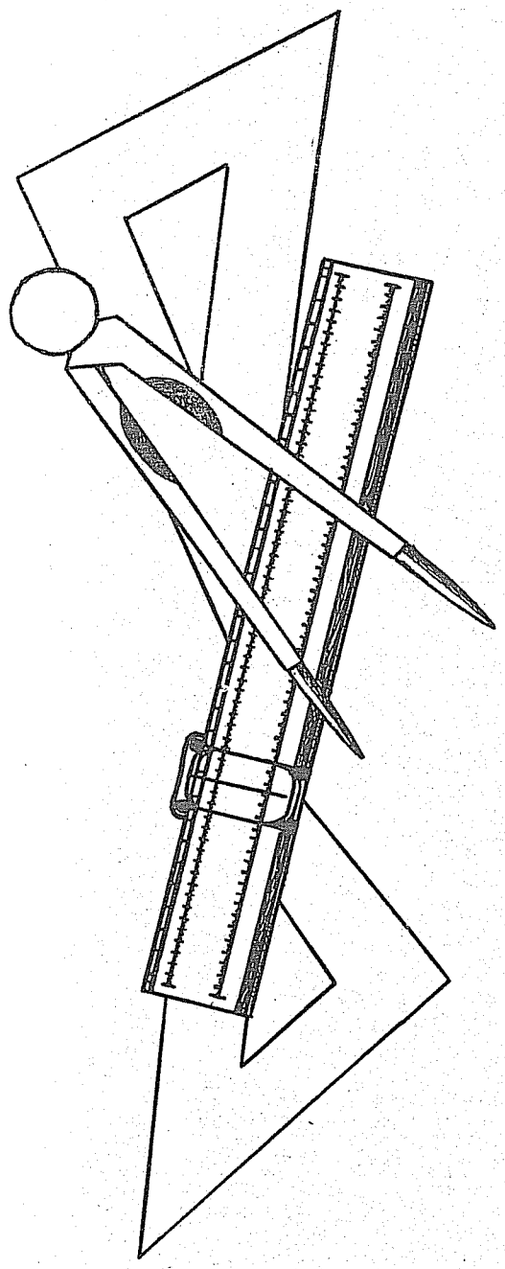
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DIVISION IN THE ELEMENTARY GRADES

J. N. C. Sharp
Co-ordinator of Mathematics
The Board of Education for the Borough of Etobicoke

Since the inception of the new approach mathematics several years ago, there seems to be a decline in student ability regarding the process of division. Many of our secondary school teachers remark that the students now entering the secondary school do not have the ability to use long division with the same degree of competence that the previous students had.

In the new approach in mathematics, the process of division relates to repeated subtraction, and students become familiar with the use of repeated subtraction at the grade two or three level. This is the stage at which they are developing the process for division. Although it is essential that an understanding of the algorithm be obtained, too often the emphasis is put on the understanding and not on the use of the algorithm that results from this understanding.

In the outline that we have prepared for use by our teachers, we look upon division as a process from Kinder-

garten through to grade 6. The following shows the stages that are used and the concepts that are developed. In the early stages more activities are involved than at the later stages, but this is true in most mathematical processes. We hope that a child leaving grade six will be able to use the division algorithm for decimals, and also be able to use short division, which seems to be a forgotten art.

KINDERGARTEN - sharing between two children

GRADE 1 - sharing among several children
- determining the amount of a share

GRADE 2 - determining the number of shares
- division and multiplication

GRADE 3 - repeated subtraction
- inverse of multiplication
- inexact division
- algorithm for one-digit divisor

GRADE 4 - the division sentence

GRADE 5 - two digit divisors
- short division
- estimation, rounding off, approximation

GRADE 6 - division of fractions
- division of decimals

KINDERGARTEN

Intuitive division through experiences in daily life situations with no reference to the concept of division.

Suggested Activities

- sharing with another child a number of cookies, candies, blocks, toys, etc.

GRADE 1

1. Continued development of division to find the amount of a share, a set, or a group through daily life situations (oral, not written).

   Suggested Activities

   - grouping children into teams to find the number on each team
   - placing chairs in rows to find the number in each row

2. Finding the amount of a share through the use of concrete material.

   Suggested Activities

   - using record discs, marbles, acorns, toy cars, unifix, pegs and pegboards, etc.

3. Finding the length of each piece if sharing a specified length between a given number of children - two children at first, then four.

Suggested Activities

- dividing a length of ribbon, wool, string, etc.

4. Finding amount of a share when portioning a volume of liquid or solid.

Suggested Activities

- pouring lemonade from a large pitcher into glasses of equal size

GRADE 2

1. Continued use of concrete material to find the amount of a share, a set, or a group when the number of shares, sets, or groups is known.

Suggested Activities

- using money

2. Using concrete material and everyday experiences to find the number of shares, sets, or groups when the amount of each share, set, or group is known.

Suggested Activities

- using toy animals, geometric solid shapes, books on shelves, paper cut outs, felt and magnetic shapes
- using a number line
- using money

3. Finding a fraction of a group

Suggested Activities

- using children, record discs, straws, chestnuts, tongue depressors, etc.

4. Relating division to multiplication to find a missing factor without reference to division

   e.g., $2 \times \bigcirc = 14$ $\bigcirc \times 3 = 12$

5. Introduction of terms - factors, product
6. Introduction of division sign to pupils ready to use it $6 \div 2 = 3$; for other pupils, use the form 2's in 10 equal

GRADE 3

1. Finding the number of equivalent subsets in a set

   Suggested Activities

   - use repeated subtraction to illustrate division and the separation of a set into subsets of equal quantity
   - using arrays to find the number of equivalent subsets in a set

2. Introduction of the concept of division as the inverse of multiplication

   e.g., $3 \times 4 = 12$ $12 \div 4 = 3$

3. Introduction of remainder where division is not exact

   Suggested Activities

   - dividing a number of children into groups for games, e.g., 17 children for 3 teams

4. Development of the division algorithm through repeated subtraction - one digit divisors only

5. Introduction to the terms and relationship between dividend, divisor and quotient
6. Relation of division to problem solving

   e.g., How many chocolate bars can we buy for
   30 cents if each chocolate bar costs 5 cents?

GRADE 4

1. Continued development of the division algorithm through use of repeated subtraction for pupils who have not progressed to the stage of understanding the algorithm.
2. Development of facility with the division algorithm for pupils able to use the algorithm
3. Finding the missing number in a division statement

   e.g., $12 \div 3 = n$

   $12 \div n = 4$

   $n \div 3 = 4$

4. Consolidation of the basic division facts up to and including $81 \div 9$.

GRADE 5

1. Extension of the division algorithm to include two-digit divisors (repeated subtraction used as an introduction only)
2. Introduction of the short method of division with one-digit divisors. Here is a suggested introduction to short

division.

Long Way			Short Way
3 8)2915 2400 (300) 515		Dividing Hundreds Think: 29 ÷ 8 The quotient is 3. The remainder is 5.	3 8)29⁵15
36 8)2915 2400 (300) 515 480 (60) 35		Dividing Tens Think: 51 ÷ 8 The quotient is 6. the remainder is 3.	3 6 8)2 9⁵1³5
364 8)2915 2400 (300) 515 480 (60) 35 32 (4) 3		Dividing Ones Think: 35 ÷ 8 The quotient is 4. The remainder is 3.	3 6 4 R3 8)2 9⁵1³5

3. Development of estimation, rounding off to enable pupils to approximate quotients and partial quotients. Emphasize four stages.
   - mental multiplication by powers of 10, and decades of powers of 10
   - reasoning about the process
   - relation to a physical interpretation
     e.g., 3250 pupils distributed equally into 8 schools, how many in each?
   - practice until process can be done with confidence and accuracy

GRADE 6

1. Division of fractions
   - several methods can be introduced
     using repeated subtraction
     using common denominators
     using number line
     using reciprocals
     using patterns (see attached sheets for Full Class Participation)
2. Division of decimals
   - see attached sheets for a developmental process

The students of grade 6 will have used the division algorithm in grade 5. This algorithm is developed as a short cut from the treatment of division as repeated subtraction. In chapter 4 of the grade 6 text, the student has continued long division by using the algorithm. It is now intended to use the same algorithm for long division involving decimals. The following sequence is suggested for the development of this method.

TYPE 1 Division by a divisor with a single digit.

(a) Evaluate $292 \div 4$

$$\begin{array}{r} 3 \\ 70 \end{array}\Big\} 73 \qquad\qquad 73$$
$$\begin{array}{r|l} 4 & 292 \\ 280 & \\ \hline 12 & \\ 12 & \end{array} \qquad\qquad \begin{array}{r|l} 4 & 292 \\ 280 & \\ \hline 12 & \\ 12 & \end{array}$$

(b) Evaluate $292 \div 0.4$

$$\frac{292}{0.4} = \frac{292 \times 10}{0.4 \times 10}$$

$$= \frac{2920}{4}$$

$$\begin{array}{r} 30 \\ 700 \end{array}\Big\} 730 \qquad\qquad 730$$

$$\begin{array}{r} 4\overline{)2920} \\ 2800 \\ \hline 120 \\ 120 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 4\overline{)2920} \\ 2800 \\ \hline 120 \\ 120 \\ \hline \end{array}$$

(c) Evaluate $29.2 \div 0.4$

$$\frac{29.2}{0.4} = \frac{29.2 \times 10}{0.4 \times 10}$$

$$= \frac{292}{4} \quad \text{see (a)}$$

(d) Evaluate $29.2 \div 0.04$

$$\frac{29.2}{0.04} = \frac{29.2 \times 100}{0.04 \times 100}$$

$$= \frac{2920}{4} \quad \text{see (b)}$$

(e) Evaluate $0.292 \div 0.4$

$$\frac{0.292}{0.4} = \frac{0.292 \times 10}{0.4 \times 10}$$

$$= \frac{2.92}{4}$$

$$\begin{array}{r} 0.03 \\ 0.70 \end{array}\Big\} \qquad\qquad 0.73$$

$$\begin{array}{r} 4\overline{)2.92} \\ 2.80 \\ \hline 0.12 \\ 0.12 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 4\overline{)2.92} \\ 2.80 \\ \hline 0.12 \\ 0.12 \\ \hline \end{array}$$

Note: The divisor in each case is a whole number.

TYPE II Division by a divisor with two digits.

(a) Evaluate $326.6 \div 71$

$$\begin{array}{r} 0.6 \\ 4.0 \\ 71\overline{)326.6} \\ 284.0 \\ \hline 42.6 \\ 42.6 \end{array} \quad 4.6 \qquad \begin{array}{r} 4.6 \\ 71\overline{)326.6} \\ 284.0 \\ \hline 42.6 \\ 42.6 \end{array}$$

(b) Evaluate $32.66 \div 71$

$$\begin{array}{r} 0.06 \\ 0.40 \\ 71\overline{)32.66} \\ 28.40 \\ \hline 4.26 \\ 4.26 \end{array} \quad 0.46 \qquad \begin{array}{r} 0.46 \\ 71\overline{)32.66} \\ 28.40 \\ \hline 4.26 \\ 4.26 \end{array}$$

(c) Evaluate $326.6 \div 7.1$

$$\frac{326.6}{7.1} = \frac{326.6 \times 10}{7.1 \times 10}$$

$$= \frac{3266}{71}$$

$$\begin{array}{r} 6 \\ 40 \\ 71\overline{)3266} \\ 2840 \\ \hline 426 \\ 426 \end{array} \quad 46 \qquad \begin{array}{r} 46 \\ 71\overline{)3266} \\ 2840 \\ \hline 426 \\ 426 \end{array}$$

(d) Evaluate $3.266 \div 0.71$

$$\frac{3.266}{0.71} = \frac{3.266 \times 100}{0.71 \times 100}$$

$$= \frac{326.6}{71} \quad \text{see (a)}$$

When the student realizes that each divisor in the algorithm

is a whole number he may then see the short cut used to obtain this type of divisor. In the final form decimal points lie in a vertical line. This is very helpful when division becomes more involved. The last column in each is the form in which division is finally done. Do not be in too much of a rush to reach this level of sophistication.

A CRITIQUE OF P1, J1

A. Tigchelaar, Co-ordinator of Pure-Applied Science
Halton County Board, Burlington

In 1966, the Department of Education issued an interim revision of P1, J1 a document which provides teachers with curriculum guidelines from Kindergarten to Grade 6 inclusive. In 1971, this document was shifted from its interim status to become the approved Program of Study for the primary and junior division of the Public and Separate Schools of Ontario.

Curriculum revision on the basis of a cyclic review of the three major curriculum division, viz. Primary-Junior, Intermediate, and Senior Division is planned by the Department based on a six year cycle, e.g. the cyclic review of the present P1, J1 is presently in process and will consist of an input stage, during which time all those concerned with the use and implementation of this document will have an opportunity to submit suggestions and criticism to the Department of Education regarding the proposed revision*.

---

*Readers are reminded that suggestions with regard to revision of P1, J1 should be forwarded to -

J. R. MacLean,
Assistant Superintendent,
Department of Education.

The curricula of the Intermediate and Senior division will also be reviewed cyclically,on the basis of one division every two years, thus in six years time, the Department will again consider P1, J1 for review. In this way each document will go through an input, planning, publication cycle of two years and approximately four years of application in the classroom.

In order that the revising committee can make intelligent and meaningful decisions, it is essential that abundant information should reach the Department of Education from the teachers in the field, so that an improved P1, J1 will result. The design of meaningful curricula requires research and this cannot be independent and apart from the teacher in the classroom who is involved with the most important and direct kind of educational research on a day to day basis.

It would, therefore, seem to be self evident that in the formation of the committees which will analyse information from the field and implement these, the classroom teacher and other educators who are directly involved with curriculum implementation should be liberally represented. This need is particularly evident in view of the criticism of P1, J1 in its present form as it relates to its philosophy and content. In this discussion, I would like to voice explicitly some of these criticisms. In so doing, I shall restrict myself to

that part of P1, J1 which deals with the proposals for teaching Mathematics from K-6.

For those who are familiar with the history of the introduction of the New Mathematics, the influence of the Nuffield Foundation project on the teaching of Elementary School Mathematics in England has had considerable affect in the schools of the province of Ontario. This, and other experimental Programs, such as the Greater Cleveland Mathematics Study and the School Mathematics Study Group in America, have added to the pressure on the educational system to carry out a number of reforms in the Mathematics program, both in content and pedagogy. That much of this reform was necessary and beneficial, cannot be denied. That some of it was misunderstood and has caused harm is equally evident.

In any event, many of the reforms introduced in our Mathematics programs follow, to a large extent, the patterns established by the experimental programs in England and America and the philosophy and content in our curricula reflects the thinking of educators in England and the U.S.A.

Because P1, J1 is a guide for Program development, and outlines specific disciplines, its use and interpretation has raised difficulties since it contains many rather general and vague philosophical and curricular statements relating to these

disciplines. For example, on page 3 (Mathematics), we read

> "No specific assignment of topics to grade levels is desirable. The placement of any topic at any particular age level of children is arbitrary."

This appears to suggest that topics may be arranged in any order and that a sequential development of Mathematics is unnecessary. On the contrary, sequence in Mathematics is essential and the arrangement of topics is in many instances determined on prerequisite experiences. To the novice in the classroom or the teachers who lack background in mathematical training, the above quote from P1, J1 is a dangerous and misleading statement. This same point of view is once more emphasized by the authors of P1, J1 (page 3) who state:

> "Sequential presentation of topics is only necessary in a broad sense, as is indicated in each of the three lists of topics. Children may work in groups at apparently unrelated tasks in mathematics, and still develop mastery of basic principles."

I would submit that the mastery of basic principles is probably best developed and more clearly evident within the con-

text of a well-planned sequentially-structured program. It is unfortunate that in the presentation of mathematical truths, the basic logical developments are frequently ignored, so that to the student, mathematics appears to be a collection of apparently unrelated methods, rules and tricks to be submitted to memory. Random selection of topics help to encourage this concept of Mathematics, and the danger becomes considerable that the end product is someone who has flitted from topic to topic, tasting and sampling, without either gaining mastery or any sense of understanding of the beauty and/or purpose of Mathematics. Children need (and frequently express a wish) to see the relevance of Mathematics to life as they know it and their early interest in Mathematics (which is considerable) can only be maintained if teaching is meaningful and interest-creating.

I would suggest that in the revision of P1, J1, very serious consideration be given to provide a more detailed and sequential curriculum guide in Mathematics for the teachers of K-6 in which each level from K-6 is dealt with individually and explicitly. As a case in point, in the present document under Numbers and Operations, teachers are asked to introduce the following topics as planned and directed activities in year 1.

(1) factors, primes and composites

(2) developing the concept of addition (and subtraction) as a binary operation, commutative and associative.

- as computation with Whole numbers
- as computation with Fractional numbers in common notation

To 2 is added the topic - as computation with fractional numbers in decimal notation for year 2.

It is my opinion that the above content description is totally inadequate as a useful guideline for the Primary teacher. In the section on Shapes and Space, under Plane Shapes, the following topics are proposed for year 1, 2 and 3.

- simple regular polygons and some irregular polygons, circle, ellipse
- edges vertices: number, relationships
- interrelationships of plane figures: triangle/square, sector/circle, and symmetry, congruency, translations, rotations, reflection

The only help the teacher gets here from P1 J1 is the statement on page 6 which says:

> "An intuitive approach is best, with children discovering properties of shapes, symmetry, similarity, then dimensions, in a progression of experiences. No introduction of formal proofs in geometry is recommended."

In reading the section on Shapes and Space, and accepting it at face value, it is difficult to decide whether this is a description of a course for the Primary, Junior, or the Intermediate division. For the inexperienced teacher this prescription is not very helpful. I question, for example, whether students are ready to deal with such topics as symmetry, congruency, rotations, translations and reflections in a meaningful way in the primary division, and whether in fact, these topics should receive anything but the lowest priority at this point in the curriculum.

Another area of concern to many teachers of Mathematics in the schools today is the lack of facility in the basic arithmetic operations (addition, subtraction, multiplication and division in the set of Whole numbers and fractions) demonstrated by many pupils. It is important that the committee responsible for the revision of P1, J1 consider making an

explicit statement with regard to the need of basic arithmetic skill as a foundation and co-requisite for the study of Mathematics. There is strong evidence to indicate that dislike for Mathematics stems from the pupil's inability to perform the basic operations of arithmetic; as a consequence the student is severely handicapped in his study of Mathematics. To expect a pupil so handicapped to like Mathematics is as reasonable as to expect a tone deaf person to enjoy music. The concept that arithmetic skills are not of prime importance in the study of Mathematics, appears to be supported by the present edition of P1, J1. On page 4, paragraph 1(a) (Mathematics) we read

> "Children should be introduced to mathematics rather than arithmetic which is just one aspect of the subject."

The paragraph in its present form is frequently interpreted to mean that arithmetic is passé and that gaining facility in the basic operation of arithmetic is somehow a non-educational activity. I cannot agree with a teaching strategy or a program in Mathematics that does not provide the child with the essential arithmetic tools to help him develop his understanding in Mathematics!

Information with regard to the implementation of the

Metric System within the next decade has been distributed by the government. The Metric System as such is not mentioned in the present edition of P1, J1. Since our present primary students will probably be in the Senior Division of Secondary School when Canada has converted to the Metric System, it is essential that teachers be made aware of the importance of developing the concepts and skills of the Metric System with their pupils. Consequently, it is suggested that the committee responsible for the revision of P1, J1 consider including, explicitly, the Metric System as part of the revised P1, J1 Curriculum.

In summary, I would recommend to the P1, J1 revising committee the following:

1. To place more explicit emphasis on the need of basic arithmetic skills within the context of a well-planned sequential Mathematics program.

2. To recognize that on the basis of present research, many students have limited capacity for abstract concepts and that the reality of their needs be recognized in the Aims and Objectives of P1, J1.

3. That some caution be advised with regard to the proliferation of topics in Mathematics with the suggestion that to develop fewer concepts in greater depth would serve the needs and interest of the majority of our students more effectively.

MATHEMATICAL NOTES

TEACHING THE SLIDE RULE

John Coenraads, Loyalist CVI, Kingston, Ont.

In the spring of this year, my level four grade 10 mathematics students expressed a considerable interest in learning how to use a slide rule. A check of slide rules on hand showed that most had broken or missing parts and the budget did not allow for the purchase of new ones. I decided then to make up a stencil for a paper slide rule which the students could construct themselves.

The scales from a standard ten inch slide rule were carefully transferred to the edge of a sheet of paper and from there, onto a master copy from which a stencil was cut. (This is a Sunday afternoon's work.) The students thoroughly enjoyed spending a period cutting out and assembling their slide rules. The cursors were made from 4cm. x 14cm. pieces of acetate and the hairlines were made by scoring a groove in the acetate, filling it with marker ink and wiping off the excess.

At first the students doubted the usefulness of their "paper slide rules" until they found that they could do almost

all of their mathematics and science problems with an accuracy of at least 1% per calculation. This fall I used paper slide rules with my grade 11 physics classes and again I found them to be a great success.

The following is a list of advantages of the paper slide rule:

1. They are virtually cost free.
2. They are made from easily available materials such as paper and acetate sheets.
3. Only a pair of scissors, some scotch tape, and a marker pen is required to assemble them.
4. Every student can have his own personal slide rule.
5. When carefully constructed they can easily give an accuracy of 1% per calculation.
6. Since only the basic A, B, C, and D scales are present, the students find the paper slide rules less confusing to read than the commercial variety.
7. Since they can easily be slipped into a notebook, students find them convenient to carry around.
8. A supply of extra ones can be kept on hand for students who forget their own.
9. A student can easily assemble a new slide rule if he should lose his own.
10. Many students eventually buy commercial slide rules once the paper variety has convinced them how useful a slide rule can be.

A FURTHER NOTE ON INSTANT INSANITY

Tom Griffiths, North Lambton Secondary School
Forest, Ontario

The following method of solution of the puzzle "Instant Insanity" may be of interest as an extension of David Wing's article in the December issue of the Mathematics Gazette (Volume 10, Number 2).

Let the colours be numbered Green 1, Red 10, White 100 and Blue 1000. The blocks can then be defined as a set of four triples, where each element of each triple is the sum of opposite faces of each cube. Hence

block 1 is (2, 1100, 1010)

block 2 is (11, 1100, 20)

block 3 is (101, 1010, 110)

block 4 is (101, 11, 1100) using Mr. Wing's blocks and order.

The object is now to arrange the triples in a matrix such that the sum of each of two columns is 2222. This is a necessary condition for solution, since each colour must appear twice only on the four pair of opposite faces.

The criterion used for solution is that at no time must a digit greater than two appear in the sum of the elements of a column of the matrix.

Locate block 1 as the "base" block (2, 1100, 1010). In block 4 the only element with zero as the last digit is the 3rd. This must therefore be located below the 2 of block 1 in order that a sum does not include the digit 3. There are two ways this can be done. They are

(a) $\begin{pmatrix} 2, & 1100, & 1010 \\ 1100, & 11, & 101 \end{pmatrix}$ and (b) $\begin{pmatrix} 2, & 1100, & 1010 \\ 1100, & 101, & 11 \end{pmatrix}$.

The totals of the columns are now

(a) (1102, 1111, 1111) and (b) (1102, 1201, 1021).

Consider possibility (a) and block 2. The element 20 can be placed only in the first column. The possibilities are therefore

$\begin{pmatrix} 2, & 1100, & 1010 \\ 1100, & 11, & 101 \\ 20, & 11, & 1100 \end{pmatrix}$ or $\begin{pmatrix} 2, & 1100, & 1010 \\ 1100, & 11, & 101 \\ 20, & 1100, & 11 \end{pmatrix}$.

The total of these are

(1122, 1122, 2211) or (1122, 2211, 1122).

The remaining block 3 would therefore have to have two elements out of the three of

(1100, 1100, 11) or (1100, 11, 1100)

which are the same. Only one of these elements appear on block 3, hence this case does not yield a solution.

Consider possibility (b). The first element of block 2

can be placed only in column 2, and then element 3 can go only in column 1 giving

2, 1100, 1010
1100, 101, 11
20, 11, 1100

as the only possibility. The totals are

(1122, 1212, 2121).

The remaining block must have two of the elements

(1100, 1010, 101),

which it has. The first column does not, therefore, yield a solution, and should be hidden.

I realise also that although this achieves the solution to the problem, it does not give the required block arrangement in detail. This can be done as follows:
Take block 1 and place it in front of you so that the opposite green faces are in the direction to be hidden. Then repeat with blocks 2, 3, and 4, hiding the column one pairs in each case.

Now arrange the blocks with column 2 faces in one of the visible directions. There will now be a total of 8 faces in this direction, those faces will include two of each colour. Fix block 1, which has a blue-white pair visible. Turn the other blocks with blue and white faces in the opposite

direction to block 1, there are 2 and 4 in fact. Now the remaining block 3 can be placed in the direction giving all four faces on each side of this direction a different colour.

The third direction can be handled now. Fix block 1 and note one of the colours on one of the two faces in our final direction. Again this colour will be repeated once more only on the other blocks in this direction. Find this block, and turn that colour to the opposite side as Block 1. The rotation will take place about the axis perpendicular to the other visible faces. Note the colour on the opposite face of this block, find it on another block and repeat till all blocks are in the required position. Thus the solution matrix is:

$$\begin{pmatrix} 2, & 1100, & 1010 \\ 1100, & 101, & 11 \\ 20, & 11, & 1100 \\ 110, & 1010, & 101 \end{pmatrix}$$

with totals 1232, 2222, 2222.

PATTERNS IN FIGURATE NUMBERS

S. E. Rowe, Box 222, Niverville, N. Y.

Figurate numbers include those numbers which can be arranged in the form of a triangle, square, pentagon, hexagon and so on. For example, if a number of objects can be arranged to form a pentagon, that number is considered to be a pentagonal number. There are some interesting relationships between the figurate numbers which we shall examine. The following are examples of figurate numbers, sometimes called polygonal numbers:

Triangular Numbers

The numbers 1, 3, 6, 10, 15, 21, 28, 36, 45, and 55 ... are called triangular numbers because the units of which they are composed can be arranged in triangles. Figure 1 shows this geometrical pattern.

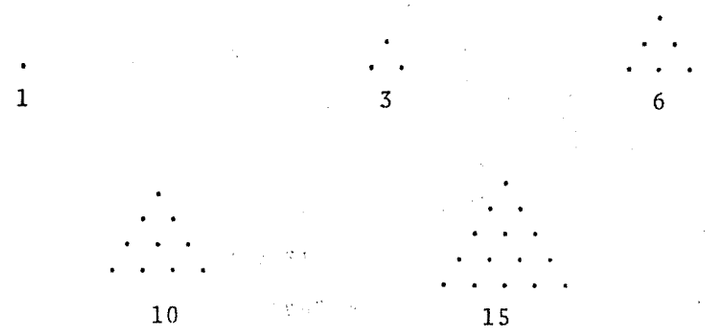


Figure 1

Note that each number is produced by adding n + 1 to the preceding number, where n is in a particular term. It is clear that the nth triangular number, T_n, is given by the sum of an arithmetic series. $T_n = 1 + 2 + 3 + \ldots n = \frac{n(n + 1)}{2}$.

Square numbers

The numbers 1, 4, 9, 16, 25, and 36 ... are called square numbers because the units of which they are composed can be arranged as squares. Figure 2 represents this:

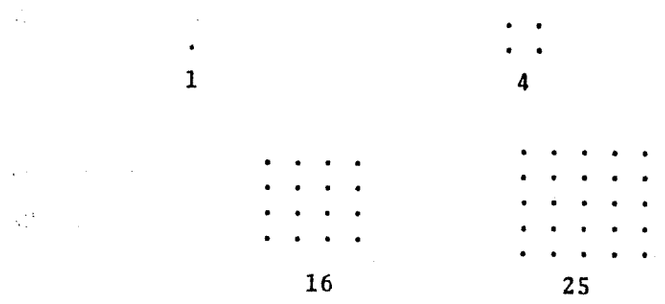


Figure 2

It is obvious that the square numbers can be formed by squaring each of the natural numbers.

Pentagonal Numbers

The pentagonal numbers 1, 5, 12, 22, 35, and 51 ... can be represented as in Figure 3. It can be shown that the general formula for finding pentagonal numbers is $n \frac{(3n - 1)}{2}$

where n is a particular term.

The pentagonal numbers are represented in Figure 3.

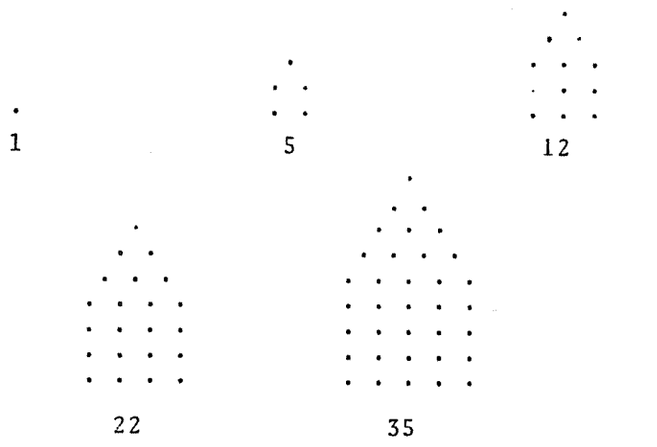


Figure 3

It is possible to develop the figures for all of the other polygonal numbers. This task will be left up to the reader.

The formation of polygonal number figures can be done by adding what is known as the "gnomon" to the previous figurate number. To form the figures of the triangular numbers, starting with 1, add the gnomon which consists of the number of points needed to make the next triangular number. That is, starting with 1 we must add 2 points to get the next triangular number and then 3 points for the next, 4 for the next and so on. The gnomon thus consists of 2 points, then

3, then 4, etc. Figure 4 shows how adding the gnomon to each previous triangular number figure forms the next triangular number figure. The gnomon is represented by "x".

```
                                  x
                          x      . x
                  x      . x    . . x
    .            . x    . . x  . . . x
    1            3      6      10
```

Figure 4

The square numbers would be formed as in Figure 5.

```
                                  x x x x
                          x x     . . . x
                  x x     . . x   . . . x
    .             . x     . . x   . . . x
    1             4       9       16
```

Figure 5

One way of forming the pentagonal numbers using the gnomon would be as in Figure 6.

```
                                  x
                                 . x
                          x     . . x
                         . x    . . x
    .                    x x    x x x
    1                    5      12
```

Figure 6

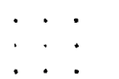
Table A

Figurate Numbers

Natural Numbers	1	2	3	4	5	6	7	8	9	10	...
Triangular	1	3	6	10	15	21	28	36	45	55	...
Squares	1	4	9	16	25	36	49	64	81	100	...
Pentagonal	1	5	12	22	35	51	70	92	117	145	...
Hexagonal	1	6	15	28	45	66	91	120	153	190	...
Heptogonal	1	7	18	34	55	81	112	148	189	235	...
Octagonal	1	8	21	40	65	96	133	176	225	280	...
Nonagonal	1	9	24	46	75	111	154	204	261	325	...
Decagonal	1	10	27	52	85	126	175	232	297	370	...

Many relationships between figurate numbers can be determined from Table A as shown above. Some of the relationships are as follows:

1. The most obvious is that a square number consists of two consecutive triangular numbers. The following Figure 7 will illustrate this.



Note that the dotted line has divided the square number (9) into two consecutive triangular numbers, 3 and 6.

Figure 7

2. The nth triangular number plus the (n + 1st) square number results in the (n + 1st) pentagonal number.

Figure 8 illustrates this.

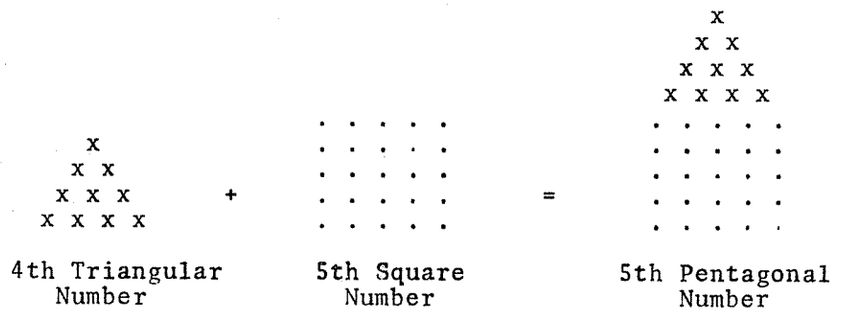


Figure 8

3. Twice the nth Triangular number plus the (n + 1) square number results in the (n + 1) Hexagonal number. The following Figure 9 represents this.

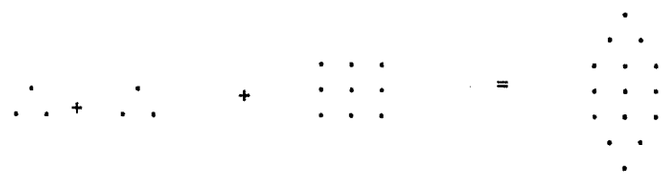


Figure 9

4. It can be seen in Table A that each number in the nth column differs by the (n - 1) triangular number. That is, each number in each column differs by $\frac{n(n + 1)}{2}$ where $n > 0$.

It is obvious that the Table itself could be generated using this property. The following Table B represents constant differences between figurate numbers.

0	1	3	6	10	...
0	1	3	6	10	...
0	1	3	6	10	...
.	.	.	.	.	...
.	.	.	.	.	...
.	.	.	.	.	...

Table B

5. When each set of figurate numbers is demonstrated under Modulo n, where n is the number of sides in the geometric representation, interesting patterns are developed. Looking at the triangular numbers in Mod 3, we notice that the pattern 1, 0, 0, 1, 0, 0, 1, 0, 0, ... develops. The pattern 1, 0, 0 repeats every 3 places. The square numbers Mod 4, form a pattern 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ... where 1, 0, 1, 0, repeats every 4 places. The pentagonal numbers, Mod 5, form a pattern of 1, 0, 2, 2, 0, 1, 0, 2, 2, 0, ... which repeats every 5 places. Each set of figurate numbers can be shown to have such patterns under Modulo n, where the pattern repeats every n places. The

usefulness of such patterns is that they could be used to check whether or not a set of numbers is triangular, square, pentagonal, etc. For example, if a set of numbers, arranged in order of size, has a Modulo 3 pattern of 1, 0, 0, one can be sure that the set of numbers is triangular.

6. It is interesting to note that certain patterns of odd numbers and even numbers occur based on the class of figurate numbers. If the class of figurate numbers forms a figure with an odd number of sides (such as triangular, etc.), a pattern of odd, odd, even, even, etc. develops. If the class of figurate numbers forms a figure with an even number of sides, a pattern of odd, even, odd, even etc. occurs. Classes of this case include squares, hexa gonals, octagons, etc. Refer back to Table A for this pattern.

After presenting the subject of figurate numbers ot his students, the teacher could expect that the students might find other relationships between the figurate numbers. It is important to the extent that it shows to the students that they can discover for themselves certain patterns which exist in mathematics. The teacher need only guide the students in discovering the patterns which exist. Many more patterns exist which I have not discussed. Let your students find some of them.

PROBLEMS AND SOLUTIONS

Edited by W. Fraser

This will be the final issue of the Gazette for which material for this section will be compiled by me. I would like to take this opportunity to thank all who have submitted either problems or solutions for their much appreciated contribution to this section. In order to get partially caught up we restrict ourselves in this issue to the presentation of some solutions.

Solutions

Problem 71.02

Solutions have been submitted by M. A. Laframboise, Detroit, Michigan, and by the proposer, Mr. E. Evans, Guelph.

The transformation $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$, with $ad - bc \neq 0$, when applied to $y^2 = 4px$ results in $(cx + dy)^2 = 4p(ax + by = 4apx + 4bpy$. An unknown λ can be added inside the square on the left side, and appropriate additions made to the right side to retain equality, leading to

$$(cx + dy + \lambda)^2 = (4ap + 2\lambda c)x + (4bp + 2\lambda d)y + \lambda^2.$$

In order that the direction determined by $cx + dy + \lambda$ should be perpendicular to that determined by the expression on the right, we choose λ so that

$$c(4ap + 2\lambda c) + d(4bp + 2\lambda d) = 0, \text{ i.e., } \lambda = -\frac{2p(ac + bd)}{c^2 + d^2}.$$

With this λ, the transformed equation becomes

$$\left[cx + dy - \frac{2p(ac + bd)}{c^2 + d^2}\right]^2 = \frac{4p(ad - bc)}{c^2 + d^2}(dx - cy) + \frac{4p^2(ac + bd)^2}{c^2 + d^2}.$$

Setting

$$\frac{d}{\sqrt{c^2 + d^2}} = \cos\theta, \quad \frac{c}{\sqrt{c^2 + d^2}} = \sin\theta,$$

we find

$$\left[x\sin\theta + y\cos\theta - \frac{2p(ac+bd)}{(c^2+d^2)^{3/2}}\right]^2 = \frac{4p(ad-bc)}{(c^2+d^2)^{3/2}}\left[x\cos\theta - y\sin\theta + \frac{p(ac+bd)^2}{(ad-bc)\sqrt{c^2+d^2}}\right]$$

From the equation in this form it is apparent that the condition that the vertex should remain at the origin is that

$ac + bd = 0$, and the condition that the length of the latus rectum remain unchanged is that $(ad - bc) = (c^2 + d^2)^{3/2}$. An infinite number of integers will satisfy the latter condition without satisfying the former, for example $a, b, c, d = 47, 4, 4, 3$.

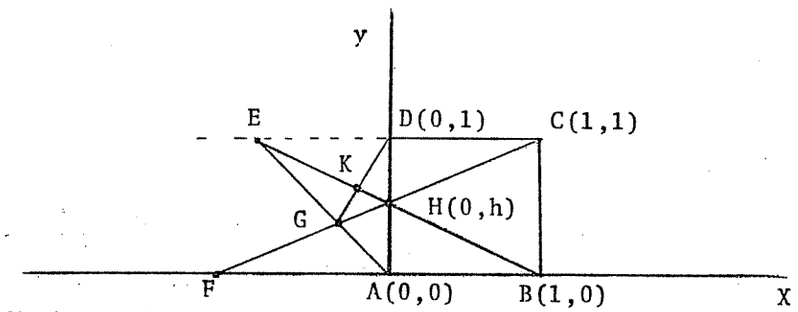
Problem 71.03

From the identity $n^4 + 4m^4 = (n^2 + 2mn + 2m^2)(n^2 - 2mn + 2m^2)$ it follows that if q is of the form $4m^4$, the number $n^4 + q$ is not prime for any natural number n. Since each integer value of n produces a value of q it follows that there are an infinite number of such values of q.

Problem 71.04

A solution for this problem was provided by the proposer, Mr. E. Evans, Guelph, who expressed the hope of seeing a solution less dependent on algebraic techniques. Solutions, all very similar, and all algebraic in nature, were submitted by E. A. French, Earl Haig S. S.; T. J. Griffiths, North Lambton S. S., and by another solver whose signature I made out to be a Mr. Asbury of Kingsgarden Road, Toronto, with apologies in the event of misreading that signature. The solution submitted by Mr. Griffiths is given as follows:

We assume that the square has sides of length 1.



We have the following equations:

$$\text{CE:}\quad y = 1$$

$$\text{BE:}\quad x + \frac{y}{h} = 1.$$

$\therefore$ the coordinates of E are $(1 - \frac{1}{h}, 1)$, and the slope of BE is -h. For AE: $y = \frac{h}{h-1}x$. For CF: $y = h+(1-h)x$.

The coordinates of G are thus $\left\{\frac{h(h-1)}{h^2-h+1}, \frac{h^2}{h^2-h+1}\right\}$.

From this it follows that the slope of GD is $\frac{1}{h}$. Hence BE $\perp$ GD.

BOOK REVIEWS

Multiple Methods of Teaching Mathematics in the Elementary School, by Charles H. D'Augustine, Harper and Row, New York, 1968

D'Augustine has written a comprehensive text intended to improve the methodologies of the classroom teacher with only rudimentary knowledge of the world of mathematics and the one who has a more complete mathematical background.

The organization and content of the text parallel that of most of the texts and programs now in use in Ontario and even the beginning teacher should have no difficulty in locating the sections related to what he wishes his class to learn at the moment. As well, D'Augustine has included topics which do not appear in most elementary school curricula except as enrichment areas (e.g. topology).

In addition to organizing his chapter contents in the same way as many of our present-day textbooks (e.g. Numeration Systems, Integers, Addition in the Set of Whole Numbers), D'Augustine has adopted a within-chapter organization that is a veritable lesson plan for successful mathematics instruction; each chapter begins with an introduction which describes some

of the background and history of what will follow within the chapter. This is followed by those definitions which pupils will have to determine, and then the terminologies and symbol systems which are most commonly used; in addition, alternative algorithms and approaches are described. D'Augustine then describes numerous readiness experiences which would prepare the pupils for the learning of the concept, and describes thorough instructions for teaching the concept. Next, he describes useful drill techniques and games which can be used to reinforce the newly acquired concepts, and story problems which invite the student to apply his new learning to real-life situations. Each chapter concludes with an exercise section where the teacher is asked to create appropriate problems which his class might use to discover or apply concepts. This latter step points out that the teacher is expected to learn as he works, not just reads, his way through the examples presented throughout the chapter, and that the test of his learning is in this form of application which is directly related to what he will or can do in the classroom.

His final chapter on classroom management techniques, testing and assessing one's teaching should be of help in subject areas far removed from mathematics.

Despite his comprehensiveness, D'Augustine has written a

book which is very pleasant reading as he develops steps which lead the teacher to discover how to teach particular mathematical areas.

D'Augustine has managed to design a book which is thorough in terms of content, but his orientation is to the child as a learning person. He emphasizes the substitution of discovery learning for 'spectator learning', the putting of the child into a situation where he is confronted with the patterns of the mathematical world and the dynamic qualities of mathematics as the pupil discovers it. He tempers this with the recognition that not all concepts lend themselves to the discovery technique in elementary schools, and reassures teachers that discovery need not be pure discovery, that there is a place in education for giving clues and prescribing.

The strongest facet of this book is its manifest belief that mathematics should be a world of discovery for the teacher as well as the pupil; the teacher is over and over again asked to work through problems, then apply the concepts he sees by formulating pupil problems or situations which will be appropriate to the given topic and will lead to be desired learning by the child. A teacher who arrives at the point where he can do this cannot help but become for it a better teacher.

Albert S. Dukacz,
St. Patrick's School, London, Ont.

Geometry: Kindergarten to Grade 13, Ontario Institute for Studies in Education, Curriculum Series 2, Toronto 1967.

It is to some extent unfortunate, although appropriate, that this book should be entitled, Geometry: Kindergarten to Grade Thirteen. This book is worthwhile reading for all those teaching mathematics and for those interested in the problems of learning. It would be of particular interest to consultants, curriculum developers, and teachers in colleges of education.

One main purpose of this book is to indicate how geometry can be restored to a healthy and revitalized relationship with other topics of the Ontario curriculum, in mathematics and other subjects. The authors are in agreement with the trend toward single courses in mathematics, rather than separate courses in algebra, arithmetic, or trigonometry but they feel that in the process there has been a tendency for geometry to be squeezed out.

Before making more specific comments concerning this book a few general comments are in order.

1. The book is very easy to read and its conciseness (119 pages) and cost ($2.50) should make it a "best seller".

2. It recommends the intuitive "interest" approach through problems significant to the student rather than the more traditional axiomatic approach. Geometry should be taught for its interesting results and as an exercise in informal reasoning.

3. Promotion in Mathematics should be determined by the student's record in mathematics, hence recommends homogeneous grouping in mathematics within an upgraded system of instruction.

4. General plan for the sequencing of curriculum experiences included the manipulation of concrete objects prior to formal development in agreement with Piaget's stages of development.

Since an upgraded curriculum is proposed, the authors outline their detailed proposals in three stages, A, B, and C which correspond to K - 6, grades 7 -10, and grade 11 - 13 inclusive. For each stage, specific suggestions are made, followed by general suggestions on the selection of topics, as illustrations.

A brief table of contents with some specific comments follows:

1. Purpose of K - 13 Geometry Committee.
2. Aims of Instruction in Geometry.

Students ask, "Why study Geometry?" - here the most compelling standard answers are summarized. The section on psychological aspects of mathematics learning (section 2.6) is excellent.

3. Major proposals and Principles.
   Proposes sound administrative changes which would result in improved mathematics education.
4. Specific Proposals for Stage A (Primary).
   Recommends that as much as is feasible, children should learn by doing and that considerable quantities of materials, games and toys should be available to the children in their own classrooms.
5. Specific Proposals for Stage B (Intermediate).
   Contains some proposals for what was named the four year and two year terminal programs. High school teachers will recognize much of this material as now existing in grades 11 and 12 courses of study.
6. Specific Proposals for Stage C. (Senior).
   Makes some long-term recommendations of geometry topics in senior grades. If students are to handle this material, greater attention will be needed to the problem of individual differences.
7. Recommendations and their Implementation.
   A number of the recommendations have been implemented but

much is yet to be done in many areas.

The authors make the statement, 'the curriculum must be fitted to the learner and not vice versa'. Speaking as a teacher of senior grade students, the mathematics population in the senior grades will be very seriously reduced if it is intended that the proposed curriculum suit the students. What are we to teach the remainder? I would recommend the approach suggested by the Department of Education in their proposed guides for planning mathematics 1 and 2. Here they divide the course into three segments,

(a) Core material - taken by all and requiring 25-30 weeks.

(b) Optional material - consists of topics which student will encounter again in later courses.

(c) Supplementary material - consists of topics which will not be encountered in later courses.

With this approach, weak students might spend a full year mastering the core material, while the better student would complete the "core" and some or all of the optional material.

I would commend this book to all those who teach mathematics if only to see what effect this report issued in 1967 has had on the teaching of Geometry in 1971.

Ronald J. Reid,
York Memorial Collegiate Institute,
Toronto.
Editorial revisions by E. Klatt,
O.I.S.E.

Mathematics and the Elementary Teacher, by Richard W. Copeland, W. B. Saunders Company, Toronto, 1972, 400 pp. $10.30.

This book is designed to provide the elementary school teacher, or interested parent, a basic understanding of some of the profound changes in the content and teaching methods that have been occurring in mathematics education for the past decade or more. Many aspects of modern mathematics programs - theory, operations, skills, applications and evaluations - are clearly presented in a simple concise language in each chapter. Many examples are provided to further illustrate and solidify concepts. Teachers and parents not familiar with terminology and concepts such as: set theory, commutative property and inverse operation - find these clearly and succinctly explained and illustrated in this text. Background history describing the development of early numeration systems as well as practical application of mathematics to everyday living situations serve to make the book interesting reading for teachers and parents alike.

The emphasis throughout the text is on teaching mathematics for understanding. This is accomplished by

(a) suggesting opportunities for exploration and induction which will enable children to discover for themselves.

(b) discussing the characteristics, essential differences, and brief histories of a variety of numeration systems,

(c) emphasizing the properties of number systems as a basis for understanding operations on natural numbers, integers, and rational numbers,

(d) developing alternative methods of performing basic operations and solving problems.

The book is not without its deficiencies. Some areas were not given enough emphasis while others were perhaps given too much. The "Integers" is a major topic introduced in the intermediate division and yet only a short chapter was devoted to this topic. No unique methods of introducing or clarifying the work with integers were suggested, while the division of integers was not dealt with at all. On the other hand, an entire chapter was devoted to "accuracy in measurement" - a topic not stressed at the elementary school level. Little information was provided in the problem solving aspect of geometry in topics such as finding perimeter, area, and volume. These have always been stressed in the elementary mathematics curriculum. I also felt that in a resource book of this type, specific reference could have been made to some of the current mathematics programs along with the strengths and weaknesses of each. This was not done at all.

In summary, I found Professor Copeland's book to be enjoyable reading because it was like taking a refresher course in the teaching of modern mathematics.

Kenneth A. Bisch,
Notre Dame Separate School,
Waterloo, Ontario.
Editorial revision by
E. Klatt, O.I.S.E.

PUBLICATIONS AVAILABLE

ONTARIO MATHEMATICS COMMISSION

1260 Bay Street, Toronto 185, Ontario

Handbook for Teachers of Mathematics in the Intermediate Grades	$2.00
Polar Coordinates by D. T. Faught	.50
Complex Numbers by J. E. Hogarth and B. J. Kirby	.50
What is Number by D. L. Mumford	.50
Mathematics K to 6 (1965 Edition)	2.00

ANNOUNCEMENT

Leadership Seminar in Mathematics

The first Leadership Seminar in Mathematics was held in Niagara Falls, Ontario in the fall of 1971. The seminar was most successful and as a result, the programme will continue as anticipated. In order to prevent confusion we have adopted a new nomenclature when referring to the seminars. Hence Phase 1 of Project 71/72 was held last fall. The same group will be invited to participate in Phase 2 of Project 71/72 scheduled for this fall. In addition a new group of teachers will enter Phase 1 or Project 72/73 during the fall of 1972.

This is an announcement to the teachers of Ontario of this event. If any teacher wishes to attend the seminar this fall he or she should contact his or her local board through the appropriate individual and indicate his or her interest. Attendance at the seminar is by invitation only and invitations are extended only to boards who request them. This is required by the nature of the programme. The deadline will be early in March and so it is suggested that any interested teacher act promptly. Further information can be obtained from any school board or the project director. Please address your queries, in the latter case, to

Dr. John Girhiny,
Project Director,
Ontario Mathematics Commission,
1260 Bay Street,
Toronto, Ontario.

Teachers may wish to discuss the seminar with those who have already attended. They are sure to find it is worthwhile and a bargain. The seminars are sponsored by O.M.C., O.A.T.M., O.P.S.M.T.F., F.W.T.A.O., and O.E.C.T.A.

The purpose of the Leadership Seminar in Mathematics is to provide a "teacher-leader" in mathematics in as many elementary schools in Ontario as possible. The staff of the seminar are all educators of extraordinary ability who have donated their time and talent in the interest of improving mathematics education in Ontario.

NOTES ON MEETINGS

Fall Conferences 1972

Joan Routledge

The Northern Ontario Mathematics Association held its fall conference on Saturday, October 2 at West Ferris Secondary School in North Bay. Over two hundred attended this conference. About 55% were secondary teachers, and 40% elementary. The programme got under way with a panel discussion on the statement "Mathematics literacy is as good today as it ever was". The panel members were Mr. Bob Zwicker from Cambrian College, Professor Madget from Laurentian University, Dr. Washington, a research scientist from Elliot Lake, and Walter Sololuk from Widdifield Secondary School, North Bay, and president of O.A.T.M.

Three of the panel members disagreed with the statement. Bob Zwicker advocated the use of computer assisted instruction to remedy the lack of essential skills. Professor Madget stated that by the time a student enters university, he has been exposed to more mathematical ideas, concepts, and terminology than ever before, but the retention and use of this knowledge is down. Dr. Washington deplored the excessive emphasis on advanced mathematics, and less and less on the basic skills. He maintained that most adults use mathematics

only for buying such things as cans of peas, and if they cannot add, subtract, multiply, and divide, they are lost. Walter Sokoluk said that he thought that the Mathematics 'revolution' had finally caught up with the content, and now the emphasis should be on teacher training. Following the statements from the panel, there was a lively discussion between the members and questioners in the audience.

There were sessions for teachers at all levels. For the Primary-Junior Division, Mrs. Margaret Stratford, Principal, Virginiatown Public School, gave a very interesting description of her school's curriculum writing project. Jeanette Bedard, Kirkland Lake Separate School Board, spoke on the topic "Math and Young Children".

At the Intermediate level, Ken Raaflaub, W. J. Fricker Senior School, North Bay, spole on "Cycling the 7 and 8 Program". Sarto Lefebvre, St. Rita Separate School, chose a timely topic, "Drills - New or Old", as did Andy Czempinski, Iroquois Falls Secondary School, speaking on "A 4 to 9 Continuum".

There were two sessions for the Senior Division - Chris Gupta, Roland Michener Secondary School, Timmins, "Fibonacci Sequence and Number Theory", and Ernie Allison, West Ferris Secondary School,North Bay,"A Program for Accelerated Students".

Following luncheon, and a lively business meeting, Jack Bell, Assistant Superintendent of Curriculum, Ontario Department of Education, gave a very interesting and informative talk on "The Metric Revolution", and its impact on the mathematics curriculum.

The executive and planning committee of N.O.M.A. are to be congratulated on the very fine programme offered at this fall conference. All who attended are looking forward to the next Northern Ontario conference.

The Ontario Association of Teachers of Mathematics Seventh Western Ontario Regional Conference was held at Sir Frederick Banting Secondary School, London, on Saturday, November 6. Dr. John R. Bramwell, Supervisor of Evaluation and Measurement, London Board of Education, opened the conference answering the question, "Evaluation for What?". As a follow-up to this talk, Merilyn Harrington, Chesley Public School, London, and Joe Kadlecik, Lord Elgin Public School, London, explained and demonstrated the intensive work being done in London to develop testing materials for the mathematics program.

There were other sessions of interest to elementary teachers. Jean Hewitt, St. George's Public School, London

spoke on "Mathematics in Advancement Classes". Bea Uttley, retired teacher from Waterloo County Board of Education, described the effect on a mathematics program of years of involvement in Cuisenaire. Rob Poile, C. C. Carrothers Public School and Giles Gauvin, Princess Elizabeth Public School, both in London, explained their individualized approach to mathematics in the Junior Division. A very interesting two-hour workshop on mathematics in an integrated programme was conducted by Judith Palmer from Running Brook Public School in Mississauga. Richard Todd of Parkview Public School, Komoka, outlined what is involved in the preparation of a local mathematics curriculum. There was a demonstration of a commercially produced system combining programmed instruction and cassette tapes, and mathematics films were shown.

At the secondary level, Walker Schofield, Oakridge Secondary School, London, suggested ways of coping with difficult classroom situations. Roy Yates, Sir Wilfred Laurier Secondary School, London, talked on the 4 Year grade 9 Mathematics. John Beggs and Bill Johnson from John F. Ross Secondary School, Guelph, described a new Grade 12 course for students headed for the C.A.A.T.'s. Objectives in teaching secondary school mathematics were outlined by R. H. Lee of Kingsville District High School. Marion McKinley, Lambton Central

Collegiate and Vocational Institute, Petrolia, talked about a new course in mathematics and culture. Dr. Clyde Reis, University of Western Ontario spoke on the topic "Some Other Transformations".

Many secondary and elementary teachers of western Ontario took advantage of this western conference. The programme offered lived up to the reputation for excellence established by conferences of previous years.

The Scarborough Mathematics teachers are hosting the three meetings of the Metro Mathematics Teachers' Association this year. The first meeting was held at Cedarbrae Collegiate Institute on Tuesday, October 26. Professor A. Rapoport, University of Toronto, gave the main address. He spoke on the topic "Mathematics - A Tool for What?". There were sessions for teachers for grade 7 to 13. Mr. B. Emes, West Hill Collegiate Institute, spoke on "The Giant Step (Grade 8 to Grade 9 Math)". At the Grade 9 level, Don Chow, Woburn Collegiate Institute, described the teaching of geometry to the general student. Rick Dermont, Sir Wilfred Laurier Collegiate Institute gave a talk and demonstration on "Programmable Calculators". This proved so popular that he was asked to repeat it at the second meeting of M.M.T.A. held at

Stephen Leacock Collegiate Institute on Monday, January 24, 1972. For Grade 11, Mr. P. W. Sherk, Cedarbrae C.I., described a "Fresh Approach to Trigonometry". Mr. B. Basiuk, Jarvis C. I. talked to Grade 12 teachers on quadratic functions. The topic for Grade 13 was "What is a Series", and was given by Dr. Dennis Russell, York University.

At the second meeting in January, the Co-ordinator of Mathematics for Etobicoke, Mr. J. N. C. Sharp, spoke to the Grade 7 and 8 teachers on "Number Patterns in Geometry". There were two panel discussions - Grade 11,"A Transformation Approach to Geometry" and "Streaming in Grade 9". At the Grade 12 level, Mr. G. I. Hill, Macdonald C. I. spoke on "The Second Year of Computer Science". Mr. W. J. Hawkshaw, Birchmount Park C. I., talked on the history of vectors to Grade 13 teachers. The main speaker at the second meeting was Professor J. M. Ham, Dean, Faculty of Applied Science and Engineering, University of Toronto. His topic was "Mathematics and Problem Solving", a topic of interest and concern to mathematics teachers at all levels.

MEETINGS

NORTHWESTERN ONTARIO REGIONAL MATHEMATICS CONFERENCE

SPONSORS: Ontario Association of Teachers of Mathematics
National Council of Teachers of Mathematics
O.S.S.T.F. Professional Development Committee
"All persons (elementary, secondary and post secondary) engaged or interested in the teaching of mathematics are invited to participate".

LOCATION: Lakehead University Centennial Building.

DATES: Friday, April 28: Saturday (A.M.) April 29, 1972.

REGISTRATION FEE: $1.00
O.A.T.M. or N.C.T.M. members - free.

O.A.T.M. MEMBERSHIPS:

Memberships ($6.00) may be purchased when registering at the conference. They include Ontario Educational Association membership and subscription to the "Ontario Mathematics Gazette".

N.C.T.M. MEMBERSHIPS:

Memberships in N.C.T.M. will also be available. A variety of benefits are included.

DISPLAYS: N.C.T.M. publications.

LUNCHEON: To be announced.

CONFERENCE COMMITTEE:

Dr. G. Vervoort, Faculty of Education, Lakehead University

Mr. D. Botly, Faculty of Education, Lakehead University

Mr. G. Ellis, Port Arthur C. I., Thunder Bay

Mr. R. Kitching, Selkirk C. & V. I., Thunder Bay

Mr. K. Hill, Fort Frances High School, Fort Frances

Friday, April 28

9:00- 9:50	Registration & N.C.T.M. publication display.
9:50-10:00	Welcome & Information.
10:00-11:30	General Session
	Dr. Steven S. Willoughby, New York University, New York Accountability: "Facts Fads and Frauds".
11:30- 1:00	Luncheon - To be announced.
1:00- 2:30	Group Sessions (choose one).
	1. Junior-Intermediate Division
	"Newspaper Mathematics"
	Mr. J. W. LeSage, Ontario Department of Education, Program Consultant, currently teaching at the College of Education, Toronto.

2. Senior Division

"A Diversified Program in Mathematics for the Senior Division."

Mr. George Scroggie, Curriculum Branch, Ontario Department of Education.

2:30- 3:30 Coffee and informal discussion period with Messrs. Willoughly, LeSage and Scroggie.

Evening Cultural Events: Random Socializing, North Shore Motor Hotel Lounge.

Saturday, April 29

9:30-10:30 General Panel Discussion and Rap Session.

Grades K-13 Title: "The Aftermath" - A searching evaluation focusing on useful and useless curriculum changes, methodology approaches and teacher training practices with special emphasis on what needs to be done.

MODERATOR: Mr. K. Hill, Fort Frances High School.

PANELISTS: Mr. H. B. Rapley, Ontario Department of Education Programme Consultant

Mr. G. Ellis, P.A.C.I.

Dr. G. Vervoort, Faculty of Education, Lakehead University

Mr. L. D. Black, Department of Mathematics, Lakehead University.

_____ Public School

_____ Separate School

10:30-12:00 Group Sessions

1. Junior-Intermediate Division

   "Newspaper Mathematics" - J. W. LeSage

   - a repeat or continuation of Friday session.

2. General Interest

   "Mathematics in New York State"

   - Dr. S. Willoughby.

N.C.T.M. Convention Saskatoon

This will be held on the campus of the University of Saskatchewan, Wednesday, Thursday, and Friday, August 23, 24, 25, and will be of interest to all mathematics teachers. Titles of topics to be considered range from "The Next Decade in Mathematics Education" through "An Approach for Enriching the Mathematics Program Using Community Resources" to "Recreational Mathematics for all". For further details write to Floyd Cousins, 14 Hoeschen Crescent, Saskatoon.

Reminder

OATM Annual Convention, Waterloo, May 12-14, 1972.
(see Volume 10, #2 for more details).

LETTER TO THE EDITOR

Dear Dr. Davison:

I would like to draw to the attention of readers of the Gazette, the existence of the OATM Curriculum Materials Committee. The purpose of this committee is to seek out material aids to teaching which have been found to be useful and to publicize these so that other Mathematics teachers may benefit. The committee is especially anxious to hear from teachers who have devised some technique, discovery exercise, or learning aid of any sort. Mr. Coenraads'* slide rule is one very appealing idea. If any Gazette reader has created something which could be shared with the other Mathematics teachers or knows someone who could be persuaded to share some useful idea, please forward these to me, as chairman of the committee, or directly to the Gazette editorial board, it is is in suitable form to be published.

R. P. Broughton,
Toronto, Ont.

* see page 160 this issue.

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