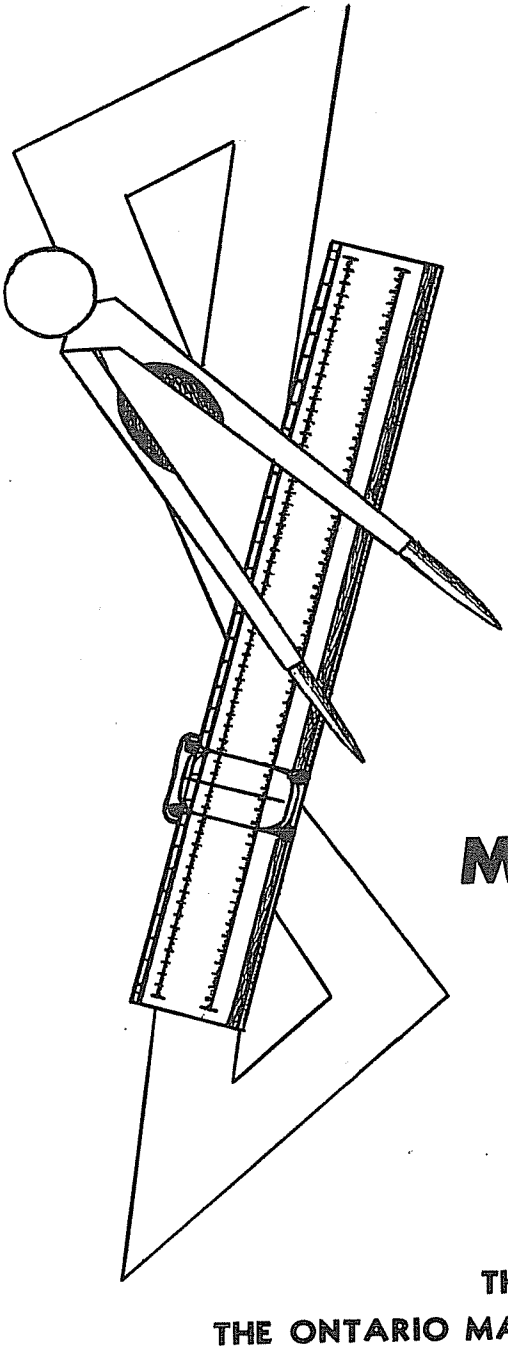


SPECIAL
ELEMENTARY
SCHOOL
EDITION
SEPTEMBER 1966

ONTARIO MATHEMATICS GAZETTE

sponsored
THE SAMUEL BEATTY FUN
THE ONTARIO MATHEMATICS COMMISSIO
THE ONTARIO ASSOCIATION O
TEACHERS OF MATHEMATI

SPECIAL ELEMENTARY EDITION OF THE GAZETTE

ACKNOWLEDGEMENTS

The Publications Committee of the Ontario Mathematics Commission are grateful to the following organizations which made donations to assist in defraying the expenses of the Special issue:

Ontario Public School Men Teachers' Federation, Professional Development Committee	- $500.00
Mathematics Department, Brock University	- 100.00
National Council of Teachers of Mathematics	- 400.00

The Editors indicate with regret the following errata:

Page 1, Line 2: replace "public" with "elementary"
Page 45: delete part in parentheses from Line 1 and add to Line 9
Page 51, Line 7: replace "arithmetic" with "authentic"

TABLE OF CONTENTS

	Page
Editorial	1
The Work of the OMC Elementary Mathematics Committee by J. Ryan	2
The Quest for an Improved Curriculum by J.R. MacLean	5
Mathematics K-6: The Interim-Updated Course in Ontario by J.K. Crossley	14
New Content and Approaches for Mathematics in the Elementary School by H.A. Elliott	17
Impressions of Mathematics Education in Elementary Schools in England by J.N.C. Sharp	32
Missing Links by W.W. Sawyer	36
Geometry in the Elementary Grades by I.J. Allen	42
The Teacher and the Interim-Updating by D.H. Crawford	49

EDITORIAL

This special issue of the Mathematical Gazette is devoted entirely to Mathematics in the Public School. Its appearance has been timed to coincide with the publication by the Department of Education of Curriculum P.1, J.1 Interim Revision Mathematics in the hope that it will provide teachers with a clearer idea of how the developments have come about, and what their implications are. The new issue may be thought of as a 3-act play. Act I consists of articles which take the reader from the past to the present, showing how the work of the Ontario Mathematics Commission led first to the formation of a K-6 Study Group and hence to the present interim-revision. Act II comprises a set of four articles, each commenting on the rationale and meaning of the new developments. Included in this collection are impressions of how similar developments in England are showing evidence of children learning actively and enjoying doing it. These articles provide as it were some kind of preview of a possible future. Finally, in Act III some suggestions for approximating this apparently heavenly state are discussed briefly.

As will be seen, many people of wide experience have contributed to this issue, and each merits your thanks. We are only at the beginning of what promises to be a very significant decade of change in the teaching of elementary mathematics in Ontario. In these days of rapid change and consequent need for readjustment let us all work together, and build surely and gradually rather than hastily and insecurely. And, in doing so, let us realize once and for all, that learning is the object of teaching and that it is a dynamic process.

As teachers, we should always be receptive to new ideas, and willing to try out new approaches. Here is our chance to put this principle into practice permanently.

D.H.C.

THE WORK OF THE OMC ELEMENTARY MATHEMATICS COMMITTEE

by J. Ryan*
Head, Department of Mathematics,
St. Patrick's College High School, Ottawa

If blame or plaudits are due any particular group regarding the upheaval in the teaching of arithmetic and mathematics in the schools of Ontario, this group is the Ontario Mathematics Commission.

It is interesting to note that the OMC grew out of the Mathematics Committee of the Ontario Teachers' Federation. As far back as 1957 and 1958 this committee was recommending some change in courses and approach for Ontario children. At that time the representatives from the elementary schools felt that the area most in need of change was possibly the grade 7 and 8 levels but that the programme in the earlier grades was sound enough. Thus it was when the Ontario Mathematics Commission of the OTF was formed and a group of teachers from universities and secondary schools together with representatives from the elementary schools and a representative from the Ontario Department of Education met for a week at Lakefield in August 1959, the emphasis was on a programme for secondary school mathematics. (Most of the topics discussed then are now topics taught in high schools even to the new grade 13 to be introduced in September 1966).

At that time there were many who felt that it would have been better to start at the early grades and work the new ideas up through the grades. However as it happened the start in the high schools made more teachers and students in the province aware of the new ideas, so much so that many elementary schools are willing and able to make changes. This is evidenced by the fact that about 90% of the grade seven classes are using a proposed new course before it becomes compulsory.

---

* Chairman, OMC Elementary Education Committee, 1964-66. Chairman, Committee Considering the Mathematics Programme (K to 6) July 1965.

When the constitution of the Ontario Mathematics Commission was drawn up, one of the main working committees formed was the Elementary Education Committee. As early as 1961 the representation of members of the Commission was adjusted to give greater voice to those directly connected with elementary school education. From the outset this committee was interested in encouraging experimentation with new ideas in the classrooms and tried to keep up to date on any work that was being done in the province. There were many committees under various Boards of Education which were trying to work out outlines of topics. Some centres were trying out Cuisenaire Rods. Some places were using series of texts from United States. It was up to this Elementary Education Committee to study all that was happening in Ontario as well as outside the province and to make recommendations to the Commission on what might be best suited for general use.

Most of the recommendations made to the Commission had to do with ways of informing the classroom teachers about the new ideas that were being talked about in school mathematics. To this end the brochure "What is Number" by D. Mumford was produced in an attempt to better acquaint teachers with the real numbers. Much time and effort, too, was spent in preparing and discussing such topics as 'Geometry' and 'Measurement'; the whole idea of terminology and symbolism was discussed thoroughly; many key topics were picked from the traditional programme to show how they could be improved; much discussion centred about the changing emphasis of mathematics in the elementary school - more towards the understanding of the relationships involved and not just their uses; the discovery approach in presenting topics of mathematics was discussed several times.

Many attempts to draw up a tentative sequence of topics which would reflect a definite progression from Kindergarten to grade 8 were made. Some thought was given to what topics could be moved down from the high school, what topics could be deleted and what topics could be introduced. The first such attempt was made in 1963. However in 1965 a sub committee

of the Elementary Education Committee did bring in a suggested outline and it was on the strength of this report that the Ontario Mathematics Commission, sponsored by the Ontario Curriculum Institute, appointed a committee to meet in Kingston for four weeks in July 1965. This committee was not to write a course of study but to study ideas and approaches to mathematics at home and abroad and to suggest possible revisions of the present curriculum for K-6. The Report of this committee was published by the Ontario Curriculum Institute in December 1965. A number of the recommendations have seen concrete embodiment in the ongoing work of the K-6 Mathematics Study Committee.

The work of the Elementary Education Committee goes on. It advises the K-6 Committee in its follow up of the report. More recently it has made recommendations to the Commission through its sub committee studying both the in-service and pre-service education of teachers. One result of this was the submission of a brief to the Minister's Committee on Elementary Teacher Education*.

It is hoped that there will never again be a stagnation in course content or approaches to the teaching of mathematics and thus there is no end in sight for the work of the OMC Mathematics Committee.

---

* The report of the Committee was published in March, 1966.

THE QUEST FOR AN IMPROVED CURRICULUM

by J.R. MacLean
Assistant Superintendent, Curriculum Division,
Ontario Department of Education

Introduction

For the past several years teachers have been almost overwhelmed by successive waves of propaganda that has generally been critical of our 'traditional' mathematics programs. Unless set notation and symbolism, non-decimal numeration, modular arithmetic and strict attention to correct terminology (such as the distinction between number and numeral) were included in our courses of study, we were not up-to-date. Our students would be ill-prepared for their responsibilities in the Space Age and we were effectively sabotaging the future of our country.

It is unfortunate that this "new math" concept has confused as much as it has changed elementary school mathematics. There are many reasons why this has occurred, but the most significant are these: the haste with which the new ideas were applied; a lack of understanding on the part of some of the mathematicians designing the programs about how children learn and what they need to learn; the general unpreparedness of teachers and supervisors which has fostered the idea that the "new" completely replaces the "old" rather than clarifying, supplementing and providing more meaningful approaches; and finally, the lack of unbiased evaluation of the newer materials and techniques.

The K-6 Kingston Study Group

Conscious of these weaknesses and yet alert to the necessity of applying new knowledge, both in mathematics and in pedagogy, to the development of new and better programs in elementary school mathematics, the Ontario Mathematics Commission, with the financial support of the Ontario Curriculum Institute, undertook the task of exploring and collating the various experiments and

approaches that are being carried out in Ontario and elsewhere. The Commission appointed ten teachers to a committee - eight directly involved in elementary schools, one from the high schools and one from the universities - and asked them to suggest possible revisions of the present curriculum and to make recommendations for the implementation of the suggested changes.

The K-6 Mathematics Study Group met during the summer of 1965 at the Royal Military College in Kingston, and produced a report which was subsequently published by the Ontario Curriculum Institute. This report has been described as one of the most significant documents to appear on the educational scene in many years. All teachers who are concerned with the teaching of mathematics are urged to read this report. Some of the recommendations of the committee have been included in the Interim Revision in Mathematics recently published by the Ontario Department of Education.

The K-6 Experimental Project

The proposals in the Report for implementing a new program have been considered by the Commission, especially item 7 ... " that a full time group be established ... to prepare teachers' handbooks and other material for selected topics in specific grades so that extensive experimental work can be carried out in the academic year 1966-67". Once more the Curriculum Institute agreed to finance a project that would involve many teachers and students throughout the province. It was decided to prepare materials for the study of Geometry in Grades 1, 2 and 3, and Graphs, Mappings and Relations in Grades 4, 5 and 6. Selected teachers from all sizes of school jurisdictions met in Toronto during July for training sessions in presenting the materials.

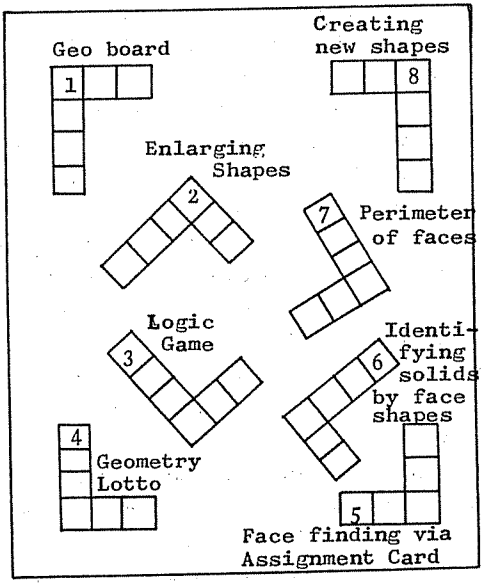
Perhaps the most vital aspect of this experimental program is the change in pedagogical techiques which in turn transform the classroom from the usual array of students sitting in desks arranged in neat rows to a virtual "laboratory for learning". Pupils are actively engaged in solving problems with physical materials that they can manipulate themselves. This approach is illustrated by an ancient Chinese proverb -

"I hear, and I forget
I see, and I remember
I do, and I understand"

Many of the ideas and techniques presented in the Nuffield Mathematics Teaching Project publications have been adapted for use in the experimenting classrooms. The writing team was greatly encouraged through the knowledge that other people in other countries were working with ideas not too divergent from its own.

The following sample lesson or more appropriately 'experience description' has been taken from the outline prepared by the writing team and presented to the teachers attending the summer course. Running through it is the central notion that children must be set free to make their own discoveries and think for themselves.

A SUGGESTED CLASSROOM ARRANGEMENT SHOWING ACTIVITY CENTRES



Lesson Topic: Examining Faces

This lesson was presented after three previous periods spent in examining, discussing and manipulating solids. Each student picks a partner to work with during the lesson period. The partners are given a piece of paper, crayons and each pupil chooses a solid from the set provided. Their assignment is to trace the flat surfaces of each solid onto a sheet of paper, attempt to select shapes that are different, and then discuss why they are different.

The class might choose the circular face to examine. One member is appointed secretary-chairman. Each pupil should draw a ring around the circular faces they have traced and suggest things that make the circle different from the other shapes. The secretary records all suggestions and the class then evaluates the chart he produces. Other faces should be examined in a similar manner before students proceed to the activity centres, indicated in the accompanying diagram.

(Activity centres are suggested as one possible way of providing for experimentation → thinking → communication which seems to be an obvious line of development for a child.)

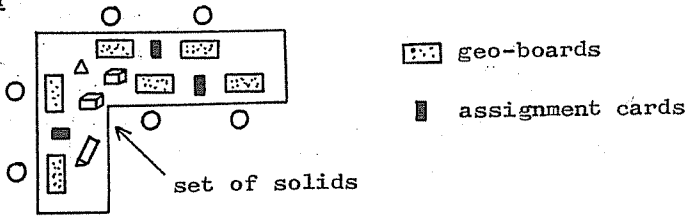
The students proceed from one activity to another, working through assignment cards, chart activities, logic games and measurement experiences. When they finish an activity they are allowed to move to any other centre provided there is an empty space. The teacher moves about the room guiding, encouraging and helping when requested. In this classroom atmosphere the teacher's role is not to stop children talking, but rather to ensure that there is something worthwhile for them to talk about. There is a place for lively discussion and the quality of the discussion will be directly dependent upon the quality of the class-teacher relationship.

Do not expect any miraculous change in the behaviour of children immediately after introducing these experience-centred classroom groupings. Initially it may be better to try fewer groups and allow the pupils time to adjust to the freer atmosphere and to develop respect for the opinions of others. This latter attribute follows smoothly the respect the teacher shows for the

student's point of view. Remember, real discussion, wherever it appears, is provoked by experience. The situation supplies the starting point; the discussion that ensues should widen the children's horizons and open up many new avenues of exploration.

Here are brief descriptions of the eight activities.

1. Geoboard



Geo-boards are square (in this case 12" by 12") boards with nails driven in at 1" intervals. Assignment cards could be similar to the following.

Assignment:

1. On your geo-board make all the faces of the triangular prism.
2. Two of the faces are alike. What shape do they have?
3. Three faces are alike. What shape do they have?
4. Count the number of sides on one triangle.
5. How many sides do you have on a rectangle?

Check your answers on the back of the card.

Students may use elastic to recreate the shapes of faces, answer the questions either orally or on paper, then check their answers.

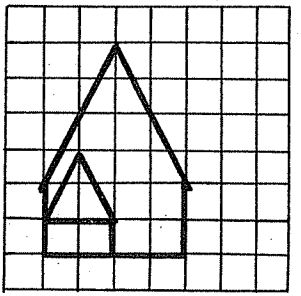
2. Enlarging Shapes

Again students work from assignment cards. They create designs using known shapes and enlarge the same design on graph paper.

Assignment Card Sample

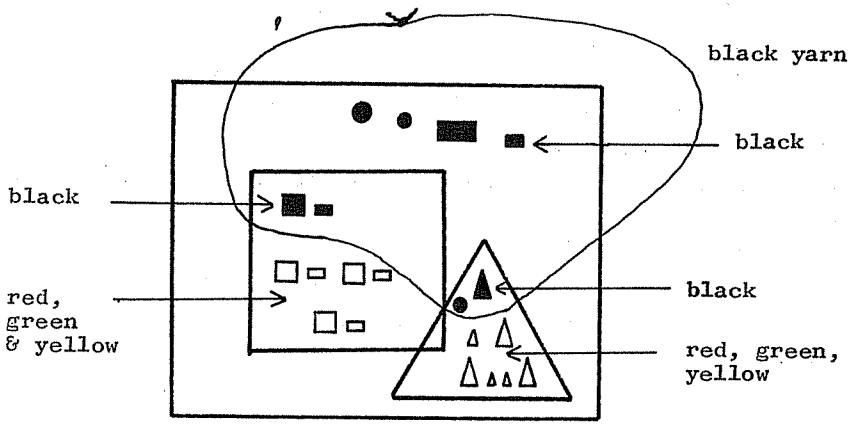
1. Use graph paper to design a house - Use a triangle and a rectangle.
2. By making the sides longer draw another house. It will be the same shape but it will be larger in size.
3. How much bigger do you think the new shape is than the first shape?

Possible Result



3. Logic Game

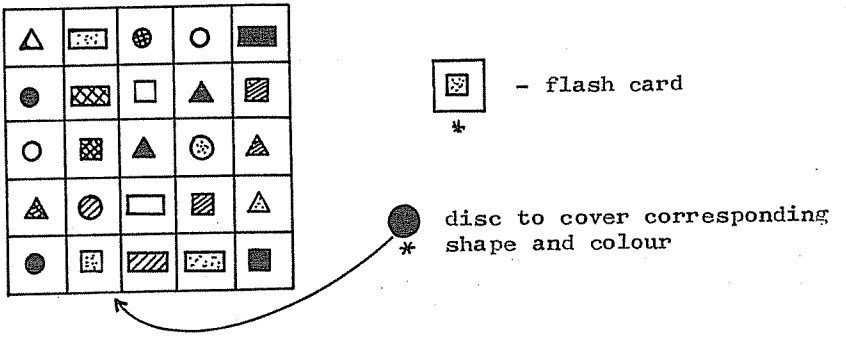
Four logic games are set out. Each set contains a large bristol board sheet with several shapes drawn on it, 4 different colours of yarn and a set of cards in the same 4 colours as the yarn. The cards may be classified according to colour (red, green, yellow or black), size (large or small) as well as shape. Students develop their own methods of placing the shapes and yarn on the Bristol board.



(All other shapes remain outside the set.)

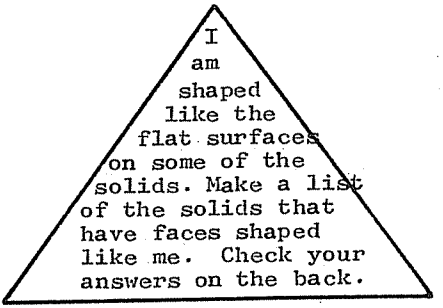
4. Geometry Lotto

This game is played much like Bingo except that colour and shape are used instead of numerals. As flash cards are held up by one student, other students cover the corresponding colour of the same shape with plastic discs.

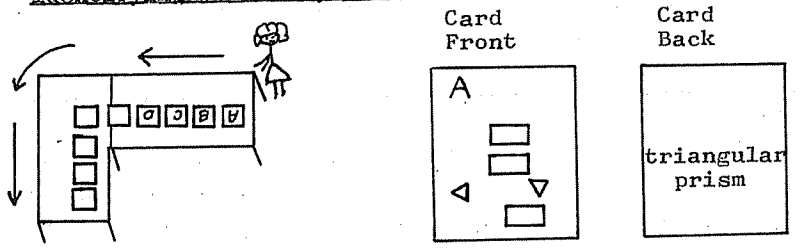


5. Face-Finding, via Assignment Cards

At this activity two people work from each assignment card. The cards are placed on the table with a set of solids.



6. Identifying Solids by Face Shape



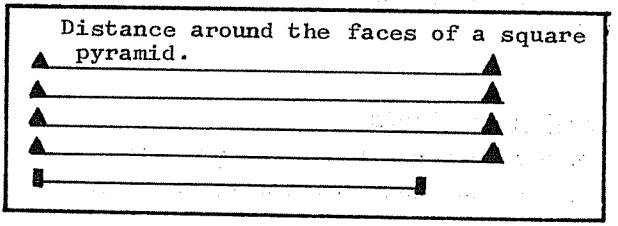
Students begin at point A and walk around the table from Card A to Card I listing the solids from which the surfaces (both flat and curved) have been traced. Then they repeat the rotation, this time checking the answers on the backs of the cards.

7. Perimeter of Faces

The pupils use yarn to find the distance around the faces of the solids, and fasten the yarn to a graph with gummed mosaic shapes.



ie.



8. Creating New Shapes

Pupils work with solids, rulers and assignment cards similar to the following:

1. Trace a square face from one of your solids.
2. Label it in this manner: 

3. How many sides has it? How many corners?
4. Join point A to point C. Can you see 3 shapes now? List them.
5. Join point B to point D. Can you find 9 shapes now? List the shapes.
6. Check your answer on the back.

Conclusion

In the 1966-67 session, the teachers trained in Toronto in July 1966 will carry the project further by experimenting with the teaching of the topics and concepts mentioned earlier. There will be three phases to the experiment, each for 10 weeks

duration. In the fall, the material will be taught for 10 weeks in Western Ontario. It will then be revised, and tried out in modified form in Eastern Ontario in the Winter term, again for 10 weeks. Finally, after a second revision, it will be taught for a further 10 weeks in Northern Ontario in the Summer term.

By means of these successive revisions, it is hoped that the feasibility of both content and approach will be put strongly to the test. All those involved look forward to the experimentation with intense interest and a fair measure of confidence in the laboratory approach.

MATHEMATICS K-6: THE INTERIM-UPDATED COURSE IN ONTARIO

by J.K. Crossley, B.A., M.Ed.,
Assistant Superintendent, Curriculum Division
Ontario Department of Education.

The appointment of the Hall Committee on Aims and Objectives of Education in 1965 undoubtedly presaged major revisions in the curriculum of the schools. The elementary grades in particular are receiving close attention as the work of the Committee proceeds, but the aims of the entire system from Kindergarten to Grade 13 are being studied also.

Any major undertaking requires considerable time, and approved recommendations of such a nature as will likely ensue from the report of the Committee will undoubtedly take more time to implement.

In this interim period, the Department of Education is assisting the process of change by updating the present courses used in the elementary schools.

The program of studies for grades 1 to 6 of the Public and Separate Schools has provided the basic guidelines for the curriculum of the elementary schools of Ontario since 1937. Minor revisions have been made from time to time, most recently in 1960, but it is remarkable how well the architects of change designed courses and set forth aims of education for young children nearly thirty years ago.

Recent developments in Mathematics have provided one cogent set of reasons for an interim updating of the Program of Studies, however. Probably the most basic of these reasons is the current belief that children should be introduced to Mathematics, rather than only the computational skills of arithmetic. For purposes of introducing a broadly-based foundation of concepts and skills to young children, it is convenient to regard Mathematics as based on the foundation stones of geometry and of arithmetic-

algebra. It is also useful to separate out measurement and relationships, since the development of skills in these two areas is of primary importance. In actual practice in classrooms, and most importantly, in the minds of children, it is imperative that as much "interweaving" of ideas as possible be done. Awareness of the unity, beauty and usefulness of mathematical ideas can best be developed when number and shape and relationships form an inter-connected whole in the mind of a child.

A second advancement in thinking about elementary mathematics has come with the increasingly widespread realization that the use of concrete materials must be the first step in the development of mathematical ideas. Just as in a sound modern reading program in the early grades, so in a sound mathematics program the ideas that children are expected to learn in symbolic or abstract form must be first discovered or comprehended in many forms and experiences in the real world - the world of sight, and odour and taste and touch, as well as of sound. Only with experiences involving the child himself, in activities where he uses several of his senses, can he build up a background of experiences from which abstract structures of mental activity, especially those that are mathematical, may be built.

Other criteria for a modern approach to elementary mathematics include the acceptance of a variety of procedures in solving problems. It is highly desirable that children devise their own methods of attack, even when learning to compute. Most standard algorithms are quite sophisticated - efficient short-cuts, in some respects - and are best arrived at after a process of discovery of principles and refinement of techniques.

It is high time, also, that recognition of the wide variations in the times at which individual children arrive at a stage of mastery of any given aspect of mathematics be reflected in the administrative procedures of schools. Again the parallel to modern reading programs is apparent.

Finally, it has become evident with the introduction of new approach programs in elementary schools, that some of the slogans and catch-phrases of time-hallowed sanctity have little validity. For instance, "flash mastery" is a contradiction of terms, if

mastery implies understanding, since instantaneous recall allows no time for thought processes to occur. "Speed and accuracy" is also an unfortunate juxtaposition of terms, since accuracy is a prime requisite of mathematical work, while speed, when demanded too early and too often from children who are trying to think, and whose reaction times vary widely, can actually become an inhibitor of accuracy.

As regards content, the most notable change is the inclusion of a planned sequence of geometric experiences beginning with the study of solid shapes in Grade 1, moving from this to the consideration of plane shapes, and then introducing at about Grade 4 the idea of a point, and various sets of points in the Euclidean plane.

A second significant innovation is the introduction of graphical representation in the very early grades with the intention of giving the pupil concrete experiences of relationships, particularly of number and quantity. Work with graphs can then become an integral part of an experience-centred curriculum, and help the child in later study of the mathematical concepts of relation and function.

In summary then, the purpose of the interim updating of the elementary mathematics course is to incorporate recent knowledge that has been gained by educators about children's learning, and about improved techniques of instruction, as well as some new content, notably in geometry and graphs.

It is hoped that the new course will be regarded as a licence to move ahead; that children and teachers will benefit from the increased freedom it allows; and that the increased responsibility placed on teachers and local supervisory people will help those concerned to improve their own competence in the teaching of mathematics to young children.

NEW CONTENT AND APPROACHES FOR MATHEMATICS IN THE ELEMENTARY SCHOOL

by Dr. H.A. Elliott
Head, Department of Mathematics
Royal Military College

Introduction

During the period from 1957 to 1964 the main impact of the revolution in mathematics appeared to have fallen on the high school course. However, teachers and parents of elementary school children began to realize that the revolution was also affecting the lower grades. In Ontario the change has officially affected Grades 7 and 8, in 1964 and 1965, but in many areas the affect had been felt in all grades.

The most talked of change early in the elementary school has been the Cuisenaire system. This system is based on the use of coloured rods as a concrete material to introduce arithmetical ideas. But in fact many other structured materials had been tried in kindergarten and Grades 1, 2 and 3 in Canada and the United Kingdom.

Two other topics most closely associated with the so-called "New Mathematics" at all levels have been the ubiquitous topic of sets and the structure of the number system. These are of course two of the items introduced in the new curriculum for Grades 7 and 8, the effects of which will first reach the high school in September next. As was the case for the new topics now being introduced into the high schools, these topics came from some of the major experiments in the U.S.A.

In some instances excessive emphasis on these approaches and topics is now receiving deserved criticism in educational journals even from some of the original proponents of the ideas. However, such criticism should not be equated with earlier criticisms from sources opposed to any change at all. It appears to be generally accepted that, where the changes were introduced

intelligently and with moderation, improvements in achievement have been obtained. Where the changes were introduced with excessive rigour and formalism at early ages difficulties were bound to occur, and have occurred.

Current re-examination of the early experiments, and new work being reported from Europe, especially the United Kingdom, suggest that now a careful evaluation of the abilities and the needs of the young pupil is being made. During 1964-65 here in Ontario a subcommittee of the Mathematics Commission undertook a study of the curriculum for kindergarten to Grade 6. This study was continued in greater depth during the summer of 1965 by a committee administered by the Commission on behalf of the Ontario Curriculum Institute, which provided the financial backing. The results of these investigations are published in MATHEMATICS, Report of the Committee considering the Mathematics K-6, Ontario Curriculum Institute, Toronto 1965.

These studies found that almost all Canadian provinces are adopting "new look" programmes in the elementary schools. Usually these are using the Canadian-published texts or one of several American series* as school texts. Several other new Canadian series are now being issued by various publishers.** All these series include more emphasis on number structure and basic concepts and less on rote learning and algorithms. Some include varying treatments of sets and functions, and also geometry. In general it may be said that a beginning is being made to teach mathematics in the elementary school and not merely arithmetic and mensuration.

---

* Patterns in Arithmetic, Chivers, Hanwell, McPherson and Smith, Holt, Rinehart and Winston, Toronto, Ontario.

Elementary School Mathematics, Brumfield et al., Addison-Wesley, Reading, Mass.

Sets, Numbers and Numerals, Cooke et al., Laidlaw Brothers, River Forrest, Ill.

Seeing Through Arithmetic, Harting et al., Gage, Toronto, Ontario.

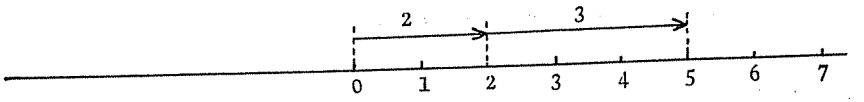
** Copp-Clark
Dent-Macmillan
Gage
Ginn
Nelson

However, even more important than the advent of these transitional or interim changes in the present curriculum is the fact that, as the committees were made increasingly aware, more far-reaching experimental studies are in progress in various countries including Canada. It is evident that a thorough study of elementary school mathematics will be made in the next few years and widely different topics will be tested at various ages. From these experiments a radically different elementary curriculum may result. Also a very different approach to the method of teaching mathematics and indeed all subjects seems likely to result from these new programmes.

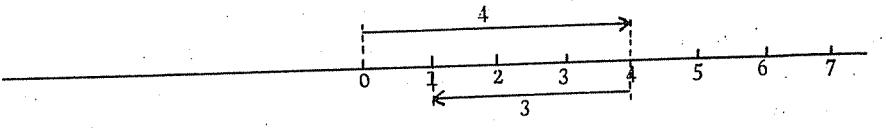
We will first look at some of the suggestions for changes in the curriculum content.

The Integers

One suggestion being made is to introduce the integers very early, probably preceding work with fractions, or at least coincident with work on fractions. In fact some teachers are finding that it is difficult to avoid the topic in class. Once the pupil has grasped the concept of number and developed ability with addition and subtraction by using blocks, rods or counters, he is introduced to the number line. On this line the natural numbers and zero are labelled and he performs addition and subtraction with its aid (a slight modification produces an addition-subtraction slide rule). For example:

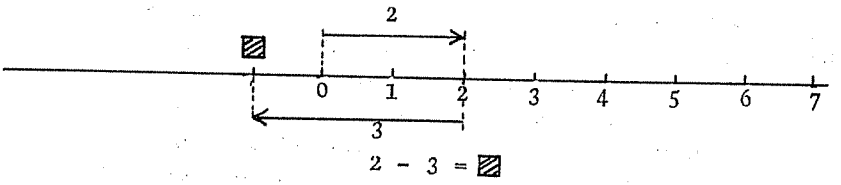


2 + 3 = 5



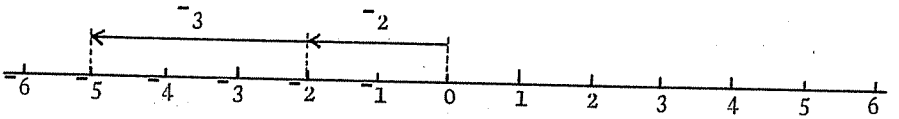
4 - 3 = 1

If the pupil is now encouraged to set his own questions he inevitably comes up with 2 - 3 = ▨ ? And he can do the problem on the line:



2 - 3 = ▨

What is the point labelled ▨ called? And why? And once the pupil has labelled the negative integers why can he not add these new numbers on the number line?



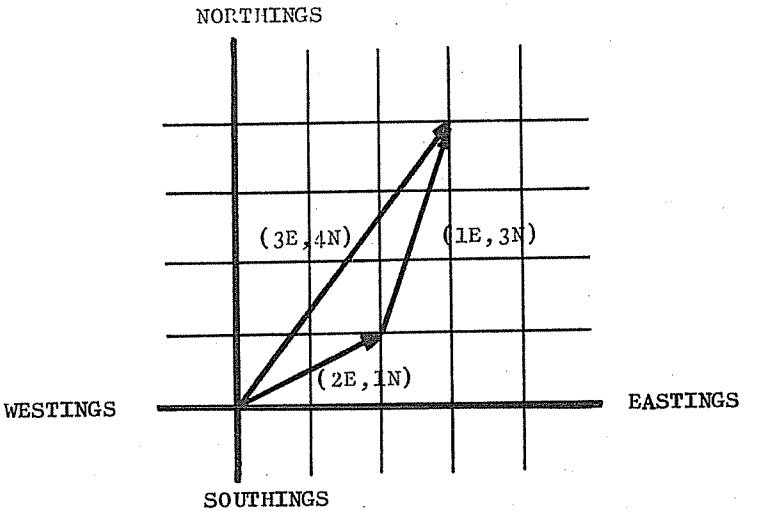
$^{-}2 + {}^{-}3 = {}^{-}5$

Of course the pupil usually does not come up with the notation $^{-}1$, $^{-}2$, etc., he speaks of numbers to the right of zero and to the left of zero; east and west of zero also seems to be a popular choice. Thus $^{+}2$ is described as 2e, and $^{-}2$ as 2w . These notations can certainly be tolerated in the early years.

Coordinates

One British program for eleven years olds who have not previously met the number line, introduced in rapid succession the number line and a rectangular coordinate system. This is rapidly followed by the introduction of two dimensional vectors as directed line segments and the coordinates are then expressed (thanks to Naval influences) as Eastings, Northings, Westings and Southings. The results are written as in the diagram.

Thus (2E, 1N) means 2 units east of the origin and 3 units north. This program claims with some justification that it is easier to teach the addition of two dimensional vectors using a triangle rule in this way than it is to teach the addition of negative numbers on the number line. The addition of two vectors is accomplished as shown below for the vectors (2E, 1N) and (1E, 3N) the result being the vector (3E, 4N) .



A year later the sufficed letters E, N, W and S are dropped and (2E, 1N) becomes (2,1) and (2W, 3S) becomes ($^{-}$2,$^{-}$3) in the more usual notations.

It can almost be a game; certainly it requires examination to see if it is more easily absorbed than some present topics and it is more useful mathematically.

Sets

Another suggestion is to use the ideas of and operations with sets in order to emphasize the meanings of concepts and operations in arithmetic. Few topics seems to arouse more heated discussion. Suppes in his series Sets and Number*

* Sets and Number, Patrick Suppes et al., Ambassador Press, Random House, New York.

introduces the braces notation and symbols for union and intersection in Grade 1. Other authors leave the symbolism and even some of the concepts until Grade 7, though almost everyone writes of sets of balls or blocks when introducing number.

It is interesting that some of the Nuffield project materials go further in the use of sets and mappings than Suppes but without any notation or symbolism. As they say 'To substitute one set of incomprehensible symbols for another, equally incomprehensible though in a different way, would represent a step backward rather than forward. To introduce new jargon which to a child is as irrelevent as the old, would not help either. At all costs we must prevent further bewilderment. All new ideas must be subjected to careful scrutiny. If they offer clarification where there was formerly confusion, if they help the child to see order and pattern within the framework of his many and varied experiences, then in all probability they will be widely welcomed.'

There seems to be no clear answer at present as to when the various topics are best introduced. This must be examined and some attempt made to make an intelligent assessment. It seems possible that a study of the union of non-disjoint sets shortly after the concept of addition has been firmly established might enhance the understanding of sets, numbers, addition and union. And similarly a contrast between multiplication and intersection may improve the appreciation of both. Such a curriculum might produce a more flexibly minded pupil in mathematics. Or would it merely produce confusion? Only careful experimentation with above and below average pupils will give the answer--if a clear-cut answer exists. But some attempt should be made to find the answer.

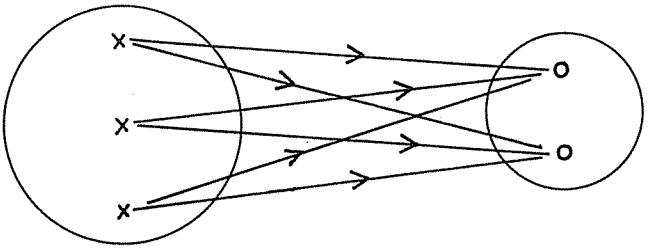
The previous paragraph has raised another bone of contention. Should "all" the properties of a number be taught at once? Cuisenaire enthusiasts say yes and claim success in showing the example that

$$6 = 3 + 3 = 4 + 2 = 5 + 1 = 2 + 2 + 2 = 2 + 3 + 1 = 6 + 0 \text{ etc.}$$

And they say also that

$$6 = 3 \times 2 = 6 \times 1 = 2 \times 3 = 1 \times 6$$

must be included. Others suggest that the traditional approach, where multiplication is delayed a year or more after the introduction of addition, is justified by the fact that multiplication is a distinct operation and not merely repeated addition. They would agree that $6 = 2 + 2 + 2$ means that 6 is three 2's in colloquial language, but this is not in principle the same as $6 = 3 \times 2$ or 2×3. Some even go so far as to suggest introducing multiplication by pairing elements in two different sets and counting the ordered pairs--by using diagrams, not using ordered pair notation!



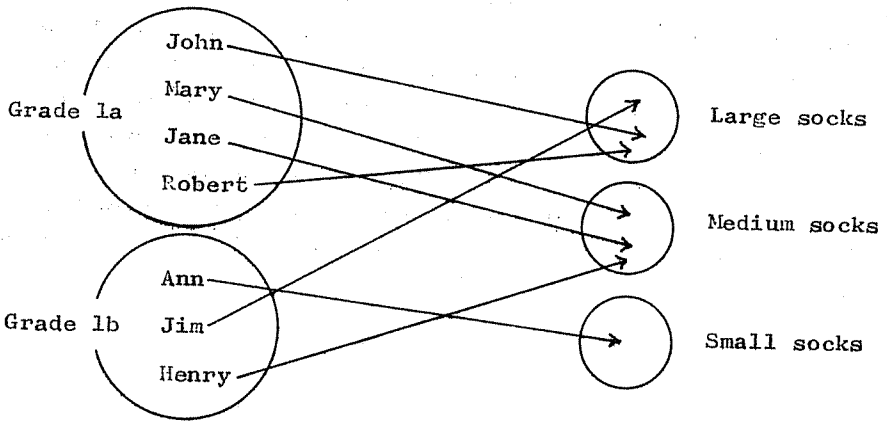
$3 \times 2 = 6$

The questions here seem to be: (1) which approach is easier at the grade level where it should be taught? (2) which helps most in later work?

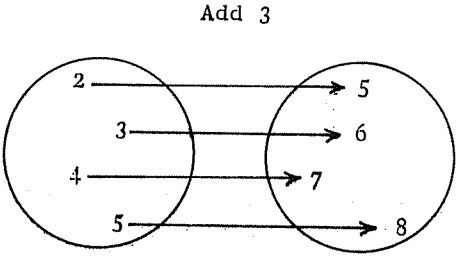
Mappings, Relations, and Functions

Mappings, relations and functions are fundamental to most mathematics at higher levels and, in fact, do exist in the earliest of mathematics and in many situations in real life. One American series introduces function by means of a "function machine" approach in Grade 5, but the British are being very

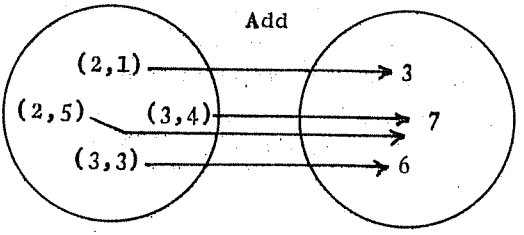
adventurous in this area. Nine year old pupils are drawing graphs and mapping diagrams, and giving written explanations of them in their own words. For example, a study of pupils and their sock sizes produced results like the following:



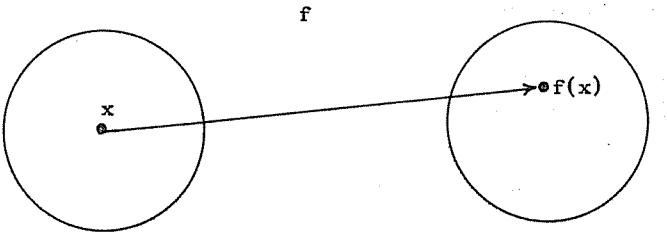
It is even suggested in the Nuffield project booklets that addition of 3 to other numbers may be mapped (at about age 6 or 7) as:



and that the pupils may even use:



and so avoid having to use the equal sign (=) before it can be reasonably carefully defined. It appears that this approach has been used in one area.



What a different view these pupils would take of the mapping $f: x \to f(x)$ compared with that of our present Grade 13 and first year university students!

Geometry

While most of the new programmes introduced at present in North America bring algebra forward into early grades of the elementary schools, most of them leave geometry until Grade 6 or 7 or even later. Suppes is an exception here as in many other ways, but even his series brings forward mainly the traditional Euclidean constructions, though some little attention is given to such topics as convexity. In the high schools also the N. American experiments have adjusted Euclid to set language but have made no radical changes. Europeans have been much more adventurous especially in high school, where admittedly they treat only a selected group of students. There now appears to be increasing activity at the elementary grades and again some English work seems worthy of attention. Emphasis is placed on three dimensional work with models as well as on two dimensional pictures. Attention is also focussed on patterns formed by several polygons and polyhedra. Experimental work, by the child, in building pin-jointed frameworks emphasizes the unique rigidity of the triangle and the subdivision of a polygon into triangles.

Non-metrical properties are also studied. In some instances experiments in the U.S.A. seem to be picking up this work.*

The new approach, which requires more attention to geometry, is planned to lead to a vector and transformation treatment of high school geometry. This means the deletion of Euclidean deductive geometry as it is now practiced in our high schools but the properties of geometrical figures are retained; only the method of obtaining the properties is changed. The new texts**prepared in New Brunswick advance part way in this new direction. An article*** in the Mathematical Gazette summarizes these views:

> "Construction is the basic experience for understanding spatial relations and free play with 3-dimensional shapes and pliable material precedes analysis..... In two dimensions, pattern is the dominant theme. Just as actual handling, rearranging and comparison of things is the essential feature in learning about number, so geometrical transformations and their implications are learned through movement. Reflection, translation, rotation and stretch are in the minds of teachers as basic forms of movement.
>
> The value of these transformations is brought out very strongly when they are used to make patterns... Work which stirs the imagination as this does, and gives deep satisfaction with what has been achieved through an understanding of form, is creating a liking for mathematics among Primary children which should carry them forward into the Secondary stage with a keen desire for more."

Some purists may say that this is art not geometry, but a careful examination of this approach and its development in high school, is very necessary. No one who has read the unit on transformations in the plane in the new Grade 13 mathematics course, or studied vectors or linear algebra at university, can doubt the desirability of such an approach to geometry from Grade 1 onwards.

---

* Geometry Concepts in Grades K-3, Virginia Felder, Arithmetic Teacher, Vol. 12, No. 5, May 1965.

** Mathematics 7,8,9 (experimental edition), Crawford, Rosenberg, Glover; W.J. Gage Co., Toronto (3 volumes).

*** What can be taught in Primary Schools, E.M. Williams, Mathematical Gazette (London) Vol. XLVII No. 36 December 1963.

Perhaps it is true as someone has said, that a child in Grade 1 has more appreciation of three dimensional geometry than a university student after three years of high school Euclid and two years of analytic geometry and trigonometry--mathematically he lives in flat land!

Approach and Method

Although we have digressed once or twice into approach and method in the previous paragraphs, we have only done so to clarify the content of the curriculum under discussion. As the recent updating bulletin from the Ontario Department of Education indicates the whole approach to mathematics in school needs extensive revision. This also is very evident in the work of the Nuffield project and in the experiments in France. The booklet I do - I understand from the Nuffield project sums up this new attitude in its title. The approach must be child centred and every individual child must be actively engaged both mentally and physically in the learning process all the time. Listening, except in small doses and in an almost individual basis, to the teacher is not involvement or activity. The teacher must create the environment in which the child can develop and discover the principles of mathematics and science, and in which the child wishes to express himself and talk about his work and achievements. It is amazing how much language facility, both verbal and written, exists in connection with mathematics and science if the child has to explain what he did or why he thinks something has happened as it did.

The emphasis in the new programmes in mathematics as in science, language, social studies and indeed all subjects is in individual work or work in small groups. Only in this way can the child develop normally and not merely be moulded into a parotting ventriloquist's doll who merely repeats what his master says. The team from the Ontario Curriculum Institute which was sent to visit the Nuffield project schools in May 1966 was most impressed to see that this individual approach was quite practicable with classes of 35, 40 or more and in

old and often inadequate buildings. It also appeared to work well with severely underprivileged and handicapped children; in fact the difference between the results of the traditional and new approaches is more marked with such children than with children from privileged areas and homes.

Most of the individual or group instruction is carried on by means of assignment cards. It is quite typical for some members of a class to be studying English, others History, others Art and others mathematics or Science all at the same time. Under such circumstances it is of course impossible for the teacher to prepare a "lesson" for the next day. The children are setting the lessons very largely on their own from day to day. The teacher must have prepared ahead, at least in outline, the progress and some key assignment cards for several weeks. The day by day task of the teacher is to adapt and modify the detail of the study for the various groups of children and individuals. This is often done right on the spot by direct discussion in which the child takes a major part. As is stated in one of the Nuffield reports (unpublished to date) "Letting them work for themselves does give time to go round and sort out at least some individual problems, and most children respond to the trust and excitement of making their own disoveries instead of having to 'take it from teacher'."

In this new approach to teaching method the quiet classroom cannot exist. Even between adults discussion in silence is impossible (unless it is done in writing--a painfully slow and inadequate method) and between children arguments often become heated. However in spite of the lack of silence, the noise level in the classroom is quite bearable. The most interesting aspect of the classroom under such conditions is the power of concentration of most of the children most of the time, they seem wholly absorbed in their work. The teacher may leave the classroom and the noise level does not change. Visitors may enter and the children hardly look up from their work. The teacher may be closely involved in the work of one group of children and the others are working just as hard. Many disciplinary problems due to boredom and inattention appear to vanish under such conditions.

It is also true that the tidy classroom of thirty desks neatly arranged in 5 columns of six rows does not exist in such novel situations. The desks, or better still small tables must be arranged in suitable groups, or individually, for the various assignment situations which arise. Informal groupings of different arrangements seem to be best--though of course they may change from day to day. Preferably different areas of the room are allocated priority in the different subjects so that mathematics and means of measurement and experiment will occupy one corner. In this area will be various sets of concrete and structured materials for number concepts, shapes, models and drawings for geometry, materials for making models and graphs, and measuring instruments of various kinds. Note that class sets of rods or other teaching aids are not needed under such circumstances--rather a large variety of aids such as Cuisenaire rods, Unifix blocks, Stern equipments, Dienes structured blocks, and even bottle caps, small coloured blocks and tiles are available in turn to each child if he wishes to use them.

Conclusion

Obviously with the impact of both new content and new approach in our school mathematics major changes in the knowledge and attitude of the child will result. Hopefully, and this seems to be occurring, we will produce generations of children who no longer hate mathematics but instead enjoy the subject. However, to encourage this the methods of testing and evaluation have to be changed. It appears that Piaget and his colleagues at Geneva are cooperating with the Nuffield project in Great Britain on the problem of testing and evaluation. Here in Ontario the Ontario Institute for Studies in Education will be actively engaged in co-operation with the new experiments. Such new tests will avoid "the craze for speed" in testing procedures, and test competence in thought and the grasp of principles. However in spite of any external tests the only true judge of the pupil's attainment, except the pupil himself, will be the responsible classroom teacher. With the new approach methods and with the new content it is hoped that each teacher

will be able to give a valid honest opinion of the child's progress.

What of the future in this experimental environment? It is too early to know in detail. Most of the teachers actively engaged in the project are convinced of its desirability and worth. Others remain sceptical. Certainly some topics in the present curriculum will have to be deleted or postponed but care will be needed so that only the superfluous is deleted and the less essential is postponed. In making decisions on this we should bear in mind the following statement of Dr. Bondi, a leading astronomer.

> "The particular significance of this for my topic today (Mathematics, the Universities and Social Change) is that it is an acknowledge fact that mathematics becomes more difficult to learn the older one gets. A branch of mathematics that has not been mastered by the age of twenty-five is not very likely to be mastered later. Perhaps newer teaching methods will help us, and no doubt necessity will be a great spur. If we expect, as I think we all must, that in future most people will have to change jobs several times during their working lives, then mathematics presents a rather special problem.
>
> From my argument it seems that we must teach mathematics to many more people than we are doing at present. I don't think the situation is quite so desperate because the particular difficulties about learning mathematics later in life seem to lie in fairly well identifiable directions. Manipulative handling is a terribly important thing for the working mathematician, but I do not think there is any particular difficulty in learning this later in life. No, the difficulty lies in appreciating highly abstract and strange ideas. It is the fact of abstraction and of appreciating abstract arguments that seems to suffer, particularly as we advance beyond our twenties. And this is a much smaller part of the subject. Of course, we do not know how many of these things can be "put on ice". Suppose you teach a youngster highly abstract ideas, so that he really

gets hold of them, he then doesn't use them, or perhaps even think about them, for twenty or thirty years, would he then be able to get used to them again and to work with them far more quickly than somebody who had not been taught these ideas earlier in his life?
I don't know the answer to this, but I rather suspect that if you really got hold of the particular abstract ideas, it is a little like having learned to ride a bicycle or to swim. If you are out of practice for twenty years, it will, nevertheless, come back to you rather quickly. So I hope that it may be possible to lessen the solidifying, rigidifying effects of the importance of mathematics for a changing pattern of tasks during a single working life by a grounding in abstract mathematics at an early age."

Although the above remarks were addressed mostly at high school and university level, we must remember that in elementary school we are supposedly <u>educating children in ideas</u> not merely training inefficient even if accurate arithmetic computers.

IMPRESSIONS OF MATHEMATICS EDUCATION IN ELEMENTARY SCHOOLS IN ENGLAND

by J.N.C. Sharp,
Mathematics Consultant,
Etobicoke Township Board of Education

During the last two weeks in May, through the cooperation of the Etobicoke Board of Education and the Ontario Curriculum Institute, I visited several elementary and secondary schools in the southern part of England. The arrangements for the trip in England were made by the Schools Council and were designed to give our group an insight into the Mathematics being taught experimentally at different levels by three different organizations. Here I wish to report only on the work being done in the elementary schools by the Nuffield Project.

The Nuffield Project is primarily concerned with the work done in elementary schools and most of the elementary schools that we visited were those that were under the direction of The Nuffield Project of which Dr. Geoffrey Matthews is the Director. This project is designed to emphasize the discovery approach to Mathematics and is very much student centred. It has been in operation for about three years and the present plans are that the experimental work should be completed by 1969.

There are two things to emphasize in designing a new course. One is the content of material, and the other is methodology. These two are very closely related and our observations indicated this fact. Much of the instruction in the primary grades is done via a laboratory method. The students are separated into groups anywhere from two to five in a group, and they work on projects as a group. These projects integrate science with mathematics. The children are given freedom to explore further into any project, and there did not seem to be any time limit placed upon them for

the completion of a given project. When one project was completed, they reported their work to the teacher and immediately began on another project. It occurred to me that a great deal of preparation is necessary for the teacher. However, during the laboratory period, the teacher was free to discuss things with us, and the children worked on their own. This method of approach did emphasize the discovery process and, in many cases, the children were doing work far beyond the work we do in Ontario at the corresponding age level. The class would have probably thirty-five or forty projects which would be passed around from group to group. They stayed in their own age group, and at the end of the year went up to the next level. For each project, a card was used which would indicate the equipment which must be taken from the supplies. It also indicated a few questions that would be asked and the rest was left to the ingenuity of the pupil. The alert pupil would do the project in shorter time than others, and would probably have shorter methods to complete the project. It did, however, allow the pupils to proceed at their own individual rate. This discovery approach integrates very well with the inductive process and it was evident that at a later stage many of these projects could be extended and made the basis for generalizations and inductive speculation.

In one infant school in Bristol, the class was called a family unit. The pupils in the class were equally divided into ages five, six and seven. This was a type of vertical streaming. At the end of the year, the seven year olds move on to the next group, and a new group of five year olds would be entered. This allowed for a fair amount of continuity from year to year where two-thirds of the pupils each year were "repeats" from the previous year. In this method of teaching, it was not necessary to have full class sets of any of the visual aids that were used or class sets of the text books. It also allows for a much greater variety of equipment that can be stored in limited storage space. Much of the equipment is homemade and the cost is negligible. The ingenuity of teachers is necessary to find situations within the child's experience where simple relationships exist and then set the stage for the child to study them.

If you were to ask a teacher formerly whether their students were better in problems or computation, their reply would undoubtedly be "computation". The feeling is that with less emphasis on computation,it is put in its proper prospective and the children learn to attack problems in a much more meaningful way. Computation is secondary. It was gratifying to see, in many cases, the reading ability, and the ability to discover methods of solving problems among children ages five, six and seven. Students learn to solve problems by practice and then must be allowed to try methods on their own.

In most of the cities we visited in England, teacher centres have been established which have a sample of all the visual aids and equipment used in the schools. The teachers come in groups, usually one or two from each school, during school time to take instruction in the use of the materials and the methods involved. There is a great deal of exchanging of ideas -- something which I think we often keep to ourselves. Further instruction is given on the B.B.C. by the use of films.

In this type of setting, the regimentation of pupils is decreased, the individual differences are catered to, and the learning is a student centred situation. Although in each room there is considerable talking by the pupils, no one appeared to be disorderly or wasting time. Some of the projects even involved the students going out into the courtyard which they are free to do. However, at no time did they take advantage of this situation.

The type of teaching discussed above is becoming more widespread each year. It is not a method that our teachers would adopt overnight. We need to experiment with new methods slowly; we need to discuss these with our fellow teachers and co-ordinators; and above all, we need to integrate these ideas slowly with our regular teaching. We must try to create attitudes in our classes and not try to teach the pupils that the factual knowledge that we convey to them is final and unalterable. Our teachers must realize that the attitudes of our

society are changing and progressing and we cannot stand still but we must continually examine our curriculum content and our teaching methods.

MISSING LINKS

by

W.W. Sawyer
Department of Mathematics, University of Toronto

What decides whether a subject is pleasant or unpleasant to learn? Many factors of course lie entirely outside the subject matter. A child who likes a teacher may find every subject taught by that teacher enthralling. But it is also possible for a child to be attracted or repelled by the nature of a subject itself.

Most children, and indeed most adults, find subjects repellent which involve great attention to apparently arbitrary detail. In English, spelling offers many examples. If all spelling were phonetic (as in some languages), if all words behaved like h-a-t hat, lessons in reading and spelling would become very much pleasanter. The English language scores, of course, in its treatment of gender, with men masculine, women feminine and things neuter - in contrast to French, and German, where the student spends an inordinate amount of time trying to remember what sex to accord to objects such as feet, hands, doors, houses and trees.

In short, most people like a subject where only a few ideas have to be memorized and everything else follows logically from these.

Now arithmetic is such a subject par excellence. It is certainly much simpler than spelling. Only ten symbols, 0 to 9, have to be memorized as against the 26 letters of the alphabet. These symbols are combined to form larger numbers,

such as 216, in a simple and systematic way. There is nothing in arithmetic to correspond to the uncertainty in reading whether C means the sound S or the sound K. The other ideas involved in arithmetic - such as adding, subtracting, multiplying, dividing, equality, larger or smaller, and fractions - are really very few in number. How then does it come about that in most countries of the world arithmetic has the reputation of being the subject that only the most brillian students can cope with?

Students who do well in arithmetic and who continue to do well in other branches of mathematics almost invariably follow the procedure described above. They grasp a few ideas and see that the rest follows from these. But textbooks in many countries do not present it this way. Rather it is a mystery which can only be coped with by memorizing innumerable rules - which of course few children are capable of doing correctly. In this way a simple and attractive subject is artificially changed into a difficult and repulsive one, and generations of teachers grow up in the belief that it is so.

I start then from the assumption that all children should do at least as well in arithmetic as in any other subject, and try to examine where things start to go wrong.

One common reason is the attempt to build on an idea that is not there. I remember in my own schooldays a music teacher writing a series of sol-fa symbols on the board and saying, "Sing that". I had no idea which note corresponded to which symbol. Nowadays children are taught some simple little song, which rattles between (say) doh and so. Then, when this tune is thoroughly familiar, they are told the names of the notes used. In this way the symbols become meaningful.

Every idea, however obvious it may seem to an adult, has to be learnt by a child. I once watched an intelligent little girl, eighteen months old, spend half-an-hour trying to put a half-dollar into a tiny bottle, the mouth of which was perhaps a quarter of an inch across. It was obvious (to adults) the coin would never go in. It is only by many such experiences that a child comes to associate any meaning with a sentence

such as "That is too large". Without these experiences, "too large" remains as mysterious to the child as sol-fa remained to me in school singing lessons.

One of the most important things for a teacher to know is what ideas each child in the class has succeeded in forming. To take a fairly advanced example, it is quite common to find children being taught rules for work with fractions without any check being made that each child understands what fractions represent. I would never teach formal work with fractions until a child had demonstrated, in several different ways, that the concept of fraction had been grasped - "Please give me three-fifths of this piece of paper; please show me three-fifths of an inch on the ruler; please fill this jar three-fifths full of water." The great value of apparatus in a classroom is not only that it teaches the children ideas but also that it allows the teacher to observe, from the children's actions, which ideas each child has or has not yet.

The main lesson to be learnt from the work of Piaget and similar investigators is that one should never take it for granted that any concept, however "obvious", is in fact possessed by children. To take a very well known example, children do not necessarily realize that you can neither increase nor decrease the number of objects in a collection by rearranging the objects.

Some people have formed the misconception that Piaget's work means that we should not teach children arithmetic until they are older. But there is no miraculous process by which the meaning of an arithmetical process is suddenly revealed to a child when his birthday arrives. We do not grow wiser simply by growing older; we grow wiser by learning from experience. Our task is to see that children have a wealth of experiences from which they can extract the ideas they need for arithmetic and for other branches of mathematics.*

* An admirable source of suggestions for activities and experiences, based on the work of many teachers, is the booklet "Mathematics in Primary Schools", obtainable from Her Majesty's Stationery Office, York House, Kingsway, London W.C.2. Available in Canada through the British Information Services, 200 University Ave., Toronto 1, Ontario.

Failure to establish ideas and to build on them covers in fact the great majority of things that can go wrong in our attempts to teach arithmetic. It may be instructive to consider the question, "What is 7 x 8 ?" - which many children cannot answer correctly - and see what ideas are involved.

First of all there is the meaning of the question. A child obviously cannot give the answer (except by rote) without understanding the question. One way of testing the children's understanding, which has found favour on both sides of the Atlantic, is to ask <u>them</u> to compose a problem to illustrate 7 x 8. If a child suggests "Each of 7 cats had 8 kittens; how many kittens altogether?" this would seem to show that the meaning of multiplication was appreciated. Or we can ask a child to <u>demonstrate</u> the meaning. A useful piece of apparatus - well known, but not found in all classrooms - is a board with 100 hooks on it, arranged in a square - 10 rows of 10 hooks. If a child hangs 7 rows of 8 rings on these hooks, that shows understanding of the question. (So would 8 rows of 7 rings, or 7 groups of 8 rings arranged in any manner.) In a certain sense, the demonstration even answers the question "What is 7 x 8 ?" "It is the number of rings on that board." But this is not quite the form of the answer that most teachers are hoping for. So we clarify our intention by asking, "What is the usual, standard way of specifying a number?" "It is by giving it as a certain number of tens and a certain number of units." "Can you arrange the rings on the board so as to show the number in tens and units?" The student rearranges the rings, and finds they form 5 rows of ten and 6 units. The answer therefore is 56.

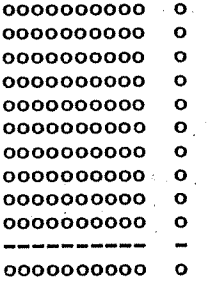
Thus two basic ideas (understanding the meanings of multiplication and numeration) and the manipulation of suitable apparatus is sufficient to produce the answer. Admittedly the method is primitive, but understanding of arithmetic often means simply ability to return to the primitive. Further it is most important for students to realise that when you have forgotten a result in arithmetic,

you can work it out for yourself from the basic ideas.

An interesting piece of research could be undertaken by teachers of arithmetic, namely, to record incorrect answers to questions, and see how often these indicate that children have no idea at all of what they are talking about. For example, suppose a child says that 4 x 7 is 42. In terms of our apparatus, 4 x 7 means 4 rows of 7 rings. This is clearly less than 4 complete rows of 10 rings, so the answer must be less than 40 . A mathematician does not simply memorize exact answers. If he makes a slip, he recognizes it by some such thought as, "No, it couldn't be that; that's too big." Being able to picture the question automatically saves one from making certain errors.

It is more important that children should learn to think out an answer slowly than to parrot an answer quickly. In this century, if you really want the answer to a question in arithmetic, you use a calculating machine. The job of human beings is to <u>understand</u>, so that they will recognize when a machine has gone wrong (which machines do) and produced a ridiculous answer.

Of course, we do not wish our students to stay with primitive methods (though, when in doubt, they must always be able to go back to the primitive). When I was about 8 years old, a boy called Sammy Wilson sat next to me in school. If Sammy wanted 11 times 11 , he would write out 11 groups of 11 ones, and count them. This was not only slow; he always made a mistake in his counting and got an answer such as 117. Now he had learnt something; he understood the meaning of 11 times 11 . If the teacher had cared to investigate <u>why</u> he produced answers like 11 x 11 is 117 (which she did not), she would have discovered what he was doing and could have taken him forward from there. She might, for instance, have pointed out that he could arrange his 11 rows of 11 like this



There are 10 rows above the line, and one row below, making eleven rows. In each row there are ten objects before the space and one object after it. It is not necessary to count. We can see a solid block of 100 objects, 10 rows of 10. (This picture of 100 should be made extremely familiar to all students.) We can see two groups of ten in the picture - one horizontal and one vertical; finally there is a single unit in the bottom right-hand corner. In all, one hundred, two tens and one unit, or 121. Moreover this picture could lead very easily to the usual procedure for multiplication. Above the line we see one hundred and one ten, making 110; below the line, one ten and one unit, making 11. The whole array thus represents

$$\begin{array}{r} 110 \\ \underline{\ 11} \\ 121 \end{array}$$

which is what we should write if we were doing 11 x 11 by a standard formal method.

It is not possible in an article of this length to sketch the entire process by which students can learn to think their way to the methods and results of arithmetic. Perhaps the examples above will serve as a sample to show that such a process is possible.

GEOMETRY IN THE ELEMENTARY SCHOOLS

by

I.J. Allen,

Supervisor of Mathematics,
North York Board of Education.

Geometry is the science that studies the size and shape of objects in our world. Awareness of such concepts is basic to life, but yet this subject has been sadly neglected in our curriculum.

Geometry, as most of us remember it from our secondary school days, consisted of proving relationships which were often obvious, by using a long and tedious method which was backed by petty rigour. The argument for this was that it taught the student to reason logically, and to accept only facts which could be proven. For the able student I believe the aim was achieved. For the average or poorer student I believe it was instrumental in destroying his mathematical common sense. Thus some students gave up completely while others would write down utter nonsense for a solution, accompanied by the number of some illchosen, misunderstood theorem.

Many students were promoted from such a course only because they memorized a few standard examination theorems, were well drilled in the basic constructions, and through the goodness of heart of the teacher.

When I encourage the teaching of Geometry I am not advocating that we bring this deductive geometry into the elementary school. However, there are many ways in which we can increase a student's awareness of the size and shape of objects in the world about him. In achieving such awareness he will certainly deduce practical facts, and in his attempt to communicate with others his language will also be enriched.

Even although we live in a three dimensional world, most of us have received little or no training in three dimensional geometry. It may be due to the difficulty in perspective drawing and our enslavement to the textbook, but nevertheless, our mathematical experiences do not allow us to describe many simple objects in a meaningful manner.

Although some of the modern mathematics programmes have gone overboard with vocabulary, I believe there are some words associated with shapes that a student should be exposed to. Let me give the following example.

While visiting a kindergarten where a good deal of practical useful learning takes place, I noticed that when a child was asked to get a rectangle he would get a block like that in Figure 1. If asked to get a square the correct response was to get a block like the one in Figure 2.

Figure 1.

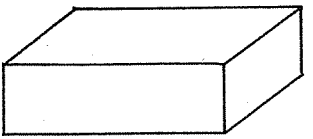
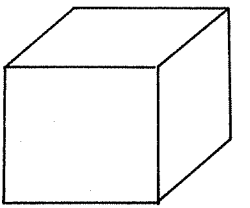


Figure 2.



Now I am not going to suggest that these kiddies be given a formal definition for these objects that could be repeated on a university exam. Nor am I critical of the teacher, who was a product of our system, for calling these squares and rectangles, because in her formal education she had not come across the proper name for the objects. However, these are very common objects with specific shapes and the problem remains, "what should we call them?" Perhaps at the Kindergarten level they should simply be called "blocks", but a fairly easy name that seems to be gaining favour is "cuboid".

After establishing a name for these objects in Figures 1

and 2, students will realize that a cuboid is quite different from a rectangle. They may note that it has 6 faces. In each case four are rectangular and two are square.

As students proceed through the early primary grades they should be given ample opportunity to handle models such as shown in Figures 1 to 11 and discuss the shapes of the faces, and count the edges and the corners. Some figures have flat or plane surfaces while others are curved. Some will roll while others will not. Commonly found objects with the same shape as these, such as tin cans, ice cream cones, boxes, oranges should be discussed and handled.

After students have had ample opportunity to discuss the shapes in their own words they may be provided with correct names for them. For example, the models in Figures 3, 4, 5 and 6 are prisms since they all have an identical top and bottom with parallelograms on all other faces. The top and bottom are called bases and the other faces are called lateral surfaces.

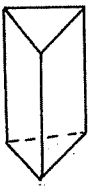


Fig. 3

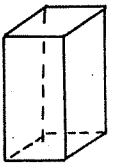


Fig. 4

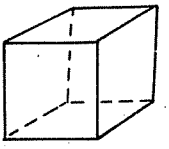


Fig. 5

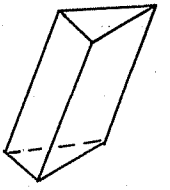


Fig. 6

Since Figure 3 has triangular bases it is a triangular prism. Similarly Figures 4 and 5 are rectangular prisms or cuboids. Since they have square bases they are also square prisms. Figure 5 is commonly called a cube since <u>all</u> faces are squares. Each model in Figures 3, 4 and 5 is a right prism. There are other forms of prisms, e.g. Figure 6 shows

an oblique triangular prism (usually called a tetrahedron). It is triangular because the bases are triangles. It is oblique because the lateral sides are parallelograms which are not rectangles.

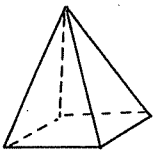


Fig. 7

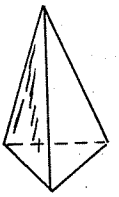


Fig. 8

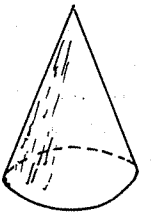


Fig. 9

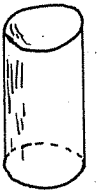


Fig. 10



Fig. 11

Figures 7 and 8 represent pyramids. Students easily recognize these because of their flat bases and triangular lateral surfaces meeting at a common point. These are also named according to their bases and thus Figure 7 shows a square pyramid and Figure 8 a triangular pyramid.

The models in Figures 9, 10 and 11 are easily recognized as a cone, a cylinder and a sphere, although it is somewhat difficult to define them at an early stage.

Once students learn to recognize these basic shapes they may be encouraged to envisage them in three different ways. For example, the cube may be a solid piece of wood. This might be called a solid cube or a cubic solid. Secondly, we may think of it as an empty box. This might be called a cubic shell or the shell of a cube. It does, however, maintain the essential features of a cube. The third type of cube might be called a cubic skeleton or the skeleton of a cube. This would consist of a model showing only the edges, but they are sufficient to outline the basic shape of the cube. After students

become familiar with these ideas they should be encouraged to look for them in the real world around them. For example a fence post is a solid cylinder, a funnel is the shell of a cone, while a T.V. tower might be a skeleton of a triangular prism.

After recognizing shapes, students should be encouraged to construct them. Even before students can handle pencils, rulers or compasses, they can construct triangles, squares, rectangles, pentagons, hexagons, etc., by using elastics and nailboards as shown in Figures 12 and 13. They can also build these simple figures from meccano sets or similar materials made from wood or plastic as shown in Figures 14 and 15. Not only does this develop their vocabulary and dexterity but they learn that the triangle is the only rigid polygon. Thus they can look for the triangle and see its constant use in the construction of playground equipment, bridges, hydro towers, and many other structures in a community.

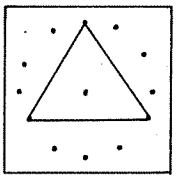


Fig. 12

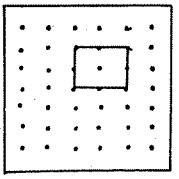


Fig. 13

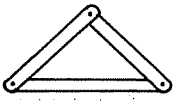


Fig. 14

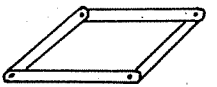


Fig. 15

After constructing two dimensional figures some three dimensional skeletons as shown in Figures 16 and 17 may be constructed using drinking straws connected by pipe cleaners or string. As a student constructs these - and they are very easy to do - he will learn the number of straws required for edges, the number of pipe cleaners required for corners and how these are to be arranged.

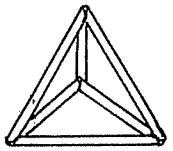


Fig. 16

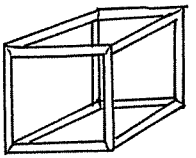


Fig. 17

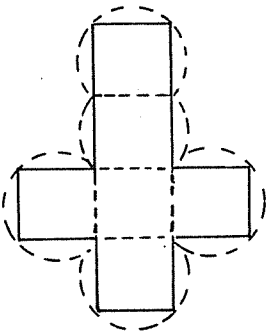


Fig. 18

Three dimensional shells can be constructed from heavy construction paper or bristol board. For example, the cube may be constructed by taping together six individual squares which were cut from bristol board. On the other hand, if the paper was marked as shown in Figure 18 it could be folded and the tabs cemented to form a cube. If students have access to solid models they can gain valuable experience by designing and making similar models from paper.

So far I have referred mainly to shapes of objects and little reference has been made to size. We realize, however, that if a door is to be constructed for a doorway it must not only be the correct shape, but must also be the correct size. This would lead to measurement. I will not attempt to discuss measurement very fully in this article. However, I believe students should have practice using arbitrary units to measure length, area and volume, before using formal units. For example, how many steps must you take to walk the length and the width of the gymnasium? How many pieces of construction paper does it take to cover a bulletin board? How many cupfuls of sand does it take to fill two different containers? While using arbitrary units students will gain appreciation of the concepts of length, area and volume and will be ready for formal units when they are introduced.

If students are involved in an active programme where they handle, discuss and make different objects I am sure they will increase their awareness of size and shape and gain a better understanding of Mathematics in general.

THE ELEMENTARY TEACHER AND THE INTERIM-UPDATING

by Douglas H. Crawford
Queen's University

Introduction

In any educational reform affecting the classroom, there needs to be cooperation between the initiators of the reform and the teachers who put it into effect. Neither can afford to ignore or manipulate the other. In my view, the classroom teacher is the crucial person in change, and so it is imperative that he be consulted, and made fully aware of the changes being contemplated.

Basic Implications

What is the significance for the teacher of the interim-updating of the K-6 mathematics curriculum?

I can see two immediate implications. They are these:

First, the elementary teacher is being asked to teach mathematics, not just arithmetic. The inclusion of a substantial amount of geometry, of work on graphs, and of emphasis on structural aspects of mathematics generally, means that the context of ideas in which the teaching is to take place is significantly wider and deeper. To teach adequately, will require a more-up-to-date understanding of what mathematics is and how its different aspects interrelate with each other.

Secondly, the definite emphasis on an experience-centred curriculum, the use of concrete material, and individual learning means that many teachers will need to revise their views on classroom organization and "lock-step" teaching and employ more flexible and varied methods in guiding the learn-

ing of their pupils.

Taken together these probably demand in many cases a re-education of the teacher, for they imply a deeper knowledge and understanding both of mathematics and the psychology of learning it.

Attitudes of the Teacher

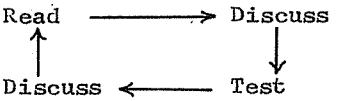
Can the average teacher cope with these demands? Probably the key factor in whether the answer is yes, or no, is the attitude to change of the individual teacher. Is he dissatisfied with his teaching? Does he strive for improvement? Is he a passive work-to-rule type? Is he in a school and system where the climate encourages change? If he is, he is likely to receive sympathetic help. If he isn't, then he may have to face the situation more or less alone.

All of us need encouragement and help when we have to grapple with new ideas and ways of operating. So I suggest that the first thing the teacher who has to face these changes should do is look for help from his principal, subject consultant, other teachers and friends - or me, if he can't find anyone else!

Now there are perhaps teachers among us who may not be convinced of the value of the suggested changes. And this is a good thing, for this should force those of us who advocate change to justify as fully as possible the recommendations we propose. Any change which does not stand the test of time in the classroom must surely be modified. The best proof of the value of recommendations is to see them working successfully under typical - or preferably poor - conditions. Too many experimental programmes - in the U.S.A. for example - rely on exceptional teachers and students.

Some Suggestions

How can the teacher prepare for teaching this updated curriculum? Here are a few suggestions. Essentially they form a kind of feedback loop, whose elements are



What should the teacher read? First, and foremost, the references at the back of Curriculum P1, J1. Of these references, the first - "Mathematics in Primary Schools" is a real gem. Three features in particular about it impress me greatly. The first is that the insistence on active learning is soundly based on research, which is cited in enough detail to be recognized as arithmetic. The second is the wealth of specific detail about how children have been observed to learn using this approach. The third is the wisdom of realizing that the best way to discover the value of discovery and experimental approaches in mathematics teaching is to discover some mathematics for yourself!

As you read this "pamphlet" (!) yourself, you will find many of your questions anticipated, and can even try the discovery approach on yourself by consulting Appendix 3. I am prepared to bet that this one book will do more to change teachers' attitudes than any other.

I hope you will read the next reference (the K-6 Study Report) fully also. It is a well set out, well-reasoned document. Naturally, it looks to the future, so don't feel over-awed by it.

In the same category, I would place "Visions in Elementary Mathematics" by W. W. Sawyer. As its title indicates, it emphasises the device of visualizing problems and ideas in mathematics. It makes absorbing reading, and is a wonderful source of excellent ideas for teaching the up-dated elementary course.

I also recommend "The New Look in Elementary Mathematics" issued by the C.E.A. Research and Information Division, and an article by myself to be published in "The Canadian School Journal" entitled "Practical Aspects of the New Mathematics". Taken together, these articles give a reasonable overall glimpse of the main developments in content here in Ontario.

Secondly, I would suggest that you view both the films

"Maths Alive" and "I do and I Understand". These films speak eloquently of the kind of classroom situation which may become the rule rather than the exception. A third film, made in Barrie by ETV and featuring good discovery and discussion teaching methods should also be seen.

Additionally, I believe that a series of 3 minute 8 mm. loop films of a discovery nature made by the National Film Board of Canada can prove helpful both in re-orientating your own thinking about mathematics, and providing situations through which pupils can experience and discover mathematics in a new way. Further details of these films can be obtained from the N.F.B. or myself.

Next winter the Department of Education will be screening a companion TV series (at the Grade 8 level) to the successful grade 7 series, and the emphasis will again be on discovery and pupil participation methods.

To sum up, then, may I suggest that you read, discuss, view, think, ask and generally put the discovery approach into effect. Inquire about the experience-centred approach which is recommended in the interim-updating. Above all, insist on seeing these approaches in action in ordinary situations. It is clear that one main means of ensuring this is to have organized in-service seminars. Press for such seminars if you feel in need of them.

In closing, let me emphasize that in my view:

1. A teacher should believe in the methods he uses. Thus, my hope is that you will become convinced that the "active learning" approach is basically valid. This does <u>not</u> mean abandoning didactive or expository methods, but rather putting minor instead of major emphasis on them.
2. Time must be available to each teacher so that he can truly assimilate new ideas and approaches. No teacher should feel pressured into changing before he is ready for it.

3. What is most important is a willingness to experiment and try out new ideas.

I hope these suggestions will help you as the central figure in change to see things in perspective so that on the one hand you will adopt a positive attitude to change yet on the other hand be critical of premature developments or pressures.

******

This special issue of the ONTARIO MATHEMATICS GAZETTE for elementary school teachers has been financed in part by a grant from the National Council of Teachers of Mathematics (NCTM). Teachers who wish further assistance in modern teaching methods at the elementary school level or who desire to keep up to date with modern trends will find the magazine "The Arithmetic Teacher" of invaluable assistance. Membership in the NCTM brings with it an automatic subscription to "The Arithmetic Teacher" (elementary level) or to "The Mathematics Teacher" (secondary level). Information about membership in the NTCM accompanies this special issue. Similar information may be obtained by writing

NTCM
1201 Sixteenth Street, N.W.
Washington, D.C. 20036

ONTARIO MATHEMATICS GAZETTE

EDITORIAL BOARD

Editor-in-Chief ---------- Dr. J. E. Hogarth,
Queen's University.

Reviews Editor ---------- Dr. D. A. Clarke,
University of Toronto.

Research Editors ---------- Dr. D. H. Crawford,
Queen's University.

and

Mr. A. W. Harris,
Althouse College of
Education,
University of Western
Ontario.

News Editor ---------- Mr. J. W. Fencott,
R. H. King Collegiate
Institute,
Scarborough.

Additional copies of the Gazette may be obtained from:

The Secretary,
Ontario Mathematics Commission,
1260 Bay Street,
TORONTO, Ontario.


PRINTED
IN
CANADA