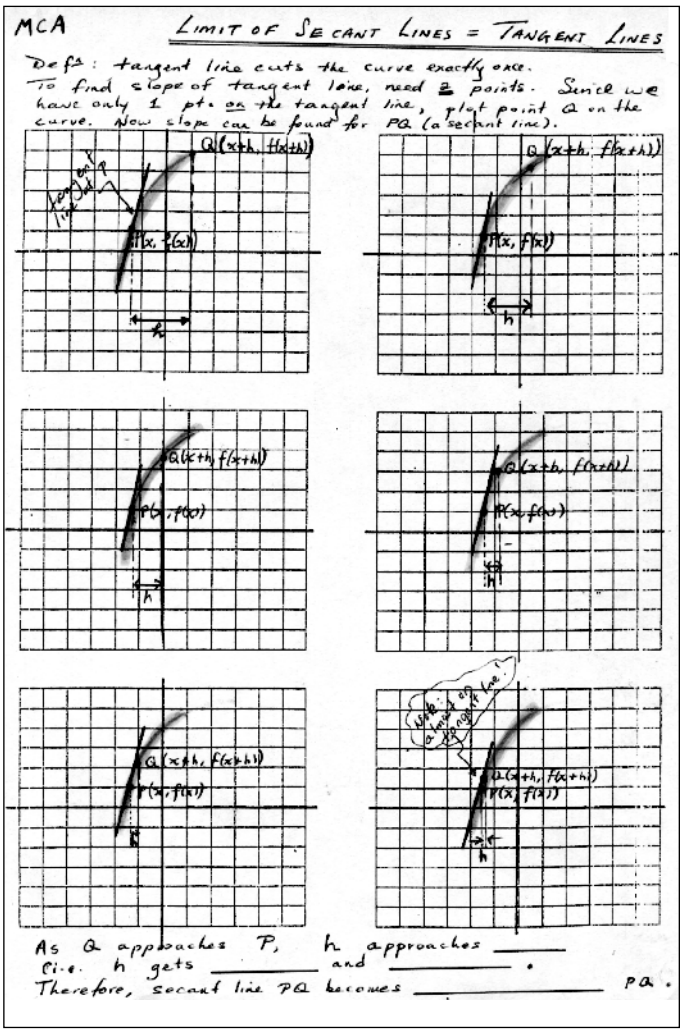
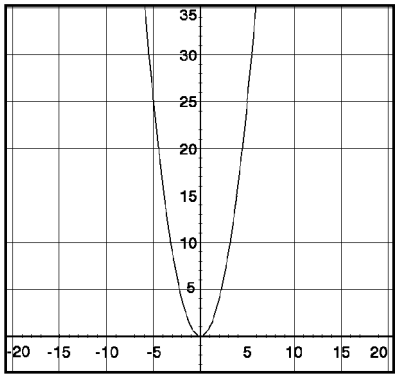


Figure 1: Traditional Teaching Approach

Finding the Slope of the Tangent Line at Point Q Using the *Geometer's Sketchpad* 4

Recall that to find the slope of a line, we need two points. To find the slope of the *tangent line* at point Q, we have only one point. So, we need another point. Call it P. We are going to determine the slope of line PQ, called the *secant line* PQ, and have the point P approach closer and closer to Q. The slope of the tangent line Q can be approximated when the point P is "on top of" Q.

1. Open a new sketch.
2. Select **Graph** in the menu bar and highlight "Show Grid".
3. To graph the function, $y = x^2$, choose **Graph** and select "Plot New Function". A calculator is displayed. Type x ^ 2. Press Okay. The function is now graphed.



4. Click anywhere on the blank screen to deselect any object. To create a point on the curve, select the curve, go to **Construct** in the menu bar and highlight "Point on Function Plot". A point will appear somewhere on the curve.
5. Repeat step 4 in its entirety since we want two points.
6. With the "**A**" tool selected in the left corner, click on each of the points constructed in steps 4 and 5. Double click on the points and label the point to the left, P, and the point to the right, Q.
7. Change the "**A**" tool back to the **arrow** tool. Be sure both points are highlighted. Go to **Construct** in the menu bar and select "Line". You've now constructed the secant line PQ.
8. To calculate the slope of secant line PQ, we need the x- and y-coordinates of points P and Q. To do that, click anywhere on the screen to deselect any object highlighted. Now, select point P. Go to **Measure** in the menu bar and select "Abscissa(x)". Click anywhere on the screen again to deselect any object highlighted. Select point P again, and in **Measure**, select "Ordinate(y)".
9. Repeat step 8 to determine the x- and y-coordinates of point Q.
10. For any two points, $P(x_p, y_p)$ and $Q(x_q, y_q)$, recall that slope is calculated using the formula, $m = \frac{y_q - y_p}{x_q - x_p}$. To calculate the slope of secant line PQ, select **Measure** in the menu bar and highlight "calculate". In the calculator display, select the bracket key (. Then, use your mouse to highlight "y_q = …", followed by the minus sign, then click on "y_p = …". Press the divide sign *outside* of the brackets. Then type the bracket key, select "x_q = …", followed by the minus sign, then select "x_p = …". Press "okay". You should now have the slope formula and the slope of the secant line.
11. Drag point Q until it is at x = 2.00. Fill in the table with the following x-values for point P: (Hint: Drag point P)

x_p	x_q	$x_q - x_p$	$m = \frac{y_q - y_p}{x_q - x_p}$
-3.00	2.00		
-2.00	2.00		
-1.00	2.00		
0.00	2.00		
1.00	2.00		
1.80	2.00		
1.85	2.00		
1.90	2.00		
1.96	2.00		
1.99	2.00		

12. As $x_p \to x_q$, that is, as $x_p \to 2.00$, what is the value of the slope approaching?
13. Let's zoom in! Do that by dragging the point (1, 0) to the right until the y-scale is 0.1. Dragging the point (0, 0) will reposition the graph on the screen.
14. Let's calculate the slope to the nearest thousandth. Click anywhere on the screen to deselect any objects. Now, select "x_p = …", go to **Edit** in the menu bar and select "Properties". Select "Value" and change precision to "thousandths".
15. Repeat step 14, one at a time for "x_q = …", "y_p = …", "y_q = …", and "$\frac{y_q - y_p}{x_q - x_p}$ = …". Be sure to deselect the object each time when repeating step 14.
16. Fill in the table. Note: You may need to reposition your point Q.

x_p	x_q	$x_q - x_p$	$m = \frac{y_q - y_p}{x_q - x_p}$
1.994	2.000		
1.997	2.000		
1.999	2.000		

17. As $x_p \to x_q$, (i.e., as $x_p \to 2.000$) the slope approaches the value of _____. The slope of the secant line PQ becomes the slope of the _____ line Q. Symbolically, $\lim_{P \to Q} m_{\text{secant PQ}} = m_{\text{tangent Q}}$.

Figure 2: Non-traditional Approach

During the fall, I presented workshops on teaching the new grade 12 advanced functions and introductory calculus using the *Geometer's Sketchpad, 4* and will do so at the provincial OAME conference in Oshawa. I would be pleased if you attend my session in May, 2003.

As a classroom teacher and department head, I am an advocate for *all* students to learn mathematics. We need to meet the diverse learning styles and needs of all students – whether they learn dominantly through spatial, linguistic, logical, or kinesthetic understanding. As a seventh year teacher, my excitement for teaching mathematics remains at an all-time high.

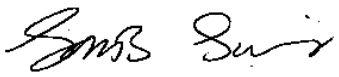
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Louis Lim
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▲ President's Message

MARGARET WARREN
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We are in our sixth year since the introduction of the new curriculum in elementary schools. The new mathematics curriculum has brought many changes to the ways that many elementary teachers teach mathematics and to the ways that many young students are now learning mathematics. I have witnessed the implementation of this curriculum through my role as program coordinator and through my role as parent of two young daughters. I see how the teachers are growing with this new curriculum. I see much more use of technology, more emphasis on communication and problem solving, and a greater understanding of the importance of estimation, prediction, intuition, and number sense. I see the five strands being taught in real world contexts and applications. I see much less isolated drill and manipulation and a greater focus on understanding how the operations work. I watch as my daughters provide reasons and explanations for their answers.

We are in our fourth year since the introduction of the new curriculum in secondary schools. The new mathematics curriculum has brought many changes to the ways that many secondary teachers teach mathematics and to the ways that many adolescents are now learning mathematics. I have witnessed the implementation of this curriculum through my role as program coordinator and more recently through my role as vice principal in a secondary school. I see how the power of visualization experienced through graphing calculators and computers is helping students with different learning styles to better understand concepts. I see how teachers are using oral and written communication as both learning and assessment strategies. I see a greater focus on investigation and inquiry and the use of performance assessments in addition to the more traditional tests and assignments. I recognize teachers are beginning to internalize and understand how modelling and the study of relationships is a central organizer for all of the courses in the new curriculum.

Although I feel fortunate to have been able to give input into the new curriculum (both elementary and secondary) at several stages, there are certainly many things that I would have done differently if I had been given any final say. Despite its shortcomings, I believe that this new curriculum has opened many doors to allow teachers to do innovative and creative things in their classrooms with students of all ages and abilities.

Most secondary teachers have probably not spent very much time reading or analyzing the elementary curriculum and most elementary teachers have probably not looked beyond what they are expected to do. There are some schools and boards that are now putting a focus on the transition between grade 8 and grade 9 – several years after the introduction and demise of the "transition years" curriculum project at the Ministry. I have had the unique experience of getting to know the entire mathematics curriculum intimately all the way from kindergarten to grade 12. It is fascinating to see the growth in both reasoning and communication skills, the sophistication of problem solving and technology use as students traverse through the years of this curriculum when it is taught and presented well. This year, I see the climax in both teachers and students as they analyze and understand limits and derivatives or design and investigate questions and issues of personal interest through the use of e-Stat. These new grade 12 courses are taking shape as they are implemented in the classroom. Teachers are excited and students are engaged.

OAME has recently embarked on a large two year project. We are crafting a statement of vision that espouses the values and practices we believe in. We are creating a video that will demonstrate this vision through vignettes of a child growing up through all the years of the new curriculum. The viewer will see snapshots of what a classroom at various grade levels looks like, sounds like, and feels like. Supporting print materials are also being developed. This will include a series of classroom ready activities – a least two per grade level – which exemplify the vision and help teachers move further along in their implementation and practice of both teaching and assessment strategies. There will be samples of student work and assessment tools.

OAME is also designing an inservice package that could be used in varied and flexible ways by boards, schools, departments, or individuals. The intent would be a series of team meetings where individuals would discuss and learn together while focusing on the OAME materials. Our goal is to have the first set of materials ready to showcase at this year's annual conference and available in print form for fall 2003. The second set of materials would be worked on the following year and ready for fall 2004. This is a big undertaking but ultimately one that will be important in shaping the growing views of OAME. We hope that it will help classroom teachers to better understand the scope and power of the new curriculum as they view snapshots from primary, junior, intermediate, and secondary classrooms. If you are interested in getting involved in piloting or assisting with the production of the materials, please contact me, your chapter representative, or one of the directors of OAME.

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Well, here's an opportunity for you!

Editor
Ontario Mathematics Gazette

The current *Ontario Mathematics Gazette* Editor is completing his term June, 2003. OAME needs a dynamic person to continue to provide mathematics educators with a quality and comprehensive journal.

OAME is currently working on a Vision Statement and would like to involve the new editor, as soon as possible, in helping to determine what this new vision will look like in this journal. As well, you may help to define the role of the editor. This could be through sharing the role with another or developing a team of people to assist with the duties of the editor.

OAME is open to your ideas!

Please consider this position for a two year term as Editor. The position commences with the September 2003 issue but there is also the opportunity to work with the current editor on the June, 2003 issue.

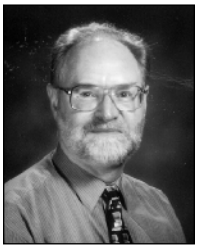
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pearson

▲ Technology Corner: Using Matrices on the TI-83 Plus in Grade 12

FRED FERNEYHOUGH
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Fred Ferneyhough is Head of Mathematics at Central Peel SS in Brampton and is technology consultant for the McGraw-Hill Ryerson texts in Ontario. He is an avid user of, and advocate for, the use of technology in the mathematics classroom.

Both the Data Management course and the Geometry and Discrete Mathematics course contain expectations that refer to the use of matrix algebra. In the MGA 4U course, one specific expectation refers to using matrices, with and without the use of technology, to perform row reduction operations. When teachers in the Peel DSB first attended a T^3 Institute in 1997, Steve Martinello showed us how to use the matrix commands to solve systems of equations. What I liked about his approach was that it avoids the "black box" fears that many of us have about using technology.

The biggest dilemma with solving systems of equations without technology occurs when our students try to do the arithmetic involved in the manipulations. Even the best student (or teacher) may fully understand the algorithms for solving systems, but can easily make an arithmetic error that ruins their solutions.

The TI-83 Plus graphing calculator is a great way to teach matrices and solve systems of equations, regardless of your approach to the topic. If you want to use the calculator to develop the ideas behind solving systems, the device allows you to do that. In my mathematics department, we often teach the methods and then turn to the technology to cut out the drudgery. Each teacher and department should decide what skills they value. Either way, I recommend using this device.

When matrices are introduced, the keyboard can be used as a matrix to get students used to the terminology and positions. Vince DeLisi at Texas Instruments says that he had his students find the keys by referring to them by row and column number. For example, the VARS key that we use in grade nine for regressions is located at row three, column four. By getting students used to this approach from the beginning, they develop a notion of matrices long before it is studied as a topic. I have students tell me where they can find the MATRX button (row four, column 1). Note that it is different on a TI-83 Plus than on the older model.

Let's move on to solving a system of equations:

$1x + 2y + 1z = 12$

$2x - y + z = 5$

$3x + 1y - 2z = 1$

To set up a matrix, press the MATRX key, move over to the EDIT menu, and then down to the name that you would like to use, and press ENTER.

```
NAMES MATH EDIT
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

You can have up to ten different matrices defined in memory. If you run a RESET program when your calculator is started, be sure that the program clears matrix variables to avoid nasty surprises.

The default size of a matrix is 1 by 1. The first thing that you have to do is to enter the dimensions. Hit ENTER after you key in each value. The size of the matrix will change on the screen.

```
MATRIX[A] 3 ×4
[0              ]
[0              ]
[0              ]
```

After entering the dimensions, the cursor will jump to row one, column one. Enter the coefficients for the system. Each time that you press ENTER, the cursor will jump to the next position in the same row. After entering the final value in a row, the cursor jumps to the beginning of the next row.

When you are finished, return to the home screen, call up the MATRX menu again and hit ENTER on the name of your matrix.

```
[A]
  [[1 2  1  12]
   [2 -1 1  5 ]
   [3 1  -2 1 ]]
```

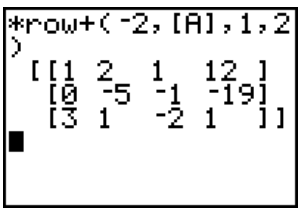
Now, we are going to use the calculator to do the row reduction operations.

In the MATRX menu, there is a set of sixteen mathematical operations.

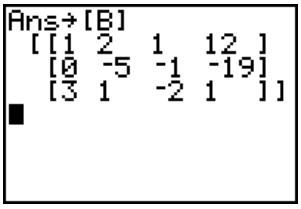




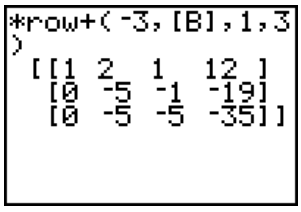
In the first step of the solution, we want to eliminate the "2" in row 2, column 1. We will use the matrix command "*row+(-2, [A], 1, 2)". This will multiply row 1 of matrix [A] by –2 and add the result to row 2. The command is entered in the home screen. It is worth mentioning here that many teachers feel that this introduces a different problem to the student. A case can be made that learning to use the commands on the calculator only substitutes one problem (the syntax on the machine) for another (their carelessness in arithmetic). It has been my experience that students pick up on syntax of commands much quicker than we do and that experience often overcomes these difficulties.



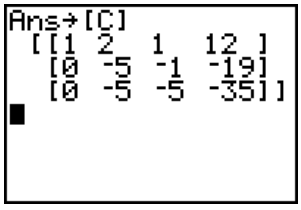
Notice that the second row has been updated. The result is shown on the screen but is not stored in matrix A. I find it best to store each successive result in a new matrix (it's easier to trace work back when necessary – and you just know that it's going to be necessary!) To do this, press STO [B].



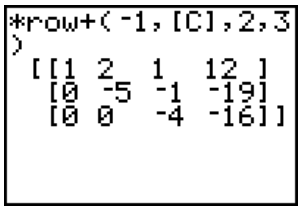
Now, we want to eliminate the "3" in row 3 column 1. The command shown multiplies the first row of matrix B by –3 and adds it to row three.



Store this result in matrix C.



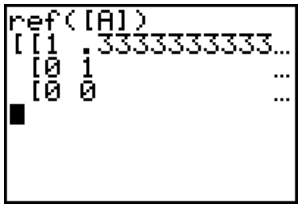
Finally, we want to eliminate the "-5" in row 3, column 2. To do this, we multiply row 2 by –1 and add it to row 3.



The result could be stored in matrix [D] or solved from the results on the screen. From row 3, it is apparent that z = 4. Back substitute into row 2 to solve for y and then to row 1 to solve for x.

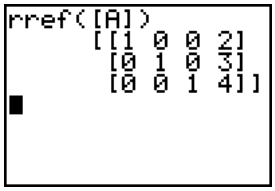
If my objective in teaching the operations is to get my students accustomed to the row reduction manipulations, then this approach allows that and gets past the arithmetic problem. Once students are accustomed to the steps in matrix manipulations, we may want to evaluate them in some manner and then move on to the built-in commands. I would recommend that we use the following commands in situations where we want to find the solutions to systems of equations and use the results to interpret the situations geometrically.

In the matrix mathematics menu is a command that will convert a matrix to reduced echelon form with the digit "1" on the main diagonal and "0" under the main diagonal. The only problem with this command is that the coefficients are expressed in decimal form. However, these are our strongest students so we hope that decimals don't scare them.



To see the other columns, scroll to the right and the missing values will be displayed. You may want to use this command to get students used to solving the system once it is in reduced form.

Finally, there is also a command that converts a matrix to row reduced echelon form.



From this screen the solution (x, y, z) = (2, 3, 4) is obvious. However, using this command requires absolutely no understanding of the manner in which the solution was obtained.

Another advantage to using the TI-83 Plus is that it allows us to use systems as big as we want. We might never use the maximum size, but it does allow us to consider systems of four and five equations without having to worry about the number of calculations that we have our students perform. ▲

▲ Happy Anniversary, Fibonacci!

BY: BARNABAS HUGHES
CSU NORTHRIDGE

Reprinted with permission from The California Mathematics Council Communicator, *September, 2002, Volume 27, Number 1, page 27.*

Eight hundred years ago in 1202, Leonardo de Pisa (Fibonacci) finished writing his compendious *Liber Abbaci* or *Book on Calculating*. The modern printed copy runs to nearly 400 pages with 43 lines to the page; imagine writing that by hand! The text covers all of the practical arithmetic, geometry, and algebra of the times, together with more than enough theory to support the applications.

Fibonacci was born in Pisa, Italy, about 1170. As a teenager he accompanied his father to Algiers where his father was a customs agent for Pisa. Leonardo easily added Arabic to Italian and Latin, the languages he already knew. He became fascinated with mathematics, particularly because the Algerians knew more mathematics than his teachers in Pisa. Wishing to improve and increase his knowledge, Leonardo travelled throughout the Mediterranean world, spending time in Alexandria (Egypt) and Constantinople (now Istanbul, Turkey). The local scholars shared their knowledge with him and, after his return to Pisa, he gathered everything he learned into *Liber Abbaci*. He became a famous teacher, mathematician, and occasional guest of Emperor Frederick Barbarossa. Leonardo died about 1250.

Although we usually associate Fibonacci with the problem about the growth of a rabbit population and the resulting famous sequence, his book contains many other interesting problems. Two of his many entertaining problems are useful for arousing students' interest and giving them projects to investigate.

Problem 1

Is it possible for the sum of two numbers to equal their product? Some students will quickly notice that 2 + 2 = 2 x 2. But are there other possibilities? Fibonacci shows us how to find them. First, choose any whole number and divide it into two numbers that add up to the chosen number. Then divide the chosen number by each of the two parts. Next add the two quotients together, then multiply them together. Behold! Their sum equals their product. For instance, 12 = 4 + 8. Therefore, $\left(\frac{12}{4}\right)+\left(\frac{12}{8}\right)=\left(\frac{12}{4}\right)\times\left(\frac{12}{8}\right)$. Have students work in small groups and use their calculators to select numbers and follow the above steps to show that this method works. It is not difficult to prove that this method works using algebra.

This exercise provides a bonus for teacher and students. Many of the quotients will be numbers with long decimals. When these are added together, their sum is a number followed by .9999.... Teachers can either explain why .9999... equals 1.0 or discuss rounding up.

Problem 2

Remind your students of the Pythagorean Theorem and that $3^2 + 4^2 = 5^2$. The group (3, 4, 5) is called a Pythagorean triple, and the two numbers (3, 4) are a Pythagorean pair. Fibonacci shows us how to construct more pairs, *the sum of whose squares is 25* ! We begin by considering another Pythagorean triple (8, 15, 17). First multiply each of the smaller numbers by 5 (the square root of 25 in the initial triplet). Then divide each product by the largest number, in this case 17. The sum of the squares of the quotients will be 25. For example: $\frac{5\times 8}{17}=\frac{40}{17}$ and $\frac{5\times 15}{17}=\frac{75}{17}$. The sum of the squares of the two quotients, $\frac{40}{17}$ and $\frac{75}{17}$ is 25. Try this method with the triplet (7, 24, 25).

Now reverse the process. If you know a Pythagorean pair, the sum of whose squares is 25, how do you find the triplet from which the pair was constructed? Fibonacci offers these two pairs, $\left(4\frac{27}{37}\text{ and }1\frac{23}{37}\right)$ and $\left(4\frac{56}{61}\text{ and }\frac{55}{61}\right)$, without telling his reader from which triple they came. Let your students investigate. If they get stuck, tell them to go back to the example above and study it to discover the method of construction. ▲

Tribute to Ron Scoins

July 19, 1939 to October 13, 2002

On October 17, 2002, Mount Zion Lutheran Church in Waterloo was filled with people whose lives were touched by this cheerful, optimistic, devoted, and caring leader to relatives, friends, and mathematicians who attended from across Ontario. Ron, in his 64th year, passed away after a valiant fight against cancer.

Ron graduated from Seaforth District High School and the University of Western Ontario. He earned his Master of Mathematics from the University of Waterloo in 1973. At that time, he was mathematics department head at the new Bluevale Collegiate Institute in Waterloo. In *Twenty-Five Years and Counting*, a history of the Grand Valley Mathematics Association (GVMA) compiled by Ed Baumgart, early highlights are "founding chairman Ron Scoins of BCI in Waterloo, announced the cost of an annual membership in GVMA would be $3.00, and that more than 145 teachers had already become members. Hence the association had been able to bank a princely sum in excess of $400." In May 1974, the members of GVMA approved their affiliation as a chapter within OAME. In the same year, Ron joined the Faculty of Mathematics at the University of Waterloo. For over twenty-five years, he was Executive Secretary for GVMA, assisting in the liaison between the faculty and secondary school teachers around the province and the organization of two mini-conferences each year.

In a message to University of Waterloo friends, Dean Alan George wrote, "Ron was a key individual in our mathematics contest operations, and was one of our leaders in the development and nurturing of the wonderful relationship we enjoy with high school teachers. He was a teacher's teacher. We have lost a wonderful friend with whom it was a joy to work."

Three Eulogists, Ed Baumgart, Joanne Colbourne, and Ron Dunkley gave

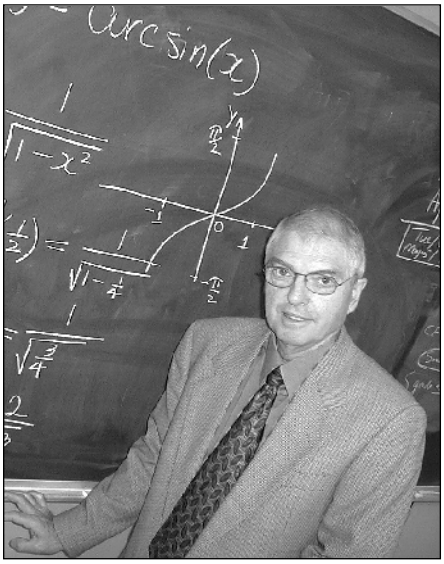


Photo courtesy of Barbara Elve

accounts of memories they had of Ron. Ed Baumgart captured the respect his family had for Ron's family. He talked about the family times, their bridge club that was just a way to-get-together, and spending time monthly doing something with close friends. Joanne Colbourne, Mayda's sister, mentioned the love the family had for Ron and the countless times he took to assist nieces and nephews.

Ron Dunkley pointed out that Ron had a gift for explaining mathematics. "Students were first and you didn't interrupt Ron when he was helping a student." In 1999, Ron received the University of Waterloo Distinguished Teacher Award: "His students are impassioned in their accounts of his generosity, encouragement, good humour, and constant support. Many who had a fear of mathematics report that they grew to love it because of Professor Scoins." Everyone could relate to the Ron who was described by the Eulogists. Ron gave effortlessly and never expected a thank you – a sign of a genuine humanitarian.

In 2001, Ron was recognized by the Ontario Confederation of University Faculty Association for "excellence in teaching". A citation prepared for the award describes Scoins, in part, as "the author, co-author or editor of numerous…educational materials: six volumes of *Problems, Problems, Problems; Problems and How to Solve Them*, and *Can you Pass These Tests?*"

Jack Weiner, in his letter of support for this award, wrote: "Teachers come to his workshops because he is an excellent, entertaining, and engrossing speaker. More importantly, they come for all the things, tangible and intangible, that he gives them to take back to their schools and classes – specific ideas and techniques, but also enthusiasm, love of mathematics, love of teaching, and the excitement at witnessing student success." Jack worked on many committees with Ron and these committees sometimes dealt with contentious issues: "Ron, far better than anyone else I have ever known, brings people to common ground. He brilliantly zeroes in on the most important issues and helps find consensus." Ron was also a contributing author to mathematics textbooks for senior high school.

When I met Ron Scoins in 1973, I was starting to tutor mathematics in the area and Ron was a true mentor in advising me about students' needs. In 1993, he encouraged and helped me get started to tell others about the perspectives of mathematics education from a professional tutor's viewpoint.

Outside the mathematics classroom, Ron loved golf, cross-country skiing, board games with friends, KW Symphony Pops concerts, and committee work at his church. His wife Mayda, daughter Sandra and her family, daughter Nancy and her family, son Steve, and mother, Ina Scoins, lovingly remember Ron.

Ron, your family, friends, and colleagues in the mathematics community miss you. You leave us with cherished memories.

Written by: Frances Schatz, with thanks to Jack Weiner and Dean Murray

▲ Contrasting Geometric and Arithmetic Progression Sums



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Bonnie Litwiller teaches mathematics and mathematics education at the University of Northern Iowa. Her mathematics education doctorate is from Indiana University. She is also a Past President of the Iowa Council of Teachers of Mathematics, has served on the Board of Directors of the NCTM, and on many other NCTM committees, including chairing the 2002 NCTM Yearbook Editorial Panel.

The topics of geometric and arithmetic progressions and their sums often arise in secondary school mathematics. We present a real-world setting which gives rise to both progressions. These progressions are treated arithmetically, graphically, and algebraically.

Situation I:

Suppose that Greg earns a salary of $30 000 in his first year of employment. Each year thereafter through his 40th year of employment, he receives a 5% salary increase. For year n of these 40 years, his salary is $30\,000(1.05)^{n-1}$. His salary in his 40th year would be $30\,000(1.05)^{39} \approx \$201\,143$.

Situation II:

Suppose that Greg earns a salary of $30 000 in his first year of employment. Each year thereafter he receives a salary increase of $4388. This is $\frac{1}{39}$ of the entire increase in salary achieved over 40 years in situation I: $4388 \approx \frac{201\,143 - 30\,000}{39}$. For year n of these 40 years,his salary will be 30 000 + (n-1)(4388). His salary in his 40th year is then 30 000 + (39)(4388) ≈ $201 132, approximately the same as in situation I.

Question: What is the sum of the 40 yearly salaries that Greg has earned in each of situations I and II? Which situation do your students hypothesize will yield the greatest sum? Table I displays the yearly salaries for each of the two situations.

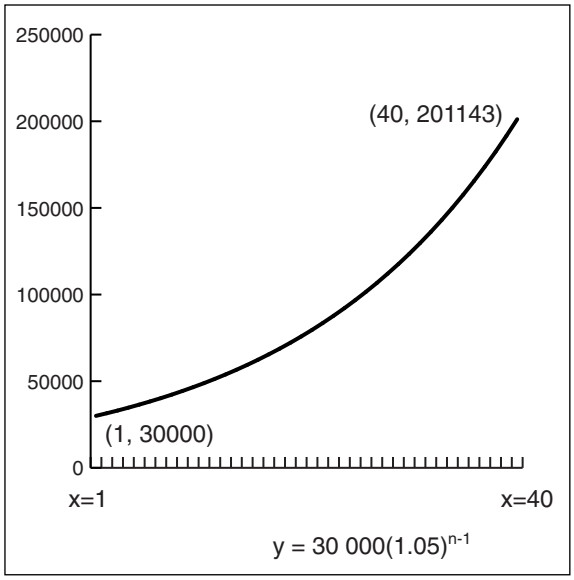
Since each situation involves a set of 40 increasing salaries, some students might suppose that a short-cut way of finding the salary totals would be to find the mean of year 1 and year 40 salaries and then multiply this mean by 40. In situation I, this method would yield $40\left(\frac{30\,000 + 201\,143}{2}\right) = \$4\,622\,860.$ In situation II, this method would yield $40\left(\frac{30\,000 + 201\,132}{2}\right) = \$4\,622\,640.$

Table 1

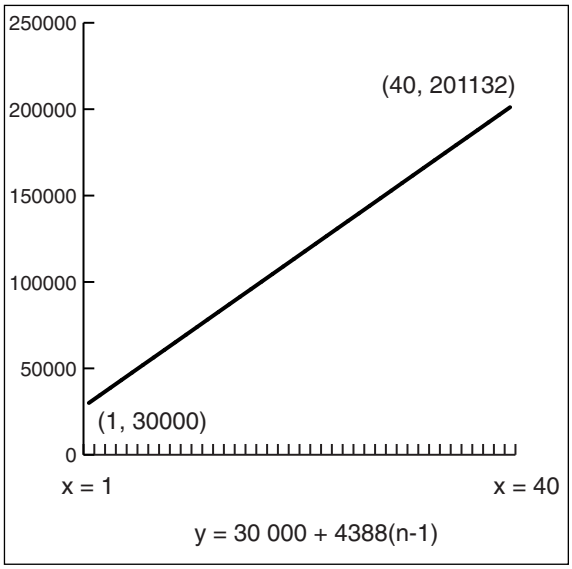
Year	Situation I:	Situation II:
	$30\,000(1.05)^{n-1}$	30 000 + (n-1)(4388)
	(Entries rounded to the nearest dollar)	
1	30 000	30 000
2	31 500	34 388
3	33 075	38 776
4	34 729	43 164
5	36 465	47 552
6	38 288	51 940
7	40 203	56 328
8	42 213	60 716
9	44 324	65 104
10	46 540	69 492
11	48 867	73 880
12	51 310	78 268
13	53 876	82 656
14	56 569	87 044
15	59 398	91 432
16	62 368	95 820
17	65 486	100 208
18	68 761	104 596
19	72 199	108 984
20	75 809	113 372
21	79 599	117 760
22	83 579	122 148
23	87 758	126 536
24	92 146	130 924
25	96 753	135 312
26	101 591	139 700
27	106 670	144 088
28	112 004	148 476
29	117 604	152 864
30	123 484	157 252
31	129 658	161 640
32	136 141	166 028
33	142 948	170 416
34	150 096	174 804
35	157 600	179 192
36	165 480	183 580
37	173 754	187 968
38	182 442	192 356
39	191 564	196 744
40	201 143	201 132

However, directly summing the 40 entries in each column of Table I yields the predicted sum of $4 622 640 in situation II, but only $3 623 994 in situation I. Why is that so?

A cursory examination of the columns in Table I reveals that although the beginning salaries are equal for the two situations and the ending salaries are approximately equal, the salaries in situation I initially grow much more slowly than those of situation II. If graphed, the yearly salaries in the two situations would yield the following graphs:



Situation I



Situation II

The area under the first graph (approximately the salary total) would be much smaller than that for the second graph. The salaries of these situations may be summed using familiar algebra formulas, which we now develop:

Situation I (Geometric Progression)

$$\begin{aligned}\text{Total} &= 30\,000 + 30\,000\,(1.05)^1 + 30\,000\,(1.05)^2 + 30\,000\,(1.05)^3 + \ldots + 30\,000\,(1.05)^{39} \\ &= 30\,000\,[1 + (1.05)^1 + (1.05)^2 + (1.05)^3 + \ldots + (1.05)^{39}] \\ &= 30\,000\,\frac{\left((1.05)^{40} - 1\right)}{1.05 - 1} \\ &\approx \$3\,623\,993\end{aligned}$$

The total of the 40 salaries is $3 623 993. This differs from the calculated sum of $3 623 944 only because the entries in Table 1 were rounded.

Situation II (Arithmetic Progression)

$$\begin{aligned}\text{Total} &= 30\,000 + [30\,000 + 1(4388)] + [30\,000 + 2(4388)] + [30\,000 + 3(4388)] \\ &\quad + \ldots + [30\,000 + 39(4388)] \\ &= 40(30\,000) + 4388\,(1 + 2 + 3 + \ldots + 39) \\ &= 1\,200\,000 + 4388\left(\frac{(39)(40)}{2}\right) \\ &= 1\,200\,000 + 3\,422\,640 \\ &= \$4\,622\,640\end{aligned}$$

In this article, arithmetic and geometric progression situations were treated arithmetically, graphically, and algebraically. Teachers and students are encouraged to perform similar analyzes in other real-world situations. For instance, suppose that the entries in Table I were decreasing from a common beginning point by arithmetic and geometric progressions.

Suggestions for the Reader and Students:

1. Use a spreadsheet in the analysis of this data.
2. Re-do the two situations using different assumptions about the initial salary and the yearly increases.
3. Find other situations in which arithmetic and geometric progressions can be developed, compared, and contrasted. ▲

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▲ Hey, It's Elementary...: Everything's Comin' Up Numbers!

LYNDA COLGAN
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Lynda Colgan has been at the Queen's Faculty of Education for four years, teaching in the B.Ed., M.Ed., and Ph.D. programs. Her primary focus is the classroom and her research is aimed at improving the mathematics learning experience of the child by supporting teachers and parents.

As I was going through my latest assortment of books, journals, and magazines, I suddenly found myself singing a familiar tune (albeit with a twist)… *Everything's Comin' Up Numbers*. After lying dormant for a few years, it seems that "numbers" are fashionably in full bloom.

In Ontario, as in countless other jurisdictions within the English-speaking world, there have been a number of reforms to elementary mathematics education. We have seen dedicated efforts to raise standards, improve student achievement, and broaden the spectrum of content to include concepts from within the areas of spatial sense and geometry, patterning and algebra, data management and probability, and measurement. There have been new textbooks that tout a problem-solving focus and new performance-based assessment instruments and tools that are aimed at weaving content strands into mathematical processes and representations.

These priorities have led to many criticisms. There are those who believe that we have not been doing our children justice because the curriculum is "a mile wide and an inch deep." Others raise the alarm bells because of strong indications that the curriculum may be neither developmentally appropriate nor conceptually based. These observations may be all too true.

There is very strong evidence to support the notion that in the first six years of school, numbers should be of primary importance. *The Early Years* study, the significant *Early Numeracy* studies currently underway in Australia, England, and British Columbia, as well as the documented success of the related *Mathematics Recovery* program, suggest that our focus, especially in the primary and early junior divisions, should be devoted to helping children seal and formalize their knowledge about numbers and numerical strategies.

The advocates of *Early Numeracy* and structured mathematics intervention programs contend that a child's elementary school mathematics education should focus on the key constructs of counting, number relationships, and grouping and partitioning. They argue that at least half the time spent on mathematics should be devoted to numbers. Numerical knowledge and strategies – counting, adding and subtracting, number word sequences and numerals, reasoning in ones and tens, and beginning multiplication and division – are now recognized as being extraordinarily significant in a child's overall mathematical development. Furthermore, since there is strong evidence to suggest that a three-year difference in number knowledge in the early years of school becomes a seven-year difference in low-achieving students after the first ten years of school, there is a need to develop strong early remediation/intervention programs for children who are experiencing difficulty.

In this column, I will report on what is known about children's numerical abilities and arithmetical development. The information comes from a variety of sources (listed in the bibliography) but relies heavily on the meta-analysis of psychological, developmental, and cognitive research completed by David Geary. His book, *Children's Mathematical Development: Research and Practical Applications,* is a must-read for everyone interested in children and mathematics.

What Research Tells Us

It is abundantly clear that all Kindergarten children have considerable numerical competencies when they enter school. This is attributable to the fact that the acquisition of mathematical skills in children appears to involve both inborn and experiential factors. Children's natural curiosity and exposure to appropriate materials, contexts and social interactions, in combination with neurobiological systems for processing numerical information, work together to ensure that some mathematics-related knowledge is acquired by all children without the need for direct instruction.

We know that infants in the first week of life know that three sets of items differ in quantity from two sets of items. The fact that new-borns are able to detect small quantities does not mean that they are sensitive to which set has more or less items, just that they are different; however, between the ages of 4 months and $7\frac{1}{2}$ months, infants can understand the effect that the addition or subtraction of one item has on the quantity of a small set. By age 18 months, infants respond to ordinal relationships, understanding that a set of 3 is more than one with two.

By age 2 years, children regularly engage in number-related activities – for example, counting fingers or toes, snacks or toys, or singing songs with number references. Although verbal counting skills develop between 2 and 3 years of age, there are several embedded concepts that must be mastered.

First, children must develop a one-to-one correspondence between number names and counted items so that each item is assigned only a single number tag. It is believed that children require as long as one year of counting experience in order to begin to associate specific number words with their mental representation of quantity.

Children need to order the number names (i.e., one, two, three, ...) and appreciate that the last number named in the count (the *cardinal* number) holds special meaning, representing the total number of items counted. When asked a "how many" question, children who do not have an understanding of cardinality will recount the set of items rather than restate the last word tag. Piaget demonstrated that the child's understanding of cardinality at this age is not mature. For instance, even if a child is presented with two rows of five toys, counts both of the rows and then states that there is the same amount in each row, he can often be talked out of that conclusion. If one of the rows is spread out and it is pointed out that it is now longer, pre-school children will often change their answer, saying that it has more toys. A child has a good conceptual understanding of counting when she sees similarities across problems. This occurs when a child knows that counting can occur from right to left, left to right, or haphazardly, with objects close together or far apart, and as long as all of the items are counted, problems will yield the same answer.

There are many other important embedded concepts. Based largely on experience, parent-child and other social interactions, the child learns that:

- each number word is unique and represents a unique quantity;
- each number represents smaller groups of other numbers (i.e., 8 can be decomposed into 6 and 2, and 5 and 3, as well as several other possible combinations);
- numbers can be used in numerical and non-numerical ways (e.g., the phone number 545-1085 does not represent the 5 451 085th phone nor a numerical value, and Bus #39 is not less than Bus #83 or greater than Bus #12).

Most children learn by rote, through the imitation of parents or siblings, their culture's number words in the correct sequence from one to ten by age 3 or 4 years. English-speaking children experience considerable difficulty with numbers greater than ten because the number words for quantities to the hundreds are *irregular;* that is, they do not map onto the underlying base 10 structure of the number. This is not the case in Chinese, Japanese, and Korean, where the words for 11 are translated as *ten one*, representing a single tens value and a single ones value. *Eleven* in English is just another number word and gives no clue to its basic unit values. To further complicate matters for English-speaking youngsters, for *teen* words, the units value is spoken before the tens value (think about *sixteen*) but for other words (e.g., *sixty-five*), the tens value is spoken first. In Chinese, 15 is simply, *ten five* and 35 is *three ten five*. These language irregularities make it easy to understand why some English-speaking children may write 51 when they hear 15. In spite of these difficulties, most children do know their number words up to 100 by Grade 2.

By age 4 or 5 years, children adapt their existing counting skills to situations and contexts that require addition or subtraction. At the same time, most children develop an understanding of commutativity and use this knowledge when solving addition problems. Without formal instruction, children tend to use a variety of strategies to solve arithmetic problems, first relying heavily on the use of manipulatives and then shifting to finger-counting, verbal counting, and finally, fact retrieval.

Arithmetic knowledge emerges from the child's natural environment. Among the first derived facts that children tend to develop are doubles (e.g., 3 + 3 = 6, 4 + 4 = 8). These serve as a basis for solving other addition problems (e.g., to solve 6 + 7, the child might retrieve the answer to 6 + 6 from long-term memory and then add 1).

Memorization of other basic facts occurs with repeated use of strategies such as counting-on and decomposition (e.g., 6 + 7 can be solved by decomposing 6 into two 3s, adding one of the 3s to 7 to get 10, and then adding 10 to the 3 or counting 10, 11, 12, 13). Even though the memorization of basic facts (direct retrieval) is not itself a strategy, it does make the solution of more complex problems quicker and less error prone.

Understanding of addition is key to other processes. It has been shown that children rely heavily on their knowledge of addition facts to solve subtraction problems and that the acquisition of multiplication skills (around second and third grade) is heavily dependent on knowledge of addition and counting; for example, adding 2 + 2 + 2 to solve 2 x 3, or counting 5, 10, 15, 20 to solve 5 x 4. Children appear to be highly motivated to learn the multiplication tables and seem to learn the basic table in less time than it takes to learn the addition table.

The solution of simple addition, subtraction, multiplication, and "sharing" problems is common before formal instruction occurs. Many six-year olds can find 45 + 3 by counting on from the larger number, just as they would to solve 5 + 3. Children appear to be able to perform more complex computations (e.g., 36 + 58) when the meaning of the number words is made obvious. The research appears to be conclusive – young children benefit from the use of manipulative models and mental strategies in the teaching of place value and in computations with large numbers. Difficulties arise, however, when intuitive strategies are devalued and children are forced to solve complex, multicolumn problems in school using standard algorithms based on trading or carrying before their knowledge is mature. In fact, it is generally agreed that it is not helpful to children who are developing place value understanding to be given the algorithm for column arithmetic before those concepts are mature.

Of course, theory always differs significantly from reality. Though the development of counting procedures, number concepts, and arithmetic learning should grow from an intuitive level to one more explicit and formal *for all children*, elementary school teachers and clinical researchers agree that large numbers of young children experience both difficulty with and anxiety about mathematics. This is in stark contrast to official estimates that suggest that only about 6% of school-aged children (70% of this group being male) have a diagnosed learning disorder in mathematics. Since a very small number children have a mathematics disability attributable to a cognitive or neuropsychological deficit, it follows that many, many children who perform poorly in math do so because of a lack of experience, poor motivation, or anxiety. Regardless of cause, both groups of children have difficulties in executing counting procedures and retrieving basic arithmetic facts even after extensive drilling. Research has shown that both groups can be helped by curricular changes that increase the child's mathematical competence; that is, a conceptual framework designed to improve basic skills and strategy-selection processes.

What Does This Mean To Us?

Even from this brief survey, it is possible to suggest many issues that need to be addressed in Ontario.

Research suggests that many children have experience with numbers in a wide assortment of contexts and are coming to school with more sophisticated math knowledge than they are given credit. A look at the Kindergarten and Grade 1 curriculum (rote counting, numeral recognition, ordinal numbers) tells us that it is not unreasonable to assume that children are being underchallenged in the first years of school, especially in the area of numbers. This could contribute significantly to lack of curiosity and motivation and a decline in mathematics achievement. Children gain intrinsic motivation from hard thinking and the knowledge that they are progressing in their learning.

Since much mathematical development occurs before children come to school, sincere efforts must be made to reach new parents so that they value and provide regular mathematics experiences, such as singing number songs and playing arithmetic games with their young children. Parents, caregivers, and day-care providers need to supply basic numerical experiences routinely in order for children to have the opportunities they need to learn. The *Count Me In* and *Count Me In Too* initiatives in Australia boast excellent parent support web sites and are outstanding vehicles and concrete supports for aligning instructional efforts at home and at school.

It is reasonable to acknowledge that there will be differences in the number experiences and conceptual understanding of children arriving at the Kindergarten or Grade 1 door. However, that door must be kept open widely so every child can completely understand every idea introduced in the math classroom. Intervention and remediation programs must be implemented as soon as teachers have diagnosed specific difficulties, in order to

"even the playing field" and nurture success. We do not need to reinvent the wheel. The success of the *Count Me In* and *Count Me In Too* studies are attributable to the fact that the curriculum is based on clinical and classroom research about how children learn math, not personal philosophies or politics.

There is an assumption that such interventions must be expensive – and they might be. The costs would not be for materials. A look at a number of reports confirm that these remediation approaches rely on simple classroom tools such as number lines, counters, popsicle sticks, ten-dot cards, and numeral cards as a starting point to replicate developmental steps and emphasize the overlapping interconnectedness of mathematics concepts and processes. But costs would be incurred in the form of dramatic changes to teacher preparation programs and extensive professional development initiatives. A principal goal is to give teachers both the mathematics content skills and pedagogical knowledge they need to know their own students as learners of mathematics. This takes time as well as support but there are precedents that can justify the investment. In Australia, thousands of teachers have been given extensive training in the stages of mathematical development and shown how to remediate by moving students along a natural continuum, advancing students' numerical knowledge and strategies, conceptually and procedurally. A fundamental component of this teacher education program is to provide the teacher with strong questioning techniques so they can elicit better student explanations and encourage students to *use what they already know to learn what they don't.*

Concluding Remarks

There is an enormous body of evidence that suggests that a child's mathematical development hinges on the complex interplay between the brain she is born with and her experiences. What neuroscience also tells us is that brain development is non-linear and that there are critical periods for developing different kinds of knowledge and skills. Clearly, educators must take advantage of the key window of opportunity that appears between birth and age 10 years because it is in this period that children are *exquisitely primed* to learn fundamental numerical concepts.

So there we have it - empirical validation of the wisdom of an early start to improving children's long-term mathematical development - and legitimization of the absolute truth in the statement, "Everything I needed to know, I learned in kindergarten."

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▲ Bedtime Stories for the Scholastically Inclined



JACQUELINE HILL

Jacqueline Hill is a teacher of mathematics at Port Perry High School. Her involvement in the mathematics education community includes Course Profile writing, Workshop Leader, Pine Ridge Math Association President, and writer for Nelson Thomson Learning. Jacqueline is dedicated to teaching all students, reaching each of them in some way. She tries to vary her instructional approaches to address the different learning styles of her students.

I first created this particular assignment four years ago. Every Friday, we start classes with nap time (grades 9 – OAC), and then follow by story time. Nap time is "lights out", with students entering the room silently and putting their heads down and unwinding for a few minutes. Story time is consumed with content-related math stories out of the volumes of stories I have. Since it was first conjured up, I have students write bedtime stories each semester. The rubric provided was a new twist for me this year.

I conference my students four times throughout the process of writing their stories. The first time occurs after they have completed the Title and Unit of Study. I have students sign up for units in the course (so each volume of stories has an even distribution of topics). The second conference contains setting, main characters, the four pieces of mathematical content they are going to cover, and the plot time line. The third conference takes place after the first paragraph is written. I usually try to have a senior English class proofread the stories when they are complete. A fourth conference is scheduled just before the students are to hand in their finished product. I have a student volunteer create the title page of the class' volume. Titles include *Bedtime Stories for the Scholastically Inclined, Volumes 1 and 2* as well as *Mother Hill's Bedtime Stories, Volumes 1 and 2*.

I have included a student's sample of work. Travis was a student in my grade 10 Applied Mathematics class who gave us permission to read, view, and use his story. Please note that there are mathematical errors in his story. So on the scoring rubric, he would have achieved a level 2 or at most 3 for his covering chapter content section.

The bedtime stories are used as part of the 30% summative assessment and given out in the latter portion of the course. Bedtime story titles have included: *Red Riding Hood and the Quadratic Visit, The Three Little Pigs and Linear Systems, The Care Bears Save the Function and Matrices,* and *Transformations Visit Cinderella at the Ball.*

Enjoy a little piece of our childhood and math content at the same time!

Bedtime Stories for the Scholastically Inclined

You are challenged to create an exciting innovative bedtime story (Friday nap time story) for the classes to come in the future after you. Your story must incorporate material from one of the units of study in the course.

Outline:

Title: _____

_____

Unit of study to be covered: _____

_____

Setting: _____

_____

Main Characters: _____

_____

Mathematical Concepts to cover:

1) _____

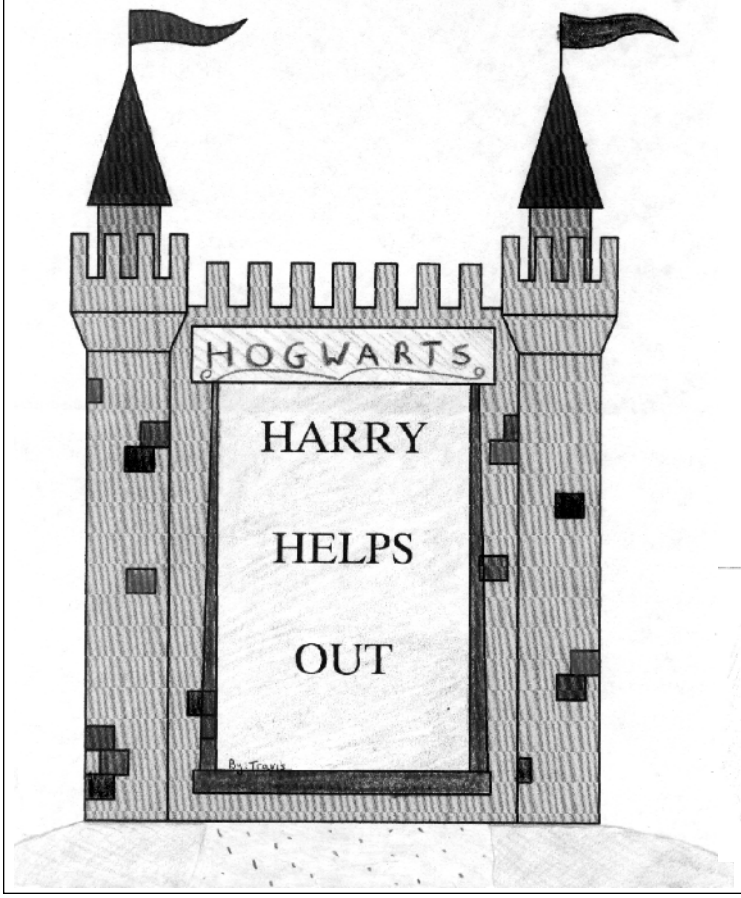
2) _____

3) _____

4) _____

Plot time line: <——————————>

Marking Scheme:				
	LEVEL 1	**LEVEL 2**	**LEVEL 3**	**LEVEL 4**
Covering Chapter Content	**5** One concept covered well (full mathematical solution modelled).	**10** Two concepts covered well or three concepts covered.	**15** Three concepts covered well or four concepts covered.	**20** Four or more concepts covered well.
Story Line	**2** Has a story line.	**3** Reads well with some errors.	**4** Reads well with only a few errors.	**5** Reads smoothly.
Presentation	**2** Written legibly.	**3** Computer processed.	**4** Computer processed and colour.	**5** Computer processed, colour, and graphics.
Grammar and Punctuation	**2** More than eight errors.	**3** More than three errors but less than eight.	**4** Less than three errors.	**5** No errors.



Harry awoke early Tuesday morning to find the dormitory blazing with sunlight. The sounds of Ron's angry grunts and groans were brought to his attention. "What's the matter Ron"? asked Harry sleepily. "Its this dumb math question that stupid Snape gave me, just because I spilled my Polly-juice potion".

"I see," said Harry. "I'm glad I was at Quidditch".

"It was not even my fault! Draco Malfoy bumped some spider legs into my potion making it overflow!" said Ron angrily.

"So what's the problem?" asked Harry.

"Well it goes like this; Y=X+1, Y=-2X+4" said Ron "and I don't know how to do this".

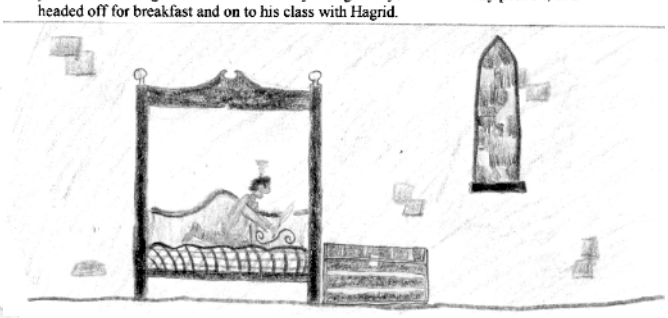
"Well, what would Hermione do?" thought Harry. All of a sudden the solution hit him. "It's substitution!" he exclaimed. Ron looked at him blankly. Harry said, "here let me show you!" With that he grabbed Ron's parchment and scribbled out the question. "I remember doing this at muggle school in Mrs. Hill's class", said Harry.

"So you actually do something useful in muggle school" joked Ron. Harry just smiled. "All you do is this;"

Y=X+1❶	sub into ❷
Y=-2X+4❷	Y=X+1
Rearrange❶ for X	-2=X+1
X=1-Y❸	-X=2+1
Sub❸ into❷	-X=3
Y=-2(1-Y)+4	-1 -1
Y=-2+2Y+4	X=-3
Y-2Y=-2+4	
-Y=2	
-1 -1	(-3,-2)
Y=-2	

"There," said Harry, "but just make sure you write it, otherwise Snape will recognize my writing and we will be in for it".

"Alright, said Ron. "Thanks, you really saved my life." Harry just winked and said "see you in Defense Against Dark arts." Harry said good bye to the fat lady portrait, and headed off for breakfast and on to his class with Hagrid.



He wanted to be there a bit early and it was a good thing he was. Harry arrived at Hagrid's cabin just in time to see some creature explode out of the front door. Hagrid wasn't far behind. "Good morning Hagrid" yelled Harry. Hagrid stopped and muttered "Harry, can you give me a hand?"

"What was that?" asked Harry.

"It was a Norwegian Ridgeback Dragon. I've always had a soft spot for 'em".

"How are we going to catch it?" asked Harry.

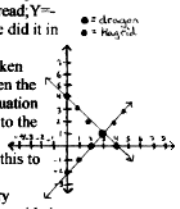
"Well," said Hagrid "they are impossible to catch if you follow them, so the trick is to intercept them".

"That will be pretty hard won't it?" said Harry.

"Yep, it will be, but I have one thing that will help us out" replied Hagrid. He gave a sly smile, pulled out his pink umbrella and muttered "appear cartesia". Right away a huge glowing Cartesian plane appeared on the ground. It was the size of the entire Hogwarts property! "Now all we need is a formula to represent me and one to represent the dragon. I was expelled before we covered any of this in Hogwarts, so I hope you can help me".

Harry stood and thought for a moment "we need two equations to do this, but I don't know which ones to use".

"Er, well, I might be able to help" said Hagrid. "A bit O magic will help". He tapped his umbrella twice and said "equations!" There was a poof of smoke and the smell of burnt plastic leaving Hagrid holding a piece of parchment. Hagrid handed it to Harry and said "I can't make nothing of it, you give it a try". Harry took the parchment. It read;Y=-X+4, Y=X-2. At once Harry recognized it. "It's the up one over one method. We did it in Mrs. Hill's math class".



" So you actually learn stuff in that, er, muggle school?" asked Hagrid, a bit taken aback. Harry was madly plotting the points while he told Hagrid what to do, when the two lines of the equations appeared on the giant Cartesian plane. "You're the equation Y=-X+4 and the dragon is Y=-2. When these lines appear you just run like mad to the place where the two lines meet. Then, when the dragon comes by, just grab it"

"Thanks so much Harry I owe you big time, but um, don't say anything about this to anyone if you, er, don't mind".

Harry grinned, then the two lines appeared and Hagrid took off in a flash. Harry watched in amusement as Hagrid did a flying superman and tackled the dragon in mid air bringing it to the ground with a crushing bone thud. There was a brief struggle. Hagrid returned panting and spitting out grass. He stuffed the dragon back into his house returning just as the students started coming to class. Hagrid's class proceeded without any trouble. Harry would tell Ron and Hermione later. He laughed at the thought of their reaction.



Harry arrived in the Great Hall for lunch. He greeted his friends and sat down beside Hermione. She had her face buried in a book, as usual.

"Arg!" she grunted. "I can't get this stupid problem!"

Harry was going to ignore her when he saw the title of her book: *Muggle Math*.

"What problem is it?' Harry asked curiously.

"It's a word problem. The only one I haven't done in the *whole* book," she replied pointing to a complex word puzzle.

"We didn't do this stuff in muggle school," she added.

Harry smiled and said, "Lucky for you I covered it in Mrs. Hill's math class." With that he snatched the book out of her hands, grabbed a piece of parchment and between mouthfuls of steak and kidney pie, Harry began to solve the question. Several servings later, the question was finished (along with the steak and kidney pie). "There!" he said, quite pleased with himself. Hermione was very surprised.

"How on earth did you did it?" she asked and Harry explained,

Y+X=83	Y+X=83
Y-X=21	52+X=83
2Y= 104	X=83-52
2Y=104	X=31
2 2	(31,52)

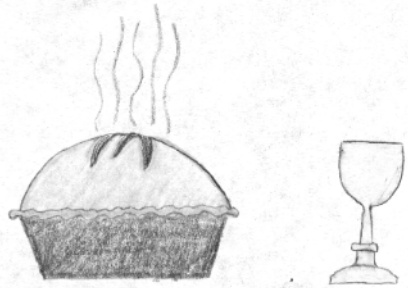
Y=52

"I can't believe I couldn't do that! It's so simple!" exclaimed Hermione.

"What ever," said Harry under his breath.

"Well thanks a bunch Harry".

"You're welcome," he replied. He turned his attention back to lunch and friends.



Later, during lunch, Harry noticed that Draco Malfoy kept looking at him smirking and giving him evil-eyed stares. "I wonder what's up with Draco" Harry asked Ron and Hermione.

"He looks more evil than ever," remarked Ron.

"If I didn't know better" said Hermione "I'd say he has a really evil plan up his crooked sleeve".

"I couldn't agree with you more, my dear," said a voice behind them. The three friends turned around and to their surprise there stood Albus Dumbledore he was smiling, but his face soon became grim.

"Harry may I see you please?" he said, then walked away.

Harry gave a puzzled glance to his friends then took off after Dumbledore. When Harry reached the door he turned around to look at his friends. At about half a glance his eyes locked onto Malfoy's. Malfoy put his finger to his throat and made a slitting motion. Harry swallowed hard, turned and followed Dumbledore to his office. Once inside, Dumbledore explained why he needed Harry.

"You see," he said, "Mr.Malfoy is up to something evil. I have observed him stealing ingredients from Professor Snape and late at night he sneaks out of Slytherin house and goes outside to the west side of the castle. He takes out his wand, taps a stone and a flight of stairs appears. He climbs them then disappears into a secret chamber. In this chamber, he is making Dark Potions that he can, and probably will, use to harm and even kill you!" Harry was taken aback, and his face expressed disbelief.

"But how can I help?" he stammered. Dumbledore replied, "the only way I can stop him is to get into the chamber. The trouble will be finding the right stone to tap. The only clue I have is this equation.

3X+2Y=-18, 3X-4Y=18

I need you to find the co-ordinates of the stone".

Harry took the parchment and thought back to Mrs.Hill's math class and unit #5, Systems of Equations.

"It's elimination," he whispered (still in shock from hearing of Draco's plan).

"Good," said Dumbledore, "then I will leave you to work your muggle magic". Harry worked diligently on the question.

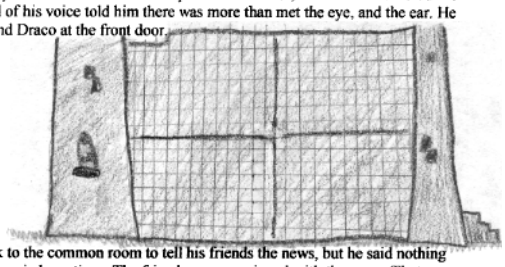
3X+2Y=-18❶	sub X=-2 into ❶
3X-4Y=18❷	3X+2Y=-18 ❶
2(3X+2Y)=2(-18)	3(-2)+2Y=-18
6X+4Y=-36❸	-6+2Y=-18
3X-4Y=18❷	2Y=-18+6
9X =-18	2Y=-12
9 9	2 2
X=-2	Y=-6 (-2,-6)

When Harry had finished, he handed Dumbledore the answer.

"Splendid," he said. "Come with me and we will put an end to Mr.Malfoy's plan." With that, they left.

They soon found themselves in front of the west wall. Dumbledore counted –2 stones across and –6 stones down. He pulled out his wand, tapped the proper stone and the staircase appeared. Harry and Dumbledore ascended the stairs. At the top, they stopped. Dumbledore turned and winked at Harry. Then in a commanding voice said, "Voldermort!" and a door magically swung open. "You stay here Harry" said Dumbledore. He entered the chamber. A moment later Harry heard the smashing of glass, and a brief struggle followed by several cracks of magic. Dumbledore came out ten minutes later, and behind him floating, bound and gagged was Draco Malfoy.

"He has had a lot of evil help" said Dumbledore. "You are extremely lucky to be alive… you probably had an hour left. He will be expelled and taken away". Harry could have jumped for joy at the news of Draco's expulsion. However, the look in Dumbledor's eyes and the sound of his voice told him there was more than met the eye, and the ear. He left Dumbledore and Draco at the front door.



Harry bolted back to the common room to tell his friends the news, but he said nothing of Dumbledore's worried reactions. The friends were overjoyed with the news. That evening Ron, Hermoine and Harry spied two Dementors escorting Draco out the front gate. They knew Draco Malfoy would never bother them again. It was a very happy moment. Through the evening and late into the night the Gryffindor house celebrated. Now, not only were they rid of a very bad wizard, but also their chances at the Quidditch cup had more than doubled! It took the Gryffindor ghost Nearly Headless Nick and Percy Weasley (a prefect) almost half an hour to get everyone off to bed. It would be a day to remember!

Up in his tower, Albus Dumbledore was deep in thought. Suddenly, he spoke to Fawks the Phoenix. "What I saw in that chamber required more magic than any child could ever have. My fears are confirmed… Voldermort lives!!!" But that's another story.



▲ Gender and Mathematics in the 21st Century

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The last twenty-five years have brought considerable progress in women's participation in mathematics at the pre-college level. However, inequities still exist in the performance of young women on college entrance exams (Groves and Cooper, 1999) with an average difference, for example, of over 30 points on the Scholastic Achievement Test-Math. Moreover, despite a doubling of the proportion of bachelor's and master's degrees in mathematics earned by women since 1950 and a quadrupling of the proportion of doctoral degrees, women are still earning about 20 percent of the doctorates in mathematics. In fact, women represent less than 15 percent of the employed doctoral scientists and engineers in computer science, mathematics, agricultural science, environmental science, chemistry, geology, physics and astronomy, economics, and engineering (National Science Foundation, 1996). Parity is still a vision for the future. Although some feminists argue that women are sensible to opt out of fields that are male dominated and hierarchical, or have the potential to involve them in work incongruent with female values such as the military (Becker, 1990; Fennema, 1996; Harding, 1986), many others condemn this under-representation despite more than two decades of intervention.

As our research on gender and mathematics has matured, it has evolved from deficit models ("sex differences") to the use of feminist paradigms (Fennema, 1996). In a deficit model, people who do not do well in mathematics are identified (e.g., females) and their characteristics are contrasted with those of people who excel (e.g., males). Interventions based on such models usually try to change those who do not do well to be more like those who excel. Feminist models acknowledge that gender is socially constructed and that differences are not biologically determined. Many scholars have learned to acknowledge and support, rather than deny, gender differences, while working to ensure that the European American male model is not regarded as the epitome to which all should strive.

"Traditional" ways of teaching mathematics that stress certainty, one correct answer, deduction, logic, argumentation, algorithms, structure, and formality may be particularly incompatible with the ways in which many women learn (Becker, 1995; Belenky et al., 1997; Jacobs, 1994). Researchers have hypothesized that a different learning style, set forth in *Women's Ways of Knowing* (Belenky et al., 1997) might account for women self-selecting out of mathematics and related careers. In this theoretical model of how women "come to know", the authors present five perspectives: *silence, received knowing, subjective knowing, procedural knowing* (separate and connected), and *constructed knowing.* In the *silence* and *received* knowing perspectives, the teacher is the expert and the student learns only by listening. In the *subjective knowing* perspective, an important one for women according to Belenky and her colleagues, knowledge develops from within, from one's own experience. Men and women seem to handle this perspective differently. While men assert their right to their opinion, women may feel it is *just* their opinion. In *procedural knowing,* the voice of reason emerges and the learner begins to evaluate the validity of arguments. However, there again seems to be a gender difference in this perspective. Men favour logic, argumentation, and rigour to evaluate validity (separate knowing), while women are more likely to use conjecture and their own and others' experiences (connected knowing). In the *constructed knowing* perspective, one integrates knowledge derived from experience.

Researchers who have interpreted this theoretical model in mathematics call for connected teaching as an approach to provide more success in mathematics for all students. Through connected teaching, a teacher can provide the learner with the opportunity to integrate knowledge that is constructed through her experience. Some elements of connected teaching include: developing students' voices; providing for first-hand experience; confirming students as knowers; stressing belief versus doubt; and supporting students in their growth toward understanding. The following table provides some ideas for the classroom teacher that can be used at all levels to incorporate connected teaching in your classroom.

Feminist theories such as *Women's Ways of Knowing* can provide a new lens through which to consider our classroom work. A recent publication from NCTM, *Changing the Faces of Mathematics: Perspectives on Gender* (Jacobs, Becker, and Gilmer, 2001), has a

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number of chapters that will challenge your viewpoint on gender equity. If you think the issues around gender and mathematics have been solved, I recommend it as a way to stimulate further thought and, hopefully, action.

Element	Connected Teaching in Mathematics
Voice	Students develop own justification Group problem solving Class discussions Teacher does not give answer Allow students to struggle for solution Focus on explanation
First-hand experience	Students draw and construct models Use visuals Use technology Students collect data Students develop and check conjectures
Confirmation of self as knower	Look for what is already known Listen to students' reasons Respect students' ideas Engage class in outside activities Have high expectations
Belief vs. doubt	Ask for explanations even when right Answer questions with questions
Support vs. challenge	Validate present understanding while providing challenges

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▲ "I Don't SEE It"



TARA TASKER

Tara Tasker is an elementary teacher who loves mathematics. She is currently pursuing her M.Ed. at Queen's University.

Last year, I decided to teach summer school. Having taught grade two, I thought it might be a leap to teach grade seven during the summer and I doubted that I would be hired. I did get hired, not for grade seven, but for grade ten academic math. I was excited to teach something so new and different. I reviewed the text and the curriculum and felt prepared for the job. I knew that most of my students were either upgrading a low mark or making a second attempt at getting their grade ten credit. What I was not prepared for was their fear of fractions. They persistently asked if they could use decimals instead of fractions. They all griped and complained about fractions. I could also see their fraction weakness when rearranging equations and finding slopes on the Cartesian plane. I taught some remedial fraction concepts, only to be bombarded with, "I don't see it."

In the fall, I went back to my elementary school to teach grade three. While preparing long range lesson plans, I realized that I would be starting my students down their own fraction road. That made me wonder what happened to all my grade ten summer school students that made them so scared of fractions and what I could do to prevent that fear. Why did they persistently say, "I don't see it" instead of "I don't understand it" or "I'm not grasping the concept?" It was then that I realized they didn't "SEE" it. They needed more time with fraction manipulatives to SEE and understand fraction concepts.

Teaching fractions is one of the more difficult units in the Junior and Intermediate grades because of several underlying problems. The first problem is providing useful manipulatives that help students understand the concepts through hands-on exploration but also provide a link to the semi-concrete paper and pencil activities. The second problem is finding these manipulatives. Many schools no longer have the budgets to provide basic supplies, let alone afford class sets of fraction manipulatives. The final problem is that many teachers were not taught to SEE fractions with manipulatives when they were in school; they were taught to memorize procedures and algorithms.

I needed to explore fraction ideas and concepts for myself and for my students. In the past year, I have attended numerous workshops and conferences and have read sections in math texts such as *Teaching Mathematics to Children* and *Elementary and Middle School Mathematics: Teaching Developmentally*. This article is a compilation of my findings and provides manipulative ideas on how to present fraction concepts so that students SEE it; however, for a more in depth explanation, I would refer the reader to the above texts.

Looking at Fractions

Fractions represent a division sentence when dividing one whole. If you look at the division symbol ÷, you can replace the dots with numbers to turn it into a fraction. The top number is the numerator and the bottom number is the denominator, but what does that mean? If we had $\frac{3}{4}$, we know that there are three quarter parts of a whole. Therefore, the numerator is the number that counts and the bottom number is the number that tells what is being counted.

There are three models that can be used with exploring fractions:

1. Area Models: These models represent the part of a fraction through the area of the whole. Examples of these include pizza slices, fraction pie pieces, pattern blocks, and paper folding.
2. Measurement Models: These models represent the part of a fraction through the length of the whole. Examples include fraction strips, number lines, and the "handy-dandy fraction helper" (shown in figure 1).
3. Counter Models: These models represent a fraction as a number of a whole. Examples of these include double coloured counters and drawings (semi-concrete).

Most of these manipulatives are already present in schools or can be easily made. Having students make the manipulatives increase their understanding. At a conference, the workshop leader said, "the individual who makes it, thinks about it."

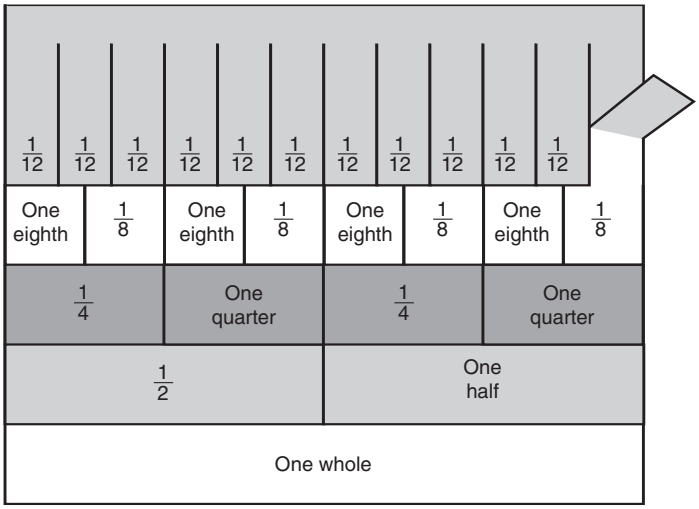


Figure 1: The handy-dandy fraction helper[1]. *The helper is put together in layers, and fraction parts are cut like fringes. You may use words or numerical representation of fractions. These may be put in by the teacher or by the students.*

Addition and Subtraction of Fractions

The algorithm for adding fractions is as follows: add the numerators and keep the denominators the same, or find a common denominator and add them together (e.g., $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$). Why does the denominator stay the same? Why do you only add the numerator? Can you hear "I don't see it" coming?

Using manipulatives allows the students to "see" that they are counting parts of a whole. Just as if they were adding 4 jelly beans with 5 jelly beans, they could count all 9 together. You are adding 2 parts with 1 part. The whole just happens to be divided into parts; specifically, 4 parts or quarters. I would suggest an area or length manipulative model to help solve these addition fraction problems.

Subtraction is very similar. If you have 7 eighths of an eight-slice pizza left over and someone takes 2 slices, how much is left? You are taking away two parts from 7 parts and the parts just happen to be eighths.

Multiplication of Fractions

When you multiply fractions, why is the product smaller, whereas when you multiply whole numbers, the product is bigger? This is a very hard concept to "SEE". A good way to demonstrate why the product of multiplying two fractions together becomes smaller is through a word problem such as:

> *You have $\frac{2}{3}$ of a pizza left over from the night before. You take $\frac{1}{4}$ of the leftovers for lunch. How much of the whole pizza do you have for lunch?*

The operation to solve this problem is $\frac{1}{4}\times\frac{2}{3}=\frac{2}{12}=\frac{1}{6}$. This problem gives clarity to the method of verbalizing the concept of why the product of multiplying two fractions together becomes smaller. We are asking to find $\frac{1}{4}$ groups of $\frac{2}{3}$, a very similar sentence when beginning to teach multiplication of whole numbers. For example, teachers may discuss 4 bunches of 3 bananas when first introducing 4 x 3 = 12. Similarly, we are asking for $\frac{1}{4}$ groups of $\frac{2}{3}$. A manipulative to use for this problem with an overhead transparency is shown in figure 2. One transparency is an area model of $\frac{1}{4}$ with the quarters divided vertically. The second transparency is of $\frac{2}{3}$ with the thirds divided horizontally. When you superimpose the transparencies, $\frac{1}{4}$ of $\frac{2}{3}$ is shown pictorially. The total number of parts (12) is shown, as well as the total number to be counted (2).

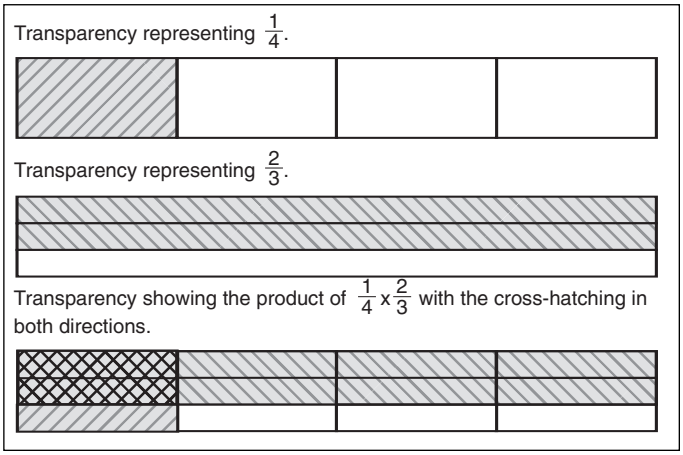


Figure 2: Transparencies to demonstrate why the product of multiplying two fractions is smaller.

Division of Fractions

Why is it when you divide fractions the dividend becomes bigger? That isn't how whole numbers work! It is very easy to teach division as a traditional algorithm; cross multiply or invert and multiply, especially after spending enough time on multiplication of fractions. However, this algorithm does not help students understand the concept, nor does it answer the question of why the dividend becomes bigger. Again, a way to

[1] *The "handy-dandy fraction helper" is a name the author's grade 6 class in Baker Lake made up together. Months after seeing something similar in the* Ellison Paper Cutter *Catalogue, the author modified and adjusted it to fit her students' mathematical needs.*

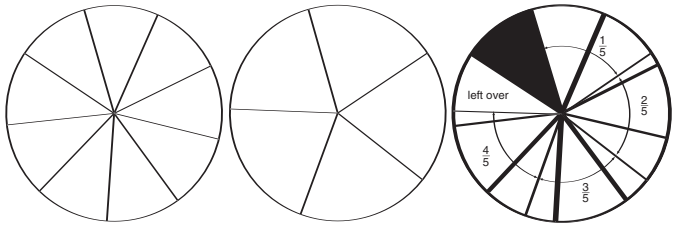
explain the division of fractions is through word problems, such as:

> *You have $\frac{8}{9}$ of a kilogram of jelly beans. The candy store fills their grab bags with exactly $\frac{1}{5}$ of a kilogram. How many grab bags can you make?*

The operation to solve this problem is $\frac{8}{9} \div \frac{1}{5} = \frac{40}{9}$, but $\frac{40}{9}$ is an improper fraction. Let's turn it into a mixed fraction. You will be able to make 4 grab bags and you will have $\frac{4}{9}$ of a grab bag left over. You can allow students to see this operation if you use the area model fraction "pie" pieces manipulative. However, instead of having the "pie" pieces, have the entire pie sectioned into ninths on an overhead transparency. With an erasable marker, colour in $\frac{1}{9}$ of the pie to indicate that it is missing. Use a second overhead transparency of a pie sectioned into fifths and lay the fifth pie over the ninth pie. Colour in one fifth at a time, until there are no more whole ninths left. This will demonstrate visually, how you are dividing $\frac{8}{9}$ into $\frac{1}{5}$.

Concluding Remarks

By the time students reach high school, they are expected to understand the concepts and algorithms of fractions. Fractions are required when rearranging



equations in algebra, for understanding slope on the Cartesian plane, and simplifying units in chemistry or physics. Teaching fractions starts in grade one, yet most students fear fractions. I wonder if that is because we forget to let our students "see" fractions or is it because we, as elementary teachers, fear fractions as well because we were never shown how to SEE them. If we are to start our students on the right track as they travel down the fraction road, then we must re-evaluate what we know and how we learned it.

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Winter green

▲ Assessment Corner: Summative Assessment Needs Summative Instruction



CHRIS SUURTAMM
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Chris Suurtamm is Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

Summative tasks are generally used for assessment purposes. However, it is important that students have opportunities to make connections between the key concepts of mathematics in instructional settings as well as in assessment settings. Teachers have been seeking and using a variety of tasks to assess student learning at the end of a unit, course, or academic year. Many teachers have included investigations or performance tasks that ask students to assimilate their knowledge and skills and to connect the important concepts that are woven throughout the course. Teachers show great perseverance since such tasks can be difficult to find, even harder to design, and can pose dilemmas in determining appropriate evaluation criteria. However, in offering these rich opportunities to students, teachers recognize that making important connections throughout, and at the conclusion of a course, adds depth to students' understanding of mathematics.

It is widely recognized that mathematical ideas do not exist in isolation from each other, and a recognition of and facility with the interrelationships among sets of mathematical ideas is a basic feature of genuine mathematical understanding. One type of connection occurs when an assessment task requires that a variety of mathematical ideas be used in the solution process, (Zawojewski & Silver, 1998).

Summative assessments are not simply a method to get a mark at the end of a course. Summative assessments afford teachers the opportunity to help students link important ideas in mathematics. These important connections exist at all grade levels. For instance, in grade 4, the concepts of comparing and ordering are shown by students comparing and ordering objects by their linear dimensions in Measurement, comparing and ordering whole numbers and decimals in Number Sense and Numeration, and sorting and classifying two-dimensional figures according to shape in Geometry and Spatial Sense. In grade 8, students use numerical and graphical analyzes of data to inform decisions in a variety of ways. They respond to numerical information in a variety of media and perform multi-step calculations in real-life situations in Number Sense, and they evaluate data and draw conclusions from the analysis of data in Data Management and Probability. In grade 11, students make connections between the notion of geometric sequences and exponential growth and between arithmetic sequences and linear relations.

At the end of a course, students should be given opportunities to make these connections before they are asked to put it all together in an assessment task. Summative tasks should include both instructional tasks and assessment tasks rather than be restricted to assessment opportunities. If mathematics assessment is to help students develop their powers of reasoning, problem solving, communicating, and connecting mathematics to situations in which it can be used, both mathematics assessment and mathematics instruction will need to change in tandem (MSEB, 1993).

Teachers cannot expect students to suddenly see the connections between components of a course unless they have been given opportunities and experiences to make connections. "The teachers' role is to provide opportunities for students to assimilate new learning and try new approaches to solving problems before being assessed for the 'big prize'" (Muri, 1998, p.91). In this way, the instructional activities help students make links, develop strategies, choose from a variety of different approaches, find appropriate models, and write, explain, or present a solution in a variety of ways.

An example of a summative unit that includes both instructional and assessment tasks for the Grade 11 University/College course (MCF 3M) is on the OAME web site (www.oame.on.ca) as part of the "Care2 Package".

This unit includes review instructional activities, assessment activities, and sample questions for a paper and pencil examination. The activities in this unit address the Overall Curriculum Expectations of the course, and together, provide a culmination to the course. However, teachers may decide to select certain activities to use for review or assessment along with their own activities (Suurtamm et al., 2001).

The unit begins with "Eric's Extreme Ballooning" which is an instructional task that asks students to examine different sets of data and to determine a suitable model. Students work in groups and are later encouraged to discuss what types of strategies they used to determine a suitable model. An overview of the entire summative unit follows:

Day / Activity	Description	Purpose
1 Eric's Extreme Ballooning	This is a group instructional activity	Instructional
2 Eric's Extreme Ballooning	Students are presented with a variety of data within the context of Hot Air Ballooning. They are required to choose from the mathematical functions presented in this course to model this data in order to solve problems.	Instructional and assessment of communication
3 Financial Decision Making	Students will explore a variety of options in making investments and determining mortgage options. Financial Applications of Sequences and Series	Assessment activity to be done in pairs
4 Review Stations	Students work through 5 stations to review important concepts and make connections between various parts of the course.	Instructional
5 Review Stations		Instructional
6 SAD	Students examine data relating to hours of daylight over a period of time. They determine a model for applying light therapy that would ensure total hours of daylight to be constant over a period of time. Trigonometry Strand	Assessment
7 Spindles and Spaces	Students explore calculations needed to determine equal spaces between the spindles of a railing. An algebraic model is required that describes the amount of space based on the number of spindles. Tools for Operating and Communicating with Functions	Assessment
Examination Questions	A "test bank" of sample examination questions. Each question is linked to curriculum expectations and Achievement Chart Categories.	Assessment

(Suurtamm, et al., 2001)

The material provides lesson plans, student activity sheets, and assessment rubrics. Naturally, there may not have enough time to work through all of the tasks. Teachers can, however, choose from these tasks for activities to be used at the end or throughout the course.

Students in all grades will benefit from work that helps them grasp the important concepts in the course. Teachers will find that tasks that are good summative assessment tasks can also be used as good instructional tasks. Students can work in groups on these tasks and exchange ways to approach the problem. Students should be able to identify and describe how the activity connects to work that they previously did in the course. Furthermore, students can exchange written solutions and give suggestions to help make the solution clearer to the reader. These tasks are not meant to imitate what the assessment task will look like but should help students build the knowledge and skills that they need to help them "sum up" their understanding.

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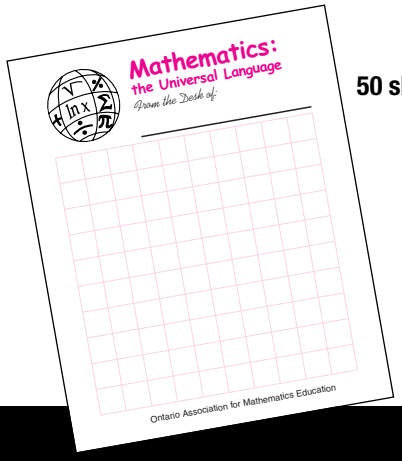
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▲ NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2003-2004

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 24, 2003, to:

Shirley Dalrymple, Past President, 31 Deerford Road, Willowdale, ON M2J 3H9
Business (905) 836-0021, e-mail: shirleydalrymple@hotmail.com

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2003, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Oshawa, May 2003.

NOMINATION FORM

I nominate: _____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Telephone: H ( ) _____ W ( )_____

Nominator's Name : _____

Nominator's Address: _____

Telephone: H ( ) _____ W ( )_____

▲ OAME/NCTM December 2002 Report



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Canadian Leadership

During August, I attended the Canadian Leadership conference in Montreal as Ontario's representative. Twenty National Council of Teachers of Mathematics (NCTM) or affiliate leaders, from most of the Canadian provinces and some nearby states, shared resources and learned what NCTM can offer affiliates. Carolyn Kieran, from the Montreal area, gave a report on behalf of the NCTM board of directors.

The mission of the NCTM is to provide the vision and leadership necessary to ensure mathematics education of the highest quality for all students. NCTM has 254 affiliates made up of 150 partner affiliates of which OAME is one. Kieran put out a special request for Canadian mathematics teachers to serve on an NCTM committee. The committees include executive, budget and finance, nominations and elections, policy and planning (equity, diversity, instructional issues, professional development), services (affiliate, conference, academy), and publications (educational materials, journals). Other task forces include assessment, e-standards, Mathematics Education Trust, *Student Math Notes*, and MATHCOUNTS to name a few. If you are interested in serving on any of these committees, send your nomination letter with qualifications and reasons to: Johnny W. Lott, President NCTM, Department of Math Sciences, University of Montana, Missoula, MT 59812.

Delegates to Leadership had to complete a 49 question e-scavenger hunt to learn what is offered via the NCTM web site: www.nctm.org. That was a most interesting exercise and certainly helped me improve my browsing skills.

Now it is your turn. Complete the following e-scavenger hunt and send your answers to me on or before January 31, 2003. At the February 2003 OAME board meeting, the publications committee will draw a winning name from the correct submissions for a certificate to purchase a popular NCTM publication.

OAME-NCTM e-Scavenger Hunt

1. What was the topic of the *Student Math Notes* for May 2002?
2. What was the theme of the *World's Largest Math Event #8*?
3. How many copies do you need to send when you submit a manuscript to the *Mathematics Teacher* journal?
4. Where and when is the next NCTM Regional conference being held in Canada?
5. What are three benefits of being an NCTM Individual Member?
6. Where will the 2004 NCTM Annual Meeting be held?
7. What are 2 of the 5 requirements you need to provide in writing to NCTM when requesting Copyright Permission?
8. What are the three different formats of *Principles and Standards for School Mathematics*?
9. What are the three types of *Illuminations* Internet-Based lesson plans?
10. What activity is found in *Reflections on Teaching for Grades 9-12*?
11. What is the theme for NCTM's 2005 Yearbook?
12. How many problems are presented in *Children Are Mathematical Problem Solvers*, a new NCTM resource?

Frances Schatz,
NCTM Representative ▲

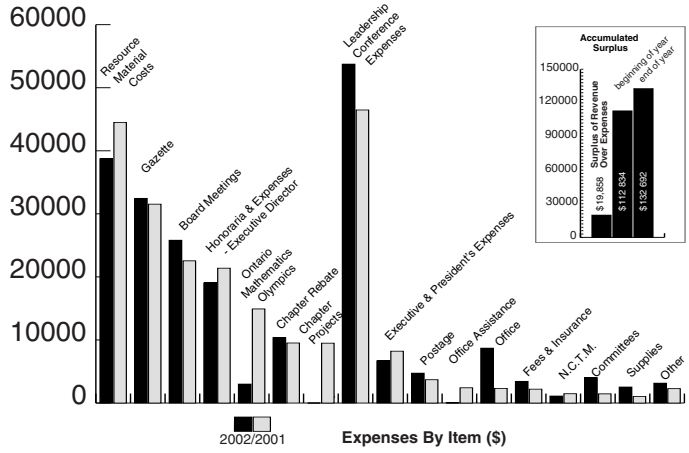
▲ OAME Financial Statement 2001/02

Notice to Readers

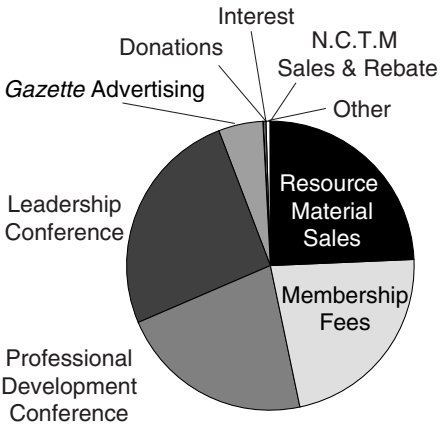
We have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 2002 and the Statement of Operations and Operating Fund Balance for the year then ended from information provided by management. We have not audited reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

London, Canada September 13, 2002
ANAS, SMITH & GRIFFIN Chartered Accountants

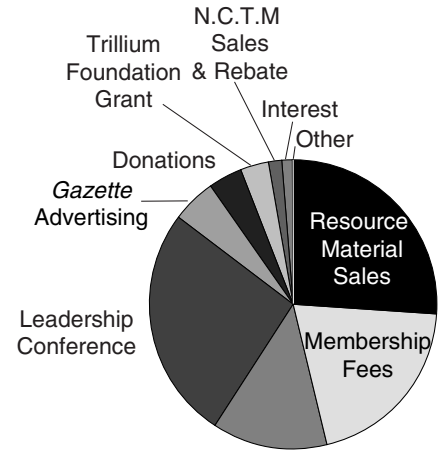
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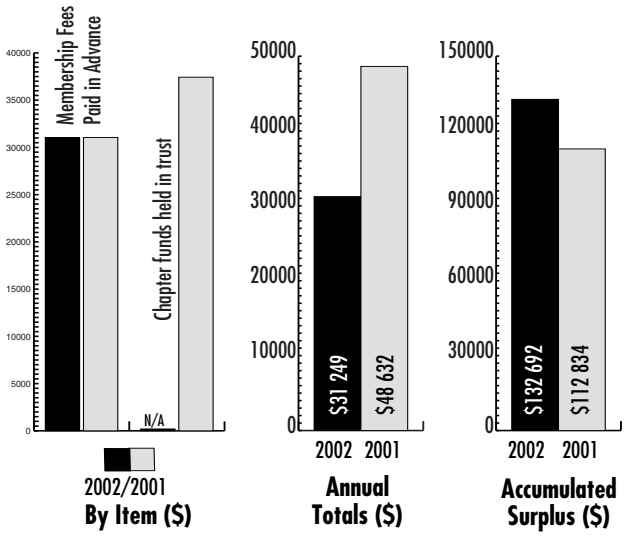


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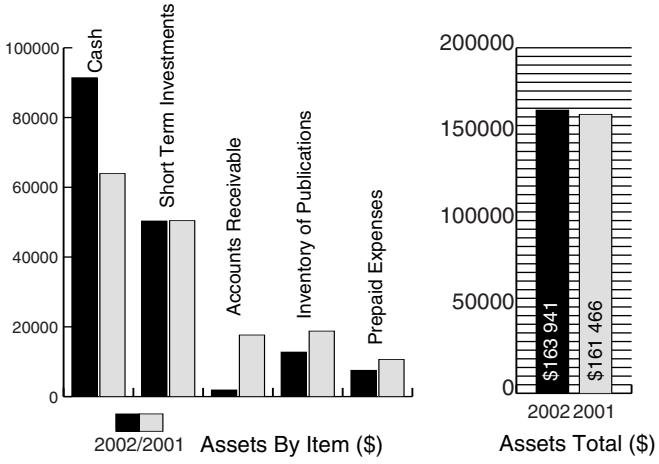


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Current Assets 2001 / 2002



▲ Photo Math – A New Way of Picturing Math

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Ron Lancaster is a middle and high school mathematics teacher, currently working throughout North America and Asia as an Independent Mathematics Consultant.

A picture is worth a thousand words is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and to promote the idea that in fact *A picture is worth a thousand numbers*. Readers can expect to find Photos with Math in every *Photo Math* column, and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.

Teachers and students are encouraged to pose mathematical questions related to their own encounters and submit these problems along with photographs to the Editor for possible publication in a future column.

Introductory Comments

For the second summer in a row, I had the good fortune of being a Leader at the highly-regarded mathematics and technology conference for middle school teachers held annually at the Peddie School in Hightstown, New Jersey. Larry Ottman (another Leader at the conference and a teacher from New Jersey) and I developed a Math Trail on the campus of Princeton University for teachers who attended the conference. The following questions are from the Trail we created.

I encourage all middle school mathematics teachers in Ontario to consider including this outstanding conference in their plans. For more information, go to www.peddie.org and follow the link to “Mathematics Conference”.

Exclusive

Photograph #1



Study the glass design above the entrance to the building at 48 Nassau Street which is similar to the one shown in Figure 1. Give a detailed description of how to construct this diagram using a ruler and compasses or using the *Geometer's Sketchpad.*

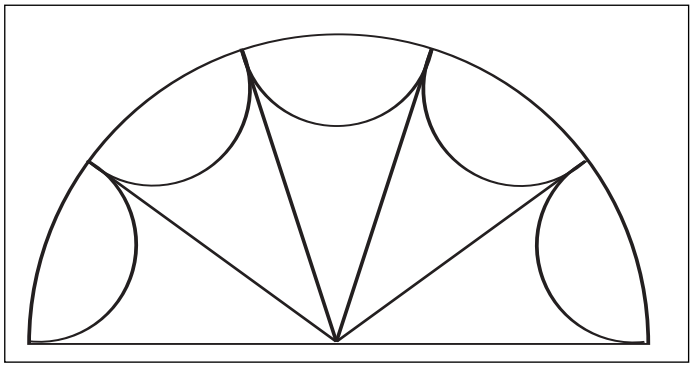


Figure 1

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Photograph #2



Study the sale sign for Talbots. What mathematical interpretation of the sign would lead to a decision not to shop at this store?

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Photograph #3



Study the base of the obelisk shown above.

(a) Starting with the top step and moving down to the bottom one, how does the perimeter of each octagon change from one step to the next? What formula could be used to give a step-by-step analysis of how the perimeter changes?

(b) From one step to the next, how does the volume of these slabs of marble change?

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▲ Union Gas/ O.A.M.E. Partnership

DAVE AND SUE HESSEY
OAME EXECUTIVES

Union Gas, together with their parent corporation Duke Energy, has a mandate to focus on three areas to help build stronger communities. These areas are: building leadership capacity in not for profit organizations, encouraging employee volunteerism, and supporting Mathematics and Science Education. Union Gas realized that O.A.M.E. values many of the same principles, that we are headed in the same direction to support Mathematics education, and that we understand that a strong foundation in Mathematics is required by Ontario's workforce. According to David Rosenbloom, Manager of Community Relations, "Union Gas is investing in math education today because it gives tomorrow's students an opportunity to succeed. At Union Gas, we recognize that a strong math foundation is essential for students looking to succeed in a global economy. Union Gas is excited about the activities that O.A.M.E. has planned for the next year and our up coming partnership."

Union Gas will donate $40 000 to O.A.M.E. in 2002 and pending final approval, $52 000 in 2003. The first installment of $20 000 was presented to Marg Warren, President of O.A.M.E. by Paul Shervill, Director Channel Management, Union Gas at the October 5th, 2002 O.A.M.E. board meeting. These donations will support the following activities over the next two years:

Developing A Vision for Learning Mathematics: JK to Grade 12

- Articulating a vision of learning
- Developing classroom ready activities and teacher guides
- Producing of a digital video
- Creating a Professional Development package and guide
- Promoting, printing, and distributing resources developed



OAME President Marg Warren and Union gas Director Paul Shervill in partnership.

Supporting Union Gas/O.A.M.E. Ontario Math Olympics 2003

Developing and presenting three provincial awards at O.A.M.E. 2003 in May:

- Union Gas Award for teaching excellence in Mathematics (elementary)
- Union Gas Award for teaching excellence in Mathematics (secondary)
- Union Gas Award for outstanding leadership in Mathematics Education

All of the members of the O.A.M.E. board of directors have been working diligently since June, 2002 to develop a vision of learning mathematics and are serving on curriculum committees at various levels from JK to Grade 12 to develop and field test classroom activities that demonstrate this vision. A video will be created in the spring that will show our vision as a continuum of learning from K to grade 11. Some of these resources will be showcased at the O.A.M.E. 2003 conference. The last component, a Professional Development package, will be developed later in 2003.

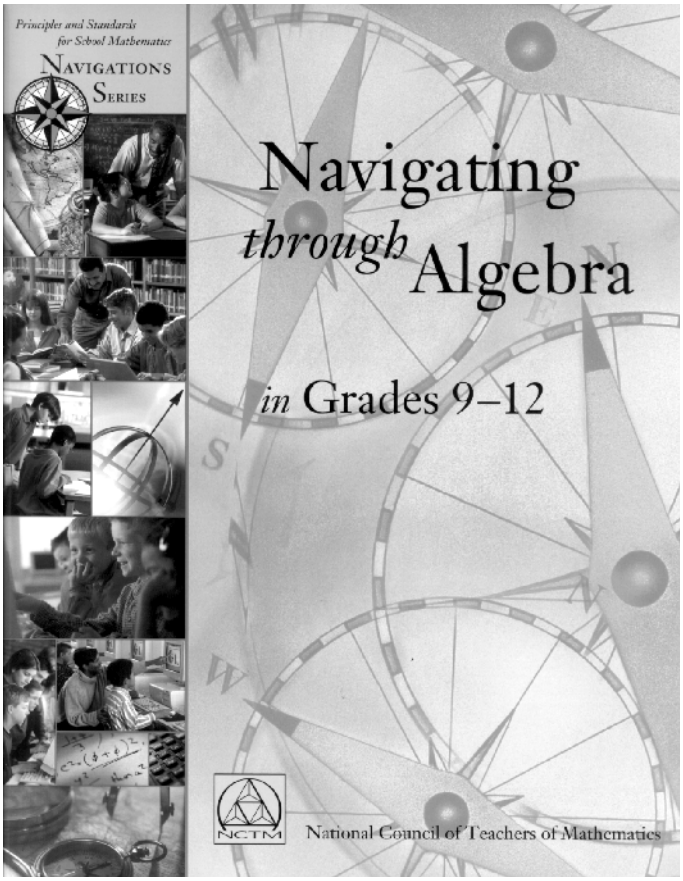
O.A.M.E. is very appreciative of the donation from Union Gas in order to provide more resources and support for Mathematics education in Ontario. ▲

▲ Book Review of *Navigating Through Algebra in Grades 9 – 12*



REVIEWED BY: JACQUELINE HILL

The National Council of Teachers of Mathematics (NCTM) has written extensively on topics of importance in Mathematics. *Navigating Through Algebra in Grades 9 – 12*, is written by Maurice Burke, David Erickson, Johnny W. Lott, and Mindy Obert. The six chapters are organized as Algebra – the Language of Process, Expanding the Notion of Variable, Expanding the Notation of Function Representation, Expanding Students Understanding of Algebraic Equivalence or Identity, Expanding Students' Understanding of Change, and Conclusion. The book has a thorough collection of black line masters, solutions, as well as an accompanying CD-Rom. The book shows NCTM's vision of how Algebra should be introduced, taught, and assessed.



Each activity is well set out for the teacher including goals, materials needed, a description of the activity, discussion to follow, and extensions. The activities should be engaging to the student as well as the teacher. The CD is a brilliant addition to the book, since many people find activities they would love to use if only they could modify them to fit their own style. This CD gives teachers that opportunity.

The book flows with the vision of investigation that the new curriculum in Ontario stresses. The book gives the scaffolding for each activity that younger students require and provides older students with a sense of security.

A teacher can use the table of contents and see where the different activities match the Ontario curriculum. In particular, Ontario teachers will have to match the activities to our Ministry Objectives.

I would give this book, and all its materials, two thumbs up for its insight, scaffolding, and generating keen interest in students. This book is available by contacting Frances Schatz, OAME's NCTM representative at frances.schatz@sympatico.ca. ▲

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education;
- specific contributions to mathematics education in the school, region, province or nation;
- demonstrates leadership in mathematics education in the school, region, province or nation;
- support of and contribution to the activities of OAME/AOEM, and
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

AWARD FOR OUTSTANDING CONTRIBUTION TO OAME / AOEM AND MATHEMATICS IN ONTARIO

This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

AWARD FOR EXCEPTIONAL AND CREATIVE TEACHING IN ELEMENTARY MATHEMATICS

The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

AWARD FOR EXCEPTIONAL AND CREATIVE TEACHING IN SECONDARY MATHEMATICS

The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

INTENDED AWARD *(please check one)*

- ❑ **Life Membership**
- ❑ **Kenneth D. Fryer**
- ❑ **Award for Outstanding Contribution to OAME / AOEM and Mathematics Education in Ontario**
- ❑ **Award for Exceptional and Creative Teaching in Elementary Mathematics**
- ❑ **Award for Exceptional and Creative Teaching in Secondary Mathematics**

Nominee's Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Nominator's Information:

_____

Name of Nominator

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

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▲ Directors' Dialogue



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This is a short version of Directors' Dialogue due to time constraints. The full version, which summarizes the October 4 and 5 board of directors meeting, is available on the OAME web site, www.oame.on.ca.

CHAPTER NEWS

CHAMP: Lindy Smith. CHAMP held its fall "Math 4U" conference at Applewood Secondary School on October 24. The evening featured workshops at the primary, junior, intermediate, and secondary levels, as well as publishers' display and dinner. From November 4 - 8, CHAMP held its Mathematics Contests for grades 7 to 11 students.

COMA: Peter Lenczewski. COMA held a fall mini-conference on Saturday, November 9 with sessions ranging from the double cohort to Math Mayhem. This is the second year that COMA has partnered with the University of Ottawa to host a conference. COMA will continue its Forum series. Plans are being formulated for an Elementary Forum in the last week of January, 2003 and a Secondary Forum planned for the second week of April, 2003. The Carleton-Ottawa Math Olympics Committee is in full swing organizing the 2003 event that will take place on Saturday, April 26.

GOLDEN: Ed Haines. Nil report.

GVMA: Paul Fennell. GVMA's fall conference took place on Thursday, November 7 at Eastwood CI in Kitchener. Future conferences include Thursday, April 10, 2003 and Thursday, November 6, 2003. Planning for OAME'2004, hosted at the University of Waterloo, has begun. Mark Wednesday, April 28 – Saturday, May 1, 2004 on your calendar!

ISOMA: Brian Raaflaub. The fall conference occurred on Wednesday, November 6 at Appleby College. Best practices were shared in round table discussions for primary, junior, middle school, and senior level courses. ISOMA continues to update its e-mail communication system with key representation from each of the independent schools. The executive would like to encourage all members to consider running for office or become an active member on a committee (contest preparation, math Olympics, conference, newsletter).

MAC^2: Greg Clarke. MAC^2 would like to acknowledge the work of the volunteers and organizing committee members who made the OAME'2002 provincial conference a huge success. On October 28, a mini-conference occurred with workshops titled, Fathom, Polydron, and Learning Carpets. Other events are in the planning stages, including a Junior Mathematics competition in the spring.

NOMA: Pauline Therrien. The annual fall conference took place on October 18 – 19, with Stewart Craven delivering the keynote address. Future plans include to encourage more elementary and French speaking members, to create a more effective communication system for the members, to assemble mini-workshops to help with the implementation of the new curriculum, and to have high schools in the region hold "feeder school" tournaments to prepare for the Ontario Mathematics Olympics in combination with the Gauss Mathematics Competition.

NWOAME: Rona Doyle. Lakehead University, in conjunction with mathematics teachers from Thunder Bay is planning a mathematics contest for high school students. The contests, to be held in mid-May, will consist of an individual test and a team component. Plans are also underway for a mini-conference in april. The chapter's OMO competitions will be held in late April.

PINE RIDGE: Jacqueline Hill. The chapter is working hard to organize OAME'2003, "Primed for Math" from May 8 – 10 at Durham College in Oshawa. The keynote speakers will be Marilyn Burns, Don Fraser, and Bill Finzer. Also, teachers are preparing grades 7 and 8 students for OMO'2003 with meaningful investigations and problem solving.

QSLMA: Robert Sherk. A fall mini conference occurred on November 7, 2002 with workshops addressing needs related to the implementation of the new K – 12 curriculum. The chapter is developing an e-mail database of members and past registrants to ensure effective communication with the area's math educators at all levels.

SAME: Sumintra Koowar. SAME held a fall conference, "Hats Off to Mathematics", on Tuesday, November 5 at Albert Campbell CI. Participants attended two workshops, browsed publishers' display, and enjoyed dinner.

SWOAME: David Petro. The chapter is slowly rebuilding its membership this fall. A mini-conference or possibly a set of workshops is planned (date to be announced). Also, planned is the local Math Olympics for next spring.

TEAMS: Pat Margerm. Nil report.

WOMA: John Ferguson. WOMA will host OMO'2003, led by Tom Griffith, on June 13 – 14. In order to encourage members to submit to *The Sine Post* (WOMA's newsletter), a limited number of $200 subsidies for those interested in attending the OAME Leadership Conference in February, 2003 are being distributed. The *Sine Post* returns to a paper version, despite its successful delivery via e-mail.

Y^{4}MA: Rosey Mastrofrancesco. Their fall conference was held on October 17 at Earl Haig Secondary School in North York. The theme was, "Mathematics for Everyone", will the keynote address by Margaret Sinclair. The chapter web site, www.y4ma.com, continues to be updated with a monthly prize raffle for new or renewing members to OAME. ▲

▲ 2002-2003 OAME Board

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