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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

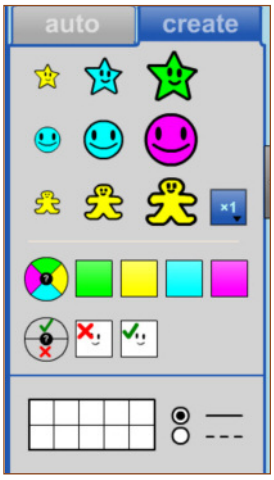
Vol. 55 #2
December 2016

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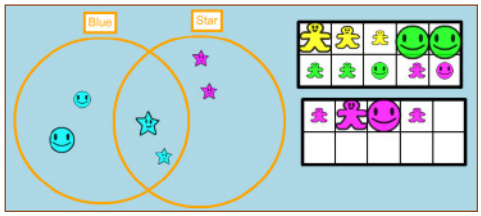




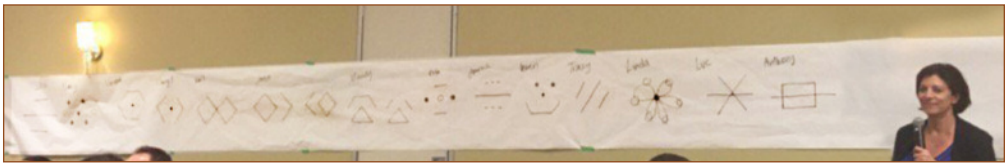
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The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Amy Lin, along with **separate files for each of the text, graphics, and/or photographs**. Please email all files to Amy Lin at **gazette@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and email address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Message



AMY LIN
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Amy Lin's background in the sciences and the arts helps explain why her passion in mathematics education is to develop both the logical and creative aspects of the subject. She is currently working on her PhD at Brock University in math education and is a Vice-Principal at Applewood Heights Secondary School. She is often asked to speak on mathematics teaching and learning around the province and beyond, and she is an author of a number of mathematics resources for students and teachers at both the elementary and secondary levels. She has initiated and successfully led many professional development projects in school boards in Ontario.

This school year, I had the privilege of teaching a pre-service math class and the opportunity to work with eager and enthusiastic prospective math teachers. Each week, as we deal with different topics, I watch as they try out math learning activities, technology, and strategies for teaching students of varying abilities, all of which will hopefully help them be successful in their future placements and encounters with students.

It did make me think about my own experiences coming out of teacher's college—and what I wish my professors had told me.

I wish I knew that my love for math and being good at math wasn't enough to teach math. Being a great math teacher is so much more than knowing how to teach the content and following a course curriculum. I learned through experience the importance of creativity, perseverance, and grit, and learned about what truly motivated students—all of that went beyond my ability to do math.

There is too much focus on structures and rules. For example, a good math lesson needs context with real-world applications, or a lesson has to be made up of three parts, or problem-based learning is about making something or doing an activity. It is these misunderstood models that can sometimes distract teachers. The focus should be on showing students how and where math is used, what is actually mathematically stimulating a child while learning, and the purpose of the math lesson. What is the important mathematics that students are actually learning, and how do I question effectively to bring that learning to the surface? It is not the structure of the lesson, the context, or the activity itself that makes this happen.

I realized after I experienced teaching a course or two that I did not have to package the curriculum into units. I also learned that all children in my classroom did not need to know the same things. Teaching math is not linear, and uniformity is what crushes creativity and thinking of our students; in other words, teaching math is messy.

Perfect lessons don't exist, and great lesson plans don't necessarily translate into great lessons. Being prepared is always a good idea, but sometimes you can't prepare for what really happens in the classroom. It is much more important to work on the effect or impact of an idea—and an idea itself doesn't mean there is a real effect. A lesson plan plays out best when your classroom environment and the culture of your classroom allows for your students to play, talk, question, and be curious.

In the early days of teaching, I was told that the purpose of assessments was to drive my instruction. It made sense to me that I would need to know what my students are excelling in and what they are struggling with to be able to provide them with the appropriate learning experiences. This doesn't mean that the teaching tasks have to look like the assessment tasks. I also have come to realize that true learning and understanding isn't linked to letters or numbers. One reason for that is that there are so many different ways to solve a problem—and sometimes I see 25 different ways—each of my students explaining in their own special way.

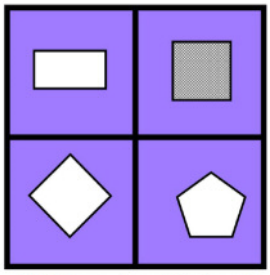
I now know that I can empower my students by not making it about grades, but to encourage students to be motivated about learning through better questions and not answers, conversations with peers, and through learning experiences that focus on their creative-thinking and problem-solving abilities. It really isn't about the teaching—it's about the learning. The learning is not accomplished by a focus on grades.

Lastly, we are more than just teachers. All of us now know that we are also first responders, counselors, moms and dads, cheerleaders, and mentors. Nothing prepares us for this. As a new teacher, I was overwhelmed with my responsibilities—but it just happened. Somehow we are there for our students for whatever they may need at the time. I want to tell my future teachers to be empathetic and to always take care of their students because every student has a story. Teachers will be surprised at that moment when they realize how rewarding it is to be a math teacher, the love they have for their students, and despite all the challenges they face and the growth they experience (and growth is ongoing), ultimately it is all worth it.

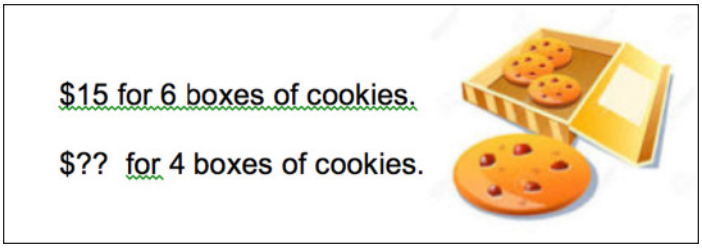
In this issue of the *Gazette*, Immaculate Namukasa takes us on her journey of using virtual and physical manipulatives in a Grade 7 and Grade 8 integers unit. Nate Banting writes about probability problems that can cultivate curiosity and conversations in the math classroom. A feature article reports on highlights at the OAME Leadership Conference, held in Toronto in early November, with Jo Boaler. There are two tributes to math educators Lorna Morrow and John Egsgard.

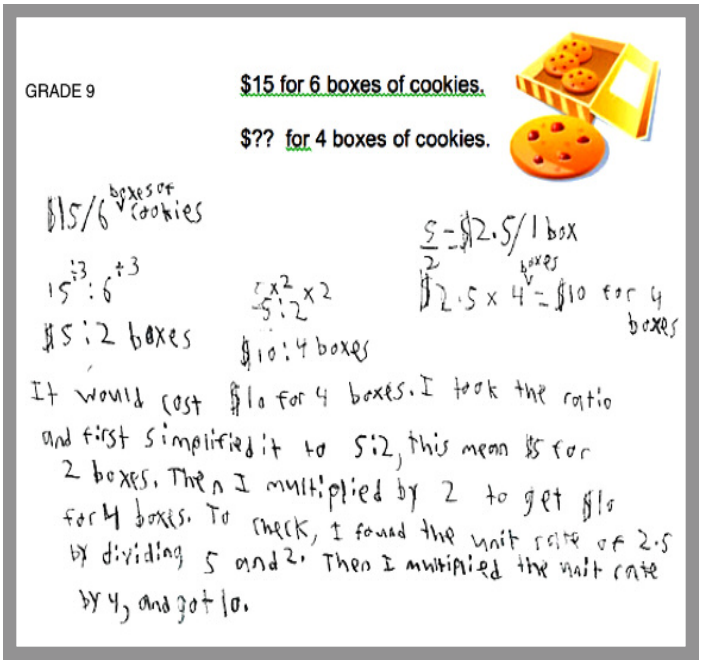
Our regular columns promise to showcase new math tools and strategies, as well as updates from the Fields Institute and NCTM. I would like to thank David Costello for taking on the MB4T column as this issue's guest columnist. Throughout this issue you will see solutions to two problems: a "Which One Doesn't Belong?" question and a problem on proportional reasoning. The questions are pictured below.

Wishing everyone a magical and safe winter! ▲



Which one doesn't belong?





▲ President's Message



JUDY MENDAGLIO
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As I write this column, a beautiful Fall is just starting to turn cold. At OAME, we are getting ready for the Leadership Conference with guest Jo Boaler, an event that I hope many of you will have been able to attend. By the time you read this, snow will have fallen, The Leadership Conference will be over, the U.S. election will be over, and we will be on the cusp of a new year.

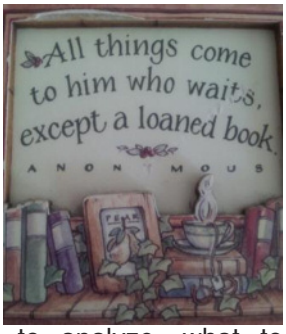
There seems to be an awful lot of activity around mathematics education in this province. I know that many of you have been searching through the data that was released by EQAO at the start of the year, identifying challenges and next steps for your own schools. Many of you have also been involved in Renewed Math Strategy meetings and workshops, developing implementation strategies for your boards and schools. Perhaps you are sharing what you learned at this summer's Grade 9 Applied Institutes in your schools. Those of you who were at the 2016 Leadership Conference may be implementing some of the strategies you learned from Jo Boaler. Many of you will be planning for your trip to Kingston in May, to attend the OAME 2017 Annual Conference. Perhaps you are waiting to hear if your proposal has been accepted. It is a busy time across the province.

In the last message, I mentioned that I enjoy reading about mathematics, and I shared an "aha" moment I had while reading Ian Hacking's *Why Is There Philosophy of Mathematics At All?*. Knowing that I will find these tidbits of wonderful ideas about how to better engage students with math in every book is one of the reasons I read them. To be clear, I am not alone in enjoying these books. In fact, I have many colleagues and friends who borrow my books, or lend me theirs, so that we can share these ideas together. I have lent so many books that have never been returned that a friend bought me this fridge magnet.

So what have I been reading lately? I have abandoned my own bedside stack of books, and have been mining the local library. A friend whose judgment I value recommended that I read *Dataclysm: Who We Are (When We Think No One's Looking)*, by Christian Rudder. Rudder is a Harvard University mathematics graduate who went on to found dating site OKCupid. OKCupid was sold to Match.com for $50 million dollars in 2011. (And to think that I was told that mathematicians never get rich!)

Rudder has gathered an amazing array of findings about our behaviour, using vast amounts of personal data mined from Google, Facebook, Twitter, and the millions of people who use OkCupid. What makes the book especially interesting is that this data did not come from a census or a survey. Rather, it is data that captures and describes the patterns in our personal lives.

I can't help but believe that within the pages of this book are many really good answers to the most popular math-student question ever: "Why are we learning this anyway?"



My teacher-eye was also intrigued by this passage from *Dataclysm*: "Numbers are tricky. Even without context, they give the appearance of fact, and their specificity forbids argument… But behind every number, there's a person making decisions: what to analyze, what to exclude,…." These statements made me think about an article I read long ago that has always stayed with me—*The Myth of Objectivity in Mathematics Assessment*, written by Lew Romagnano and published in 2001, long before *Growing Success* was released in Ontario in 2010. To me, Rudder's remarks remind me of the ongoing struggle between qualitative and quantitative data, between what is valid data and what is reliable data. Romagnano's argument that all assessments of students' mathematical understanding are subjective had me immediately re-evaluating my own assessment and evaluation practices. In the years since, I have become even more convinced that Romagnano was correct. EQAO was still new to us, and many of us went to "moderated marking" sessions. Participating in these sessions provided more opportunity for me to understand my own beliefs about the "objectivity" of math assessment in a new way: at each table, we were given explicit instructions about the purpose of each question (what was being assessed), and we were given very clear mark allocation instructions. Yet, as individual, well-experienced teachers, we came up with very different marks for the same work.

I hope that you have a great start to the new year and that 2017 is math-ful for you! ▲

Lorna Morrow Remembered

BY PETER SAARIMAKI AND HER FELLOW OAME PAST PRESIDENTS



On August 19, 2016, Lorna Morrow passed away after a brief illness. Lorna was born in 1939 in Espanola, Ontario and lived a full, varied, adventurous, eclectic, and generous life. Lorna is survived by her brothers Richard and Michael, their families, and her many friends and colleagues.

After several moves around the province, Lorna graduated from Kingston Collegiate and Vocational Institute, Queen's University, and Peterborough Teachers' College before starting teaching in a small two-room school just outside Kingston in 1960, also serving as principal. Then, in 1963, she moved to the North York Board of Education (NYBE), where she spent the rest of her career, until she "retired" in 1994, or at least that was when she left full-time employment.

In the NYBE, Lorna taught secondary and junior high before she became a K–13 math consultant, working with John Del Grande for many years. As a consultant, Lorna was noted and appreciated for the practical help and support she regularly gave to teachers, and the NYBE presented her with a citation recognizing her contribution to mathematics. Lorna worked on the Gauss Contest problems committees for the University of Waterloo's Centre for Education in Mathematics and Computing (CMEC) for many years, and encouraged teachers and students to participate in the math contests.

During these years, she also earned her Master of Arts, Master of Education, and Doctor of Education degrees from the University of Toronto and the Ontario Institute for Studies in Education. She was also giving talks, presenting workshops, developing curricular materials, and authoring articles and books. By the early 70's she was co-author with John Del Grande on several books. And that was just the start of her publishing career.



Lorna was also volunteering her time and talents to various math organizations. She was an active member and supporter of OAME from its beginnings, serving as President in 1980–1981. Lorna's workshops at mini-conferences and the Annual Conference were always well attended and much appreciated.

NCTM also benefited from Lorna's efforts as she wrote and edited many National Council of Teachers of Mathematics (NCTM) publications. She was part of the organizing team for the 1982 National Council of Supervisors of Mathematics (NCSM) and NCTM conferences in Toronto. She is the only Canadian to edit an NCTM Yearbook. Later she authored many of the NCTM Addenda series.

Lorna was also heavily involved in NCSM from its early days. She was Canada's third Director in the early 80's, was involved in conference planning and a key instigator for the first NCSM Glenn Gilbert Award being presented to John Del Grande in 1983. In addition, Lorna was part of the organizing committee for the International Congress on Mathematical Education (ICME) in 1992 in Laval, Quebec.

After "retiring" in 1994, Lorna continued with her work with CMEC and was awarded the prestigious Society of Descartes Medal in 1994. Lorna continued travelling for math conferences as well as pleasure, visiting such far-ranging places as Australia, England, Germany, Russia, Scotland, Spain, Venezuela, and Tanzania, where one summer, she spent her time training teachers.

And what did Lorna do in her spare time? Well, she "took up" hobbies. These were not just passing fancies. Lorna would tackle new projects with a passion. Her hobbies included knitting, embroidery, crocheting, rug hooking, jewellery making, silversmithing, quilting, all types of sewing, making ceramic pieces, and recently, woodcarving and soapstone carving. Lorna also had a private pilot's licence.

She enjoyed playing the piano and accordion, taken up at an early age, and was learning to play the guitar and the bagpipes. Many will remember her from early OAME conferences as part of "Johnny D and the Math Cats."

Lorna was also deeply involved in genealogy, a wonderful problem-solving environment. I first met Lorna at a problem-solving workshop for Metro Toronto consultants she ran with John Del Grande in the early 80's. She introduced me to one of Piet Hein's "Grooks," namely:

Problems worthy of attack
Prove their worth by fighting back.

Lorna will be remembered for her generosity to others, her passion for new projects, and her enthusiasm for mathematics and life. She is finally at peace after a career and life fully lived.

▲ In Memoriam

JOHN CHRISTIAN EGSGARD

March 25, 1925 – July 10, 2016



Teacher, Leader, Mentor, Friend, Husband, Father, Author, President

Local Chair for Toronto NCTM Regional Conference, 1961;

OATMP President, 1962–1963;

OAME President, 1974–1975;

NCTM President, 1966–1968;

Descartes Medallist, 1968;

CAMT President, 1969–1970;

OAME Life Member, 1986;

Prime Minister's Award for Excellence in Mathematics Teaching, 1993;

Editor of the OAME *Gazette*, 1996–1999;

NCTM Lifetime Achievement Award, 1997.

BY GREG CLARKE, JACK LESAGE, AND BILL MORRISON (WITH CONTRIBUTIONS BY NEIL EGSGARD, ALF STONG, AND MARY LOU KESTELL)

The world lost a great leader in mathematics education, and a great human being, this past summer when John Egsgard passed away. In his 91 years, John touched many lives, and was loved and respected by all—his family, friends, colleagues, and his students. Though John was a humble man, he made lasting impacts on the lives of the students he taught, as well as on the mathematics community worldwide.

A gifted and internationally acclaimed mathematics educator, John graduated from St. Michael's College School (where he was already a member of the Basilian Order), completed Bachelors and Masters Degrees in Mathematics at the University of Toronto, and was Gold Medallist in his graduating class. John was ordained into the Catholic priesthood in 1953 and continued to teach at St. Michael's College School in Toronto, where he started his career in 1950, until 1971 (with a year at Aquinas Institute in Rochester). There he was head of the Mathematics Department, Superior, and hockey coach. He then taught at Phillips Exeter Academy in New Hampshire and Phillips Andover Academy in Massachusetts in 1971/72. In Orillia, he taught at Twin Lakes Secondary School (1972–1987), with a year at the International School in Geneva, and Patrick Fogarty Secondary School (1987–1995), where he coached the ski and rugby teams.

By the 1960s, John was involved with a number of professional organizations: OATMP (Ontario Association of Teachers of Mathematics and Physics), which later became OATM, which later became part of OAME; NCTM (National Council of Teachers of Mathematics); OMC (Ontario Mathematics Commission). To date, he is the only non-American to have been president of the NCTM.

John was one of the original Descartes Medalists (University of Waterloo). He was the founding president of CAMT (Canadian Association of Mathematics Teachers). On behalf of CAMT, John attended the First International Congress on Mathematical Education, in Lyons, France in 1969. He took part in subsequent congresses up to and including the Eighth International Congress in 1996.

John was one of the founders of the newest chapter of OAME, the Mathematics Association for Cottage Country (MACC or MAC^2) in 1995, the year he retired from teaching at age 70. He shared his vast knowledge of mathematics through sessions for teachers at many MAC^2 conferences, including the very first one in 1995. As a member of the Council, his experience and insights helped to guide MAC^2 through the growing years. We were blessed to have one of the leading Canadian math educators in our chapter.

After his term as editor of the *Gazette* had ended, he served as Associate Editor as needed.

John was also the co-author of over 25 mathematics textbooks.

In the early 1970s, he met and fell in love with an American mathematics educator, Lyn McLane. John left the priesthood and, with the Pope's permission, John and Lyn married in 1973. John and Lyn shared a remarkable love, and raised their three children—Jennifer (Ashvin Malkani, Alexander, and Zachary); Erik (Kristina, Rowan, and Walker); and Neil (Honorine)—in Orillia.

John was talented, kind, thoughtful, extremely devoted to his family, and deeply in love with Lyn. He is profoundly missed.

MEMORIES OF JOHN

From Deacon Alf Stong, retired Judge from the Superior Court of Ontario, former student.

I first got to know John as Father Egsgard, the Basilian priest who taught at Saint Michael's College School in Toronto. I was in Grade 10 and he was our math teacher. He was the kind of guy who could make a most difficult subject, like trigonometry or calculus, interesting and understandable. Unlike some teachers, John's teaching ability was enhanced by his dry sense of humour, which often eluded us, but by which he was able to command our attention and respect, rather than demand it.

From Mary Lou Kestell, *Abacus* co-editor, former OAME President and Life Member.

I visited John shortly after Lyn's passing. His daughter sort of called on the math community because he had moved from Orillia to Toronto. John was famous! He was a leader in the Ontario math education community (NCTM president!!). I always felt special in the company of John and Lyn. Even though we both had attended many of the same OAME conferences, I'd never talked with him one-on-one until the day I visited his new place in Toronto. We

connected on my visit to the retirement home because I lost the love of my life seven years ago. We talked about family and shared stories of our teaching experiences. So glad I went. He was a GREAT man.

From Greg Clarke, OAME webmaster and Life Member, MAC² member, former student and colleague.

I first met John when I attended Twin Lakes Secondary School in Orillia, and I was lucky enough to have Mr. Egsgard (as I knew him then) as my Grade 9 Math teacher. He made mathematics interesting and fascinating, not only for me, but for all my classmates (which I came to appreciate later as a teacher, was an incredible challenge!). He knew exactly how to nudge students to continue to the next step, and for those of us who were "done early," knew exactly what to suggest to keep us investigating, or puzzling, or exploring mathematics. I credit John with helping to foster my love for mathematics, and my decision to major in Mathematics at the University of Waterloo.

I was graced once again to cross paths with John at the beginning of my teaching career. In my second year of teaching, I joined the staff of Patrick Fogarty Secondary School in Orillia and was now a colleague of Mr. Egsgard (who insisted I call him John). In that first year, John and I team-taught a class of 40 Grade 11 students. That hands-on approach to learning from the master himself was probably the greatest learning experience of my life. One time that year, John said something which sticks with me still, and I think summarizes perfectly his thoughts on the art of teaching: "A starting teacher starts by teaching the curriculum. A good teacher teaches the subject. A great teacher teaches the student."

As I was starting out, I, as many do, struggled to keep afloat by following the curriculum, worried about the lessons, techniques, topics that had to be covered in the next day, week, or month. John allowed me to see that eventually, with experience and reflection, I would see how all these would fit into the beautiful subject of Mathematics, and how it was all connected. He also demonstrated daily, how to spend time with each student, to learn their needs, strengths, and weaknesses so that you could help them along the path of understanding, challenge their thinking, and impart (ideally) a bit of the joy of mathematics along the way.





At an OAME conference, John presented a session he called "Teaching for 40 years, and looking forward to 40 more." John was the consummate teacher, and despite his world travels and connections with international math associations, he was, quintessentially, a dedicated teacher of mathematics, and loved it and his students.

He did get five more years, and we were blessed to have him for as long as we did.

From Neil Egsgard, son of John and Lyn Egsgard, excerpts from the funeral eulogy.

My dad, John Egsgard, was a man of faith and family, blessed with: remarkable talents in education, intelligence, dedication, and passions for wine, technology, and adventures. As a father, he provided me a model of how to be a good and caring man, a devoted father and husband, how to create adventures in life and overcome obstacles.

Dad was a man of incredible educational power. He was able to make mathematics into a subject in which any student could succeed. Dad found his philosophy to apply to students from all ranges of intelligence and motivation. It was that, "students will enjoy mathematics if they are treated with love and respect and taught at their level of intelligence so they can be successful." Treat people with love and respect; use little successes to grow big successes.

In his 91 years, Dad was making the world a better place, dedicating his life to his faith, improving the lives of thousands of students, finding and creating adventures, discovering Lyn and building a strong and loving family. Now Dad is finally home again with the woman he loves and with God.

For some online tributes to John, check out ***www.legacy.com/guestbooks/ottawacitizen/john-christian-egsgard-condolences/180634216****.*







▲ Ask Assessment Abby: Help with Student Engagement

ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

How can I use assessment to help students be better engaged with math?

Help me with student engagement,
Questioning Assessment

One of the questions we get asked a lot from teachers is, "How can we assess what students know and can do in mathematics through conversation and observation?" Well, the truth is that this is really easy, provided you give students an opportunity to act as mathematicians in math class. When students are working on a problem that they find interesting and they are working together in a collaborative effort to get answers to that interesting problem, then the teacher is freed from delivery mode and is now able to wander around, observe what mathematics the students are doing, and listen to how they are talking about the mathematics they are doing. This type of classroom dynamic provides the teacher with golden opportunities to assess the strengths and challenges of their students. Immediate feedback (from peers as well as teachers) keeps students moving forward through a rich problem. While you are observing, plan how the sharing of student solutions should happen. Students will be engaged when they see that different ideas and strategies are valued.

So where to find such problems? There are excellent resources for whatever grade or course you are teaching. Never before have teachers shared their best tasks as now. The OAME *Gazette* and *Abacus* are a great place to start looking. Check out different websites, including the OAME website (**www.oame.on.ca/**) or EduGAINS for Mathematics K–12 (**www.edugains.ca/newsite/math/index.html**). Follow @oamecounts on Twitter (which appears on our home page). Attend your local chapter conferences or the OAME Annual Conference. There are all kinds problems out there that can be used to engage students in the work of mathematicians.

Keep Math Rich with Students in Mind,

Assessment Abby▲

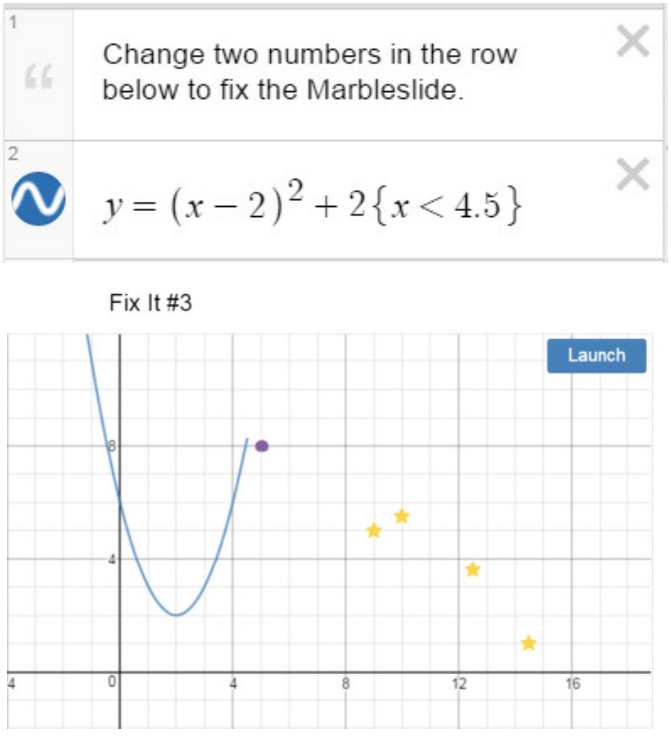
▲ Technology Corner: Desmost Marbleslides and Sort Cards



MARY BOURASSA
EMAIL: mary.bourassa@oame.on.ca
TWITTER: @MaryBourassa

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Desmos is an educational technology company that never stops innovating. They listen to teachers and create products that help teachers be more effective in the classroom. The activities they have developed have strong pedagogy in addition to being engaging and fun for students to complete. Marbleslides is no exception. The Marbleslides activities allow students to test their knowledge of a particular type of graphs (lines, parabolas, periodics, exponentials, rationals) in a series of challenges. Here is an example:

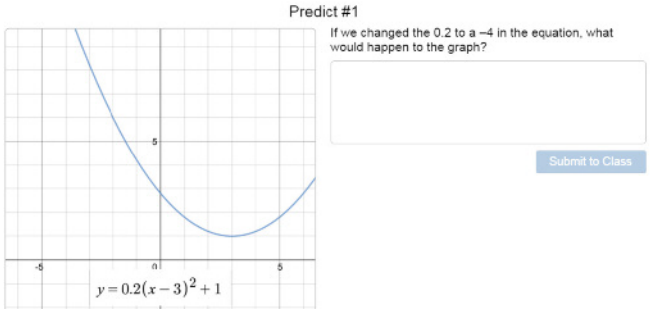


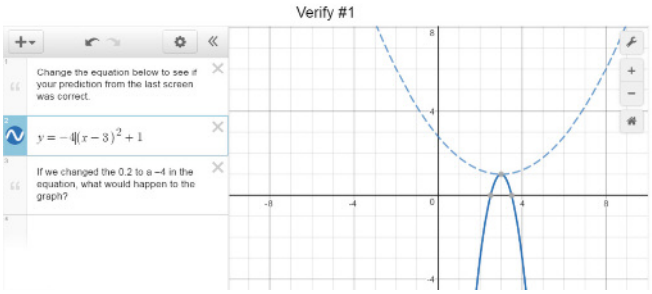
When you click the Launch button, marbles fall and hopefully go through all the stars. This is what you want to achieve:



If a student does not succeed on the first try, he or she can try again and again, changing parameters and launching as many times as needed in order to find an equation that will work.

In between the challenges, students are prompted to reflect and predict to ensure that their understanding is solid. They can test their predictions and adjust as they learn from their mistakes.

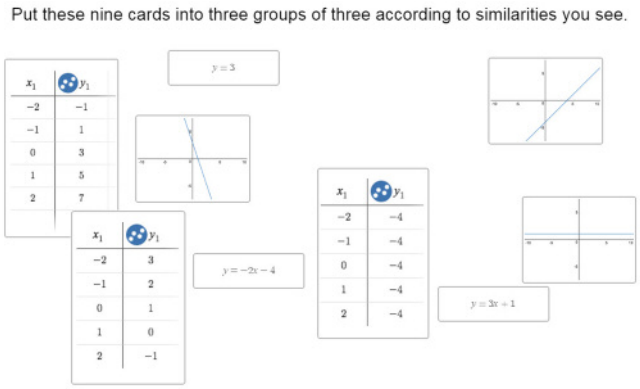


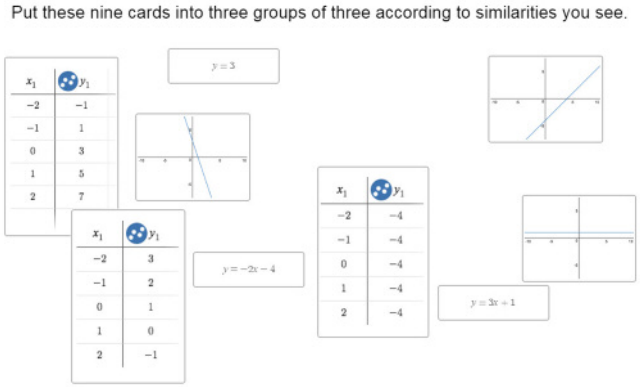


The teacher dashboard, for this and all Desmos Activity Builder activities, shows what screen each student is working on, as well as the final equation used. It also allows the teacher to pause the entire class or force every student to a particular screen (teacher pacing). You can anonymize students' names and show interesting graphs or equations to allow for a classroom discussion before moving on.

You can add a Marbleslide screen to any activity you create or edit, and can do the same with a Card Sort screen. Card Sorts allow you to add Math or Text cards, Image cards, or Graph cards. The idea is that students will sort the cards into groups.

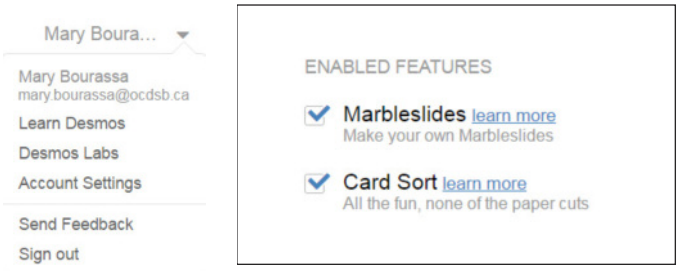
Here is an example from the Desmos activity "Card Sort: Linear Functions":





You can include an answer key so that groups of cards will appear in green (correct) or red (incorrect) on the teacher dashboard. The summary also allows you to see the most common incorrect groups or cards. Again, you can pause the class as needed to discuss the misconceptions.

In order to be able to include Marbleslides or Card Sorts in your activities, you need to turn them on. Once you have signed in on the teacher.desmos.com page, you need to click the drop-down menu next to your name at the top right of the screen. Then, choose Desmos Labs and check the boxes next to Marbleslides and Card Sort.



To keep up with all the latest developments from Desmos, I strongly recommend signing up for the Des-Blog. You can sign up to receive the newsletter via email on the **blog.desmos.com** page. The Desmos Twitter account is also a great way to ask questions, as they are always helpful and quick to respond. ▲

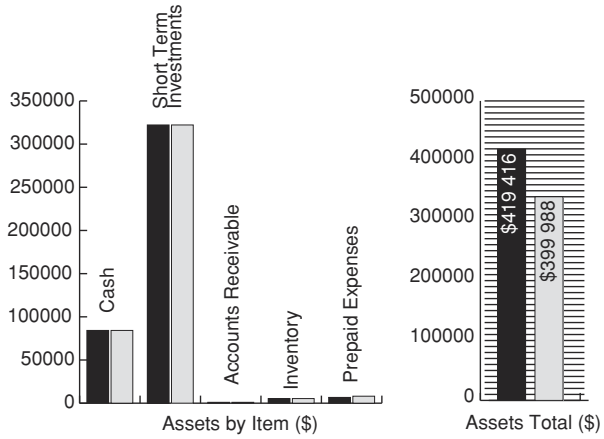
▲ OAME Financial Statement 2015-16

Notice to Readers

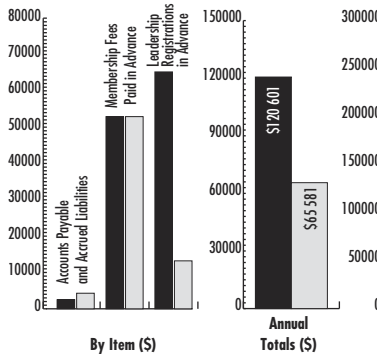
On the basis of information provided by management, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as at August 31, 2016, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario, September 21, 2016, SKRYPNYK GROUP, Professional Corporation, Authorized to pratise public accounting by CPAofO.

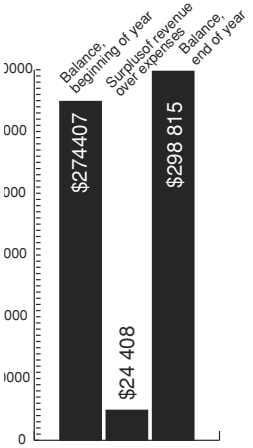
Current Assets



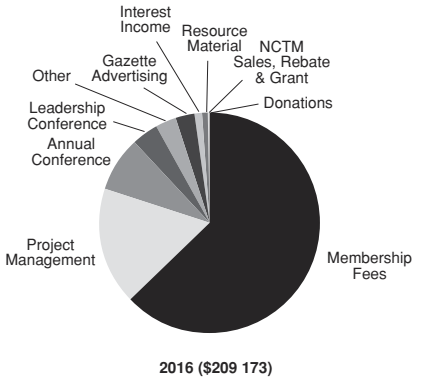
Liabilities and Fund Balance



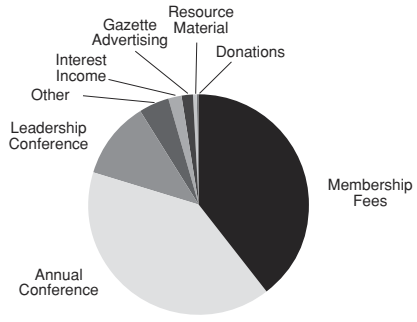
Operating Fund Balance



Revenue

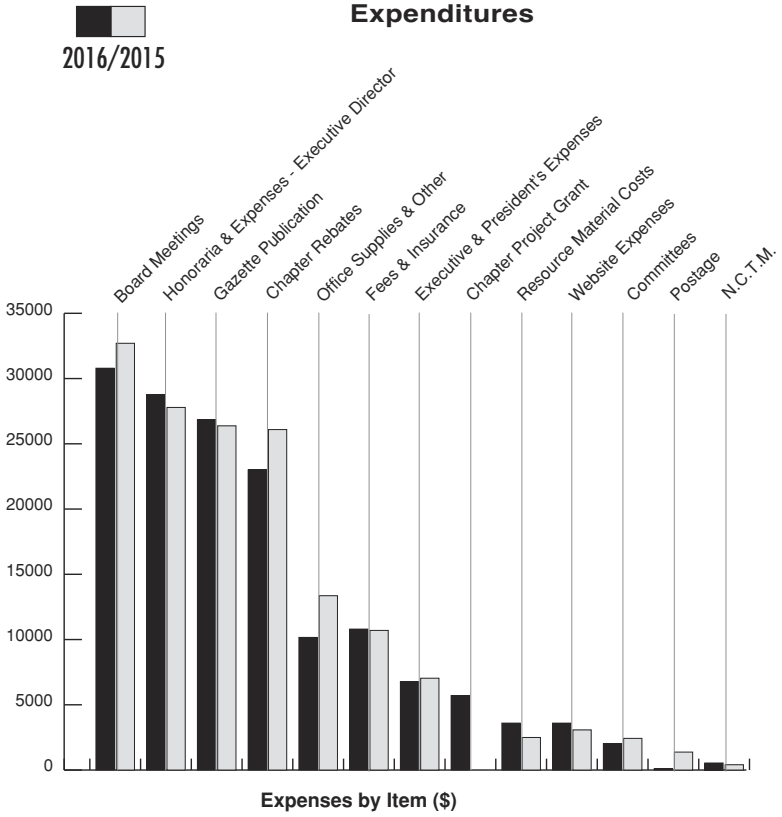


2016 ($209 173)



2015 ($261 062)

Expenditures



▲ Hey, It's Elementary: Choose a Path That Includes Math: Bringing parents and teens together for planning mathematics pathways and pursuits



LYNDA COLGAN
EMAIL: colganl@educ.queensu.ca

Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

> Less than 50 percent – The average annual percentage of secondary school graduates, across a selection of provinces, completing Grade 11- and 12-level mathematics and science courses. (*Let's Talk Science, Spotlight on Science Learning, 2013*)

In 2013, *Let's Talk Science*, a national STEM Outreach organization (housed at the University of Western Ontario), in partnership with Amgen Canada, published a startling research report: *Spotlight on Science Learning 2013: The high cost of dropping science and math (SOSL 2013)*. To prepare the report, researchers at *Let's Talk Science* and *Impakt*, a research-based global consultancy, carried out an environmental scan of peer-reviewed literature, reports, media articles, and institutional and government websites to investigate the financial, opportunity, and social costs of student disengagement from science, technology, and mathematics courses at the secondary school level. To supplement the environmental scan, interviews were conducted with stakeholders in Canadian business, innovation, media, public policy, economics, governance, and skills development. Participants contributed commentary to the initial findings of the study to help frame the "costs of student disengagement" as they applied to Canadian society, and point to recommendations to improve the percentage of secondary school students in Canada graduating with STEM courses.

The report included many disquieting findings. For example, did you know that the average annual expenditure per student in publically funded schools (from Kindergarten to Grade 12) in Canada from 2011 to 2012 (ranging from $11 360 in PEI to $22 202 in the Northwest Territories) was $12 557.00? Imagine the cost knowing that at least 20 000 Ontario students return each year for a fifth year of secondary school, *after meeting graduation requirements*. Did you know that the average cost of one semester of undergraduate university tuition or two semesters of college courses is $2790? Or that the institutional cost for each first-year Canadian college or university student who fails to progress to the second year is between $6111 and $10 800? Or this…

> *…in general, students without Grade 12 math can expect to be excluded from 40 to 75 percent of program areas, and those without Grade 12 science can expect to be excluded from 30 to 65 percent of programs at Canadian universities. Students without Grade 11 or 12 math can also expect to be excluded from half of community college programs.*

Did you know that 70 percent of Canada's jobs require a science, technology, engineering, and mathematics (STEM) education? And some of the careers that require a background in STEM education may surprise you—including being a Chef, Early Childhood Educator, Welder, Industrial Designer, and Actor. According to SOSL 2013, ALL of Canada's top jobs require a STEM background: Oil & Gas Drilling Supervisor, Head Nurse, Health Care Manager, Electrical and Telecommunications Contractor, Chemical Engineer, and Petroleum Engineer.

The report underscores the fact that by the time students apply for skilled-trades training, college, or university, it's too late. They find out that they are unprepared in prerequisite courses or lack the experience required to pursue a number of career options.

The report underscored the fact that students, their parents, teachers, and guidance counsellors need to be aware of the options and opportunities lost when they say goodbye to STEM learning in secondary school, and the financial and time costs involved of later re-engaging in STEM learning.

Then in 2015, Let's Talk Science, in partnership with Amgen Canada, published a related research report called *EXPLORING PARENTAL INFLUENCE: Shaping teen decisions regarding science education*. The results for this study emerged from the data obtained from an online survey of 805 randomly selected Canadian parents, whose children (aged 13–17) were entering Grades 7–12 in Fall 2015. The sample was balanced on a 50/50 gender split and to regional data for the parents in proportion to the census.

The good news is that the majority of parents polled

(57 percent) reported that they discussed education goals or pathways with their children often, and 51 percent said that they frequently talked to their children about career goals.

Yet while a whopping 75 percent of parents agree that STEM education is very important for today's youth, and two-thirds of parents (67 percent) agree that it's important for all children to participate in STEM education until the end of high school, only 28 percent of parents discuss the value of these courses. Why? At least one-third believe that there are enough careers for their children that do not require STEM, or that STEM courses are too difficult for their child.

The research report also highlighted this finding: *Almost nine in ten parents (88 percent) believe they have a lot or some influence on their children's education—and their children agree.* In fact, teens said that parents, by far, have the greatest influence on their educational direction, with teachers a distant second (76 percent versus 24 percent). As students try to decide what to do after high school, parents also remain a highly useful source of information, according to 94 percent of the youth surveyed.

Given the enormous influence of parents, it is important to recognize that it is largely their encouragement that will help children pursue the grounding they need to open up academic and career choices, avoid the need to make up courses after high school (due to dropped science and math), and prevent lost opportunities.

As a mathematics educator and a STEM Outreach professional, the findings of these two important research reports were deeply troubling for me. Also worrying were the numerous anecdotal reports from parents across the province who attended my workshops and seminars about *Inspiring your child to learn and love math*[1]. These concerned parents expressed enormous confusion about how to support their children's math course selections for Grade 9 and beyond. Most often, they reported overwhelming frustration because they had such limited access to resources to help them in this critically important endeavour. Perhaps of greatest concern to me was the knowledge that students and parents have a limited menu of available experts at the school level to support them in decision making. Why? In Ontario, secondary schools report an average of 381 students for every guidance counsellor (with some secondary schools reporting a ratio as high as 595 students per guidance counsellor. Moreover, only 17 percent of elementary schools report that they have guidance counsellors (People for Education, 2016). This means that the burden of support falls squarely on the shoulders of mathematics teachers, many of whom are not well positioned to offer extensive, comprehensive, and targeted advice because of other duties, responsibilities and limiting factors.

It was out of these needs that a second CODE toolkit, *Choose a Path That Includes Math*, designed for both parents[2] of teens in Grades 7 to 12 and students in Grades 7 to 12 was born. Unlike *Inspiring Your Child to Learn and Love Math*, this new toolkit (to be launched early in 2017) is entirely web based and highly interactive. Like *Inspiring Your Child to Learn and Love Math*, it is multimedia and includes videos, downloadable documents, and a range of formats, from "quick scan" to detailed.

Here is an overview of the toolkit, which is currently under development. One-half of the website will be directed toward parents. It will answer the question, *What do you need to know in order to help your child make the best decisions for the future?* This documentary-style video portion of the website will give parents an opportunity to learn about and hear what all the partners in a child's education are saying and feeling about mathematics: other parents, teachers, and adolescents. The goal is to assure parents that they are not alone in their questions and concerns about their role in their secondary-school child's mathematics learning life and decision making. In addition to a focus on math, part of this introduction will feature interviews with experts about the transition from Grade 8 to secondary school and general advice for student success throughout high school.

The second element of the parent portion of the website will be informational, and will provide the most up-to-date data and constructive material to guide parents in their efforts to support their high school-aged child. One exciting feature of this section is the interactive pathway resource. Our team is working diligently to make all math course descriptions interpretable to parents across many demographics, respecting the widely varying educational backgrounds, attitudes to mathematics, and cultural contexts. Our goal is to show various routes through the pathways by communicating the general nature of each course and illustrating the strands or big ideas from each grade with application questions (and solutions) that have been chosen specifically to evoke parents' memories, such as *I remember doing questions like that!* or *So that's why you need to know about graphs!* Questions will cover the spectrum from real-world examples about the "two-second rule" used by a driver who wants to maintain a safe following distance at any speed (the solutions will show how, as the speed of the cars increases, the minimum following distance also increases) to those about claims on cereal boxes:

> *A seven-year-old boy has a favourite treat, Super Fruity Fruit Snax. These "Fruit Snax" come in pouches of 10 snack pieces per pouch, and the*

pouches are generally sold by the box, with each box containing 4 pouches. The snack pieces come in 5 different fruit flavours, and usually each pouch contains at least one piece from each of the 5 flavours. The website of the company that manufactures the product says that equal numbers of each of the 5 fruit flavours are produced and that pouches are filled in such a way that each piece added to a pouch is equally likely to be any one of the five flavours. Of all the 5 fruit flavours, the seven-year-old boy likes mango the best. One day, he was very disappointed when he opened a pouch and there were no (zero) mango-flavoured pieces in the pouch.

As the parents move along the pathway, they will learn about the various options and course choices, and at the end of the pathway, they will see what post-secondary possibilities (trade, college, and university) are available. We will feature almost 100 diverse careers, all of which require students to keep math on their path.

The complementary part of the website will be both for teens to explore individually and for families to use together. The purpose of this more interactive component is to be inspirational and illuminative. The principal goal is to answer this question: *What do you (i.e., the 14- to 18-year-old adolescent) need to know about the importance of math for you and your life?*

It will include the interactive pathway experience, but in reverse; i.e., students select their career choice first and work back from Grade 12 to Grade 9 to see the optional routes available to achieve their goal.

There will also be fast-paced documentary-style videos that include interviews with:

1. positive role models encouraging students to keep taking mathematics through to graduation
2. people who use math daily in their careers (including a pastry chef, game developer, and YouTube personality)
3. young adults (early 20's), who share stories about why they regret dropping math early in high school (e.g., had to do a victory lap, couldn't get into the program they really wanted)
4. individuals with inspirational success stories to share (e.g., someone who is glad she kept math in her program, someone who overcame a challenge with math)

A fun element will be our "Streeter" feature, a take-off of Rick Mercer's *Talking with Americans* feature from *This Hour Has Twenty-Two Minutes*. In this segment, a person will be approached on the street by an interviewer and cameraperson. They will be asked a simple math question (e.g., about which of several 1L bottles with totally different shapes holds more). The purpose of this tongue-in-cheek component is to show that math does play a role in real life, and that the consequences of understanding such information contribute to numeracy in our society.

The last element of the toolkit, important to parents and students, will be a suite of resources about career choices, math remedial support, math enrichment, math tutorials, and an annotated list of high-quality math apps.

The design of the website will have a graphic novel feel, and the opening page to each section will feature images of the three most important people in the teaching/learning endeavour: the teacher, parent, and student (denoted by the "superman-"like letters on their clothing). A broad range of characters will be included on the many pages so that all representatives of gender, race, ability, and age are included.

I hope that mathematics educators across Ontario will be on the lookout for the launch and be enthusiastic about sharing the news so that together we can encourage many more students to keep math on their path and support parents (arguably the most important individuals of influence in children's lives) in being adequately informed about and engaged in the positive outcomes associated with a robust mathematics education.

References

Hamlin, D., Cameron, D.H., & Watkins, E.K. (2016). *Ontario's guidance counsellors: Spread thinly in an environment of growing expectations.* Toronto, ON: People for Education.

Let's Talk Science. (2013). *Spotlight on science learning: The high cost of dropping science and math*. London, ON: 2013.

Let's Talk Science. (2015). *Spotlight on science learning: Exploring parental influence: Shaping teen decisions regarding science education.* London, ON: 2015. ▲

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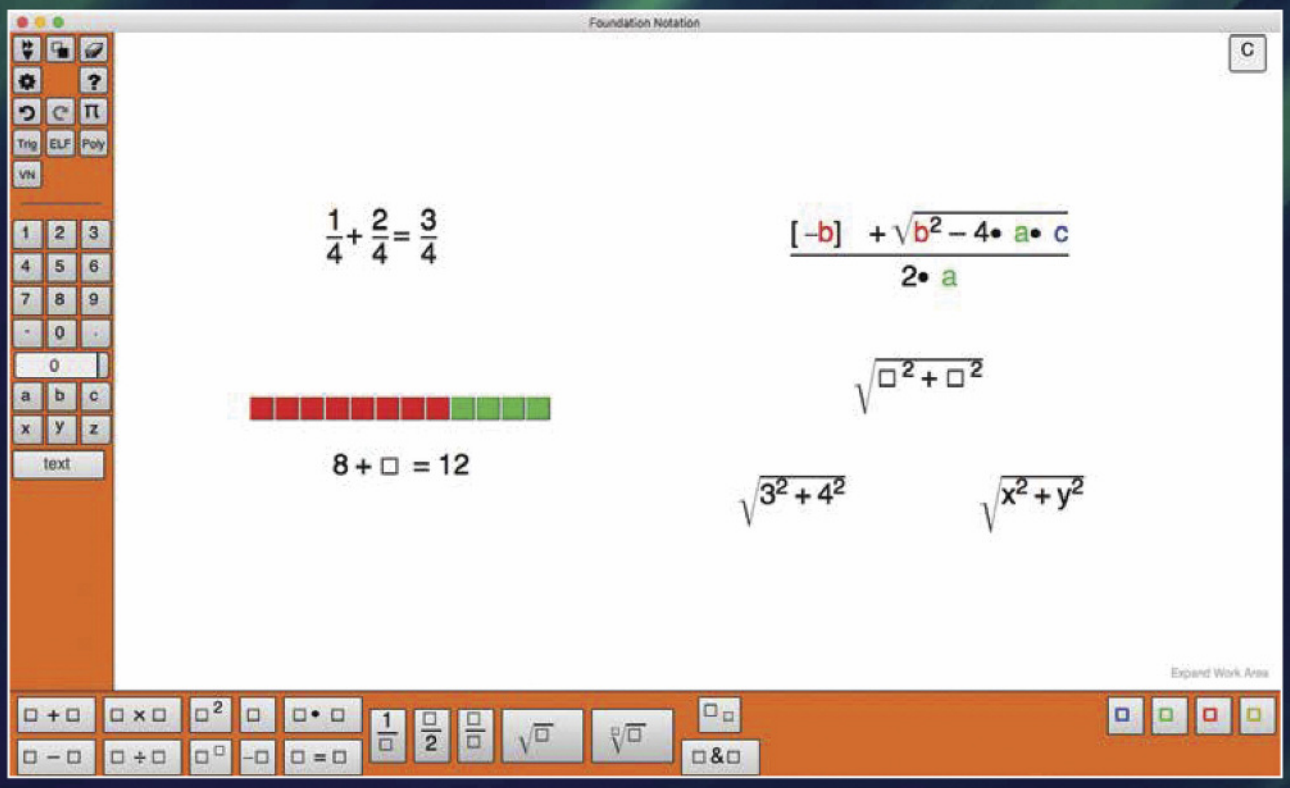
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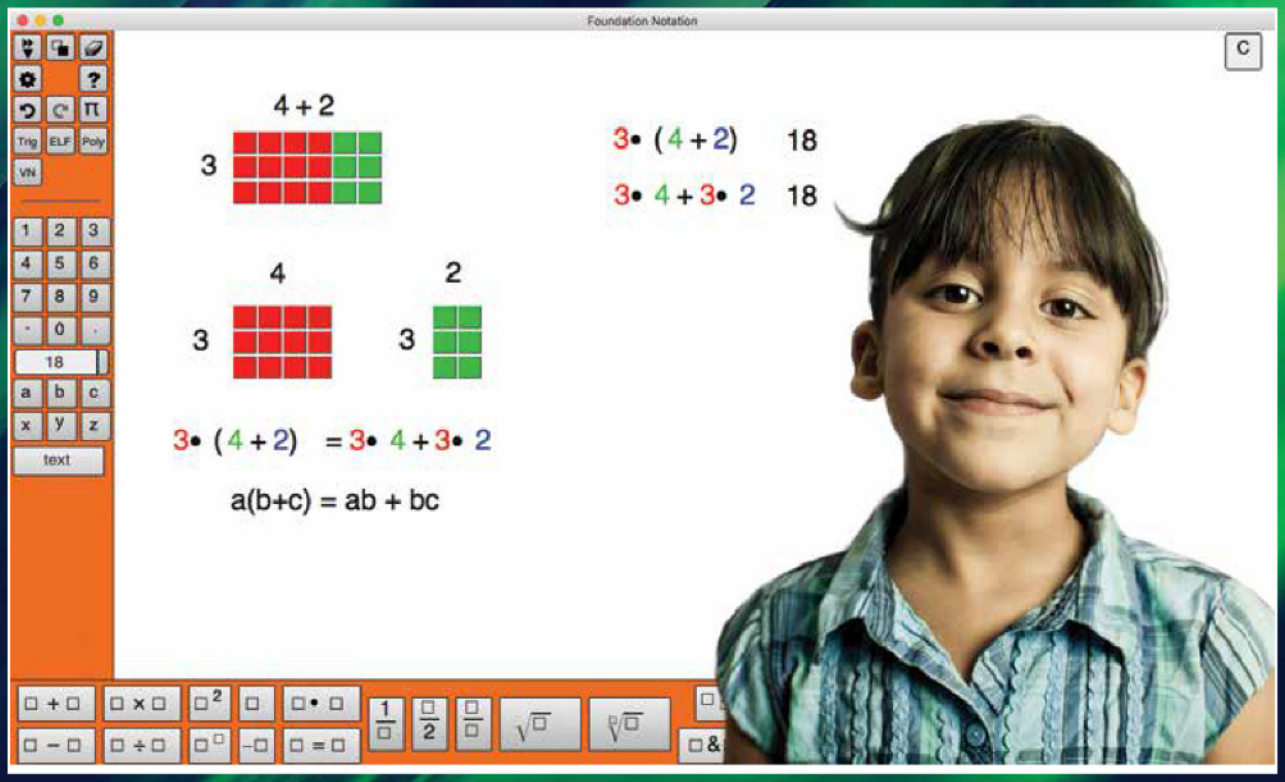
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▲ Fields Institute MathEd Forum Notes



STEWART CRAVEN
EMAIL: NUMERATECITIZEN@MAC.COM

Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

The September MathEd Forum meeting was an excellent kick-off to this year's series of monthly meetings (September 2016 to April 2017). Attendance was strong and included a significant number of elementary and secondary school teachers from the Toronto region. The program began with two audience-participation workshops, followed by a panel discussion.

Workshop A was conducted by Diane Tepylo (University of Ontario Institute of Technology). She described her workshop as follows: "Decades of research in psychology have demonstrated the importance of spatial reasoning to mathematics. As the mathematics education field moves to incorporate spatial reasoning activities into mathematics teaching, many elementary teachers struggle to see the links to their curriculum. In this activity, participants will explore a PF modelling task, where participants must intentionally move between 3-D and 2-D. After completing some of the task, participants will discuss the spatial reasoning within the task, links to [the] Ontario Mathematics curriculum, and cross-curricular opportunities within the task."

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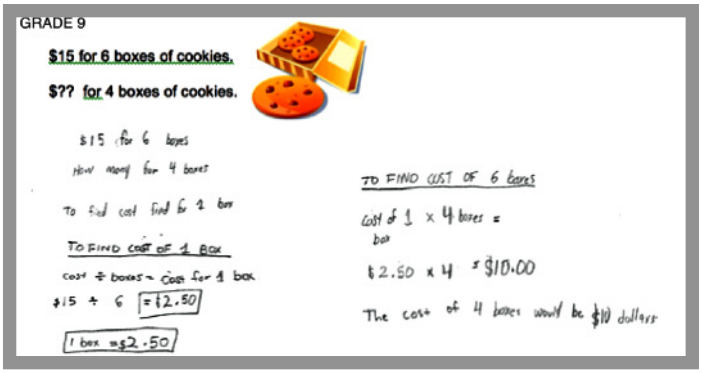
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Workshop B was conducted by Walter Whiteley (York University) and Ami Mamolo (University of Ontario Institute of Technology). Their summary stated, "We will engage with a hands-on activity that uses 3-D models and 2-D dynamic geometry software to explore a well-known geometric problem of optimization and rate of change. Through a spatial–visual approach, we explore an activity that highlights 'big ideas,' such as how properties change during optimization, why an optimum is found where it is, as well as what (global conclusions) can be inferred by comparing (local) pairs of models. The activity affords a hands-on investigation of rates of change in volume with spatial–visual reasoning (and without calculation)—leading to an enriched sense of the process and encouraging reflection and connection among different approaches."

Attendees took part in both workshops and engaged fully in the activities and discussions that followed. The afternoon session consisted of a panel discussion; Ruth Eichholtz (The York School), Sharla Falodi (Toronto District School Board), Behnaz Herbst (Toronto District School Board), and Maryem Mesrop (York University) spoke about their initiatives to embed Spatial Reasoning in their respective programs. These ranged from elementary- to university-level programs.

By the time this column is published, we will be approaching the January MathEd Forum meeting. To obtain more information about this and subsequent meetings, go to **www.fields.utoronto.ca/activities/workshops** and scroll down to the "MathEd Forum" meeting.

Throughout the year, the Steering Committee for the MathEd Forum seeks input for new themes and would appreciate your suggestions. Drop me an email at **numeratecitizen@mac.com** and I will pass your ideas along. ▲



OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary school mathematics teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancement of mathematics education in Ontario through innovation and/or research related activities.v

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary school mathematics staff which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teachingrecognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Mathematics Staff Award and Elementary School Staff Award).
4. Send the completed cover page and supporting documents by January 15, 2017 to: OAME Executive Directors, 4751 Mack Avenue, Town of Plympton-Wyoming, ON N0N 1J6, or email **EDs@oame.on.ca**.

DEADLINE: Nominations are due by January 15, 2017 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in May 2017.

▲ What's the Problem? Expanding an Ellipse



SHAWN GODIN
EMAIL: shawn.godin@ocdsb.ca

Shawn is the head of Mathematics, Business, Law, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario. He is a past member of the OAME Board of Directors, and past Editor-in-Chief of Crux Mathematicorum. *He currently writes problems for the Fryer, Galois, Hypatia, and Canadian Open Mathematics Challenge contests.*

Welcome back, problem solvers. Last time, I left you with the following problem:

If x and y are real numbers, determine the minimum possible value of the expression

$$(x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2.$$

This problem appeared as question 22 in the 2016 Fermat contest for Grade 11 students from the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. You can check out the many resources offered by the CEMC, including copies of past contests, at their website, **www.cemc.uwaterloo.ca**.

Let's play with this a bit to see what is happening. We can graph the information in a couple of ways. In Figure 1, a moveable point was created using the online graphing calculator *Desmos* (**www.desmos.com**). When you use variables as your coordinates (a, b), Desmos will ask if you would like to create sliders for the variables. You can then use the sliders to move the point, or you can just drag the point on the grid. Also, in the expression list, the expression that we want to minimize has been reproduced in terms of a and b. Now the point can be moved around the grid to get a feel for how the function is behaving and you can uncover the required minimum.

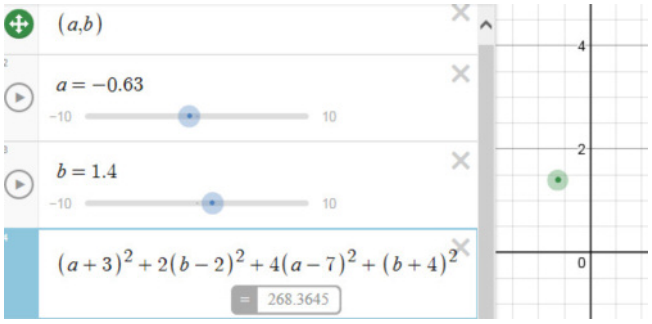


Figure 1

You can obtain a similar solution by using a spreadsheet. In Figure 2, the columns are labelled by the x values and the rows are labelled by the y values. Cell B2 contains the formula

=(B$1+3)^2+2*($A2-2)^2+4*(B$1-7)^2+($A2+4)^2.

The "$" that appears in the formula keeps a quantity constant as the formula is copied to other cells. Thus, B$1 keeps the 1 constant so that we are always drawing from the first row (the x values) and, similarly, $A2 keeps the A constant so that we are always drawing from column A (the x values). Figure 2 shows the answer that you would obtain when using a spreadsheet. When the formula from cell B1 is copied to other cells, the cell Q12 contains the formula

=(Q$1+3)^2+2*($A12-2)^2+4*(Q$1-7)^2+($A12+4)^2.

So you can see how the "$" keeps certain entries constant.

	A	O	P	Q	R	S
1	y\x	3	4	5	6	7
9	3	151	136	131	136	151
10	2	136	121	116	121	136
11	1	127	112	107	112	127
12	0	124	109	104	109	124
13	-1	127	112	107	112	127
14	-2	136	121	116	121	136

Figure 2

You can access this spreadsheet from my What's the Problem Google drive folder at **https://drive.google.com/drive/folders/0ByDlaUaj8StpanhnUWo2bEV6ZE0?usp=sharing**.

We can also use Desmos in a slightly different way to solve the problem. If we type in the equation

$$(x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2 = k,$$

as in Figure 3, again we will want to create a slider. This time, adjusting the slider produces ellipses that grow bigger or smaller, and at some point, disappear. Again, you can get a pretty good idea of what the answer is, but you must be careful to "zero" in on it.

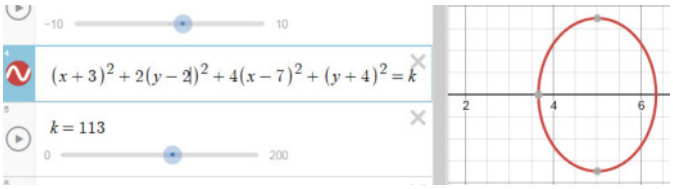


Figure 3

You can play with both of the Desmos graphs at **www.desmos.com/calculator/h7xjem2wzp**.Now let's

attack the problem with some algebra. Expanding and simplifying the original expression gives

$$5x^2 - 50x + 3y^2 + 229.$$

Completing the square on x then yields

$$5(x-5)^2 + 3y^2 + 104.$$

Since $(x-5)^2 \geq 0$ and $3y^2 \geq 0$, we see that

$$(x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2 \geq 104,$$

so the minimum, 104, occurs when $x = 5$ and $y = 0$.

We can consolidate our solutions by noting that the equation

$$5(x-5)^2 + 3y^2 + 104 = k$$

can be rearranged to give

$$\frac{5(x-5)^2}{k-104} + \frac{3y^2}{k-104} = 1,$$

which defines an ellipse with centre (5, 0) and major axis parallel to the y-axis for each real $k > 104$. When $k > 104$, the ellipse is only a single point. This situation corresponds to the solution to our problem.

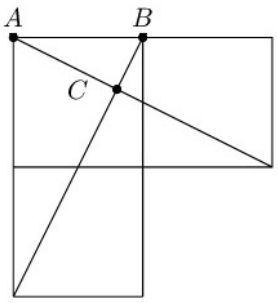
Sometimes students will want to use their brand new tool rather than sticking to more simplistic ones. It would be interesting to give this problem to an introductory calculus class to see if they want to starting taking derivatives. The problem for students in an introductory course is that they will only have been working with single-variable calculus, whereas the expression can be used to define a multivariate function

$$f(x,y) = (x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2.$$

Fortunately, we can think of this as the sum of two functions of two independent variables; hence, we can optimize the "x-function" and the "y-function" separately. I will leave this method as an exercise for your amusement.

And now for your homework:

Three unit squares and two line segments connecting two pairs of vertices are shown.



What is the area of ΔABC ?

Until next time, happy problem solving. ▲

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2017–2018

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 31, 2017, to:
Judy Mendaglio
judy.mendaglio@oame.on.ca

Include a paragraph of the nominee's qualifications with the Nomination Form. A nominee must be a member of OAME. By March 2017, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting will be online and is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2017.

NOMINATION FORM

I nominate:_____
for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Email: _____

Telephone: Home (_____)_____

Work (_____) _____

_____

Nominator's Name : _____

Nominator's Address: _____

_____

Email: _____

Telephone: Home (_____)_____

Work (_____) _____

▲ MB4T (Mathematics by and for Teachers): Teaching Place Value Through the Operations



DAVID COSTELLO
EMAIL: dacostello@edu.pe.ca
TWITTER: @dr_costello

David is the elementary numeracy leader for the province of Prince Edward Island. In this role, he supports teachers, coordinates elementary math services, leads a team of math coaches, and is responsible for professional development and learning for teachers of Kindergarten through Grade 6. Previous to this, David worked as an elementary teacher, math coach, math consultant, and elementary curriculum leader for the English Language School Board of Prince Edward Island. David also instructs university courses in the areas of mathematics methods and problem solving, facilitates professional learning sessions for schools and school boards, and shares his research and expertise in understanding math as a literacy at various conferences locally, nationally, and internationally.

Representing concepts and ideas can look different to a variety of teachers and students. And, it is in this representation that conceptual understanding can be supported or hindered. In many boards, districts, and provinces, teachers are expected to work within a yearly plan, whereby concepts and strategies are compartmentalized (e.g., patterns, measurement, operations, place value). These yearly plans, based on units of study, have been developed to ensure all mathematics curricular outcomes are addressed throughout the year. While this instructional practice has been applied for decades, there exists a dichotomy in regards to research emphasizing the interconnectedness of mathematical concepts and strategies. Such an approach to instruction has limitations, such as isolating learning, and may jeopardize student mathematical understanding.

We need to think outside the box and come back to the question, "How do we conceptualize mathematics?" To address this question, I will attempt to propose how a holistic approach to math, supported through connections of concepts and strategies, can lead to a strengthening of student mathematical understanding. What I have chosen to write about in this article is how we can represent the four operations differently. I believe that by representing operations through place value, students will see the importance of judging the reasonableness of their responses when adding, subtracting, multiplying, and dividing, and will not see place value as an isolated four-week unit of study. Instead, place value will be considered through the operations, therefore embedding the concept of place value into the daily work of students instead of in isolation.

Before continuing too much further, I want to state what I mean by place value. For me, place value is a standardized approach to the number system based on ten. While I do understand that place value is part of the larger field of decomposition of numbers, I will use this definition of place value for this article.

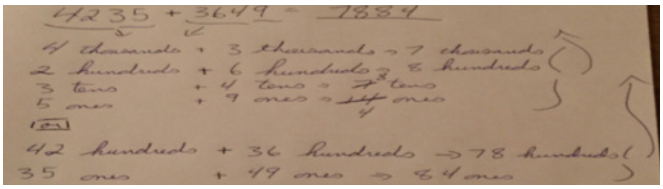
Traditionally, we have seen place value taught via the following prompts, questions, and/or tasks:

- How many tens, hundreds, thousands are in the following numbers…?
- What is the value of the underlined digit…?
- Given the following digits, create two numbers whereby one is much smaller than the other….
- How many times greater is the first number than the second number…?

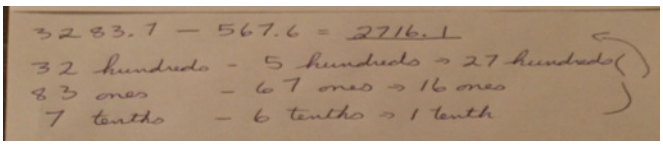
Taking this one step further, we can approach place value with more open-ended questions that encourage thinking and learning. We need to provide students with opportunities to see the importance of place value, and how it is of significance when working through the four operations for the purpose of judging the reasonableness of their response. For the purpose of this article, I will provide an overview of how to represent place value through the four operations.

Operations

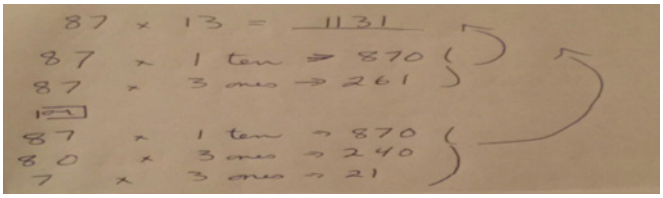
When working with the four operations in the classroom, I use problems that provide a context, a problem to engage with, and a purpose. However, for this article, with brevity being important, I will focus on the actual procedure of the operation, not identifying an operation or relating the response back to the problem. In addition to this, I would like to preface that for the following problems, it may be useful to approach a place value by the number of units (e.g., 8000 is 8 x 1 thousand) or expanded form (e.g., 345 = 300 + 40 + 5).



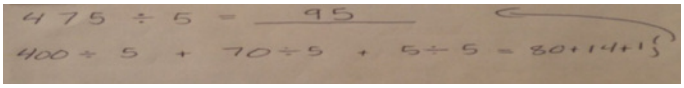
Addition



Subtraction



Multiplication



Division

Discussion

In the four work samples above, there is evidence as to how place value was applied to solve the operations. In each of the problems, the numbers were decomposed into units (ones, tens, hundreds, thousands, tenths) for the purpose of strengthening place value while adding, subtracting, multiplying, and dividing. We are solving problems by place value rather than procedure. In this way, students continue to develop their understanding of numbers versus rote process of an algorithm.

By using place value as the lens through which we add, subtract, multiply, and divide, we not only strengthen our understanding of numbers, but also provide a platform from which to judge the reasonableness of our response. Too often, when place value is separated from operations, students work through a problem without realizing the suitability of their response. Place value is the means by which students make meaning of their work, both during and after.

When we teach in a manner that integrates place value and the operations, we no longer have to rely on meaningless statements, such as "line up your decimal," "drop a zero," or "borrow a one." Although statements such as these help students to arrive at a correct response, they do nothing to develop student understanding of number.

Conclusion

While the strategies demonstrated for the four operations are not new, it is important to take note of how to apply a place value lens to the operations. By integrating place value and operations, the interconnectedness of mathematical concepts and strategies are strengthened through instructional planning. We must go beyond spiralling mathematical concepts and strategies throughout the year. Instead of compartmentalizing concepts and strategies, mathematics must be thought of, and worked with, in an integrated manner so that students do not run the risk of having isolated and segmented learning experiences.

Place value does not have to be taught and worked with as its own unit of study. We can link concepts and strategies together so as to provide linkages that students can rely upon for understanding. Not only will this support place value, it will also support the four operations by incorporating place value as a source of reasonableness. Consider how students, when working with place value and the four operations in an integrated approach, can apply a variety of strategies to problem solving self-monitor as they think and work. ▲

INTERESTED IN EXPERINCING MORE ABOUT HOW MATH SHAPES OUR WORLD?

Participate in 2017 International Mathematical Modeling Challenge (IM²C)

The IM²C is an international contest that challenges **high school** students to apply mathematical tools and concepts on **real world problems** in biology, engineering, finance, medicine, etc.

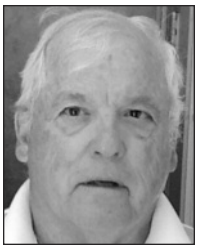
Teachers can register a team of **up to 4** students from the same school. Each team works over **5 consecutive days** to solve a practical problem and submit solution online.

The next Canadian contest is open to **all public, catholic and private high schools in Ontario during March 13 – April 26, 2017**

im2c-canada.math.yorku.ca

CAIMS SCMAI · EFiC Inc. · FIELDS science · YORK UNIVERSITÉ UNIVERSITY

▲ OAME/NCTM Report



TODD ROMIENS
EMAIL: todd.romiens@oame.on.ca

Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

This column will include talk about the *Affiliate News*, which helps bring information to all affiliate groups concerning the foci of NCTM. As well, I will look at the dialogue around the importance of EQUITY in the mathematics community and how NCTM is addressing this very serious issue.

As you probably already know, and as I recently spelled out in this column, OAME is an affiliate group to NCTM. As such, we receive the *Affiliate News* to update all affiliates on events that are occurring throughout the realm of NCTM. I bring this up because there is an opportunity at each annual NCTM conference for a delegate from OAME to attend the affiliate meeting, usually held at the conference before the sessions take place. This is an opportunity for any affiliate group to bring items that are important to the group and all NCTM members, to be discussed, voted on, and perhaps passed at this meeting.

Items discussed in the *Affiliate News* provide general information, especially designed for leaders of affiliate groups to increase awareness and ensure communication with NCTM. It also attempts to provide information of interest to all members of the mathematics world, and which will be a valuable source for affiliate groups.

NCTM has recently taken a new look at their role in equity for the mathematics curriculum. They have joined forces with a group called TODOS: Mathematics for All and have issued a statement that defines social justice as:

> …a key priority in the access to, engagement with, and advancement in mathematics education for all youth. A social justice stance requires a systematic approach that includes fair and equitable teaching practices; high expectations for all students; access to rich, rigourous, and relevant mathematics; and strong family/community relationships to promote positive mathematics learning and achievement. Equally important, a social justice stance interrogates and challenges the roles power, privilege, and oppression play in the current unjust system of mathematics education—and in society as a whole.

NCTM has adopted this definition and has vowed to "question and reassess their equity stance, actions, and language." The feeling is that even though they have been advocates of equity and have published a great deal about it ("The Access and Equity Principle" in *Principles in Action*, for example), they recognize that most of the work focused on standards, curriculum, instructional practices, and assessment, and too often addressed these issues in decontextualized ways that frequently ignored the experiences of children's lives.

The new NCTM statement on equity says:

> The NCTM Board has officially reframed its equity work to focus on Access Equity and EMPOWERMENT to capture the official constructs of students' mathematical identities, sense of agency, and social justice. The Board has modified its strategic priorities to reflect this framing of NCTM's scope to include more than just access and equity.

In order to accomplish this, NCTM has embarked on a number of initiatives to help make this stance a reality, including the following:

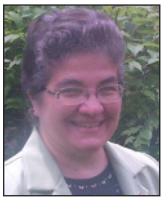
1. NCTM has partnered with other mathematics' organizations, including TODOS and NCSM.
2. NCTM has committed itself to monitoring its language to ensure that students are positioned as having assets and not deficits.
3. Access, Equity, and Empowerment will be the focus of NCTM's 2017 Innov8 Conference in Las Vegas (November 15–17, 2016).
4. The 2018 Annual Perspectives in Mathematics Education will address issues surrounding access, equity, and empowerment. The volume's working title is *Rehumanizing Mathematics Teaching and Learning for Students Who Are Latina and Black*.
5. A Curriculum Resources Collaboration Centre on **NCTM.org** is being developed with the Math Forum. On this site, educators will be able to share and collaborate on lessons and instructional challenges.

The NCTM Board and staff are engaging in their own professional learning on issues surrounding access, equity, and empowerment. This will also include professional development to look at the impact of microaggressions in the classroom in an effort to better understand the issues.

The NCTM ANNUAL CONFERENCE next year is in San Antonio, Texas from April 5–8. I hope you can attend. ▲

Provincial Digital Learning Resources – What's New? The Set Tool







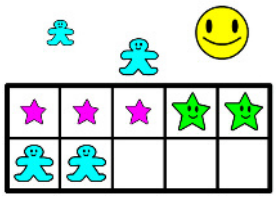


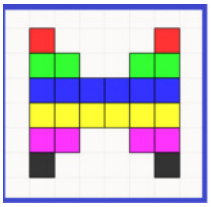
ROSS ISENEGGER
EMAIL: ross@isenegger.ca
AGNES GRAFTON
EMAIL: agrafton@bhncdsb.ca
MARKUS WOLSKI
EMAIL: markus.wolski@gmail.com
GREG CLARKE
EMAIL: gpclarke@smcdsb.on.ca

Ross (Near North District School Board), Agnes (Brant Haldimand Norfolk Catholic District School Board), Markus (Bluewater District School Board), and Greg (Simcoe Muskoka Catholic District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS and mathies.ca projects.

The Set Tool

As of September, you will notice two new tools live on mathies.ca: the Set Tool and Colour Tiles for desktop. These new tools provide a rich site for the exploration of topics from all divisions.



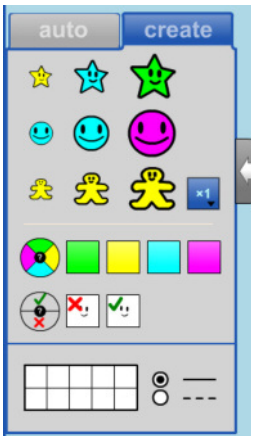


The original version of the Set Tool was based on the Partitioning Sets Tool. It has been replaced with a much more robust learning tool that allows the user to have more control over the various attributes and more sophisticated interactions with the objects.

The new Set Tool allows for up to four attributes to be considered:

1. Shape: Star, Circle, and Gingerbread
2. Size: Small, Medium, and Large
3. Colour: Green, Yellow, Blue, and Pink
4. Face: Shown and Hidden

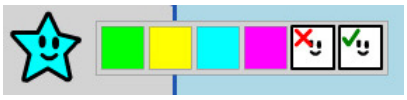
The Create Tab – User-Generated Sets



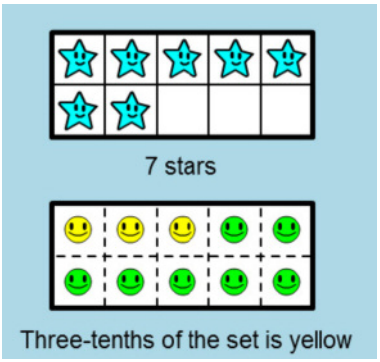
Nine cloner shapes representing all the possible shapes and sizes are presented in the first section of the panel, together with the multiplier which allows the quick creation of one, two, five, or ten copies of a shape. The nine shapes have faces shown and are randomly coloured.

You can change the colour and face of the cloners, using the buttons in the second section. To randomize an attribute, press the button with the question mark.

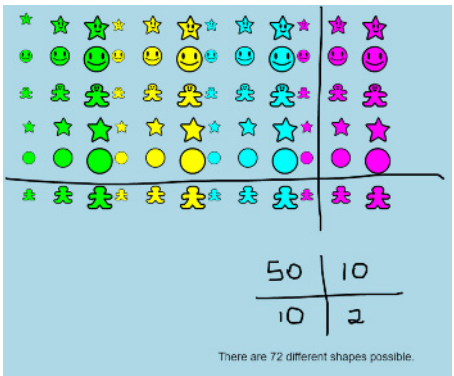
You can also change an individual cloner by clicking on it and selecting the desired colour and face.



Drag a ten frame into the workspace from the third section of the panel. If you use dashed internal divisions, it is called a frame of tenths—the full frame representing a whole in that case.



Students could be asked to create and count all the shape possibilities.



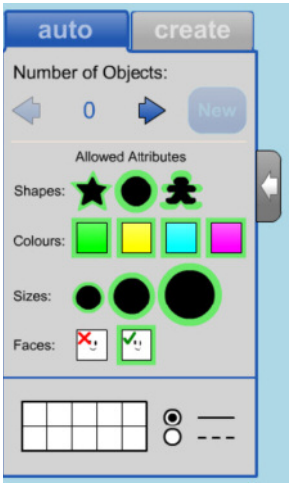
To create the above, one of each of the nine cloners is dragged into the workspace maintaining 

the 3 by 3 array. This group is then selected and coloured green, using the Configure Selected Shapes button.

It is then copied to create the yellow, blue, and pink possibilities. Then the whole lot is copied, positioned, and the smile wiped off their faces (see below for an investigation that uses the Copy and Configure Selected Shapes buttons in more detail).

Students in later grades might calculate the total number of possibilities, using 3 x 3 x 4 x 2. This construction allows students to see the 3 times 3 to start, then the multiplication by 4 as the colours are considered, and then the doubling when "facedness" is addressed. The eight copies of nine elements form a 6 by 12 array, which is decomposed into friendly numbers in order to determine the product. The annotation tools help to make the thinking visible.

The Auto Tab – Randomly Generated Sets



Type a number or use the decrease/increase buttons to force the set to have a specific number of objects. Click the New button in the first section of the panel to create a random set, using the allowed attributes from the second section. Simply toggle attributes on and off in the second section before pressing New to control which attributes are included in the randomized set. Notice that you can drag ten frames from the auto tab as well.

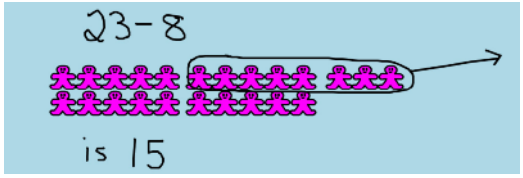
Examples Using the Set Tool



- Students might be asked to create sets of a certain size, like 10. They can use the Scramble Position button, which we liken to kicking the desk leg, to work on Conservation of Quantity; 10 stays 10!
- Simple patterns can be created and extended.

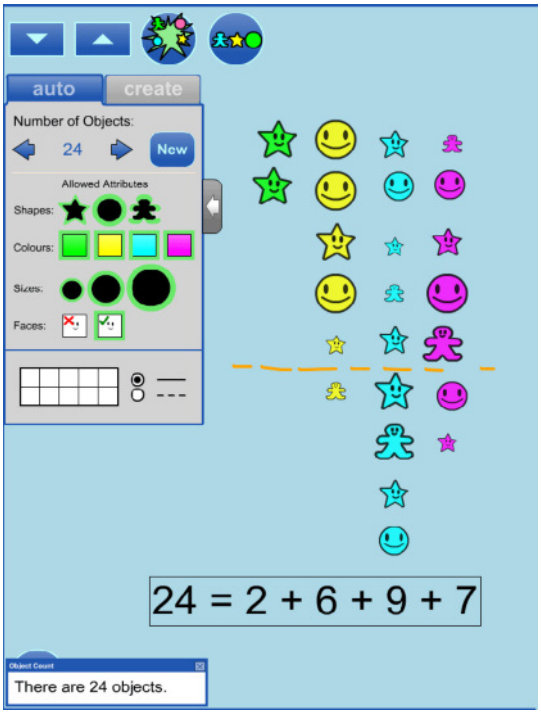


- Operations can be modelled.



The multiplier allows multiple shapes to be dragged onto the workspace in one action. Here, the multiplier was set to 10, and in two drags, 20 gingerbreads are placed in the workspace. Next, the multiplier was set to 5, and 5 more were added. Two were selected and deleted, using the recycle bin, leaving 23. The annotation tool is used and the take-away meaning of subtraction illustrated. The three groups of 5 that are left are counted to get the difference. Notice how anchors of 5 and 10 help solve this problem.

- Young students are often asked to sort sets by attribute. The auto tab allows for random sets to be created quickly. Here, 24 objects are created automatically and the student has sorted them by colour.

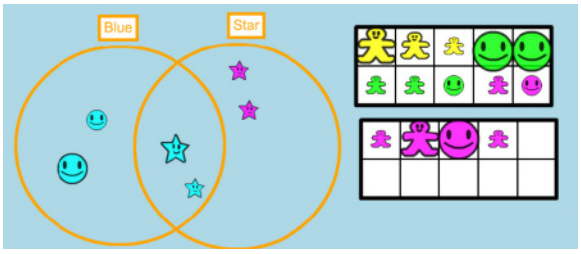


This student's sort is a pictograph on its head!



The Count button can be used as a check, which is especially useful when students have created the set themselves.

- Later, students might use a Venn Diagram to sort based on two attributes. The Venn Diagram is created using the oval and text box annotation tools accessed from the pencil icon.



Here a student creates 20 random shapes and then classifies them by the two attributes—blue and star—dragging them to their proper position in the Venn Diagram. The ten frame is useful for organizing and counting the 14 objects that are neither blue nor stars. Students will get a feeling for variability as they compare their results to those of other students, and realize that although the colours are equally likely to occur; they do not often occur equally. Here we would expect 6 or 7 stars (20 divided by 3 possible shapes), with each colour represented 1 or 2 times (4 colours evenly distributed among the 6 or 7 stars). Instead, we get four stars, none of which are yellow or pink.

A Grade 12 student could calculate some of the theoretical probabilities. For example, the probability that an object is placed outside the rings is $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$. This means that you would expect to have only 10 objects outside, which is consistent with the observation that both blues and stars are underrepresented in this sample (4 blues compared to 5 expected, 4 stars compared to 6.67 expected).

Undo/Redo

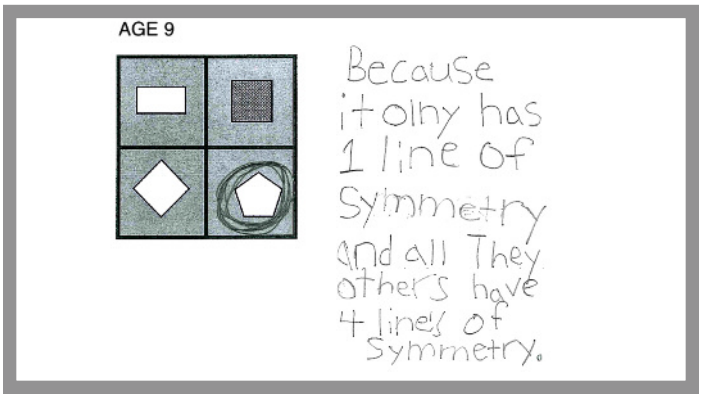


You can use Undo and Redo in the Set Tool, Colour Tiles, Money Tool, and others. This feature is not only useful for backtracking when a misstep is made, it enables students to demonstrate their work from start to finish. They press Undo until they are at the start of their solution, and then press Redo repeatedly, explaining each step.

The Love of Money/Argent

Take some time to explore the enhanced Money Tool (version 2.1.3, June 2016) and its offspring: the Set Tool and Colour Tiles.

Why not try the Money Tool in French? Simply press the Language button, and not only does the text change, but so does the format of the numbers.





Feedback and Future Requests

Please feel free to send us your feedback about any of these tools, using the Feedback Form button inside the Information Dialog, accessed from the i button.



You can also send your comments to **WhatsNew@oame.on.ca**. If you have ideas about nice uses for Money, Colour Tiles, or the Set Tools, we would love to receive them and include them on the Support Wiki, which is a good place to learn more about our tools. You can share worked examples on Twitter, using the hashtag #ONmathies. To be among the first to find out about latest digital tool developments, sign up for our email list at http://mathclips.ca/WhatsNewEmailList.html. You could hear, for example, when the mobile versions of the Set Tool and Colour Tiles are published. ▲

▲ In the Middle: Just Your Basic Walk Around Town!



CARLY ZINIUK
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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and AP Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very involved in using real-life data to engage her students in solving problems.

When the weather in September is so nice, it is often hard for our students (and us!) to abandon the beautiful outdoors for the reality of paper-and-pencil mathematics. I have frequently taken my students on wonderful explorations, "math trails," in places like the National Gallery in Ottawa or downtown Toronto's CBC Building, and the Roger's Centre. Public art has always been a great source of interest for these trails for me, but last year, our Grade 9 classes headed to Stratford for three days to see some plays, and I offered to put together a math trail downtown. So with my own two boys, 12 and 14, we got a chance to see some plays with them and think about math. Once we were done, I realized that the trail that we had created would work regardless of location.

This trail adapts very well to any middle school class. We then used all the things students had collected in class throughout the year to do topics including Pythagorean Theorem, geometric relationships (including parallel lines), slope, circle measurements, and 3D geometry. Our timing was about 2 hours for the data collection and 1 hour back in class to dump their photos online and save them. I have included examples of some of my students and my own children's photos for you to look at the kind of things they found!

Teaching notes:

- We had one teacher with ten students. The students will be able to choose where they want to go, and they need to find ten different locations in their morning experience. We separated the groups and hopefully, dispersed them over the trail starting locations for the morning. This is an activity best done with fewer, not more, students.
- The focus of the day needs to be on making sure students collect the data and the pictures; however, the students should take some time to stop, consider their route, relax, and spend time together. They will also need to take a lot of photos and measurements from each site so they can use their learning later in math.
- Your job will essentially be to make sure they get the information they need and that they stay on task as much as possible. The goal for them is to get good photos and measurements and sketches. Try to ensure that every student has a copy of the sketch for himself or herself (rather than relying on someone else to have it).

Student trail information:

For this adventure, you will need to find ten different mathematical sites.

For each site, your group will need to collect:

- a photograph
- a sketch with all measurements recorded and labelled in centimetres for lengths and degrees for angles
  - You may need multiple sketches for three dimensions, if appropriate/necessary.
  - Your sketches don't need to be to scale, but approximate proportions with good measurements will be very helpful.
  - If you cannot get the measurements (too high, too big, too far away, not safe to collect), please consider finding another site.
- a note about where the photo was taken/where the site is (near what intersection, which landmark, which building or location)

It is highly recommended that each of you record your own paperwork and share all site photos while you are in Stratford. These will be used for learning back at Bishop Strachan School, and it will be frustrating to everyone if you do not have all the information you need when you need it.



- A rectangular area surrounded by a closed perimeter



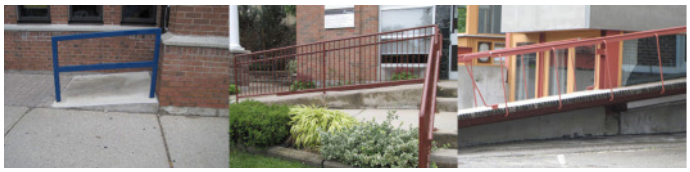
- A prism that is not rectangular



- The circumference of a tree trunk



- A staircase. Ideally, a side view to measure the height, width, and length of the stairs



- A ramp. Ideally, a side view to measure the height, width, and length of the ramp
- A rate of change (Hint: Look for units that include things like “per” or “/.”)

Students can find these in speed signs, in stores (Timbits), or at the boat rentals – think creatively!



- At least three different possible quadratics

A parabola is the shape of the curve determined by the basic quadratic equation, $y = x^2$. Parabolas have a single minimum or single maximum, like the following. You may only see one side, but a full parabola is symmetric horizontally.



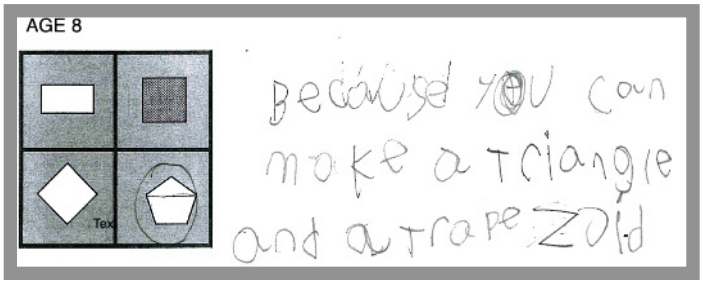


- A geometric design that contains at least one set of:
  - parallel lines
  - perpendicular lines
  - non-parallel, non-perpendicular, but still intersecting lines
  - a right-angled triangle

You can have one picture for each property or one picture that contains all or some of them. You will need both length and angle measurements for these!

- At Rocky Mountain Chocolate Factory, find the prices of multiple scoops of ice cream and varied kinds of cones. Please record this information in a table.
- A circle where you can clearly see the centre and radius (or diameter)

If you do not have these items by the end of your Stratford experience, you will need to take a photo and measurements of these items at home. Doing them in Stratford with your friends is so much more fun! ▲



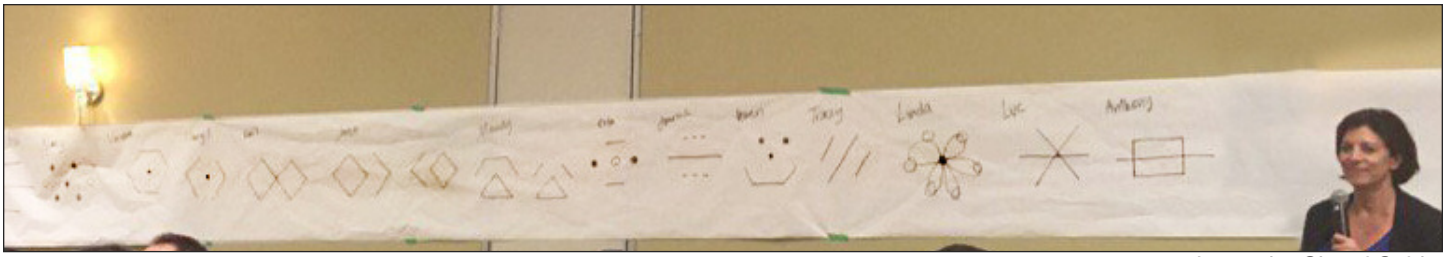


Image by Cheryl Snider

▲ Report from OAME Leadership 2016: A Day with Jo Boaler

MATTHEW OLDRIDGE

"There is no wall," Jo Boaler told 350 OAME members and educators at the Pearson Convention Centre in Brampton on Friday, November 4. We all have stories about hitting "the wall" in math class. Maybe our wall was algebra, or maybe it was calculus we crashed into at 100 kilometres per hour. (Calculus is often "the wall"!) A key message in Boaler's mindset mathematics work is that there is no need for a "wall." We can all learn mathematics to the best of our ability, there are no "math brains," mistakes are valuable, and Carol Dweck's notion of mindset can and should underpin all our work as mathematics classroom teachers.

"Mindsets change learning," Boaler stated, recounting how Carol Dweck's work inspired her own. Judging by all the copies in the room, among the 350 strong educators and OAME members, her own book, *Mathematical Mindsets*, is inspiring many in the current moment.

Introducing math concepts is important, but changing beliefs about mathematics is much more important. Beliefs such as, "everyone can do math to the highest level," which is one of YouCubed's (Stanford's and Ms. Boaler's very successful website) seven positive norms of the math classroom.

Ms. Boaler's brain-based math learning messages are perhaps what she is most known for. Famously, she highlights in her work how "mistakes make the brain grow," through increased synaptic action. When you realize you have made a mistake, your brain grows as you consider your mistake, and, presumably, attempt to correct it. Boaler's brain-based math ideas are also somewhat controversial, and she has drawn criticism in some quarters for how she has used cognitive research (see interview, inset, for how Ms. Boaler addresses critics of her work). She even spoke of how she was disparaged in the British Parliament for speaking against times tables!

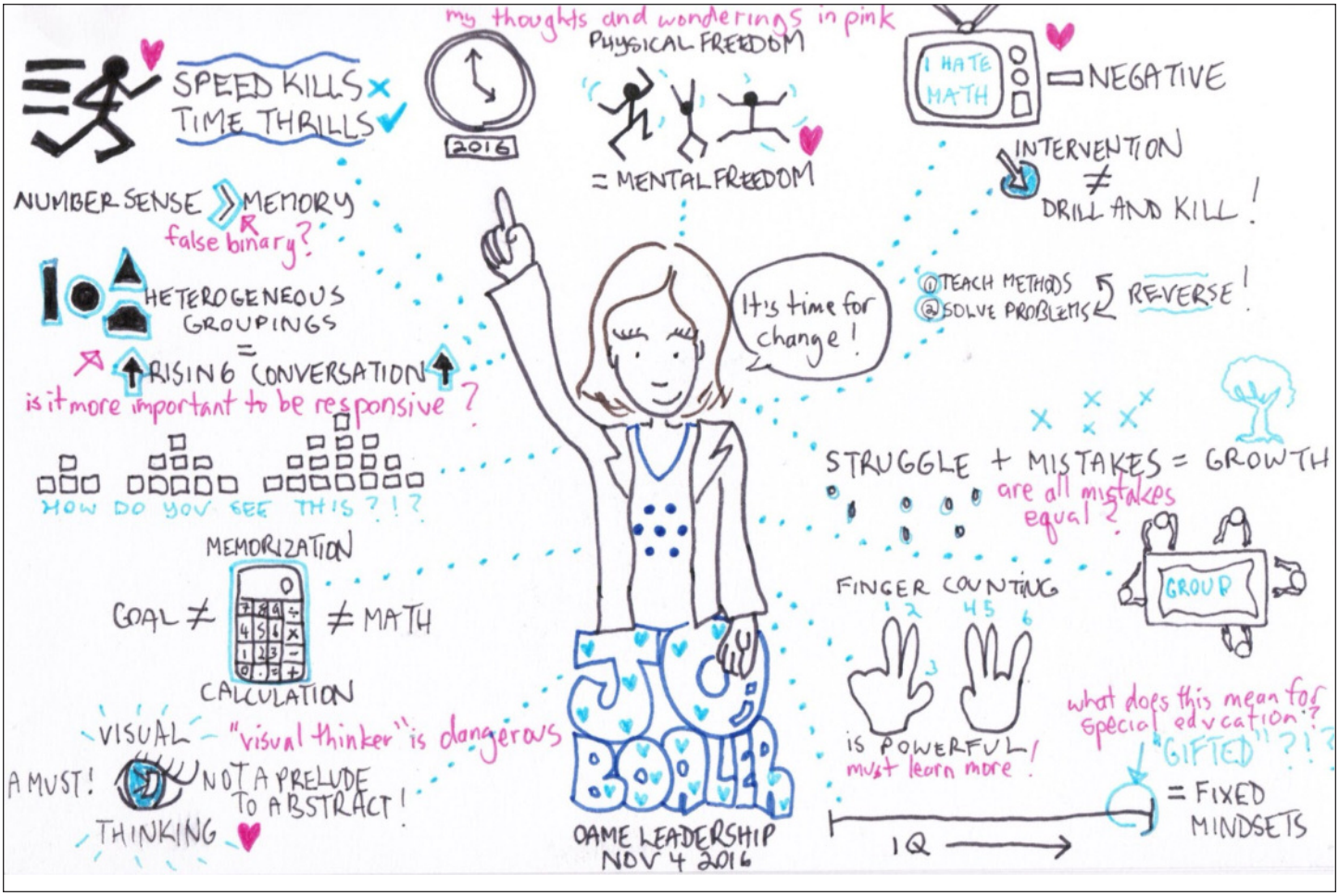
"It's important to know some math facts. Maybe. We have our phones." These are the sorts of challenging and debatable statements that Boaler is known for. This is a tricky one, and we've all thought about it. For Boaler, fluency with numbers is much more important than what she calls "blind memorization," and she nicely addresses the difference between fluency and memorization (see interview, inset). I think most of us would say that it is important to know math facts—it's just how we learn them that matters. Humans are best at thinking, and machines are best at calculating. Conrad Wolfram's TED talk, which makes this kind of claim, was used to support Boaler's thoughts on fluency and memorization.

As Ms. Boaler laid out the research on fluency, she told the room that having strategies to deal with numbers is what is most important. For example, one might say, "My use of strategies, through number talks, has helped me to understand the relationships between numbers." Park and Brannon, in an MRI study on brain pathways, called this "training the approximate number system." She also cited a study by De Lazer and others, which used fMRI to differentiate learning by strategies and learning by drill.

"Memorizers are the lowest achievers," is another of Boaler's famous claims. For Boaler, the highest achievers in mathematics are those who see it as a set of connected ideas. Thinking slowly, thinking deeply—this is what we should aspire to in our classrooms. Images of "Mad Minute" worksheets drew a sort of "we've all been there" chuckle.

As always, when you get a bunch of math teachers actually doing math, there is a buzz and hum in the room. We engaged with interesting tasks throughout the day. Starting with an interesting growing pattern, we explore our different ways to "see" how it grows. Mathematics should be visual is one of her key messages, and she drove that home by leading us through her famous 7-dot number talk, as well as with 15 unique ways of "seeing" in the room! The centrepiece of the afternoon was "Jo's favourite problem," a quadratic growing pattern we extended backwards to the "-1st" term.

Math should be visual was another key message. Visual math is not a prelude to abstraction. Boaler took us through



Sketchnote by Royan Lee

her new (and again, somewhat controversial, idea), that using fingers in math class is not "babyish." Visualization of our fingers is key to mathematical development.

It was a very inspiring and thought-provoking day. Three hundred and fifty educators left feeling inspired and invigorated by Boaler's interesting work on mathematical mindsets. Below are Jo Boaler's answers to the two questions I had the chance to ask her, (I almost had a full interview the night before, but lost airline luggage conspired against us!)

Boaler on Fluency Versus Memorization

Somebody who is fluent, I would say, is somebody who has number sense, who is able to effectively use numbers, feels comfortable with numbers, and who is able to adapt and use them. You can memorize numbers, and not have fluency, just have memorized numbers. So, it's better to be really comfortable with numbers and be able to use them in flexible ways, and I would say that's part of fluency. It doesn't mean you have committed things to memory. You can, by using numbers flexibly, still... have them committed to memory, but having remembered things is different from blind memorization. So, people who are fluent have many things that they have in their memory, but it doesn't mean they've learned them through blind memorization.

Boaler on the Math Wars, and Facing Critics of Her Work

The people who are coming after the researchers... the researchers know what works. We've done research, we've worked in classrooms. The teachers know, the researchers know what works. The people who are coming after us generally don't know. They may be mathematicians, but maybe they have never really engaged in thinking about learning, and what helps with learning, and how students learn, they're just going from what they think works. Some people are doing that very aggressively, so instead of going for conversation, they just attack, and that's not good for public discourse. ▲

▲ Manipulatives and Analogies for Teaching Integers: An Example of a Grades 7 and 8 Unit



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Immaculate Kizito Namukasa is an associate professor at the University of Western Ontario. Her current research interests in mathematics education include mathematical thinking and activity, mathematics teacher education, non-routine problem solving and learning environments, complexity research framework, and critical mathematics education. Namukasa is also interested in globalization and internationalization of schools and schooling in general.

Mathematics learning materials that learners could touch, see, hear, manipulate, and even program, are crucial in teaching mathematics. The challenge of selecting materials that best meet curriculum objectives and teaching goals is magnified by the profusion of learning materials available, especially the more recent touch-screen tablet and smartphone applications. Conversations among educators and teachers about manipulatives, especially materials accessible via digital devices, appear to lag behind the rate at which the availability of mathematics materials is growing. This paper shares selected virtual materials, together with physical materials, in teaching integers, and shares this in the context of a unit on negative integers.

The Collaborative Process

In a collaborative inquiry, two researchers (both mathematics education researchers at a university), two graduate student researchers (both classroom teachers on leave from teaching to pursue graduate education), and a teacher (with over ten years teaching experience in both junior/high and high school grades), jointly planned and reflected on a unit on integers to be taught in a combined Grades 7/8 class. The general intent of the collaborative inquiry was to engage both the teaching and research perspective in designing a more refined teaching approach for a unit that utilized both physical manipulatives and virtual manipulatives.

Integers in the Curriculum

Negative integers are not introduced until Grade 6 in several jurisdictions. In the Ontario curriculum, negative integers are now introduced in the Grade 7 Number Sense and Numeration strand, and have applications in the Grade 8 Number Sense and Numeration strand, the Grade 8 Patterning and Algebra strand, as well as the Grade 8 Geometry strand.

Grade 7 students are taught introductory concepts, such as to "represent, compare, and order numbers, including integers; demonstrate an understanding of addition and subtraction of fractions and integers" (Ontario Ministry of Education [OME], 2005a, pp. 99–100).

- "identify and compare integers found in real-life contexts;
- "add and subtract integers, using a variety of tools."

In Grade 8, students consolidate and apply their knowledge of integers when they "solve problems involving… integers, using a variety of computational strategies" (OME, 2005a, pp. 111–112). Specifically,

- "to use estimation when solving problems involving operations with… integers…; to represent the multiplication and division of integers, using a variety of tools."

Further, in the Grade 8 Patterning and Algebra strand, learners explore patterns and relationships concepts that involve integers, specifically to:

- "evaluate algebraic expressions with up to three terms, by substituting… integers for the variables (e.g., evaluate $3x + 4y = 2z$, where $x = 1/2$, $y = 0.6$, and $z = –1$); they solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a "balance" model" (OME, 2005a, p. 117).

Integer concepts and operations are also applied in the Geometry and Spatial Sense strand, specifically when learning location and movement concepts such as:

- in Grade 7: "plot points using all four quadrants of the Cartesian coordinate plane" (OME, 2005a,

[i] *Several of the apps seen in the results were found on both iTunes and Google Play stores. Aside from being tagged integer multiplication, a good number of these apps did not have much to do with learning integer multiplication, let alone learning mathematics. Of the apps specific to integer multiplication involving negative integers, a majority were designed for building procedural proficiency of integer multiplication. Only a few were designed for conceptual understanding of integer-multiplication concepts. Some apps tagged integer multiplication were also applicable to other integer concepts. A third-party app analytic, such as App Annie, returns more results for integer multiplication because it is not limited by a store's retrieval method, such as featured or top apps.*

p. 104); in Grade 8: graph the image of a point, or set of points, on the Cartesian coordinate plane after applying a transformation to the original point(s) (i.e., translation; reflection in the *x*-axis, the *y*-axis, or the angle bisector of the axes that passes through the first and third quadrants)" (OME, 2005a, p. 115).

In the Grade 9 Academic course (MPM1D), Principles of Mathematics, and in the Grade 9 Applied course (MFM1P), Foundations of Mathematics, students learn to:

- "simplify numerical expressions involving integers and rational numbers, with and without the use of technology" (OME, 2005b, p. 30).

They also learn to "add and subtract polynomials with up to two variables," "multiply a polynomial with a monomial" of the same variable, "expand and simplify polynomial expressions," "solve first-degree equations," "rearrange formulas involving variables in the first degree," and "solve problems that can be modelled with first-degree equations" (OME, 2005b, p. 30). This all involves negative integer operations in the context of expressions and equations.

In Grades 11 and 12, for instance, learners:

- represent, solve, and solve problems that involve quadratic equations and exponential functions involving negative integers, say, exponents and bases, as well as connect graphs and equations of exponential functions containing integer exponents and bases (Functions, Grade 11 University Preparation, MCR3U)
- represent, and solve problems that involve quadratic equations and exponential functions, say, involving integer values, as well as connecting graphs and equations of exponential functions containing integer exponents and bases (Functions and Applications, Grade 11 University/College Preparation, MCF3M)
- connect graphs and equations of quadratic relations with expressions containing integer exponents (Foundations for College Mathematics, Grade 11 College Preparation, MBF3C)
- evaluate logarithmic expressions; for example, recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number…, and evaluate simple logarithmic expressions. Sample problem: Why is it not possible to determine log(10) (– 3) or log(2)0? Explain your reasoning. (Advanced Functions, Grade 12 University Preparation, MHF4U)
- solve problems, using mathematical models and derivatives that involve negative exponents (Calculus and Vectors, Grade 12, University Preparation, MCV4U)
- solve exponential equations algebraically and graphically (Mathematics for College Technology, Grade 12 College Preparation, MCT4C)
- solve exponential equations containing integer exponents (Foundations for College Mathematics, Grade 12 College Preparation, MAP4C)

Thus, in Grades 11 and 12, students further explore applications of negative integers in the context of function (quadratic, exponential, logarithmic, polynomial, rational, and trigonometric) concepts, calculus and vector concepts, and data-management concepts. For instance, when students simplify, solve, evaluate, and model functions and data sets, they come across the following negative integer objects: negative functions; functions passing through negative points; functions with negative exponents; negative regions of a function; negative instantaneous rates of change; Cartesian representations of vectors and planes involving negative *x*, *y*, or *z* values; data sets with negative correlations and negative standard deviations. Only two high school courses exclude negative-integer concepts. Understanding and proficiency with integer concept and procedures, respectively, at the elementary grades is important for university and college preparation mathematics courses.

Manipulatives for Teaching Negative Integers

The teacher selected the Grades 7/8 integer unit as the focus of the inquiry. Her rationale was that the integer unit is one of the most difficult middle grades units to teach in ways recommended by current curricula reforms. Many teachers, due to this complexity, fall back to teaching this unit "by the rules." The rules, such as the rule for adding and subtracting integers, are not easily mastered and are difficult to recall by the students. The lead teacher had, in the past years of teaching, adapted several approaches identified to assist in explaining quantity relationships and operations on negative numbers, but a couple of approaches had not been without difficulties. She observed that some of the teaching materials resulted in misconceptions among students. It is very common, for instance, to emphasize the difference between the subtraction sign and negative sign—"minus" being an operation performed on two numbers, whereas "negative" is an operation performed on a single number. This emphasis is not necessary, however, when negative integers are taught using carefully selected manipulatives and analogies. Also, it is common for the rules of operations on integers to be stated for the learners as a list of rules applying to the

different scenarios. For example, the rule adding integers would have a list of at least three rules, including one for adding a negative and a positive integer: To add a positive and a negative integer, you subtract the absolute values and keep the sign of the integer with the largest absolute value. On top of these three rules, a learner would have to master several others for subtraction, multiplication, and division of integers. Mastering several rules without understanding how they come into place is difficult for a majority of students.

Minus is just a sign for "opposite" direction, whether we use it as a subtraction sign or a negative sign. If students are taught to treat "negative" "as a mathematical notation" for "opposite to a number on the number line," then it is more accessible to understand why – (5) equals 5. It means "opposite of which is 5." The distinction between subtraction sign and negative sign all but disappears when students meet with polynomial algebra in high school. The rules of simplifying polynomials are then formulated only in terms of addition: to subtract polynomials, change the signs of ALL of the terms being subtracted. Change the subtraction sign to addition of a negative quantity. Follow the rules for adding signed numbers. Seeing negative numbers as opposites to counting numbers, and emphasizing the notation of negative to indicate a number opposite to another on a number line, is in line with the understanding of addition as the inverse or opposite of subtraction. As well, it is in line with historical development of numbers.

Seeing negative numbers as tools (e.g., as fictitious or false solutions to equations) evolved as early as the seventh century.

For example, we cannot solve the equation $X + 2 = 1$. There is simply no natural number which, when added to 2, will produce the number 1. In order to do this, we need more numbers. So we introduce the negatives of the numbers we already have. This means that for each natural number X, we introduce a new number Y having the property that $X + Y = 0$. The number Y is called the negative of X. Once this has been accomplished, and negative numbers have been introduced as tools to aid in solving equations, we have the set of numbers ...–4, –3, –2, –1, –0, +1, +2, +3, +4, ... The numbers written in this infinite list are called the integers. (The solution of $X + 2 = 1$ is now $X = –1$.) (King, 1992/2006, pp. 63–64)

So the "–" (minus) sign with regards to numbers is a mathematical notation for "opposite to 0 on a number line." But thinking about negative numbers as quantities on which mathematics operations of collecting (adding and subtracting), as well as expanding or shrinking (multiplying and dividing) can be performed, evolved much later in the eighteenth century. Both the use of negative numbers as tools as well as quantities is useful when manipulating expressions, variables, and functions in Grades 9 and 10 algebra courses.

The lead teacher, in a manner aligned with instructional reform in Ontario, designed a unit that utilized both physical manipulatives and virtual manipulatives. Several manipulatives, representations, and analogies are shared in textbooks and web resources (such as CLIPS, NLVM) for teaching integer concepts and operations: coloured chips, the number line, the elevator analogy, the emotions analogy, height above and below sea level, the balloon and sandbag analogy, debt and credit, temperature measurements, the application of positive and negative electrical charge (or protons and neutrons), and so on. The teacher designed a concrete and a virtual manipulative based on the coloured-chips manipulative to be used in the unit. In the next section, we share details about the manipulative. The unit design and individual lessons in the unit may be found at **http://bit.ly/2bHWYz2**.

Physical Manipulative, Virtual Applets, and Apps

Several articles (e.g., Peterson, 1972; Skip, 1990) and online forums (e.g., "negative × negative = positive" at **MathForum.org**) explore the use of physical, virtual, and visual strategies, among others, for teaching meanings and operations of negative integers. Kamii, Lewis, and Kirkland (2001), for example, speak about teaching more difficult concepts such as subtraction, fractions, and integers, and maintain the need to teach these concepts with meaning. More recent conversations focus on the use of Information Communication Technology (ICT)-based technology (e.g., interactive whiteboard and computer games) to make difficult topics more accessible to learn. Selected apps such as Interactive Integers by TicTapTech and Integer Multiplication by Ventura could be used in the teaching of integer addition and subtraction, and integer multiplication, respectively.

Ms. LT, the lead teacher, drew inspiration from the history of integers, mathematical analysis, and use of analogy when considering a specific manipulative to make learning integers less intimidating for students. For hundreds of years, Chinese administrators, engineers, and mathematicians would perform all basic arithmetic operations with a bundle of counting rods and a counting board. Ancient Chinese rod calculus works on the principle of addition, of collecting objects. The process of addition involved mechanically moving the rods without the need of memorizing an addition table. The rods representing the addend number were placed at the bottom of the counting

board, the rods in the augend – at the top. The rods in the addend at the bottom were moved to the top, into the augend area. The rods in the augend (top area of the counting board) changed throughout the addition, and the rods in the addend (bottom area) eventually "disappeared."

For the Chinese, in contrast to present-day accounting, red rods represented positive numbers and black represented negative numbers. Black and red rods were used for commercial and tax calculations, where the black, erased off, cancelled out the red when placed in the same area. Subtraction on the counting board was handled as the reverse of addition. Five take away three (5 – 3) was treated as 5 + 3 (Yan & Shiran, 1987).

The strategy used in the manipulative invented by the Lead teacher, Ms. LT, is a simplified variant of Chinese rod calculus. The manipulative is a set of integer counters and counting board as shown in Figure 1.

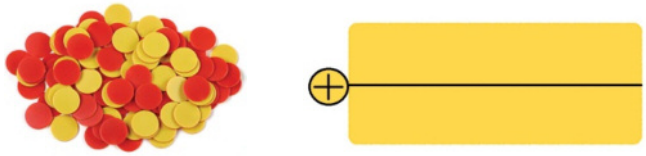


Figure 1: *Ms. LT's integer manipulative, counters, and a counting board*

This virtual manipulative, a version of this physical manipulative that Ms. LT designed, is shown in Figure 2 and is available online at **http://scratch.mit.edu/projects/20893892/**. She designed it, using a very friendly block-based coding program, Scratch, that students could also learn to use. Instead of yellow tiles, she used blue tiles in the virtual manipulative to match the colour being used. As shown in Figures 1 and 2, the counting mat is divided horizontally into two parts, the upper being a space for counters representing augend and lower a space for counters representing addend.

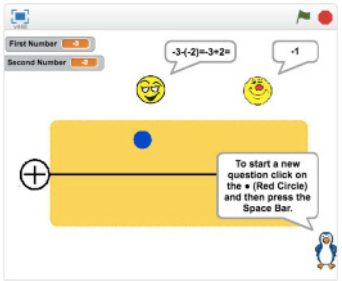


Figure 2: *Ms. LT's app designed using Scratch*

Example 1: To model 3 + 5, place 3 red counters in the upper part of the board and 5 in the lower part of the board. Then you move addend (lower) counters into the augend (upper) space. Since the counters are of the same colour, the answer is 8 red counters (3 + 5 = 8). Figure 3 shows a screen capture of Example 1 using the virtual manipulative.

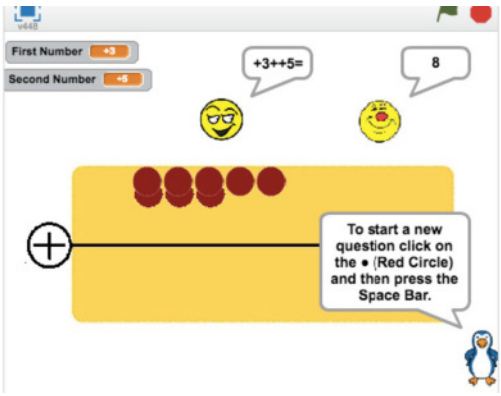


Figure 3: *Positive 3 plus positive 5*

The concrete counting board has a "+" sign, indicating that all problems must eventually be represented as addition, collection of objects, expressions. When the problem begins as a subtraction question on the virtual board, the next step is for the virtual board to represent it as an addition question. We return to this in Example 3.

Example 2: To model 3 + –5, on the physical board, you place 3 red counters in the upper part of the board and 5 blue counters in the lower part of the board. Negative and positive numbers of the same absolute value are additive inverses (1 + 1 = 0, –2 + 2 = 0, and so on). To perform the addition, you move addend (lower) counters into the augend (upper) space. A red counter and blue counter "cancel" each other, as they are additive inverses (identity pairs) and you are left with two red counters 3 – 5 = –2. Figures 4 to 6 demonstrate this process on the virtual manipulative.

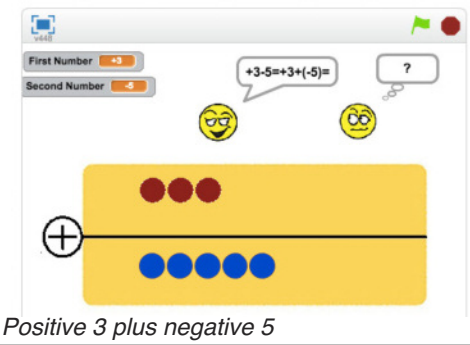


Figure 4: *Positive 3 plus negative 5*

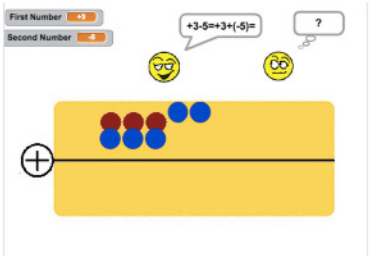


Figure 5: *Moving the addend (lower) counters into the augend (upper) space*

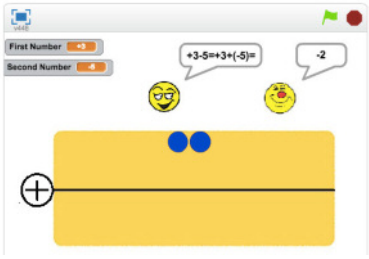


Figure 6: *Removing identity pairs*

Note that to model 3 – (–5), Example 3, on the physical manipulative, you rewrite the expression with a hidden "+"—3 + – (–5)—and recall that – (–5) means opposite of (–5), which is just 5. Then 3 – (5) = 3 + – (–5) = 3 + 5. See Figures 7 to 11. Learners are reminded that the virtual manipulative app is programmed to: treat "negative" "as a mathematical notation" for "opposite to a number" and so change – (–5) to 5; and treat "take away" in terms of combining sets, as with addition: "To subtract, change the subtraction sign to addition of the opposite, negative quantity. Then follow the rules for adding integers."

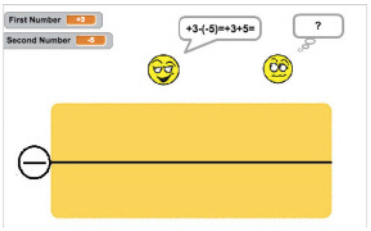


Figure 7: *Solving a subtraction question, positive 3 minus negative 5*

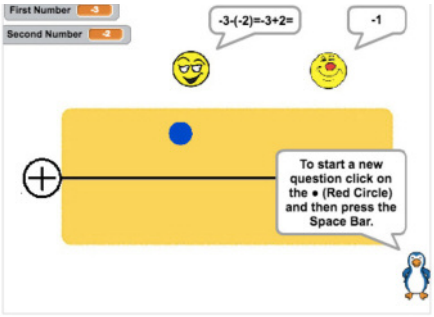


Figure 8: *Rewriting the expression in terms of an addition question, collecting terms*

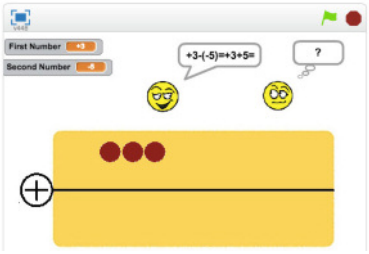


Figure 9: *Modelling the first augend*

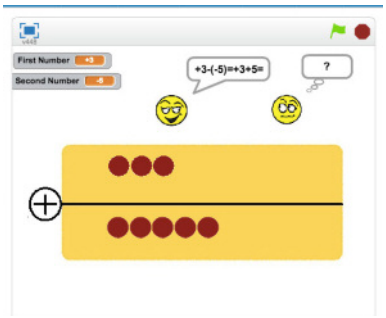


Figure 10: *Modelling the addend*

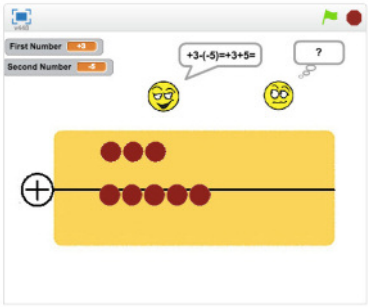


Figure 11: *Collecting addend and augend*

Reflections on the Teaching of the Unit

At the end of the unit, the researchers conducted an interview with the lead teacher on the implementation of the unit. They reflected on the unit's design, how it was taught, the adjustments that were made, the observed success of the unit, feedback on students' learning, and what to change the next time the unit is taught. Here are the highlights of the reflection.

The lead teacher in this study, for instance, built on the concept of whole-number operations and their varied meanings, and on the linear model of numbers on a number line, together with the motion geometry concept of reflection. She prompted her students to imagine a new space of negative numbers as the imagined opposites of whole numbers resulting from a vertical line passing through zero.[ii] The analogy of a mirror line showing reflections along the

[ii] *"Imagined" here is used in a manner similar to its use on imaginary numbers when referring to complex numbers at the university level. Complex numbers introduce the roots of negative numbers, the imagined roots of negative numbers.*

vertical line was helpful to explain negative numbers as opposites of positive numbers on the number line. She also explained negative numbers to have both magnitude and direction. The teacher also shared with the students the story that mathematicians had to agree on negative and positive symbols, as well as the conventions that apply to the system of integers. Further, when selecting real-life contexts, the teacher saw it helpful to make connections to some contexts such as debt, fortune, and misfortune. The teacher not only avoided the use of lists of rules when teaching negative-integer operations, but she also made connections for the students between negative integers as a concept explored in number sense and numeration and as a concept explored in patterning and algebra, geometry, and when manipulating expressions, variables, polynomials, and functions in Grades 9 and 10 algebra courses.

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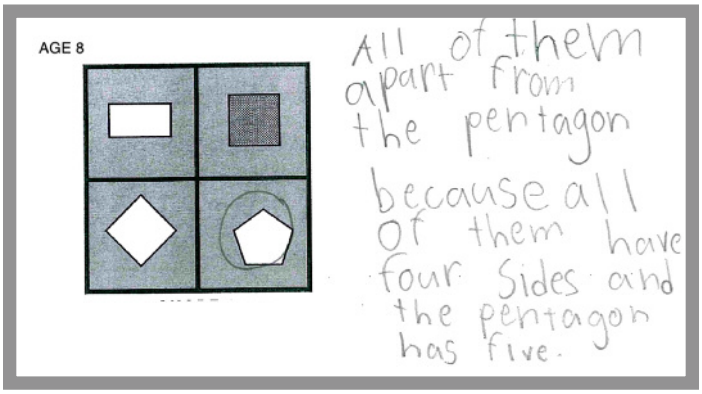
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▲ Auston Matthews and Monty Hall



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I am a huge fan of the Toronto Maple Leafs. I am a card-carrying member of Leafs Nation—the collective of people who would side with Sittler over Ballard, still blame Kerry Fraser for what he did in 1993, and carry the hollow, haunting memories of May 13, 2013. On my childhood streets, I played the role of Doug Gilmour, while my best friend alternated between Mats Sundin and Darcy Tucker. Anyone who follows the National Hockey League (NHL) knows that recently the Leafs have lost some of their lustre—but not for me, not now, not with Auston Matthews. The Leafs drafted the future superstar with the first overall pick in the 2016 NHL Entry Draft; he is phenomenally talented, and will no doubt become the face of the franchise. This article is not an exercise in blind faith, but rather a sobering reminder of what could have been, because the draft is an exercise in counter-intuitive probability. By examining one of the most famous problems in the history of probability—the Monty Hall problem—the goal is to appropriate the unlikelihood that allowed to us (I use plural pronouns in the belief that my fate and the Leafs' fate are wholly complicit) to procure the right to draft Matthews, as well as appreciate the benefit of counter-intuitive problems of probability in the mathematics classroom.

The NHL Draft Lottery Process

The draft lottery determines the order in which the teams that did not qualify for post-season play will select new prospects to add to their organization. Under the current format in the NHL, all 14 non-playoff teams have a chance at winning the lottery and getting the first overall selection. The lower the team finishes in the standings, the greater the probability that the team wins the right to select first overall. The probabilities established by the league are listed in Table 1.

Team	Final position in league standings	Total number of combinations (/1000)	Probability of drafting first overall	Percentage chance of drafting first overall
Toronto Maple Leafs	30th	200	.200	20.0%
Edmonton Oilers	29th	135	.135	13.5%
Vancouver Canucks	28th	115	.115	11.5%
Columbus Blue Jackets	27th	95	.095	9.5%
Calgary Flames	26th	85	.085	8.5%
Winnipeg Jets	25th	75	.075	7.5%
Arizona Coyotes	24th	65	.065	6.5%
Buffalo Sabres	23rd	60	.060	6.0%
Montreal Canadiens	22nd	50	.050	5.0%
Colorado Avalanche	21st	35	.035	3.5%
New Jersey Devils	20th	30	.030	3.0%
Ottawa Senators	19th	25	.025	2.5%
Carolina Hurricanes	18th	20	.020	2.0%
Boston Bruins	17th	10	.010	1.0%

Table 1: *Pre-lottery statistics for the 14 eligible teams*

Actually, the top three picks are all lottery picks. That is, each of the first three selections is determined by the draft lottery. In order to do so, 14 balls numbered 1 to 14 are placed into a lottery machine, and four balls are drawn, irrespective of order. This results in 1001 possible four-ball combinations ($_{14}C_4$ = 1001). One combination (11, 12, 13, 14) is eliminated by designating it as a redraw. The result is 1000 equally probable outcomes that are then assigned to eligible teams according to Table 1.

First, the combination is chosen for the winner of the first overall pick, which we now know was (5, 6, 8, 13) and belonged to the Leafs. The balls are then all replaced and another four-ball combination is drawn for the second pick. If the second combination belongs to the winner of the first lottery, the balls are replaced and combinations are drawn until a unique team wins the second overall pick. The process is repeated as such for the third pick. Most of the media attention is focused on the probability of winning the first pick, but the probability of a team winning the second overall pick (or third overall pick) can also be calculated, deepening the probabilistic appeal of the entry draft by serving as an interesting ancillary problem. Probabilities of drafting second overall are given in Table 2.

Team	Final position in league standings	Percentage chance of drafting first overall	Percentage chance of drafting second overall
Toronto Maple Leafs	30th	20.0%	17.5%
Edmonton Oilers	29th	13.5%	13.1%
Vancouver Canucks	28th	11.5%	11.4%
Columbus Blue Jackets	27th	9.5%	9.7%
Calgary Flames	26th	8.5%	8.8%
Winnipeg Jets	25th	7.5%	7.8%
Arizona Coyotes	24th	6.5%	6.9%
Buffalo Sabres	23rd	6.0%	6.4%
Montreal Canadiens	22nd	5.0%	5.4%
Colorado Avalanche	21st	3.5%	3.8%
New Jersey Devils	20th	3.0%	3.3%
Ottawa Senators	19th	2.5%	2.7%
Carolina Hurricanes	18th	2.0%	2.2%
Boston Bruins	17th	1.0%	1.1%

Table 2: *Percentage chance that each eligible team had at the second overall pick*

Note: Percentage chance of drafting second overall taken from Fox (2016).

How the draft was won. The draft is a televised event every year. To maximize the suspense (and television ratings), the results are revealed by listing the resultant draft order in reverse. Positions 14 through 3 are revealed, and then the second pick is skipped to reveal the winner of the top pick (and the services of Auston Matthews). In 2016, two teams remained: the Winnipeg Jets (originally finishing 25th with a 7.5% chance at the first pick) and your Toronto Maple Leafs (originally finishing 30th with a 20.0% chance at the first pick). The rest is history. Naturally, I was extremely nervous as the winner was about to be revealed; after all, decades of mood swings hung in the balance. In the moment, however, I never realized just how nervous I should have been. Not until a friend and I were recapping the events and he said, “I was really nervous at the beginning, but as teams were eliminated, I just knew. By the time it was just us and Winnipeg, I’d take the coin flip—I knew we had it.” And thus, the Auston Matthews Problem was born: Knowing the order of picks 3 through 14 (Table 3), what was the probability that the Leafs should have won the first overall pick? I went through the rest of the day thinking about the possible solutions, each of which seemed probable, intuitive, yet contradictory. I didn’t arrive at the solution—to be presented later—until I framed the Auston Matthews alongside another famous problem, the Monty Hall problem. Here, I claim that the Monty Hall problem and the Auston Matthews problem are mutually informative. That is, features of each problem offer insight into the solution of the other.

Further, I claim both have value in the mathematics classroom.

Team	2016 Draft Position
Toronto Maple Leafs	1st or 2nd
Winnipeg Jets	1st or 2nd
Columbus Blue Jackets	3rd
Edmonton Oilers	4th
Vancouver Canucks	5th
Calgary Flames	6th
Arizona Coyotes	7th
Buffalo Sabres	8th
Montreal Canadiens	9th
Colorado Avalanche	10th
New Jersey Devils	11th
Ottawa Senators	12th
Carolina Hurricanes	13th
Boston Bruins	14th

Table 3: *The 2016 draft order prior to the reveal of the last two positions*

The Auston Matthews Problem

This article assumes that you are a Leaf fan (you're welcome). The problem is as follows: There are 1000 randomly distributed combinations according to the frequencies in Table 1. Two hundred (20%) of those combinations result in your team winning the first overall pick. If positions 14 through 3 have already been revealed, and your team is one of the final two teams eligible for the number one pick, what is the probability you win the number one pick? It may be that you already have an answer and requisite justification in mind. In order to expand on the counter-intuitive nature of the problem, three perfectly sensible answers are provided below.

The 50/50s. This was the reasoning provided by my friend when he said that the final reveal was a "coin flip," and it is quite simple. When there are two teams remaining, one of two things can happen—either you can win the pick or the other team wins the pick. This is a sample space of two events. Only one of those events is favourable, and so P(Win) = $\frac{1}{2}$. It makes sense.

The 7.5/20s. People in this camp follow a similar line of reasoning, but they contend that the 50/50s do not account for which other team is remaining at the end of the draft lottery. (Remember, in our case, it was the Winnipeg Jets with a 7.5% chance at the first pick, or 75 total combinations). Because all of the previous teams have been eliminated, the choice is between the two remaining teams. However, their chances are not equal; your team has 200 combinations and the other (in this case, the Winnipeg Jets) has only 75. Therefore,

$$P(Win),\ \frac{200}{200+75} = \frac{200}{275} = \frac{8}{11},$$

much greater than $\frac{1}{2}$. It makes sense.

Both of these solutions rely on shrinking the field as teams are eliminated. Knowing that certain teams cannot win eliminates them from the sample space of possibilities. It needs to be restated that restricting the sample space seems like a logical pathway; the process is included in the study of conditional probability in MDM4U courses (Ontario Ministry of Education, 2007) and is performed when drawing the lottery balls. The calculation of any team's probability of drafting second overall provides an interesting problem-within-the-problem, but it comes with implications. That problem revolves around the fact that if a team wins a pick, it becomes ineligible for another top pick. Consider the percentage chance that the Leafs would have won the second overall pick (see Table 2). That calculation was conducted by assuming that the Leafs did not win the first overall pick, and that each other team (one by one) did win the pick, eliminating those teams' combinations from the sample space, and then recalculating the new probability that the Leafs (still retaining their 200 combinations) would win the second draw.

Let "*Xn*" be the number of winning combinations assigned to Team *Xn*, $2 \le n \le 13$, $x \in$

$$P(\text{Leafs Win 2nd Pick}) = \left(\frac{x_2}{1000}\right)\left(\frac{200}{1000-x_2}\right) + \left(\frac{x_3}{1000}\right)\left(\frac{200}{1000-x_3}\right) + \ldots + \left(\frac{x_{13}}{1000}\right)\left(\frac{200}{1000-x_{13}}\right.$$

$$= \frac{200}{1000}\left(\frac{x_2}{1000-x_2} + \frac{x3}{1000-x_3} + \ldots + \frac{x_{13}}{1000-x_{13}}\right)$$

$$= \frac{200}{1000}\left(\frac{135}{865} + \frac{115}{885} + \ldots + \frac{10}{990}\right)$$

$$= .175, \text{or } 17.5\%$$

Although the second overall selection is outside the scope of the Auston Matthews problem, the format is deceptive. After the 14th overall pick is revealed, it appears that team no longer has an impact on the outcome of who gets the first draft pick. If they are all eliminated from contention, it follows that the 50/50s and 7.5/20s gain credence. Again, it makes sense.

The 80/20s. People in this camp believe the probability that the Leafs win the first overall selection is 20%. This can come across as uninformed. How, after all that has unfolded, can they possibly claim that the probability is unchanged? The answer resides in the chronology of events. When selecting who would draft first overall, the first combination

chosen won. At that time, all 1000 combinations remained equally probable. Phrased differently, every team was still very much in contention for the number one pick at the time of selecting the number one pick. It seems like the losing teams are eliminated because the teams are revealed in reverse order, but it is only hindsight that allows us to eliminate them. While it seemed as though the Leafs were gaining influence with each passing elimination, their chances, in fact, were always 20% throughout the reveal process: $P(Win) = .20$ (or 20%). This means that the probability of the Leafs losing the number one pick also remained constant throughout the process: $P(Loss) = 1 - P(Win) = 1 - 20 = .80$ (or 80%). In the end, the entirety of the 80% chance of loss was represented by one team—the Winnipeg Jets. If all of this sounds outlandish, assume for a moment that the reveal process was flipped. Assume that the Deputy Commissioner of the NHL began the draft lottery show by revealing the winner of the number one selection. Perhaps more intuitively, we can now see the Leafs "own" 200 of 1000 combinations: $P(Win) = .20$ (or 20%). Revealing the results in the same way they were created sheds some important light on the situation. It makes sense.

The Monty Hall Problem

Researchers and recreational mathematicians alike have parsed apart the Monty Hall problem in an attempt to analyze its intricacies (e.g., Barbeau, 1993; Franco-Watkins, Derks, & Dougherty, 2003; Rosenhouse, 2009) and document the psychological effects at play while solving it (e.g., Gilovich, Medvec, & Chen, 1995; Krauss & Wang, 2003). The problem is as follows: There are three doors. A desirable prize (typically, a car) is hidden behind one random door, and the other two doors hide undesirable prizes (typically, goats). You choose a single door (say, Door #1) in hopes of revealing the car, but before the door is opened, Monty Hall opens another door (say, Door #2), which *always* reveals a goat. Regardless of your initial choice, there will always be a goat to reveal. Monty Hall then asks if you would like to stay with your original choice (Door #1) or switch to the other door that remains closed (Door #3). What is the probability of winning the car if you stay with your original choice? In the context of the problem, you never "win" a goat. There are only two possible outcomes: winning a car and losing a car. Two categories of familiar reasoning are detailed here; they represent two popular logics, not an exhaustive list.

The 50/50s. This is the identical "coin flip" reasoning presented earlier, except the teams from the Auston Matthews problem are now doors, and the first overall pick is now a car. When you are asked to make a decision, there are only two possible outcomes. From the sample space of two doors, there is only one that is hiding a favourable outcome, and therefore, $P(Car) = \frac{1}{2}$. It makes sense.

The 67/33s. This line of reasoning plays on the fact that any subsequent decisions in the game rely on the initial decision—the choice of Door #1. Regardless of which door you choose, Monty will reveal a goat, so while it may seem like he is acting randomly, he is not. The elimination of teams in the Auston Matthews problem did not affect the original conditions of the draft lottery for the number one pick, and Monty's elimination of a losing door does not affect the initial choice—Door #1. Few people would argue that you would have a one-in-three chance of winning if the game were to choose a door and Monty immediately revealed if it hid a car or not. Obviously, in that case, $P(Door\ \#1\ Car) = \frac{1}{3}$. That means $P(Door\ \#2\ Car \cup Door\ \#3\ Car) = \frac{2}{3}$. Eliminating Door #2 *after* you have made a choice does not spontaneously cause the probability of the car being behind Door #1 to increase; no prizes are being shuffled and no choices are being remade. That means that $P(Door\ \#1\ Car)$ remains $\frac{1}{3}$.

The entirety of $P(Door\ \#2\ Car \cup Door\ \#3\ Car) = \frac{1}{3}$ has *condensed* and is now represented by Door #3—the only other remaining door (Figure 1). When you chose Door #1, the probability that you chose the door hiding the car was $\frac{1}{3}$. Every option from the $\frac{2}{3}$ probability that you did not choose the door hiding the car has been condensed into a single door—Door #3. The probability that the car is behind *that* door is now $\frac{2}{3}$, or approximately 67%. This means $P(Door\ \#1\ Car) = \frac{1}{3}$, or approximately 33%. The probability of winning a car if you stay with your original choice is $\frac{1}{3}$. It makes sense.

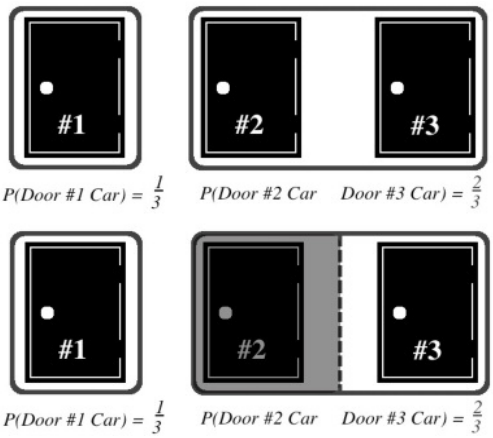


Figure 1: *P(Door #2 Car Door #3 Car) condensed into one door—Door #3*

The Problems as Mutually Informative

Considered separately, the two problems spark considerable debate. Examined together, each offers insight into the other through their structural similarities. Imposing key characteristics from one problem onto the other clarifies interpretive possibilities. For the 80/20s and 67/33s, these amalgams will further support their cause. Others are left to debate whether these new possibilities have provided any logical weight against their arguments.

Monty Hall informs Auston Matthews. Let us, for a moment, impose a characteristic from the Monty Hall problem onto the Auston Matthews problem. Redefine the outcomes into two categories instead of the original 14 categories, where each team winning the lottery was considered to be a separate outcome. Now, the Leafs only win if they win the first overall selection. If any other team wins, it is categorized as a loss. Essentially, the 13 possible non-Leaf teams all result in the same outcome, a loss, so they are treated as the same—much like all the goats are created equal to Monty Hall. Projecting this binary vision of winning and losing onto the Auston Matthews problem—rather than entertaining 13 unique ways of losing—allows us to envision how the probability is condensed in the same vein as the Monty Hall problem. That is, I do not care which team is left standing alongside the Leafs, only that there is one non-Leaf team remaining. We know that $P(Win) = \frac{200}{1000}$, = .20 or 20%, represented by the 200 combinations assigned to the Leafs. It follows that the $P(Lose) = \frac{800}{1000}$, = .80 or 80%, represented by the 800 votes attributed to the other teams.

The top image in Figure 2 is a familiar partition of outcomes; each of the 1000 dots represents a four-ball combination. The top two rows (200 total dots) represent combinations assigned to the Leafs. The remaining rows (800 total dots) represent combinations assigned to other teams. At the time of the reveal, the draft order had been set. It is not being reshuffled—only revealed. Exposing teams drafting lower in the order does not shrink the sample space of the original draw; however, it also never eliminates any of the Leafs' combinations. The bottom image shows how the *guaranteed* elimination of non-Leaf teams condenses the probability of losing into the combinations held by the remaining team (Winnipeg's 75 votes pictured), while the $P(Win) = \frac{200}{1000}$ = .20 remains represented by the original 200 combinations. Clumping the combinations into 2 groups as opposed to 14 affords the image of condensing probability to the Auston Matthews problem, further supporting its solution—$P(Win) = \frac{200}{1000}$ = .20 , or 20%.



Figure 2: *P(Lose)* = $\frac{800}{1000}$ *condensed into the 75 combinations of the Winnipeg Jets*

Auston Matthews informs Monty Hall. Let us, for a moment, impose a characteristic from the Auston Matthews problem onto the Monty Hall problem. A door is chosen and the host opens all but one other door, revealing a goat each time. All of the original mechanisms of the problem remain the same. At that point, the contestant is still asked whether he or she would like to stick with his or her original choice or switch to the one door remaining closed. The only difference, imported from the Auston Matthews problem, is that instead of three initial doors, there are 1000—mimicking the 1000 possible four-ball combinations from the draft lottery. Now, you choose a door (say, Door #1) and Monty Hall opens up 998 of the remaining 999 doors to reveal goats (Figure 3). He then asks if you would like to stick with your original choice, where P(Door #1 Car) = $\frac{1}{1000}$, or switch to the one remaining door (say, Door #34, in honour of Auston Matthews), which now holds the condensed probability.

$$P\left(\overline{Door\#1\,Car}\right) = 1 - P(Door\#1\,Car) = 1 - \frac{1}{1000} = \frac{999}{1000}$$

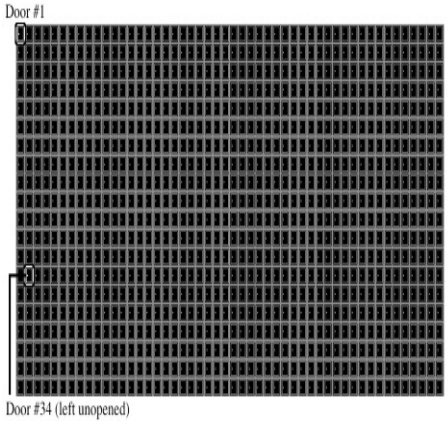


Figure 3: $P\left(\overline{Door\#1\,Car}\right) = \frac{999}{1000}$ *condensed into the one door—Door #34*

Now, the clear (and maybe even intuitive) choice is to switch to Door #34. This result can be extended to any number of "n" doors ($n \geq 3$, $n \in \mathbb{N}$). The first door is chosen (say, Door #1). Monty Hall opens "$n - 2$" doors to reveal "$n - 2$" goats. In each general case, you should switch to the remaining door because

$$P\left(\overline{Door\#1\,Car}\right) = 1 - P(Door\#1\,Car = 1 - \frac{1}{n} = \frac{n-1}{n}$$

This probability will never be smaller than when $n = 3$, the original Monty Hall problem.

$$P\left(\overline{Door\#1\,Car}\right) = 1 - P(Door\#1\,Car = 1 - \frac{1}{3} = \frac{2}{3}$$

In other words, regardless of "n," your chances of winning the car if you stay with your original choice are never greater than $\frac{1}{3}$. We can now see that as "n" gets smaller, the intuitive result clouds itself in the desire to restrict the sample space instead of condense the probability, but switching to the solitary door left by Monty becomes more and more intuitive as the number of original doors increases—a feature imported from the Auston Matthews problem.

Final Comments

It is my belief that examining these two problems in tandem constitutes a better understanding of both problems, and, in turn, that a better understanding exposes the subtleties of probability as a domain of study. As a corollary, I contend that debating the subtleties of probability is a crucial piece in a robust education in probability. The results reported here typically garner two types of responses, both of which are *undoubtedly* positive. First are those, like myself, who find the dual analysis of the Auston Matthews problem and the Monty Hall problem as mutually informative. This is positive because, hopefully, they have begun to develop, as I did, a more informed stance when tackling counter-intuitive problems of probability. Second are those who contend that the previous analysis is misguided and malformed. This is still a positive outcome worth cultivating in a classroom because these people (or students) have arrived at this conclusion after an investigation of their existing understanding of probability. This interrogation is surely a valued result for the teaching of mathematics. On top of that, this conviction is positive because they, like my friend, will find themselves much less stressed watching draft lotteries and game shows. Heaven knows it is already stressful enough to be a Leafs fan.

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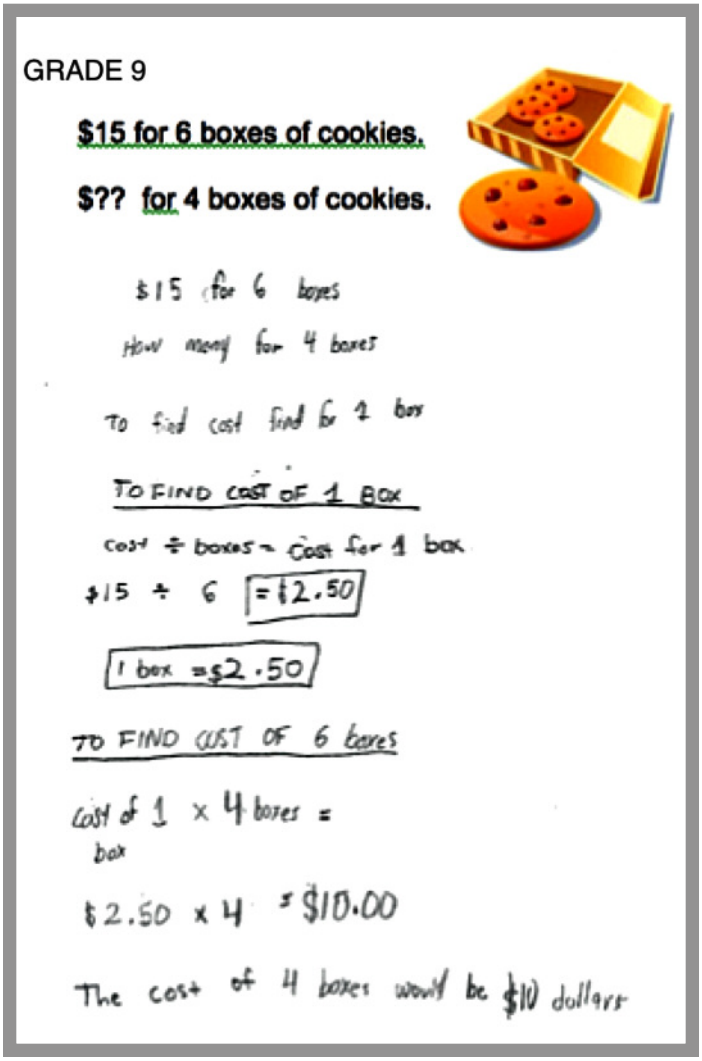
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