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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

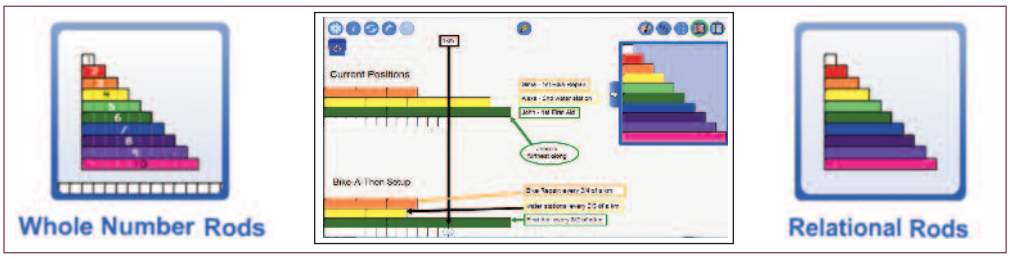
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 54 #3
March 2016

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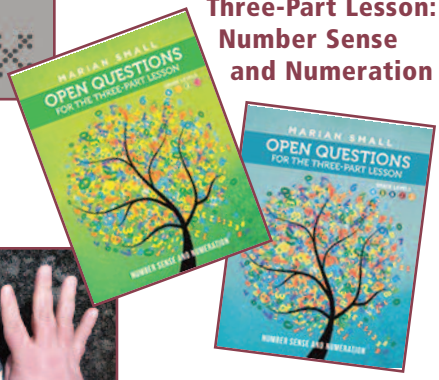
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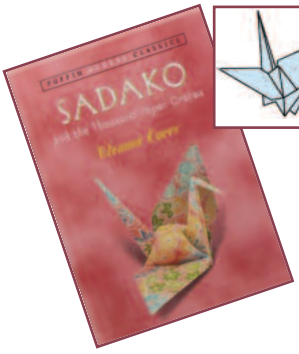
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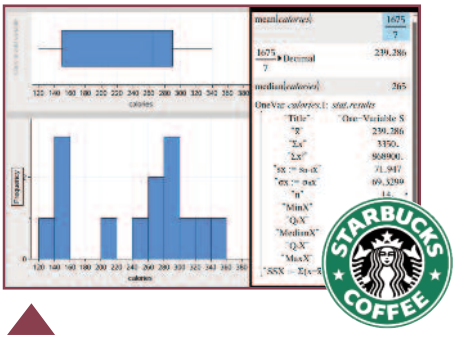


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OAME Wins NCTM Affiliate Journal Publication Award 2016!

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Develop **Geometric** and **Spatial Thinking** in **K–2**

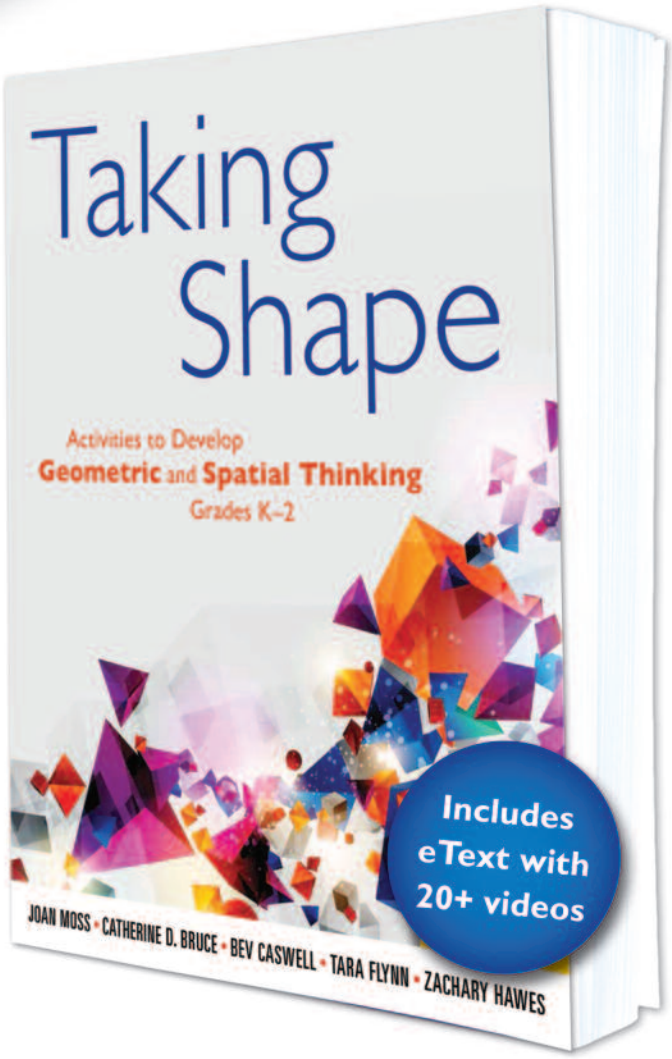
Research suggests spatial reasoning abilities are strongly linked with overall mathematics success

LEARN

- What is **spatial reasoning/thinking**?
- The **importance** of developing spatial reasoning in early childhood
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APPLY

- Easy-to-implement **lesson plans** based on playful pedagogy
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A Professional Learning Service that helps to extend and embed the learning will be available Fall 2016

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Dan Jarvis, along with **separate files for each of the text, graphics, and/or photographs**. Please email all files to Dan Jarvis at **dan.jarvis@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and email address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Editor
▲ **Dan Jarvis**
Email **dan.jarvis@oame.on.ca**

Associate Editors
▲ **Anne Yeager**
Email **anne.yeager@oame.on.ca**
▲ **Marilyn Hurrell**
Email **mhurrell@tbaytel.net**

***Abacus* Co-Editors**
▲ **Mary Lou Kestell**
Email **marylou.kestell@oame.on.ca**
▲ **Kathy Kubota-Zarivnij**
Email **kkz@oame.on.ca**

Copy Editing
▲ **Gitta Berg**
Email **gitta.berg@sympatico.ca**

Design and Production
▲ **Penny Clemens,** Graphic Designer
Email **pennyydezign@gmail.com**

Printing & Binding
▲ **Pole Printing,** Box 69, 1-89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager
▲ **Robert Sherk**
4366 Snider Rd., Verona, ON K0H 2W0
Home (613) 374-1515
Email **robert.sherk@oame.on.ca**

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Message



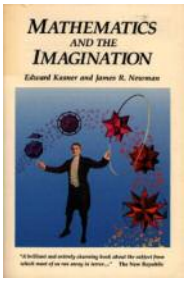
DANIEL H. JARVIS, O.C.T., Ph.D.
EMAIL: dan.jarvis@oame.on.ca
http://faculty.nipissingu.ca/danj/

Dr. Daniel Jarvis is Professor of Graduate and Mathematics Education in the Schulich School of Education at Nipissing University, North Bay, Ontario. His research interests include instructional technology, integrated curricula, and mathematics of the workplace.

Mathematics and mathematics education both possess rich, lengthy, and oft overlapping histories, in which those of us involved in mathematics teaching and math education research continue to play an ongoing part. Indeed, the *Gazette* originally began, and still remains, a publication that celebrates both areas.

Some of you will perhaps be familiar with the work of the late American mathematician, mathematics historian, and New York lawyer, James Roy Newman (1907–1966). During and after World War II, Newman held several positions in the United States government, including Chief Intelligence Officer at the U.S. Embassy in London, Special Assistant to the Undersecretary of War, and Counsel to the U.S. Senate Committee on Atomic Energy (during which time he helped draft the Atomic Energy Act of 1946).

In 1940 Newman co-authored (with Edward Kasner) *Mathematics and the Imagination,* in which they identified the mathematical concept of a very large but finite number



called a "googol" (10^{100}, or a 1 followed by 100 zeroes), a term apparently coined by Kasner's 9-year-old nephew Milton Sirotta when asked to think of a word that would adequately represent such a large number. When asked about an even larger number, the "googolplex" (10 raised to the power of a googol, or a 1 followed by a googol of zeroes), the young boy aptly described it as a "one, followed by writing zeroes until you get tired." More recently, we've seen the addition of an even greater number to the contemporary lexicon, namely the "googolplexian" (10 raised to the power of a googolplex, or a 1 followed by googolplex of zeroes).

The Google Company (note the spelling modification of "googol" that the founders chose for general ease of use) headquarters, which were moved to Mountain View,



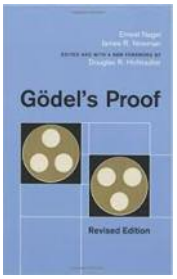
California in 2004, and which looks more like an amusement park than an office complex, is affectionately known by employees as the "Googleplex."

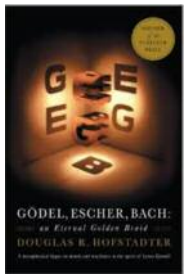
In 1942, Newman wrote *The Tools of War*, which was an illustrated examination of warfare. In 1948, he published *The Control of Atomic Energy*. In 1955, he wrote *What Is Science?*, and in 1956, he published *The World of Mathematics* (attractive original boxed set shown here), a 4-volume series covering many branches of mathematics and representing a 15-year effort by Newman to collect what he felt were the most important essays in the field of mathematics. Newman referred to his expansive collection as "a small library of the literature of mathematics from A'hmose the Scribe to Albert Einstein."



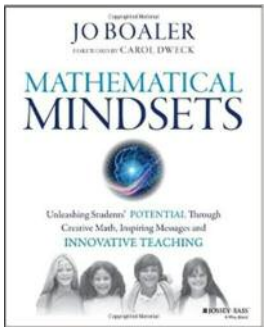
This classic series includes fascinating essays by Descartes (Cartesian coordinates), Whitehead (mathematical logic), Weyl (symmetry), Dedekind (irrational numbers), Heisenberg (uncertainty principle), Turing (computer intelligence), Boole (set theory), Eddington (group theory), Turnball (great mathematicians), Huxley (young Archimedes), Jeans (mathematics of music), Clifford (exactness of mathematical laws), Poincaré (mathematical creation), Birkhoff (mathematics of aesthetics), Newman (Gödel's proof), Russell (mathematical creativity), Von Neumann (*The Mathematician*), Leacock (*Common Sense and the Universe*), Morse/Kimball (*How to Hunt a Submarine*), Warner (*Geometry in the South Pacific*), Carroll (*What the Tortoise Said to Achilles and Other Riddles*), and Hardy (*A Mathematician's Apology*). The set is often praised as being suitable for any level of mathematical skill, and, owing to its popularity, has been reprinted several times, and is now freely and fully available for reading online at the *Internet Archive*.

Newman also wrote *Gödel's Proof* (1958), with co-author Ernest Nagel, presenting the main results of Gödel's incompleteness theorem. This book inspired Douglas Hofstadter to take up the study of mathematical logic, and to write his famous work *Gödel, Escher, Bach: Eternal Golden Braid* (1979). The prolific polymath, Newman, also penned *Science and Sensibility* (1961), *The Rule of Folly* (1962), and *The Harper Encyclopedia of Science* (1963).

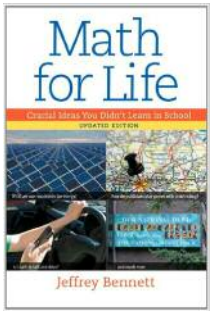




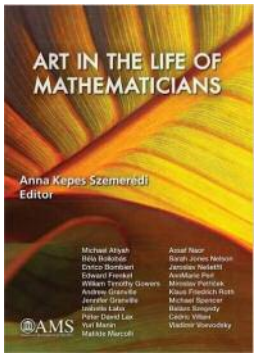
Fast forward six decades, and three other math-related works that I've been enjoying recently are perhaps worth sharing. For the elementary teacher of mathematics, Jo Boaler's newly released book, *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (2016), presents her philosophy of teaching, based on extensive research with children, and dealing with topics such as brain research, growth mindsets, rich mathematical tasks, equity, the power of mistakes and struggle, and assessment.



Secondary school teachers will no doubt be challenged by Bennett's eye-opening text, *Math for Life: Crucial Ideas You Didn't Learn in School* (2012), which includes chapters such as *Statistical Thinking, Managing Your Money, Understanding Taxes, Energy Math, The Mathematics of Growth,* and *Getting 'Good at Math.'* His poignant Chapter 3 epigraph attributed to the "father of Futurism," H.G. Wells, certainly caught my attention: "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write." Wide-ranging and thought provoking, this unique book helps educators to recognize the kinds of practical mathematical perspectives and critical-thinking skills needed in everyday life, and from which any informed citizen can benefit.



Tertiary-level educators (or any math enthusiast who is enamoured by the Newman volumes mentioned above) may find the edited collection by Anna Kepes Szemerédi, *Art in the Life of Mathematicians* (2015), to be of particular interest. In highlighting over 20 colourful stories of mathematicians who are also accomplished artists (e.g., musicians, photographers, painters, dancers, writers, filmmakers), Szemerédi explores the perceived roles that mathematics and the arts have played in the diverse lives of these multi-talented individuals.



The March 2016 issue (Volume 54 Number 3) of the *Gazette* includes three articles, ten regular columns, the

Abacus insert focusing on the elementary panel, along with several other special features.

Betty Long and Deborah Crocker in their article, *The Good Luck Paper Crane: Paper Folding and Mathematics Activities*, provide directions for folding paper cranes, and ideas for mathematics activities using the paper cranes and children's literature. Specifically, the authors highlight the touching story of Sadako Sasaki, the Japanese girl who was exposed to radiation from the atomic bomb dropped on Hiroshima during World War II, and who subsequently attempted to fold 1000 paper cranes in order to obtain her secret wish (Coerr, 2004).

In *Mathematics and Special Education: Two Teachers in One Classroom*, Jacob Speijer, Connie Gray, Sandra Peirce, and Gail Doherty present the results of a board-wide professional learning project, for which the primary goal was to leverage the complementary instructional strengths of the regular classroom teacher (RCT) and the special education resource teacher (SERT) within the classroom context. Anticipated benefits (reduced stigmatization, mathematics program integrity, increased opportunities for individual help, and greater differentiated instruction and assessment) and observed program results are discussed at length, as are recommendations for future implementation of similar programming, based on their co-teaching experiences.

Meth Devendra, building upon one of his previous and related *Gazette* articles (exponential functions and tangents, V54N1), provides algebraic, geometric, and practical examples of *Orthogonal Curves*, i.e., the situation where the tangent lines to two curves at the point of intersection are perpendicular.

Regular columns include the following highlights: OAME President, Tim Sibbald (President's Message) discusses the numeracy–literacy connection, and draws attention to two new books by authors Roberts (regarding Conway's mathematical contributions) and Angus (regarding Attawapiskat and social-justice issues, including education); Todd Romiens (OAME/NCTM Report) addresses the controversy surrounding the American Common Core State Standards for Mathematics (CCSSM), specifically highlighting the mathematical content standards and mathematical practices (process standards); Shawn Godin (What's the Problem?) explores the "nifty number 9," including finger multiplication; Assessment Abby (eponymous) focuses on assessment of the Mathematical Process expectations; and Markus Wolski, Agnes Grafton, Ross Isenegger, Greg Clarke, and Kathleen Corrigan (Provincial Digital Learning Resources) describe the "Whole and Relational Rods" tool, designed to help students explore number concepts, develop reasoning for various operations, and foster proportional reasoning skills.

Ann Kajander (MB4T) looks at teaching and learning strategies for introducing the first two of the four possible cases of dividing integers; Lynda Colgan (Hey, It's Elementary) discusses rich tasks, big ideas, and a classroom activity in which students analyzed Northern Ontario bear family composition statistics as compared with those of humans; Mary Bourassa (Technology Corner) explores Number Talks, introducing two rich online resources created by Tranchmontagne (Number Talk Images) and Nguyen (Math Talk); and Carly Ziniuk (In the Middle) presents the second part of "Starbucks" Math, in which she focuses on data-analysis activities using technology. In the *Abacus* insert, co-editors Mary Lou Kestell and Kathy Kubota-Zarivnij continue their Volume 54 focus on relational thinking, here focusing on multiplication.

Volume 54 Issue 3 also includes several special features: (i) two letters to the editor, one from Louis Lim in which he pays personal tribute to the late Frances Schatz, and a second by Jack Weiner in which he thanks the co-author of the Schatz tribute; (ii) a review of the 2-volume book set entitled *Open Questions for the Three-Part Lesson: Number Sense and Numeration* (2015) by author Marian Small; (iii) three Ontario mathematics education researcher profile highlights; and (iv) further excerpts from Lerman's *Encyclopedia of Mathematics Education* (2014) regarding a variety of contemporary math education issues.

As Editor, I am very pleased to here announce to the OAME membership that the *Ontario Mathematics Gazette* has been awarded the National Council of Teachers of Mathematics (NCTM) Affiliate Member Publication Award 2016 for outstanding journal publication. Founded in 1920, NCTM is the world's largest mathematics education organization, with 80 000 members and more than 230 Affiliates throughout the United States and Canada. This will be the sixth time that the annual Journal Publication Award has been given out, and the first time that a Canadian Affiliate member has won the award. NCTM describes the award criteria as follows:

> The Affiliate Services Committee wishes to recognize the outstanding work of NCTM Affiliates in producing excellent journals (a peer-reviewed periodical in which scholarship relating to mathematics education is published) and newsletters… to keep their membership informed. These publications should keep their membership apprised of the latest happenings in mathematics education, provide great ideas for improving the teaching and learning of mathematics, and provide networking opportunities for members. Judging will be based on content, accessibility, and relevance.

Owing to its unique structure, the *Gazette* represents a truly team effort by a large group of committed mathematics educators and enthusiasts in Ontario. I would specifically like to thank our *Abacus* co-editors, Mary Lou Kestell and Kathy Kubota-Zarivnij; our two Associate Editors, Anne Yeager and Marilyn Hurrell; our copy editor, Gitta Berg; our graphic designer, Penny Clemens; our advertising manager, Robert Sherk; and our printers at Pole Printing. I would be remiss if I did not also recognize the dedicated service of all the past editors of the *Gazette* and *Abacus*; the many contributors of regular columns, articles, and special features; our dedicated team of reviewers; and you, the OAME member, whose ongoing financial support for, and interest in, our journal ultimately made this distinction possible. I am very pleased to be able to accept this award on behalf of the organization at the upcoming NCTM annual conference, "Building a Bridge to Student Success," to be held in San Francisco, California in April 2016.

P.S. Although I am not among those who identify as die-hard "Trekker" fans, I would nonetheless like to correct a mistake that was made in the December issue where I most unforgivably referred to the Star Trek "tricoder," rather than using the proper term, "tricorder" (i.e., an abbreviation of the device's full name, the "TRI-function reCORDER," referring to the device's primary functions of sensing, computing, and recording). How very terrestrial of me.

References

Bennett, J. (2012). *Math for life: Crucial ideas you didn't learn in school.* Boulder, CO: Big Kid Science.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* San Francisco, CA: Jossey-Bass.

Coerr, E. (2004). *Sadako and the thousand paper cranes.* New York, NY: Puffin Books.

Hofstadter, D. (1979). *Gödel, Escher, Bach: Eternal Golden Braid.* New York, NY: Basic Books.

Kasner, E., & Newman, J.R. (1940). *Mathematics and the imagination.* New York, NY: Simon & Schuster.

Lerman, S. (Ed.). (2014). *Encyclopedia of mathematics education.* London, UK: Springer.

Nagel, E., & Newman, J.R. (2008). *Gödel's proof* (Rev. ed.). New York, NY: NYU Press. (Original work published 1923)

Newman, J.R. (Ed.). (1956). *The world of mathematics.* New York, NY: Simon & Schuster.

Small, M. (2015). *Open questions for the three-part lesson: Number Sense and Numeration, Grades K–3.* Oakville, ON: Rubicon.

Small, M. (2015). *Open questions for the three-part lesson: Number Sense and Numeration, Grades 4–8.* Oakville, ON: Rubicon.

Szemerédi, A.K. (2015). *Art in the life of mathematicians.* Providence, RI: AMS. ▲

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Luis Radford, Laurentian University

EMAIL: lradford@laurentian.ca

Projects: Dr. Radford's research interests cover both theoretical and practical aspects of mathematics thinking, teaching, and learning. An over-arching emphasis of his work is a sustained commitment to being involved with classroom teachers and students. As a sociocultural theorist, he draws upon Vygotsky's historical–cultural school of thought and the epistemology of Ilyenkov, and his current work focuses on the elaboration of an approach in which the problem of teaching and learning is formulated around the concept of "alterity" by Lévinas and Bakhtin. In such an approach, he notes that "learning is conceptualized as a social, political, and transformative process through which the students critically encounter other (historical and contemporary) voices and perspectives. The formulation of learning as a cultural and historical process, where knowing and being are mutually constitutive, leads to a non-utilitarian and a non-instrumentalist conception of the classroom and education. Entrenched in unerasable ethical concerns, the classroom appears as a space for the growth of intersubjectivity and the nurturing of what I call the communal self." He has served as Chair of École des sciences de l'éducation de l'Université Laurentienne, and is also the Chair of the International Study Group on the Relations Between the History and Pedagogy of Mathematics. In 2011, Dr. Radford was the recipient of the Hans Freudenthal Medal, awarded by the International Commission on Mathematical Instruction (ICMI), recognizing a major cumulative program of research.

Selected Publications:

Radford, L. (2015). Of love, frustration, and mathematics: A cultural–historical approach to emotions in mathematics teaching and learning. In B. Pepin & B. Rösken-Winter (Eds.), *From beliefs and affect to dynamic systems: (Exploring) a mosaic of relationships and interactions* (pp. 25–49). Advances in Mathematics Education. New York, NY: Springer.

Radford, L., & Sabena, C. (2015). The question of method in a Vygotskian semiotic approach. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 157–182). New York, NY: Springer.

▲ Letters to the Editor



December 31, 2015

Dear Editor

It is with deep sadness to learn of the passing of a great friend and colleague, Mrs. Frances Schatz. The mathematics education community has definitely lost a talented educator. I treasure the time I worked with and learned from Frances.

Frances was the *Abacus* editor and NCTM representative when I first met her on the OAME board of directors in 1997. As a member of the publications committee, I enjoyed my time at the board meetings, and especially enjoyed sitting beside Frances. She was a great supporter when I expressed interest in serving as the *Gazette* editor. We were a great team as *Abacus* and *Gazette* editors, and collaborated on an article to seek submissions for our publications. She cared so much for the organization.

Since I stepped down as *Gazette* editor in 2003, Frances and I remained in contact via our annual Christmas card greetings. I believe we last saw each other face-to-face at the OAME'97 conference, while our last email correspondence was March 6, 2015, when she informed me her granddaughter is attending a school close to my previous school. She was always proud of her family and kept busy with many projects, including being a children's writer. She was a woman of great character and boundless energy.

My 15 years as a mathematics teacher were very happy ones. OAME was a huge part of my professional growth. It is through OAME that I met many wonderful educators. I thank Frances for her friendship and for being a trusted colleague and mentor.

Sincerely,

Louis Lim
OAME *Gazette* Editor, 1999 to 2003
Vice-Principal, Bayview Secondary School,
York Region District School Board
louis.lim1@gmail.com ▲

February 10, 2016

Dear Editor:

While I was correctly listed as an author of the Frances Schatz tribute (OMG, December 2015 issue), I would like it noted that Frances' daughter, Anita Crump, co-wrote the article. We are both very pleased to see that the *Gazette* continues to thrive.

Jack Weiner
OAME *Gazette* Editor, 1992-1996
jweiner@uoguelph.ca ▲

▲ President's Message



TIM SIBBALD
EMAIL: timothys@nipissingu.ca

Numeracy has a place alongside literacy. Nowhere is this clearer than in March! As pi-day (aka pie-day) approaches—this year with accurate rounding! 3/14/16 (March 14, 2016). This also happens to be STEM-able, as it is Einstein's birthday. For an English connection, pi-day is followed by the illustrious "Beware the ides of March" (March 15). One should be wary of the ides this year because of the pie that one consumed celebrating mathematics the previous day. If you are teaching calculus, you could goad your students with talk of celebrating "e" day. E day would be 2.7, or February 7th, but this has now passed. With a little stretch of the imagination, have students consider 2.71, or when the 71st day of February would be (i.e., 71 days after January 31st). You can always round it if you would rather celebrate on Tuesday, April 12 instead. For non-calculus classes, perhaps 4/8/16, April 8, is a day for a geometric sequence party. The message here is clear—that there is a time to simply have fun with our beloved subject. It may be whimsical, but sometimes that is the best way to connect with students.

I would like to acknowledge that the *Gazette* and *Abacus* have won the NCTM Affiliate Recognition of Outstanding Publication Award. This reflects the hard work of our editors: Dan Jarvis, Mary Lou Kestell, and Kathy Kubota-Zarivnij. It speaks to the quality of editorial work that they do. It also acknowledges the work of the behind-the-scenes production team, the reviewers of articles, and the ongoing supply of articles that our membership provides. The *Gazette* and *Abacus* are a cornerstone of our organization and it is wonderful that they have been acknowledged with this award.

Recently I read a pair of books that juxtaposed two sides of education. The spirited, intriguing side of mathematics is brought to light in Siobhan Roberts' latest book (Roberts, 2015). For me, personally, this was a chance to reminisce about many mathematical topics that arose in my formative years. I had not been aware of the close ties between Conway and Martin Gardner (who wrote a regular column in *Scientific American* that is well worth seeking out). Nor was I aware that Conway's "Game of Life" led to Wolfram and his associates devising a "game" in the same style, called Rule-110. These are not games in the usual sense,

and are better thought of as a geometric starting point and iterative rules to make the next geometric pattern in the sequence. What I found intriguing about the Game of Life, in my youth, was the vast variety of behaviours that can arise. It always seemed to have a biological or ecological flavour as patterns evolved. Roberts' writing allowed me to revisit this and see how Rule-110 was proved to be a universal computer—it can be set up to perform a calculation through iteration.

The Collatz Conjecture

Pick a number and use it to begin a sequence where: Rule 1. If the current value (N) is odd, the next value is 3N+1. Rule 2. If the current value (N) is even, the next value is N/2.

Example: 23, 70, 35, 106, 53,…

The conjecture is that regardless of the number you pick, the sequence will reach one.

I could go on about Roberts' latest book because there is such a rich vein of mathematics in which Conway was involved, such as the discoveries of sporadic groups with mind-boggling sizes, not to mention Conway's questioning whether the Collatz conjecture is solvable. Suffice it to say that the book is very readable and filled with nuggets.

On the challenging side of education issues, Charlie Angus' latest book (Angus, 2015) provides insight into the issues that Attawapiskat has faced, with an emphasis on the last decade. While this book does not specifically deal with mathematics education, it does address the fundamental need for quality and stability as a foundation for effective education programs. While Charlie Angus is a political figure, the book is overwhelming in terms of evidence that exceeds simple political rhetoric. It is an interesting read because it challenges the assumption that we have equitable, or near equitable, education in the province. There is also a brief mention of EQAO data that adds clear evidence of inequity because it is disingenuous to consider anything less than an equitable desire of all parents to want the best for their children. Overall, the book points to a very clear aspect of education that one could use to examine social justice within our province. In this respect, the book opens the door on an opportunity to develop some math topics with an Ontario-based social-justice theme.

It is that time of year, when chapters are running OMO events for Grade 8 students. This contest holds a distinction in my mind for being principled on gender equity and development of teamwork in mathematics. The efforts of all involved are to be applauded because of the larger message that the event carries. That is not to diminish other competitions, but it is to acknowledge that the OAME has embedded its own principles in the approach. This is also the last *Gazette* edition before the annual conference, "Leap Into Math," which is being organized by MAC^2 in Barrie. I would like to wish the organizers every success and encourage members to check the details at **www.oame2016.ca**.

References

Angus, C. (2015). *Children of the broken treaty: Canada's lost promise and one girl's dream.* Regina, SK: University of Regina Press.

Roberts, S. (2015). *Genius at play: The curious mind of John Horton Conway.* New York, NY: Bloomsbury Publishing. ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Inquiry-based mathematics education (IBME) refers to a student-centered paradigm of teaching mathematics and science, in which students are invited to work in ways similar to how mathematicians and scientists work. This means they have to observe phenomena, ask questions, look for mathematical and scientific ways of how to answer these questions (like carrying out experiments, systematically controlling variables, drawing diagrams, calculating, looking for patterns and relationships, and making conjectures and generalizations), interpret and evaluate their solutions, and communicate and discuss their solutions effectively. The role of the teacher in such a setting is different to traditional teaching approaches . . . [T]he teacher's role includes making constructive use of students' prior knowledge, challenging students through effective, probing questions, managing small group and whole class discussions, encouraging the discussion of alternative viewpoints, and helping students to make connections between their ideas."

Dorier, J.-L., & Maass, K. (2014). Inquiry-based mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 300–304). London, UK: Springer.

▲ Ask Assessment Abby: Assessing Math Processes



ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A^3 is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I am looking for clarification with respect to the mathematical processes. More specifically, how might I assess (report on) the process expectations? How do I embed the processes and the Achievement Chart in my evaluation?

Help me with Mathematical Processes
Connecting Processes and Assessment

While the Mathematical Processes expectations are unlike Overall Expectations insofar as they do not require reporting, they do constitute the essential skills of mathematics. Since the Math Process expectations are the only expectations that appear, basically unchanged, throughout the Ontario Curriculum Mathematics program Grades 1–12, I agree with your desire to track a student's progress through these skills.

Assessing mathematical processes can be a complex process, so it is no wonder you have questions. The complexity was highlighted in the *Gazette* more than ten years ago by Dr. Christine Suurtamm (2004), who suggested that, as some processes may be overlooked by paper-and-pencil tasks, **using a variety of alternative strategies is valuable. These can include rich tasks, observations, and conversations, as well as effectively employing anecdotal feedback.** These suggestions align with our current focus in curriculum documents and *Growing Success*.

So when we use a variety of strategies in assessment, how might we assess and report the process expectations? The Achievement Chart opens up different possibilities for integrating the processes, particularly in the *Thinking, Application,* and *Communication* categories. With the Thinking category, for instance, we may assess critical- and creative-thinking skills through problem-solving activities, in which students reflect (on the reasonableness of a solution), select appropriate tools and computational strategies, reason and prove by justifying their thinking, make connections between concepts and procedures, and represent and communicate their thinking. I would encourage you to check out Ministry-developed resources at the EduGAINS website for ideas, including a generic rubric that integrates the processes and Achievement Chart categories. At this site, you can also find a document for integrating assessment and the processes, which includes look fors, sample questions, and sample feedback to consider when integrating mathematical processes. Further resources are available on this site for assessment as and for learning.

Keep Math Rich with Students in Mind,
Assessment Abby

If you have any questions or comments for Assessment Abby: **assessmentabby@oame.on.ca**

Growing Success: **www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf**

EduGAINS – Math Processes: **www.edugains.ca/newsite/math/mathprocesses.html**

Suurtamm, C. (2004). Assessment corner: Assessing mathematical processes – a complex process. *Ontario Mathematics Gazette, 43*(1), 7–9. Retrieved from **www.oame.on.ca/main/files/gazettefiles/SelectedGazArt73.pdf** ▲

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Douglas McDougall, Ontario Institute for Studies in Education, University of Toronto

EMAIL: doug.mcdougall@utoronto.ca

Projects: Dr. McDougall's research program focuses on mathematics teaching and learning, the professional learning of teachers, technology-supported learning, and academic leadership. Doug currently serves as Associate Dean, Programs (2015–present) in the Department of Curriculum, Teaching and Learning at OISE/UT.

Selected Publications:

Stoilescu, D., McDougall, D., & Egadawatte, G. (2015). Teachers' views of the challenges of teaching grade 9 applied mathematics in Toronto schools. *Educational Research for Policy and Practice, 15*(2), 1–15.

Martinovic, D., McDougall, D.E., & Karadag, Z. (Eds.) (2012). *Technology in mathematics education: Contemporary issues.* Santa Rosa, CA: Informing Science Institute.

Provincial Digital Learning Resources – What's New? Whole and Relational Rods (mathies.ca)





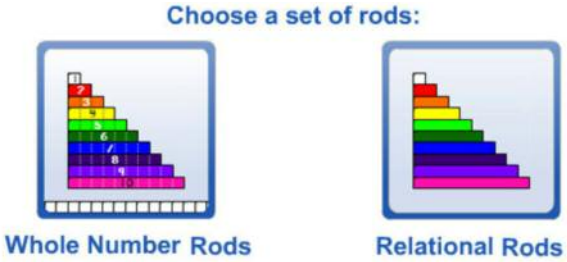


MARKUS WOLSKI
EMAIL: markus.wolski@gmail.com
AGNES GRAFTON
EMAIL: agrafton@bhncdsb.ca
ROSS ISENEGGER
EMAIL: ross@isenegger.ca
GREG CLARKE
EMAIL: gpclarke@smcdsb.on.ca
KATHLEEN CORRIGAN
EMAIL: kcor@rogers.com

*Greg (Simcoe Muskoka Catholic DSB), Ross (Near North DSB), Agnes (Brant Haldimand Norfolk Catholic DSB), and Markus (Bluewater DSB) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS and **mathies.ca** projects. Kathleen (formerly with Simcoe County DSB) also consults on these Ministry projects.*

If you have visited the tools section of **mathies.ca**, you may have seen both the Relational and Whole Number Rod tool available for desktop computers. We are currently in the process of updating these tools and are hoping that by the time this article appears, they will also be available as free mobile apps for both Android and iOS devices. If not, rest assured they are coming very soon!

Starting with the Tool



Relational Rods are rectangles of various lengths, where each rod can be evenly subdivided by the smallest square rod.

Whole Number Rods use the same rectangular lengths as Relational Rods, but numeric labels and dashed divisions can be seen on each rod.

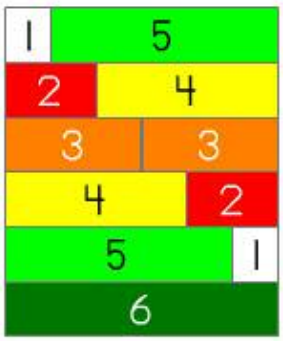
Physical versions of these tools have been used in classrooms for years because of the powerful way they support the exploration of number concepts, help develop reasoning for various operations, and foster proportional reasoning skills.

Recently both physical and virtual versions of Relational Rods were featured as part of the "Paying Attention to Spatial Reasoning" series (**www.edu.gov.on.ca/eng/literacynumeracy/LNSPayingAttention.pdf**).The following video on Composing and Decomposing Numbers (**http://learnteachlead.ca/videos/composing-and-decomposing-overall-description/**) shows both the concrete and virtual tools in action.

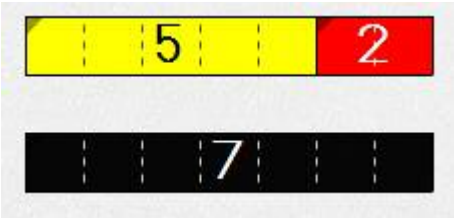
Representing Numbers and Concepts

There are many ways that numbers and concepts can be represented using Relational and Whole Number Rods.

1. Compose and decompose numbers. (Below is the decomposition of 6.)

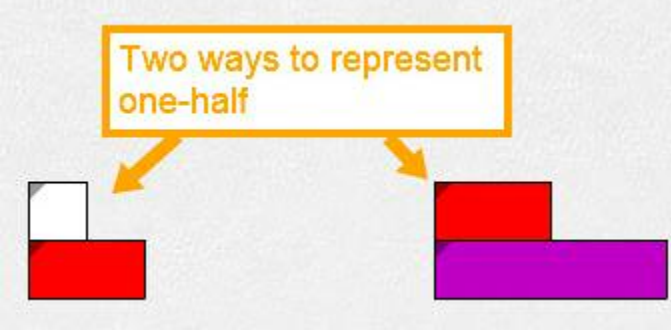


2. Line rods up in "trains" to visually model adding and subtracting.

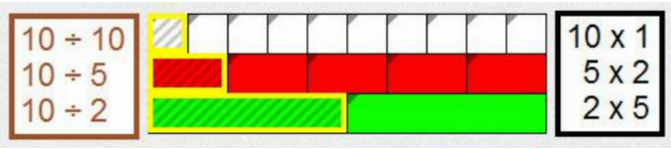


3. Create fractions dynamically, using any base as a whole.

   The model on the left uses the red rod as the whole, while the model on the left (see below, on next page) use a purple rod. This flexibility in proportional reasoning is important to understanding fractions.



4. Explore factors and multiples.



How might the picture show each product and quotient?

How else might the picture be interpreted?

The above examples are just a few of the ways you can visually represent mathematical concepts and reinforce particular operations. For additional ideas about how to use relational rods, visit **www.edugains.ca/resources/LearningMaterials/ManipulativesSupport/TipSheets/Manipulatives_RelationalRods.pdf**.

Customizing the Tool

The buttons at the top of the workspace allow you to customize the tool in many ways. Here is a list of the buttons and what they allow you to do:



Settings
Allows the hiding and showing of the unit "ruler" at the bottom of the workspace and also lets you reset all settings to the mode in which you started.



Information
Access a link to a wiki that contains examples and more information, as well as copyright information.



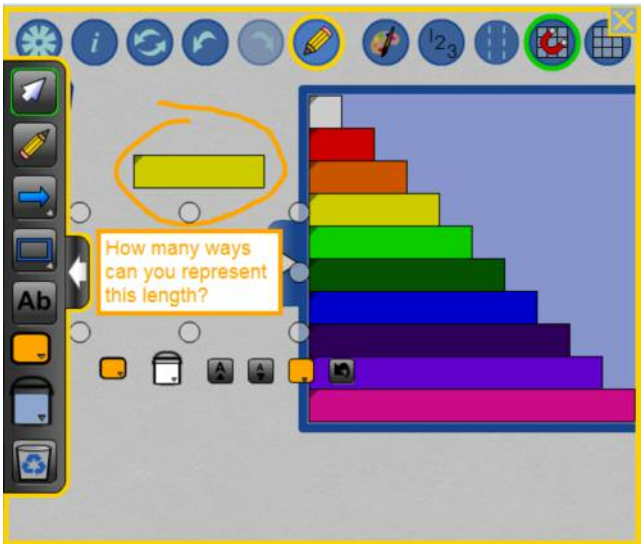
Reset
Delete all your work and go back to the starting screen.



Undo/Redo
Step backwards or forwards through your actions with the tool.

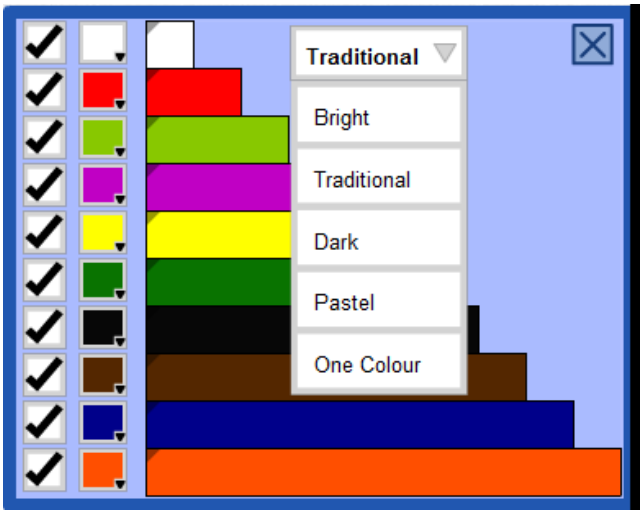


Annotation Tool
Enter "annotation mode," which allows the user to use drawing, text boxes, and shapes to highlight parts of the tool, explain thinking, or record information.





Colour Palette
Set the colour of individual rods or the colour scheme for the entire set of rods. The "traditional" palette allows the digital tool to more closely match the colours of the concrete manipulatives used in classrooms. It is also possible to hide some of the rods in order to work with a restricted set.





Numbered Labels
Show or hide the numerical labels on the rods.



Dashed Separations
Show or hide the dashed separations on the rods.



Snap to Grid
This mode is on by default and will line up any dragged rod with the edges of other rods. Turning this off allows the free placement of rods.



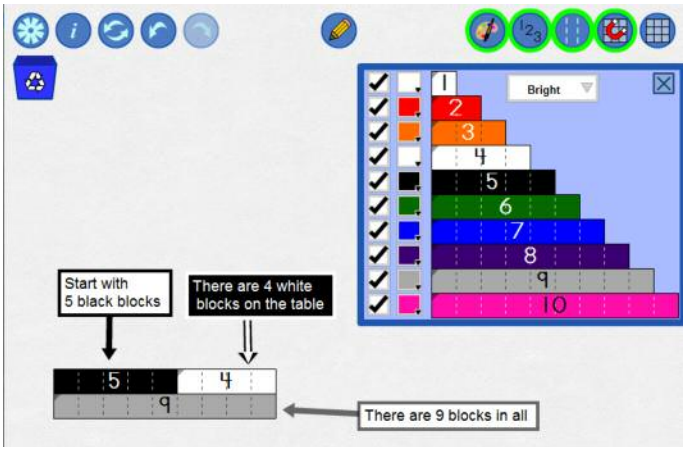
Grid
Show or hide a grid in the background.

Example Problems

Here are some examples that use the Relational Rods to solve the following problems:

Whole Numbers

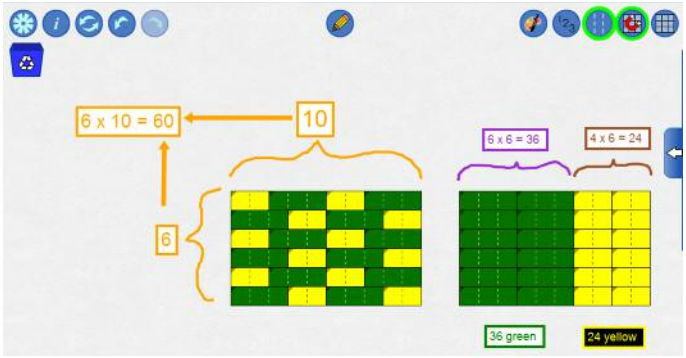
There are 9 building blocks on a table. Some are black and some are white. If 5 are black, how many white building blocks are there on the table?



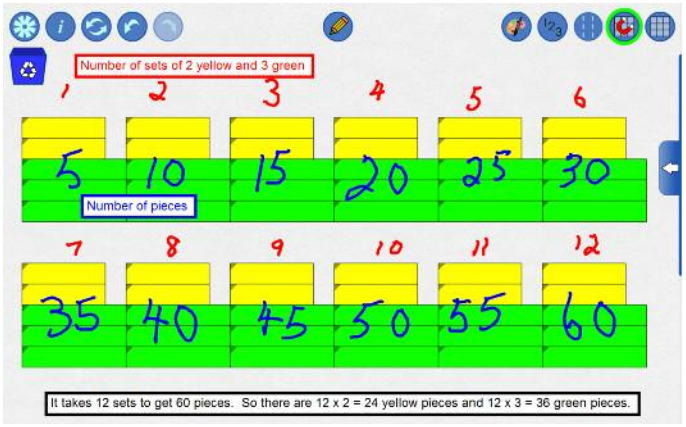
Ratios of Area

Paul's grandmother is making a quilt. For every 2 yellow pieces in the quilt, there are 3 green pieces. If there are 60 pieces in the quilt, how many are yellow and how many are green?

Solution 1:



Solution 2:

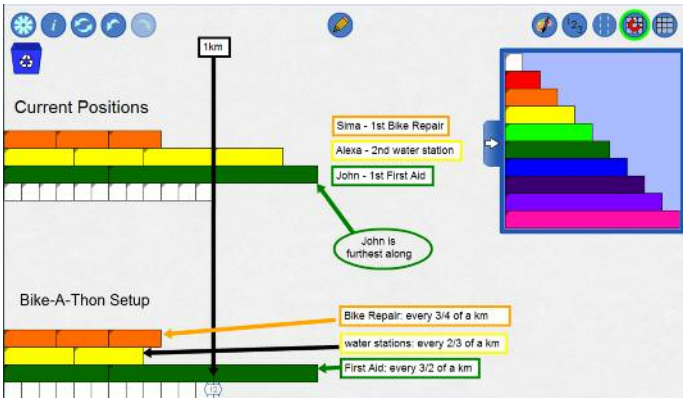


Comparing Fractions

Students are participating in a bike-a-thon to raise money for cancer. To help the participants have a successful bike-a-thon, the organizers have set up stations along the course.

- Cyclists will find water stations every two-thirds of a kilometre.
- There will be a first-aid station every three-halves of a kilometre.
- Bike repair stations will also be available on the course every three-fourths of a kilometre.

John has reached the first first-aid station.
Alexa is at the second water station.
Sima is at the first bike-repair station.
Who is furthest along the course?



Feedback and Future Requests

If you end up using the Relational Rods on any of the available platforms, please feel free to send us your feedback, using the Feedback Form button inside the Information Dialog. You can also send your comments to WhatsNew@oame.on.ca. You can share worked examples on Twitter, using the hashtag #ONmathies. To be among the first to find out about latest digital tool developments, sign up for our email list at **http://mathclips.ca/WhatsNewEmailList.html**. ▲

▲ MB4T (Mathematics by and for Teachers): Dividing Integers (Part 1)



ANN KAJANDER
EMAIL: ann.kajander@lakeheadu.ca

Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, have written and published a new book for teachers called Mathematical Models for Teaching: Reasoning without Memorization.

As with integer products, the signs of integer quotients can be "told" to students via rules. However, this is to be avoided, as all too often, students wrongly generalize such rules to other calculations, for example, saying that -3 – 5 must be 8 because "two negatives make a positive." Alternately, many of these rules are quite straightforward to generate with understanding, using manipulatives, and this method, as discussed in previous columns, also gives students something to fall back on if they forget the rules.

There are four possible combinations of positive/negative numbers when dividing. The positive divided by a positive situation doesn't involve any negatives, so really there are only three new cases of integer division. Two of these cases will be discussed here, leaving the more difficult one (positive divided by a negative) for the next issue. Of course, the need for integer-division development could also be avoided completely by rewriting a question such as (– 6) ÷ 2 as (– 6) × ½, and using the rules for multiplication as established in the previous column. However, we will resist this temptation here—although, of course, the integer-division outcomes can be double-checked to align with fraction-multiplication outcomes.

Models of Division

The most important background students need to understand integer division is a conceptual understanding of whole-number division. In particular, students need to know the partitive, or "equal sharing," model of division, as well as the measurement, or "repeated subtraction" model. For example, to model the sharing of 6 candies between 2 people, we might use counters to show the role of the 2 as "splitting" the 6 into 2 parts as shown to follow, which is the partitive model of division.



On the other hand, if the context is a pile of 6 matching socks that we wish to pair up, then the measurement model makes more sense:



Here, the role of the 2 is that we have sets or groups of 2.

These two interpretations of division allow the development of the following two cases to be done quite easily, using coloured counters.

Negative Divided by a Positive

A group of negative counters can be easily divided by a positive number, using the partitive interpretation of division. For example, (-6) ÷ 2 can be thought of as "6 negative counters split into 2 equal groups." Other than the fact that the counters are now negative values, the operation of division here is completely familiar from whole-number methods.

Negative Divided by a Negative

This case sounds hard initially, but is also straightforward as long as we choose the right division interpretation. For example, the question (-6) ÷ (-2) can be thought of as "how many sets of 2 negatives are in a group of 6 negatives," which is the measurement or repeated subtraction idea. We can even use a sort of repeated addition concept, in which we start from zero and count up how many groups of (-2) we need to get to (-6). Since we are adding up these groups of (-2) to reach 6, it makes sense that the answer is 3. Either way, it's clear we need 3 sets; in other words, the outcome is positive. Here is a model:

Start with 6 negative counters.

Now count groups of 2 negatives.

Since there are 3 such groups of (-2), the answer is 3. Note that this is positive 3.

Other examples can be provided until it is clear that these sign results always work in the same way.

Thinking Ahead

The final case, that of a positive number divided by a negative number, is the hardest to establish with models and reasoning. In the next column, several possible methods will be offered. The reader is invited to think about this case between now and then! ▲

Orthogonal Curves



METH DEVENDRA
EMAIL: meth_devendra@yahoo.com

Meth Devendra is an Ontario-certified teacher working as the Program Chair of the Math & Science Department at Boren Sino Canadian School in Jiangmen City, China. He holds a PhD in mathematics from the University of Dundee, Scotland.

In this article, we look for orthogonality of two curves. We are looking for the tangent lines at the point of intersection to two curves to be perpendicular. This is a follow-up of an earlier article, where we searched for parallel tangent lines when two curves touch. See "Exponential Functions and Tangents," *Ontario Mathematics Gazette*, Vol. 54 #1, September 2015.

The question, then, is to ask why teach intersection of two curves? In relation to this article, why are we particularly interested in the orthogonal (perpendicular) intersection of two curves? In real life, there are many situations that require us to look for orthogonal intersections. If you are studying a map or when you are driving on a road, you may see a road going right across. If you are a welder or a carpenter, you may want to connect two structures at 90°. If you are building a fence or bricklaying a wall, you will be interested in orthogonal intersections. In a math classroom, we try to simulate these life experiences in a safe environment and add a content-rich experience. We use pencil and paper and technology to understand orthogonal intersection of two curves. We will consider several cases of intersections.

Case 1: Power Function

Main result: Let us consider functions $f(x) = x^n$ and $f(x) = -x^n + K$, where $n(\geq 2)$ is an integer and a K parameter. Then the two curves $y_1 = x^n$ and $y_2 = -x^n + K$ intersect orthogonally at $\left(\frac{1}{n^{\frac{1}{n-1}}}, \frac{1}{n^{\frac{n}{n-1}}}\right)$ when $K = \frac{2}{n^{\frac{n}{n-1}}}$.

Let us suppose that the two curves intersect when $x = x_0$. Then $x_0^n = -x_0^n + K$, giving $2x_0^n = K$.

Let us compute the slope values next. Throughout this article, m_1 will be the slope value of the first curve y_1, and m_2 will be the slope value of the second curve y_2.

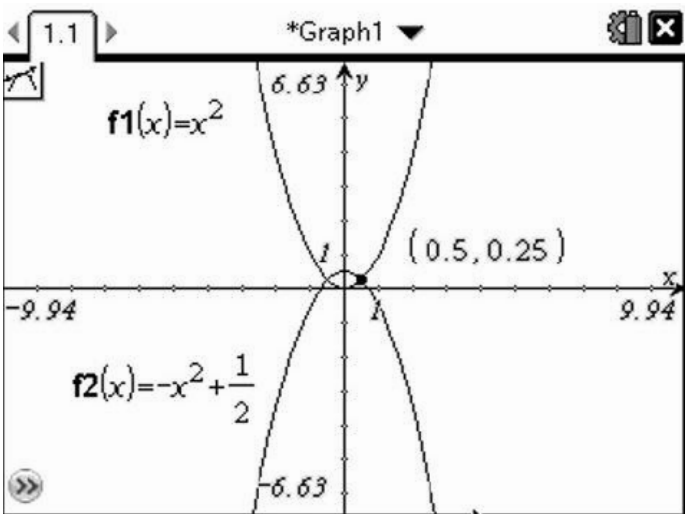
Differentiation gives $\frac{dy_1}{dx} = n\,x^{n-1}$; $m_1 = n\,x_0^{n-1}$ and $\frac{dy_2}{dx} = -n\,x^{n-1}$; $m_2 = -n\,x_0^{n-1}$.

Now for orthogonality, we use the property that $m_1\,m_2 = -1$ to get the condition $-n^2x_0^{2n-2} = -1$. Solving this equation, we get $x_0^{n-1} = \frac{1}{n}$, and $x_0 = \sqrt[n-1]{\frac{1}{n}} = \frac{1}{n^{\frac{1}{n-1}}}$.

We next use this value to determine the value of the parameter: $K = 2\,x_0^n = 2\,\frac{1}{n^{\left(\frac{n}{n-1}\right)}}$.

Then the two curves $y_1 = x^n$ and $y_2 = -x^n + K$ intersect orthogonally at $\left(\frac{1}{n^{\frac{1}{n-1}}}, \frac{1}{n^{\frac{n}{n-1}}}\right)$ when $K = \frac{2}{n^{\frac{n}{n-1}}}$.

Example: Consider the two parabolas given by $y = x^2$ and $y = -x^2 + K$. Then we can use the above result and $n = 2$ to find the orthogonal intersection at $\left(\frac{1}{2}, \frac{1}{4}\right)$ when $K = \frac{1}{2}$.



Case 2: Exponential Functions

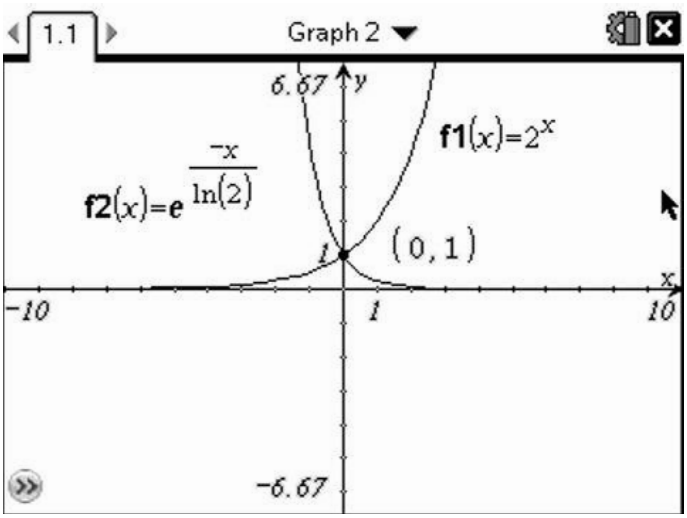
Main result: Let us consider two exponential functions $f(x) = a^x$ and $f(x) = b^x$ with $a,b > 0$, $(\neq 1)$. Then the two curves $y_1 = a^x$ and $y_2 = b^x$ intersect orthogonally at (0,1) when $(\ln a)\,(\ln b) = -1$.

Differentiation gives $\frac{dy_1}{dx} = a^x \ln a$; $m_1 = \ln a$ and $\frac{dy_2}{dx} = b^x \ln b$; $m_2 = \ln b$.

Now for orthogonality, we use the property that $m_1\,m_2 = -1$ to get the condition $(\ln a)\,(\ln b) = -1$.

Example: $y_1 = 2^x$ and $y_2 = b^x$. We use $a = 2$ and the condition $(\ln a)\,(\ln b) = -1$ to find the value of b. Then, $(\ln 2)(\ln b) = -1$ gives $b = e^{\frac{-1}{\ln 2}} = e^{-\log_2 e} = 2^{-(\log_2 e)^2} \approx 0.23629$. Thus, the two exponential curves $y_1 = 2^x$ and $y_2 = e^{\frac{-x}{\ln 2}}$ intersect orthogonally at (0,1).

We can verify at $x = 0$, $m_1 = \ln 2$ and $m_2 = \frac{-1}{\ln 2}$, giving $m_1 m_2 = -1$.



Case 3: Exponential Function and Power Function

Main result: Let us next consider an exponential function $f(x) = b^x$, $b > 0$, $(\neq 1)$ and a power function $f(x) = a x^n$. Then the two curves $y_1 = b^x$ and $y_2 = a x^n$ intersect orthogonally at the solution of the equation $(n \ln b)\, b^{2x} + x = 0$. Let x_0 be the solution to the above equation, then $a = \frac{b^{x_0}}{x_0^{\,n}}$.

For orthogonality, we consider $m_1 m_2 = -1$. Differentiation gives $\frac{dy_1}{dx} = \ln b\, b^x$, and $\frac{dy_2}{dx} = nax^{n-1}$.

Then $(b^x \ln b)\,(nax^{n-1}) = -1$.

For the intersection, we set $b^x = a x^n$.

We eliminate parameter a from the above two equations, to get $(b^x \ln b)\left(n\frac{b^x}{x}\right) = -1$, to get $(n \ln b)\, b^{2x} + x = 0$. Finally, if x_0 is the solution to the above equation, then $a = \frac{b^{x_0}}{x_0^{\,n}}$.

Example: $y_1 = e^x$ and $y_2 = a x^3$.

Then for orthogonality, $(e^x)\,(3ax^2) = -1$.

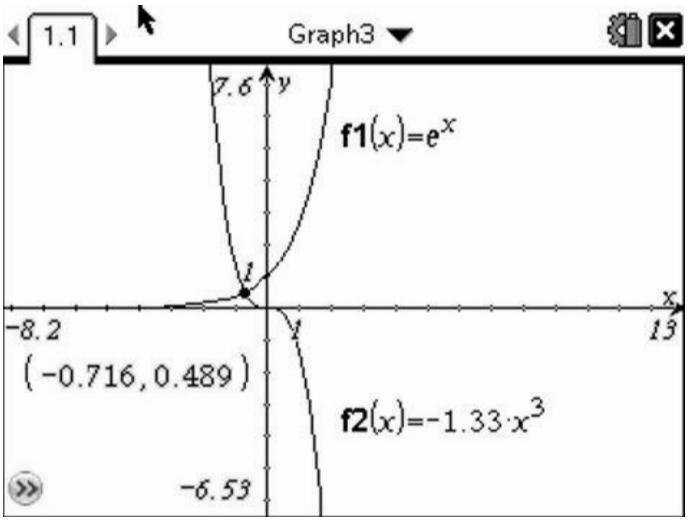
For the intersection, we set $e^x = a x^3$.

Then eliminating parameter a we get $3\, e^{2x} + x = 0$.

Let x_0 be the solution to the above equation. The closed-form solution of this equation uses Product Log function, which is beyond the scope of this article. We find a numerical approximation to get $x_0 \approx -0.7162024$. Then $a = \frac{e^{x_0}}{x_0^{\,3}} \approx -1.329994772$.

We can estimate at $x = -0.7162024$, $m_1 = e^{-0.7162024} = 0.48860426068802$, and $m_2 = 3(-1.329994772)(-0.7162024)^2 = -2.0466460072422$, and giving $m_1 m_2 \approx -1$.

Then the two curves $y_1 = e^x$ and $y_2 = a x^3$ intersect orthogonally at $x_0 \approx -0.7162024$ when $a \approx -1.329994772$.



Case 4: Logarithmic Function and Power Function

Main result: Let us next consider a logarithmic function $f(x) = \log_b x$, $b > 0$ $(\neq 1)$ and a power function $f(x) = a x^n$. Then the two curves $y_1 = \log_b x$, and $y_2 = a x^n$ intersect orthogonally at the solution of the equation $n \ln x + (\ln b)^2 x^2 = 0$. Let x_0 be the solution to the above equation, then $a = \frac{-\ln b}{n x_0^{\,n-2}}$.

For orthogonality, we consider the product of derivatives to be equal to –1.

Differentiation gives $\frac{dy_1}{dx} = \frac{1}{x \ln b}$ and $\frac{dy_2}{dx} = nax^{n-1}$.

Then $\left(\frac{1}{x \ln b}\right)(nax^{n-1}) = -1$.

For the intersection, we set $\log_b x = a x^n$.

We eliminate parameter a from above the two equations to get $\left(\frac{1}{x \ln b}\right)\left(n\frac{\log_b x}{x^n}x^{n-1}\right) = -1$, to get $n \log_b x + x^2 \ln b = 0$.

Changing base, we get the equation $n \ln x + (\ln b)^2 x^2 = 0$. Let us set x_0 as the solution to the above equation, then $a = \frac{-\ln b}{n x_0^{\,n-2}}$.

Example: $y_1 = \ln x$ and $y_2 = a x^2$.

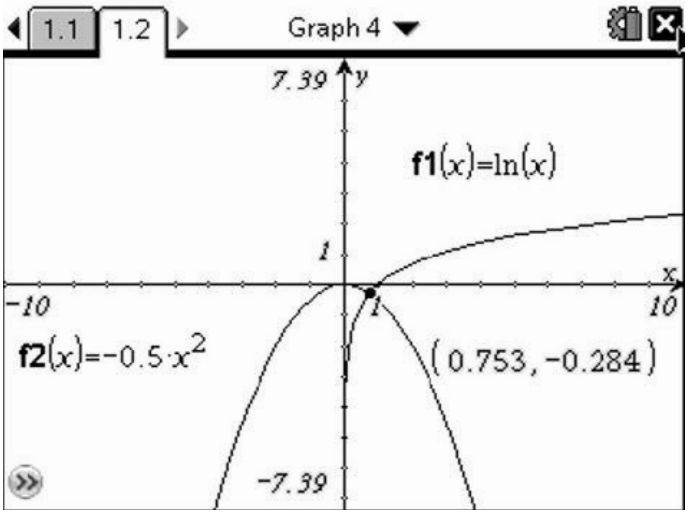
Then for orthogonality, $\left(\frac{1}{x}\right)(2ax) = -1$, gives $a = \frac{-1}{2}$.

For the intersection, $\ln x = \frac{-1}{2}x^2$ we get $2 \ln x + x^2 = 0$.

Let x_0 be the solution to the above equation. We find a numerical approximation to get $x_0 \approx 0.75308916497967$. Then $a = \frac{-1}{2}$.

We can estimate x = 0.75308916497967, $m_1 = \frac{1}{0.75308916497967}$ = 1.3278643028912, and $m_2 = 2\left(\frac{-1}{2}\right)$ 0.75308916497967 = – 0.75308916497967, giving $m_1\, m_2 \approx -1$.

Then the two curves $y_1 = \ln x$ and $y_2 = \frac{-1}{2}x^2$ intersect orthogonally at $x_0 \approx 0.75308916497967$.



Case 5: Two Logarithmic Functions

Main result: Let us consider two logarithmic functions $f(x) = \log_a x$ and $f(x) = \log_b x$, with $a,b > 0$, ($\neq 1$). Then the two curves $y_1 = \log_a x$ and $y_2 = \log_b x$ intersect orthogonally at (1, 0) when (ln a)(ln b) = –1.

Since both curves pass through the point (1, 0), we have intersection, and we need to check orthogonality at $x = 1$ Let us compute the slope values at the point of intersection.

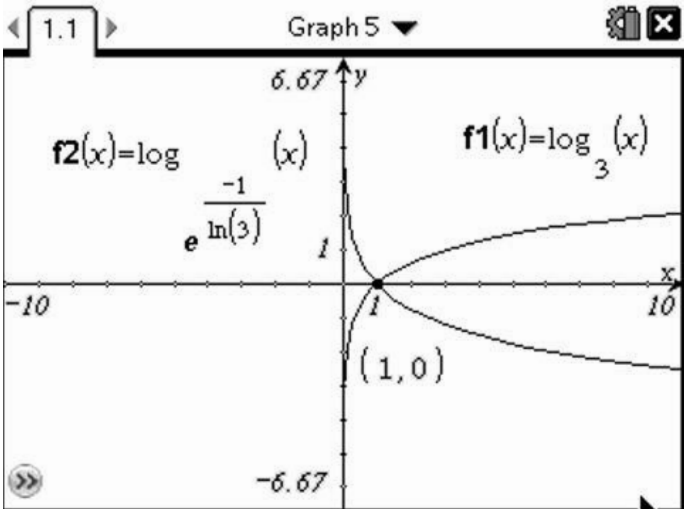
Differentiation gives $\frac{dy_1}{dx} = \frac{1}{x \ln a}$, $m_1 = \frac{1}{\ln a}$ and $\frac{dy_2}{dx} = \frac{1}{x \ln b}$, $m_2 = \frac{1}{\ln b}$.

Now for orthogonality, we use the property that $m_1\, m_2 = -1$ to get the condition (ln a)(ln b) = –1.

Example: $y_1 = \log_3 x$ and $y_2 = \log_b x$. We use $a = 3$ and the condition (ln a)(ln b) = –1 to find the value of b. Then (ln 3)(ln b) = –1 gives $b = e^{\frac{-1}{\ln 3}}$. Thus, the two logarithmic curves $y_1 = \log_3 x$ and $y_2 = \log_{e^{\frac{-1}{\ln 3}}} x$ intersect orthogonally at (1, 0).

We can verify at x = 1, $m_1 = \frac{1}{\ln 3}$ and m_2 = –ln 3, giving $m_1\, m_2 = -1$.

> *"I had this rare privilege of being able to pursue in my adult life, what had been my childhood dream."* Andrew Wiles



It is possible to use the technique illustrated here to find orthogonality of other classes of functions, and we leave it to the reader to enjoy making more connections. We can also encourage our students to find the angle at intersection, using $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$. All these cases arise out of my students' curiosity, and as teachers, we should encourage our students to ask questions and explore extensions beyond textbook material. ▲

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▲ In the Middle: Starbucks Math (Part 2)



CARLY ZINIUK
EMAIL: carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and AP Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very involved in using real-life data to engage her students in solving problems.

This is the second piece of a two-part activity looking at the math of Starbucks. This is primarily a data analysis to connect students to the experience of looking at data, using technology. The solutions presented used TI-NSpire and TI-84+, but the activity can be easily adapted to have students use many different technologies (web-based online included) or even by hand.

The primary learning goal for this activity is to identify the relationships between ingredients in their "favourite" beverages. Since many middle school students consider Frappuccinos from Starbucks to be a treat, and not really something they need to think about in terms of their nutrition, this is actually helpful to have them explore, using mathematics.

The activity starts with a collection of the data from the Starbucks Canada website and a comparison to what is available on the U.S. site. It also has students investigate the daily recommended intake for a teenager for each nutrient, and what a Frappuccino's percent of that daily recommended intake would be.

This activity extends heavily into the content of the MPM1D and then even into the MDM4U course, but can be readily adapted to solely Grades 7 and 8 by removing some specific content at the latter questions. The research questions at the end provide great extension and enrichment opportunities for students to explore these topics, which they may not even touch on until Grade 12. Some middle school students love thinking about themselves doing Grade 12 content—and why not!

1. Why do you think this information is required to be available to the public?
2. Under what circumstances might the actual amount of each ingredient (in grams or milligrams or kilocalories) really matter?
3. What do you notice is different between the two presentations of information? Why do you think they differ?
4. With your partner, select a single Frappuccino that you might be interested in trying (or one you buy regularly). Compare the nutritional information for your choice in both the American and the Canadian sites. Is there any difference? Why do you think the differences occur?
5. Using your two websites find ten Frappuccinos that are available in both Canada and the United States. Both you and your partner should each complete the following chart for the ten Frappuccino types. Make sure you have all the characteristics (e.g., whipped cream, milk type where possible, size of drink) the same for all options, so you can compare them fairly.

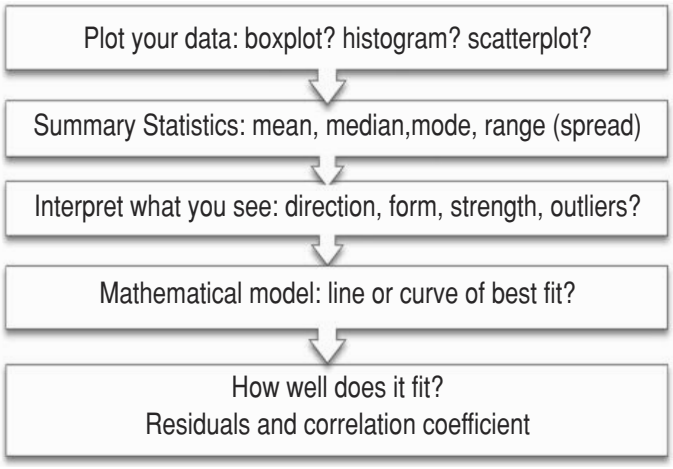
Frappuccino Type	Total Energy (kcal)	Fat (g)	Carbohydrates (g)	Fibre (g)	Sodium (mg)	Protein (g)	Caffeine mg)

A *scatterplot* is a graph that shows the relationship between two sets of numeric data. The points in a scatterplot often show a general pattern or trend. From the pattern or trend, you can describe a relationship, if it exists. Statisticians (like you!) consider three features of scatterplots: *direction*, *strength*, and *linearity*.

A *line of best fit* is a line that approximates a trend for the data in a scatterplot. It passes through as many points as possible, with the remaining points grouped equally above and below the line and spread out along the line, rather than concentrated at one area.

Interpolating involves a point within the data span of the explanatory variable (in mathematical circles, called the domain). When the value of the explanatory variable falls outside the spread of the recorded data, it is called *extrapolating*.

You may want to think about looking at a collection of data with this set of steps. This will frame your learning. As you work through this, you might want to make some notes about what that presentation tells you about the nutrition information of the ten Frappucinos you have selected.



6. Consider each nutrition factor (for each column in the chart). What do you notice about each column? Express these in a graph (a histogram or boxplot).
7. Determine the summary statistics (mean, median, range, and others!) for each column.
8. Research the daily recommendation for teenagers for each nutrient. Then, considering your research above, what percent of that daily recommendation would a Frappuccino (the average one or the one you selected before) be? Use this chart to help organize your learning. *This varies significantly for males and females, size, and activity level—so you can decide based on your own factors, or just choose an approximate based on the centre of the range provided.*

Nutrient	Daily Recommendation for Teenagers	Frappuccino's Percentage of Daily Recommendation
Total Energy (kcal)		
Fat (g)		
Carbohydrates (g)		
Fibre (g)		
Sodium (mg)		
Protein (g)		
Caffeine (mg)		

9. What do you notice about a single Frappuccino as a source of snack nutrients?

   Now, consider two of the columns in your chart that might be related. Ask a question that interests you about this data by comparing two variables. For example:

   - Do the kilocalories mostly come from sugar? So, is there a relationship between the amount of energy and the amount of carbohydrates?
   - So they put more salt in when there is more fat in the drink? So, does the amount of sodium increase with the increase in fat?

   Choose your two variables: _____ versus _____
10. Given your question, which is the explanatory (independent) variable and which is the response (dependent) variable?
11. Create a scatterplot of your data. Make a sketch of the data as you see it. Be sure to label your axes carefully and appropriately with a slight scale.
12. Does your scatterplot show a definite relationship? If so, describe the relationship (increasing or decreasing, steep or shallow). Does the scatterplot reveal an obvious pattern?
13. Do the points seem to describe a straight line (linear) or a curve (non-linear) or neither (no relationship)? Why did you choose the model you did?
14. Use the software to draw an approximate line of best fit for this data. (On TI-NSpire, this is called creating a *moveable line*.) What does the line tell you about the relationship?
15. Use the software to create a line of best fit based on the data. Can you have it create a curve of best fit as well? What is the difference in these? Which has the best fit?
16. Determine the "measure" of how well a line fits the set of data.

    The correlation coefficient r is a measure of how well the line fits the data. Your technology may also display the correlation coefficient r and coefficient of determination r^2.

    When $r = 1$, the fit is perfect. What would this mean? The closer r is to 1, the closer the line fits the data.

    Is our line a good fit, and if so, why?
17. Does the relationship support your hypothesis? If not, can you revise your original hypothesis or pose a reason why not?
18. You can also try to see whether you will obtain similar results for some different food items from the two Starbucks websites. Does your relationship hold for the food items as well?

Technology and Research Learning Extensions

19. Two variables can have a relationship or no relationship. If there is a relationship, that relationship can be described as linear or non-linear. These two references explain more about how scatterplots work.
    - Statistics Canada has a description of the thinking about Scatterplots: Relationships and Strength. **www.statcan.gc.ca/edu/power-pouvoir/ch9/scatter-nuages/5214827-eng.htm**

- BBC Bitesize has a description of how scatterplots and correlation can be understood. **www.bbc.co.uk/schools/gcsebitesize/maths/statistics/scatterdiagramsrev1.shtml**

20. You may be curious about how to know how "good" the fit is for the line technology built for you. For a linear model (regression), technology will create a measure of this goodness of fit called the correlation coefficient. If you want to explore what this is, some suggestions for research include the following:
    - NCTM lets you plot points on a graph, then approximate the line and test your approximation. **http://illuminations.nctm.org/Activity.aspx?id=4186**
    - Rossman Chance has a similar build feature, but allows you to use sample data or create your own. **www.rossmanchance.com/applets/RegShuffle.htm**
21. What is the difference between *continuous variables* and *discrete variables*? Check out: **www.bbc.co.uk/schools/gcsebitesize/maths/geometry/measureshirev1.shtml**

    Here is a quick summary of how to use these:
    - When the variables are continuous, you can connect your data points on a scatterplot. (Memory Aid: **C**ontinuous = **C**onnect).
    - When the variables are discrete, you do not connect your data points on a scatterplot. (Memory Aid: **D**iscrete = **D**on't connect).

Look at your Starbucks investigation. What kind of variables did you collect there?

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Beverly Caswell, Ontario Institute for Studies in Education, University of Toronto (OISE/UT)

EMAIL: beverly.caswell@utoronto.ca

Projects: Dr. Caswell currently serves as Director for the Robertson Program for Inquiry-Based Teaching in Math and Science (**www.oise.utoronto.ca/robertson/About_the_Program.html**) at OISE/UT. Beverly works collaboratively with educators to design inquiry-based learning environments for children that promote equity and a deeper understanding of

Continued on page 36

▲ In the Middle: Starbucks Math (Part 2 Answers)

1. Some medical conditions expect people to know the specific amount of ingredients/nutrients in food and drinks. Many people also want to track what they are eating in terms of hidden sodium or calories for other related issues like high blood pressure and weight management.
2. Diabetes, for example, requires that people watch precisely the amount of sugars they consume.
3. The Canadian page is a fixed, non-adjustable pdf, which doesn't allow you to adapt the drink much at all, so you can't add whipped cream or change to soy milk. The Canadian page shows all of the drinks at one time in a chart form. The U.S. page allows you to look at more drinks, but only one drink at a time, so you cannot compare. It is much more consumer friendly to understand and see, whereas the Canadian version is very, very, very small, and you have to zoom in to read anything. Part of the reason they differ is because the nutritional information recommended in Canada is different from that in the U.S. It is generally understood that Canada has stricter rules on things like food additives, and so products do differ, and what is reported has to differ as well.
4. When choosing Double Chocolaty Chip without whipped cream and with whole milk, the only thing that differs is the calories.

Nutrient	Canada	U.S.
Total Energy (kcal)	290	300
Fat (g)	8	8
Carbohydrates (g)	53	53
Fibre (g)	2	2
Sodium (mg)	250	260
Protein (g)	6	6
Caffeine (mg)	15	15

UPCOMING MATHEMATICS EDUCATION CONFERENCES

Ontario Association for Mathematics Education (OAME)
Georgian College, Barrie, Ontario
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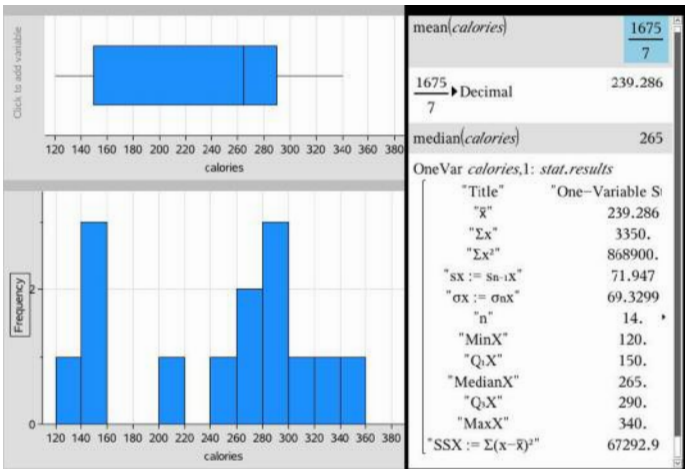
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University of Arizona, Tucson, Arizona
November 3-6, 2016

	A type	B calories	C fat	D carbs	E fibre	F protein	G sodium	H caffeine
1	mocha	290	4	61	1	4	220	110
2	pumpkinspice	300	3.5	71	0	5	270	100
3	pumpkinlight	150	0.5	31	3	6	240	100
4	coffee	240	3	50	0	3	220	95
5	javachip	340	7	67	2	5	250	110
6	caramellight	150	0.1	33	0	3	200	90
7	mochalight	150	1	33	1	4	200	95
8	strawberries	260	4	53	1	4	190	0
9	vanillabean	280	4.5	56	0	4	230	0
10	greentea	320	4.5	63	2	5	220	70
11	caramel	280	3.5	60	0	3	220	100
12	coffeelight	120	0.1	26	0	3	210	95
13	cinnamondolce	270	3	57	0	3	300	90
14	doublechocolaty	200	4.5	37	2	4	180	15

type

Example of summary statistics and graphical representation for calories



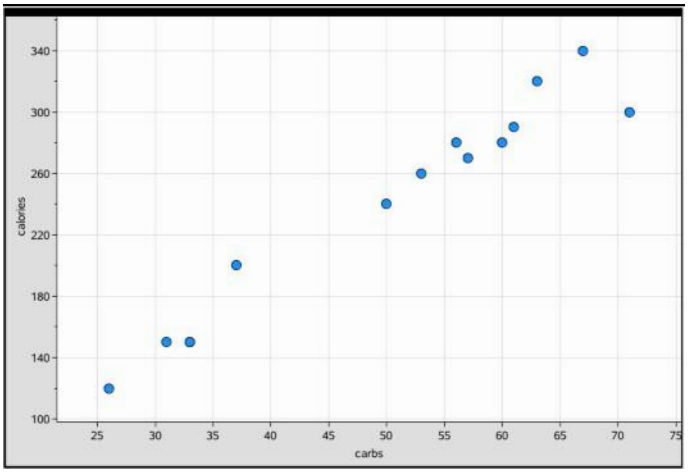
Using the data from the Mocha Frappuccino and Health Canada sources

	A type	B calories	C fat	D carbs	E fibre	F protein	G sodium	H caffeine
1	mocha	290	4	61	1	4	220	110
2	pumpkinspice	300	3.5	71	0	5	270	100
3	pumpkinlight	150	0.5	31	3	6	240	100
4	coffee	240	3	50	0	3	220	95
5	javachip	340	7	67	2	5	250	110
6	caramellight	150	0.1	33	0	3	200	90
7	mochalight	150	1	33	1	4	200	95
8	strawberries	260	4	53	1	4	190	0
9	vanillabean	280	4.5	56	0	4	230	0
10	greentea	320	4.5	63	2	5	220	70
11	caramel	280	3.5	60	0	3	220	100
12	coffeelight	120	0.1	26	0	3	210	95
13	cinnamondolce	270	3	57	0	3	300	90
14	doublechocolaty	200	4.5	37	2	4	180	15

type

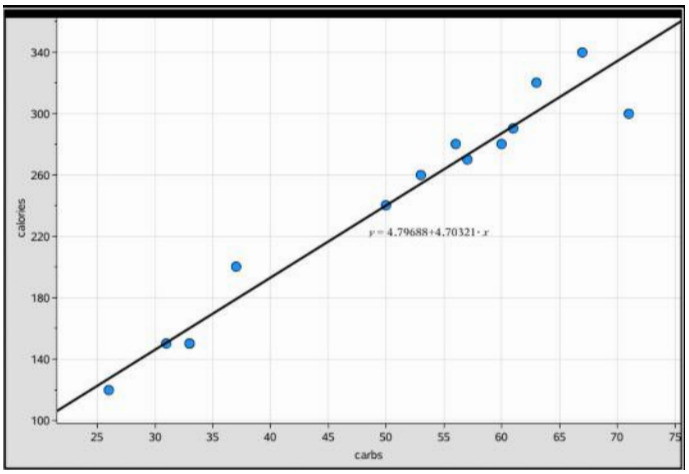
Nutrient	Daily Recommendation for Teenagers	Frappuccino's Percentage of Daily Recommendation
Total Energy (kcal)	2000 (varies for boys, girls, and activity level)	290/2000 = 14.5%
Fat (g)	65	4/65 = 6.2%
Carbohydrates (g)	300	61/300 = 20.3%
Fibre (g)	25	1/25 = 4%
Sodium (mg)	2400	220/2400 = 9.2%
Protein (g)50	(46 for girls, 56 for boys)	4/50 = 8%
Caffeine (mg)	85 (for 12-year-olds)	110/85 = 129%

9. One Frappuccino is one-fifth of all your carbohydrates of the day! And almost a tenth of all your sodium! It is a lot of your total energy as well (1/7th). If you have a caffeinated one (like most of them are), you are having more caffeine total than is recommended—by a lot. That means no other chocolate, tea, cola drinks, or anything else for the day and you are still over your recommended caffeine.
10. Looking at the relationship between carbohydrates and calories

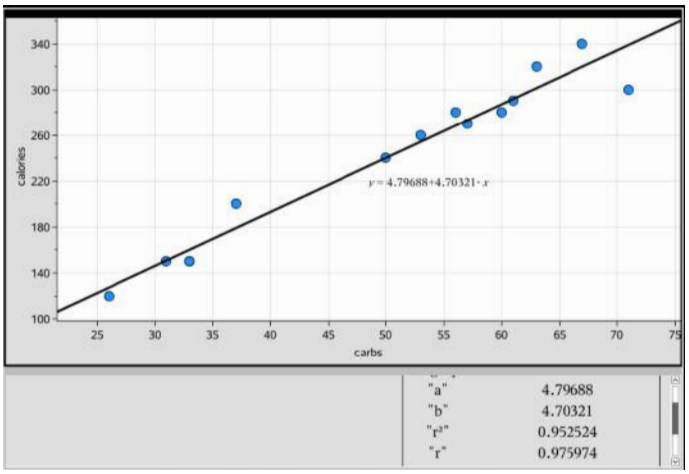


11. This is definitely an increasing relationship that is linear. As the carbohydrates go up, the calories go up. In Frappuccinos, it seems like most of the increase in calories comes from increasing carbohydrates (sugar!).
12. It definitely appears more linear than any other shape. It has some parts at the end that are less on the line, but most of them are pretty close to straight.

"Some mathematician, I believe, has said that true pleasure lies not in the discovery of truth, but in the search for it." *Leo Tolstoy*



As an extension, it is interesting to note that every increase of one gram of carbohydrates contributes to an increase in 4.7 kCal. You can see this in the slope of the line. A little research determines that protein and carbohydrates contribute about 4.1 kCal/g, which is pretty close to this value. The *y*-intercept of approximately 4.8 is likely contributed by the fat in the drinks.



The fit of this line is very strong. Looking at the results here and the graph, you can see that the correlation coefficient and the coefficient of determination are both in the high 90% range.

There is a similar, but less strong, relationship between fat and calories, and similarly between fat and carbohydrates. ▲

"No employment can be managed without arithmetic, no mechanical invention without geometry." *Benjamin Franklin*

▲ Hey, It's Elementary: Rich Tasks and Big Ideas... Then and Now



LYNDA COLGAN
EMAIL: colganl@educ.queensu.ca

Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

I began a six-month sabbatical on January 1, 2016—a wonderful opportunity to go back through files and take stock of what is a "keeper" and what is best bound for the recycling heap. As I worked my way through drawers and boxes of articles, first drafts, half-written pieces, and one-page brainstorm doodles, I came across what I believe is a wonderfully rich Grades 7/8 task: *A Population Mystery*.

The task was developed in collaboration with my former Scarborough Board colleague, Peter Harrison, for our work in professional development with Intermediate teachers, in part, as our response to a presentation by Dr. Rafaella Borasi (now Dean of the Warner School of Education at The University of Rochester) at the 1997 Canadian Mathematics Education Study Group Conference at Lakehead University in Thunder Bay, Ontario.

At the presentation, Dr. Borasi emphasized the fact that "…professionals are rarely, if ever, presented with a well-defined problem and expected to apply known mathematical models to come up with an objective solution. Rather, the task is most often presented to them in the more vague and open-ended form of a 'problematic situation.'"

Dr. Borasi's comment resonated with Peter and myself, and over many long hours, we discussed the inauthentic nature of so many of the "real world" problems that Grades 7/8 students faced in class, and the shallowness of many of the conceptual demands that were placed on student expectations.

Our resolve to generate a suite of rich tasks was galvanized when we read Howard Gardner's 1998 letter to *The Harvard Education Review*. In his eloquent piece, Gardner described what he believed to be the mathematical skills, knowledge, and values that should be emphasized in schools:

> …adults need to be able… to make prudent decisions in light of the laws of probability;… individuals should understand the nature of scientific enterprise: what is a fact, a theory, a hypothesis, an experiment. Individuals should be able to understand a newspaper report about a new medical treatment, evaluate the evidence on which it is based, and decide whether to believe it.

Gardner went on in his letter to argue that our goal, as mathematics educators, should be to teach and measure the complex web of the "big ideas" of mathematics that are at the heart of mathematical thinking and informed decision making.

Gardner went on to describe "big ideas" as the connecting concepts and processes that go beyond content: mathematical reasoning, mathematical modelling, problem solving, and communication, using multiple representations. Gardner's definition of "big ideas" was the notion of the family of conceptual connections that empower students both to see mathematics as an integrated whole and to make thoughtful connections among related mathematical notions and purposeful applications to other domains.

In response to Drs. Borasi and Gardner, Peter and I set our sights on developing problematic situations for middle school students that reflected Gardner's words, i.e., "to give students a sense of what it means to understand a topic in detail, and one hopes, whet their appetites for continuing to learn and understand deeply for the rest of their lives."

This particular task was designed to show that learning tasks in the Data Management and Probability strand need not be limited to spinners, dice, playing cards, and bingo chips. Our goal was to challenge students to interpret the probability that surrounds us daily by illustrating the importance of justifying real-world conclusions, using mathematically defensible arguments.

After an introduction to the task, its "big ideas," and a description of the curriculum expectations for the task and context, what follows is a "teaching episode"—the task is presented as interactive discourse among teacher and students and students and students.

Overview of the Activity

In this problematic situation, students attempt to solve a population mystery based on statistical information about human populations and bear populations. They are given the data in the form of a set of graphs, and they are required to analyze the information, looking for clues that will help them to evaluate the validity of a researcher's conclusions about a bear population. To develop their argument, students must apply probabilistic arguments to interpolate and extrapolate values/decisions from the visual data, and use both experimental and theoretical probabilities in concert with other factors, to guide their conclusions.

Big Ideas

Based on Gardner's contention that graphing, statistics, and probability are extremely important domains because of the fact that we are living in an information age, we elected to turn to the various forms of print media to determine how information is categorized, summarized, and displayed. We presented our problematic situation in such a way that students would be required to apply and integrate skills and knowledge from graphing (displaying information) with those from statistics (discussing data numerically) and probability (the mathematics of outcomes).

With much large-scale assessment data suggesting that while students are able to make direct readings from graphs and tables, but unable to articulate relationships among data, we sought, through this problematic situation, to challenge students to examine graphical displays, using a qualitative lens, e.g., looking for symmetry and other patterns that can form and inform quantitative arguments.

To develop their argument, the students must assess the strengths, weaknesses, and biases of samples and data-collection methods; critique the ways in which the statistical information and conclusions are presented; and evaluate decisions based on probability that will require a combination of theoretical calculations, experimental results, and subjective judgments.

Prior Knowledge

This problematic situation assumes that students will be familiar with the numerical definitions of probability (including conversions among decimal, percent, and fractional forms) and the strategies for listing all the possible outcomes for an event (i.e., probability paths, tree diagrams).

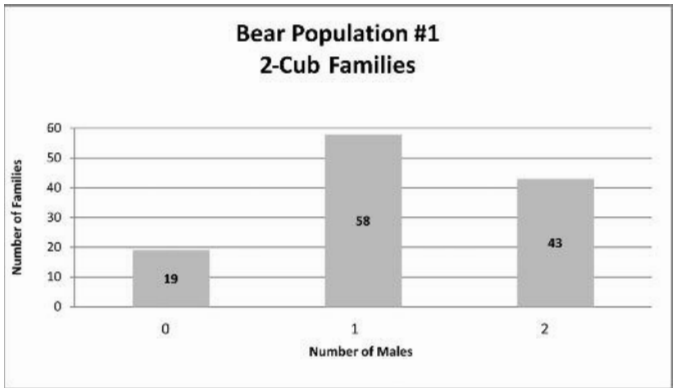
The Classroom Context

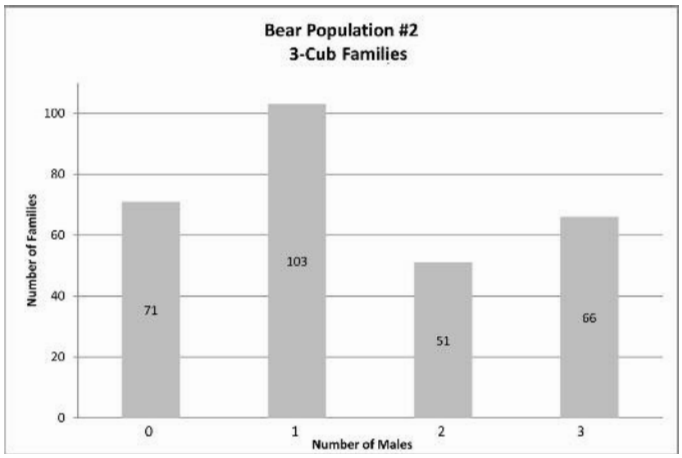
Our teacher, Ms. B., believes, as did Socrates, that good teaching occurs when one instructs, delights, and moves another. Her math classroom was energetic: one in which students regularly worked collaboratively. Ms. B. was enthusiastic about challenging her students with this genuine dilemma, and she and her students were happy to have us become temporary members of their classroom.

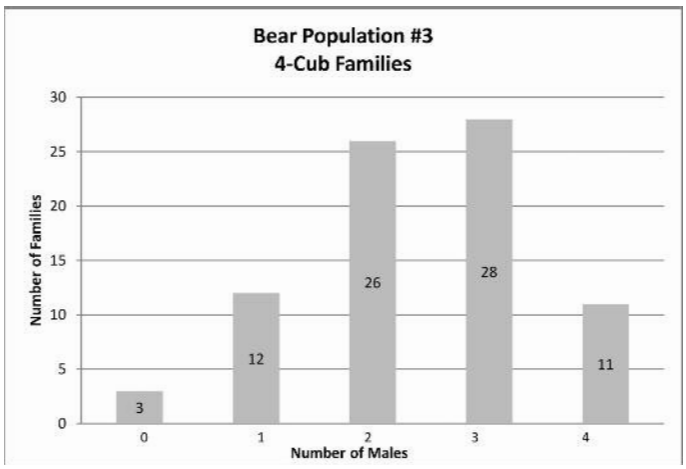
The Problematic Situation

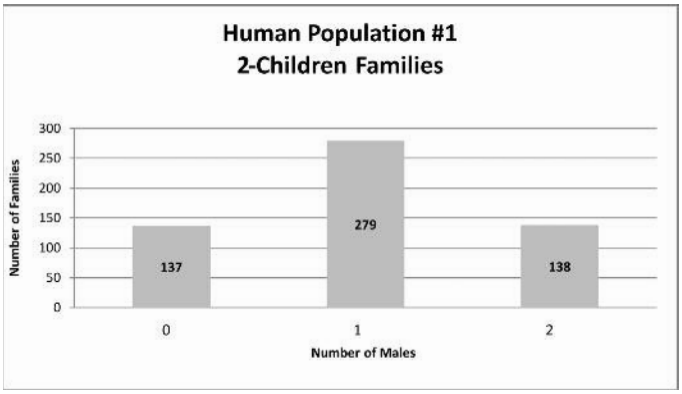
A team of researchers in Northern Ontario has been tracking bear families and keeping track of the number of male cubs in each family. They think that they have collected

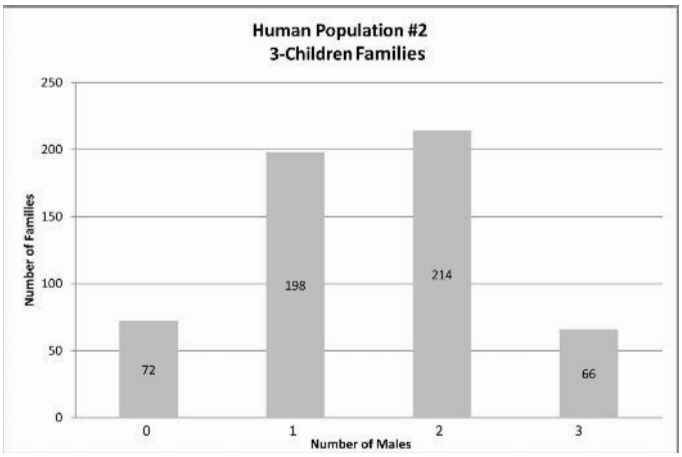
some rather odd findings. To help understand their data, they have also assembled some similar data for humans. They surveyed a series of human families with similar numbers of children and made a note of the number of boys. All of the results are shown in graphical form. The researchers concluded that the probability that a mother bear will give birth to a male cub is different from the probability that a human mother will give birth to a son.

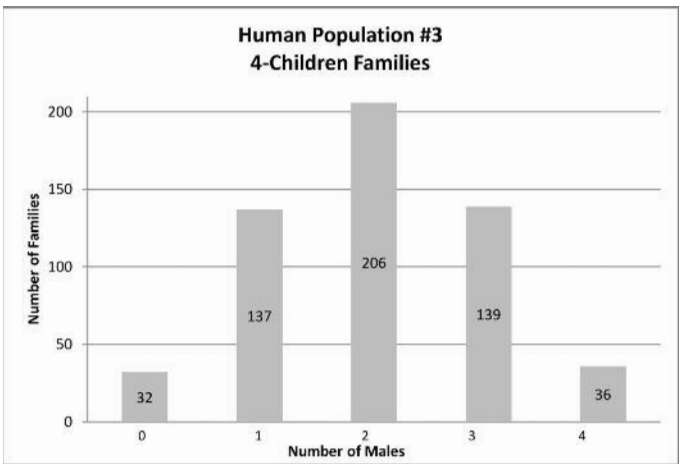












Ms. B. These graphs certainly contain a lot of information. I'd like someone to give us a quick summary of the story that these graphs tell.

Anne I'll try. We have six bar graphs. Each one is about either bear families or human families, and they only tell us about the number of male offspring—either kids or cubs—and they are paired off so that the bear and human families with the same number of offspring are next to each other.

Ms. B. Thanks, Ann. Why is it important for the graphs to be side by side?

Ian It makes it easier to compare. If we look at the second row, we have all the families with either 3 cubs or 3 kids. You can see right away that there were 15 bear families with no male cubs and 72 human families with no male kids.

Ms. B. Good point, Ian. But what if you look at the overall shapes of the graphs instead of focusing on the numbers? What stories do the graphs tell then?

Ian The human graphs are easier to describe. They're pretty balanced. The bear graphs are a little lopsided. I'm not sure what that means though.

When no one offers an answer immediately, Ms. B. suggests that they look at the graphs again and then discuss Jeff's point with the other students in the pod. After about five minutes, she calls the class's attention.

Ms. B. Well, what do you think?

Hoda I think that you'll always have the same number of boys and girls because it's all about chance. When a baby is born, it will either be a boy or a girl. It's a 50-50 chance thing.

Ms. B. Jeff, you started the conversation about "balance" in the graphs for humans. Who can add a little more description to the shape of the graph?

Mollie I think it is important that the graphs are symmetrical. There's a hump in the middle. The shorter bars are off to the end and the taller bars are in the middle.

Ms. B. How is this conversation like another probability conversation that we had in class recently?

Stefan The coin-toss experiment! When we tossed two coins, we found out that we got one head and one tail more often that either two heads or two tails. So, instead of heads and tails, we have boys and girls.

Ken Are you saying that the chance of tossing a head is the same as getting a boy in the family?

Ms. B. At this point, we need facts to support our ideas. Where can we find evidence to help us understand this situation?

Ann I think we need to make a tree diagram for families with two babies.

Ms. B. Great suggestion, Ann. In pairs, I'd like you to construct a tree diagram for Graph #2—human families with two children. Is there a pair who would be willing to draw their tree diagram on chart paper to help us in our discussion later? Thanks. Hoda and Jeff have volunteered.

Ms. B. gives the students about 12 minutes to work on the task and then calls for the class's attention as Hoda and Jeff display their tree diagram.

Ms. B. Jeff or Hoda, would either of you please walk us through your tree diagram?

	# OF MALES	0	1	2			
TWO CUBS	REPORTED	19	58	43			
	EXPECTED	19	58	43			
	# OF MALES	0	1	2	3		
THREE CUBS	REPORTED	15	71	103	51		
	EXPECTED	15	69	103	52		
	# OF MALES	0	1	2	3	4	
FOUR CUBS	REPORTED	3	12	26	28	1	1
	EXPECTED	2	12	28	28	10	

Jeff I'll start. This is the same as the coin-toss experiments. If you have one coin, there are two options: heads or tails. If there is one child, there are two possible outcomes: boy or girl. If there are two coins, you have four possibilities: two heads, two tails, or one head and one tail, and vice versa.

Hoda Then we remembered that we also used fractions with our tree diagrams for the coin-toss experiments, so we did the same thing here, and then we calculated percents too.

Jeff The graph tells us that there are 550 families in all. Just add up the numbers on the bars. 25% of the families should have zero boys. And they do—one-fourth of 550 is 137.5.

Stefan This experiment was fixed. The theoretical and experimental probabilities match perfectly.

Ms. B. Good point, Stefan. To test the scientists' conclusions, we'll need to create similar trees for 3- and 4-children families and check out the numbers.

Ms. B. asks the students to form groups of four. One pair in each group is given responsibility for the 3-children families, and the other pair constructs the tree diagram for the 4-children families. She asks them to check each other's work. After 15 minutes, she calls for the class's attention. There is general agreement that the numbers on the human graphs are very accurate reflections of what probability would predict. A lively discussion ensues about how "real" the data is because the students feel that the human families are "too perfect."

Ms. B. Great discussion. Thanks to everyone for participating. Now let's turn our attention to the graphs of the bear families.

Mollie Well, those graphs sure aren't perfect. The bear graphs all slant in favour of boys. If you look at the 3-cub families, there are 51 families with 3 males and only 15 families with 3 females.

Tony How do you know that, Mollie?

Mollie If you look at the graph, you can see that there are 15 3-cub families with zero male cubs, so that means the family must have all female cubs. And in the 71 3-cub families with 1 male cub, there must be 2 female cubs.

Ms. B. Thanks for clarifying, Mollie. I'm sure that there were others in addition to Tony who had the same question.

Ann There sure are more male bear cubs. I wonder why. Does that mean the male cubs are stronger and have a better chance of surviving in the wild? Or maybe it's not 50-50 for bears like it is for humans. Maybe it's 60-40 or 70-30.

Ms. B. We are missing a lot of information. All we have are the graphs and the information that they contain. We know the graphs are skewed toward male cubs. So, what are our next steps?

Hoda I think Ann is right. We can't use 50-50 chances for bears.

Jeff So you mean we have to use trial and error when we do the calculations and then see how close our experimental numbers are to the actual numbers?

Hoda Ms. B., is that right?

Ms. B. We won't know unless we try. I think the suggestion is good. We have to conjecture about our probability: 55%, 60%, 65%, 70%, and so on, and then compare our results with the data that we have in the graphs. So, for homework, Pod A will work with 40% and we'll continue along the alphabet to Pod H trying 75%. I'm looking forward to your findings. See you tomorrow.

The Next Class

Ms. B. We are going to take a few minutes so that everyone in each pod can compare their solutions with the others in your group. When your pod is satisfied that they have a solution that everyone can explain, we'll move into expert groups to compare solutions for all the probabilities from 55 to 75.

Ms. B. circulates around the classroom to be sure that each pod has a correct solution, and provides support and help as required. When she is confident that the expert groups have come to consensus about the best possible solution, she asks for a volunteer to present to the class. Ken, Stefan, and Mollie come to the board and record their table.

Ms. B. Thanks to our three volunteers. Who is going to report?

Mollie I'll start. I started by making a tree diagram. When I created the tree for a 4-cub family, I got 16 possibilities, like FFFF, MFFF, MFMM, and MMFM. I noticed that four of my arrangements had 3 males and 1 female. I tried working with 40% probability for a male and started doing the calculations. <insert chart>

Ken We weren't really sure what to do when we had our numbers. But then Stefan turned back to the coin-toss experiments from last week and we remembered that we had to multiply the probabilities together.

Stefan To get the expected number of 3 males, we multiplied 0.4 x 0.4 x 0.4 x 0.6 x 4 x 80 and got 12.3. Too small.

Mollie So if 40% was too small, we knew it had to be another number. The group that tried 60% was almost perfect. They multiplied 0.6 x 0.6 x 0.6 x 0.4 x 6 x 80 to get 27.6.

Ms. B. Are there any questions or comments?

Eugene I didn't do the problem that way. I looked at the graph for 3-cub families, and calculated the total number of cubs and the total number of males. There are 15 families with no males, so 0. There are 71 families with 1 male, so that's 71 male cubs. There are 103 families with 2 males, so that's 206 more male cubs. Last, there are 51 families with three cubs; that's 153 male cubs. So in all, there are 0 + 71 + 206 + 153, or 430, male cubs. Then I found out how many cubs in all. That's 240 families with 3 cubs each, or 720. The probability that a cub is male is 430/720, which is about 60%. I tried it for all the graphs, and my method works.

Ms. B. So we have two different approaches that lead us to the same solution. So, what should we say to the researchers about their conclusion?

Hoda We can tell them that their graphs do tell us that the probability of a mother bear cub is 60%.

Ann But I'd like to tell them that their graphs don't tell the whole story without knowing information about the number of cubs born in the litters, compared to the number of cubs in the family when the tally was taken. Maybe more newborn female cubs die because of poor health or a predator.

Ms. B. Interesting. That leads right into your final task, which is to write a report for the researchers, critiquing the presentation of their results and their conclusion, and suggesting ways to improve the accuracy of their study and the media reports about their study.

Peter and I hope that you found this dialogic format, based on classroom field tests of one of our attempts to formulate a rich task, from nearly 20 years ago, to remain a highly illuminative example of the role of "rich learning tasks" and "big ideas" in teaching and learning mathematics today.

And now I'll return to the next box of files. Who knows if another treasure awaits? ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Over the past several decades, changes in perspective as to what constitute statistics and how statistics should be taught have occurred, which resulted in new content, pedagogy and technology, and extension of teaching to school level. At the same time, statistics education has emerged as a distinct discipline with its own research base, professional publications, and conferences. There seems to be a large measure of agreement on what content to emphasize in statistics education: exploring data (describing patterns and departures from patterns), sampling and experimentation (planning and conducting a study), anticipating patterns (exploring random phenomena using probability and simulation), and statistical inference (estimating population parameters and testing hypotheses)... The main sources of students' difficulties were identified as: facing statistical ideas and rules that are complex, difficult, and/or counterintuitive, difficulty with the underlying mathematics, the context in many statistical problems may mislead the students, and being uncomfortable with the messiness of data, the different possible interpretations based on different assumptions, and the extensive use of writing and communication skills."

Ben-Zvi, D. (2014). Data handling and statistics teaching and learning. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 137–140). London, UK: Springer.

▲ What's the Problem? Nifty Number Nine



SHAWN GODIN
EMAIL: shawn.godin@ocdsb.ca

Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. He is a past and present member of the OAME Board of Directors and is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem.

Welcome back, problem solvers. Last time, I left you with the following problem:

Pick any whole number between 20 and 100, inclusive. Add the digits together and subtract the sum from the original number. Finally, add the two digits from your last result and concentrate really hard.

You have probably seen this problem, or a variation of it, at some time or another. This problem came from *The Magic of Math* by Arthur Benjamin. Arthur Benjamin is a mathematics professor at Harvey Mudd College in California. He has an entertaining TED Talk video, where he performs mental arithmetic that is worth checking out.

This is a great one to give to the kids, have them run through it, and see their surprise when you announce that their number is 9. You or your students may have seen similar "number-predicting tricks" that can be explained with algebra. A common theme is shown in Figure 1.

Step	Example	Algebra
Pick a number	17	x
Add 13	$17 + 13 = 30$	$x + 13$
Double the result	$30 \times 2 = 60$	$2(x + 13) = 2x + 26$
Subtract 8	$60 - 8 = 52$	$2x + 26 - 8 = 2x + 18$
Divide result by 2	$52 \div 2 = 26$	$(2x + 18) \div 2 = x + 9$
Subtract the original number	$26 - 17 = 9$	$x + 9 - x = 9$

Figure 1

Unfortunately, for the problem we are considering, we are combining a number with its digits, and the final result is the sum of the digits of our calculation. If x is the number, how do we deal with its digits? The spark of insight needed is that we don't use a variable to represent our chosen number; rather, we use two variables to represent the digits. If we use T and U to represent the tens and units digits, respectively, then our original number is $10T + U$ and the sum of the digits

is $T + U$. So when we subtract the sum of the digits from the original number, we get $(10T + U) - (T + U) = 9T$, that is, a multiple of 9. Since our original number was between 20 and 100, inclusive, that forces T to be anything from 2 to 10, inclusive. Thus, the result is a two-digit multiple of 9. It is a well-known fact that the sum of the digits of the first 10 multiples of 9 add to 9 (see Figure 2).

1 x 9 = 9	0 + 9 = 9	6 x 9 = 54	5 + 4 = 9
2 x 9 = 18	1 + 8 = 9	7 x 9 = 63	6 + 3 = 9
3 x 9 = 27	2 + 7 = 9	8 x 9 = 72	7 + 2 = 9
4 x 9 = 36	3 + 6 = 9	9 x 9 = 81	8 + 1 = 9
5 x 9 = 45	4 + 5 = 9	10 x 9 = 90	9 + 0 = 9

Figure 2

The fact that the sum of the digits of the first 10 multiples of 9 is 9 is used in the "finger-multiplication trick" shown in Figure 3. To multiply 9 by a single-digit number, hold your hands out in front of you with your fingers extended. To calculate 6 x 9, for example, you would count from the left to the sixth finger, then bend it over. Your answer is staring you in the face: the number of fingers to the left of the bent finger is the tens digit (5), and the ones to the right are the units digit (4), so 6 x 9.



Figure 3

Now, the really interesting thing at this point is not that the answer to our problem is 9, but why it should be so. The answer lies in our number system. Since our number system is base 10, one more than 9, we get some interesting things happening. For example, take a minute and calculate the remainder when 314 is divided by 9. Did you notice the connection between the digits of 314 and the answer?

Doing the long division, you would find that 314 divided by 9 is 34, with a remainder of 8. If you thought about it, you may have noticed that 3 + 1 + 4 = 8. This isn't a coincidence. If I write 314 out in long form, we would see that $314 = 3 \times 100 + 1 \times 10 + 4$. Thus, $314 = 3 \times (99 + 1) + 1 \times (9 + 1) + 4 = (3 \times 99 + 1 \times 9) + (3 + 1 + 4)$, where the expression in the left-hand bracket is a multiple of 9 (why?).

If you take any number and add its digits together, the new number you end up with leaves the same remainder when you divide by 9. Thus, if we continue the process, repeating it until we get a single digit, we will end up with the remainder when you divide by 9, or 9 if the number is a multiple of 9.

At one point during his TED Talk, Professor Benjamin asks volunteers to multiply a given four-digit number with a three-digit number of their choice. He then asks them to tell him all of the digits, in any order, except one. He then proceeds to tell each person which digit they had left out. The key to his trick is that the original number, supplied by professor Benjamin, is a multiple of nine. For example, if the given number was 5121, where 5 + 1 + 2 + 1 = 9, he knows that whatever you multiply by will be a multiple of 9. So, if you picked 347, you would get 347 x 5121 = 1776987. Suppose you then told him the digits were 1, 7, 7, 7, 8, 9, omitting the 6, he would mentally calculate 1 + 7 + 7 + 7 + 8 + 9 = 39 and he knows the digits' sum to a multiple of 9. The closest one is 45, which is off by 6, which is your number. Alternately, when he got 39, he could have calculated 3 + 9 = 12 then 1 + 2 = 3, so the number would be what he needed to bring it back to 9. This idea of the remainder that a number leaves is so important that Carl Gauss, one of the greatest mathematicians of all time, introduced the notation for it. We would write $314 \equiv 8 \ (mod\ 9)$, which reads 314 is congruent to 8 modulo 9, which means that (314 – 8) is divisible by 9.

Here are a couple of problems that are related to the original problem:

i. Take any three-digit number whose digits are decreasing (for example, 952). Then reverse the digits and find the difference between the two numbers. If you add the digits of this number, you get 18.

ii. Begin as in i. above, but at the end, instead of adding the digits, reverse the digits again and add the two numbers, and you always get 1089.

Both are problems that should be entertaining to students and also provide a good exercise in algebra to discover why they work.

And now for your homework.

I have three cards: one has a circle on one side and a square on the other; the second has circles on both sides; and the third has squares on both sides. I put the cards into a hat, have you stir them up and pick one. You show me one side, for example, a square. I say, "Since the other side is either a square or a circle, there is a 50 percent chance of me choosing correctly. Would you be willing to wager even money that I can guess what is on the other side?"

Until the next issue, happy problem solving. ▲

▲ Technology Corner: Number Talks



MARY BOURASSA
EMAIL: mary.bourassa@oame.on.ca
TWITTER HANDLE: @MaryBourassa

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

In December, Pierre Tranchmontagne (@Pierre_Tranche) created a website to collect number-talk images. This, similarly to the Which One Doesn't Belong website (**http://wodb.ca/**), is the result of a number of these types of images being posted on Twitter and someone saying that there needed to be a website to house them.

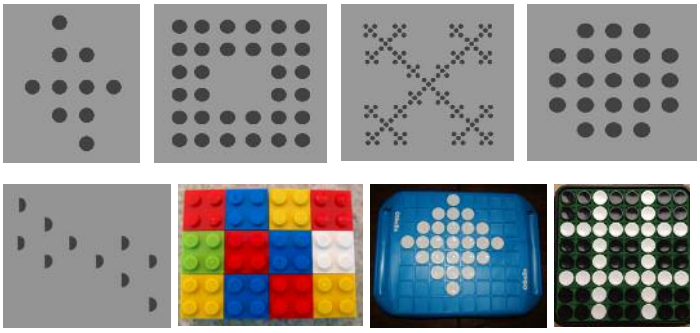


I learned about number talks after taking Jo Boaler's online course a few years ago. They are great warm-ups that help to develop students' number sense, see patterns in different ways, and learn to reason numerically. There are lots of websites about number talks and many books, including *Making Number Talks Matter* by Cathy Humphreys and Ruth Parker.

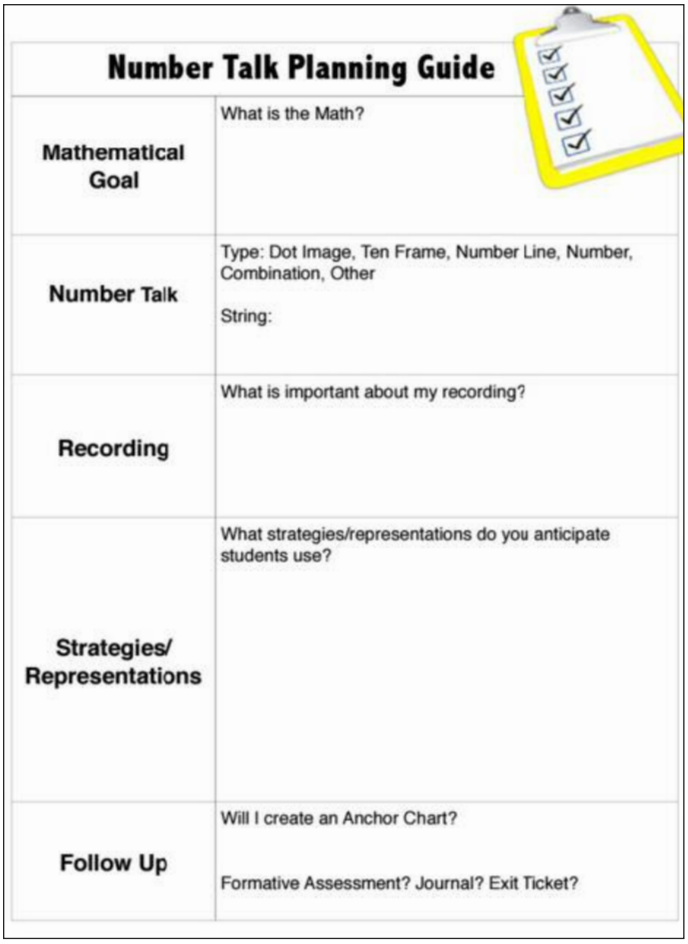
The site, like Pierre, is bilingual and is described as "a collaborative project dedicated to gathering interesting images to be used as a launching point for Number Talks." It is broken up into dots, photos, and fractions dots.

The site focuses on images that can lead to great mathematical discussions in class. Asking questions such as "How many dots do you see? How did you count them?" lead to students seeing different strategies for finding answers to questions involving addition, subtraction, multiplication, and division. Talking about how they see each image lets students develop their ability to articulate and defend their ideas, and exposes them to new ideas, as others students may see different patterns in the images.

Here are some samples images:



The Resources tab includes a helpful, downloadable planning guide from Kristin Gray (@MathMinds).

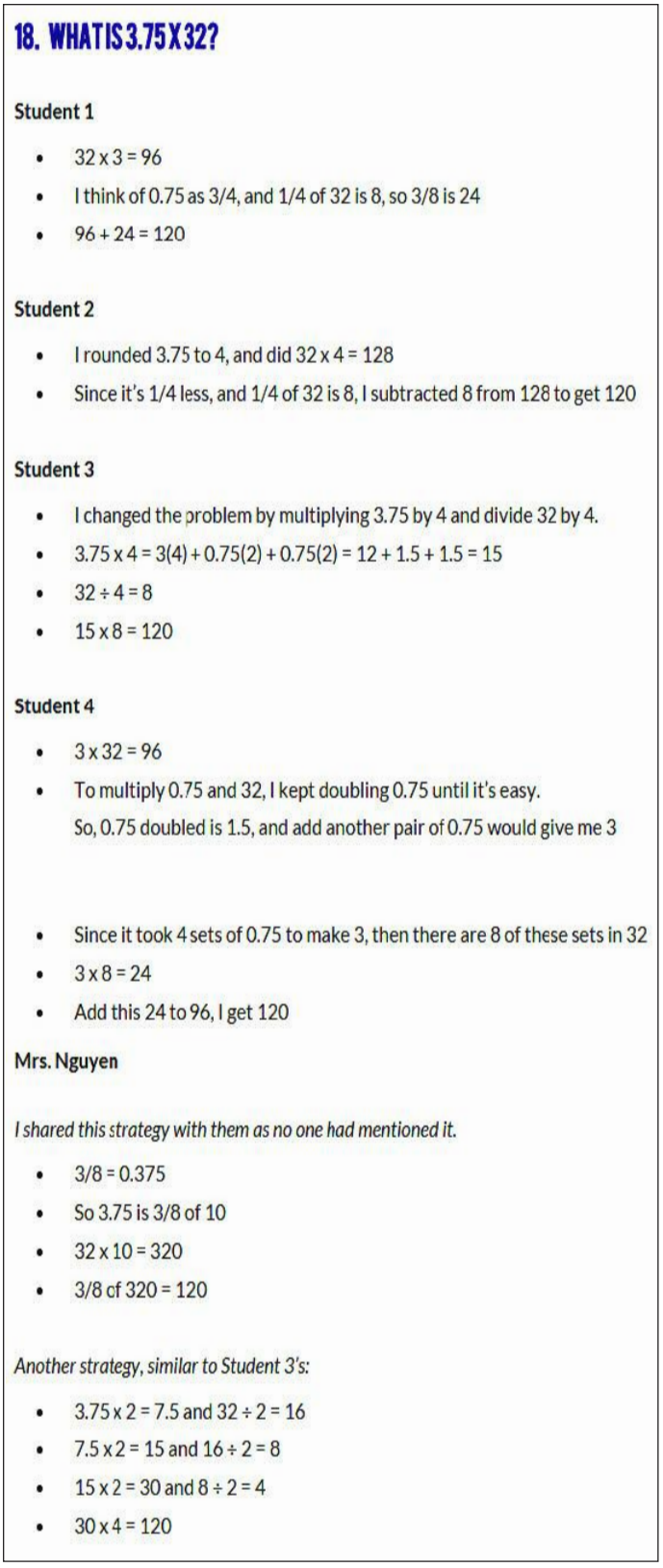


Number Talk Planning Guide

Mathematical Goal	What is the Math?
Number Talk	Type: Dot Image, Ten Frame, Number Line, Number, Combination, Other String:
Recording	What is important about my recording?
Strategies/ Representations	What strategies/representations do you anticipate students use?
Follow Up	Will I create an Anchor Chart? Formative Assessment? Journal? Exit Ticket?

The website also has a submission form, so please consider making your own pattern or taking pictures of interesting patterns you see and sending them in.

Another wonderful resource around number talks is from Fawn Nguyen (@FawnPNguyen), creator of the Visual Patterns website (**www.visualpatterns.org**). The Math Talk website (**www.mathtalks.net**) has number talks and pattern talks accompanied by actual student responses. Here is a sample:



18. WHAT IS 3.75 X 32?

Student 1

- 32 x 3 = 96
- I think of 0.75 as 3/4, and 1/4 of 32 is 8, so 3/8 is 24
- 96 + 24 = 120

Student 2

- I rounded 3.75 to 4, and did 32 x 4 = 128
- Since it's 1/4 less, and 1/4 of 32 is 8, I subtracted 8 from 128 to get 120

Student 3

- I changed the problem by multiplying 3.75 by 4 and divide 32 by 4.
- 3.75 x 4 = 3(4) + 0.75(2) + 0.75(2) = 12 + 1.5 + 1.5 = 15
- 32 ÷ 4 = 8
- 15 x 8 = 120

Student 4

- 3 x 32 = 96
- To multiply 0.75 and 32, I kept doubling 0.75 until it's easy.
  So, 0.75 doubled is 1.5, and add another pair of 0.75 would give me 3
- Since it took 4 sets of 0.75 to make 3, then there are 8 of these sets in 32
- 3 x 8 = 24
- Add this 24 to 96, I get 120

Mrs. Nguyen

I shared this strategy with them as no one had mentioned it.

- 3/8 = 0.375
- So 3.75 is 3/8 of 10
- 32 x 10 = 320
- 3/8 of 320 = 120

Another strategy, similar to Student 3's:

- 3.75 x 2 = 7.5 and 32 ÷ 2 = 16
- 7.5 x 2 = 15 and 16 ÷ 2 = 8
- 15 x 2 = 30 and 8 ÷ 2 = 4
- 30 x 4 = 120

The Teachers page on this site has a fairly comprehensive list of resources around number talks, as well as detailed explanations of how Fawn runs math talks in her classroom. This insight is very valuable as you begin using number talks. Enjoy! ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Although mathematics has been taught and learned for millennia, not until the past century or so have the nature and quality of teaching and learning mathematics been studied in any a serious manner… Only during the nineteenth century, however, as national educational systems were established and the training of teachers moved into colleges and universities, did people begin to identify themselves as mathematics educators and begin to conduct research as part of their scholarly identity… As mathematics educators began to study children's mathematics learning and thinking, they increasingly recognized that laboratory studies present a restricted view of those processes… Children do most of their learning of mathematics in school classrooms along with other children, and their thinking about mathematical concepts and problems is much influenced by others, including their teacher… The subject matter of research studies has broadened to include such topics as the school mathematics curriculum, assessment in mathematics, the education of mathematics teachers and their professional development, the socio-political context of learning and teaching mathematics, teaching mathematics to students in special education programs, and the politics of mathematics education. The methods used to conduct research now go well beyond experimentation to include case studies of teachers and students, surveys of attitudes and beliefs, and ethnographies of cultural practices."

Kilpatrick, J. (2014). History of research in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 267–272). London, UK: Springer.

▲ Fields Institute MathEd Forum Report



STEWART CRAVEN
EMAIL: numeratecitizen@mac.com

Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

Our meeting on November 28, 2015 was an excellent example of why I rarely miss a MathEd Forum meeting. The following summaries provide only a hint of what is in store when you attend MathEd Forum talks and take part in the ensuing discussions.

The following are brief summaries of the presentations from that meeting:

- Carly Rozins (Ph.D. Candidate in Applied Mathematics at Queen's University)

  Title: *Becoming a Computational Thinker*

  Description: What is computational thinking and how does an undergraduate student become a computational thinker? The speaker used a math problem (accessible to high school students) to discuss: 1) the presentation of the problem, and 2) how the setting in which the problem is presented can influence how the student approaches the problem. This example was then used to suggest problem designs that can stimulate computational thinking.

- George Gadanidis (Professor of Education at Western University)

  Title: *Computational Thinking in the Elementary Classroom*

  Description: Professor Gadanidis outlined computational thinking in the elementary math classroom by giving a number of examples ranging from geometric patterns to the binomial theorem. See more at **www.researchideas.ca/coding.html**.

- Chantal Buteau (Professor of Education at Brock University), Ramona Rat (student at Brock University), and Eric Muller (Professor Emeritus at Brock University and a Fellow of the Fields Institute)

  Title: *Thinking in "Code" and Coding for Thinking: A University Implementation*

  Description: In this presentation, the speakers described the programming-based mathematics work of undergraduate student Ramona and presented an analysis of her learning experience. Since 2001, three MICA (Mathematics Integrated with Computers and Applications) courses have been designed and implemented to help students develop proficiency in programming for mathematical investigation, simulation, modelling, and visualization.

- Sheila Rhodes (Instructor at the Faculty of Education at UOIT)

  Title: *Programming with LEGO Robotics*

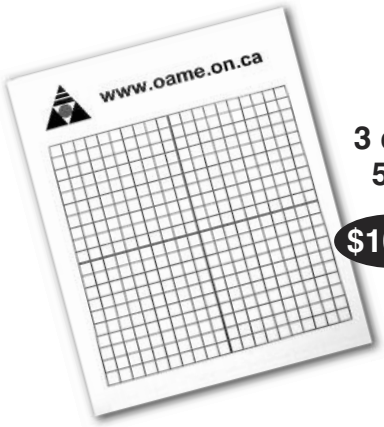
  Description: Ms. Rhodes demonstrated the ways in which computational thinking can be used to program LEGO robots (i.e., classroom robotics kits).

Each of the above presentations was delivered enthusiastically. The presenters used excellent examples and concrete materials to powerfully demonstrate their ideas.

After each talk, the audience got involved in questions and answers, and discussions spilled over into the lunch break and carried on after the meeting. This is how all of us should be learning!

The Steering Committee for the MathEd Forum seeks input for new themes throughout the year and would appreciate your suggestions. (Drop me an email at numeratecitizen@mac.com and I will pass your ideas along.) Agendae and information can be found at **www.fields.utoronto.ca/programs/mathed/forum/**. ▲

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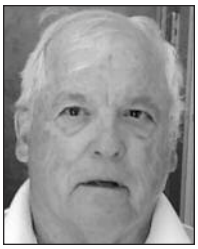
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▲ OAME/NCTM Report



TODD ROMIENS
EMAIL: todd.romiens@oame.on.ca

Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

You have probably heard of the attempt to implement the Common Core State Standards for Mathematics (CCSSM) across the United States. This is an educational initiative that details what K–12 students should know in mathematics at the end of each grade. It seeks to establish consistent educational standards across each state, and to ensure that students graduating from high school are prepared to enter credit courses at two- or four-year college programs or enter the workforce.

President Obama is clearly in favour of this endeavour, and often tries to claim credit for it, but the development of Common Core was well underway before he took office in 2009. NCTM has issued a favourable position statement.: "The widespread adoption of the Common Core State Standards for Mathematics presents an unprecedented opportunity for involvement in mathematics education in the United States."

The mathematics standards include both mathematical content standards and mathematical practices (process standards) outlining what each student should know and be able to do at the end of each grade.

The mathematical practice standards mandate eight principles be taught:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

These are adapted from the NCTM standards document, and should be taught at every grade level from K–12. How this is done is left to the local implementation of the standards.

Here is the full description of principle number 6 from above:

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they use, and are careful about specifying units of measure and labeling axes. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In elementary grades they give carefully formulated explanations.

The mathematics content standards are organized into a number of domains. At each grade level, there are several standards for each domain, organized into clustered and related standards. Examples of the domains for elementary school are as follows:

1. Operations and algebraic thinking
2. Number and operations in base 10
3. Measurement and data
4. Geometry

High school domains for algebra include the following:

1. Seeing structure in expressions
2. Arithmetic with polynomials and rational functions
3. Creating equations
4. Reasoning with equations and inequalities

All domains are broken down with more detailed descriptions of what they entail.

NCTM provides a number of resources designed to help teachers carry out the expectations of the Common Core. There are lessons, journal articles, books, professional development, and online content, organized by level available. They emphasize effective instruction strategies in all grade levels, for understanding concepts as well as assessment of those concepts. You can access these by going to "NCTM Common Core Resource Collections" on the NCTM website, which also has a detailed correlation of the Common Core Standards with the Ontario curriculum.

Formal assessment of the Common Core Standards initiative is scheduled to take place now, during the projected implementation. As I mentioned in June's column, two consortiums are creating assessment items: the Partnership for Assessment Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium (SBAC). Each group has a different assessment strategy and has different support from states. PARCC has 19 states following, while SBAC has about 20. Obviously not all states have opted into a plan. Some states are working together to create a common, universal assessment, while

others are working with PARCC and SBAC to develop their own assessment.

There is still a lot of controversy surrounding the implementation of these standards and their assessment. Some feel very strongly that the benefits will far outweigh the negatives, while others suggest there is too much government intervention in the educational process.

A distinguished professor from Michigan State University, William Schmitt, wrote: "In my view, rather than a fragmented system in which content is a mile wide and an inch deep, the new curriculum offers the kind of mathematics instruction we see in top achieving nations." While Louis C.K., a parent, says: "My kids used to love math. Now it makes them cry. Thanks standardized testing and the Common Core." Stay tuned.

Finally, I would just like to add my congratulations to all of those who are involved in the production of the *Gazette*, regarding the recent news of the NCTM Journal Publication Award. Well done! ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Cultural diversity in mathematics education is a widely used expression to discuss questions around why students from different cultural, ethnic, social, economic, and linguistic groups perform differently in their school mathematics... With changes in communication, technologies, and unprecedented levels of migration, cultures have become increasingly complex, connected, and heterogeneous. One of the major impacts on education has been a substantial change in the cultural and ethnic composition of the school population. Schools and classrooms become places where teachers, students, and parents are exposed to and have to respond to many types of cultural differences. For many these differences are resources enriching the learning opportunities and environments. For many others, diversity is experienced as a problem, which is reflected in school achievement."

De Abreu, G. (2014). Cultural diversity in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 125–129). London, UK: Springer.

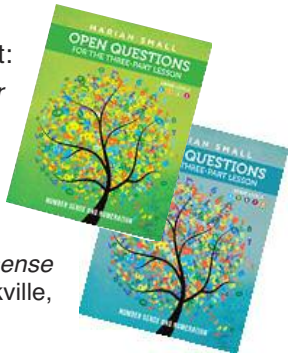
▲ Book Review: Open Questions for the Three-Part Lesson: Number Sense and Numeration

REVIEWED BY DANIEL JARVIS
EMAIL: dan.jarvis@oame.on.ca

Book review of the 2-volume set:

Small, M. (2015). *Open questions for the three-part lesson: Number sense and numeration, grades K–3*. Oakville, ON: Rubicon.

Small, M. (2015). *Open questions for the three-part lesson: Number sense and numeration, grades 4–8*. Oakville, ON: Rubicon.



In her new series *Open Questions for the Three-Part Lesson: Number Sense and Numeration* (Grades K–3, 4–8) author, Dr. Marian Small, provides a rich variety of "open questions" for the three parts of the problem-solving lesson, namely *Minds On* (getting started), *Action* (during the learning experience), and *Consolidate* (following the activity).

The adjective "open" appears to here refer to the idea of both "open-ended" questions, where multiple solutions are possible, given a shared starting point or specific mathematical question; and to "open-entry" questions, wherein students are often given a final answer and then challenged to generate different situations or number contexts that might lead to the initial target. For example, from the Grade 1 section dealing with the content area of "dividing objects into parts" from the green-cover volume (pp. 40–42):

- *Minds On* Open Question: Is one-half big or small?
- *Action* Open Question: Use square tiles to show that a single square tile could be one-half, one-third, or one-fourth of something.
- *Consolidate* Open Question: You cut a shape into fourths. Do you think the fourths have to look the same?

To further demonstrate the strategy, here is another example set of "open" questions from the Grade 7 section dealing with "percent" from the blue-cover volume (pp. 191–193):

- *Minds On* Open Question: A percent of a certain number is 35. What could the percent and the number be?

- *Action* Open Question: Investigate the difference between adding in the sales tax before calculating a discount versus taking the discount first and then adding sales tax. Explain the reason for the difference you found.
- *Consolidate* Open Question: If you know 20% of a number, what other percents of it are easy to figure out? How would you figure the other percents out?

This approach takes some getting used to as a teacher, but one can clearly see the benefits of student-generated questions/scenarios within given parameters, thereby unleashing creative thinking and more deeply reinforcing the main learning goals by virtue of comparison and peer discussion. This is classic *working backwards*—in order to more confidently move forward with shared understanding—at its pedagogical finest.

Number Sense and Numeration is one of five strands, or curriculum content groupings, within the *Ontario Curriculum* (2005), and these two volumes provide ample coverage of all of the main subtopics within this broader category, such as: quantity, counting, subitizing, estimating, decomposing, operations (add/subtract/multiply/divide), ordinal numbers, skip counting, decimals, ratios, percentages, fractions, prime numbers, integers, exponents, factors, multiples, square roots, and proportional reasoning.

These books lend themselves well to a variety of teaching contexts and goals, a list of which would include: problem-based learning sessions with students working in pairs or small groups for the "Action" portion, and in a larger whole-class group for "Minds On" and "Consolidate"; for assessment purposes, either individually (teacher–student) or in small groups (teacher–group interview); and as a repository of stimulating and relevant ideas for math journaling, homework extensions, or enrichment activities.

There are also a number of dynamite bonus features in this series: (i) each content area begins with clustered Specific Expectations from *The Ontario Curriculum: Mathematics* (2005), providing the elementary or middle school (i.e., Primary/Junior/Intermediate Divisions in Ontario) teacher with a sample approach of how to plan more effectively; (ii) all questions are followed by sample student responses, allowing teachers to be better prepared for what they may expect to hear; (iii) visual icons are used to categorize certain types of questions (e.g., money sign, manipulatives like Pattern Blocks) for ease of reference; and (iv) a Curriculum Index at the back of each volume provides instant linking of *The Ontario Curriculum: Mathematics* (2005) expectations with associated open questions and page numbers. The two volumes are spiral bound and printed on extra heavy paper, providing extra durability and longevity, especially if being shared among teachers.

As a university instructor who has worked with well over a 1000 pre-service B.Ed. teacher candidates over the past 14 years, and who has already benefited immensely from exposure to Dr. Small's insightful teaching strategies and philosophy, and from the use of her highly effective textbook, *Making Math Meaningful to Canadian Students, K–8* (2012), I view the advent of this exciting new *Open Questions* series as a true gift to experienced and neophyte teachers (and teacher educators) alike. I look forward to seeing other strands.

References

Ontario Ministry of Education. (2005). *The Ontario curriculum, grades 1–8: Mathematics* (Revised). Toronto, ON: Queen's Printer for Ontario. Retrieved from **www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf**

Small, M. (2012). *Making math meaningful to Canadian students, K–8.* (2nd ed.). Toronto, ON: Nelson. ▲

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▲ Mathematics and Special Education: Two Teachers in One Classroom



JAY SPEIJER
EMAIL: jspeijer@scdsb.on.ca

Jay is currently assigned as an Innovation Resource Teacher for the Simcoe County District School Board. In his spare time, he co-authors mathematics and physics textbooks for McGraw-Hill Ryerson and Nelson, respectively.



CONNIE GRAY
EMAIL: cgray@scdsb.on.ca

Connie is with the Simcoe County District School Board. She has been working as a Special Education Consultant and Integrated Technology Consultant for several years, and has been working with the Math and LD project team for the last four years.



SANDRA PEIRCE
EMAIL: speirce@scdsb.on.ca

Sandra is an Instruction and Assessment Facilitator with the Simcoe County District School Board. Her main interest is in supporting educators and students of Grades 9 and 10 Applied Mathematics, through the Collaborative Inquiry process, and in cross-panel professional learning with teachers of Grades 7 to 10 Mathematics.



GAIL DOHERTY
EMAIL: gdoherty@scdsb.on.ca

Gail is with the Simcoe County District School Board. She has been working in the field of Special Education for several years in both the elementary and secondary panels.

"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is." John von Neumann

Introduction

The whole co-teaching concept fundamentally changes my teaching practices. Actually, it makes it more fun. Grade 9 Mathematics Teacher

Picture a typical Grade 9 or 10 Applied Mathematics classroom. The challenges facing the Regular Classroom Teacher (RCT) are many, including: student disengagement, significant learning gaps, and a high number of students with Individual Education Plans (IEPs), specifically for students with a Learning Disability (LD). Despite the prevalence of resources designed to diagnose, remediate, engage, and instruct learners in the applied pathway[1], classroom teachers continue to voice frustration in trying to meet the broad range of needs of the students in front of them.

Now imagine that same mathematics classroom with immediate support from a Special Education Resource Teacher (SERT) available 100 percent of the time. How could this improve the level of personalized instruction available to students? How could this open doors to broaden instructional and assessment practices? What might the impact be on student engagement and achievement? These are the questions that our school board explored in the second semester of 2014–2015.

Rationale

A student diagnosed with an LD is typically supported with specific teaching strategies and/or accommodations as documented in his or her IEP. Traditional high school SERT models of support typically involve the withdrawal of students from the regular classroom which, while allowing for individualized instruction, can also sometimes result in stigmatization and loss of rich learning opportunities. While these issues are not exclusive to Grades 9 and 10 applied mathematics, our data analysis prompted this area of focus for our pilot project.

A primary goal of this professional learning project was to leverage the complementary instructional strengths of the RCT and the SERT completely within the classroom context. Anticipated benefits included reduced stigmatization, mathematics program integrity, increased opportunities for individual help and greater differentiated instruction and assessment. Another important focus was to understand specific students' strengths and needs and to target instructional strategies based on them.

[1] *See, for example, Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM), Closing the Gap, Critical Learning Instructional Paths Supports (CLIPS), in addition to several materials produced by commercial publishers, specifically targeting the needs of applied learners in mathematics.*

Project Overview

This project was offered to all secondary schools in our board, with 100 percent choosing to participate. Schools were funded, pro-rated by student population, for either one, two, or three sections of SERT time completely allocated to the in-classroom support of either a Grade 9 or 10 applied mathematics class.

To successfully implement this initiative, some cultural challenges were anticipated. For some RCTs for whom co-teaching was unfamiliar territory, the notion of having a second teacher present in the classroom can create anxiety. Another concern was to ensure that the SERT be fully invested in co-planning and co-teaching and not be relegated to a passive supporting role that might otherwise be served by an Educational Assistant (EA) or peer tutor. To address these issues, a key goal maintained throughout the professional learning component of the project was to apply various co-teaching models, strategically chosen to increase student engagement and achievement.

Professional Learning Component

All participating staff were provided release time to attend five days of professional learning, jointly facilitated by the board's Special Education, Program and Innovation, and Student Success departments, and supported by the Ministry of Education[2]. Content of these sessions was largely drawn from *Supporting Students with Learning Disabilities in Math*, an implementation resource based on research findings by the York Catholic District School Board (see Figure 1):

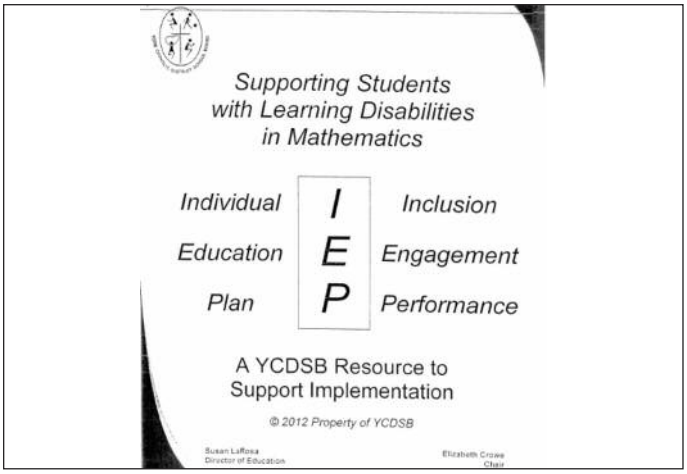


Figure 1. *Supporting Students with Learning Disabilities in Math, York Catholic District School Board.*

[2] *The professional learning sessions included* Supporting Students with Learning Disabilities in Mathematics*, Adobe Connect Series, Ministry of Education Curriculum and Assessment Policy Branch.*

The main goals of these sessions were to develop an increased understanding of the student Strengths and Needs Profile related to learning in mathematics by performing a Collaborative Analysis of Student Learning (CASL) and to explore various co-teaching models: one teach and one observe, team teaching, parallel teaching, alternative teaching, one teach and one assist, and station teaching. Teacher teams were encouraged to choose co-teaching models based on time-sensitive and content-sensitive factors, such as the needs of the learners, the nature of the mathematics and the relative instructional strengths of the teaching team members, and to vary the co-teaching models according to students' response to instruction. During the early stages of implementation, many teaching teams defaulted to the one-teach, one-assist model; however, as the program developed, more teams were reporting success with some of the other models, such as team teaching, alternative teaching, and parallel teaching, the latter of which provided particularly rich opportunities for differentiated learning groups.

Participants examined profiles of students having learning deficits in the cognitive domains and processes of verbal comprehension, perceptual reasoning, memory, processing speed, visual motor integration, and executive functioning. RCTs and SERTs then collaborated to co-create a Strength and Needs Class Profile (see Figure 2).

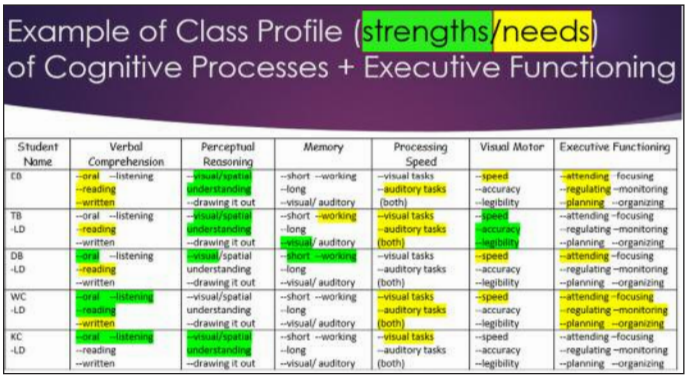


Figure 2. *Sample Strengths and Needs Class Profile.*

To complete this profile, school teams drew information from IEPs, Psycho-Educational Assessments, Strengths and Needs Committees[3], report cards, and other sources of assessment data.

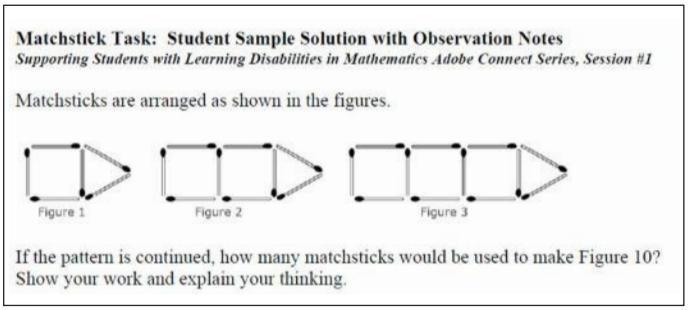
In order to better understand the cognitive barriers

[3] *In Simcoe County District School Board, a Strengths and Needs Committee (SNC) is a problem-solving team. Through an ongoing, collaborative process, the team, in consultation with students and parents/guardians, provides strategies and assistance for supporting individual students. Core members of the committee can include: parents, guardians, student, principal, vice-principal, RCTs, and SERT.*

encountered by a student with an LD, teachers were taken through a variety of math classroom activities that simulated the experience of having an LD. Through these simulations, teachers developed a better understanding about how a student may struggle with a particular task, and just as importantly, steps that could be taken to reduce or eliminate the barriers. Leveraging a student's strength while accommodating the need was encouraged as an important guiding principle of strategy selection. A key professional learning outcome was that, when the personalized strategies and accommodations were made available to the students with special needs, the students were more likely to engage in the mathematical thinking and work completion expected by the teacher.

In between professional learning sessions, teacher teams were asked to administer a rich mathematics task (see Figure 3) to one or more students with a learning disability in their classroom. The CASL process provided an opportunity for the teams to better understand the student's strengths and needs by:

- closely observing the student in a mathematical problem-solving setting
- assessing the efficacy of teacher-selected instructional strategies
- documenting and analyzing student-selected problem-solving strategies



Matchstick Task: Student Sample Solution with Observation Notes

Supporting Students with Learning Disabilities in Mathematics Adobe Connect Series, Session #1

Matchsticks are arranged as shown in the figures.

Figure 1 Figure 2 Figure 3

If the pattern is continued, how many matchsticks would be used to make Figure 10? Show your work and explain your thinking.

Figure 3. *Sample task for Collaborative Analysis of Student Learning (CASL).*

The task and the resulting student work were then moderated by school teams through these lenses. Through detailed reflection on student products, observations, and conversations that resulted, teacher teams were able to plan more deeply informed instructional next steps.

Other components of professional learning were dedicated to addressing local needs and providing co-planning time. All participants were enrolled in a Google Classroom (see Figure 4). This interface provided a means to share materials such as classroom activities, journal articles, and web links, and to facilitate ongoing communication between participants and facilitators.

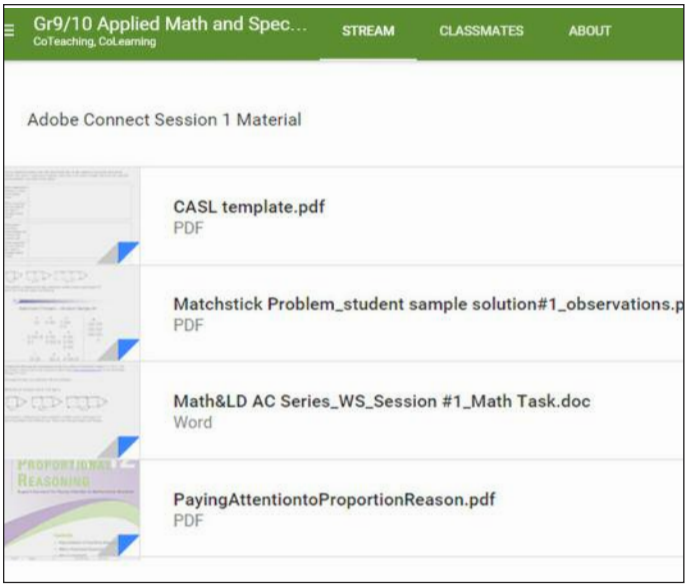


Figure 4. *A Google Classroom was set up for all participating teachers to facilitate the sharing of resources and ideas.*

Results and Feedback

Teacher feedback from this initiative has been largely positive, with many expressing a desire for the program to continue. Distinct from other support models (e.g., EAs, peer tutor), the RCT and SERT routinely shared the responsibility for making important instructional and assessment decisions in the classroom. In a Google Forms exit survey, teachers identified themselves as either a "Math Teacher," a "SERT," or "Both," the latter comprising a relatively small number. Professional learning outcomes for participating teachers were observed to differ, based on the teaching role of the participant.

RCTs without Special Education assignment (i.e., "Math Teachers") experienced the greatest learning about students with learning disabilities. Sixty-four percent of Math Teachers responded "Very Much ++" or "Definitely +" to the question, *How has involvement in this project affected your understanding of learning disabilities?*, while 82 percent responded similarly to the question, *How has involvement in this project affected the teaching of students with learning disabilities?*

By way of contrast, SERTs without a math assignment experienced the greatest learning about teaching mathematics. Sixty-seven percent responded "Very Much ++" or "Definitely +" to the question, *How has involvement in this project affected your understanding of mathematics curriculum?*, while 89 percent responded similarly to the questions, *How has involvement in this project affected your understanding of teaching strategies for all students?* and *How has involvement in this project affected your understanding of assessment in mathematics?*

The dedicated opportunities for school-based co-planning was consistently identified as a strength of the professional learning program by both cohorts of teachers. Seventy-eight percent of all respondents chose "Very Much ++" or "Definitely +" to the question, *How has involvement in this project affected your understanding of co-teaching models?* Specifically, completion of a class strengths and needs profile was found to be beneficial by 73 percent of all responding Math Teachers and 89 percent of all responding SERTs. The following are excerpts of feedback obtained from various stakeholders in this initiative.

Student Quotations

I don't know which our regular math teacher is; we just have two teachers.

I came into the course thinking that I only needed a 51, but two and a half, three weeks in, I realized that I could do it and I was having a good time with it… I felt like I was supported and I ended up resetting my goal to 80 instead of 51.

Teacher Quotations

My teaching practice has improved because I have a much better understanding of math strategies, three-part math lesson, and how to support teachers when they are supporting my students with IEPs.

While we have always co-planned grades, we feel that we were able to address the IEP concerns more in depth.

There has been a growth in assessment methods—beyond just testing them on what they know.

At the beginning of the semester, we heard a lot of "I am not good at math," and now students are not saying that, as they are seeing success within the classroom.

Principal Quotations

I was impressed with the growth of both teachers (RCT and SERT) as I witnessed innovative practices, purposeful use of technologies, three-part lesson, student interaction, modelling of different strategies… UDL (Universal Design for Learning).

Going forward, I am planning to continue the project with purposeful staffing, to propagate learning.

Sustainability

Assigning two teachers to teach one section has required a significant investment of money, time, and professional learning. Recognition of the challenges faced in Grades 9 and 10 applied mathematics courses prompted this investment by our board at this time. With notable gains achieved in the areas of student engagement and achievement, and in professional instruction and assessment, our school board is committed to exploring models of sustaining RCT and SERT co-teaching, both with and without funding. An overarching goal of this project is the re-examination of the role of the SERT, not strictly in direct support of mathematics, but in all subject areas. Under current funding and workload conditions, it is not likely feasible for SERTs to spend 100 percent of their time in classrooms in a co-teaching capacity. Identification, Placement, and Review Committee (IPRC) procedures, IEP development and implementation, and ongoing liaising with stakeholders, including students, parents/guardians, RCTs, and administration, are all important professional activities that compete for the SERT's time. The results of this project, however, have prompted our board decision makers to ask, "What if?" What if some of these competing responsibilities were removed or streamlined? Do the benefits outweigh the costs? The promising results of this pilot project strongly suggest that this is worthy of further exploration. ▲

Continued from page 18

mathematics and science concepts. She is an Associate Member of the Centre for Urban Schooling at OISE and one of the founding coordinators of OISE's Inner City Option, Initial Teacher Education. She completed her doctoral research on equity in mathematics (Supervisor: Dr. Indigo Esmonde), and is dedicated to working with students typically underserved by the educational system. Dr. Caswell is currently involved in a mathematics professional development project in collaboration with Dr. Joan Moss and educators from the Rainy River District School Board and the Seven Generations Education Institute.

Selected Publications:

Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (under review). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a seven-month intervention. *Cognition and Instruction.*

Moss, J., Hawes, Z., Naqvi, S., & Caswell, B. (2015). Adapting Japanese lesson study to enhance the teaching and learning of geometry and spatial reasoning in early years classrooms: A case study. *The International Journal on Mathematics Education, 47*(3), 1-14.

▲ The Good Luck Paper Crane: Paper Folding and Mathematics Activities



BETTY B. LONG
EMAIL: longbb@appstate.edu

Betty Long teaches mathematics content courses for elementary and middle school pre-service teachers at Appalachian State University in Boone, North Carolina. She is very involved in professional organizations that promote excellence in mathematics education.



DEBORAH A. CROCKER
EMAIL: crockerda@appstate.edu

Deborah Crocker teaches mathematics content courses for elementary, middle grades, and high school pre-service teachers at Appalachian State University in Boone, North Carolina. She is interested in appropriate uses of technology and incorporation of problem solving in mathematics classrooms.

The folding of origami paper cranes is often connected to a Japanese legend that says folding a thousand paper cranes is good luck. The good luck varies from being granted a wish to eternal good luck. Because of this legend, paper cranes are often gifts for family and friends on special occasions.

One of the best-known stories involving paper cranes is the one about Sadako Sasaki, a Japanese girl who was exposed to radiation from the atomic bomb dropped on Hiroshima during World War II. Her story and her attempt to fold a thousand paper cranes is told in the book *Sadako and the Thousand Paper Cranes* by Eleanor Coerr.

In this article, we will share directions for folding the paper cranes, and ideas for mathematics activities, using the paper cranes and children's literature. We will begin with directions for folding the paper crane. While folding, emphasize vocabulary such as *isosceles triangle, square, parallelogram,* and *kite*.

A set of ideas for mathematics activities follows the directions for folding. They can be used for Grades 5–8. Spatial visualization, symmetry, computation, and graphing are some of the mathematical concepts in the activities. One of the activities incorporates the children's literature book, *Sadako and the Thousand Paper Cranes*, by Eleanor Coerr. *The Paper Crane* by Molly Bang, another children's literature book with an easier reading level, can be used to introduce folding paper cranes to your class. While folding the paper crane, mathematics vocabulary should be emphasized. Shapes and symmetry can be discussed after each fold.

Materials Needed: square pieces of paper (origami paper or construction paper)

Emphasize vocabulary as you fold.

Folding the Paper Crane

If you use origami paper that is coloured on only one side, start with the coloured side down. After the first fold, the coloured side will face out.

1. Fold the square top to bottom (horizontally). Keep folded side at the top.
2. Find the midpoint of the folded edge and put a faint crease there.
3. Fold the top right corner down (from the folded edge) to the bottom edge. Be sure all the edges end up flush at the bottom of the paper.

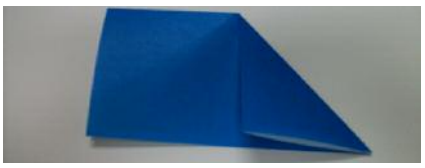


Figure 1. *Steps 1, 2, and 3*

4. Turn over the paper, keeping the folded edge at the top, and repeat step #3.



Figure 2. *Step 5*

5. At this point, you have the outline of an isosceles triangle (which could be opened up like a hat).
6. Open the hat and flatten it so that you can make a crease on the opposite sides from where the original creases are. NOTE: It will now look like a square.

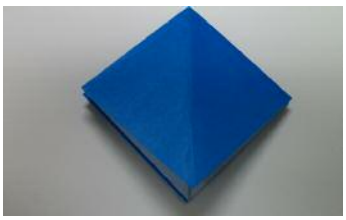


Figure 3. *Step 6*

7. Hold the square so that the open ends are toward you and the folded edges are at the top.
8. Fold the right-hand corner of the top layer over to the centre crease so that the open edges of the square will line up with the centre crease.

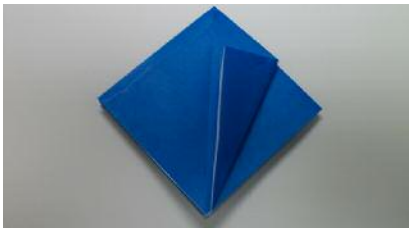


Figure 4. *Step 8*

9. Fold the left-hand corner over to the centre crease so that the open edges of the square will line up with the centre crease.
10. Turn the square over.
11. Repeat steps #8 and #9.
12. Now your paper looks like a kite.

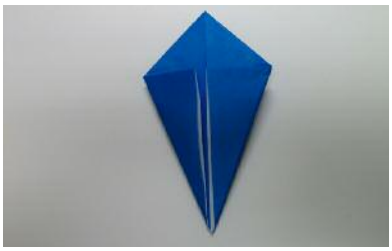
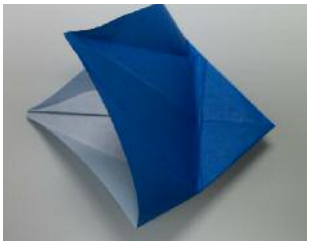


Figure 5. *Steps 9–12*

13. Fold the top of the kite (an isosceles triangle) down to make a crease, but then put it back up.
14. Turn the kite over, do the same thing, and put it back up (in other words, a double fold).
15. Pull the inside piece of paper up to the top and flatten it along the existing creases. Now it looks like a parallelogram.



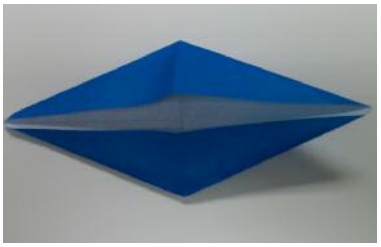


Figure 6. *Step 15*

16. Turn it over and do the same thing to the back.
17. Now the parallelogram has legs at the bottom.
18. Do the ice cream cone fold. That is, take the corner of the top layer on the right and fold to the middle. Take the corner on the left and fold it to the middle.

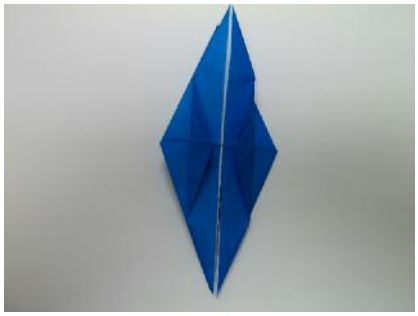


Figure 7. *Step 18*

19. Turn it over and repeat step #18.

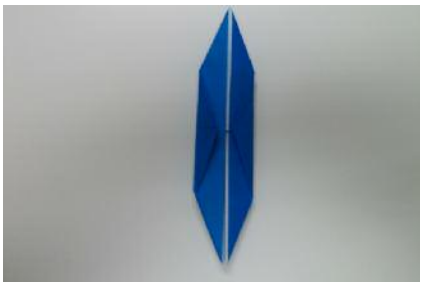


Figure 8. *Step 19*

20. Pull the right-hand leg up so that it is in between the top right part of the paper. Notice there is a crease at the point where you fold.
21. Do the same thing with the left-hand leg.





Figure 9. *Steps 20 and 21*

22. Now pull out one of the legs far enough to make the crane's tail, and pull out the other leg far enough to make the crane's neck.



Figure 10. *Step 22*

23. Fold down a little bit of the neck to make the head and beak, and curl the wings over your finger and roll them to make them point down.

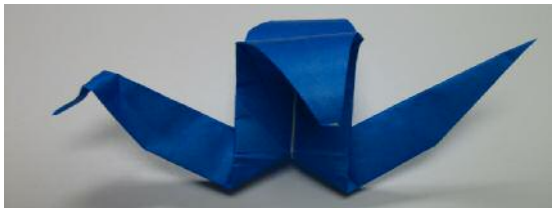


Figure 11. *Step 23*

24. If you pull on your crane's neck and hold the tail, it should "fly"!

You can get directions with pictures online at **www.origami-fun.com/support-files/origami-crane-print.pdf**.

Activities Using Paper Cranes

1. Read the book *Sadako and the Thousand Paper Cranes* by Eleanor Coerr (ISBN 0-606-02067-5) or, for a slightly easier reading level, *The Paper Crane* by Molly Bang (ISBN 0-688-07333-6).
2. Learn to fold a "good luck" paper crane. (Even if yours does not come to life as it does in *The Paper Crane*, it may bring you good luck!)
3. Discuss the various geometric shapes you get as you fold the paper cranes.
4. Discuss any symmetry as you fold.
5. In *Sadako and the Thousand Paper Cranes,* her friend Chizuko visits her in the hospital and folds a paper crane out of gold paper. Chizuko tells Sadako, "If a sick person folds one thousand paper cranes, the gods will grant her wish and make her well again. Here is your first one." Sadako dies before folding one thousand cranes. She folded six hundred forty-four. After her death, her classmates folded another three hundred fifty-six cranes, and a thousand cranes were buried with her.

   In 1958, a statue of Sadako was unveiled in the Hiroshima Peace Park. Because of the story of Sadako, each year children around the world make thousands of paper cranes and send them to be placed under the statue on Peace Day, August 6.

   Have students prepare a "plan" for folding 1000 paper cranes in a class of 25 students (or whatever the size of your class might be). Students can time how long it takes them to make 5 paper cranes. The times for each student in the class can be averaged, and an estimate for how long it might take to make 1000 paper cranes can be found. Tell them that their plan for making 1000 paper cranes should take from 10 to 30 days to complete. Have students explain why their plan is the best one possible. Have students make a graph of their plan over the 10 to 30 days. The graph can show the cranes made by each student over time or the total made by the class over time.
6. For more information about events, Sadako, and Peace Day, look at the Internet site **http://en.wikipedia.org/wiki/Thousand_origami_cranes**. You will find links to other interesting sites.

References

Bang, M. (1985). *The paper crane.* New York, NY: Greenwillow Books.

Coerr, E. (2000). S*adako and the thousand paper cranes.* New York: Scholastic, Inc.

Origami Crane Instructions. (n.d.). Retrieved from **www.origami-fun.com/support-files/origami-crane-print.pdf**

Thousand Origami Cranes. (n.d.). Retrieved from **http://en.wikipedia.org/wiki/Thousand_origami_cranes**

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DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

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4751 Mack Ave.
Plympton-Wyoming N0N 1J6
Business & Fax: (519) 899-4203
eds@oame.on.ca

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North Bay, ON
tim.sibbald@oame.on.ca

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paul.alves@oame.on.ca

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Stillwater Lake, NS
bill.otto@oame.on.ca

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Oshawa, ON
sandra.jean.price@oame.on.ca

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Brampton, ON
amanda.laxton@oame.on.ca

COMA: Melissa Black
Kanata, ON
melissa.black@oame.on.ca

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Grimsby, ON
pamela.chun@oame.on.ca

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Guelph, ON
vincent.glavine@oame.on.ca

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Toronto, ON
charlotte.aust@oame.on.ca

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Barrie, ON
paulene.washington@oame.on.ca

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Sudbury, ON
denise.filipovic@oame.on.ca

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corrine.russell-pritoula@oame.on.ca

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Pickering, ON
lori.yee@oame.on.ca

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Smith Falls, ON
kathy.pilon@oame.on.ca

SAME: Freda Liu
Toronto, ON
freda.liu@oame.on.ca

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Windsor, ON
christine.sasso@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: Katelyn Beecroft
London, ON
katelyn.beecroft@oame.on.ca

Y⁴MA: Chi Hang
East York, ON
chi.hang@oame.on.ca

DIRECTORS – TERM EXPIRING 2016

Jill Lazarus
Pembroke, ON
jill.lazarus@oame.on.ca

Bart Vanslack
Toronto, ON
bart.vanslack@oame.on.ca

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jung.choi-perkins@oame.on.ca

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anne.yeager@oame.on.ca

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Kanata, ON
patricia.kehoe@oame.on.ca

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bill.lundy@oame.on.ca

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Todd Romiens
Windsor, ON
todd.romiens@oame.on.ca

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Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Dan Jarvis
North Bay, ON
dan.jarvis@oame.on.ca

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