Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Marian Small at **marian.small@gmail.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARIAN SMALL
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In my role as an author and a consultant, I have had the opportunity to work in about 35 school boards in the province, sometimes with elementary teachers and sometimes with secondary teachers. What strikes me is the similarity of our work and not the differences between teaching at those two levels.

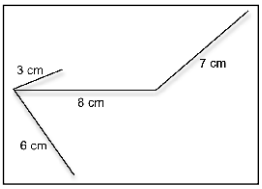
All of us are encouraging students to stretch their thinking, to build connections, to see past the rules to gain a deeper understanding of what mathematics is and how it works, and to enjoy mathematical "play." Often the very same problems that challenge younger students challenge older ones.

Recently, I gave a talk where I shared some problems that might pique student curiosity. Perhaps some of these would be useful to you. I would invite you to try these out with students and send in their thinking on the questions. We will publish a few of the more interesting approaches to the problems.

1. Choose three numbers in the table on the right to add. None can be in the same row or column as the two others. Add them. What do you notice? Why does it happen?

180	230	280
150	200	250
120	170	220

2. Look at this device. Imagine that the various segments are hinged. What lengths can you measure "exactly"?



3. Is there a Friday the 13th every year? How can you be sure?
4. Suppose you want to be able to pay exact change for any price from 1¢ to 99¢. What is the fewest number of coins you need no matter the price what are they?
5. Each student records his or her ten-digit phone number. Rank the choices below from most likely to least likely considering all the phone numbers in the class.
   a) The final digit is 0.
   b) The second digit is greater than the fourth digit.
   c) The first digit is a 4 or 5.
   d) The sum of the ten digits is more than 20.
6. a) A picture that looks like something real is created with 16 base ten blocks. What does it look like and how much is it worth? OR
   b) A picture that looks like something real is created with exclusively quadratic functions on a graph. What does it look like and what are the equations? ▲

▲ President's Message

DAVID ZIMMER, PRESIDENT
E-MAIL: davidzimmer@xplornet.com



Unanticipated Consequences

This semester, I have the privilege of working with a class of Junior–Intermediate teacher candidates at the University of Ottawa. The 43 students represent a wide spectrum of past experiences with school mathematics. Some were quite successful and others not so much. They come from different provinces and countries and have experienced varied forms of instruction. They represent a wide range of ages; some are old enough to have school-aged or older children, and others are recent university graduates.

As a follow-up to a class in which we began the discussion of assessment in mathematics, I asked the students to write a reflection journal entry in which they recounted two personal experiences with assessment in a mathematics class. One was to be a pleasant memory and the other not.

Their candour was refreshing. Please read these very personal reflections and draw your own conclusions about the impact a teacher's approach to assessment, particularly in mathematics, can have on a student's life.

> *One of my earliest math memories comes from Grade 2.... In this memory, it was the end of the school year and we were doing work that we would learn again in September. I was given a sheet of paper with multiplications on it (2 x 2 = ___) and asked to write the answers. As far as I remember, I hadn't actually been taught what an "x" sign meant, so I just added up all the numbers on the sheet and handed it in. I remember getting it back and being shocked. It was the first time I had ever gotten something back where I got 0, and the paper was filled with red marks. I remember we were given time to go over our answers and figure out the mistakes we made, but I genuinely didn't understand what I did wrong because I genuinely didn't understand the "x" symbol.*

> *The... positive experience occurred in Grade 10. I had a teacher who was very approachable. I felt very comfortable in his class, and every time I asked a question, feeling a little stupid at times, he would say, "There is no such thing as a stupid question." His patience was relentless, not just with me, but with everyone. I did not have many problems with Grade 10 math, but on those few occasions when I did, I would stay after class and speak to him. I remember him being so encouraging and telling me that he was certain I would do well.*

> *When I got to university, I took first-year Calculus from a wonderful professor who had a dynamic manner in class. However, I got behind in my homework and did poorly on the second test. I should have asked for help, but I had no experience struggling with math and I did not know how to get help. In retrospect, I wish I had sought out some tutoring, or worked with someone on the homework, to sort out the issues. I need to talk out a solution in order to remember how to do it and I did not get the opportunity to do that.*

> *There was one math test that I wrote in Grade 10 that I recall as... a positive experience. I had been sort of struggling in the class. The teacher had noticed this and went to great lengths to help me to understand the material. She used to meet with me every day after school and she would go over and over the problems with me. She was very patient, helpful, and supportive. When it came time to write the second unit test, I did it without any problems whatsoever and got a really good mark. I was proud that my hard work had paid off and continued going after school for extra help.*

The unfortunate thing about this math-reflection assignment is that it made me realize that I remember the negative experiences much more vividly than the positive ones. I have to be very conscious of this in my teaching career, to be aware that a positive comment and guidance in the right direction is far more valuable than a big red "X" on a test.

▲ Technology Corner — Modelling Data

MARY BOURASSA
E-MAIL: mary.bourassa@ocdsb.ca

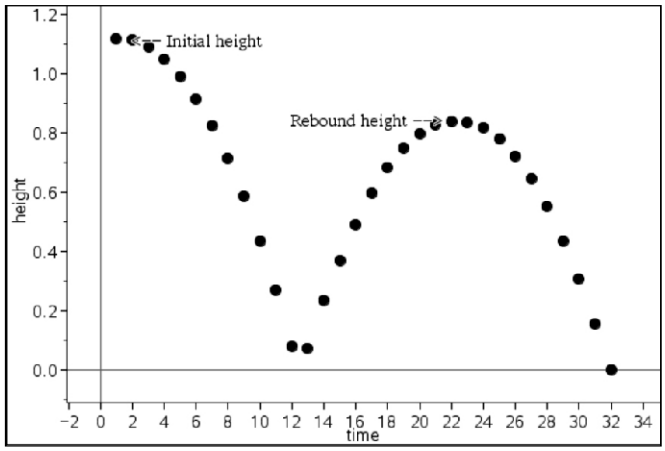


Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Reading through the Ontario mathematics curriculum, one observes many expectations containing the words "Through investigation, using technology…" and "Collect data that can be modelled as a <*insert type*> function through investigation with and without technology, from primary sources, using a variety of tools, or from secondary sources." What follows are ideas for activities that either provide data or allow students to collect it. Examples cover some polynomial functions, exponential functions, and sinusoidal functions, with the goal of providing teachers with at least starting points, if not entire activities, for these types of expectations.

Linear data is fairly easy to come by. Tried-and-true examples include hand span versus height, unit conversions, hours worked versus income, weight versus drug dosage, and certain distance versus time questions.

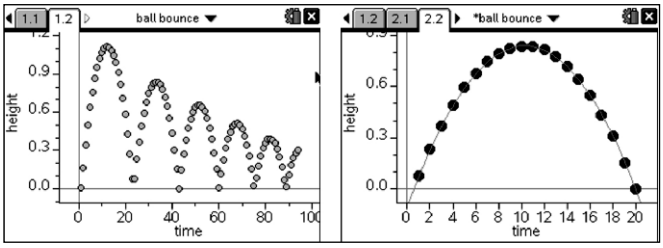
A fairly simple linear data-collection activity is based on rebound height. Start the activity by setting the context, for example: "Isabelle wants to know if she drops a ball outside her window on the third floor, will it bounce high enough for Noah to catch it standing at the second floor window directly beneath her?" To complete this investigation, students release a ball from different heights and record the rebound heights of the first bounces over many trials. The ratio of rebound height to the initial height should be fairly consistent. Using a CBR/CBR2 motion detector above the ball with the Ball Bounce application works well, as students don't have to eyeball the rebound height; as a result, the data collected is more accurate. Students then have to develop a mathematical model based on their data and use their model to predict the rebound height of their ball released from a given height. Follow-up questions could include, "What factors may influence the accuracy of your prediction?" and "Do you think the type of ball used affects the outcome?" If you can find a location in your school where students can actually test their model, so much the better! Although a linear function fits this data well and gives good results, an exponential model makes more sense if you plot the local maximum values (top of each bounce) as the ball continues to bounce.



Another activity involving linear data collection is Tying Knots. In this activity, students determine the relationship between the length of the rope and the number of knots that have been tied with the rope. This activity is very rich. Giving different groups ropes of the same thickness, but different lengths, will produce lines with the same slope, but different *y*-intercepts. Giving other groups ropes of the same length, but of varying thickness, will produce lines with different slopes. Students can even determine the relationship between the circumference of the rope and the slope (this is no longer linear) so that they could answer the question at the end of this scenario: "Mr. Emmell just moved to a new house and has a very big old tree in the backyard. He decides to put up a rope swing for his kids, but does not know how much rope to buy. He knows that the tree limb he wants to use is 18 m high and he wants to put knots every 0.5 m. The rope that he buys has a circumference of 10 cm. How much rope does he need to buy?"

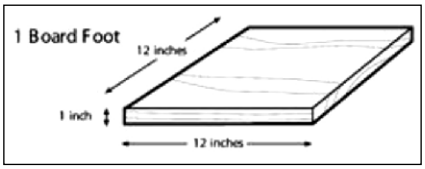


The classic quadratic data-collection activity is the ball bounce. A CBR/CBR2 records the height versus time of a dropped ball; you can then cut the data to display a parabola which students can model.



Another quadratic activity deals with lumber and the data is supplied to students (not collected):

The Forest Service has reported that one of its employees, Tim Lovitt, has found the largest ponderosa pine in the United States. The tree is located in the Shasta-Trinity National Forest in Northern California. It is 240.5 feet tall and the trunk is almost 8 feet wide.



The amount of usable lumber in a tree is measured in "board-feet." One board-foot is a piece of wood that is 12" by 12" by 1" thick.

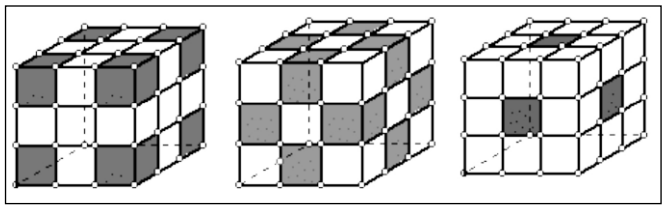
This table relates the number of board-feet of lumber in a collection of ponderosa pines to the diameter of each tree.

Diameter(inches)	Board-Feet
17	190
19	250
20	320
23	570
25	710
28	1130
32	1230
38	2520
39	2590
41	2940

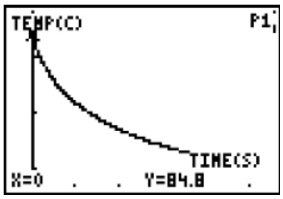
Plot these points, with diameter as the independent variable.

Determine an equation to model this data and use it to calculate the number of board-feet for the largest ponderosa pine found.

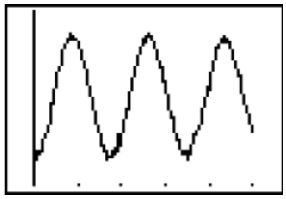
The painted-cube activity is one of my favourites, as different questions based on the same scenario yield different types of data; it could be data that is constant, linear, quadratic, or cubic. A large cube, made of 1 x 1 x 1 cubes, is dipped into a can of paint. Students have to determine how many of the small cubes have paint on no faces, one face, two faces, and three faces. They do this by modelling the situation with 2 x 2 x 2 cubes, then 3 x 3 x 3 cubes, and so on.



Manipulatives such as cube-a-links are really key to this activity being successful, as they allow students to visualize the larger cubes. Graphing technology is also very helpful, as it allows students to quickly try various models.



Population data is often used to show exponential growth; exponential decay can be modelled by collecting temperature data. Instead of letting a cup of hot liquid cool over time, this activity can be done much more quickly by inserting a temperature probe in hot liquid until it reaches the temperature of the liquid, then removing the probe and starting the experiment. It only takes about 3 minutes for the probe to reach room temperature and a beautiful curve is produced.



Sinusoidal data abounds on the Internet, from the phases of the moon to tide data. E-STAT even has a sinusoidal activity called Marriages by Month! If you are interested in having your students collect sinusoidal data, you might try the Tuning Fork activity. This uses a microphone to collect the sound waveform. I prefer to run this activity by collecting the sound created by students blowing into a glass bottle instead of using tuning forks. The data produced is very good and students love doing this. Other sinusoidal activities include the Hula Hoop (also known as Walking in Circles and Wagon Wheels) and collecting data on a pendulum or even on a swing.

Modelling real data may seem overwhelming, but it is truly worth the effort. Students understand and remember these activities and their results and also see that mathematics is all around us. ▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Outstanding contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Kenneth D. Fryer Award* recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

*This award was established in memory of the late Kenneth D. Fryer and is co-sponsored by the Grand Valley Mathematics Association.

Elementary School Staff Award For Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK-8) whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca** or contact the Executive Directors for an electronic version (**EDs@oame.on.ca**). You may include a 1-page letter of recommendation.
2. Provide up to 3 letters of support, of no more than 2 pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific award.
3. Include the nominee's Curriculum Vitae of up to 5 pages. (C.V. not required for Kenneth D. Fryer award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2011 to: OAME Executive Directors, 1652 Chester Dr., Caledon Village, ON, L7K 0W6, fax: 519-316-0123, or email: **EDs@oame.on.ca**

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

DEADLINE: Nominations are due by January 15, 2011 to the Executive Directors (contact info above).

The awards will be presented at the Annual OAME Conference in May 2011.

▲ OAME/NCTM Report December 2010

LAURIE MOHER
E-MAIL: laurie_moher@kprdsb.ca



Laurie is Numeracy Consultant, JK–8 for the Kawartha Pine Ridge DSB. She has served on the Board of Directors of OAME for over 12 years.

This school year, the National Council of Teachers of Mathematics (NCTM) professional development focus is "Geometry: Constructing and Transforming Perspectives." This focus is intended to help educators broaden their perspectives on geometric thinking and to encourage them to investigate how geometric thinking can be developed across all the grades.

In keeping with the focus on geometry, NCTM recently released the third book in its high school series—*Focus in High School Mathematics: Reasoning and Sense Making in Geometry*. This new resource will accompany the algebra and statistics and probability resources previously released. In addition to these volumes, NCTM is currently developing two other books. One will have a focus on ensuring that all students have opportunities to participate in reasoning and sense making as part of their daily classroom activities. The other volume will have a focus on the importance of integrating technology into reasoning and sense-making activities.

In addition to the publication of the Reasoning and Sense Making volumes, NCTM has committed to developing other resources to support educators. These resources include reasoning and sense-making tasks; video clips; a PowerPoint presentation; and a teacher, student, and family guide. Resources will be added as the year progresses. The initial resources are available at **www.nctm.org/hsfocus**.

NCTM has also released three new books entitled *The Teaching and Learning Mathematics* series. The books are intended for administrators, elementary school teachers, and secondary school teachers. Each book contains a number of articles geared toward the target audience that show how ideas from research can be used to improve mathematics teaching and learning. Each book can be purchased in a print format, in an e-book format, or as individual articles for download. NCTM has posted the table of contents, preface, and an excerpt from each publication that can be viewed at NCTM's online store.

Illuminations Spotlight

Do you need a student recording sheet with number lines? Do you need a master of the pattern block shapes? Perhaps you need some graph paper? Check out NCTM's Dynamic Paper tool. This tool allows you to create the above resources and more, all to your own specifications. It then allows you to save what you have created to a PDF file to use with your students or a JPEG file to use in other applications. The Dynamic Paper tool can be found at **illuminations.nctm.org/ActivityDetail.aspx?ID=205** or by searching the Illuminations website for Dynamic Paper.

Pre-K–2 Lesson: What Should I Measure Next? How About Me?

This lesson sequence combines measurement and geometry concepts. In the first session, students use yarn to measure lengths such as arm span and height. As they work, students notice similarities and differences between the arm spans and heights of their peers. They use reasoning skills as they discuss their findings. In the second session, students place the yarn strips from the previous session onto paper and draw around them to create a map of their body. They use reasoning skills as they assemble and draw their body maps and then use directional and positional language to describe their "Map of Me."

Grades 3–5 Lessons: Geometry in the World of Art

In this series of six lessons, students build and draw geometric objects and investigate a variety of geometric concepts. In two of the lessons, they use a virtual geoboard. Three of the six lessons focus on Russian artist Wassily Kandinsky and the techniques he used. In the culminating task, students create their own piece of art, combining their new knowledge of geometry and Kandinsky's techniques.

Grades 6–8 Lessons: Algebraic Transformations

This is a sequence of two lessons in which students extend their understanding of the commutative and associative properties as applied to standard operations for integers to sequences of geometric transformations.

In the first lesson, students explicitly extend their understanding of the concepts of identity, inverse, commutativity, and associativity. Geometric moves are performed using an applet, and the results of sequenced pairs of those moves are analyzed. The second lesson is a continuation of the exploration and analysis of moves.

Grades 9–12 Lesson: Reflect on This

In this lesson, students investigate reflections, using hinged mirrors. Using the mirrors, students use multiple reflections to create regular polygons. They then examine the relationship between the number of sides in a regular polygon and the measure of the angle between the mirrors. A short demonstration video is included with the lesson. The lesson might also be appropriate for students in earlier grades. ▲

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▲ What's the Problem? Getting to the Root of the Problem

SHAWN GODIN
E-MAIL: shawn.godin@ocdsb.ca



Shawn is the head of Mathematics, Technology, and Family Studies at Carrie Wilson Secondary School in Orleans, Ontario. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back, problem solvers! In the September column, I mentioned that I am trying to share some problems with specific curriculum ties. The problem that I left you with was finding a rational approximation to $\sqrt{2}$. I had envisioned, depending on the age and sophistication of the student, that there would be elements of square roots, decimals, fractions, equations, sequences, and patterns that would come up, as well as lines and slopes. Let's see what can be done with this problem.

As a starting point, it might be a good idea to bring out a calculator to find out what $\sqrt{2}$ looks like as a decimal. Calculating we get $\sqrt{2} = 1.41421356...$

From the decimal digits of $\sqrt{2}$, we can get the following approximations to $\sqrt{2}$.

$$\frac{14}{10} = \frac{7}{5} \qquad \frac{141}{100} \qquad \frac{1414}{1000} = \frac{707}{500}$$

We can see that at each stage, the denominator of the lowest term fractions are growing by factors of 2, 5, or 10. Students could first consider why this is true. Then they might notice that if the denominators are $5n$, the numerators are slight adjustments to $7n$, making the ratio close to $\frac{7}{5}$, but incorporating additional precision. For example, 100 = 5 x 20, but 141 = 7 x 20 + 1.

There are other ways to think about $\sqrt{2}$ as well. For example, if $\frac{a}{b}$ is a rational approximation for $\sqrt{2}$, then a^2 is about $2b^2$. Thus we can find approximations to $\sqrt{2}$ by looking for perfect squares that are almost double other perfect squares.

Collection 1

	n	n_sqrd	dbl_n_sqrd
=	caseindex	n^2	$2n^2$
1	1	1	2
2	2	4	8
3	3	9	18
4	4	16	32
5	5	25	50
6	6	36	72
7	7	49	98
8	8	64	128
9	9	81	162
10	10	100	200

Collection 1

	n	n_sqrd	dbl_n_sqrd
=	caseindex	n^2	$2n^2$
11	11	121	242
12	12	144	288
13	13	169	338
14	14	196	392
15	15	225	450
16	16	256	512
17	17	289	578
18	18	324	648
19	19	361	722
20	20	400	800

I created the above tables using Fathom. From the tables, we see some good candidates for *a* and *b*. Since

$2 \times 2^2 + 1 = 3^2$ $\quad$ $2 \times 5^2 - 1 = 7^2$

$2 \times 7^2 + 2 = 10^2$ $\quad$ $2 \times 12^2 + 1 = 17^2$

candidates for $\frac{a}{b}$ are:

Rational Approximation	Decimal Equivalent	Approximate Difference from $\sqrt{2}$
$\frac{3}{2}$	1.5	8.58×10^{-2}
$\frac{7}{5}$	1.4	1.42×10^{-2}
$\frac{10}{7}$	$1.\overline{428571}$	1.44×10^{-2}
$\frac{17}{12}$	$1.41\overline{6}$	2.45×10^{-3}

Notice that the second entry was exactly the first approximation we got from looking at the decimal digits of $\sqrt{2}$. Also, other than the third entry, the approximations get better. The third came from when double a square was 2 away from another square, whereas all the others were off from a square by only 1. Notice, too, that the last entry is a better approximation than $\frac{141}{100}$, even though it is using much smaller numbers.

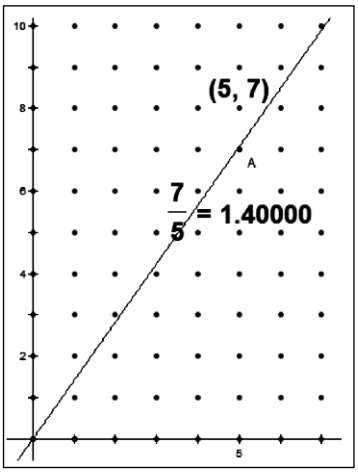
Let's concentrate on when double a square is one away from another square. If you extended your table, you would find that the first five approximations that come from these numbers are

$$\frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}$$

The last two are included, since $2 \times 29^2 = 41^2 + 1$ and $2 \times 70^2 = 99^2 - 1$.

The last entry gives an approximation that matches $\sqrt{2}$ to four decimal places! If you take a close look at the fractions above, the numerators and denominators each form a sequence with a fairly straightforward recurrence relation. See if you can find it. You can check my wiki for the answer. (**Hint:** Similar to Fibonacci.)

Let's go off on a different path. Consider the graph of the line with equation $y = \sqrt{2}x$. It has slope $\sqrt{2}$, which we can approximate with the slope formula using points that are almost on the line. We know for a fact that (0,0) is on the line, so then if (x,y) is a point close to the line, then $\frac{y}{x} \bullet \sqrt{2}$. The graph below, created using the Geometer's Sketchpad, shows one of the approximations.



The sketch, along with directions for how to create it, are also on my wiki.

Last of all; let's perform a little algebraic sleight of hand. Since we are trying to approximate $\sqrt{2}$, then if $x = \sqrt{2}$, $x^2 = 2$. Thus,

$$x^2 + x = x + 2$$
$$x(x+1) = x + 1 + 1$$
$$\frac{x(x+1)}{x+1} = \frac{x+1}{x+1} + \frac{1}{x+1}$$
$$x = 1 + \frac{1}{x+1}$$

If we consider this as a recurrence relationship, that is

$$x_n = 1 + \frac{1}{x_{n-1} + 1}$$

using any positive number as our first term, the sequence converges to $\sqrt{2}$. As a matter of fact, the only values that won't converge are ones where $x_n = -1$ for some *n*. It turns out these values are those listed below.

$$-1, -\frac{3}{2}, -\frac{7}{5}, -\frac{17}{12}, -\frac{41}{29}, -\frac{99}{70}, \ldots$$

Hmmmmm…. Interesting!

Suppose we took the equation $x = 1 + \frac{1}{x+1}$, which is satisfied by both $\sqrt{2}$ and $-\sqrt{2}$, and substituted it into itself, we would get

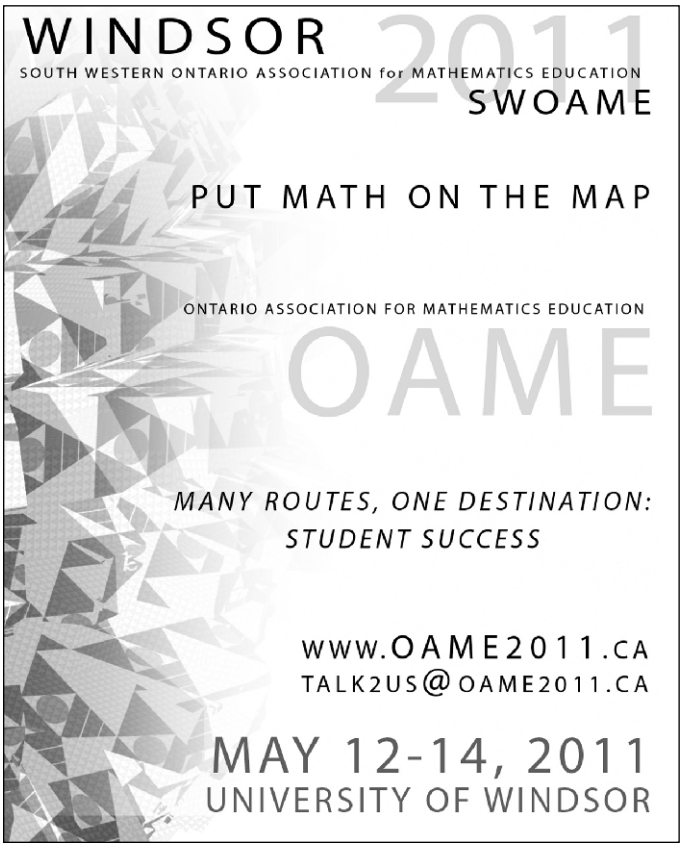
$$x = 1 + \frac{1}{x+1}$$
$$= 1 + \frac{1}{1+1+\frac{1}{x+1}}$$
$$= 1 + \frac{1}{2+\frac{1}{x+1}}$$

If we continued this process *ad infinitum*, we would get

$$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\frac{1}{2+...}}}}$$

which is called the simple continued fraction expansion of $\sqrt{2}$. If you end the fraction at one of the twos and work the fraction back to an improper fraction, it will be one of the values from our list.

OK, now it is time for your homework. A square is constructed on an 11 by 11 pin geoboard. What are all the possible areas for the square? Until next time, happy problem solving! ▲



▲ Data Rocks – Why Learn with Statistics?

YVES SAINT-PIERRE, STATISTICS CANADA
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Yves Saint-Pierre has worked at Statistics Canada in many capacities since 1990: questionnaire design, data editing and imputation, analysis, and publication. He has been sharing his knowledge with schools since 2002.

The year 2010 saw a focus on statistics in the news. The Canadian Census garnered quite a bit of media attention, which spurred discussion on the pros and cons of why we need reliable data in our lives. The launch of the first-ever UN-proclaimed World Statistics Day on October 20 put further spotlight on official statistics and their use in countries around the world. While my contributions to this column have often been about particular resources on StatCan's website, I'd like to take this opportunity to review the purpose behind Statistics Canada's Education Outreach program.

The Education Outreach program at Statistics Canada formally began in the late 1980s, when one of our senior managers was giving a lecture in a university economics class. The economics professor was using American data for teaching. When asked why, he reported that Canadian data was not affordable or accessible, so the professor used free U.S. data and divided it by ten, reasoning that Canada had about a tenth of the population of the United States.

The rest is history. Over the years, the Education Outreach program spread from the university level to include high schools and even elementary schools, as a greater demand for electronic data became apparent. The mandate of our program has changed from simply providing access to and distributing data to a focus on encouraging statistical skill development so that students learn to use and understand data.

At Statistics Canada, we believe that every citizen needs to understand statistical concepts. Statistics are used in all kinds of fields: health, social and economic research, sports, entertainment, etc. What's more, in an information-rich society, citizens often have to make decisions on what to buy or what policies they want their governments to pursue. Getting a grip on those issues often requires the evaluation of statistical information.

This is the basis for our focus on statistical literacy in a new resource called "Statistics Are Important!" at **www.statcan.gc.ca/edu/edu06/edu06f/edu06f_0003-eng.htm** (or look on the Student page under Find).

And, as Professor Geoffrey Roulet from Queen's University notes, "After basic arithmetic, which we all use for our regular financial transactions, statistics is the branch of mathematics with which we have most contact."

What is Statistical Literacy?

Our new Learning Resources section, *Statistics Are Important!,* explains that there are many elements to the definition of statistical literacy. We have contributions from international experts in the field: mathematics educators, statisticians, even from the chief economist at Google, Hal Varian, who writes:

> *"I keep saying the sexy job in the next ten years will be statistician— The ability to take data—to be able to understand it, to process it, to extract value from it, to visualize it, to communicate it—that's going to be a hugely important skill in the next decades, not only at the professional level, but even at the educational level for elementary school kids, for high school kids, for college kids."*
>
> Here are two other excerpts from the experts.
>
> *Statistical literacy is more than drawing graphs and calculating means. It is understanding tools such as these when they appear every day in the news media accompanied by the need for interpretation of and decision-making on topics as wide-ranging as politics, health, town planning, environmental control, unemployment, sport, science, and even attendance at cultural events. [Jane Watson, Mathematics Education professor and co-author of* Statistical Literacy at school: Growth and Goals*, 2006]*
>
> *Statistical literacy brings a range of thinking and practical skills that include: knowledge; comprehension; application; analysis; synthesis; and evaluation. It enables a feel for data, including being able to support an argument with evidence, but also being aware of the variety of interpretations that are possible from those data. A statistically literate person will apply common sense to problems and appreciate that information that is gleaned from data will have uncertainty attached to it. [Neville Davies, Director of the Centre for Education, Royal Statistical Society, Plymouth University, United Kingdom]*

Why learn with official statistics?

- Real data makes the learning so much more relevant, and the availability of statistics on a great number of subjects allows students to pretty much pick the topic that they like.
- Students can also draw from a variety of data sources, as there is often more than one survey dealing with one subject.
- Official statistics come from a credible source!
- Official statistics also come with a lot of documentation, or what we call metadata. It is Statistics Canada's policy that all of our data releases point to the methodology of the survey from which the data came. The metadata includes the target population, survey design, the sampling method and sample size, notes on data quality, etc.

Statistical Learning Resources

Remember that you can find many learning resources at **www.statcan.gc.ca/learningresources** > Teachers > Mathematics. Once you've chosen the level that you teach, there are three tabs to the mathematics page: Key Resources, Lessons, and Data. Among other things, you'll find in the Key Resources is one called Statistics: Power from Data, an e-textbook on all things in a survey.

Please tell your students about our new student page (**www.statcan.gc.ca/learningresources** > Students). We conducted a survey of students and then held some focus groups to find out what students were looking for. The new design is based on their feedback. ▲

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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2011–2012

Each year, elections are held for a President-Elect, a Vice President (3-year term), and 6 to 9 Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 14, 2011, to:

Beverly Farahani, Past President,
Kingston CVI, 235 Frontenac St., Kingston, ON K7L 3S7
E-mail: bamineur@lakeheadu.ca

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2011, the Nomination Committee will publish a slate of candidates on the website, including qualifications, for consideration by the membership. Voting will be on the website, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Windsor, May 2011.

NOMINATION FORM

I nominate _____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest: ☐ Grades JK–6 or ☐ Grades 7–12

Nominee's Address: _____

_____

E-mail: _____

Telephone: H (_____) _____

Telephone: Home (_____)_____ Work (_____)_____

Nominator's Name : _____

Nominator's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____ Work (_____)_____

▲ Hey It's Elementary – Math Hits Prime Time with the *Prime Radicals*

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy!

It will come as no surprise to read a statement that says, "Television is one of the most prevalent media influences in children's lives." In fact, children are fascinated by TV. According to *Kids' Take on Media*, a survey conducted in 2003 by the Canadian Teachers' Federation, watching TV is a daily pastime for 75 percent of Canadian children, both boys and girls from Grade 3 to Grade 10; and, according to a Statistics Canada survey, Canadian children between 2 and 11 watched 15.5 hours of television per week in Fall 2000.

It will also come as no surprise that the degree to which children are impacted by TV is a function of many factors: what they watch, how much time they spend in front of a TV, their age and personality, whether they watch alone or with adults, and whether their parents talk with them about what they see on TV.

Many educators have received professional development about the potentially negative effects of television-watching on children, and will be aware that empirical research is contributing to a growing consensus that children may be vulnerable to television's violent images and messages. Three specific potential responses to media violence in children have been identified through rigorous research. These are:

- **Increased fear—also known as the "mean and scary world" syndrome**

  Children, particularly girls, are much more likely than adults to be portrayed as victims of violence on TV, and this can make them more afraid of the world around them.

- **Desensitization to real-life violence**

  Some of the most violent TV shows are children's cartoons, in which violence is portrayed as humorous—and realistic consequences of violence are seldom shown.

- **Increased aggressive behaviour**

  This can be especially true of young children, who are more likely to exhibit aggressive behaviour after viewing violent TV shows or movies.

Recently, substantive attention has been paid to a number of other research studies that report on other potentially negative consequences correlated to television watching. Some studies claim that television adversely impacts learning and school performance because television-watching time cuts into (or eliminates) the time children need for activities that are deemed critical for healthy mental and physical development: playing, reading, exploring nature, learning about music, or participating in sports. An ancillary finding is that television-watching (a sedentary activity) is a significant factor in childhood obesity. According to the Heart and Stroke Foundation of Canada, almost one in four Canadian children between 7 and 12 is obese—a statistic that these researchers attribute to the fact that time spent in front of the TV is often at the expense of more active pastimes.

What is less publicized in the popular press (the vehicles through which most people learn about research findings) are the studies that show that television- watching appears to have many benefits for children. For example, television, because of its ability to create powerful touchstones, enables children to share cultural experiences with others and learn important values and life lessons. Shared viewing gives family members of all ages an opportunity to spend time together, and because television programs may explore controversial or sensitive issues, it may be easier for parents and kids to discuss them within a context. Parents can use TV as a catalyst to get kids reading—following up on TV programs by getting books on the same subjects or reading authors whose work was adapted for the programs. And television can be a fun and educational part of a child's daily routine.

Research suggests that television has a role to play in children's cognitive development. It is known that children's viewing is governed by the novelty of the visual stimulus and that the production quality of high-quality children's programming, built on features such as movement, visual complexity, cuts, pans, and zooms, leads to "active"—not "passive" —television-watching by children.

And the theory of "active" television viewing suggests that attention is linked to learning and comprehension and may be a catalyst to engage children in "unplugged" learning. When kids see their favourite characters engaged in fun learning games, they want to join in the activity. When the visual stimulus of active learning activities led by beloved characters on screen transfer to active, "unplugged" activities at the kitchen table, playground, skateboard park, or garden, educational psychologists agree that television can contribute to growth in content and process knowledge and experience.

This long introduction is a very abbreviated summary of what I have learned about children's television over the past three years—an enormously steep learning curve, especially when one is on the other side of the television screen, i.e., creating and developing a children's show. Here is the story:

For about six years, I had the privilege of writing a biweekly, full-page column about mathematics in *The Kingston Whig-Standard*. It was a wonderful opportunity to write about the mathematics of the sculpture and frieze patterns in the architecture around town. My editors indulged two columns about my beloved Black Labrador Retriever, Brigg, one of which featured the transformational geometry of pawprints in the snow at the dog park, and the other, his uncanny ability to catch a ball, using what research calls "doggy calculus." Over the years, there were about 120 columns, and I was fortunate enough to have my articles collected by a local documentary movie maker, who took them to Ottawa to be reviewed by Hoda Elatawi, a Senior Producer with the General Assembly Production Company (GAPC), and developer of the award-winning children's program, *Spellz*.

Hoda contacted me with the wonderful news that there was a children's TV show in "them thar" columns, and so *The Prime Radicals* was born.

The Prime Radicals is a 26-episode series of 15-minute shows geared to children aged six to eight, based on my math columns. The series will air in Ontario on TVO beginning in January[1]. *The Prime Radicals* was developed in conjunction with TVO Kids and in association with Knowledge Network, SCN, and ACCESS Alberta, and with the financial participation of the Canada Media Fund, The Shaw Rocket Fund, The Bell Broadcast and New Media Fund, and the Ontario Media Development Corporation (Screen Content Initiative Program).

The format of the show is live action and is centred around three characters to present real scenarios in a way to make math easy and fun. The show follows two cousins, Kevin and Alanna, and their Uncle Norm [a crazy and goofy character (think of a Maxwell Smart/Inspector Clouseau hybrid)]. Uncle Norm is always getting into trouble and has some problem to resolve. The kids, of course, use math to save the day (and Uncle Norm's bacon).



Uncle Norm is played by veteran comedic actor Norman McQueen (who is also an English teacher with the Ottawa-Carleton District School Board). Alanna Bale is a theatre student at Ryerson University, and first-time actor Kevin Wang is the acknowledged "math whiz" of the cast.

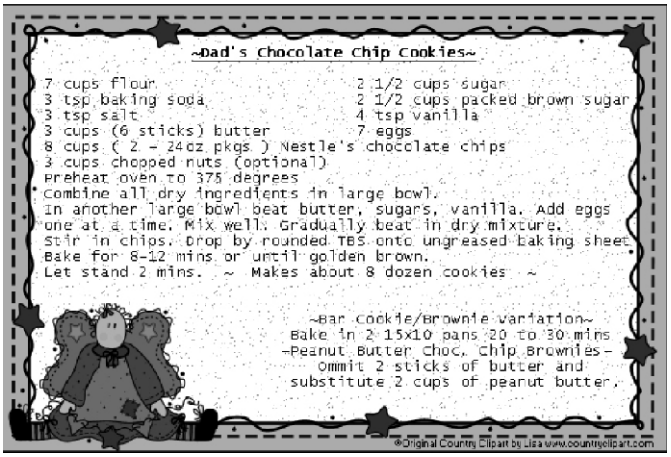
The problems Uncle Norm faces lead to opportunities to learn about coordinate grids, growing and shrinking patterns, volume, non-standard measurement, the base 10 system, symmetry, right angles, fractions, and codes and ciphers. All episodes are correlated to the Ontario Mathematics curriculum and are complemented and supplemented by ancillary activities on a companion website.

Each episode begins with an "opener" which sets the scene. After establishing the problem, the kids tell Uncle Norm that they know an expert who can help. This leads to a field trip, during which Uncle Norm and one of the kids meets with an interesting person who uses math daily—for example, the official timekeeper at the Peace Tower in Ottawa, a musician who uses fractions to compose scores, a fire fighter who reads maps to locate specific addresses, an astronomer who documents stars in the sky, a sprinter who excels in the 100-metre dash, an engineer who builds strut bridges, super-cool skateboarders, or CBC gardening expert Ed Lawrence.





[1] *The show will also air on educational stations in British Columbia, Alberta, Saskatchewan, and Manitoba.*



After learning the math to solve the problem, Uncle Norm and the kids apply their new knowledge and generate a tangible product that illustrates the central concept: a pot of chili, braided rug, planter box, kaleidoscope, or bracelet. The "big idea" is then reinforced via a virtual field trip on a CSI-like screen that features fast-paced visuals of Emperor Penguins, cube vans, million dollar coins, and barleycorns. After this, Alanna leads a small lesson on a craft that the children at home can make to reinforce the major content points. The show closes with an interaction between Uncle Norm and Kevin and Alanna to conclude the episode and lead into a song that reinforces the content in an engaging and fun way. All songs were composed by Andrew Huggett, Creative Director and Music Composer at GAPC, and are sung by 12-year-old Ottawa powerhouse, Ayda Khan. They will be available for download as MP3s on *The Prime Radicals* website. Ayda was featured in 2009 on the **Oprah.com** website as one of the top child artists. In the winter of 2009, Ayda was in Harlem at the Apollo Theatre, giving New Yorkers a taste of what's to come.



I am very proud of *The Prime Radicals*. It is the product of an amazing team of extraordinary people who are deeply committed to children and their learning through alternate "classrooms." I sincerely hope that The Prime Radicals will find its way into homes and classrooms across most of the country, given that it is a substantive and fun way to open doors to and conversations about mathematics for children and their families.

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▲ In the Middle – From Start to End: Maximizing Instructional Time in the Intermediate Classroom

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AND JUNG-CHOI PERKINS
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Caroline Rosenbloom is currently on secondment as an Elementary Coordinator teaching mathematics in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board and Education Officer at EQAO. Caroline is co-author of Investigating Mathematics Using Polydrons.



Jung Choi-Perkins is presently an Instructional Leader in Mathematics and Numeracy for the Toronto District School Board. She teaches a Primary/Junior Math AQ course and sits on the OAME Board of Directors. She has experience teaching mathematics in Grades 7–9.

We are well into our school year and have now created a mathematical environment in our classroom. When we walk in, we see walls covered with interesting puzzles, bookshelves full of math storybooks, games available to sign out on weekends, and math tools in bins readily available when needed. When we walk in, we hear pairs or triads discussing math problems, questions being asked, and students articulating their mathematical thinking.

Now that we've created this environment, how do we ensure we are utilizing instructional time effectively?

A video study from the TIMMS report had given some insight as to how time was typically spent within mathematics classes in the United States, Japan, and Germany (Fig 2) (Martinez, 2001). Japanese lessons featured a more active learning environment where students worked on problems, struggled with the problems, and then articulated their thinking by sharing different representations of the solution. Lessons from the United States and Germany were more teacher-led, where teachers provided instructional steps, examples, and opportunities to do seat work that were similar to the questions done in class. So how do we structure our class to incorporate a more student-centred lesson?

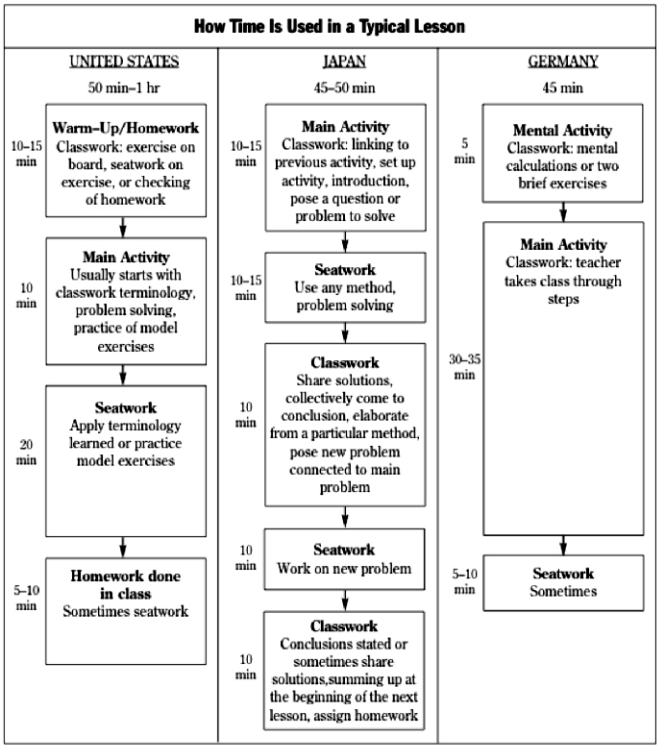


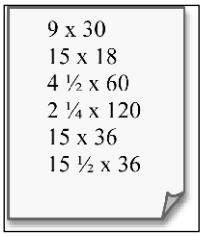
Fig. 2 Uses of time in classrooms in the United States, Japan, and Germany (USDE 1997, p. 57)

Here in Ontario, many are familiar with the three-part lesson format. TIPS4RM (Ministry of Education, 2010) refers to the parts as:

- Part 1 – Minds On
- Part 2 – Action!
- Part 3 – Consolidate Debrief

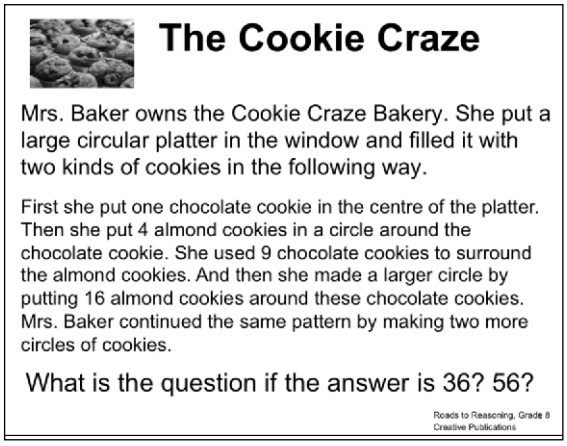
Part 1: Minds On "*Activating Prior Knowledge*" ~ 5–10 minutes

The *Minds On* is the hook to activate prior knowledge that is needed for the task with which students will be engaging in Part 2 of the lesson. A math string is one example of a *Minds On* task. For example, a question might be:



How could you mentally represent earlier calculations in the string to help you figure out later ones?

A *Minds On* can also be a problem. In one classroom, students were given the Cookie Craze problem, shown here from Krulik and Rudnick (2002). Clickers were used to respond to the problem. Discussions followed about how the mathematics was generally represented but further discussed later in *Part 3: Consolidate Debrief*.



The Cookie Craze

Mrs. Baker owns the Cookie Craze Bakery. She put a large circular platter in the window and filled it with two kinds of cookies in the following way.

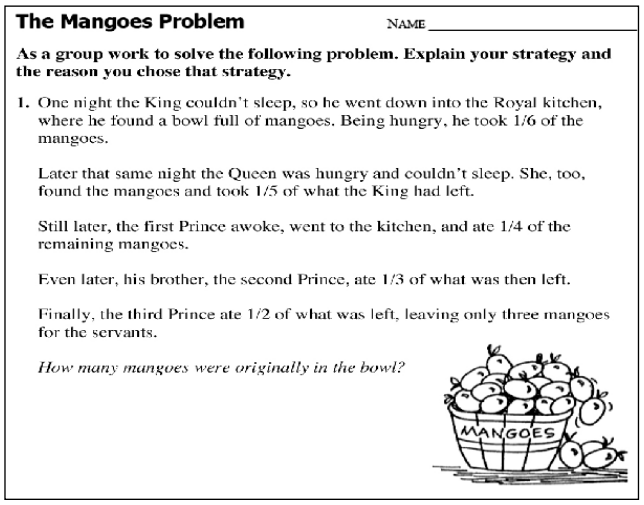
First she put one chocolate cookie in the centre of the platter. Then she put 4 almond cookies in a circle around the chocolate cookie. She used 9 chocolate cookies to surround the almond cookies. And then she made a larger circle by putting 16 almond cookies around these chocolate cookies. Mrs. Baker continued the same pattern by making two more circles of cookies.

What is the question if the answer is 36? 56?

Reads to Reasoning, Grade 8
Creative Publications

Part 2: Action! "The Investigation" ~ 15–20 minutes

The *Action*! is the part of the lesson during which students explore and investigate a new concept. Students can work in pairs or triads to solve a problem. Same-ability groupings allow every student to have a voice and be actively engaged in the process. Often when students are partnered with another student at a different level, the stronger, more assertive student tends to do the work. In this case, a problem from the NCTM Illuminations site was used.



The Mangoes Problem NAME _____

As a group work to solve the following problem. Explain your strategy and the reason you chose that strategy.

1. One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

   Later that same night the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

   Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

   Even later, his brother, the second Prince, ate 1/3 of what was then left.

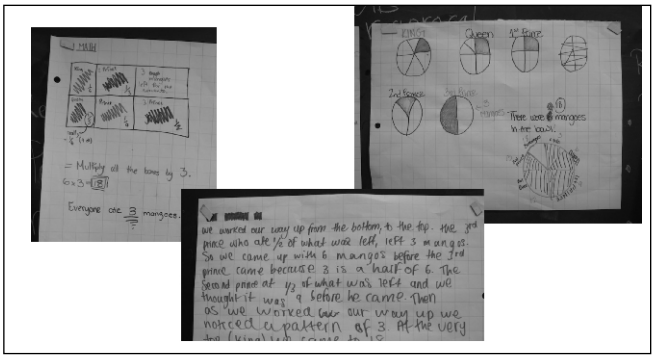
   Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

   How many mangoes were originally in the bowl?

A variety of manipulatives, such as fraction rings and circles, are provided so students can choose tools appropriate to their learning style. Chart paper and markers are used to record students' thinking when they solve the problem.

Part 3: Consolidate Debrief "Summarizing the Learning" ~ 20–30 minutes

The purpose of the *Consolidate Debrief* is to connect the mathematical concepts to the actions students took in Part 2. Students summarize their learning by sharing their strategies, comparing and contrasting solutions, identifying common misconceptions, and raising other math questions that came out of the lesson. This consolidation can be done through a math congress, where students present and justify their work to their peers.



Questioning is a large and important part of the *Consolidate Debrief*. Robert Marzano (2001) describes instructional strategies that increase student learning. The following chart connects some of these strategies to actions taken during a math congress.

Instructional Strategies	During the congress…
Identifying similarities/differences	Ask: *"What is similar about these solutions? What is different?"*
Summarizing	Ask: *"Can someone describe what you think this group did to solve the problem?"* and *"What did you do when you got stuck?"*
Non-linguistic representations	Ask: *"How does the concrete representation connect to the algebraic expression?"*
Co-operative learning	During the task, students work in same-ability pairs/triads to actively involve all learners. These groups present together to justify their solutions.
Providing feedback	Teachers facilitate discussions, providing feedback and encouraging students to also provide feedback to solutions they see.

Now that we have a structure for our lesson, how can we support each other in our schools to have consistency among all our classes? Administrator support is key to making this happen. Scheduled time is needed during the day for teachers to plan together and teach together. Here are some suggestions that can help make this happen:

- **Pairs of teachers co-teach together.** This should be inquiry based, reflective, and collaborative. Teachers can take turns assuming the role of lead co-teacher and can teach either one of their classes together or both classes together. Co-teachers have "live time" professional discussions about what they are observing and doing and make collaborative decisions to best meet the needs of the students. The teachers are learning from each other as their students are learning from them.
- **Collaborative planning.** Teachers of a grade, division, or subject meet and plan lessons that include trying out the task/activity/problem and anticipating what student responses might be. The group could consider a focus for their professional learning, such as "questioning" or time on task for each portion of the three-part lesson. The focus is on planning and instruction and moves the community of teachers in a school along their continuum of learning.
- **Professional learning communities.** This professional learning community can take place right in the classroom. Teachers could have colleagues (teachers, administrators, consultants, coaches) come to co-teach with them. Other teachers can observe this co-teaching session and be involved in the debriefing and planning for the next lesson. The co-teaching model should be ongoing, whether it be once or twice a week or once a month.

Happy planning!

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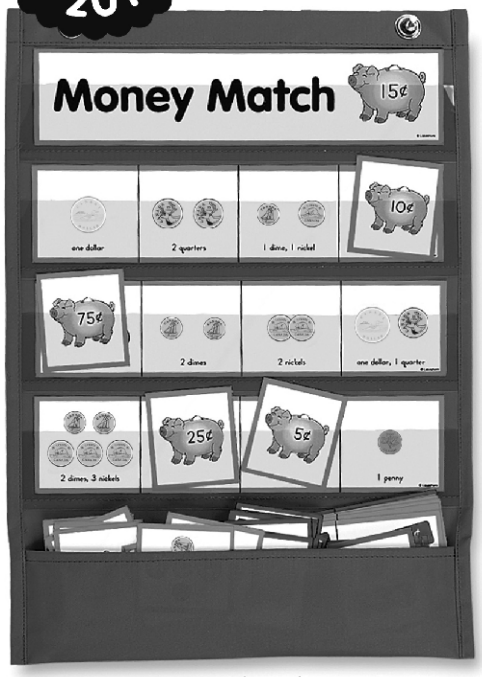
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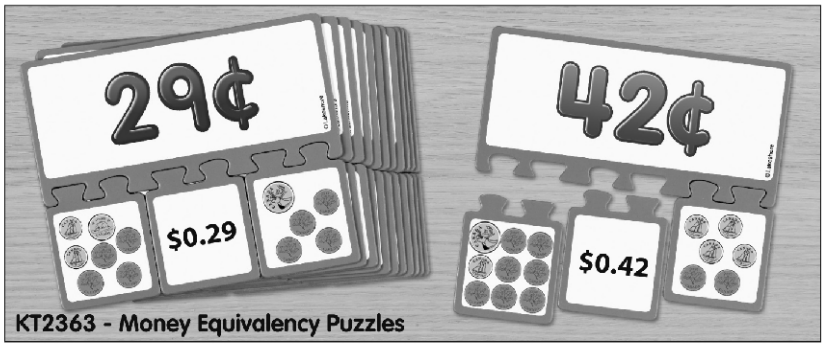
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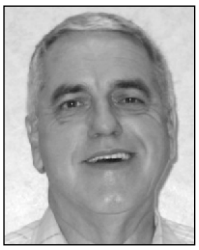
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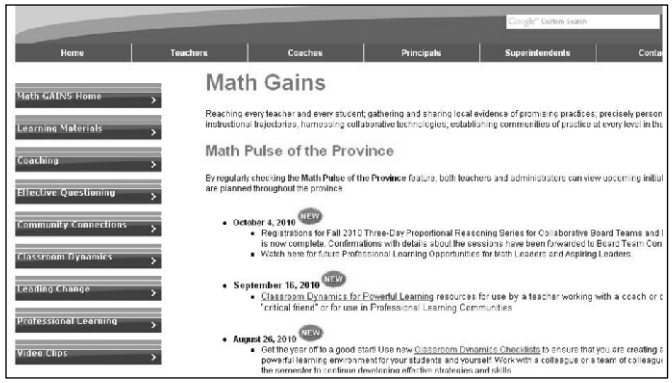
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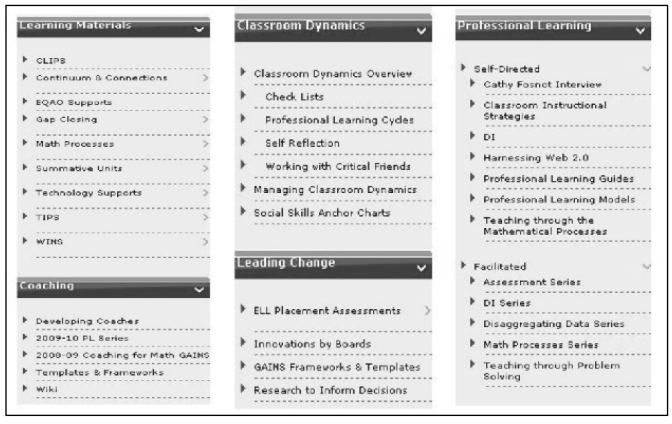
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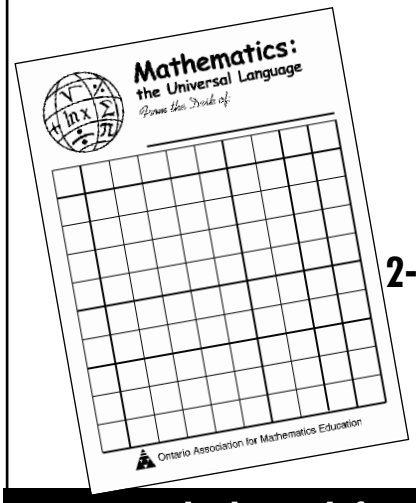
- Teachers, and new teachers in particular, will find the materials under ***Classroom Dynamics*** particularly useful.
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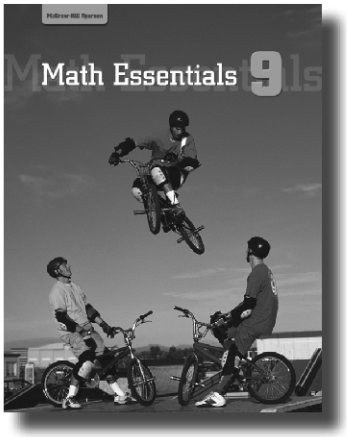
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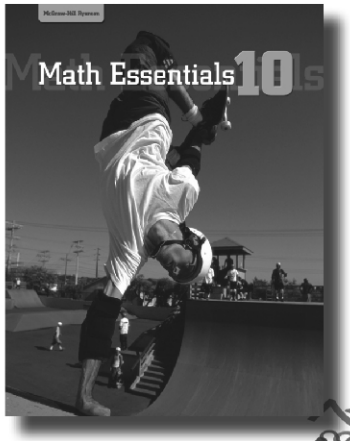
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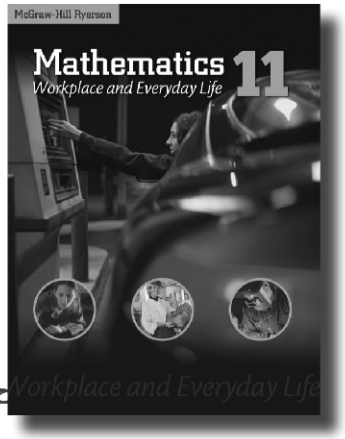
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▲ Why Look at the Mathematical Processes: One School's Journey

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Having worked in education for the last 18 years as a math teacher, department head, and vice-principal, Deidre is currently Principal at Centre Dufferin District High School in Shelburne.

As a secondary principal, I was recently involved in the Leading Student Achievement (LSA) project funded by the Literacy and Numeracy Secretariat. It focused on professional learning for principals and teachers that improves student achievement. The LSA project uses a process called the Teaching Learning Critical Pathways (TLCP), where teachers get together over a series of meetings to examine student data, make a plan to improve student performance, act, observe, and then reflect on the process. Having seen LSA previously focused only in elementary education and on literacy skills, I chose to focus on mathematics education and brought together a team of secondary math teachers from multiple grades to test the process.

The team began by looking at EQAO data. Closely examining multiple-choice questions on which students scored poorly suggested that students were not spending time reading, thinking about, or solving the multiple-choice questions. It appeared they were choosing the first answer that showed any relationship to the numbers found in the problem. Questionnaire data indicated that students did not enjoy mathematics and saw it as a mystery. Classroom data revealed students were focused on completing questions quickly in as few steps as possible. The data left the team thinking about one question: How can we get students to stop, reflect, and think more about the problems they are solving?

The current Ontario math curriculm outlines seven key *Mathematical Processes* (problem solving, reasoning and proving, reflecting, connecting, selecting tools and computational strategies, representing, and communicating) that could improve teaching and learning in mathematics, especially for students at risk or experiencing math anxiety. We were interested in exploring whether a focus on these processes helps students move from being receptors of knowledge to disciplined thinkers of mathematics.

After looking at each of the mathematical processes, the team chose to focus on explicitly teaching the process of reasoning and proving as a way of improving a student's ability to reason through a problem. Four different classes of students from Grades 9 to 12 were asked to reflect on their thinking by answering five questions (see Table 1: Thinking Questions). The questions, designed by the team, revealed students' thinking after solving a rich problem on whatever content they were currently studying (e.g., linear relations, geometry). The questions reflected the teachers' ideas about what a student should be asking himself or herself when solving a rich problem. The questions became a pre-test before any explicit teaching took place.

Table 1: Thinking Questions

1. What were you thinking about the information in the question? Explain.
2. This question reminds me of....
3. What did you anticipate were possible solution(s) to the problem? Justify.
4. Describe the different strategies that came to mind to solve this problem.
5. What convinced you that your answer was correct? Be specific.

After students answered the pre-test questions, the team marked them using teacher moderation and a four-level rubric (Appendix 1) that the team created. The moderation process was new for these teachers and proved to be very challenging. It was through this examination of student work that the team built consensus around the rubric and decided what reasoning and proving at each level needed to look like.

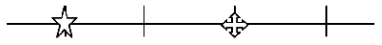
For example, the Grade 9 class pre-test required students to solve a linear relationship problem, where they were asked to choose between two different car services. Students created an equation for each service, made a table of data, graphed, and explained when each service was better. From the responses, it became clear that most students were thinking about the problem, but they did not communicate that they had a clear process on how to use the information in the problem to get to a final solution; they were simply convinced they were right. Question 1 asked students what they were thinking about the information in the problem. Students typically scored Level 1 on Question 1 of the rubric, either because they did not interpret the question to mean that they should indicate the relevant and irrelevant information in the problem or communicate how they might need to manipulate the information (such as adjust units), or because they were really not able to do so. Some student

responses were: “I was thinking of how far I would be traveling,” “I was thinking about how far away the airport was,” “I was thinking of making a graph to use a visual explanation.” The second question, with better results, required students to connect to a similar problem previously studied either from class or from their homework. The third question, anticipating the answer, proved more difficult for students. Several students simply made no prediction or said they thought it was “even.” A few students knew “one plan was better at a certain time,” but most did not predict when the change would occur. Since students had not been instructed in using multiple strategies to solve a problem before the project, listing different strategies in Question 4 proved challenging for most students. The final question, requiring students to justify their answer, had most students respond “because it looked right” or “because it makes sense.” Students did not connect to their prediction or use an alternate strategy to verify their solution.

Scores from the pre-test were posted on a data wall (See Table 2: Data Display) and used as a base measure, with each teacher choosing a few students from their class to watch. The selection of students to track was at the discretion of the teacher, but all chose a strong, average, and weak student, as well as one or two students they thought had the potential to improve. Students were then instructed in those steps of reasoning and proving suggested by the five questions. Teachers took every opportunity to model the steps by “thinking out loud” as they solved problems with their students. For example, teachers tried to solve problem in their lesson, using an alternate strategy, and encouraged student to solve problems more than one way.

As I visited classes during the teaching time, I repeatedly heard teachers say, “How else could we solve this problem?,” “Is there another method we could use to help convince ourselves the answer is correct?” Teachers found they were using several common prompts for each question and began to record these when they taught. As the project unfolded, the team made a poster (Appendix 2) listing the five questions, with the prompts they developed for each, similar to what they had been modelling through their “think out loud lessons.” The poster in an early form (without graphics and on 8½" by 11" plain paper) was displayed in classrooms and referred to during teaching. The rubric teachers used on the pre-test to assess student work was shared with students and used by both the teachers and students to help critique the thinking behind the solutions to problems with multiple-step responses.

Table 2: Data Display

Student Name	Data Graph	Questions #1
S15 ###############		
S16 ###############		

At a later date toward the end of the course, after the explicit instruction on reasoning and proving had taken place, the five questions were administered again (a post-test) to measure students’ reasoning and proving on a rich task in whatever content area they were currently studying. The team used the same rubric as before to score student responses. The selected students mentioned earlier showed improvement by moving up at least one level for most of the five questions. Teachers had set a goal of helping each student to move up one level from wherever they began on the pre-test and hence named the final poster: *Take It to the Next Level*. The greatest student improvement was noted in Question 1 and Question 5.

For example, the Grade 9 class was working on geometry during the post-test. The problem consisted of two rectangular prisms with dimensions given. The task was to find which would hold the most liquid, comment on the cost to produce the prisms, and explain if there was a more optimal container. For Question 1, most students made specific reference to the formulas for surface area and volume and the dimensions given. Units were noted and extra information about the context of the problem was identified as irrelevant. Students did well again on Question 2, and this time, had real predictions for answers to Question 3. Most students predicted one shape had “greater volume because its dimensions are closer together. That would make it closer to a cube, which is always the greatest volume.” Listing multiple strategies in Question 4 was not as challenging for students the second time. Students were making connections to their prediction and their calculations for Question 5. For example, students mentioned they were convinced because “answers for previous questions were always the most cube-like prism” and that they “both had the same surface area, but one held more than the other.”

The biggest learning in the project was the teachers’ realization that teaching the thinking behind reasoning and proving was necessary. The team was originally surprised by their students’ low scores on the pre-test

and that the students did not intuitively follow the steps suggested by the questions. Teachers felt the thinking required to reason through a problem had been implied in their teaching, and thus were dismayed that it required an explicit explanation.

The project was successful at improving student's reasoning and proving skills, but the project also had a huge impact on teacher learning. Misconceptions about what students were thinking when solving problems were uncovered; teachers experienced the value of collaboration. Following a process that focused on student work changed the nature of conversation in the group to teaching and learning, and a need for improved instructional practice was realized.

Reflecting on the process they went through, one teacher on the team said it was "focused and purposeful; affects how math is taught," and another said, "There is a lot more to thinking going on than I realized—it opened my eyes." Remembering the discussion in the meetings, one teacher said, "The conversation relates to student learning and teacher practice"; another said the process had a "positive effect on teacher discussion." While thinking about their practice, one teacher said, "I have not been teaching alternative ways to find a solution—made me stop and find other ways"; another said "effect on teacher piece is huge."

Because students were tested on different content on the pre-tests and post-tests, their success on the actual tasks could not be compared. Only improvement on their reasoning and proving skills was recorded. This preliminary study of thinking skills indicates there is value in explicitly teaching the thinking behind reasoning and proving, but how this teaching improves academic performance on the mathematics content is the next question for the team. What impact would explicitly teaching **all** the mathematical processes have on student metacognition, understanding, and performance?

It is believed that students' awareness and regulation of metacognition can impact their ability to master mathematical problem solving and apply their problem-solving skills in different situations. "Without effective metacognitive awareness and regulation, students will not be able to recognize or use their knowledge to help them solve a problem when they get stuck, and they typically persist with inappropriate strategies that lead nowhere" (Goos, Stillman & Vale, 2007, p. 37). Students can develop this self-regulation in learning in three ways: through experience, direct instruction, and through practice (Paris & Paris, 2001). The project was about the impact of direct instruction in metacognition, through teachers helping students develop an awareness of their thinking, and teaching students how to regulate their thinking with the help of the mathematical process: reasoning and proving.

Some would say that the goal of mathematics education is to move beyond delivering mathematical facts and procedures to students and instead to move to fostering in students an appreciation for mathematics and an ability and desire to learn how to apply mathematical skills in real-life problems (Schoenfeld, 1992). Adding metacognitive instruction to whatever strategies teachers are using to teach the content of the mathematics curriculum helps to develop students who think more deeply and are disciplined in their thought. Teaching reasoning is not a new idea, but may be one that is not practised. The team made an assumption that students know how to reason through problems. After going through the TCLP process, they discovered that they were incorrect. The process of examining their data, observing their students, and reflecting on their teaching led them to see the need for explicitly teaching how to think and reason through a rich task. The process helped put student learning at the forefront and the desire to develop disciplined mathematical thinkers.

Acknowledgements

The success of this project is a reflection of the dedication of a team of hard-working math teachers at Centre Dufferin District High School: Chris Sales, Mike Delaat, Leah Pressy, Greg Heaslip, and Dave Langman. This project would not have been possible without guidance and support from our board curriculum consultant, Rob Rosie, and our OPC representative, Bob Jackson. LSA projects are made possible by the Literacy and Numeracy Secretariat and all three principals' councils in Ontario.

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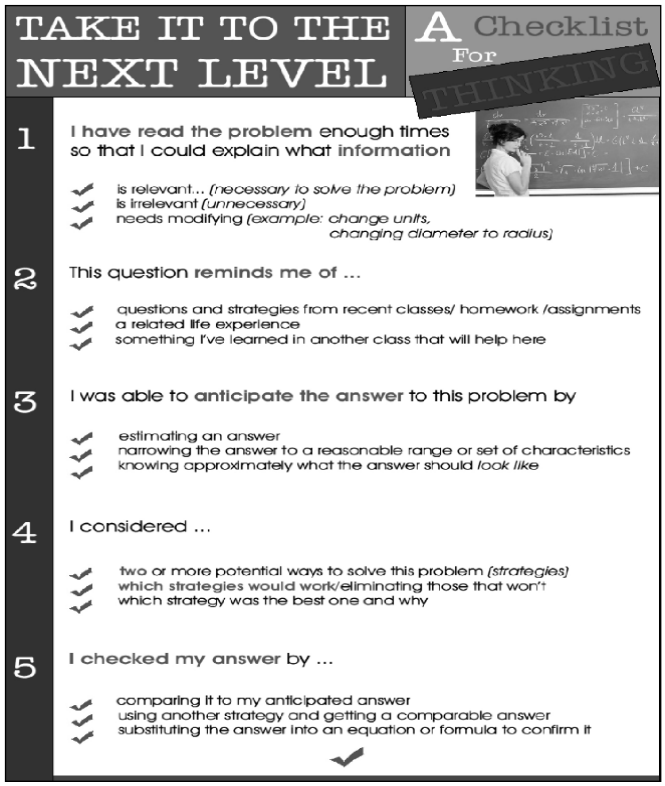
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Appendix 1 Rubric for Measuring Thinking after Solving a Multiple-Step Problem

What were you thinking about…	R	1	2	3	4
What were you thinking about the information in the question? Explain.	Blank Unrelated to the question	Demonstrated understanding of little of the relevant information, irrelevant information, and information that needed manipulating.	Demonstrated understanding of some of the relevant information, irrelevant information, and information that needed manipulating.	Demonstrated understanding of most of the relevant information, irrelevant information, and information that needed manipulating.	Demonstrated understanding of all of the relevant information, irrelevant information, and information that needed manipulating.
This question reminds me of....	Blank Unrelated to the question	Makes vague reference to a question done previously or to related experience.	Makes some reference to a question done previously or to related experience.	Makes specific connections to a question done previously.	Makes specific connections to a question done previously, describing (naming) the strategy used.
What did you anticipate were possible solution(s) [answer(s)] to the problem? Justify.	Blank Unrelated to the question	Shows little evidence of estimating or anticipating.	Shows evidence of estimating or anticipating, with some reference to the question.	Shows evidence of estimating or anticipating, with specific reference to the question.	Shows evidence of a reasonable estimate or anticipated result, with specific reference to the question.
Describe the different strategies that came to mind to solve this problem.	Blank Unrelated to the question	Gives an alternate strategy inappropriate to the question or only one strategy given.	Gives evidence of considering an appropriate alternate strategy.	Gives evidence of considering an alternate strategy with explanation.	Gives evidence of considering an alternate strategy with explanation, and justifies the choice.
What convinced you that your answer was correct? Be specific.	Blank Unrelated to the question	Little evidence has been provided that the solution was checked.	Some evidence has been provided that the solution was checked.	Clear evidence has been provided that the solution was checked.	Evidence has been provided that the solution was checked in more than one way.

Appendix 2 A Checklist for Teaching Thinking during a Multi-Step Problem



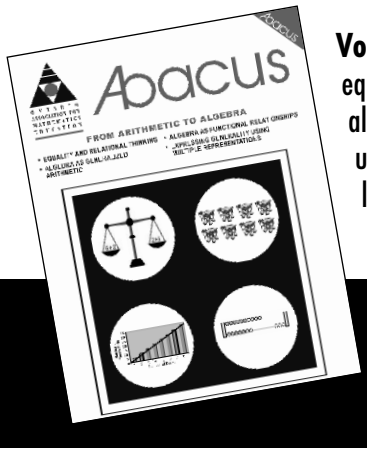
Poster developed by the LSA team at Centre Dufferin District High School: Chris Sales, Mike Delaat, Leah Pressy, Greg Heaslip, Dave Langman, and Deidre Wilson. ▲

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▲ Project Idea: A Case Study – The Application of Statistical Analysis Skills, Using Google Sites and Statistics Canada

JELENA LONCAR
E-MAIL: jloncar08@gmail.com



Jelena began her career at the secondary level, but has since moved into the post-secondary world teaching mathematics.

This Case Study allows students to think critically about the use and purpose of statistical calculations studied in the Grade 12 Data Management course. Students apply these tools to a topic of their own interest.

The learning objectives of this project are multi-fold:

- for students to gain a deeper understanding of statistical parameters
- for students to be able to determine which data is most important and be able to apply statistical tools to make appropriate conclusions
- for students to use their mathematical expertise to prove a conclusion
- for students to learn to navigate through the Statistics Canada website
- peer reflection; mathematically critiquing other student work

Ultimately, this can lead to higher-quality projects equipping students with research and analytical skills required for their post-high school endeavours.

The Logistics

How It Works

1. The Case Study was comprised of two components: The Analysis and The Discussion.
2. This work was to be presented online using **https://sites.google.com/**. The teacher created an account for the class website and "invited" each student via email to join. The teacher had "owner" access. Students had "collaborator" access.
3. Students were assigned a partner. Each pair then presented its Case Study by creating a web page on the class website in Google Sites.
4. Students were given a choice of six topics. The given CANSIM number was provided so that students did not need to bother with data mining and could focus solely on the analysis (Appendix I). As this was most students' first Case Study in mathematics, guiding questions were provided to help with the analysis (Appendix II).
5. To assign topics, each pair chose a random number from a hat. The pair with #1 was the first pair to choose a topic.
6. Ideally, each group should have a different topic. If this is not possible (due to numbers or interest), two class websites can be created within one account—one for Group A and the other for Group B. Each pair in each of the groups had a different topic.

Grading

This was a summative evaluation. Although students worked in pairs, each student was graded individually.

To track student contributions to the project, I used the "Revision History" feature of Google Sites. This allowed me to follow which changes the student had made (and at which time... excellent for meeting deadlines!). For this to work smoothly and effectively, it is critical that teachers introduce their students to Google Sites and have each student ready to go with a working account at the start of the assignment. I found that it takes at least 24 hours for students to receive their "invitation" to the site. This creates problems if students take too long to register and will slow down their progress in completing the assignment. Make sure you do this well in advance!

If it is a major issue for a particular student to get their account working, you may alternatively have a pair of students use one account for completing the assignment, but have students sign their name at the end of the sections that they contributed to the project. As with any assignment, this will definitely take some honesty on the students' part.

The rubric used for assessment can be found in Appendix III.

Deadlines

The discussion portion of the assignment relies on the completion of the analysis. Hence, I have created two deadlines: the analysis is due three days prior to the due date for the discussion. In my experience, this has left enough time for a good thread of discussions to occur. Usually students would have a back-and-forth commentary one evening (just like on Facebook!). A

quick turnaround between the two deadlines also allows room for a test at the end of the unit.

Internet Security

Google Sites has multiple privacy features. I had my settings so that the sites are not available to the public domain and could only be accessed by the students with a password.

Having "owner" access as a teacher allows you flexibility with the program. Students will have "collaborator" access, which does not allow for as much control over the site.

Internet Access

As this activity relies heavily on the use of computers and the Internet, I had class time for students to complete this project. In my experience, it was sufficient to have three to three and a half days of classes to work on this assignment toward the end of the unit. This usually allows enough time for students to complete their work in school (especially important if students do not have ready access to the Internet outside of school).

Advantages and Teaching Points

Advantages of Using Google Sites

- Revision History feature: This feature allows you as a teacher to track your students' activities, so you know which students have been posting and when. This will help hold students accountable for meeting deadlines and will help teachers maintain a record of student submissions.
- Comments feature: This creates a great forum for student-led discussions of math.
- Documents feature: This allows students to work with Word and Excel online, and then easily insert files or images to Google Sites.
- Privacy Settings: This feature allows teachers full control over what can be seen on the World Wide Web.

Teaching Points

- Poke around Google Sites to familiarize yourself with the program and additional features that might be useful for your purposes.
- Remember, the initial email you send to students to "invite" them to join the Google Sites class website takes about 24 hours for students to receive. Do this well in advance so that each student has a working account prior to the start of the assignment. I used student email accounts that were given to them by the school.
- Some instruction is needed to give students an overview of the site and some useful features. Beyond this, the program is very user friendly and students have been quick to pick it up.

This Case Study project can be adjusted to meet any teacher's needs. It can be used with different grade levels, as well as in different subject areas. I found that students enjoyed this assignment; it allowed them to work on the computer, personalize their pages with pictures, and participate in discussions… all while doing math!

Following is sample work of the first part of one group's work. All graphs have been inserted using the Excel feature in "Documents" or using Statistics Canada. Student names have been blocked out. The "Comment" section has been omitted here to conserve space.

Appendix I: CANSIM Tables

E-commerce	CANSIM: tables 358-0135 to 358-0138
Listening to the Radio	CANSIM: tables 503-0001 to 503-0005
Household Spending	CANSIM: tables 203-0001 to 203-0020
Travel	CANSIM: tables 427-0001 to 427-0006
The Movies	CANSIM: table 501-0010
Motor Vehicle Sales	CANSIM: table 079-0003

Appendix II: Guiding Questions

First, the general questions are listed for each group to answer, then the specific ones for each topic.

A. General Questions

i. Define any terminology that may need clarification (e.g., e-commerce, "shelter" category for household spending).

ii. Who is being studied? (e.g., age, sex, region).

iii. Which time frame (in years) are you using for your analysis? Is this data longitudinal or cross-sectional?

iv. Pose questions to your classmates for discussion.

E-commerce

i. Who are the heaviest online shoppers? How much are they spending? What are they buying?

ii. Are consumers browsing the Internet ("window shopping") or are they buying?

iii. What is the trend in shopping on the Internet? Percentages of increase over time?

iv. Is e-commerce helping the economy? How?

v. What are the concerns of consumers regarding e-commerce?

The Radio

i. Who are the heaviest radio listeners?
ii. What are they listening to? Programming (i.e., CBC Radio)? Types of music?
iii. Why are they listening to the radio? News only? Traffic?
iv. What are the trends? To which demographic do these trends belong? Percentage increases/decreases for each?
v. What are consumers listening to other than radio? What other trends are increasing in listening to music? Is this good for the economy? What types of long-term trends can you foresee?

Household Spending

i. How much are households in your region of choice spending? Percentage increases/decreases between years? Trends?
ii. What accounts for the largest share in spending? Trends? Percentages? Standard deviation among categories of spending?
iii. Any increases in spending in particular categories? Any decreases?
iv. How do you think these trends impact the economy? Do they?

Travel

i. Which countries are the most popular travel locations?
ii. What is the most popular mode of transportation to each of these destinations?
iii. Changes in travels over time? Percentages?
iv. Reasons for travels?

Is this good for the economy?

The Movies

i. How has attendance changed in movie theatres? Drive-ins?
ii. Has an increase in attendance necessarily translated to increases in profits for each?
iii. Which region goes to the movies most often? Least often?
iv. Which types of movie theatres are seeing an increase in attendance?

Motor Vehicle Sales

i. Have car sales changed over the years? How?
ii. Compare manufacturing region sales.
iii. Compare provinces.
iv. Suggest reasons for trends. You may access other databases to support your claims.

Appendix III

The following rubric was be used in grading this assignment. The descriptions under each Level are as described in the Ministry documents.

The Case Study	Level 1	Level 2	Level 3	Level 4
Introduction: Does the introduction provide all necessary background information required to understand what the analysis will contain? Clearly state what exactly you are investigating.				
Analysis: Is the analysis thorough? Does analysis contain all relevant calculations and graphs needed to make a complete analysis of the topic? Is mathematical vocabulary used? Are proper tools/skills applied to analyze the topic? All topics mentioned in the introduction have been discussed.				
Conclusion: State conclusion clearly. Is the conclusion backed up by the mathematics presented? Do you agree with the evidence? Determine an extension question that would be interesting to consider for future study.				
The Discussion				
Discussion Question: A discussion question is suggested for classmates to comment on.				
Comments on Other Topics: There is a minimum of one comment on two different topics analyzed by others. Comments constructively critique the analysis, backed up by mathematics/evidence in the data.				
Comments on Own Topic: Comments on your topic are managed well. Responses are timely, address points made by the reader, and are backed up by mathematics/evidence in the data.				

THE ASSIGNMENT

Following are the specific instructions to students. These, along with the marking scheme, are posted on the main page of the class website.

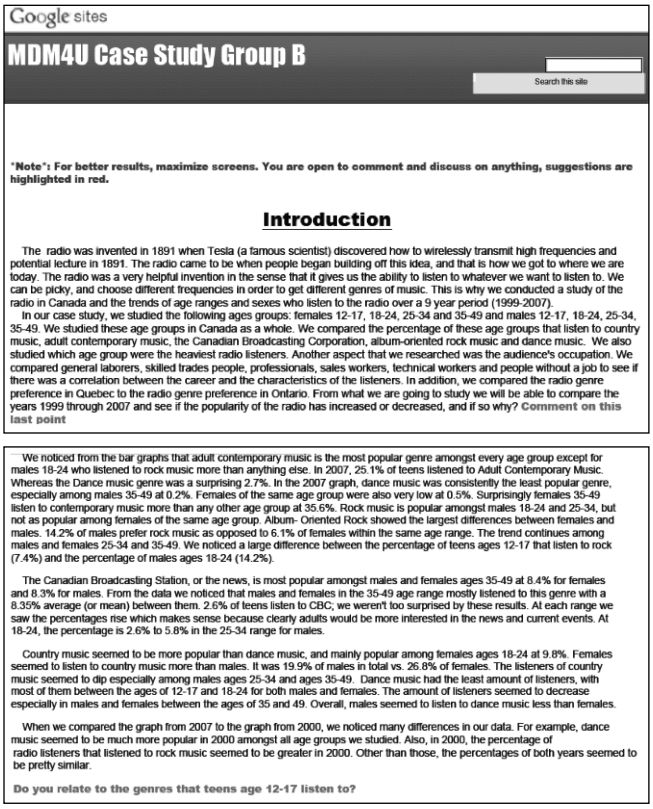
PART I: THE ANALYSIS. Students are required to prepare a report, based on the following instructions:

1. Write an introduction or background for the topic of your choice.
2. Present an analysis. This will be based on the data you have. Include relevant calculations (regression, standard deviation), graphs, etc., to come to one or more conclusions. Use the questions given to guide you through. You must answer all questions asked. Answer in paragraph form.
3. Provide a conclusion to your analysis that briefly summarizes your results. Do you agree with the analysis? Comment on this, critiquing the data if you do not agree. Do you need to consider other variables that are not included in the data sets given? Anything else?

4. Mathematical language and vocabulary studied must be used (distribution types, correlation, etc.)

PART II: THE DISCUSSION. In addition to the analysis, students are also required to participate in a discussion on the topic, using the "Comments" section of each web page. Here are the instructions that are given:

1. Read a minimum of two other topics which your classmates have studied.
2. Provide a comment on a minimum of two topics. You must use mathematics to back up your opinion. This may include your critique of their mathematics, or the introduction of new ideas.
3. Follow through with the thread of comments.



Google sites

MDM4U Case Study Group B

Search this site

"Note": For better results, maximize screens. You are open to comment and discuss on anything, suggestions are highlighted in red.

Introduction

The radio was invented in 1891 when Tesla (a famous scientist) discovered how to wirelessly transmit high frequencies and potential lecture in 1891. The radio came to be when people began building off this idea, and that is how we got to where we are today. The radio was a very helpful invention in the sense that it gives us the ability to listen to whatever we want to listen to. We can be picky, and choose different frequencies in order to get different genres of music. This is why we conducted a study of the radio in Canada and the trends of age ranges and sexes who listen to the radio over a 9 year period (1999-2007).

In our case study, we studied the following ages groups: females 12-17, 18-24, 25-34 and 35-49 and males 12-17, 18-24, 25-34, 35-49. We studied these age groups in Canada as a whole. We compared the percentage of these age groups that listen to country music, adult contemporary music, the Canadian Broadcasting Corporation, album-oriented rock music and dance music. We also studied which age group were the heaviest radio listeners. Another aspect that we researched was the audience's occupation. We compared general laborers, skilled trades people, professionals, sales workers, technical workers and people without a job to see if there was a correlation between the career and the characteristics of the listeners. In addition, we compared the radio genre preference in Quebec to the radio genre preference in Ontario. From what we are going to study we will be able to compare the years 1999 through 2007 and see if the popularity of the radio has increased or decreased, and if so why? **Comment on this last point**

We noticed from the bar graphs that adult contemporary music is the most popular genre amongst every age group except for males 18-24 who listened to rock music more than anything else. In 2007, 25.1% of teens listened to Adult Contemporary Music. Whereas the Dance music genre was a surprising 2.7%. In the 2007 graph, dance music was consistently the least popular genre, especially among males 35-49 at 0.2%. Females of the same age group were also very low at 0.5%. Surprisingly females 35-49 listen to contemporary music more than any other age group at 35.6%. Rock music is popular amongst males 18-24 and 25-34, but not as popular among females of the same age group. Album- Oriented Rock showed the largest differences between females and males. 14.2% of males prefer rock music as opposed to 6.1% of females within the same age range. The trend continues among males and females 25-34 and 35-49. We noticed a large difference between the percentage of teens ages 12-17 that listen to rock (7.4%) and the percentage of males ages 18-24 (14.2%).

The Canadian Broadcasting Station, or the news, is most popular amongst males and females ages 35-49 at 8.4% for females and 8.3% for males. From the data we noticed that males and females in the 35-49 age range mostly listened to this genre with a 8.35% average (or mean) between them. 2.6% of teens listen to CBC; we weren't too surprised by these results. At each range we saw the percentages rise which makes sense because clearly adults would be more interested in the news and current events. At 18-24, the percentage is 2.6% to 5.8% in the 25-34 range for males.

Country music seemed to be more popular than dance music, and mainly popular among females ages 18-24 at 9.8%. Females seemed to listen to country music more than males. It was 19.9% of males in total vs. 26.8% of females. The listeners of country music seemed to dip especially among males ages 25-34 and ages 35-49. Dance music had the least amount of listeners, with most of them between the ages of 12-17 and 18-24 for both males and females. The amount of listeners seemed to decrease especially in males and females between the ages of 35 and 49. Overall, males seemed to listen to dance music less than females.

When we compared the graph from 2007 to the graph from 2000, we noticed many differences in our data. For example, dance music seemed to be much more popular in 2000 amongst all age groups we studied. Also, in 2000, the percentage of radio listeners that listened to rock music seemed to be greater in 2000. Other than those, the percentages of both years seemed to be pretty similar.

Do you relate to the genres that teens age 12-17 listen to?

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Supporting Algebraic Thinking: Prioritizing Visual Representations

RUTH BEATTY
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Dr. Ruth Beatty is an Assistant Professor at Lakehead University (Orillia Campus), teaching pre-service Primary/Junior candidates. Her research focuses on students' development of algebraic thinking from Grade 4 to Grade 9.

For a number of years, mathematics education researchers have been promoting the introduction of algebraic concepts in elementary and middle schools (Warren & Cooper, 2006; Greenes et al., 2001; Kieran & Chalough, 1993; Kieran, 1992, 1991). There is a belief that an early introduction to algebra may ease the transition from arithmetically based instruction in elementary school to formal algebraic instruction in high school (Kieran, 1992). As part of this new initiative, mathematics education researchers have been investigating how young children can think algebraically when working with patterns (Yerushalmy & Shternberg, 2001; Noss & Hoyles, 2006; Mason, 1996; Lee, 1996).

In this article, I outline the results of a three-year research study that tracked the development of algebraic thinking in students from Grade 4 to Grade 6. Although I was working with elementary students, the concepts they considered were well beyond the current Ontario elementary curriculum expectations and incorporated concepts formally taught in the intermediate and senior grades. In the following sections, I introduce a sequence of activities that support students—from working with linear growing patterns, patterns whose terms grow by a constant value, to solving linear equations.

One of the goals of my work has been to introduce students to various representations of linear relationships. Understanding the connections among representations underpins the ability to predict changes in one representation based on transformations of the other.

The final sequence of lessons can be roughly divided into three parts. Part 1 includes unordered tables of values integrated with activities involving linear growing patterns. Part 2 introduces another visual representation,

graphical representations. In part 3, students compare lines on a graph as precursory understanding for solving equations of the form $ax + b = cx + d$. These three parts were designed to help students transition from the expectations outlined in the Ontario junior and intermediate curriculum to the expectations outlined in the senior curriculum.

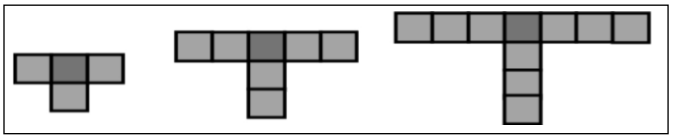
Grades 4-8	Grade 9
• recognize, describe, generalize patterns • use graphs, tables, and verbal descriptions to recognize relationship • understand variables as changing quantities • represent patterns algebraically and make predictions	• identify properties of linear relationships • investigate relationships between two variables • understand the characteristics of a linear relationship • connect representations—tables of values, graphs, and equations • understand the formula $y = mx + b$

In the sections below, I will review some of the well-documented difficulties students encounter as they develop their algebraic reasoning. I will then outline the components of the curriculum developed in response to these difficulties.

Patterns and Algebraic Thinking

In elementary curricula, algebraic understanding is introduced through working with patterns. Patterns offer a tremendous opportunity for students to explore some fundamental algebraic concepts. For example, patterns support students' abilities to generalize. Patterns also offer students a way of concretely exploring the idea of systematic variation between two sets of data.

The expectation is that working with patterns will support students to discover and articulate mathematical structure. Students working with patterns are expected to be able to describe the pattern, extend the pattern, and identify the underlying structural rule (generalize).



For instance, in the pattern above, most students are able to recognize and describe the pattern as "you add three more light square tiles each time." Students are also able to extend the pattern— the next "T" would have one dark square tile and four light square tiles on each of the three sides. The difficulty arises when students are asked to predict how many square tiles would be needed far down the sequence. For the tenth term, students might rely on drawing the 4th, then the 5th, then the 6th, and so on, adding one square tile to each arm as they go. Asking for the 100th term highlights the problem of this method. Students also cannot devise a general rule that would allow them to predict the number of square tiles for any term of the pattern. The problem with the rule "add three each time" is that for any term number, you need to know the number of square tiles in the preceding term. This is known as recursive reasoning. With recursive reasoning, only the variation in one data set—in this case, the number of tiles—is considered.

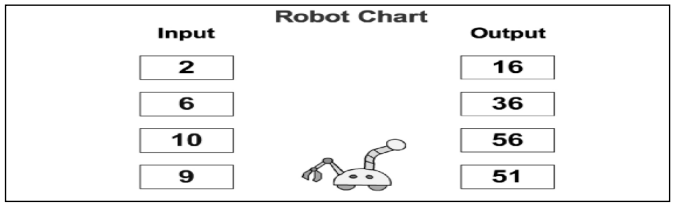
The pattern above can also be represented using an ordered table of values. This is the strategy that many students use to enable them to consider both the term number and the number of tiles at each iteration of the pattern.

Term Number	Number of Tiles
1	4
2	7
3	10

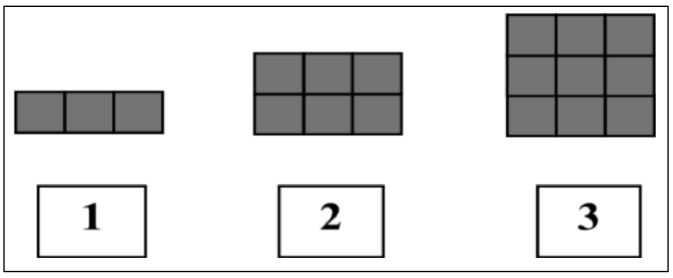
Ultimately, students learn strategies to figure out the number of tiles from the term number, but the focus of the study was to develop those strategies with conceptual understanding.

Curriculum Part 1

The activities in this part of the curriculum, (i.e., the lesson sequence developed) introduce students to linear relationships via unordered tables of values and linear growing patterns. Unordered tables of values are introduced using a "function machine" type activity (Willoughby, 1997) called the Robot Game. Teachers and students take on the role of a robot that transforms input numbers to output numbers by following a linear rule of the form $y = mx + b$. Students call out input numbers at random, which ensures that the resulting "robot chart" is unordered. Students are thus forced to look across the two data sets (input and output numbers) in order to determine the pattern rule. The rule is expressed as, for example, "the output number is equal to the input number times five plus six" and written as output=input x 5 + 6—which is structurally similar to $y = mx + b$.

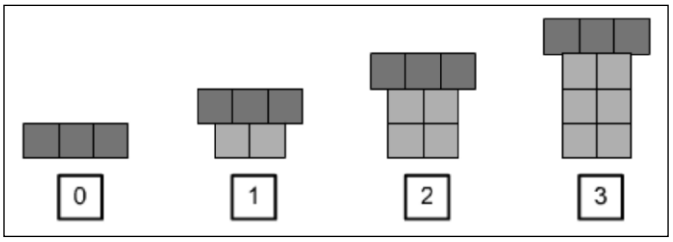


This numeric pattern activity is interspersed with activities involving linear growing patterns. Students are first introduced to the pattern below, built with square tiles and including "position cards" at each iteration.



In this activity, students can describe and extend the pattern as "add three more." However, when asked for the number of tiles for the 10th position, most students correctly answer that there would be 30 tiles because 10 times 3 is 30. Students make the connection between the ordinal position number of each term of the pattern, and the number of tiles in that term. The position number is not a label, it is a quantity—the position numbers represent one set of data and the tiles represent the other set of data, and the pattern rule articulates how the two sets co-vary. In this case, unlike the case where recursive reasoning is used to determine the number of tiles for a far term or position, or for *any* position, students can use the pattern rule that describes the relationship between the position number and the number of tiles—"the number of tiles is equal to the position number times three."

Students then work with more complex patterns like the one below. Here, the pattern rule can be articulated as "the number of tiles is equal to the position number times two plus three" and written as "number of tiles = position number x 2 + 3."



The multiplier, x 2, is represented by the light tiles that grow by 2. The constant, +3, is represented by the three dark tiles that "stay the same" at each position. Notice that at the "zeroth" position, the multiplier is represented by 0 tiles, and the constant by the three dark tiles.

By engaging in a number of Robot Game and pattern-building activities, students learn that linear rules can be expressed as the relationship between input and output numbers in unordered tables of values, and as the relationship between the number of tiles for each position of a linear growing pattern. They are able to consider particular instances of a linear growing pattern and formulate a general rule. Students also make connections between the Robot Game activities and the pattern-building activities, and the different ways to express the rules, so that "output = input x 3 + 2" is linked to "number of tiles = position number x 3 + 2." The students recognize that both are representations of the underlying mathematical rule. The following is an excerpt from a transcript of a classroom lesson, during which a Grade 6 student explained the connections he saw between the Robot Game and building linear growing patterns.

> *In pattern building, the input part is the position number and the output part is like the pattern you build. So in the middle is the operation or the rule you have to use. So it's kind of like the Input/Output (Robot Game). In the Input/Output you have to use the Input, do the rule and then you get the Output. The same with this one (pointing to pattern), you have to use your position number, do the rule, and get your answer and make the pattern.*

Graphs and Algebraic Thinking

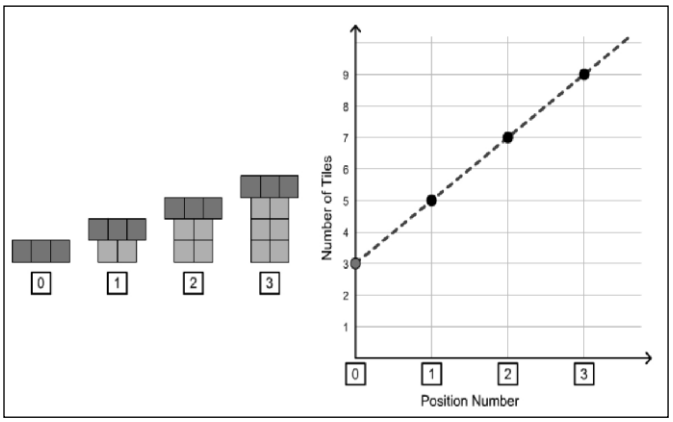
Numerous researchers have documented the difficulties students encounter when considering graphical representations of linear relationships (e.g., Evan, 1998; Moschkovich, 1996, 1998, 1999; Brassel & Rowe, 1993; Yerushalmy, 1991). Many students find it difficult to connect the algebraic equation $y = mx + b$ with the graph and do not realize that the multiplier of the independent variable is responsible for the angle (or slope) of the line, and that the constant is responsible for the *y*-intercept (Bardini & Stacey, 2006). Students find it difficult to predict how changing the multiplier or the constant in the equation will affect the graph. For instance, most students predict that the line for the equation $y = 5x + 5$ will be both higher and steeper than the line for the equation $y = 5x$; students are unaware that these two parameters (m and b) are independent (Moschkovich, 1996). Because the two domains of solving algebraic equations and constructing and interpreting linear graphs are often taught separately, students often treat the algebraic and graphical representational domains as if they were independent.

Another difficulty some students have is that they tend to focus on individual points (like the *y*-intercept),

but do not recognize the line as a representation of linear growth (Leinhardt, Zaslavsky & Stein, 1990).

Curriculum Part 2

When engaged in activities from the second part of the curriculum, students learn how to construct linear graphs based on their understanding of pattern rules. At this point, the students use only the upper-right quadrant of the coordinate plane, since the pattern rules include only positive integers for both the multiplier and constant. The position number cards of the growing patterns are mapped onto values along the *x*-axis. The total number of tiles in each position is represented by values along the *y*-axis. One point on the *y*-axis represents the number of tiles that would be at the "zeroth" position of a growing pattern, the value of the constant, which is graphed as the *y*-intercept. When constructing the graph, students are given rules, such as "number of tiles = position number x 2 + 3," and asked to build geometric patterns based on the rule. They are then asked to calculate the total number of tiles at each position of the pattern and draw a dot on the graph to represent how many tiles are at each of the position numbers built. Students therefore graph "ordered pairs"—position number, number of tiles—without being explicitly asked to do so.



Students then build and create a graph of the 0th, 1st, 2nd, and 3rd position for three pattern rules that have different multipliers and the same constant ($y = x + 1$, $y = 3x + 1$, $y = 5x + 1$), and for rules that have similar multipliers and different constants ($y = 3x + 1$, $y = 3x + 3$, $y = 3x + 5$). For each set of rules, students are asked to predict what the graph would look like (e.g., parallel lines, lines with different steepness). The students build all three patterns and graph them, and note similarities and differences between the pattern rules, the patterns, and the graphs, both within and between the two sets of rules. Thus, students are given the opportunity to explicitly consider how a change in one representation, the numeric/symbolic rule, affects both the linear growing pattern and the graph.

In the following transcript, a student discusses the graph of pattern rules that have the same multiplier, but different values for the constant. The student is able to reason about the connection between the two parameters of the pattern rules and the parallel lines on the graph.

> *The lines are parallel to each other and that was my prediction—that they would have the same steepness, but have different heights. The multiplier decides the steepness. If the multiplier is big, like times 9, then the line goes steep, but if it's lower, like times 3, it's not going to be that steep. The constant decides where the line starts—so, like the height of the line on the graph. If you have the same multiplier, all the lines have the same steepness, and the constant decides the height of the line.*

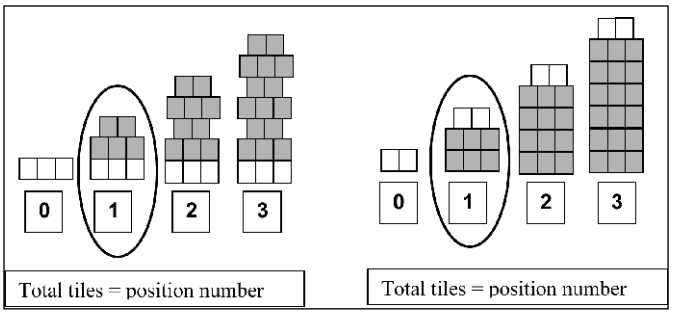
Students realize that graphs model the rate of change of the growing patterns, and that the higher the multiplier, the more tiles are added to each successive position in their pattern, and the steeper the slope of the line. Students also developed an understanding that the constant is represented by "where the line starts" on the *y*-axis (*y*-intercept), since the number of tiles at the "zeroth" position of a pattern is graphed at the vertical axis, and only the value of the constant is represented (Beatty, 2007).

Curriculum Part 3

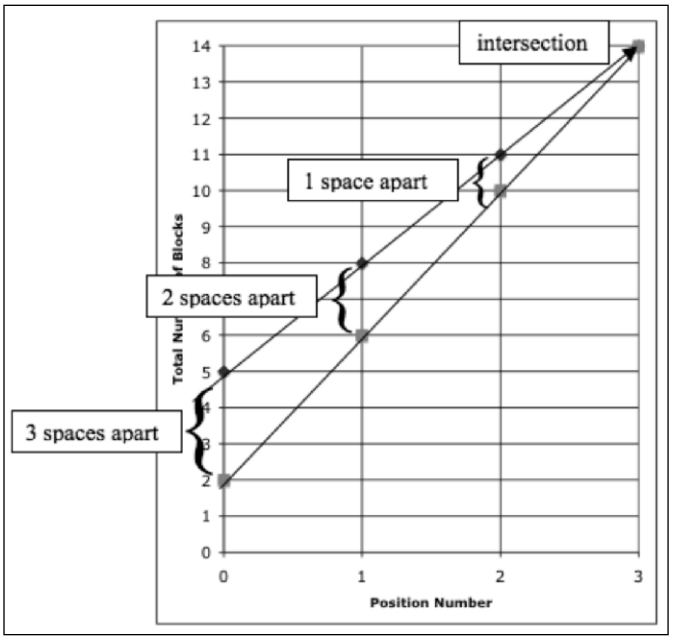
In the third part of the curriculum, to provide a conceptual background for solving equations of the form $ax + b = cx + d$, the lesson sequence is designed to support students to think about intersecting lines as representing the relationship between two linear rules.

Students are given pattern rules and asked to predict the position at which both rules would have the same number of tiles. They are then asked at what *x*-value on the graph the lines would intersect.

When comparing two rules, such as $5x + 3$ and $6x + 2$, students initially identify that both rules would have 8 tiles at position 1 (if they were linear growing patterns) and that this would be represented by lines that intersect at (1,8) on the graph.



Students then discover that when considering any two pattern rules that have a difference of 1 for the multipliers, the difference between the constants is the *x*-value at which the lines will intersect. For instance, $4x + 2$ and $3x + 5$ have *y*-intercepts that are 3 spaces apart on the graph, and the lines come together by one space each time.



Knowing how far apart 2 rules "start"—i.e., based the values of the constants represented by the *y*-intercepts—and knowing the rate at which the lines "come together" on the graph—i.e., knowing that if the multipliers differ by 1, the lines come together by one space at each successive *x*-value—allows for the prediction of the *x*-value at which the lines will intersect.

The students further generalize this understanding, realizing that you can work out the point of intersection if you know where any lines start—the difference between the constants—and compare the rate at which they come together—by comparing the value of the multipliers. For example, $3x + 6$ and $5x + 2$ start out 4 spaces apart on the graph (the *y*-intercepts are 4 spaces apart), but come together by two spaces at each successive *x*-value. Therefore, you can figure out that the lines will meet, or intersect, by *x*-value 2.

This understanding can be used to solve equations of the form $ax + b = cx + d$. Students are given rules in standard algebraic notation and asked, "At what position would the lines for these rules intersect?" At this point, the graph is no longer the site for problem solving, but instead, is used as a tool for checking and justifying solutions. For example, John was given the equation $2x + 16 = 5x + 1$ and asked to predict the point of intersection, which is, in effect, solving for *x*.

John: Well, I would see that the difference between these two (the multipliers $2x$ and $5x$) is 3 and that the difference between these two (constants +16 and +1) is 15. I know that 15 divided by 3 is 5, so I think it's going to intersect on the 5th position.

Teacher: How would you check?

John: Try it out. So 5 x 2 is 10 plus 16 is 26, and 5 times 5 is 25 plus 1 is 26.

Teacher: What does it mean when you get 26 for both rules?

John: Um, that's the amount of each pattern and that's where they would intersect.

Teacher: But I thought you said they intersect at 5?

John: The 5th position! [pointing to the *x*-value 5 on the graph] At the 5th position, they would both equal 26 tiles—that's like the number they would intersect on. So it would be [drawing a dot on the graph at (5, 26)] they would both end up there.

In his explanation, John integrates multiple representations by making a reference to pattern building, that at the "5th position, they would both equal 26 tiles," and the graph by stating that "they would both end up" at (5, 26).

If we think of $ax + b$ and $cx + d$ as two pattern rules for which the value of the multiplier and the constant are different, then the solution to the equation $ax + b = cx + d$ is analogous to finding the *x*-value at which the lines of the two rules will intersect (or the position number at which both patterns will have the same number of tiles). To determine how far apart lines "start" on the *y*-axis, students find the numeric difference between the values of the constant, or $(d - b)$. To find the rate at which they "come together," students find the difference between one multiplier and the other $(a - c)$. To find the position number (x), they divide "how far apart they started" by

"the rate at which they come together" or $x = (d - b) \div (a - c)$. Students demonstrate a conceptual understanding of why they carry out the operations of subtraction and division, rather than just memorizing an algorithm.

Conclusion

Current research indicates that there is a need for instructional models that help students understand the connections among representations of linear relationships (e.g., Greenes et al., 2007). The instructional sequence described in this paper has been shown to support Grades 4, 5, and 6 students in developing their understanding of generalizing, mathematical structure, and the connections among representations of linear relationships. Results indicate that these young students are capable of engaging in algebraic thinking beyond current provincial elementary curriculum expectations and can consider concepts from the intermediate and senior mathematics curricula.

In particular, the prioritization of visual representations seems to support students' emergent understanding of linear relationships. Understanding a concept presupposes the ability to recognize that concept in a variety of representations and the ability to handle the concept flexibly within different representational systems. By utilizing students' understanding of patterns, the connections between equations and graphic representations were made as transparent as possible. Students were able to see how changing a pattern rule resulted in changes in the linear growing pattern, which then led to the ability to predict how transforming a rule would also transform the graph of that rule. This then led to an initial understanding of comparing lines on a graph and using that understanding to solve equations $ax + b = cx + d$.

Preliminary results indicate that this may also be a useful approach for students in Grades 7 and 8 (Bruce, Ross, Beatty & Sibbald, 2010). However, more research is required to determine whether an approach to teaching linear relationships that integrates multiple representations, and prioritizes visual representations, can support the transition of middle school students to formal algebraic instruction.

On the OAME website, CLIPS materials based on the ideas in this article are available for use with students.

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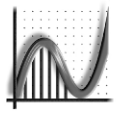
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▲ Billy and the SMART Board™

WILLIAM LUNDY
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Bill is a special-assignment teacher attached to the ITS department of the Hastings and Prince Edward District School Board. His areas of interest are interactive whiteboards and software for mathematics education.

(The principal character, Billy, is a fictional composite of several classroom teachers who have collectively had these revelations. The focus of the article is on how a number of teachers, as personified by the singular 'Billy,' leveraged certain features of interactive whiteboard software to create learning artifacts for later review, reflection, and analysis in order to provide feedback to students and shape future instruction.)

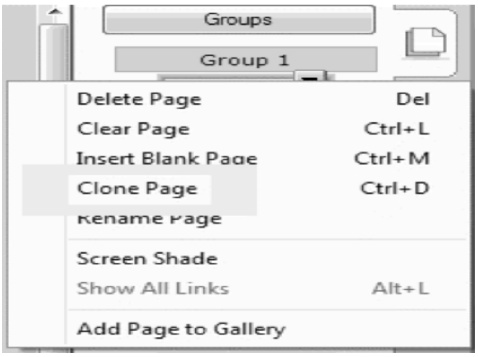
When Billy's SMART Board™ was installed in his room, he was ecstatic. Finally, he could bring the world into his classroom! The Winter Olympics were about to happen, so he quickly found out how he could bring in streaming video to keep his class up-to-date, access and use "Google Earth" for virtual field trips to the Winter Olympics venues, create a collaborative spreadsheet of results with his students, and study Olympic-themed graphs with some cool lessons he found on the "SMART Exchange." But he became interested in how to use technology to collect information for assessment purposes.

One day, Billy was working with a group of Grade 3 students, looking at ways to represent given amounts of money. He decided to let groups of students make up the amounts of money they wished to represent to keep the question open. To further challenge their thinking, he asked each team to represent its money amount using two different sets of coins.

His learning goal was that students would recognize that there are many ways to represent numbers, in particular, that there were many ways to represent a given amount of money. He created a one-page SMART Notebook™ file, with all of the coins shown (each one once) down the left-hand side of the page. He applied the infinite clone tool to each coin image, so that he could call up as many coins as needed.



He decided it would be useful to duplicate that page. This would yield a page or two for the "minds on" portion of his three-part lesson and his demonstration to the class, as well as an extra page for unexpected events. At the time, he didn't consider that the students would show their solutions on the board as well!



So, he clicked on the thumbnail of that one page (he went to the page sorter, clicked the thumbnail, and accessed the drop-down list). He used the option to "Clone" that page to make an exact duplicate of his first page, with all of the coins in the clone also having the infinite clone option he'd previously set up. This he repeated once again so that he had three identical pages. After saving the file, he was ready to begin.

When the class was gathered, he used page 1 to recall previous learning. Students named the coins, their absolute values (in cents), and their various relative values (e.g., "two nickels have the same value as a dime"). He then went to page 2 to demonstrate the problem for that day. A brief discussion yielded the amount $4.88 as the target value for the problem. Using a "think aloud" technique, he carefully dragged out clones of a couple of toonies, three quarters, a dime, and three pennies. Then he asked the class for suggestions about another way he could represent the same amount. Based on suggestions from various people, he again dragged out various coins to total $4.88, adding aloud as he went.

After a brief pause to reiterate the directions to students, he showed page 3 to them and challenged them to choose an amount of money to represent, and to do so in two ways. Students returned to their desks and worked in pairs with their plastic coin sets to build their solutions.

It hit Billy as they began working: *"Why not clone 'page 3' as many times as I have groups?"* This would then allow each group to show its solutions on the white board. This he quickly did, then moved to supervise and observe student work and conversation. During his math congress, each team had its own opportunity to show its work. Students were delighted they could do the manipulating themselves, doubly so, when they were able to sign their names to their "own" pages. Billy promised them he would print out hard copies of their pages for their portfolios. Excitement that day knew no bounds!

At the end of the period, another revelation struck Billy: *"What if I saved this file so I have an electronic record of what we did today?"* This he did, by simply using *File > Save As...*, and adding a suffix to the original file name with a new date. He reasoned that calling up this particular version of the file next time he taught that group would help students recall the previous day's efforts, and provide a nice bridge into the succeeding lesson.

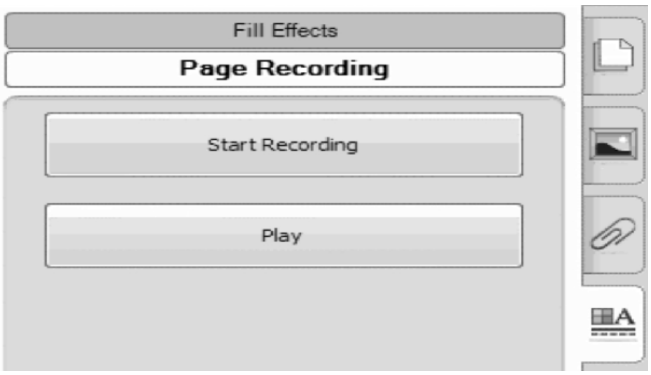
That evening, when planning the exact details of the next lesson, it struck him that it seemed most groups solved the problem as he'd hoped; however, there was a nagging feeling that he couldn't shake. Yet another light flashed on in his mind: "*Why not access the file I made today to be sure?"* He was able to quickly do so, and in reviewing the students' answers, realized what was nagging him: most had endeavoured to find answers involving the fewest number of coins. While this was fine, he wondered if they had really challenged themselves enough. So, he opened his original file, and removed a few key coins (saving this revised version with its own suffix). This revised file would be used the next day to provide the challenge he'd hoped would have occurred the first day.

What did this have to do with assessment? Because Billy was able to save the students' work quickly and easily, he was also able to access it efficiently and use the artifacts to drive the focus for the next period's instruction and activity. In reviewing his artifacts, he was looking to see how well the students were applying the notion that money amounts had many representations. Here, his assessment was not geared to any particular individual, but to the group as a whole; nonetheless, the

fact that he was able to save and recall artifacts meant his approach was driven by formative assessment.

A little while later, Billy was asked to work with a group of intermediate-level students who needed practice simplifying simple linear equations. It struck him that the white board had some recording tools he could use so that he could obtain artifacts of not only the students' products (i.e., the completed solutions), but also their processes. Being able to capture both the steps taken, as well as the final solutions, without his having to look over their shoulders, would enable him to focus his efforts on facilitating conversation with students. Reviewing the recordings could wait until later that day.

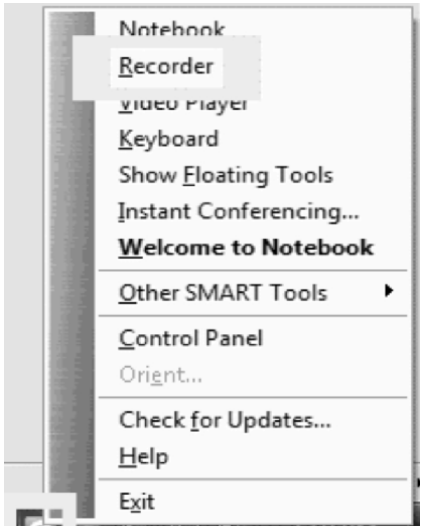
After opening up a blank SMART Notebook™ file, he clicked the "Properties" tab for page 1. There, he had access to a page recorder at the click of a button. After briefly describing the steps for recording a page to the students, he asked for a pair to come up to the board, select an equation, and solve it on the board. The only thing each pair of students had to remember to do was tap the "Start Recording" button before beginning to write, then tap "Stop Recording" when finished (as well as add a new blank page for the next pair).



The class went well: pairs went to the board in turn, each pair recording its entire process on a new blank page. Billy was able to spend his entire time coaching the students at their seats; aside from occasional glances at the white board to see how things were going, he was able to ignore what was being produced and captured on the board. At the end of the period, he saved the file for later assessment for learning.

That afternoon, after dismissal, he retrieved the file, and reviewed each pair's work. Because the "Page Recorder" tool works within SMART Notebook™, everything was in one easy-to-find place. The Notebook file had a recording on each page of the file, one for each team. He opened each page and critically reviewed what he saw. He noted that one pair, Liam and Will, had a clear, efficient process, but showed a need to take better care with simple calculations. That point was duly noted for mention to them during the next class. Many of the teams had arrived at solutions without showing all steps necessary to clearly show their processes. This Billy also noted, for general discussion with the group as a whole.

Here, again, is an example of how Billy was able to capture artifacts of student learning for later review and timely follow-up with students. The only thing lacking was audio: Billy realized that seeing the process was only part of the picture necessary to fully assess what the students were thinking. The addition of audio to the recording process would allow him to capture math talk, as well as see what was going on.



By clicking the "SMART icon" in his system tray (down by the clock) he accessed a pop-up menu, with one feature (the second form the top) being "Recorder." The recorder operated separately from SMART Notebook™ and produced (by default on his system) a .wmv file, playable in one of several media players available to him.



He plugged a long jack cord into the computer running his board; the microphone was hung from the projector arm, so as to be above the students' heads as they worked.

The next day, he was able to provide feedback based on his review of the previous day's artifacts. The only new twist in the class was the use of the recording tool. Each group was shown how to access the "Recorder" from the system tray, and how to save their own recordings. On this occasion, things progressed sufficiently quickly that there was time to play some of the recordings with the students still present.

Having picked one .wmv file at random (or, at least as randomly as possible under the circumstances), he began playing the media file. Within seconds, the students whose video was being shown were asking for a re-recording session, having noted (without teacher prompting) a couple of glaring errors. Billy facilitated a discussion about what errors they noted, and how they could address those errors next time around. Thus, using a media file containing the math talk, as well as the visual record of the process, students were afforded the opportunity to self-assess.

So, in the course of one week, Billy had used his interactive white board tools to produce static, video, and audio/video artifacts of student process and products. This enabled him to efficiently:

- obtain current assessment information about both groups and individuals, so that his student data were reflective of a variety of undertakings in non-paper-pencil fashion
- provide timely feedback to students
- have a comprehensive artifact to share with colleagues

When discussion of the *Growing Success* document began in his school, Billy was already equipped with a few tools to help applying several of its fundamental principles. Having existing skill using SMART Notebook™ capture tools to create learning artifacts helped him provide advice to others in supporting the implementation of this foundational directive.

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▲ Sator Squares

KATIE BARRON WITH DAVID WESSEL
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Katie is a Grade 11 student at St. Mary Secondary School in Cobourg. She was a Grade 10 student when her curiosity led her to her discovery of palindromic patterns.



Dave Wessel has been a math teacher for the past 17 years, and is currently teaching at St. Mary Secondary School. Katie was one of his Grade 10 academic math students last year.

The I in this article is Katie. Katie wrote this article with the support of her teacher, Dave Wessel.

When I was younger, my father once showed me something called a Sator Square. The Sator Square is a grid that runs 5 by 5 and has repeating letters so that words can be read forwards, backwards, up and down. It will always read the same sequence of five words, such as SATOR, AREPO, TENET, OPERA, and ROTAS. The earliest known Sator Square was found carved on the walls in the ruins of Pompeii, which indicates that the Sator Square has been in existence since at least 79 AD. The following is a diagram of a Sator Square.

The Sator Magic Square

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

As soon as I saw this square, I wondered if I could create one of my own, and after a few half-hearted tries, I accepted the fact that I was unable to. Years later, I was in a science class and finished my work early so I had a chance to fool around with the square. I had the idea that, instead of blindly substituting in *letters*, I could use *numbers* instead, just to get a better view of the pattern. As I put in the numbers one, two, three, four, and five for the first row, I began to see a pattern, and as I finished off the square, it had become quite clear—the numbers worked in "shells" radiating outward from the centre and repeating in various patterns within the square. The following is the basic numerical square I developed.

1	2	3	4	5
2	6	7	8	4
3	7	9	7	3
4	8	7	6	2
5	4	3	2	1

The shaded portions of this square indicate vowels and white squares indicate consonants; if you follow this rule, your palindromic square will always be "pronounceable."

Making Your Own Square

The following steps are useful when making your own palindromic square.

1) Examine the basic numerical square above and choose one letter for each number, following the checkerboard pattern for vowels and consonants. To make it easier, start with the centre row and column and work outward. You may repeat some letters, but you could end up with some strange-looking words.
2) Substitute in your own letters and what results will work perfectly!

For example, the square shown below started with the palindrome CIVIC, upon which the words ATIVE, EVITA, FACED, and DECAF were built.

F	A	C	E	D
A	T	I	V	E
C	I	V	I	C
E	V	I	T	A
D	E	C	A	F

Of course, not all of these words are found in *Webster's Dictionary*—ATIVE is a suffix, and EVITA is the name of a person and a musical—but sometimes such alternative thinking is needed to complete an effective palindromic square!

Challenge yourself by trying to fill your square with as many known or "real" words as possible! Palindromic squares are a fun and easy way to kill time or impress your friends. You can even use these squares to brainstorm name ideas for a pet or a project. Try and find as many patterns as you can! ▲

▲ Caribou Online Mathematics Contests

SANTO D'AGOSTINO
E-MAIL: sdagostino@brocku.ca

The Brock University Mathematics Department is pioneering a series of innovative, free online junior mathematics competitions running under the banner of The Caribou Online Mathematics Contests. Intended for children attending Grades 3 to 8, the goals of the Caribou Mathematics Contests are to:

- provide challenging math activities for interested students,
- enhance the appreciation of math and science among children, and
- strengthen students' preparation for future math courses.

The first competition was held in February 2009 for students attending Niagara schools. Caribou is flourishing, and its success has led to its expansion in 2009/10 throughout Ontario. Beginning in October 2010, the Caribou contests were available across Canada.

On each of six contest days per year, three contests are held (one for Grades 3/4, one for Grades 5/6, and a third for Grades 7/8). The Caribou website compiles detailed data on participation, including how often students work on practice tests. There is a spike in participation in practice tests in the days before a competition. Thus, by having multiple contests per year at each grade band, the frequency of practice is increased, making Caribou more effective at stimulating children to enjoy solving mathematics problems.

The rapid success of the online Caribou Mathematics Contest is attested to by the many positive testimonials from teachers across Ontario.

Caribou's unique and positive attributes are listed below.

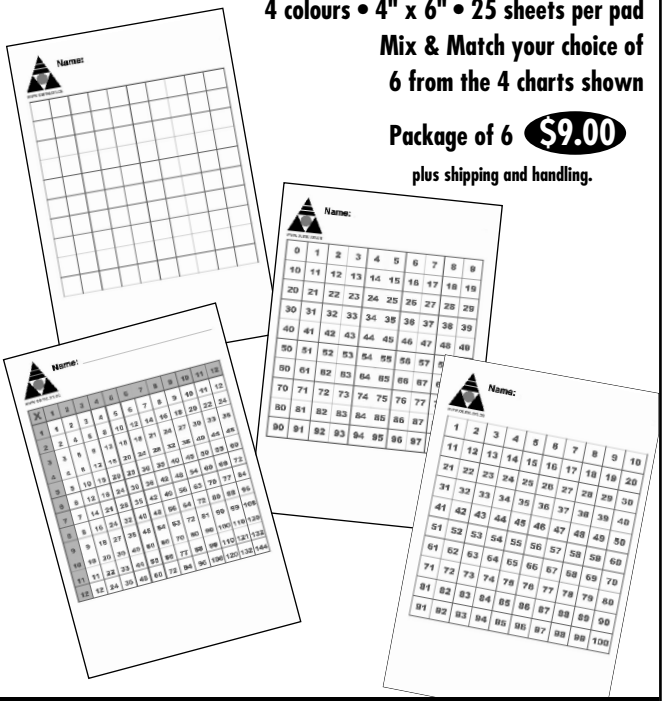
- The contest is absolutely free for participants and their teachers and schools.
- Free public access to all previous contests is available; students can take any of the previous contests online for practice and receive immediate feedback on each question once the contest is completed.
- The contest runs six times per year at each grade group; the high frequency has already stimulated some schools to start math clubs, and certainly stimulates more students to practise more math.
- Each test is offered simultaneously in both English and French.
- The test is marked automatically, with detailed statistics available (including a listing of the top 50 students) on the evening of each contest day.
- The contest dates are posted before the beginning of each school year, allowing teachers to plan their participation in advance; only a few minutes are required from each participating teacher, in addition to supervising students during the hour that they take the contest.
- Home-schooled students may also participate at local public libraries.

For further information about the online Caribou Mathematics Contest, please see **www.brocku.ca/caribou/**. ▲

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OAME Financial Statement 2009/10

Notice to Readers

On the basis of information provided by management, I have compiled the balance sheet of the Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathematiques) as at August 31, 2010 and the statement of operations and operating fund balance for the year then ended. I have not performed an audit or a review engagement in respect of these financial statements and, accordingly, I express no assurance thereon. Readers are cautioned that these statements may not be appropriate for their purposes.

London, Ontario, September 8, 2010, PHILIP J. GRIFFIN Chartered Accountant, Licensed Public Accountant

