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Ontario Mathematics Gazette

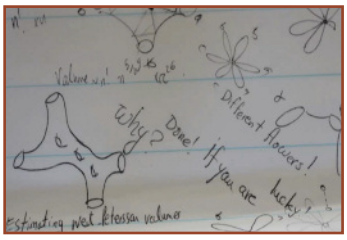
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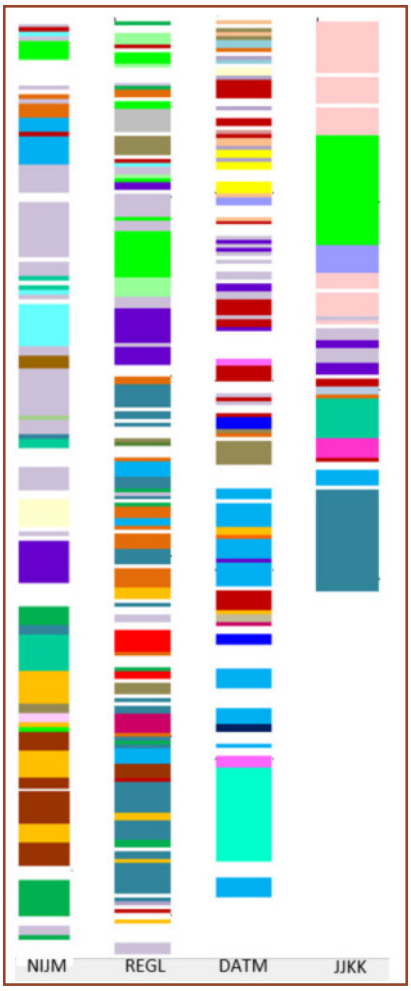
Vol. 56 #2
December 2017

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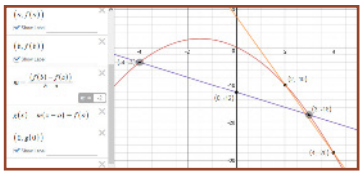


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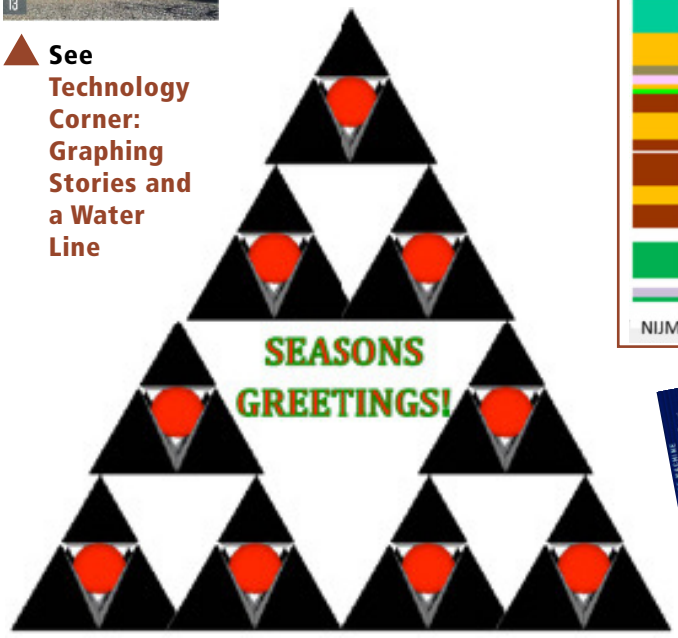


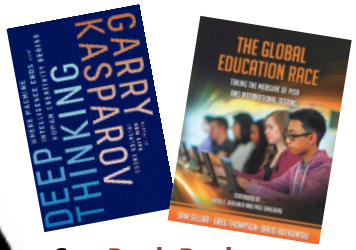
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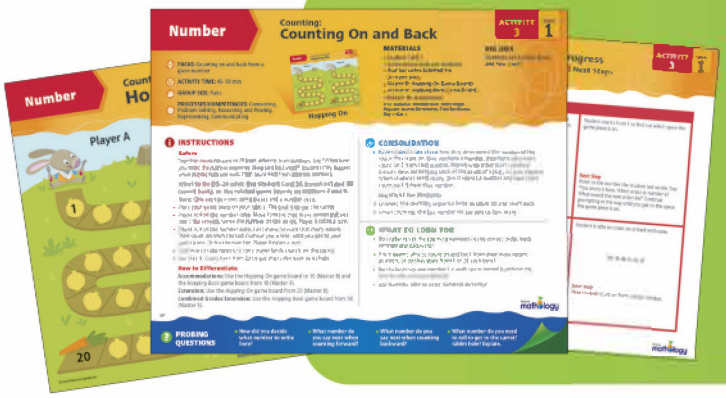


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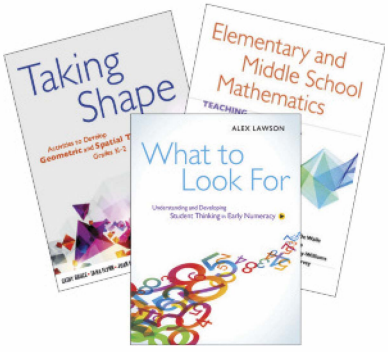
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The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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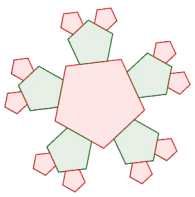
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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
EMAIL: gazette@oame.on.ca

Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.*

As teachers, there are no end of details we consider to be effective in the classroom. We wrestle with issues that reflect a variety of human conditions that our students bring through the door. Teachers take a breather now and then to refocus, to remind themselves of the important work they do, and why they pursued teaching in the first place. I do not think these notions surprise anyone who has spent time in the classroom, and I wish you a restful holiday season, where you may find time to contemplate the mathematics of cones as you look on a Christmas tree.

What is surprising is the way math educators go about expanding their content repertoire. The *Gazette* is ripe with ideas for good teaching, and OAME membership is a voice that promotes quality content instruction. One concern I have had over the years is with the larger field of mathematics. It is a nagging concern about how modern content migrates from its discovery to the K–12 classroom. What bothers me is the difficulty in coming up with an example from the field of pure mathematics. Even in Applied mathematics, examples are challenging to find. There are a few examples, such as fractals, as mimicked with the *Seasons Greetings!* Sierpinski triangle tree on the cover (Figure 1), but examples seem relatively rare.



Figure 1: *Sierpinski-inspired holiday wishes*

Over the years, I have asked whether this is a reflection of how forward thinking or abstract mathematics is—I am not sure. Certainly much of mathematics fits both criteria. However, when I began teaching, as a science teacher, I found plenty of connections to science research and much of it was about abstract scientific concepts—subatomic particles, gravity waves, and genes were all abstract concepts and forward thinking. That has led me to wonder if mathematics is simply lacking communication channels or the capacity to articulate mathematical research ideas to teachers.

There are problems in mathematics, however, that are unsolved and accessible. An example arose while reading an

article about Maryam Mirzakhani. It revealed a straightforward, yet unresolved, problem asking when billiard ball motion in a polygon has a repeating path (see **www.nature.com/nature/journal/v549/n7670/full/549032a.html?foxtrotcallback=true**). This is an unusual example of a possible way to connect with work done by active mathematicians. It is unfortunate that we do not see mathematics research provide more avenues for teachers to connect with their work.

I am delighted to see an article about Maryam Mirzakhani, as she was a modern woman exploring groundbreaking mathematics. Being the first woman to win the Fields medal is something well suited to classroom discussion and exposition. Further, knowing that the Fields medal is named after a Canadian (Figure 2) should be part of every Canadian's math education.

While on the topic of groundbreaking people, the movie *Hidden Figures* (20th Century Fox, 2016) is fascinating and brings the curriculum orientation about inclusion into focus. It speaks directly about the role of Black women within NASA and reminds us of the reality of that time. Coincidentally, the movie demonstrates that times change, and in this case, the way the computer was transforming the mathematical workplace. They had to address the onset of the computer in the workplace, and teachers continue to wrestle with the place of technology, knowing it has value, but that one has to be circumspect about its usage. Certainly *Hidden Figures* is a movie worthy of watching, if you have a few hours over the holidays.



Figure 2: *Charles Fields (from **www.fields.utoronto.ca/aboutus/jcfields/index.html**)*

I thought I would share a couple of photos that are mathematically interesting. The first is from the summer and raises the simple question, "How big is the bug?"

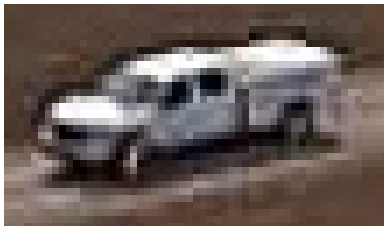


Encourage students to explore different interpretations of "big." The antennae are clearly long, but is the length of the antennae enough to convey "big-ness"? What size of triangular box would fit the bug? I understand from Oriana Pokorny, a colleague at Nipissing University, that this is an Eastern Sawyer Beetle, more commonly referred to as a "pine bug." She advises that their mandibles can provide a nasty bite, if they are provoked. Perhaps the effort to heed that advice should consider the size of circle the beetle could make, if it were defending itself—a different conception of "big-ness."

The second photo is of a wind turbine taken from atop a neighbouring wind turbine. As these machines have become more commonplace in Ontario, so has employment to sustain them. It is an example of how teachers, in the past, prepared students for careers they did not necessarily know would exist.



It is also an opportunity to exercise some math skills, and while I have mentioned some mathematics of photos beforehand (see Issue 53(4), p. 25 in the archives), I chose to take a different approach to the wind turbine photo because I had information about it. Specifically, I was inclined to see what I could determine, using what I knew about the photo, rather than a thorough examination of the math of camera photos.



To start, the truck at the bottom of the turbine, also shown enlarged, is a Chevrolet 3500 that, according to the Internet, has a wheelbase of 167.7 inches. By counting pixels—the little blocks that make up digital photographs—I found 24 pixels make up the wheelbase. I concluded one horizontal pixel is approximately 7 inches, or 17.75 cm, and that allows me to scale details in the vicinity of the truck, like the diameter of the base of the tower (which also happens to be 24 pixels, making the diameter 167.7 inches, or 4.26 m).

In the wind turbine photo, the shadow of the tower is 132 pixels long, and from the base of the tower to the top of the nacelle (the box on top of the tower) is 521 pixels. The tower is actually 70 m high, which, using proportional reasoning, makes the shadow 18 m long. The blades are 295 pixels and 295/521 x 70 m gives their length as 39.6 m long. At this point, it might be worthwhile using a hallway to mark that out to appreciate it is longer than a football field!

Since this photo was taken on September 12, the sun was 10.28 days away from the Fall equinox, thus the earth was tilted so the sun was directly overhead 2.57° north of the equator. The photo was taken at approximately 42.63° north, so the shadow forms an angle with the ground of approximately 50°. Unfortunately, as nifty as this calculation is, and it could be measured in a schoolyard, it is problematic in the photograph because the angle that the camera was pointing is unknown. The shadow length in the photograph can appear substantially shorter than the actual shadow on the ground because of the viewing angle. This brings the adventure to an interesting

challenge question: What can and cannot be determined? Since the photo was taken from the top of a neighbouring, and identical, wind turbine, can the distance between the two turbines be determined or not?

Before introducing the rest of this issue, there are two requests to convey. First is a call for volunteers to help with the OAME 2018 conference (Figure 3). This is well suited to pre-service teachers or responsible high school students who wish to earn community service hours and are in the area of Humber College. Anyone wishing more details should contact Margaret Quinn at **margaretquinn8@ gmail.com**.

Figure 3: *Volunteers needed*

The second request is for an *associate editor* for the *Gazette*. This is part of a team where the associate editors give oversight of draft articles and columns with an eye for mathematical accuracy, looking for improved wordings to clarify meaning, and to help keep an eye on consistency of content with the vision of the OAME. If you would like details about this role, please contact Tim Sibbald at **gazette@oame.on.ca**

In This Issue

Jill Lazarus brings the *President's Message*, and it is interesting to see the remarks in the *Executive Directors' Report* about the role of the President. It is coming around to the time when nominations can be made for the board of directors and the executive. If you are thinking that this may not be for you, then please approach your colleagues and encourage them. Sometimes it takes a supportive colleague to make one realize that they are ready to consider such options.

The regular columnists bring a raft of interesting ideas as usual. Carly Ziniuk (*In the Middle*) shows how photos taken on a math trail can encourage student understanding of how mathematical ideas fit with real-world shapes. Assessment Abby, the place to send your assessment questions, suggests resources to help support different ways of addressing proportional thinking. Jacqueline Hill returns as a *Gazette* columnist as the NCTM representative, and in the *NCTM Report*, she challenges us to earn Prizes! Shawn Godin (*What's the Problem?*) considers a question from a math contest, and looks at the design of the question, which facilitates instructional options for using the question. Judy Mendaglio brings us the first female Fields medal winner. Jacqueline Hill and I bring book reviews—if you know of a book you would like to review, please contact me (**gazette@oame.on.ca**).

The Grade 9 Applied project returns with a focus on professional learning communities. This is an interesting shift from their article in the last edition. The Provincial Digital Learning Resources team also brings a follow-up article as they delve into digital tools to support teaching multiplication and division of fractions, and point out many useful resources. Mary Bourassa (*Technology Corner*) gathers multimedia tools that engage students in "telling graph stories."

In *Hey, It's Elementary*, Lynda Colgan gives an interesting picture of the current challenges that math educators face. I found myself revisiting a spreadsheet (Figure 4) I had tucked away. I think the graph helps in the interpretation of EQAO data because it includes the historical results for Grades 3, 6, 9 Applied, and 9 Academic. Note that EQAO has not reported results from 2015 because of issues surrounding the testing that year. I will pose two questions about this data: Firstly, why did Grade 3 and 6 begin to decline at the same time rather than staggered by three years? Second, why hasn't the decline in Grades 3 and 6 shown up directly in Grade 9? As editor, I invite you to engage these questions or ask others in letters to the editor.

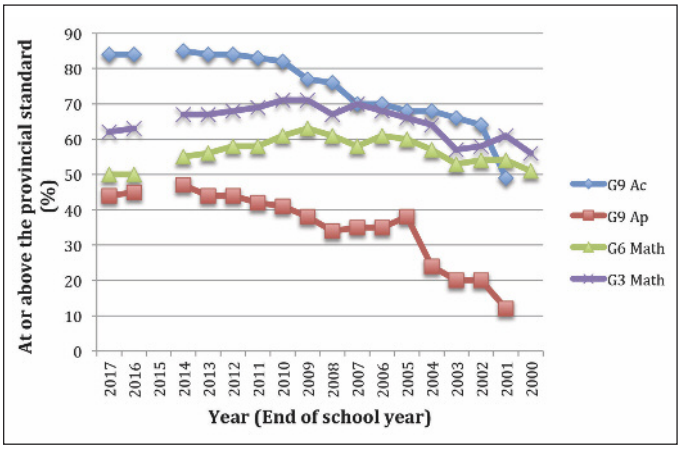


Figure 4: *Historical Provincial EQAO results (based on **www.eqao.com**)*

In terms of articles, I am pleased to see Alayne Armstrong bring her work to the *Gazette*. Her use of tapestries to show the complexity of student conversations is intriguing. The article is a very brief summary of her doctoral dissertation.

Jeff Gardner explains a technique for developing rational function questions that are designed to facilitate instruction. It is an interesting approach that will be useful for senior math teachers.

Again, the *Gazette* has the benefit of the many article authors and columnists sharing their ideas and expertise. I also would like to acknowledge the ongoing behind-the-scenes work of Anne Yeager, Gitta Berg, and Penny Clemens—without them, the *Gazette* would not have the professional sharpness we all enjoy.

Lastly, best wishes, on behalf of the *Gazette* team for the holiday season!▲

President's Message



JILL LAZARUS
EMAIL: Jill.Lazarus@oame.on.ca

Jill Lazarus, the current President of OAME, is a high school teacher in the Renfrew County District School Board. She is also completing a PhD in Education at the University of Ottawa. Jill has a range of interests in mathematics education, including curriculum design and implementation, student communication in mathematics, educational technologies, classroom assessment, and professional learning.

I am writing this message a little over a month into a new school year, after engaging in much reflection on the current mission and goals of the OAME. As Judy Mendaglio, the Past-President of the OAME, highlighted in the September 2017 issue of the *Gazette*, "[...] our organization has undergone explosive growth in a very short time" (p. 7). This growth is exciting and has prompted us to reconsider the current and future goals of the OAME. As Judy also mentioned, this work has already begun.

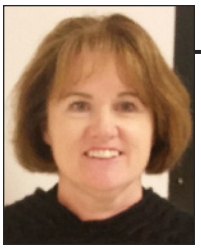
During a meeting in August 2017, the OAME Executive reflected on our current and long-term goals in connection to our mission statement, objectives, and vision, which are identified on the OAME website (**www.oame.on.ca/main/index1.php?code=principles**). One goal that emerged involved the need to support membership and to cultivate relationships with OAME Chapters, school boards, the Ontario Ministry of Education, post-secondary institutions, and other organizations. With the current capabilities of technology, we are now exploring ways to use technology to support members. This conversation will continue this year, and we will be seeking input from members.

The OAME Board of Directors has also been working to prepare for the upcoming 45th annual OAME Conference, "Infinite Possibilities," which will be held on May 3–4, 2018 at Humber College in Toronto. Please visit the conference website at **www.oame2018.ca** for up-to-date information. Full registration will begin on February 1st. Mark your calendar because it is going to be another exciting event. So make sure to save your spot and register early.

Reference

Mendaglio, J. (2017). Past-President's retrospective. *Ontario Mathematics Gazette, 56*(1), 7. ▲

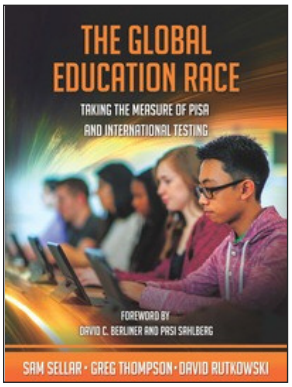
A Review of *The Global Education Race: Taking the Measure of PISA and International Testing*



REVIEWED BY JACQUELINE HILL

Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a past president of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the OAME Gazette for a number of years.

Sellar, S., Thompson, G., & Rutkowski, D. (2017). *The global education race: Taking the measure of PISA and international testing.* Edmonton, CA: Brush Education Ltd. See **www. brusheducation.ca/books/the-global-education-race**.



This book is an informative read and will be useful for classroom teachers, consultants, university professors, and anyone interested in the global education of students to gain an understanding of the larger context of standardized testing. The authors did their homework before writing this book. They took into account both narrow and wide scopes of testing globally. Their reference list is extensive. At 96 pages, the book is a starting place for educators to improve their understanding of testing, as well as a conversation starter for those already immersed in educational issues. The authors speak in depth about rankings, tests, validity, politics, and helping policy-makers find the right track. They include a discussion about the inception of PISA all the way to the data from the 2015 testing. This book is an appropriate candidate for a book study group of consultants and principals in education. It is well suited to anyone who wishes to gain some insight into the big ideas of standardized testing and international comparisons. ▲

▲ Executive Directors' Report

LYNDA AND FRED FERNEYHOUGH
EMAIL: eds@oame.on.ca





Lynda and Fred Ferneyhough have been the Executive Directors of OAME since 2010. They taught in the Peel District School Board for over 30 years and had a three-year stint in the United Arab Emirates. Both of them served OAME as Chapter Representatives for CHAMP, Directors, and Vice-Presidents. During their career, they were Department Heads, and authors for McGraw-Hill Ryerson. They are both certified as instructors for Texas Instruments (TI), and Fred continues to coach for TI in the United States.

In our role as Executive Directors, we are grateful for the many volunteers who make this organization run so well. For this issue of the *Gazette*, I want to focus on those individuals who have served as President of OAME.

In our organization, the President serves a one-year term in that role, but also serves one year as President-Elect prior to their year in office and one year as Past-President. This gives those individuals a year to prepare for their role and a year thereafter to assist and advise their successor. When we compare our organization to similar mathematics organizations in Canada, we find that many of our counterparts require their president to serve for two years. At the moment, the SPaRC committee is considering the wording of a motion that would extend the term of the President of OAME to two years.

Over the 40 years I have been involved with OAME, I have seen some amazing math teachers serve as the presidents of OAME, almost a "Who's who?" of people in our profession. We, as an organization, have been blessed with amazing individuals who have been willing to lead and lay out a vision for OAME. They share that vision by planning a leadership conference to set the tone for their year in office. Frequently, Presidents organize professional development during board meetings in order to convey their vision to the elected directors and chapter representatives.

Over the years, there have been legal, personal, and political challenges that have been handled in a professional manner by the Presidents, with help from the executive and board. The individuals and boards have learned from these situations, and from time to time, the constitution has been amended accordingly. The upside is that the organization grows and learns through these challenges, and OAME remains steadfast in its vision.

So, you might well ask, "Why would anyone want the job of President of this organization?" I have found that teachers are by nature giving people. We give our time and our talents and we want to share our passion for mathematics. Our Presidents take that attitude to the extreme and have the privilege of working with an incredible group of board members, and the opportunity to share their vision for mathematics education in Ontario. What we have heard from many of the recent individuals in this role is that one year is just not enough, that you just find out how to get things done and then are moved along to Past-President.

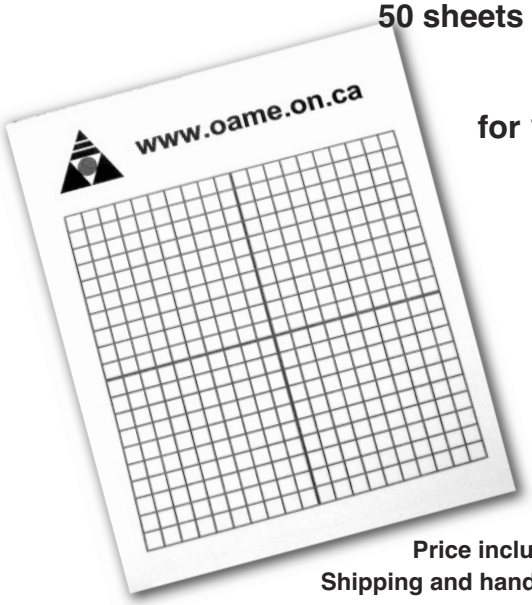
From my perspective as an Executive Director, I am very thankful that so many talented, intelligent, and wise people have stepped forward to serve as President of OAME. Check out the archive section of the OAME website for a listing of all of the amazing people who have served as president of OAME.▲

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▲ MB4T (Mathematics by and for Teachers): A Cornucopia of Important Concepts across the Grades



TIM SIBBALD
EMAIL: timothys@nipissingu.ca

Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and a past-president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University.*

As teachers, we focus on good instruction and strive to enact it. However, most of us remain convinced that there is still work to be done. Certainly anyone who has participated in lesson study, reflective practice, or other collaborative activities will realize that there is often rich dialogue to be had. In this column, a few details are explored with an eye to the conceptual implications for instruction.

Pi – The concept

Many students think π is a magic number. It certainly is and has a huge range of properties to support that view. However, what teachers need to consider is that many students think the magic is due to an instruction *faux pas*, where the number is pulled out of thin air. It is incumbent on teachers to point out that students can determine the approximate value of pi through calculation. As one example, consider that pi can be estimated using the area of a circle. The units for area are square units, so a radius measured in cm leads to an area in cm^2. These units suggest the area is a multiple of the square of the radius: $A = br^2$, where *math teachers know* $b = \pi$, but why ruin the surprise for students? Instructionally, we can measure the area by counting squares and similarly measure *r.* Then it becomes feasible to approximate pi!

The measurement process does not need to be in centimetres, which were only used to argue for the area being a multiple of the square of the radius. In Figure 1, an estimate is shown that uses squares of arbitrary size; in fact, they are shown as they might appear if drawn freehand. In the figure, counting the horizontal diameter is 8 squares, so the radius is 4 squares. The squares inside the circle can be counted in a variety of ways; I counted from the top left and left to right (Figure 1, numbers inside the circle). When I got half the circle filled, I decided I could use symmetry to argue the other half should be the same. The count of 16 squares inside the circle has missed the partial squares around the edge of the circle. For these, an estimate was made of how much of a full square needed to be added on. For the part square to the right of the square with 16 written in it, an estimate of only 0.9, or 90 percent, of the square inside the circle was made and added to the 16 to get 16.9. The next value of 17.4 adds 0.5, or 50 percent, of a square onto the 16.9. This was done until the area of all part squares in the semicircle was estimated, giving a total area of 23.3. The estimate of the area of the whole circle is 2 x 23.3 = 46.6.

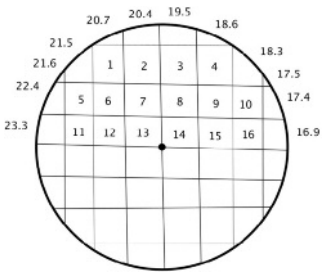


Figure 1: *Estimating the area of a circle*

Since $A = \pi r^2$ and the area and radius have both been estimated, it is possible to use those values to calculate pi. Substituting the estimates gives $46.6sq^2 = \pi(4sq)^2$ and an approximate value of 2.9 for pi (with no units). Now you know that is not the true value of pi, but the point is that this can be demonstrated freestyle in a classroom. When I have done this in Grades 10 and 12 classes, and similarly in classes of teacher candidates, there is a stunned sense that pi is more accessible than they previously thought.

The freehand approximation is out by 8 percent, and that sets up an opportunity to talk about ways to improve the estimate. During freehand classroom demonstrations, I have always arrived at values between 2.8 and 3.3. On one occasion, I achieved 3.1, which was somewhat unfortunate because it diminished the discussion of improving the estimate. Improvements can entail using a larger circle on graph paper. Students can also count only squares that are fully inside the circle to get an underestimate of the area and then include full border squares to get an overestimate, while using the average of the two values as their estimate. Ultimately, increasing the number of horizontal and vertical lines gets one closer to the real value of pi, but it becomes tedious to count individual squares. To address this problem, students can draw rectangles inside the circle as an efficient way to count many squares quickly.

Fraction language

Fractions are a challenging topic for many students and we, as teachers, hope that instruction is providing the best support possible (see additional resources for some current Ontario ideas). Since fractions have several distinct definitions (Small, 2013), it is important to have models that highlight the details of each definition. For example, parts of a whole require a model that can be disassembled into equal

pieces, such as segments of an orange. Contrast that with a fraction being part of a collection, such as the cats among the pets in a house or the number of perch among the total number of fish caught while fishing. The ratio, another definition of a fraction (Small, 2013), emphasizes two parts and, while the whole remains evident, it is de-emphasized. Ratios are well suited to Cuisenaire rods or pattern blocks.

Something for you to consider is how you provide language that supports the structure that defines a fraction. In the fraction $\frac{3}{5}$, we name the "3" as the numerator, and we name the "5" as the denominator, but how many teachers give a name to the slanted line, known as a *solidus*, that is also part of a fraction? This is not to be confused with the horizontal line in a fraction, such as $\frac{4}{3}$, which is called a *vinculum*. With ratios 1:2, there is a *colon* separating the values. The consideration, in this case, is whether there is a pedagogical value in explicitly naming all three parts of a fraction. Clearly having different terminologies—solidus, vinculum, and colon—is potentially confusing. Personally, I mention the vinculum to draw attention to the fact that the numerator and denominator are tied together. I do not, generally, use the solidus because I choose not to confuse the concept by changing the language. However, a colon is a colon, and that is what I call that part of a ratio. Pedagogically, I feel that providing language draws the three parts of a fraction together. This addresses a common problem where students focus on the two numbers, without recognizing they are connected, to define the value of the fraction.

While I feel there is value in naming all three parts of a fraction, I know that view is not universal. Some teachers feel it detracts from the focus on the conceptual numerical meaning of a fraction. Perhaps fractions are difficult enough without introducing more terminology. If you try it, perhaps let readers know with a letter to the editor.

Pythagorean conceptual details

An opportunity arrives when working with students on the Pythagorean theorem. Students learn that $c^2 = 25 \Rightarrow c = \sqrt{25}$, which is correct because the Pythagorean theorem is used to derive a length, and by definition, a length is non-negative. However, this is a very early opportunity to introduce the concept of a *domain*. It is the concept of a domain that clarifies that we are only interested in solutions with a certain property. There are two solutions: $c = 5$ or $c = -5$ but the second doesn't satisfy the requirement that lengths are not negative. The importance of adding this detail is to facilitate early preparation for learning about quadratics, where both solutions are required, and for learning about domains that are formally addressed in Grade 11.

A similar issue arises when there is a direct benefit derived from including units in calculations. Why not show the Pythagorean theorem with sides measured in metres? For example, $c^2 = 25m^2 \Rightarrow c = \sqrt{25m^2} = \sqrt{25}\sqrt{m^2} = 5m$ shows that the theory gives a hypotenuse with a numerical length in metres. Some care is needed when introducing this to students, as they can confuse the units with the use of variables. It may be beneficial to have students use two colours, putting the primary calculation in one colour, and the unit portion in a second colour. This will help students appreciate that the units are not algebra, but a definitive standard for measurement.

Algebra defies order of operations

Consider an example of the distributive law: 2 (5 + 7) = 2 x 5 + 2 x 7. An important observation is that it allows one to alter the order in order of operations. In this example, instead of 5 + 7, the distributive law allows 2 x 5 and 2 x 7 to be done before the bracketed addition. That may not seem like an important point, but when it comes to 2 (w + 7) = 24, the distributive law gives two ways to tackle the equation. Importantly, we cannot determine $w + 7$ because w does not have a definitive value. An option is to divide both sides by 2 to get $w + 7 = 12$. Alternatively, applying the distributive law leads to $2w + 14 = 24$, and that gives a second way to make progress. Having the option to defy order of operations is particularly important later in the curriculum, when solving equations like $2(w + 3) = w$.

Algebra contains many examples that allow one to defy the order in order of operations. Consider $(x + 2)^2 - 4x$, where the bracket cannot be evaluated unless you know the specific value of x. Luckily algebra allows us to perform binomial squaring (the proper name of the process that is commonly associated with the acronym FOIL) and change the order as $x^2 + 4x + 4 - 4x = x^2 + 4$.

The message is simple: order of operations may be an important standard for communications, the Internet, and banking, to be consistent when they cross national borders, but algebra provides multiple approaches so that order of operations does not have to hold up the simplification process.

The importance of choosing the order

Another important detail with order of operations is to make sure that students understand that BEDMAS is actually BE(D&M)(A&S) and that one has to parse the calculation. The fact that division and multiplication can be done in any order (D&M) is important for calculations such as:

$$21\times\frac{12}{7}=\frac{21\times12}{7}=\frac{252}{7}=36$$

$$\textit{or}\ \ 21\times\frac{12}{7}=\frac{21}{7}\times12=3\times12=36$$

The latter calculation is conducive to mental math because the numbers remain relatively small.

Students who have only learned the acronym BEDMAS are confused when it is pointed out that in 2^{1+3}, one has to add ("A") before evaluating the power ("E"), and A does not come before E in BEDMAS! The fact is that the algorithm is recursive and one has to determine what to evaluate first—a process known as *parsing* in computer science. Applying BEDMAS implies the exponent has to be determined and then, within the exponent, the BEDMAS order is applied once more. Similar to the distributive law allowing a change in the order, this example shows how exponent laws allow a change of order, $2^{1+3} = 2^1 2^3 = 2 \times 8$, but in this case, BEDMAS is restored, rather than defied!

Taking slope to new heights

Slope is a concept that requires showing how equivalent fractions correspond, geometrically, to different rises and runs. However, the triangles that emerge are similar and that preserves the ratio that is the slope. This concept is fundamental to the definition of right-angle trigonometry ratios. The conceptual challenge for experienced teachers is to bring equivalent fractions, slope, similar triangles, and the tangent ratio into a single united coherent picture. In my experience, there is time pressure due to the need to extend trigonometric functions for acute angles to obtuse angles in order to support the full extent of the sine and cosine laws. Of course, that also meets with the issue of using the unit circle to define trigonometric ratios so that the generalization for all angles arises without addressing the changing domain. The conceptual details of the unit circle as opposed to using slope, similar triangles, and domain extension can be argued at great length. It is one of the few places, in my opinion, where there is clear value in each of two conceptually distinct approaches.

The introduction of quadratics

A final, more challenging conceptual detail arises when quadratics are introduced as if they are a line with an added square term. That is $y = ax^2 + mx + b$. Yet it is rare to hear teachers talk about having a very small and the quadratic approximated as a line. There is an important conceptual issue where the coefficients a, m, and b are interconnected when properties of the quadratic are described. They are independent in the sense that you can change any one value. However, if you wish to change the underlying properties of the quadratic, such as the vertex position or the roots, then it requires changes to more than one of the quadratic coefficients. The same is not true when using the representation $y = a(x - h)^2 + k$ that has vertex (h, k) and vertical stretch, compression, and/or reflection defined by a. Similarly, $y = a(x - r_1)(x - r_2)$, which explicitly shows the roots, while a has the same shape determination. In both of these forms, the parameters are directly linked to underlying properties, and one can independently change the underlying characteristics by changing individual parameters. For example, the vertex can be moved horizontally (h changes) or vertically (k changes). Similarly, the shape can be changed (a changes). However, the same cannot be said for the polynomial form.

It would be worthwhile to consider how each form of the quadratic ends up approximating a line when the coefficient of the squared term is small. Consider that a line does not have a vertex and it only has one x-intercept. How does that happen by shrinking the coefficient of the squared term?

Conclusion

As teachers, we continually polish our content presentation and instructional approaches. However, the conceptual richness often presents us with more options than we can easily digest. That is, in my opinion, why collaborative opportunities can be such rewarding experiences for teachers. It is an opportunity to share the richness that we see. It is also why there is so much value in regular professional reading such as the *Gazette*. The cornucopia of ideas presented here represents some of the finer points for a variety of topics, but they are far from the only finer points. Truly, there is more exploration to be done!

Reference

Small, M. (2013). *Making math meaningful to Canadian students K–8.* (2nd ed.). Toronto, ON: Nelson.

Additional Resources

See **www.edugains.ca/newsite/math/fractions.html** and **www.mathclips.ca** for more Ontario-based ideas about fraction instruction. ▲

WANTED

The *Gazette* editorial team has an opening for an associate editor. This requires a good eye for details, such as wordings that can be improved, grammar improvements, and proper use of punctuation. Knowledge of APA6 is useful, but not essential.

Interested? Email the editor and, using formal English, please articulate your interest.

▲ In the Middle: A Math Trail That Lasts All Year Long!



CARLY ZINIUK
EMAIL: carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

In the December 2016 issue, I wrote about taking our students to Stratford for a Math Trail that could be adapted for any middle school classroom. On that trip, the students were each given the task of taking ten photos, illustrating concepts like Pythagorean theorem, slope, and angles formed from transversals and parallel lines.

Throughout the school year, I was able to extend this Math Trail, by using the images they had taken, across the learning cycle for instruction, consolidation, and feedback. Photos like those you see here, however, do not have to be taken on a field trip. Students could take photos in their school or at home for a single topic or lesson starter. The examples here demonstrate student work and assessment opportunities. They also include Desmos (**www.desmos.com**) and TI-Nspire™ with both Data and Geometry. The focus is Grades 9 and 10 courses, but could easily be adapted to middle or senior high school courses with varied technology options.

After the Math Trail, I had my students upload the photos to a Google Drive folder, which we then used for many different learning opportunities, including quizzes, journal entries, and posters in our classroom. Because there were many images, I could model a variety of concepts with their photos in direct-instruction lessons and smaller group activities, where I would have previously used a diagram or a word problem. I found that having a shared drive, where all of the images were accessible for all of the students, gave another opportunity to reinforce how math is found everywhere and how each of their classmates saw mathematics in many different places.

At home, in preparation for or conclusion of a lesson, each student would take a photo to demonstrate the key concept. If they did not have an image, or their photo did not work well to show the concept, the expectation was that they would take a new one before the next class. It was a simple starter to have students show their photo to a partner or demonstrate it to the class. I could also quickly use their images for reinforcement, practice quizzes, or even for peer feedback. Similarly, it was easy for me and for them to come up with practical questions connected to the content motivated from their real-life pictures. We even used their images to determine whether any of "their" ramps would be difficult for manual wheelchairs to navigate. Taking the time to research ramp ratios led to great conversations and a new perspective on the importance of calculating slope.



Name: _____

Date: _____

Length of leg: _____

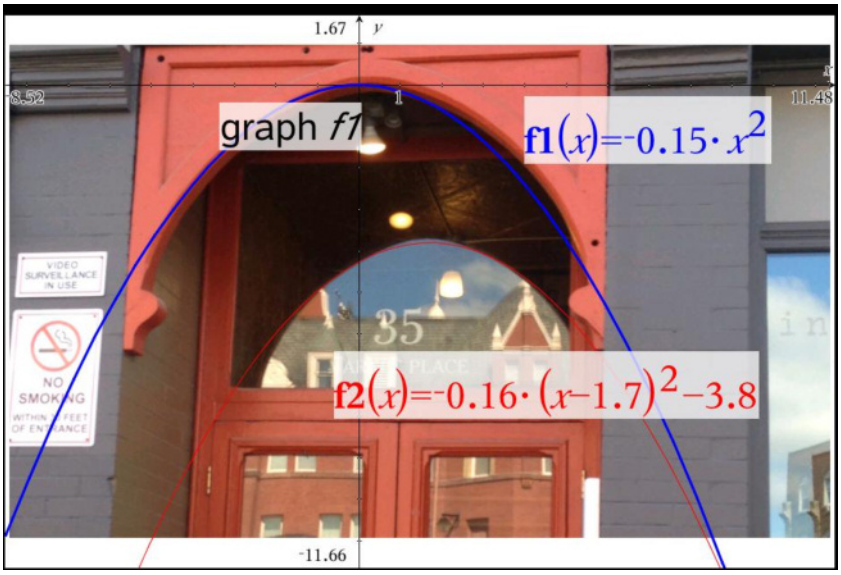
Length of leg: _____

Length of hypotenuse: _____

Demonstration of Pythagorean theorem:

_____

_____



Name: _____

Date: _____

For each of these quadratics, find the:

- vertex
- roots
- transformation

(The picture is from TI-Nspire™ and shows what TI-Nspire™ software can produce.)





Name: _____

Date: _____

Identify three-angle properties that you find with your sets of parallel lines.

What are the slopes of your parallel lines?





Name: _____

Date: _____

Demonstration of perpendicular line slope property:



Name: _____

Date: _____

Calculation of circumference, using radius:

(Note: Our Grade 10 teachers really liked this for equation of a circle, where they could move the graph paper and thus the centre!)



Name: _____

Date: _____

Create a table for a linear equation that demonstrates this rate of change as slope. What would the graph look like for your table?

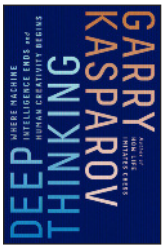
I developed the template below, which worked well in class and could conveniently have the photos swapped out for a quiz or for students to use in their notebooks or journals to record visual connections to their learning. This also contributed to decorating our classroom and for use in student–parent conferences to make their thinking and learning visible to their parents/guardians and to each other.

Now I have an extensive bank of some good images for use in my math class, and readers may find it useful to consult the Photo Math columns in the *Gazette* archives that Ron Lancaster contributed to use in the future for this kind of activity. This year, our Grade 9 students are heading to downtown Toronto to look at public art with a mathematical lens. I hope to be able to extend our start of the year trip throughout the entire school year, and perhaps you can do the same with your classes! ▲

▲ Book Review: How Human Thought and Machine Thought Collide: Deep Thinking

REVIEWED BY TIM SIBBALD

Kasparov, G., & Greengard, M. (2017). *Deep thinking: Where machine intelligence ends and human creativity begins*. New York, NY: PublicAffairs.



Reviewing a book that is about chess for a math educator publication seems a little unusual. However, the unique positioning of Kasparov to speak to the interplay between computer technology and human thinking has far too many insights for math teachers to simply be ignored.

As world chess champion from 1985 to 2000, Kasparov faced the major developmental phase of chess computers—both programs and hardware. He was the first world champion to lose a game to a computer and subsequently, in a story that is unsettling, he was the first to lose a multi-game match to a computer. The unsettling aspect is that the role of the humans involved in running the computer side of the match becomes unclear.

It is not necessary for students or teachers to know how to play chess to read this book, though there are some remarks that chess players will find interesting. Certainly some comments made about particular openings are conducive to getting out a chess board and looking up the moves. They are also conducive to thinking about how one could program a computer to measure the strength of a position or the change in strength offered by a particular move. Some commentary about this is quite amusing, such as the connection between grandmasters sacrificing their queens strongly correlating to their wins. Early computer programs that relied on databases of many games inadvertently would give up their queen because the program associated this with winning. In this regard, Kasparov points to the need to understand why moves are being made. Math teachers will relate to his promoting conceptual understanding of chess moves.

The promotion of understanding of chess moves points to the need for computer programs to have strategic algorithms, rather than simple brute force. However, within this book's timeline, computer power increased markedly and the interplay of strategic and brute force programming is evident. While brute force clearly has led to huge gains, it also has clear limitations, and that is indicative of a human flair for balancing different approaches to the game. With the highest-performing computers, it is surprising that they have issues with computer crashes, and the book leads one to wonder what role the non-programmed events surrounding the computer may play in the ultimate outcome.

As human and computer chess players head into the future, Kasparov sees players developing at younger ages because of their endorsement of a technology that can improve their performance. He also points to discoveries by computer chess programs that players were unaware of. However, it is also evident that competitions that allow humans to use computers during the event have created a new activity called informed chess. In this way, computers are enriching the game for humans, rather than diminishing it. As a whole, this book makes one wonder how human ingenuity was able to develop such a high level of play before the computer was invented, not to mention how chess will continue to grow as computers facilitate gaining deeper insights.

While this is not a math book, it is very much about thinking and the development of concepts. It will be a good read for any student who has a basic knowledge of chess, and good food for thought for math teachers because of the parallels with math education. ▲

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▲ Ask Assessment Abby: Reasoning Strategies

ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I don't understand how to write different questions to get at different types of student reasoning. For instance, I know that students reason proportionally in different ways and that they need to be flexible in their reasoning instead of being tied to one strategy. How can I tell if they are using multiple strategies in their reasoning?

> "Proportional reasoning is a lynchpin for future success in mathematics and science"
>
> (Carney et al., 2015, p. 141).

It is often difficult to pry a student away from a method that he or she has come to regard as tried and true. Look at these two questions:

Number of cookies	4	
Cost ($)	13	39

Number of cookies	4	
Cost ($)	12	42

The first is designed to be perfect for proportional reasoning, using scaling. "To get from 13 to 39, I multiply by 3, so I have to multiply 4 by 3 to get 12 cookies for $39."

The second question looks the same at first glance, but has had the numbers changed so that it is an unfamiliar multiple of 12 that results in 42. This encourages the student to find a different route to an answer. "I see that 4 x 3 is 12, so the relationship between cookies and cost is multiplication by 3. Therefore, the relationship between cost and cookies is division by three. "Since 42÷3 = 14, I can get 14 cookies with $42." This is functional perspective on proportional reasoning.

The way you formulate questions and the numbers you select affect the ways students will go about solving a problem. Also, adding context to the problem is a powerful way for students to see a problem differently; for example, the use of cookies in the earlier examples provides a scenario for developing a picture. Knowing the purpose of the question and the thinking you are trying to make visible should guide you in writing questions that will demonstrate evidence of that thinking.

"Recognize that mechanical procedures for solving proportions do not develop proportional reasoning and that students need to be flexible in their thinking and acquire many strategies" (adapted from Van de Walle, 2001, p. 262).

Try some questions from the following websites:

- **www.wouldyourathermath.com/**
- **www.mathbeforebed.com/**

Alternatively, you can try to make "Two Truths and One Lie" questions, like those found at **www.mrorr-isageek.com /tag/two-truths-one-lie/**.

Resources

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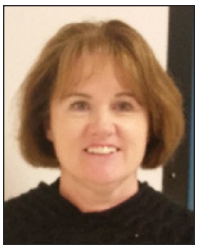
▲ Call for *Gazette* Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2018 issue.

Job shadowing available now.

For more details, please contact Executive Directors:
Fred and Lynda Ferneyhough
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▲ OAME/NCTM Report



JACQUELINE HILL
EMAIL: jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a past president of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the OAME Gazette *for a number of years.*

Firstly, I would like to thank Jill Lazarus for writing about the NCTM annual conference in the gap between the outgoing NCTM representative and myself. I would also like to thank Todd Romiens for his great work as the NCTM representative (as well as his co-editor position for the OAME *Gazette* leading up to the year 2000). Thanks to the long line of NCTM representatives before me. Laurie Moher continues to contribute to mathematics education in the province of Ontario. John Kersley was an NCTM representative as well as a board member for OAME. John always had classroom-ready activities to share at each board meeting. The late Frances Schatz brought many NCTM resources to the OAME board meetings. This gave the OAME members a first-hand preview and chance to purchase these products. Special thanks also go to Frank Schatz for being the strength in the partnership to haul resources to each board of directors meeting. As the OAME's representative to NCTM, I will take the advice and suggestions from the OAME board of directors and executive as to what they would find beneficial.

As a way to get you connected to the NCTM organization, I have taken a page from Frances Schatz's book of tricks and have provided an e-scavenger hunt for PRIZES!!

Complete the following e-scavenger hunt and send your answers to me on or before January 31, 2018. At the March 2018 OAME board meeting, the publications committee will draw two winning names from the correct submissions. Each will receive a certificate to purchase a popular NCTM publication.

Scavenger Hunt Questions

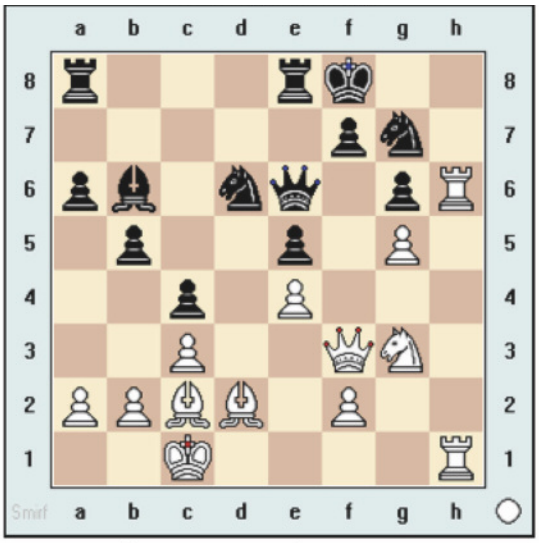
1. What does NCTM stand for?
2. When was the NCTM founded?
3. Approximately how many members does the NCTM have?
4. Who is the current NCTM President?
5. What is the philosophy statement of the NCTM?
6. The OAME has a charter official number as an NCTM affiliate. What is the number?
7. List three benefits of joining the NCTM.
8. In 2002, how many affiliates did the NCTM have? (Hint: Check out Frances Schatz's NCTM December report for the answer.)
9. Where is the NCTM annual meeting and exposition being held? (Hint: April 25–28, 2018 is the date.)
10. List a classroom resource or research article or professional development piece you would consider using in your practice in the next few weeks. Justify your selection.

Happy answer seeking!

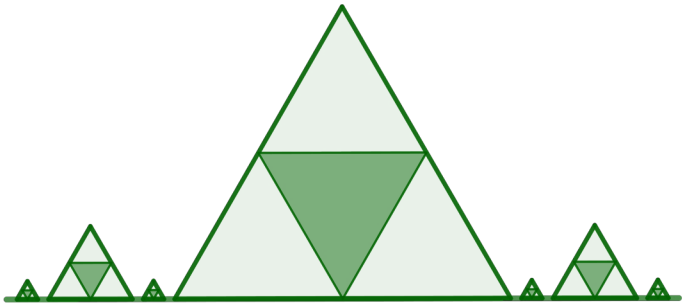
Remember to send your answers to me care of my OAME email address (as listed at the top of the column). ▲

Kasparov vs. Alexander Karman, 2000

White to move and reach checkmate in 3 moves (or less).



From **www.chess.com**



Provincial Digital Learning Resources – What's New? Exploration of Fraction Operations



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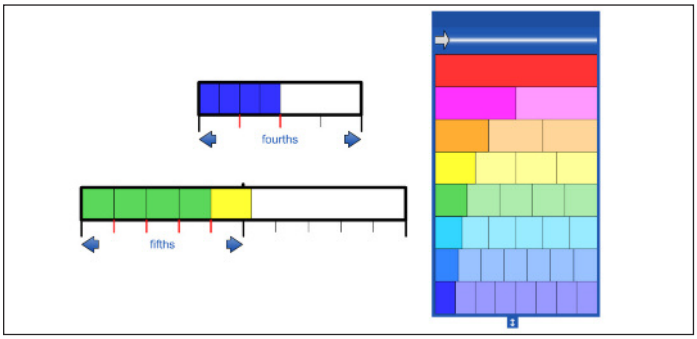






Agnes (Brant Haldimand Norfolk Catholic District School Board), Ross (Near North District School Board), Markus (Bluewater District School Board), and Greg (Simcoe Muskoka Catholic District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the digital resources found at ***mathies.ca****.*

In the September 2017 issue of the *Gazette*, some of the features of the Mathies Fraction Strips tool were explored. The tool was used to represent, compare, and order fractions, as well as think about different ways to visualize equivalent fractions. The exploration of fraction operations focused on how the tool might help students develop an understanding of what it means to add and subtract fractions. In this issue, the exploration of fraction operations will continue by considering how powerful visual representations can support the understanding of multiplication and division.



Multiplication of Fractions

Students often have the misconception, based on their experience with whole numbers, that multiplying always results in a larger quantity. Modelling questions, using the Fraction Strips Tool, will help students understand that this is not always the case.

Students should be encouraged to estimate an answer before calculating the result. Unfortunately, students sometimes fail to appreciate the value of estimation. Perhaps this is because they see it simply as an exercise in rounding. Rather, students should be encouraged to think about what size of an answer would be reasonable, given the values and operation in the question. Some questions to consider might be:

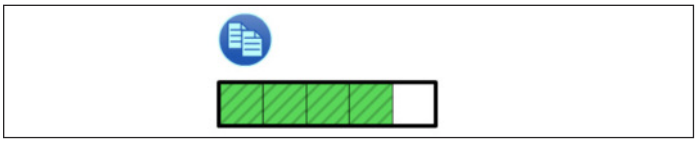
- How do I expect my answer to compare to various benchmarks? (e.g., one-half, one whole)
- How do I expect my answer to compare to the values in the question? Why?
- What answer would be much too large?
- What answer would be much too small?

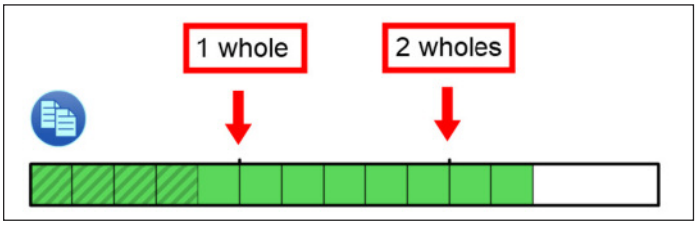
Students who are confident in their ability to use their number sense skills to predict an answer prior to calculating are more likely to look for errors if their calculated answer does not match their prediction.

Multiplying a Whole Number and a Fraction

Multiplying a fraction by a whole number can be thought of as repeated addition and modelled by making the appropriate number of copies of the fraction. For example, $3 \times \frac{4}{5}$ can be thought of as 3 groups of $\frac{4}{5}$. Students might estimate that this product will be somewhere between 2 and 3, since $\frac{4}{5}$ is a bit smaller than 1.

Represent $\frac{4}{5}$, then press the copy button twice to model a total of three $\frac{4}{5}$.





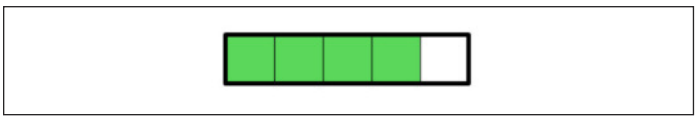
Now, name the result. So this product is either 12/5 (count the number of green one-fifth pieces: 1 one-fifth,

2 one-fifths, 3 one-fifths, etc.) or $2\frac{2}{5}$ (which is visually supported by the black tick marks at the end of each whole).

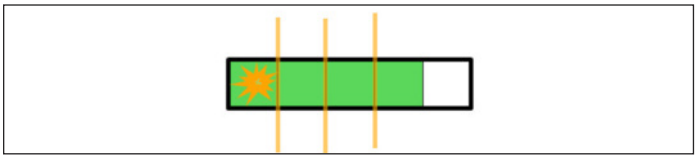
Multiplying Proper (or Improper) Fractions

One way to calculate $\frac{1}{4}\times\frac{4}{5}$ is to think about this expression as $\frac{1}{4}$ of $\frac{4}{5}$. When thinking about estimating products of proper fractions, students will come to understand that, in this case, multiplication actually results in a smaller result, since you are finding a part of some other value.

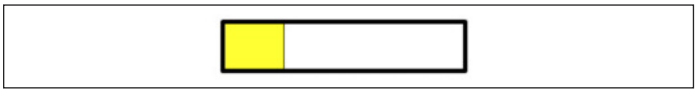
Start by representing $\frac{4}{5}$.



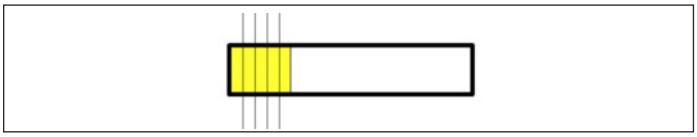
To determine one-fourth of something, you need to partition the quantity into 4 equal parts. In this case, that is straightforward, since the fraction four-fifths is made up of 4 one-fifth pieces. So $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{5}$, which is 1 green piece.



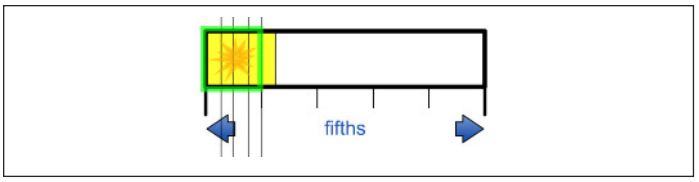
Multiplication is commutative, so $\frac{4}{5}\times\frac{1}{4}$ gives the same result, but modelling this requires us to think differently as we determine $\frac{4}{5}$ of $\frac{1}{4}$. Start by representing $\frac{1}{4}$.



To determine $\frac{4}{5}$ of this quantity, we need to partition it into 5 equal parts and then select 4 of them. The Vertical Comparison Bar, together with our spatial reasoning skills, allows us to create this partition.



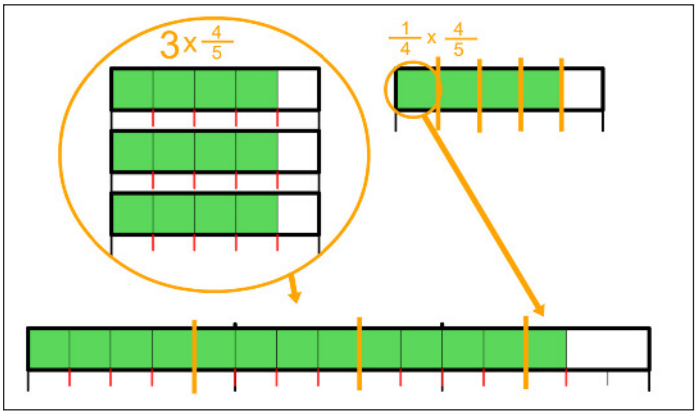
How big is each of these pieces? There are 4 one-fourths in the whole (black rectangle). If each piece is partitioned into 5 equal pieces, then there will be 20 equal pieces in 1 whole. So, each piece is 1/20. We need 4 of these pieces, so we have 4/20, which simplifies to $\frac{1}{5}$ as expected.



Modelling the multiplication of various pairs of fractions can lead students to better understand why the standard algorithm works.

Multiplying Mixed Numbers and Fractions

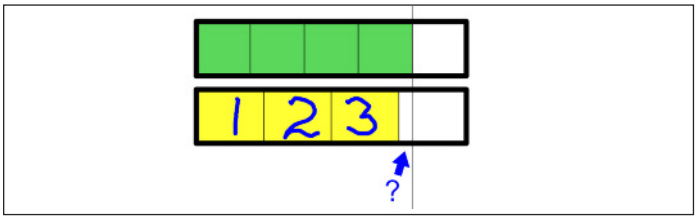
Combining these ideas allows us to model products such as $3\frac{1}{4}\times\frac{4}{5}$ by using the distributive property to break this product into partial products. First determine $3\times\frac{4}{5}$, then determine $\frac{1}{4}\times\frac{4}{5}$, and add these two results together.



We commonly think of $\frac{4}{5}$ of a quantity as the result of splitting the quantity into 5 equal pieces and combining 4 of them. However, we can also think of $\frac{4}{5}$ as 4 wholes shared equally among 5. Here, then, we could have created 4 copies of $3\frac{1}{4}$ and taken $\frac{1}{5}$ of the result. Four copies of $3\frac{1}{4}$ combine to make 13, and $\frac{1}{5}$ of 13 is $\frac{13}{5}$, as above.

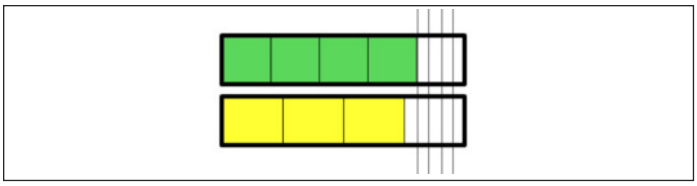
Division of Fractions

One way to calculate $\frac{4}{5}\div\frac{1}{4}$ is to start by representing $\frac{4}{5}$, then think about how many one-fourth pieces fit in $\frac{4}{5}$.

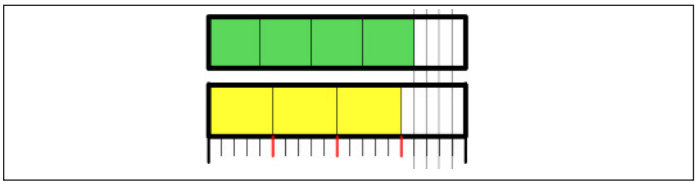


Using the vertical comparison bar to draw a line at the end of the $\frac{4}{5}$ green strip helps us to see that the answer is 3 and a bit. What fraction of a one-fourth piece is this bit?

We employ the equal partitioning idea described above to figure this out. Copy additional comparison bars and place them so they partition the white space at the end of the yellow strip into roughly equal pieces.

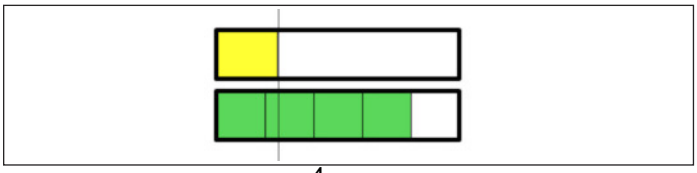


It looks like the extra bit is $\frac{1}{5}$ of a one-fourth piece, so the quotient may be $3\frac{1}{5}$. This visual representation helps students to give meaning to the numeric answer. There are 3 and $\frac{1}{5}$ yellow one-fourth pieces in $\frac{4}{5}$. Turning the rulers on helps ensure that we have partitioned the one-fourth piece accurately.



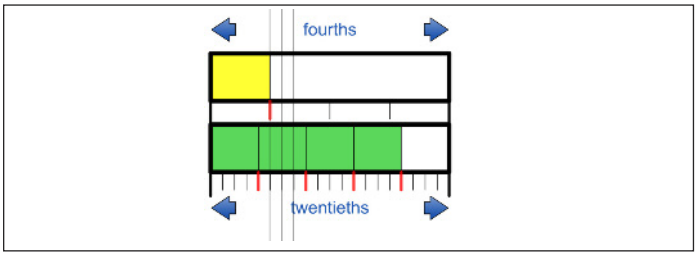
As teachers, we know that division is not commutative, so we would expect $\frac{1}{4} \div \frac{4}{5}$ to give an entirely different result. It would be interesting to see whether or not students are surprised by the different answer.

It is helpful to think about this division slightly differently, since $\frac{4}{5}$ is larger than $\frac{1}{4}$. This means that only some portion of the $\frac{4}{5}$ piece will fit inside $\frac{1}{4}$. So, the question we need to explore is how much of a $\frac{4}{5}$ piece fits inside a $\frac{1}{4}$ piece?



Only a portion of the $\frac{4}{5}$ piece will fit, so we know the answer will be less than 1. In fact, visually, it looks like the answer will be just a bit larger than $\frac{1}{4}$ of the $\frac{4}{5}$ strip. Again, the question is, how big is this little bit? That is, what part of a one-fifth piece is this extra bit?

Equally partition the one-fifth piece. Use the rulers to convince yourself that you have done this accurately.



The extra bit looks like it is one-twentieth of the whole or, more importantly, for our purposes, one-fourth of a one-fifth piece. This means that there would be 16 such pieces in the green four-fifths strip altogether. So, the yellow one-fourth piece is made up of $\frac{5}{16}$ of the green four-fifths strip, four-sixteenths from the first piece, plus one additional sixteenth; that is, $\frac{5}{16}$ of the green $\frac{4}{5}$ strip fits in $\frac{1}{4}$.

Division is definitely not commutative, but comparing the two quotients is interesting: What is the relationship between $3\frac{1}{5}$ and $\frac{5}{16}$? What do you notice when you write the first as an improper fraction? What is the product of these two fractions?

Have you ever thought of using common denominators to divide? Let's revisit the same two division questions, using this strategy. See Figure 1.

The question of how many of one quantity fit into another is the standard "goes into" division situation. We should not be surprised in the first column that the answer is $16 \div 5$, or 16/5, and in the second column, that the answer is $5 \div 16$, or 5/16. Thinking of a fraction as a quotient, the answer to a division question is an important way we use fractions.

Other Fraction Supports

Teaching and learning about fractions in a meaningful way can certainly be challenging! Here are some other Ontario Ministry of Education resources that support the effective development of fraction concepts.

Gap-Closing ePractice Activities

Visit **www.mathies.ca/additionalSupports.html** to access ePractice resources. Topics include representing, comparing, and operating with fractions. These digital interactive activities are designed to provide practice opportunities for students and support conceptual understanding by providing detailed feedback. The operation activities illustrate two different ways of thinking about each operation.

Fractions Learning Pathways

A research informed framework found at **www.edugains.ca/newsite/DigitalPapers/FractionsLearningPathway/index.html** is a helpful planning tool for teaching fractions. It includes a range of field-tested tasks that have proven to be effective in the teaching and learning of fractions in Ontario schools.

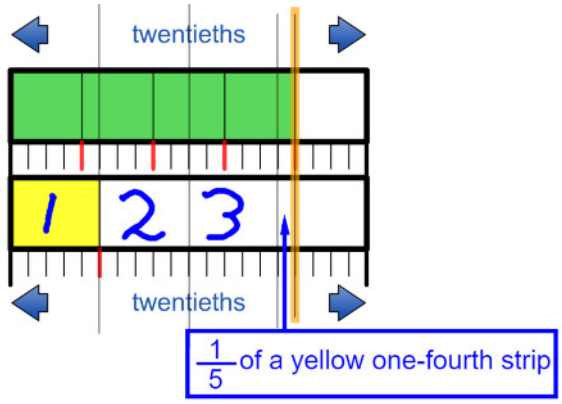
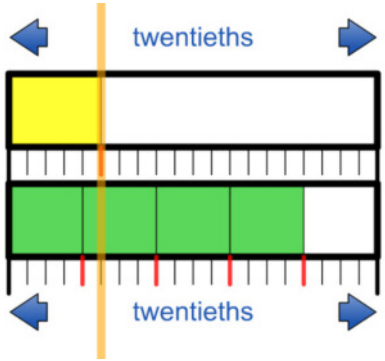
	$\frac{4}{5} \div \frac{1}{4}$	$\frac{1}{4} \div \frac{4}{5}$
Step 1:	Represent both one-fourth and four-fifths.	
Step 2:	Rename these fractions, using the same fractional unit (in this case, twentieths). As illustrated in the diagram, • $\frac{1}{4}$ is equivalent to $\frac{5}{20}$ • and $\frac{4}{5}$ is equivalent to $\frac{16}{20}$.	
		
Step 3:	Consider: "How many five-twentieths fit in sixteen-twentieths?"	Consider: "How many of the sixteen-twentieths fit in the five-twentieths?"
Step 4:	The diagram shows that 3 yellow pieces plus $\frac{1}{5}$ of a yellow piece will fit, so the answer is $3\frac{1}{5}$ yellow one-fourth pieces.	In fact, only 5 out of 16 of them fit, so the answer is $\frac{5}{16}$ of the green four-fifths strip.

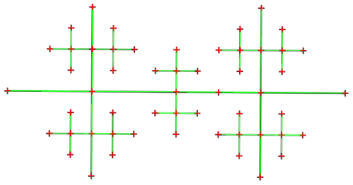
Figure 1

Ontario Fractions Educational Research

A growing body of research connected to the teaching and learning of fractions can be found at **www.edugains.ca/newsite/math/fractions.html**. Resources found here include the Paying Attention to Fractions document, as well as a collection of Research Summaries outlining key ideas for teachers to consider when planning and delivering lessons such as the use of Purposeful Representations and ideas about how to build understanding of Unit Fractions.

Feedback and Future Requests

Please feel free to send us your feedback about any Mathies tool, using the Feedback Form button inside the Information Dialog. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.





You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #ONmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **www.mathclips.ca/WhatsNewEmailList.html**. Watch for announcements regarding exciting improvements to annotations, including new drawing objects (rulers, number lines, grids, hops, and polygons), as well as the ability to copy and rotate all annotation objects. These improvements will show up first as updates to existing tools, and then in the two new apps in development. Stay tuned! ▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary school mathematics teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancement of mathematics education in Ontario through innovation and/or research related activities.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary school mathematics staff, who foster collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

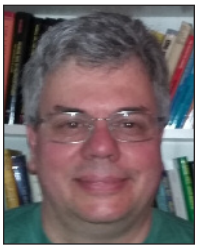
Elementary School Staff Award for Exceptional and Collaborative Mathematics Teachingrecognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Mathematics Staff Award and Elementary School Staff Award)
4. Send the completed cover page and supporting documents by January 15, 2018 to: OAME Executive Directors, 4751 Mack Avenue, Town of Plympton-Wyoming, ON N0N 1J6, or email **EDs@oame.on.ca**.

DEADLINE: Nominations are due by January 15, 2018 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in May 2018.

What's the Problem? Parabolic Products



SHAWN GODIN
EMAIL: shawn.godin@ocdsb.ca

Shawn teaches and is a department head at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. In the last issue, I left you with the following problem:

> A line intersects the parabola with equation $y = -x^2 - 3x$ at points with $x = 2$ and $x = 4$. Determine the y-intercept of the line.

This problem was inspired by question #3 from the 2017 Hypatia contest from the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. You can access additional great contest problems, and other mathematics and computer science resources, from the CEMC website, **www.cemc.uwaterloo.ca**. I had originally read about this problem in *The Grapes of Math: How Life Reflects Numbers and Numbers Reflect Life* by Alex Bellos. Alex has a few mathematically themed books that are well worth a read.

As to the problem at hand, I don't know what I was thinking. This seems to be a fairly straightforward application from Grade 10. Substituting $x = 2$ and $x = 4$ into the equation yields the points (2, –10) and (4, –28). We can then calculate the slope of the line

$$m = \frac{-28-(-10)}{4-2} = \frac{-18}{2} = -9,$$

so the equation must be of the form $y = -9x + b$. Substituting either point into the equation, we discover that the y-intercept is $b = 8$.

Hmm, nice, a little exercise, but not much of a problem. It is strange that the two x coordinates were 2 and 4 and the y-intercept was 8 = 2 x 4. I wonder if that is just a coincidence. If we go to Desmos (**www.desmos.com**), we can graph the items from the original problem and obtain the graph in Figure 1.

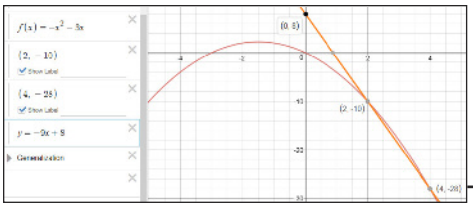


Figure 1: *Original problem in Desmos*

Now let's see what happens for two arbitrary, but different, points. By asking Desmos to plot the point (a, $f(a)$), where we have defined f as the original quadratic function (see Figure 1), Desmos will prompt us to create a *slider*. This allows us to vary the values of a, the x coordinate, and we would see the point move on the parabola. We can similarly create a point (b, $f(b)$).

We want to create a line through these two arbitrary points, which requires a calculation of the slope. As Desmos includes a calculator, we can type in

$$m = \frac{f(b)-f(a)}{b-a},$$

and Desmos will calculate the slope for us and keep its value as the variable m. We can now use the fact that if a line with slope m passes through a point (x_1, y_1) then the equation of the line, called the point-slope equation of the line, is $y - y_1 = m(x - x_1)$.

We can rearrange this formula and use one of our points to get $y = m(x - a) + f(a)$.

When we put all of this information into Desmos, we get two points on the parabola that we can control by sliders, and the line that goes through them. If we use function notation, and define our line as $g(x) = m(x - a) + f(a)$, we can also plot the point (0, $g(0)$) to obtain the y-intercept of our line, and we will get something like the plot in Figure 2.

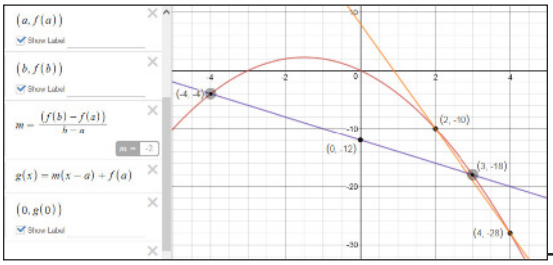


Figure 2: *Line through two arbitrary points and its y-intercept*

As a and b are varied, we will see that each time, the y-intercept is (ab, 0) as anticipated. Great! Let's see if we can use a little mathematics to prove that this actually has to be the case.

From our notation, we have

$$\begin{aligned} m &= \frac{f(b)-f(a)}{b-a} \\ &= \frac{-b^2-3b+a^2+3a}{b-a} \\ &= \frac{(a-b)(a+b+3)}{b-a} \end{aligned}$$

This means that $m = -(a + b + 3)$ as long as $a \neq b$, in which case, the points are not unique. Thus, the equation of the line will be of the form $y = -(a + b + 3)\ x + B$ for some real number B. Since $(a, f(a)) = (a - a^2 - 3a)$ is on the line, if we substitute these points coordinates into the equation, it yields

$$\begin{aligned} -a^2 - 3a &= -(a + b + 3)a + B \\ -a^2 - 3a &= -a^2 - ab - 3a + B \\ ab &= B \end{aligned}$$

Hence, the y-intercept is the product of the x coordinates of the two points. We can deal with the case where the two points are the same, using calculus to determine the slope of the tangent, and you will find it works in this case as well. I will leave that for you as an exercise.

The next big question that should be coming to mind is, "What other parabolas share this property?" To answer this question, we can set our original function to $f(x) = Ax^2 + Bx + C$, and we would calculate the slope of the line between the two arbitrary points to be $m = A(a + b) + B$. Since we want the line to have the same property, we can set its y-intercept to be ab, substitute the point $(a, f(a))$ into the equation, and when the dust clears, we get $C = (A + 1)ab$.

Remember that we wanted the conditions on the coefficients of our quadratic, A, B, and C, so that the "multiplicative y-intercept" property would be true for any two points. What that means is that A, B, and C are fixed values, to define a particular quadratic function, while a and b are variable values, which could be any two points on a particular parabola. The only way the condition $C = (A + 1)ab$ can be true for all possible values a and b is if $C = (A + 1) = 0$.

Since B does not turn up in this relationship, it shows that it could be *any* value. Hence, any quadratic function of the form $f(x) = -x^2 + Bx$ will have the desired property for any B.

I would encourage you to try to build this general model in Desmos and ascertain that the property will hold for this family. You can check out my Desmos sketch containing the models for the diagrams in this column and the final family at **www.desmos.com/calculator/cipllss6nf**.

I think it is important for students to ask those "What if…?" and "Why does this happen?" and "What's next?" type questions. Encourage them to think about where a problem leads them and give them the opportunities to explore that next step.

Now for your homework (you are not getting off that easily!):

You have a number of 1 by 1 squares and 1 by 2 rectangles. You want to create a 1 by 10 rectangle. In how many ways can you do this?

Until next time, happy problem solving! ▲

▲ Remembering Maryam Mirzakhani



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Judy Mendaglio considers herself a "slow mathematician," who imagined herself an arts person rather than a math person—until a spark ignited her passion for mathematics. That spark was the recognition that, while the outside world may view mathematics as a set of tools, the world of mathematicians sees mathematics as a world of imagination and creativity. Judy is currently the Past President of OAME and Chair of the Steering Committee of the Fields Institute's MathEd Forum.

The year 2014 was a very important year for many math teachers in Ontario. Why? Well, 2014 was the last time the Fields Medals were awarded. (The next time will be 2018, and then 2022. Do you discern a linear growing pattern?) The maximum number of medals, four, were awarded that year. One recipient was Manjul Bhargava, a young mathematician who was born in Hamilton, Ontario. That is exciting to Ontario math teachers. But the most exciting thing about the 2014 Fields Medal awards announcement was that, for the first time since its inception, a medal was awarded to a woman. At the International Congress of Mathematicians, held in Seoul, South Korea, Maryam Mirzakhani became the first woman to ever win a coveted Fields Medal, the highest honour a mathematician can receive. It was awarded for her groundbreaking work in structures that arise in natural ways in various areas of mathematics and physics, and that have what Maryam called "rich geometries." Her work was notable in how it crossed the boundaries that define various fields within mathematics, offering previously unseen connections.

The world of mathematics lost Maryam Mirzakhani on July 14, 2017, when she succumbed to the cancer that she had been fighting since before she accepted the Fields Medal in 2014. She was 40 years old.

Maryam Mirzakhani was a mathematician who never expected to become one. "I wasn't always very excited about math. I was more excited about reading novels and I thought I would become a writer one day." However, the beauty and creativity of mathematics eventually pulled her away. Maryam saw the solution to a math problem as an unfolding story. "There are different characters, and you are getting to know them better." She did, however, see a difference between the narratives in books and the narratives in math

problems: unlike characters in literature, math problems are more like real people in that they change as you get to know them better. “Things evolve, and then you look back at a character, and it’s completely different from your first impression.”

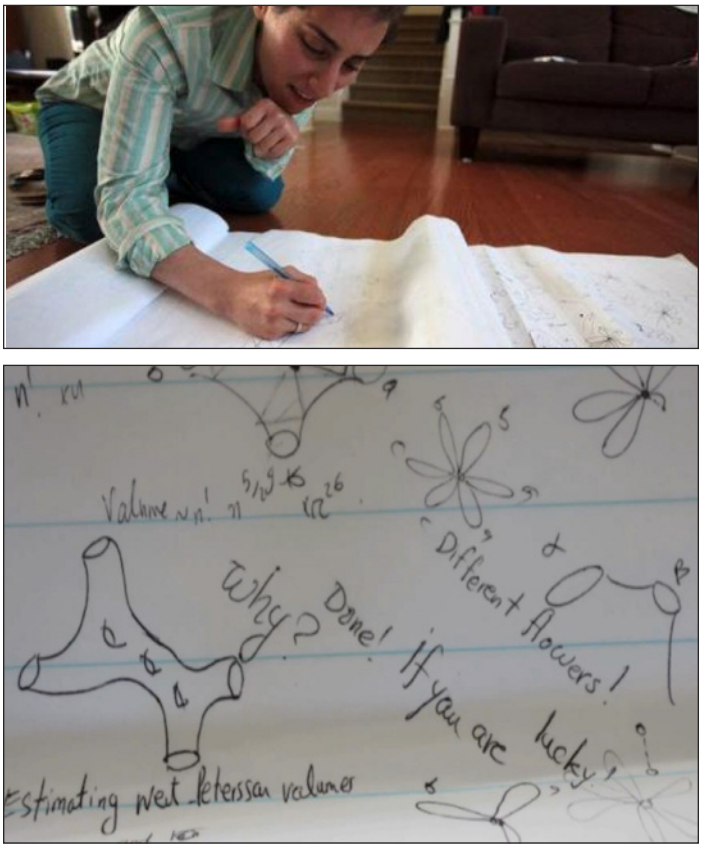


Figure 1: *Maryam Mirzakhani working on a “doodle.”* (Source: **www.mathunion.org/general/prizes/2014/**)

As difficult as it may seem, Maryam Mirzakhani considered herself “a slow mathematician” in more ways than one. One of her first teachers actually characterized her as lacking in aptitude! Her childhood classmate and life-long friend, Professor Roya Beheshti, remembered back to when Maryam, age 11, was not good at math, despite being a top student in every other subject. She recalled when Maryam, receiving another disappointing mark on a test at the end of the school year, tore up the test in frustration, saying that that was it for her and math, and that she was not even going to try to do better. Thankfully, she came back the following year with renewed confidence and a changed attitude, so, as they say, the rest is history. Maryam was also “slow” in the way she solved problems. Maryam’s work was very visual. She is perhaps best known for her doodles. It would not be unusual to find Maryam kneeling on great sheets of paper covering her floor, drawing “doodles” to release hidden patterns. Her doodles helped her better understand the nature of the problems she tackled and to identify the relationships that could be found if you continued to look. She viewed these doodles as allowing her to stay connected with the problem. She felt that to focus on the details would result in disconnection. In fact, her young daughter, seeing her mother on the floor, working away at a math problem, would say that her mother was painting again (see Figure 1).

Maryam went on to study mathematics in Iran, and moved to Harvard to complete her PhD. She was an Assistant Professor at Princeton and then moved to Stanford University.

To what can her enormous success in the world of mathematics be attributed? According to those who knew her, Maryam had the attitude that she would always finish what she started, but perhaps more importantly, she *could* finish what she started. She had that kind of mindset. Many math educators were introduced to Maryam through Jo Boaler’s book *Mathematical Mindsets* (2015). In the book, Boaler tells of Maryam reflecting on her work: ”I like crossing the imaginary boundaries people set up between different fields—it’s very refreshing... there are lots of tools, and you don’t know which one would work. It’s about being optimistic and trying to connect things” (p. 183). As math educators in Ontario, we cannot help but see these words reflected in our efforts to spiral our programs and to teach through the Mathematical Processes in order to allow our students to experience the richness of mathematics. As we strive to connect the mathematics that lies within our curriculum strands and units, we can be certain that we are supporting our students in the very best ways, whether they are “slow” or not.

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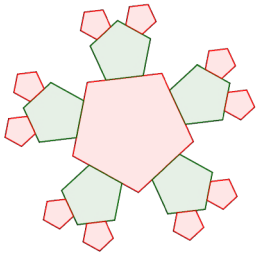
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▲ Hey, It's Elementary: The Best of Times or the Worst?



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In *A Tale of Two Cities* (1859), Charles Dickens wrote, "*It was the best of times, it was the worst of times….*" The full quotation, from the opening of his book is:

> It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way—in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.

Fast forward to 2017… what "times" does math education find itself in?

Without doubt, we are in a time of rapid technological advancement and fundamental social change. When I consider my BEd candidates at Queen's, I am acutely aware of the fact that they will not be taking notes in my classroom using paper and pencil—they prefer to record the lesson on a laptop, tablet, or phone. My students expect that every lesson will be documented using presentation software (PowerPoint or Prezi) and then posted to the Internet, usually in advance of the class. They submit assignments electronically and receive feedback and grades the same way.

My colleagues in mathematics education leadership positions across the province talk about professional development sessions conducted virtually, using live video-conferencing software. Participants sit comfortably in their offices while presenters use interactive whiteboards, video, and web-conferencing features to share ideas and resources, engage in debate and dialogue, and record meetings for purposes of accountability.

My teaching colleagues in local school boards tell me that they are expected to maintain class websites that record not only homework and upcoming tests, but short explanatory videos about math content (e.g., addition by place value or the grid model for multiplication of multi-digit numbers). They work diligently to maintain open lines of communication and transparency about instruction, assessment, and growth mindset, often using social media technologies such as Twitter.

In elementary and secondary classrooms across Ontario, students interact with dynamic geometry and coding software, spreadsheets, and graphing calculators (hand-held or virtual), and at home, they use the Internet to seek support (e.g., TVO's Math Homework Help), or play games that extend the classroom experience with tangrams, pentominoes, or base-10 blocks, to name but a few examples.

At the same time, at all levels, mathematics educators have been charged with addressing EQAO data that shows a continued eight-year decline in achievement. Only 50 percent of Grade 6 English-language school students achieved the provincial standard for mathematics in 2015–2016, compared with 61 percent in 2009–2010; only 63 percent of Grade 3 students met the provincial standard for mathematics in 2015–2016, compared with 71 percent in 2009–2010 (EQAO, 2014; 2016). In 2017, only 62 percent of students met the provincial standard at Grade 3, and for the second consecutive year, one-half of the province's English-language Grade 6 students failed to meet the provincial standard.

More worrying is the fact that between Grades 3 and 6, EQAO data shows that 21 percent of students who met the standard in Grade 3 did not meet the standard by Grade 6, while only 4 percent of students who did not meet the standard in Grade 3 rose to meet the standard in Grade 6. Over 25 percent of Ontario's English-language school students achieved below the standard in both Grade 3 and Grade 6. The 2017 data shows nearly identical results, with a worrisome increase of 1 percent for students who achieve below the standard for both Grade 3 and Grade 6.

The significance of these results is clear: EQAO analyses have consistently shown that students who do not meet the provincial standard in Grade 3 or 6, or both, are much more likely to experience difficulties in Grade 9. Further, the OECD states, "…proficiency in mathematics is a strong predictor of positive outcomes for young adults, influencing their ability to participate in post-secondary education and their expected

future earnings" (p. 6). Stinson (2004) discusses the history of mathematics as a gatekeeper for "economic access, full citizenship, and higher education," traditionally disadvantaging under-represented groups such as girls and children living in poverty (p. 11). He urges us to examine the ethics of "gatekeeper mathematics," and to use mathematics education as a means of empowerment, rather than stratification (p. 16).

To this end, educators have been seeking ways to improve engagement in mathematics, sustain interest and enrolment in mathematics, and showcase the possibilities of mathematics-focused careers. In most cases, this has been attempted by highlighting all of the interesting and practical ways mathematics is applicable in the real world. What better way to pique and sustain students' interest? By illuminating the widespread use of mathematics in everyday life, it is anticipated that students will decide that this is a subject that they must study. Clearly, mathematics is important in everyday life, and that importance is increasing. Obviously, it is beneficial to study math to achieve career and life goals.

Making math relevant is commonly seen as essential. Consider the following open-ended linear patterning activity for Junior Division students: *At a new restaurant in town, there are 24 square tables. One chair is placed on each side of a table. How many customers can be seated at this restaurant?* In the Intermediate Division, as part of the unit on linear relations, a teacher might ask students to take on the role of being a representative for a cellphone company, whose job it is to clarify different cellphone plans. The teacher explains that the company has three plans, and a customer wants to know how to decide which plan will save her the most money. The teacher asks the student "sales reps" to determine which plan has the lowest cost, given the number of text messages a customer is likely to send. We tell our students about the mathematics and coding that operates *Asimo*, The Honda Humanoid Robot and the many mathematically skilled individuals who were on the team that constructed and programmed him, hoping to light the fire of possibility and continued study.

We work diligently to make mathematics interesting as a way to support learning and reinforce the reasons for learning and doing math. Yet, the best of times may have put us squarely into the epicentre of the worst of times, for we find ourselves in a paradoxical position: as the importance of mathematics in everyday life increases, its use by the average consumer, and by professionals, has decreased significantly. We must ask ourselves if a navigator in today's world would solve right triangles to calculate his position at sea, or if a contemporary bridge designer would compute an integral by hand to satisfy engineering design requirements. We must ask ourselves when "Joe Public" might, in reality, ever need to do a mental calculation to add a 15 percent tip. We have apps to help us to make financial investments and remove the burden of household budgeting: a differentiated "bing" from our smart phone informs us of our account balance or deficit.

At no time has there ever been a greater need to provide opportunities for students who wish to broaden their access to higher mathematics—there is an enormous need for mathematicians who work behind the scenes, developing the models to explain the outcomes of clinical trials for potential cancer treatments or concretizing models of climate change through augmented reality sandboxes. Concomitantly, there has never been a time when it has been more important for students to be able to evaluate and interpret data that is presented by a calculator, computer, media, or professional (e.g., one of the first questions people diagnosed with cancer may ask a physician is, "What's the chance of survival?" Understanding statistics becomes very important because a doctor communicates them to estimate a patient's prognosis, or chance of recovery, and determine treatment options).

We are facing imminent curriculum reform in Ontario once again. One hopes that those who are in a position of power understand and appreciate the fact that technological discoveries have had, and continue to have, an enormous impact, not only on learners of mathematics and their teachers, but also on those who work out of sight to improve the quality of life for many people, and those who are the beneficiaries of such efforts. It is time that it was recognized that computers, the Internet, and the various gadgets that have become an integral part of our life, are all made possible by mathematics and its applications; and, that associated with these inventions have changed the type of mathematical skills essential to the end-user.

At this crossroad juncture, it is time to look at emerging topics and changing needs. It is time for hard and honest discussions about why students need to study mathematics and what mathematics they need to study. Perhaps the time is ripe to debate a technique like long division. No one disagrees that the teaching of long division has been perfected over the course of centuries. But does it have practical application? I'll close by saying that it is rare to see anyone using long division today. The best of times, indeed.

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Technology Corner: Graphing Stories and a Water Line



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Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Ten years ago, Dan Meyer created a set of graphing stories for his students and blogged about them at **blog.mrmeyer.com/2007/graphing-stories/**. In 2012, he teamed up with the good folks from BuzzMath (**www.netmath.ca**) to create the Graphing Stories site (**www.graphingstories.com**). The concept is simple: students are shown a 15-second video and they must create a graph to represent the story.

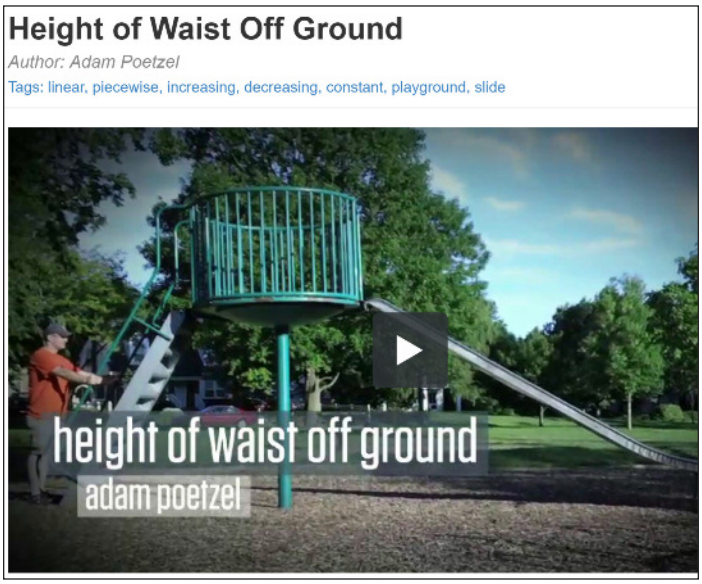


Figure 1: *Opening screen for a story*

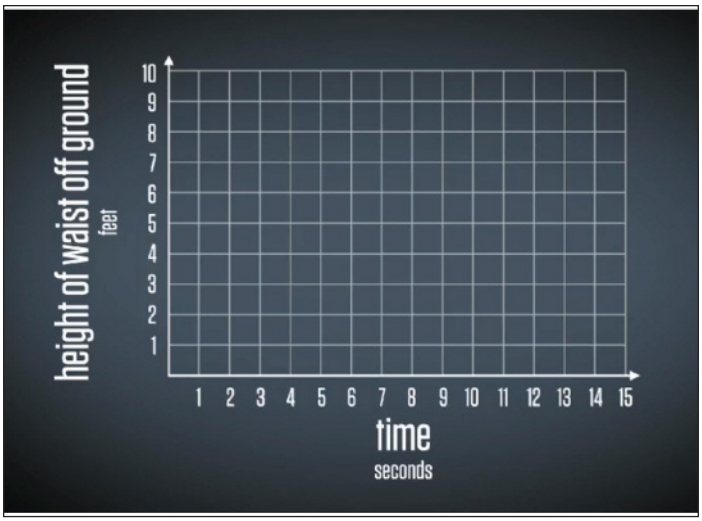


Figure 2: *Example of graph for the story*

There are 21 videos to choose from. You can select from linear, parabolic, periodic, piecewise, or step graphs, and choose whether you want to look at constant, increasing, or decreasing graphs. You can also download each of the videos.

The opening screen for a video is shown in Figure 1. The picture, along with the title, provides a setting for the story. After the opening screen, a graph, Figure 2, is provided for students to construct their ideas. The units and labels are given on the graph.

The website provides a tab to download the student template handout shown in Figure 3. At this point, the provision of the graph provides a scaffold, and students can complete the scale and label the vertical axis.

The video then plays once at regular speed. Figure 4 shows a sequence of still frames from the video, with the time displayed in the lower left corner.

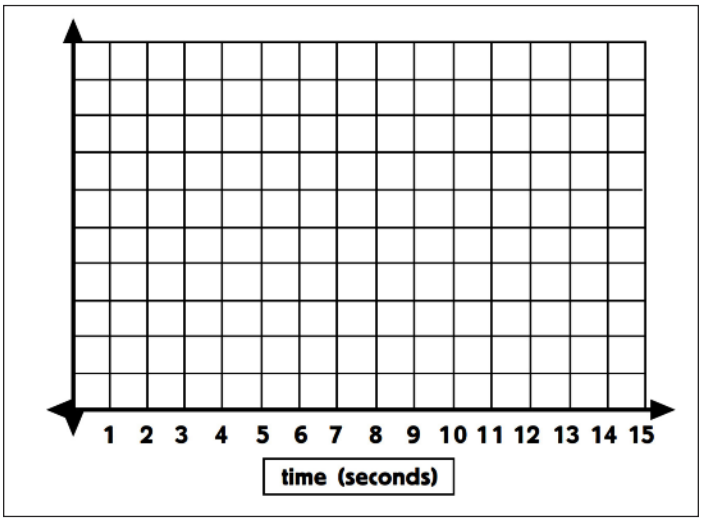


Figure 3: *A student template from the website*

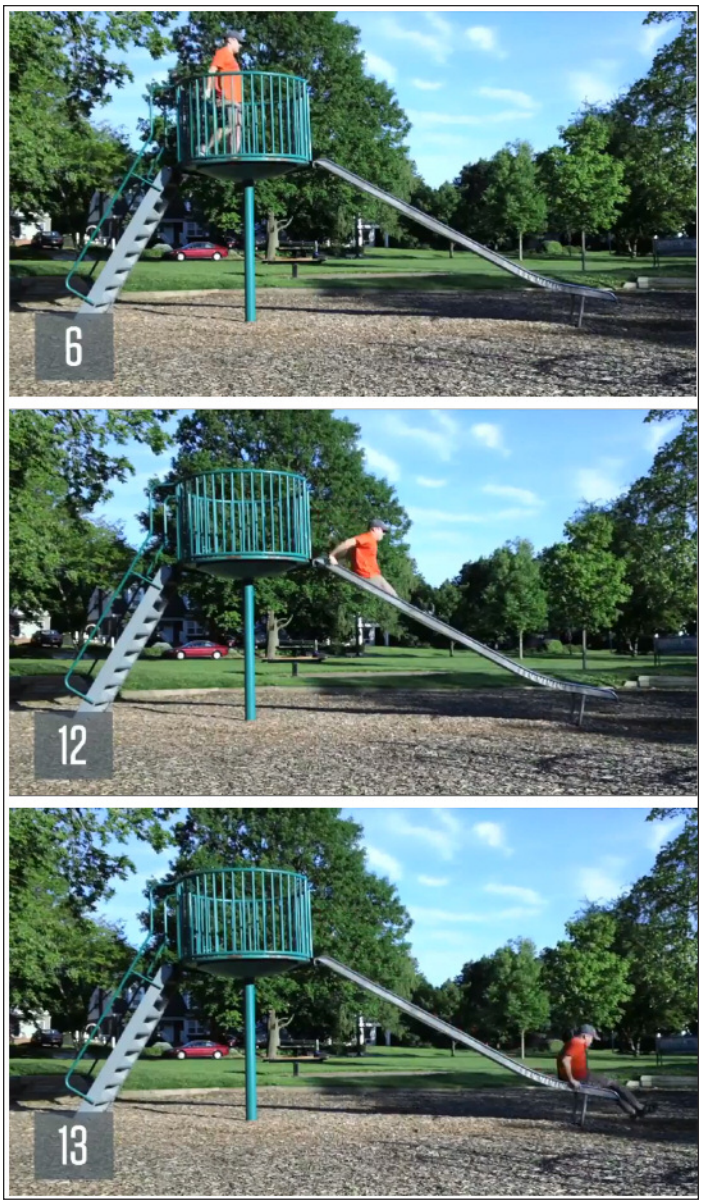


Figure 4: *Sequence of still frames from video*

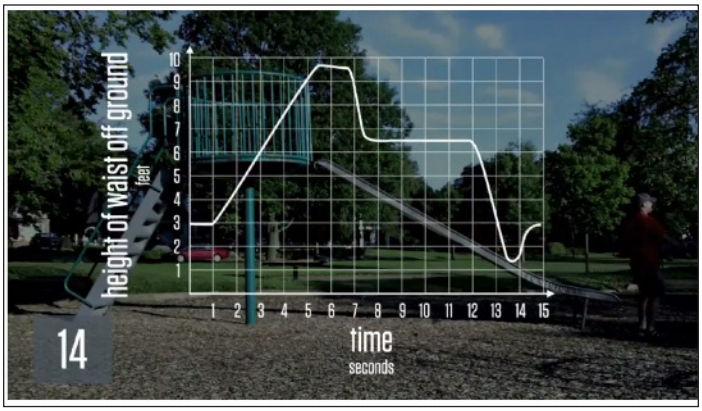


Figure 5: *The graph superimposed on the video setting*

The video then replays at half speed, giving students the opportunity to improve the connections between the video and the graph they are creating. Teachers should also feel free to pause the video as needed throughout this process.



Figure 6: *A graph of the story*

Following the second playing of the video, there is a section called “the answer.” The graph is superimposed on the video (Figure 5), so the relationship between the two is clear before the graph alone is displayed (Figure 6). Teachers can rewind and replay the answer as needed, to make sure all students are seeing how the variables relate to each other.





Figure 7: *Video and the start of a graph of the story*

Graphing stories provides an opportunity to initiate classroom discussions. Is the data continuous or discrete? If a section is linear, is it a direct or partial variation? How would you figure out the slope of a linear section? Can you interpolate parts of the graph if you know other parts? Errors that students make while creating their graphs foster rich conversations and beneficial learning opportunities.

Another example involves a runner going from home plate to 1st, 2nd, then 3rd base, before returning to home plate. There is an instructional opportunity to incorporate the Pythagorean theorem and delve into the finer details of the graph. Some of the screens are shown at the bottom of the previous page and top of this page, but I leave it to you to think about what the remainder of the graph would look like.

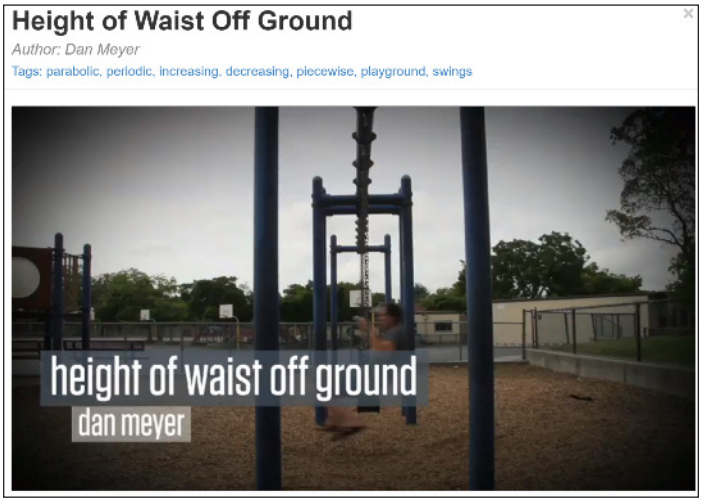




Figure 8: *A video involving a swing*

I will share one more example involving sinusoidal functions. Dan Meyer is featured on a swing in this video (Figure 8). He swings back and forth a number of times, and then he throws a twist at the end, when he jumps off the swing.

Once students get the idea behind graphing stories, they can even create their own, as Jaime Holmes had her students do. The video shown in Figure 9 is from her Twitter feed.

Further to Graphing Stories, Desmos created the Water Line activity (Figure 10), available at **www.teacher.desmos.com**. It shows a glass of water being filled and has students graph the height of the water versus time. They can play back the video to compare their graph to the correct one, and can see the precise point on the graph that corresponds to the height of the water. This is repeated with different shapes of glasses, and then students are challenged to create their own glass, which they add to the class cupboard (a collection made by the class) to challenge other students in the class.



Figure 9: *Video showing students can make their own stories*

The teacher can see an overlay of all the students' graphs (Figure 10), which provides timely feedback. It also facilitates asking what the glass would look like for an incorrect graph.

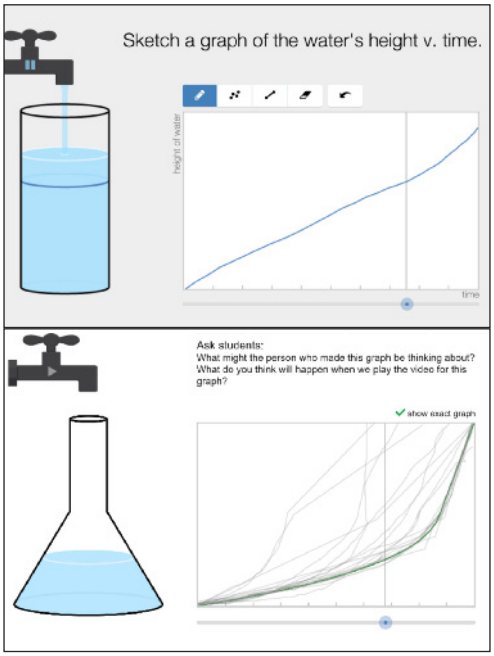
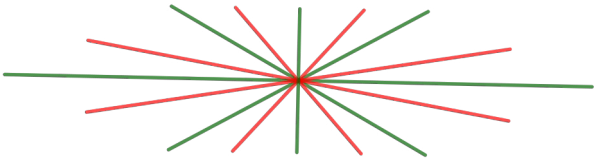


Figure 10: *Displaying a class set of graphs*

It is important for students to have a solid conceptual understanding of what a graph represents, and both the Graphing Stories videos and Water Line activity connect real-life concepts to graphs. They do this in a pedagogically sound environment that happens to be a lot of fun for students. ▲



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> *"I think we are better together, if there is a motto, and I would even extend this to my online professional learning community that I have, I'm way better with them"* (OAME Grade 9 Applied Project Participant, 2015).

The OAME Grade 9 Applied Mathematics Project was launched in October 2015 with ten school-based Professional Learning Communities (PLCs) from across Ontario. The PLC schools represented a wide variety of settings, ranging from small, rural schools to large urban schools. The needs of the teachers and students varied as well. Over the two years of the project, the design and focus of the PLC work evolved through the work of the participants, the research team, and the steering committee supporting them. In an earlier article (Suurtamm, Lazarus, & McKie, 2017), we discussed what we learned about teaching in ways that address the needs of Grade 9 applied students. In this article, we describe what was learned over the two years with respect to professional learning.

Each school-based PLC was comprised of an administrator, a Special Education Resource Teacher, a Student Success Teacher, Grade 9 Applied Teachers, and a Board or School Math Lead. The composition of many of the PLCs evolved, however, to include Grade 7, 8, or 10 teachers, and other support personnel. During the two years of the OAME Grade 9 Applied Project, PLC participants were given time, support, and encouragement to pursue their own professional learning. The Ontario Ministry of Education identified the following goals for the project:

- Increase teacher knowledge of the curriculum, both in terms of mathematics content knowledge and mathematics knowledge for teaching;
- Identify effective teaching and learning strategies;
- Share implementation strategies with stakeholders;
- Foster teacher leadership in mathematics; and, ultimately,
- Increase student achievement in Grade 9 Applied Mathematics.

Research on Professional Learning

The Grade 9 Applied Project was intentionally designed to correspond, as closely as possible, with current research on effective professional learning, professional learning communities, and effective teaching practices in mathematics. There is no one-size-fits-all formula for designing professional development for mathematics teachers, as each school or community of teachers has its own individual goals and its own specific learning environment (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). However, there are some particular components that researchers have found to be effective, regardless of the setting (Loucks-Horsley & Matsumoto, 1999). When considering professional learning models to support teacher learning and enhance classroom practice, many components need to be taken into account. Through an extensive review of literature on supporting the professional learning of mathematics and science teachers, Loucks-Horsley and Matsumoto (1999) identified several important components of effective professional learning that are supported by a variety of researchers over decades of work:

- **Curriculum-based** (Cohen & Hill, 1998; CPRE, 1998) – Professional learning should help to support teachers as they focus on understanding the curriculum, both the content and intent of the curriculum.
- **Focuses on student thinking** (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996) – Professional learning should focus on student thinking. This includes observations of student thinking during co-teaching or lesson study; examining student solutions of mathematics problems; viewing and analyzing video recording of student thinking. This helps to focus attention on the results of shifts in teaching practice on students' mathematical thinking.
- **Collaborative, within supportive school and district environments** (Elmore & Burney, 1997; Little, 1993) – There is a great deal of research on the positive role of teacher dialogue and collaboration on professional learning. Teacher dialogue and collaboration are seen as vitally important aspects of encouraging changes in classroom practice (e.g., Fullan, 2007; Hargreaves, 2009; Lachance & Confrey, 2003). Teachers derive support, motivation, and direction from one another.
- **Aligns with other elements in the system, e.g., assessment, curriculum, administrative support** (Cohen & Hill, 1998; CPRE, 1998; Kahle, 1999) – It is important that teachers receive similar messages from educational policies, curriculum documents, professional learning, and practitioner and research literature. In other words, coherence in the educational system helps to support teachers.

All of these components were considered in designing the professional learning in the OAME Project. Professional learning communities are productive spaces for teacher dialogue and collaboration, and thus, developing,

Area of Focus	Actions	Resources
Use of rich tasks	• Defined the construct of rich tasks and how to implement them in a lesson • Created a rich task resource bank • Debriefed lessons following the incorporation of a rich task to learn what was effective and what wasn't with various groups of students • Examined student work from rich tasks together	• "The Border Problem" from the *Connecting Mathematical Ideas* book by Boaler and Humphreys (2005) • The use of the Smith and Stein (2011) resource, *Five Practices for Orchestrating Mathematical Discussions*
Focus on assessment	• Mapping rich tasks to the curriculum • Rethinking assessment of student work on vertical surfaces • Assessing through observation and conversations with students • Creating success criteria as a PLC and making them explicit with students • Using EQAO sample questions as "checkpoints" during work with rich tasks	• Peter Liljedahl's (2016) *Thinking Classroom* (**www.peterliljedahl.com**) • Student-generated screencasts, using apps such as *Show Me* or *Explain Everything* • Online quizzing tools such as *Kahoot* or *Quizzlet*
Creating a positive environment for learning mathematics	• "Focus on growth" mindset • Use of videos and activities	• *Mindsets in the Classroom* (Ricci, 2013) and *Mindsets, the New Psychology of Success: How We Can Learn to Fulfill Our Potential* (Dweck, 2006) • Videos from Boaler (**youcubed.org**)
Going deeper with the mathematics curriculum	• Working across the grades, inclusion of Grades 7 and 8 teachers from board • Spiralling, reorganizing the curriculum, scope, and sequence • Using the curriculum as the foundation of their work and discussions as PLCs	
Going deeper with algebraic thinking	• Shifting focus from algebraic skills to algebraic reasoning • Using manipulatives to uncover algebraic meaning in structures	"The Border Problem" (Boaler & Humphreys, 2005)

Table 1: *Focus and Actions of PLCs*

supporting, and sustaining PLCs were at the core of this project.

Focus of the PLCs

Before beginning to describe the professional learning of the PLCs, it is useful to provide an overview of the types of pedagogical practices and inquiries that were engaged in. Table 1 provides a summary of some areas of focus the members of the groups chose, some of the actions they undertook to address these areas, and resources they found helpful.

PLC Learning

The Grade 9 Applied project provided teachers with time to grapple with different ways of teaching in a supportive environment. Our research indicated that their professional learning was supported in a number of ways. For instance, participants reported different ways that their perspectives on mathematics, on mathematics teaching and learning, and on the curriculum shifted during the work in the PLCs. They suggested several ways their learning was enabled. As discussed below, some of these ways include the structure of the PLC, the professional learning models used by the PLCs, and the structure of this project itself.

Structures of the PLC

Several participants suggested that the diversity of the school PLC team and their different roles helped to contribute to the PLC members' learning. Each of the participants brought different knowledge and experience to the discussions. For instance, the Special Education Resource Teacher or Student Success Teacher might have had different perspectives toward mathematics or different views on ways to support students than the teachers of Grade 9 Applied mathematics might have had. The administrator role was critical to establishing credibility, coherence, a vision, and a means for sustainability. Administrators were often able to support the work of PLCs by ensuring that particular structures were in place to assist the PLCs in meeting, by carefully considering the placement of the Grade 9 Applied classes in the school timetable schedule, and by supporting the continued work of this project, even beyond the two-year time frame. As PLC participants, administrators gained insight into the mathematics education world. Many of the administrators became more involved in mathematics education activities, including the OAME conferences and other professional learning opportunities that were new to them.

The structure of the PLC encouraged a distributed leadership model that seemed to help to facilitate participant learning. Many participants commented about feeling that they had a safe environment to share their thinking. School PLC members talked about the importance of respectful dialogue. Participants reported that learning was "messy" and there were tensions at times. Although participants within a PLC might have been working on the same thing, such as a task to try out in their class or a new assessment method, school PLC members were working on them in different ways. Those differences posed challenges at times, but also helped to push the participants' learning further. There were also moments of discomfort. For instance, the administrator in one PLC stated that she was fine with leading a PLC focused on literacy, but uneasy at first about leading a PLC focused on mathematics. However, the discomfort eased as the project progressed. PLCs emphasized that although they might have a common theme to their work, it was important to offer flexibility for teachers to implement ideas in their own ways.

PLC Models of Working

Many of the professional learning models that were suggested to the school PLCs during this project helped draw attention to the four effective components of professional learning outlined above. For instance, in the case of lesson study (or when co-planning or co-teaching), participants in a PLC could work with the curriculum, research, and resources to design a lesson that would address the curriculum and then implement it in different classrooms. They might also observe student learning during implementation, debrief their observations, and then work together to make adjustments to the lessons. This type of model incorporates collaborative work, focuses on the curriculum and the continuum of learning, and pays attention to student thinking. Book studies offered another model that incorporated research-based methods, connections to classroom practice, and included samples of mathematics tasks and student work. Book studies can help to bring a "knowledgeable other" to the table, provide curricular ideas to test out in the classroom, incorporate student thinking to examine, and stimulate collaborative discussion. Other suggested ideas included collaborative task design, working on tasks to anticipate student responses, and examining student work together.

Although the research team suggested different types of professional learning models, each PLC determined what they would work on and the ways that they would work. The PLCs engaged in a variety of activities within their meetings that included:

- classroom observations
- co-teaching, co-planning
- lesson studies
- book studies
- co-designing new tasks
- moderated marking
- examining data
- anticipating students' responses

- examining students' thinking through responses to tasks
- participating in interviews with students
- examining video recording of lessons and/or students' thinking
- discussions with teachers from other grades or schools

Depending on the PLC focus and needs, each meeting might have included a variety of these types of activities. For instance, a book study might emerge to address an aspect of the school PLC's current work. One PLC started a book study on *Connecting Mathematical Ideas* (Boaler & Humphreys, 2005), when the researcher recommended the book after noticing that the PLC was interested in exploring algebraic thinking. Other school PLCs chose books on growth mindset when that was their focus. Book studies on *Five Practices* (Smith & Stein, 2011) emerged as participants started to wrestle with facilitating rich tasks in their classrooms.

A great deal of time was spent in co-planning, followed by classroom observations and/or sharing student work, and debriefing after the planned lessons were implemented. Respectful dialogue was essential, and with all PLC activities, a strong connection between the work of the school PLC and the classroom was required.

Structure of the Project

The structure and responsiveness of this project was a key component of the PLC participants' learning. Several participants suggested that there was great value in the sustained nature of the project. As one participant explained, "I would really like to see more projects like this that are longer in nature… This is nice. It just shows dedication to one concept and to be able to see it through" (Grade 9 Applied Teacher, 2015).

Another aspect of the project that facilitated the school PLC participants' learning was the structure and responsiveness of the project as a whole. The monthly meetings of the research team and the frequent meetings of the Steering Committee provided the opportunity to share what was happening in the individual PLCs, and allowed for the emergence of new structures and ideas. For instance, during Year 1, the research team, in their monthly research team meetings, noticed the similarities between the activities and foci of the different PLCs. Although the researchers could see the connections, they realized that the PLCs themselves would benefit by having the opportunity to connect with other PLCs who were working on similar things. The research team, along with the Steering Committee, decided that a Summer Institute to gather all the PLCs together for two days would be beneficial to the school PLCs. This allowed for the cross-pollination of ideas to enhance the learning, as well as provide planning time for the PLCs before starting their next school year. Several teachers mentioned that learning from the other PLCs played a big role in shifting their understanding of teaching and learning in the Grade 9 Applied classroom. Some PLCs even visited with other school sites to share and learn from each other. As one participant explained:

> I think that's it's nice that we were able to cross-pollinate with other schools at the anchor sessions… because this is where that came from. This came from [Grade 9 teacher] seeing the spiralling… at the Summer Institute, and so this is where the idea was born, and so I think that the cross-pollination this project has provided really got what we needed into [our school board] (Instructional Coach, 2016).

Recommendations for Professional Learning Communities

Many things have been learned from this two-year OAME Grade 9 Applied Collaborative Project. Working with many dedicated educators in the PLCs from across the province provided a great deal of insight. The following recommendations for supporting Professional Learning Communities that are focused on improving mathematics teaching and learning emerged from our research. These recommendations align with, and extend, prior research about structural and social conditions necessary for effective PLCs (Kruse, Louis, & Bryk, 1994; Roberts & Pruitt, 2009):

- **PLC membership** – The diversity of membership of the PLC enhanced the learning. The team was not only comprised of teachers, but also of a Principal or Vice-Principal who could support the team's work. The inclusion of a Special Education Teacher and/or Student Success Lead provided a different perspective to the math teachers' perspectives and also enhanced the Special Education or Student Success Teachers' understanding of current approaches to mathematics teaching and learning.
- **Distributed leadership and teacher-directed learning** – The most successful PLCs were those that worked on what teachers wanted to work on and that had a distributed leadership, rather than a top-down approach. Although it is important to have someone manage the logistics of the PLC, the work of the PLC should primarily be teachers engaged in collaboration and dialogue that is focused on ideas, tasks, and student work. It is necessary to directly connect to classroom practice. In other words, the learning should constantly be moving in and out of the classroom.
- **Be responsive to provide models of sound professional learning** – The Research Team and the Steering Committee provided the PLCs with information

about models of sound professional learning (e.g., book clubs, lesson study, co-planning, examining student thinking). However, they also reinforced the use of these models by suggesting books, articles, tasks, and activities throughout, when PLCs were ready for the idea.

- **Provide resources to spark new ideas** – The PLCs in this study were provided with access to research, books, monographs, use of Twitter to follow other math educators, workshops, and webinars, which were suggested in a timely manner to spark new ideas when it seemed appropriate. PLCs often benefit from new ideas injected throughout their journey.
- **Be ready for challenges** – When teachers talk, differences in perspectives and practices emerge. Establishing a culture of respectful dialogue and a safe place to take risks is essential. Many of the aspects of a positive classroom culture, where students are able to take risks and make mistakes, are also critical to a positive PLC culture.
- **Recognize that change takes time** – Be sure to plan for sustained learning. Much like in a classroom environment, everyone is at a different place in his or her journey, and perhaps each member of the PLC has a different destination. Respect and patience are important.
- **Provide opportunities for interactions** – PLC members should have opportunities to interact with one another and with members of other PLCs. We structured interactions in a variety of different ways. PLCs leaders met together, Principals met together, and participants from different PLCs worked together, thus creating a network of PLCs.
- **Build in support** – Support needs to be available at many different levels – the PLC helps to support classroom teachers as they take risks and try out ideas, but Principals, Lead Facilitators, and Math Board Leads also need ways to feel supported.

Acknowledgements

The authors would like to acknowledge the extensive work and dedication of the educators who participated in the school-based PLCs and made this project and study possible. We would also like to thank the Ontario Ministry of Education, the Ontario Association for Mathematics Education, and the Ontario Mathematics Coordinators Association for their support. Finally, we would like to acknowledge the full research team, who took part in the research that led to this article. The research team included the three authors and Alison Goss, Kelsey Knowles, Dr. Martha Koch, Ina Lazarescu, Ellen Morrison, Jimmy Pai, Brenna Quigley, and Dr. Timothy Sibbald.

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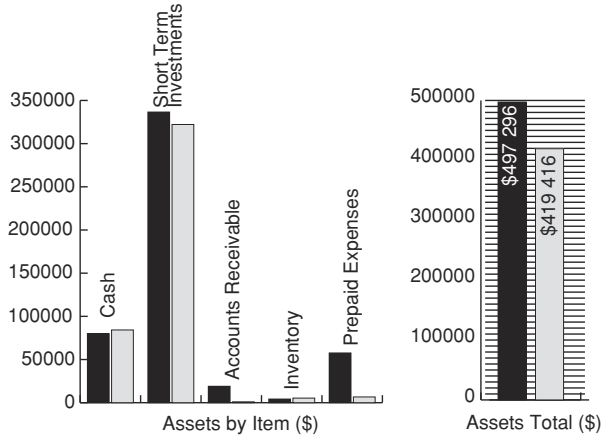
▲ OAME Financial Statement 2016-17

Notice to Readers

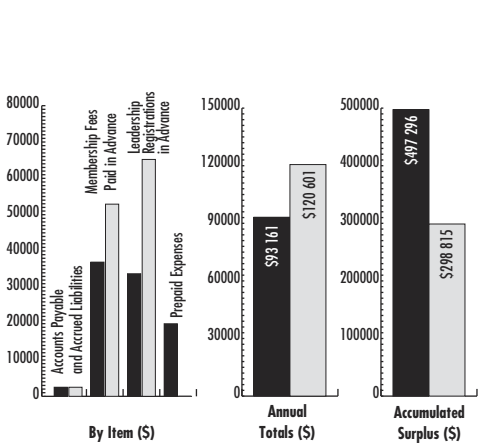
On the basis of information provided by management, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as of August 31, 2017, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario, September 12, 2017, SKRYPNYK GROUP, Professional Corporation, Authorized to practise public accounting by CPA of O.

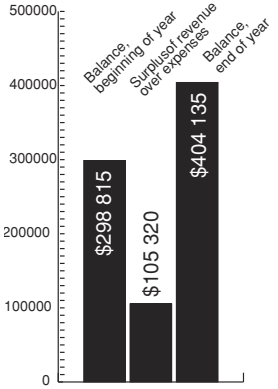
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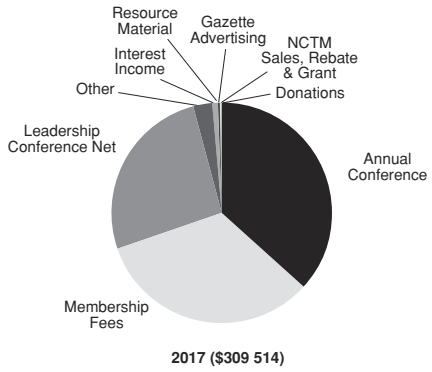
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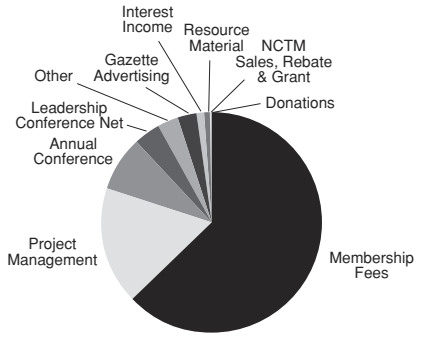
Operating Fund Balance



Revenue

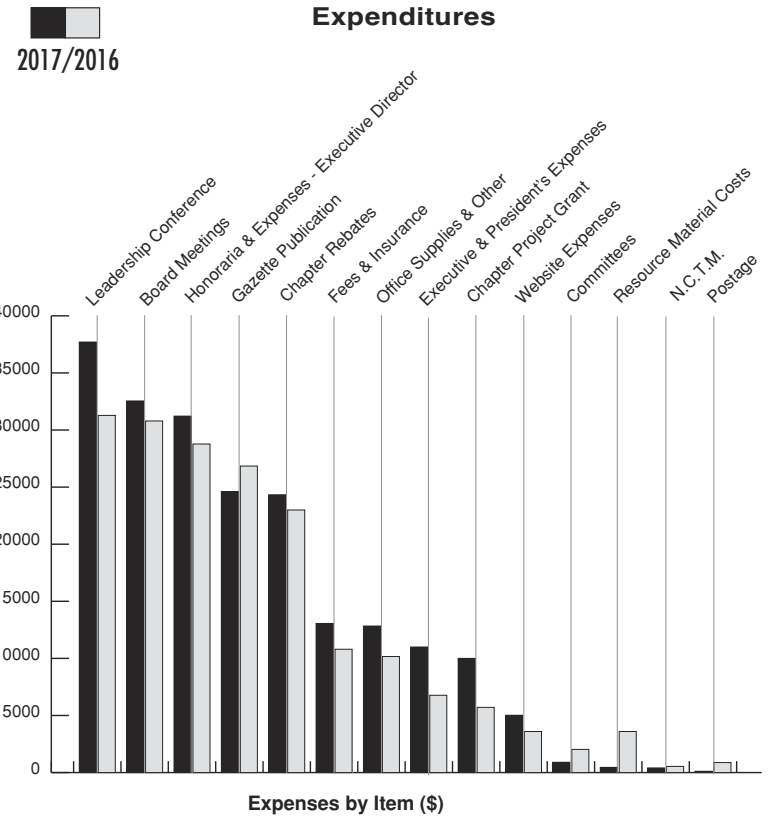


2017 ($309 514)



2016 ($209 173)

Expenditures



▲ Professional Learning: Moving from Bricks to Bricolage



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Alayne Armstrong is an assistant professor in the Faculty of Education at the University of Regina. She taught in the Coquitlam School District, British Columbia, for almost 20 years.

As teachers, we know that we should teach from where the students "are at," but mathematics is a subject that somehow seems to rise above the personal. Instead, we hear about students needing a strong foundation to be successful in math. Continuing with that metaphor, it is seen as a subject that is built up, brick by brick, and that we need to be careful about what bricks we offer students if they are going to be able to build a stable structure.

This building metaphor is a limited one, however. Perhaps we would do better to treat math more like a story, something students can be the author of, in much the same way that they can author their assignments in other subjects like language arts or social studies.

Authors often describe how stories can seem to write themselves. Once a character is put into a situation, conflicts arise and the plot emerges from there. Think of Shakespeare's *Romeo and Juliet*. Romeo Montague crashes the Capulet's party (problem – his family is feuding with them) and falls in love with Juliet Capulet (problem); Tybalt notices that Romeo is an uninvited guest and fights Romeo's friend Mercutio, killing him (problem); in response, Romeo fights and kills Tybalt (problem) and must flee the kingdom and leave Juliet behind (problem); so the story goes. Look at all these problems—you'd think we were in math class!

What happens with the conflicts in *Romeo and Juliet* is similar to what happens when students tackle a rich mathematical task. As they begin to work, they realize there are aspects that they don't yet fully understand and need to know more about, so they pose problems. The problems they pose trigger other problems, and so on.

This was the basis of my doctoral research study (Armstrong, 2013). I compared four groups of Grade 8 students as they worked on the same mathematics task, to see what kind of problem-posing patterns developed.

The Task: The Bill Nye Fan Club Party

The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.

If the presents are opened in 5-minute intervals, starting at 1:00 p.m., the last gift will be opened starting at 5:35 p.m. How many club members are there?

For this task, the students had to determine how many time intervals of five minutes there are during the gift-giving, which also equals the total number of gifts (56). Then, given that each member brings presents for everyone else, but not for himself or herself, 56 = (number of members) x (number of members –1). Eight club members each bring seven gifts. All four groups arrived at this answer.

In introducing the task to her students, Mrs. Shug reminded them that they would need to be able to explain their solutions clearly to their peers later in the class, and then led a class discussion to clarify what the term "interval" meant. To be honest, I thought the task was going to be straightforward and there would be little for me to observe, but I was wrong. There were 31 different problems that the groups posed about this task (Figure 1). I assigned specific colours to each problem and then used these colours to highlight the discussion of each of the problems in the transcripts of the groups' sessions. I then shrank the transcripts to create "tapestries," the idea being that a tapestry is something like a story graph—you can just glance at it to get an idea of the "plot" that a group is following. Even with a black-and-white version of these tapestries (see Figure 2 or **nsuworks.nova.edu/tqr/vol22/iss6/13** for a colour version), what is striking is just how different the patterns are from one another, providing visual evidence of the complexity, fluidity, and unpredictability of the groups' discussions.

Going back to *Romeo and Juliet*, there are lots of plays, movies, and books that focus on a pair of star-crossed lovers, including *West Side Story* and *High School Musical*. Even though each text has the same starting point and (usually) the same conclusion, the different authors have each created a unique storyline. In a similar fashion, the tapestries display how each group authors its own storyline/solution to the Bill Nye task, by posing problems that are specific to their particular needs and interests. Almost half the problems posed are unique to specific groups. Even the few problems that are posed by all four of the groups are posed at different times, discussed for different lengths of time, and re-emerge at different points during the conversation.

What can we as teachers take from this?

No matter how carefully teachers frontload students with strategies and concepts, there is no guarantee that this is what students will use when working on a task. In fact, teachers can't be sure what students are going to make use of. Having only a limited experience with algorithms and procedures, students tend to draw on their own informal experiences, performing bricolage (Lévi-Strauss, 1966) by using whatever tools and ideas they have available to deal with the task at hand.

Take, for instance, the posed problem, "Is it square roots?" In their regular lessons outside of the study, Mrs. Shug's classes had been learning about square roots. The groups in the study were aware that the Bill Nye task was not related to their regular lessons, yet two of the groups still posed, "Is it square roots?" This resulted in rich discussions about what square numbers are, how to find their roots, when to round decimals, etc. Another group posed the problem, "How can we use the 24 clock?" They were drawing on an experience with a previous task in the study that involved converting time from the standard 12-hour a.m.–p.m. clock to the 24–hour clock. It was an unusual way to find the number of intervals, but later in the session, the group compared this time-conversion method to another method, which involved physically counting around the classroom wall clock. This in turn generated a discussion about whether calculations were necessarily always better than a method where you could physically see what you were counting.

That so many problems were posed and discussed suggests how important it is that group communication strategies are valued in the mathematics classroom. The students in this study were at a school where the teachers had engaged in much in-service work over the years about co-operative learning strategies. Mrs. Shug had been careful to encourage her classes to develop and maintain effective group work routines, the evidence being how students were willing to risk voicing and trying unusual ideas (such as the time-conversion method), and how both small-group and full-class discussions were respectful, with students asking their peers for clarification, and arguments being backed up by evidence.

Finally, the tapestries provide visual evidence of how posed problems emerge and re-emerge. This might be the biggest take-away of all about teaching math from this particular study: you can't just address a mathematical topic with your class once and be done with it, giving students a conceptual brick to add to their mathematical pile. It would be more helpful to treat mathematics as a story that students weave together over time, where we as teachers can help to "pull" threads to students' attention, highlighting the ways these strands connect with one another.

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Colour	Problem posed (generalized)
Lavender	Do we use time and divide by 5 [number of intervals]?
Medium blue	What about if everyone brings x gifts each?
Purple	Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Deep red	How many people are there?
Slate blue	What are the factors of [particular value]? (For example, "What are the factors of 56?")
Lime green	What is meant by an interval?
Olive green	Do all members give to everyone?
Goldenrod	Do they also bring gifts for themselves?
Orange	Does everyone bring the same number of gifts?
Sky blue	How many gifts are there?
Brown	What if there are [particular number] people?
Green	How do we think outside the box?
Teal	Is it a square root?
Fuschia	Why did we get [particular value]?
Dark pink	How long does it take to open all the gifts?
Light purple	Can they take breaks in between opening gifts?
Pale yellow	Does it start at 1:00?
Gray	What is a tournament?
Red	What if it's an exchange?
Light green	How long does it take to open one gift?
Forest green	Can't we just count the number of people?
Lilac	How many gifts does each person bring?
Coral	How many gifts are opened in an hour?
Gold	Is another group's answer right?
Sage	Can they bring partial gifts?
Pink	What if someone doesn't get a gift?
Dark blue	How do we know if we're right?
Blue	What if there are [particular number] people and gifts?
Peach	Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Light blue	How can we use the 24-hour clock?
Yellow	Can they open gifts at the same time?

Figure 1: *Colour-coding chart*

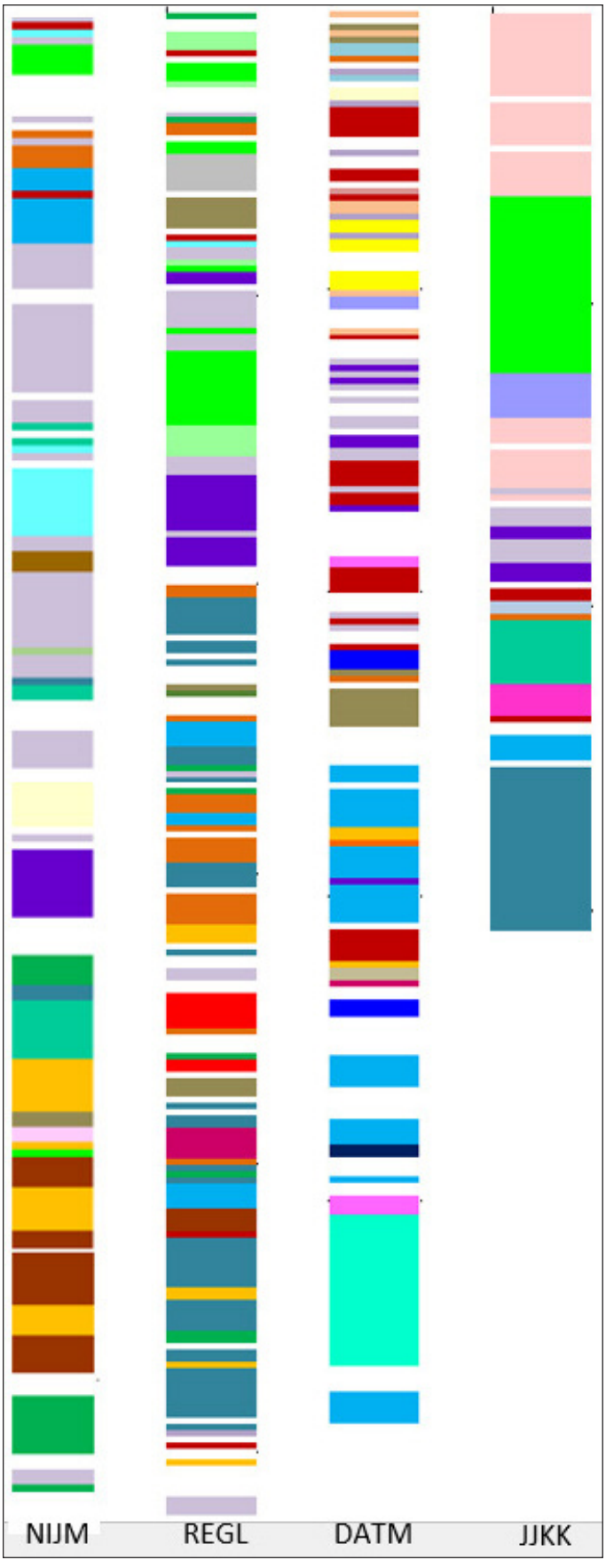


Figure 2: *Tapestries for the four groups* ▲

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2018–2019

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 31, 2018, to:
Jill Lazarus
jill.lazarus@oame.on.ca

Include a paragraph of the nominee's qualifications with the Nomination Form. A nominee must be a member of OAME. By March 2018, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting will be online and is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2018.

NOMINATION FORM

I nominate:_____
for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Email: _____

Telephone: Home (_____)_____

Work (_____) _____

_____

Nominator's Name : _____

Nominator's Address: _____

_____

Email: _____

Telephone: Home (_____)_____

Work (_____) _____

A Method for Creating Sums of Rational Expressions with Quadratic Denominators

JEFF GARDNER
EMAIL: jgardner@quickclic.net

Jeff started teaching mathematics (and computer programming) in 1985. He has taught in Ontario with positions in both private and public schools, and in alternative and traditional programs. He is currently teaching at Sherwood Secondary School in Hamilton.

In my Functions, University Preparation (MCR3U) classes, we spend time on questions for computing the sum of rational expressions, such as the following problem:

Simplify $\frac{8}{x^2+3x-10}-\frac{3}{x^2-6x+8}$ *to lowest terms.*

This addresses the expectations labelled "Determining Equivalent Algebraic Expressions."

One way to simplify the expression is to start by factoring the denominators:

$$\frac{8}{(x+5)(x-2)}-\frac{3}{(x-4)(x-2)}$$

Then determine a common denominator and express each term as an equivalent fraction with that denominator:

$$\frac{8(x-4)}{(x+5)(x-2)(x-4)}-\frac{3(x+5)}{(x-4)(x-2)(x+5)}$$

The expression simplifies algebraically

$$\frac{8x-32-3x-15}{(x+5)(x-2)(x-4)}-\frac{5x-47}{(x+5)(x-2)(x-4)} \quad x\neq -5,\ 2,\ 4$$

But, what if the numerator $5x - 47$ were, say, $5x + 20$? Then:

$$\frac{5x-20}{(x+5)(x-2)(x-4)}=\frac{5(x-4)}{(x+5)(x-2)(x-4)}=\frac{5}{(x+5)(x-2)} \quad x\neq -5,\ 2,\ 4$$

Here the possibility of simplification urges students to check that their solution is in lowest terms. If we want students to examine their work and to make sure they have simplified their work as much as possible, then we must give them questions that encourage them to do it. Designing such a question was not trivial. However, it is possible to construct these types of expressions that can simplify to a lower degree. This article investigates one such method.

Question

When is the sum $\frac{d}{ax^2+bx+c}+\frac{h}{ex^2+fx+g}$ equal to $\frac{M}{Jx^2+Kx+L}$, where a, b, c, d, e, f, g, h, J, K, L, and M are integers, and a, d, e, $h \neq 0$? It is also to be understood that $ax^2 + bx + c \neq s(ex^2 + fx + g)$, for any scalar s.

Method

(1) Arbitrarily choose two distinct first-degree linear polynomials.

(2) Choose two arbitrary integers d and h so they are not in the same ratio as the coefficients of the linear polynomials in Step 1 (if this step is not followed, the simplified numerator will be an integer, not a polynomial).

(3) Simplify the polynomial expression to find the common factor to complete the "question."

Example 1

(1) Choose two arbitrary and distinct linear polynomials: $(x + 6)$ and $(x - 1)$

(2) Choose two arbitrary integers d and h: In this example, the ratio of the coefficients in Step 1 is 1:1, so choose any values which are not in a ratio of 1:1; for example, let $d = 3$ and $h = 4$.

(3) Simplify the polynomial expression:

Given

$$\frac{3}{(x+6)(\textit{common binomial})}+\frac{4}{(x-1)(\textit{common binomial})}$$

Find a common denominator:

$$\frac{3(x-1)}{(x+6)(\textit{common binomial})(x-1)}+\frac{4(x+6)}{(x-1)(\textit{common binomial})(x+6)}$$

Simplify algebraically:

$$=\frac{3x-3+4x+24}{(x+6)(x-1)(\textit{common binomial})}$$

$$=\frac{7x+21}{(x+6)(x-1)(\textit{common binomial})}$$

For the rational expression $=\frac{7(x+3)}{(x+6)(x-1)(\textit{common binomial})}$ to be reduced further, the *common binomial* can be assigned as $(x + 3)$ in order to cancel with the numerator. This will give the rational expression $\frac{3}{(x+6)(x+3)}+\frac{4}{(x-1)(x+3)}$ with quadratic denominators.

The question presented to the students would have expanded denominators and be presented as:

Simplify to lowest terms and state the restrictions of

$$\frac{3}{x^2+9x+18}+\frac{4}{x^2+2x-3}$$

A sample student solution might be:

$$\frac{3}{x^2+9x+18}+\frac{4}{x^2+2x-3}$$
$$=\frac{3}{(x+6)(x+3)}+\frac{4}{(x+3)(x-1)}$$
$$=\frac{3(x-1)}{(x+6)(x+3)(x-1)}+\frac{4(x+6)}{(x+6)(x+3)(x-1)}$$
$$=\frac{3x-3+4x+24}{(x+6)(x+3)(x-1)}$$
$$=\frac{7x+21}{(x+6)(x+3)(x-1)}$$
$$=\frac{7(x+3)}{(x+6)(x+3)(x-1)}$$
$$=\frac{7}{(x+6)(x-1)} \quad x\neq -6,\ 1,\ -3$$

There is an associated algebraic identity that can facilitate generating d and h values when $ax^2 + bx + c$ and $ex^2 + fx + g$ are equated and simplified (i.e.,

$$\frac{-b+\sqrt{b^2-4ac}}{2a}=\frac{-f+\sqrt{f^2-4eg}}{2e}$$

with a, $e \neq 0$). In this example, the result gave $d = \frac{3}{7}s$ and $h = \frac{4}{7}s$, for any scalar s. If we set $s = 7$, then we get $d = 3$ and $h = 4$ (which happen to be the arbitrary values chosen in Step 2, above). Although the algebra to find the identity is challenging, the investigation is not beyond the skills of stronger Grade 11 or 12 students, if a teacher wishes to use it as enrichment. (If you wish more details, please contact the author.)

Example 2

(1) Choose: $(2x - 3)$ and $(3x - 1)$

(2) The ratio of the coefficients in Step 1 is 2:3. Let $d = 6$ and $h = -4$.

(3) Giving

$$\frac{6}{(2x-3)(\textit{common binomial})}-\frac{4}{(3x-1)(\textit{common binomial})}$$
$$=\frac{6(3x-1)}{(2x-3)(\textit{common binomial})}-\frac{4(2x-3)}{(3x-1)(\textit{common binomial})}$$
$$=\frac{10x+6}{(2x-3)(3x-1)(\textit{common binomial})}$$

Choosing a d and h that share a common factor (i.e., are not relatively prime) leads to a factorable *common binomial* (i.e., we got $(10x + 6)$, which factors to $2(5x + 3)$). Therefore, letting the common binomial be $(5x + 3)$ yields the final answer:

$$=\frac{2(5x+3)}{(2x-3)(3x-1)(5x+3)}$$
$$=\frac{2}{(2x-3)(3x-1)}$$

Thus, using the common binomial $(5x + 3)$ would make the question:

Simplify to lowest terms and state all restrictions of

$$\frac{6}{10x^2-9x-9}-\frac{4}{15x^2+4x-3}.$$

Creating examples such as those discussed provide factoring applications for both the Functions, University Preparation (MCR3U), and Advanced Functions (MHF4U) courses, as well as interesting domain and range applications. The types of questions described above address the Advanced Functions (MHF4U) expectations regarding Combining Functions ("…determine functions that result from the addition, subtraction… of two functions… describe some properties of the resulting functions…"). They can also lead to a robust discussion in the Polynomial and Rational Functions expectations ("…identify and describe some key features of the graphs of rational functions, and represent rational functions graphically") by contrasting the graphs of this type of function when it simplifies with when it does not. A graphical comparison is shown in Figure 1. This also demonstrates that students can use graphing to check whether they have the answer in lowest terms or not.

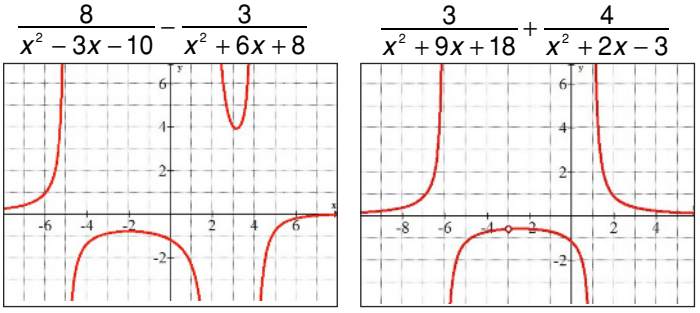


Figure 1: *The left graph (introductory example) is the typical result that does not simplify and has three vertical asymptotes. The graph on the right (Example 1) has only two vertical asymptotes because the (x + 3) term cancels, which results in a discontinuity. The discussion of the discontinuity would also set up nicely for left- and right-hand limits in Calculus and Vectors (MCV4U).*

This approach enables the creation of questions that oblige students to keep an eye out for opportunities to simplify their algebra. The method suits the overall theme of Grade 11 with problem solving involving algebraic functions. It also fits with graphing functions in Grade 12 and can be used when discussing left and right limits.

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