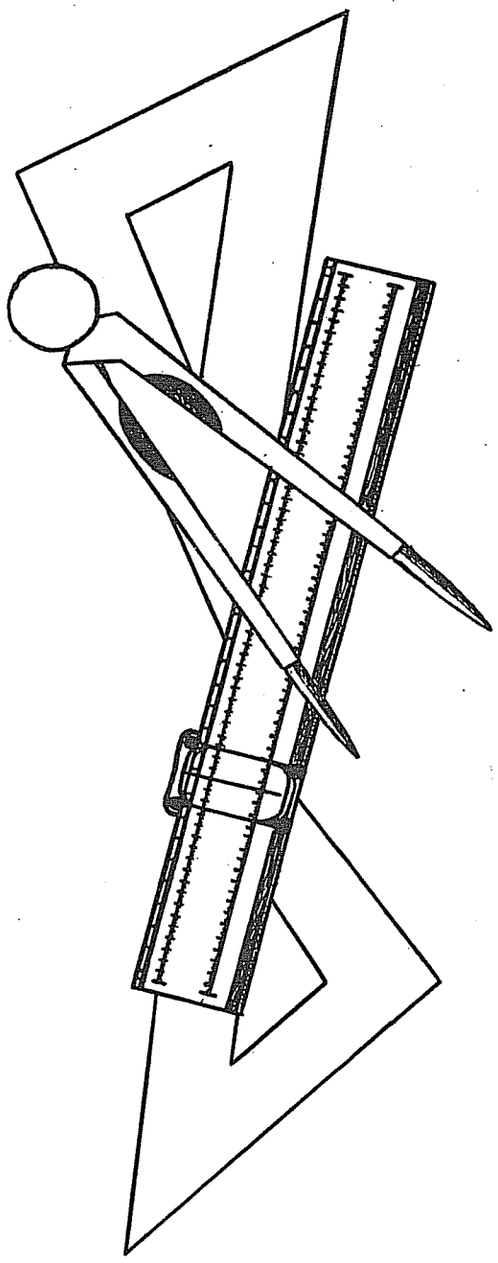
VOLUME 11
NUMBER 1
SEPTEMBER 1972



ONTARIO MATHEMATICS GAZETTE

sponsored by
THE SAMUEL BEATTY FUND
THE ONTARIO MATHEMATICS COMMISSION
THE ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS

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EDITORIAL

Dr. T. Davison and his board of editors have relinquished the task of editing the Gazette. Dr. J. S. Griffith and Dr. W. Eames of Lakehead University have taken over the editorship. We would like to thank the previous editors especially Dr. T. M. K. Davison for their work and congratulate them on maintaining a high standard in the Gazette.

Lakehead University has a Department of Mathematical Sciences with approximately 1200 students (unfortunately mainly in service courses) and fifteen faculty. We are very interested in liaison with the schools and Dr. Griffith has prepared a rough draft of suggested applications of the computer to high school topics that is being distributed to schools in our region. Dr. G. Vervoort has pioneered a highly successful course of mathematics for elementary school teachers. The largest area of concern that my colleagues in the university have about the school system is the manner in which fluency in mathematical manipulation seems to have been replaced by a rather vague impressionistic view of mathematical and physical ideas. It is certainly good for students to have a wide view of the ideas and place of mathematics in the world, but some method of generating sufficient acquaintance with the elements of arithmetic, algebra and geometric manipulations to avoid students making trivial mistakes that hinder their understanding of advanced mathematics needs to

be found. Why do university students make mistakes in adding fractions, have difficulty in determining algebraic factors, cannot use trig. functions correctly, cannot use log tables, etc.? Perhaps drill exercises should occupy a portion of each week, with emphasis on speed and accuracy. The rest of the week could be devoted to more advanced topics. The lack of uniformity of curricula within schools forces us to teach to the least common denominator in the first year, devoting time to topics which some students are (or consider themselves to be) familiar with, and consequently tend to turn off their attention. Suggestions and comments are welcomed, and some will hopefully be published in a new section entitled 'Educational forum'.

The purpose of the Gazette is unchanged:- "The promotion and advancement of mathematics and mathematics education in primary and secondary schools, and in community colleges". In the March 1970 issue, T. Davison wrote:-

"Most of the recipients of the Gazette teach mathematics at one of the above levels. We select material accordingly.

Main Articles. We seek discussive and expository articles reflecting the content or mode of current mathematics or mathematics teaching.

Mathematical Notes. We seek short papers which give new insights, new proofs of old theorems, applications (in any sense of the word) of standard material, which are suitable for classroom

presentation or for student projects.

Problems. We seek easily stated problems, particular problems in that part of modern mathematics which is now finding a place in the 'new' curricula.

Book Reviews. We seek reviews of textbooks and collateral material. Critical as well as informative reviews of textbooks are welcomed, especially those based on classroom use. Thus reviews of established texts are acceptable, not only of new books.

Notices of Meetings. Since the Gazette is a medium for the exchange of information, we wish to inform our readers of relevant meetings and conventions. Thus we welcome announcements of such.

Notes on Meetings in Mathematical Education. We seek reports of such meetings.

It has been pointed out to me that there has recently been a tendency in recent issues to stress the philosophical aspects of mathematics education with a consequent decrease in mathematics itself. Some of you may be in favour of this tendency, others opposed. Let me know your ideas remembering that it is from our subscribers that most material comes, so criticism should be constructive with a source for articles of the type you want to see suggested.

J. S. Griffith
W. Eames

THE EXPANSION OF PASCAL'S TRIANGLE INTO n DIMENSIONS

Thomas R. Bull

In my project I only encountered two sidetracks. The first, and the most difficult to untangle, was involved with the equation which expressed the value of a coefficient as the value of up to three other coefficients. I kept trying to prove an incorrect equation. Once I realized this, the first sidetrack was over.

The second sidetrack involved in the expression of the general equation for the value of a coefficient using pi and sigma notation. At times the lower limit would become greater than the upper. When I found no solution in references, I modified the notation by using the "is less than" sign to control the size of the lower limit.

References: Several references gave no information, but I will list them anyway:

Algebraic Structures and Probability by Elliott, Fryer, Gardner & Hill

The Calculus by William L. Shaaf

Advanced Engineering Mathematics by Erwin Kreyszig

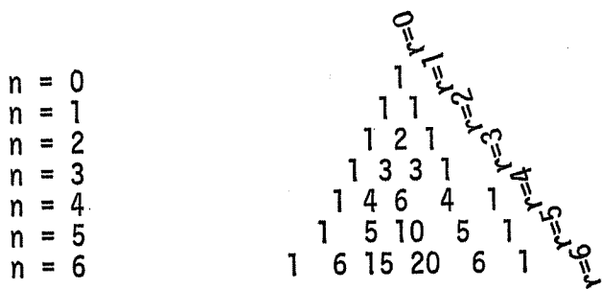
My findings began at about the middle of page four. I have duplicated some previous work, that I know of. I still know of no previous work which expresses the value of a coefficient of a term in an expanded polynomial as the sum of up to as many other

coefficients as there are variables in the polynomial.

I have also proven all my findings.

PASCAL'S TRIANGLE

Pascal's Triangle is an arrangement of numbers that look like this:



The r's and n's are used to classify the columns and rows, and they serve another purpose which will be described later.

The sum of all the numbers in each row is equal to 2^n. The sum of the numbers in the previous row is one-half of the total of the numbers in row n, or 2^{n-1}. "n" represents any row. Each number in Pascal's Triangle is equal to the sum of the two numbers above it to the right and left diagonally. I will prove these on the next sheet.

There are two uses of Pascal's Triangle. It gives the coefficients of any binomial expanded to the nth degree, i.e. $(a+b)^n$. In this case n represents the degree of the binomial, and r represents one of the exponents in each term. The other exponent is, obviously, n-r.

The other application is in probability theory. In this case n means that a total of n events will take place. The

r means that one event may take place r times, e.g. a coin turns up tails r times. The other event takes place n-r times. The probability of the event taking place is the coefficient with the proper values for n and r over the total of all the numbers in the row, e.g. if we flip a coin twice the probability of a head and tail turning up in any order is $\frac{2}{4}$, or $\frac{1}{2}$. This only applies when there are only two different variables, e.g. heads and tails.

Each coefficient is equal to a combination, i.e. the number of ways that a certain number of two kinds of objects can be arranged.

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

THE PROOF OF PASCAL'S TRIANGLE

Each coefficient in Pascal's Triangle is a combination because the coefficient is the number of ways each term can be written without the use of exponents. The coefficient of x^2y^2 is six. We can write x^2y^2 as xxyy, xyxy, yxxy, yyxx, yxyx and xyyx. There are six ways of writing it. This holds for all terms, if you wish to check it.

Here is the proof that each coefficient is the sum of two other coefficients:

$$C(n,r) = C(n-1,r)+C(n-1,r-1)$$

$$\frac{(n-1)!}{r!(n-r-1)!} + \frac{(n-1)!}{(r-1)!(n-r)!} = \frac{(n-1)!(n-r)+(n-1)!r}{r!(n-r)!}$$

$$= \frac{(n-1)!(n-r+r)}{r!(n-r)!} = \frac{n!}{r!(n-r)!}$$

Now I will prove that the sum of all the numbers in each row is 2^n.

$$C(n,0)+C(n,1)+C(n,2)+\ldots+C(n,n)$$

$$= \sum_{r=0}^{n} C(n,r) = \sum_{r=0}^{n} 1\ C(n,r) = (1+1)^n$$

$$= (2)^n = 2^n$$

The above is a fairly sufficient proof of Pascal's Triangle. Since this is intended only as a review of Pascal's Triangle I have not dealt with the proof too rigorously. The proof of my generalization of Pascal's Triangle will be more rigorous.

THE PURPOSE OF MY PROJECT

Pascal's Triangle can only be used when there are only two variables being used, either in algebra or probability.

I decided that working on a similar footing as Pascal's Triangle I could expand its use so that it could be used for any number of variables. In other words I decided that I would generalize Pascal's Triangle.

THE DEVELOPMENT OF MY PROJECT

I tried many various ways of expanding Pascal's Triangle into three dimensions. I finally arrived at what you see here. I then decided to make a model of Pascal's Triangle expanded

into four dimensions. In order to display it in three dimensions it had to be dismembered. I could only make a mathematical model of my generalization of Pascal's Triangle. I will show it later

In order to arrive at a general equation for my project I examined some more special equations. The special equations that I examined had to do with the value of each coefficient being equal to the sums of other coefficients. I developed the three variable equations myself. The equations are:

1 variable: $\frac{n!}{n!} = \frac{(n-1)!}{(n-1)!}$

2 variables: $\frac{n!}{r!(n-r)!} = \frac{(n-1)!}{r!(n-r-1)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$

3 variables: $\frac{n!}{r!s!(n-r-s)!} = \frac{(n-1)!}{r!s!(n-r-s-1)!} + \frac{(n-1)!}{r!(s-1)!(n-r-s)!} + \frac{(n-1)!}{(r-1)!s!(n-r-s)!}$

which can be proven:

$$\frac{(n-1)!}{r!s!(n-r-s-1)!} + \frac{(n-1)!}{(r!(s-1)!(n-r-s)!} + \frac{(n-1)!}{(r-1)!s!(n-r-s)!}$$

$$= \frac{(n-1)!(n-r-s)+(n-1)!r+(n-1)!s}{r!s!(n-r-s)!}$$

$$= \frac{(n-1)!(n-r-s+r+s)}{r!s!(n-r-s)!} = \frac{(n-1)!n}{r!s!(n-r-s)!}$$

$$= \frac{n!}{r!s!(n-r-s)!}$$

ADVANCEMENTS IN MY PROJECT

I realized quite soon that the number of variables

involved would determine the number of dimensions in the expansions of Pascal's Triangle. An expansion of Pascal's Triangle for d variables will also hold for any number of variables less than d. To see this examine my models.

Now I will explain how I got to my general equation which states that the value of each coefficient can be expressed as the sum of d other coefficients, as well as a simpler equation giving the value of the coefficient. d is the number of variables. First, let me say that in Pascal's Triangle each coefficient can be considered to be in a triangle. The value of this coefficient is the sum of the two coefficients in the triangles at the apex of the previous triangle. In the same way each coefficient in the three dimensional tetrahedron each coefficient can be considered to be in a tetrahedron. The value of the coefficient in this tetrahedron is the sum of the coefficients in the three tetrahedrons at the latter tetrahedron's apex. For the four dimensional expansion each coefficient can be considered to be a hyper-tetrahedron. The value of this coefficient is the sum of the coefficients at its hyper-apex. In the case of the five variable expansion each coefficient can be considered to be in a hyper-hyper-tetrahedron. The value of each coefficient is the sum of the five coefficients at its hyper-hyper-apex. If we look at the physical model of each case it appears that at times the value of a coefficient is the sum of less than d other coefficients. This is true.

If we look at the mathematical model we see that the value(s) of the absent coefficient(s) works out to be zero. Also, at the apex of the physical model it appears that the 1 comes from nowhere. This can be explained using the mathematical model.

Before I show the beginnings of my general equations I will explain what all the variable indices mean. d stands for the number of variables in the expansion. r_1, r_2, r_3, etc. stand for r, s, t, etc., respectively. They stand for the same thing that r did in Pascal's Triangle, except that there are more for each dimension. Where a stands for any natural number, there is one less r_a than there are variables.

A MATHEMATICAL MODEL OF THE GENERALIZATION OF PASCAL'S TRIANGLE

The following formula is my rudimentary general equation for the values of each coefficient:

$$\frac{n!}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!} = \frac{(n-1)!}{\prod_{a=1<d}^{d-1} r_a!\left(n-1-\sum_{a=1<d}^{d-1} r_a\right)!}$$

$$+\frac{(n-1)!}{\prod_{a=1<d}^{d-2} r_a!\,(r_{d-1}-1)!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!}+\frac{(n-1)!}{\prod_{a=1<d}^{d-3} r_a!(r_{d-2}-1)!(r_{d-1})!\left(n-\sum_{a=1<d}^{d-1} r_a\right)}$$

+ ... for d terms

It is, in more refined form:

$$\frac{n!}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!}$$

$$= \sum_{j=1}^{d} \frac{(n-1)!}{\prod_{a=k<d}^{d-j} r_a! \prod_{a=d-j+1<d}^{d-j+1<d} (r_a-1) \sum_{a=d-j+2<d}^{d-1} r_a! \left(n-1- \sum_{a=1<d}^{d-j+1<d} r_a - \sum_{a=d-j+1<d}^{d-j+1<d} \right) \over (r-1) - \sum_{a=d-j+2<d}^{d-1} r_a \Big)!}$$

In this equation ignore all the terms where the lower limit in the pi and sigma notation is greater than the upper. I have devised a system to show this in the notation. Greater or less than signs are used in expressing the limits. This can be seen in the above equation.

The sum of all the coefficients with the same value for n is d^n.

When we apply this generalization to the probability aspect of the generalization, the probability of an event, or sequence taking place is:

$$\frac{n!}{\prod_{a=1<d}^{d-1} r_a! \left(n - \sum_{a=1<d}^{d-1} r_a \right)!} \Big/ d^n$$

This is my model of the generalization of Pascal's Triangle.

PROOF OF MY MATHEMATICAL MODEL

$$\sum_{j=1}^{d} \frac{(n-1)!}{\prod_{a=1<d}^{d-j} r_a! \prod_{a=d-j+1<d}^{d-j+1<d} (r_a-1)! \prod_{a=d-j+2<d}^{d-1} \left(n-1- \prod_{a=1<d}^{d-1} r_a \sum_{a=d-j+1<d}^{d-j} (r_a-1) - \sum_{a=d-j+2<d}^{d=1} r_a \right)!}$$

$$= \frac{(n-1)!\left(n+\sum_{a=1<d}^{d-1} r_a - \sum_{a=1<d}^{d-1} r_a\right)!}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!} + \frac{}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!}$$

$$\sum_{r_a=0}^{n} \frac{n!}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!} = \sum_{r_a=0}^{n} \frac{n!}{\prod_{a=1<d}^{d-1} r_a!\left(n-\sum_{a=1<d}^{d-1} r_a\right)!}$$

by a simple transformation we get

$$\left(\sum_{k=1}^{d} 1\right)^n = (d)^n = d^n$$

Now I will prove that the above stated equation for the value of each coefficient is, in itself, true. In the proof we multiply together the coefficients of expanded binomials. There is one coefficient for each term. The value for n is reduced from one term to another because each time a certain number of each term is removed there is a greater chance of picking out all of one type of the remaining terms, until it is one.

$$C(n,r)\cdot C(n-r,s)\cdot C(n-r-s,t)\cdot\ldots\cdot) = \prod_{j=1}^{d-1} C\left(n-\sum_{a=1<j} r_a, r_j\right) = \prod_{j=1}^{d-1} \frac{\left(n-\sum_{a=1<j}^{j^*} r_a\right)!}{r_j!\left(n-\sum_{a=1<j} r_a - r_j\right)!}$$

it can be seen, by cancelling out from term to term, and changing the dummy symbols:

$$\frac{n!}{\prod_{a=1}^{d-1} r_a! \; n - \sum_{a=1}^{d-1} r_a !}$$

THE MODELS

The model entitled Pascal's Triangle expanded into three dimensions is just that. It follows the equations set up in my general model. The large number on each tetrahedron is a coefficient of one term of the trinomial to the nth degree. n is the number in the top line enclosed in brackets. The numbers in the bottom line are two of the three exponents in the term, the other is easily found. Here is how you can find the probability of an event which allows three different, equally likely, things to happen.

Say you want n of these smaller of these events to take place, then you would look to the nth level of the tetrahedron, and add all the coefficients. Now find the coefficient whose two exponents are the same as the number of times you want two of the lesser events to take place. Place this coefficient over the sum of all the coefficients, and this is the probability of the entire event.

The model entitled a representation of Pascal's Triangle expanded into four dimensions, follows the general equations for four variables. It is really only a picture, because it does not show how we got these coefficients from the sum of other

coefficients, directly. Each number in this expansion is a coefficient of a polynomial of four terms expanded to the nth degree. Again n is the number in the top row in the brackets, the bottom three are the exponents of three of the terms, the other can again be found easily. The same basic method is used to find the probability of an event, except there are four variables. Remember this time each tetrahedron has the same value for n.

Note how the sides of each tetrahedron is a portion of the tetrahedron in one less dimension.

SYMBOLISM

$$C(n,r) = \binom{n}{r} = \frac{n!}{n!(n-r)!}$$

$$\binom{n}{r,s,t,\ldots} = \binom{n}{r_1,r_2,r_3,\ldots} = \frac{n!}{r_1!r_2!r_3!\ldots(n-r_1-r_2-r_3-\ldots)!}$$

$!$ is the factorial sign, e.g. $4! = 4 \times 3 \times 2 \times 1$

$\sum$ is the summation sign

Π is the product sign.

CONCLUSIONS

I have made a general mathematical model for my generalization of Pascal's Triangle. It does the same things that Pascal's Triangle does, except my generalization works for any number of variables. I have proven the general model, and it works.

Finally, I know of no previous work in this specific

area.

Thomas Bull was in grade 9 (Woodbine Jr. High School) and in September will be in grade 10 (George S. Henry Secondary School). His project won:-

1st prize (Mathematics and Computer) 1972 Metro Toronto Science Fair

Best Junior Project (Grades 7-9) 1972 Metro Toronto Science Fair

NASA award

1st prize Junior Project (Grades 7-9) Physical Sciences 1972 Canada Wide Science Fair

with several other awards. His hobbies include war games, rowing, stock market, politics, languages, art, history, ping pong, religion, collecting, reading, badminton, thinking ...

DOWN WITH SILLY DIVISION

Joan Routledge, Master Teacher - Mathematics
York County Board of Education

In the April 1970 edition of the Ontario Secondary School Mathematics Bulletin, Dr. K. D. Fryer had the following mini-editorial.

> "Last year I smiled as my friend John Del Grande, Co-ordinator of Mathematics for the Borough of North York, told me of his frustrations over his daughter's problem with "the new division". Last week I asked me grade seven daughter to divide 456 by 2. This was her solution.

```
2 | 456 | 223
    446
    ---
     10 |   5
     10
     --
      0 | 228
```

> It is time that steps were taken to have the silly division method demonstrated above down-played or eradicated from the elementary schools. Use it once or twice if you must, for the sake of "understanding" but then get on with the practical method. When last seen my daughter was trying to divide decimals using this method. Try it!

Surely one of the fundamental practices of pedagogy stressed at all Colleges of Education is that you

should not teach something that has to be untaught. Maybe such practices haven't filtered down to the Teachers' Colleges yet. Or maybe no one has told them that such teachings have to be untaught. Let's tell them.

Let's start a campaign.

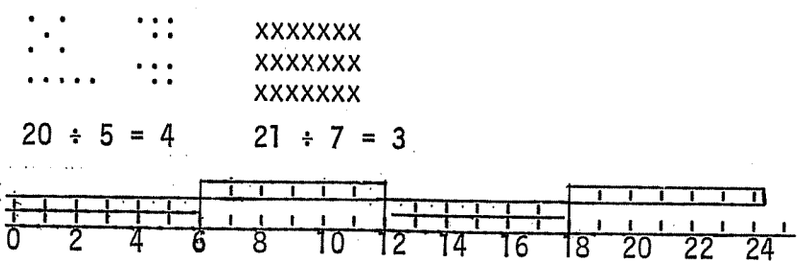
DOWN WITH SILLY DIVISION!"

Many elementary teachers will say a hearty amen to Dr. Fryer's editorial. The "new division" was supposed to overcome many of the difficulties children encountered in attempting to master the traditional division algorithm. The same type of child is having just as much trouble with the "new" method as he would have had with the old. And what is more alarming, a number of above average students are entering the intermediate grades with very inefficient methods for dividing by a one-digit as well as a two-digit divisor. Only a few students have sufficient mastery of "short" division to be able to perform it mentally.

Since the introduction of "new" mathematics, elementary teachers have had to introduce several new topics into the curriculum - sets, geometry, numbers in other bases, equations in problem solving, to name a few. At the same time they have been pressured into throwing out the textbook and workbooks and writing their own programs geared to the needs of the individual children under their care. They are advised to use workshop techniques

and laboratory methods. Ideas and processes must be discovered and explored, and the child's own methods accepted. Drill on basic facts and practice sessions for building and maintaining computationsl skills have been down-played, even outlawed. Experienced teachers who know that repetitive practice is essential in order to establish skills have had to go underground with their drill.

The division algorithm can be mastered only when all the skills involved in it have been mastered. First of all, it is essential that the child understands thoroughly what is meant by division. Kindergarten and primary teachers take advantage of every situation where division is required. Jelly beans are shared equally by some children. They find out how many children could have three sheets of construction paper from a pile. Older children demonstrate division with sets of objects, with arrays, and with line segments on a number line. A number sentence is written to describe these pictures



There is little to be gained by stressing the division method shown on the right. The student should be made aware of the fact that it is much more efficient to use his multiplication-division facts. In the second example, he should think of the product on the six-times-table that is nearest but less than 32. Then the five sixes or thirty can be subtracted all at once. He must be skilled in subtraction, or he is in trouble!

```
5 | 20 |
  |  5 | 1
  | 15 |
  |  5 | 1
  | 10 |
  |  5 | 1
  |  5 |
  |  5 | 1
  |  0 | 4
```

```
6 | 32 |
  |  6 | 1
  | 26 |
  |  6 | 1
  | 20 |
  |  6 | 1
  | 14 |
  |  6 | 1
  |  8 |
  |  6 | 1
  |  2 | 5
```

```
5 | 20 |
  | 20 | 4
  |  0 |
```

```
6 | 32 |
  | 30 | 5
  |  2 | 5
```

```
6 | 496 |
  | 480 | 80
  |  16 |
  |  12 |  2
  |   4 | 82
```

```
9 | 3026 |
  | 2700 | 300
  |  326 |
  |  270 |  30
  |   56 |
  |   54 |   6
  |    2 | 336
```

When tens of sixes are involved as in the first example above, the student can use his multiplication tables as he did before. He thinks of the product on the six times table that is nearest but less then 49 tens. That is 48 tens or 480, the product of 80×6. In the second example, the student should see that there are not one thousand nines in three thousand, but there are at least one hundred nines in thirty hundred. Twenty-seven hundred or three hundred nines is the closest that he can get. The student has to be encouraged to do division questions with the fewest

possible steps.

The advantage in placing the partial quotients down the side instead of on the top is not appreciated fully until two digit divisors are introduced. This method allows for under estimations of partial quotients without having to erase in the body of the algorithm.

```
68 | 2788
   | 2040   30
   |  748
   |  680   10
   |   68
   |   68    1
   |    0   41
```

```
68 | 2788
   | 2720   40
   |   68
   |   68    1
   |    0   41
```

```
68 | 2788
   |  680   10
   | 2108
   |  680   10
   | 1428
   |  680   10
   |  748
   |  680   10
   |   68
   |   68    1
   |    0   41
```

Students should be discouraged from using the very inefficient method demonstrated in the third example.

Many students find it less confusing to place the digits for the quotient in their correct place above the dividend, especially if they use a quadrille ruled workbook. If they are taught this method when two-digit divisors are introduced, they may need a method for adjusting for underestimated partial quotients until they become skillfull in estimating. The second example below illustrates this.

```
      409
76 | 31084
     304
     684
     684
       0
```

```
      408
76 | 31084
     304
     684
     608
      76
      76
       0
```

The students will need to be able to perform the traditional algorithm shown above before they are asked to divide by decimal fractions. If they are skilled in using this traditional algorithm, they will find no difficulty in understanding and performing short division on paper, and then mentally, short division is very useful in removing common factors to reduce fractions to lower terms.

$$2\overline{)456}\quad 228$$

Let's join Dr. Fryer's campaign. DOWN WITH SILLY DIVISION.

CLASSROOM NOTES

CONTIG

R. Wigle, Chairman, Secondary Advisory Committee,
Ontario Mathematics Commission

PURPOSE

To provide practice in the four basic operations using the counting numbers (1, 2, 3, ..., 12).

To provide practice in the use of the commutative, associative, and distributive properties.

METHOD

Preparation - Prepare a playing board like the one pictured on page 24. You will need a supply of markers which may be used to cover the numbered squares on the playing board for each group of 2 or more students playing the game. You will need to make 3 dice from wooden cubes. One die contains the numbers 1 to 6, a second contains the numbers 4 to 9, and the third the numbers 7 to 12.

Rules of the game:

1. Two or more players may play CONTIG.
2. To begin play, each player in turn rolls all three dice and determines the sum of the three numbers showing. The player with the smallest sum begins play. Play then progresses from left to right.
3. The first player rolls the three dice. He must use one or

two operations on the three numbers shown on the dice. He is then allowed to cover the resulting number on the board with a marker. When he has finished his turn, he passes the dice to the player on high right. He may not cover a number which was previously covered.

4. To score in CONTIG, you must cover a number on the board which is adjacent vertically, horizontally, or diagonally to another covered number. One point is scored for each adjacent covered number.

When a player rolls the dice and is unable to produce a number which has not already been covered, he m t pass the dice to the next player. If he incorrectly passes the dice believing he has no play when he does, in fact, have a play, any of the other players may call out his mistake and cover the number as their own.

A cumulative score is kept for each player. A player is eliminated from further play in a game when he fails, in three successive turns, to produce a number which can be covered. When all players have experienced three successive failures to produce a coverable number, the game ends. The player with the highest cumulative score wins.

CONTIG

Use three dice: one numbered: 1 to 6; one numbered: 4 to 9; one numbered: 7 to 12.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
15	16	17	18	19	20	21	22	23	24	25	26	27	28
29	30	31	32	33	34	35	36	37	38	39	40	41	42
43	44	45	46	47	48	FREE	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79	80	81	82	83
84	85	86	87	88	89	90	91	92	93	94	95	96	FREE
97	98	99	100	101	102	103	104	105	106	107	108	109	110
111	112	113	114	117	120	121	126	128	130	132	135	136	140
143	144	147	150	153	154	156	160	162	165	168	175	176	180
189	192	196	198	200	210	FREE	216	220	224	225	231	240	243
245	250	252	256	264	270	275	280	282	288	294	297	300	308
315	320	324	330	336	350	352	360	378	384	385	396	400	405
420	432	440	450	462	480	486	495	504	528	540	576	594	648

VARIATIONS:

1. Use a one minute egg timer to time the turn of each player. This will tend to speed up the game.

2. Allow for any player to challenge an opponent if the opponent does not choose the number which will score the maximum

number of points. The challenger should then receive the difference between the number of points scored by the chosen number and the greatest number of points which could have been scored.

3. For a faster game, allow only five turns for each player. The player with the highest score at the end of the fifth round would be the winner.

4. Play as a solitaire game and attempt to score as many points as possible before experiencing three successive unsuccessful rolls of the dice.

CONTIG

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	44	45	48	50	54	55
60	64	66	72	75	80	90	96
100	108	120	125	144	150	180	216

Use three standard dice.

CONTIG WITH FRACTIONS

Use two dice: one marked: 1/6, 1/5, 1/4, 1/3, 2/5, 1/2; one marked: 1/2, 3/5, 2/3, 3/4, 4/5, 5/6

$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{3}{25}$	$\frac{1}{8}$	$\frac{2}{15}$	$\frac{5}{36}$	$\frac{3}{20}$	$\frac{4}{25}$
$\frac{1}{6}$	$\frac{3}{16}$	$\frac{1}{5}$	$\frac{5}{24}$	$\frac{2}{9}$	$\frac{6}{25}$	$\frac{1}{4}$	$\frac{4}{15}$	$\frac{5}{18}$
$\frac{3}{10}$	$\frac{5}{16}$	FREE	$\frac{8}{25}$	$\frac{1}{3}$	$\frac{7}{20}$	FREE	$\frac{3}{8}$	$\frac{2}{5}$
$\frac{5}{12}$	$\frac{13}{30}$	$\frac{4}{9}$	$\frac{7}{15}$	$\frac{12}{25}$	$\frac{1}{2}$	$\frac{8}{15}$	$\frac{11}{20}$	$\frac{5}{9}$
$\frac{7}{12}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{19}{30}$	$\frac{13}{20}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{3}{4}$	$\frac{23}{30}$
$\frac{4}{5}$	$\frac{5}{6}$	$\frac{17}{20}$	$\frac{13}{15}$	$\frac{9}{10}$	$\frac{11}{12}$	$\frac{19}{20}$	$\frac{29}{30}$	1
$\frac{31}{30}$	$\frac{21}{20}$	$\frac{16}{15}$	$\frac{13}{12}$	$\frac{11}{10}$	$\frac{17}{15}$	$\frac{23}{20}$	$\frac{7}{6}$	$\frac{6}{5}$
$\frac{37}{30}$	$\frac{5}{4}$	FREE	$\frac{13}{10}$	$\frac{4}{3}$	$\frac{3}{2}$	FREE	$\frac{8}{5}$	$\frac{5}{3}$
$\frac{9}{5}$	$\frac{15}{8}$	2	$\frac{25}{12}$	$\frac{9}{4}$	$\frac{12}{5}$	$\frac{5}{2}$	$\frac{8}{3}$	3
$\frac{16}{5}$	$\frac{10}{3}$	$\frac{18}{5}$	$\frac{15}{4}$	4	$\frac{25}{6}$	$\frac{9}{2}$	$\frac{24}{5}$	5

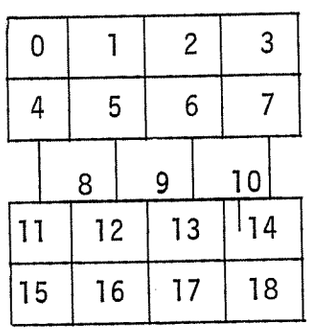
CONTIG

Use with three dice: two numbered -6 to -1; and one numbered 1 to 6.

-72	-66	-60	-55	-54	-50	-48	-45	-44	-42	-41
-40	-39	-38	-37	-36	-35	-34	-33	-32	-31	-30
-29	-28	FREE	-27	-26	-25	-24	-23	FREE	-22	-21
-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
2	3	4	5	6	FREE	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	FREE	36	37	38	39	40	FREE	41	42
44	45	48	50	54	55	60	64	66	72	75
80	90	96	100	108	120	125	144	150	180	216

CONTIG

Use only addition and subtraction with three regular dice.



CONTIG

Use two regular dice and the four basic operations.

1	2	3	4
5	6	7	8
9	10	11	12
15	16	18	20
24	25	30	36

TWELFTHS

H. Allen, Queen's University Faculty of Education

EQUIPMENT:

Two dice, each bearing the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{5}{12}$. (Opposite faces total $\frac{7}{12}$.)

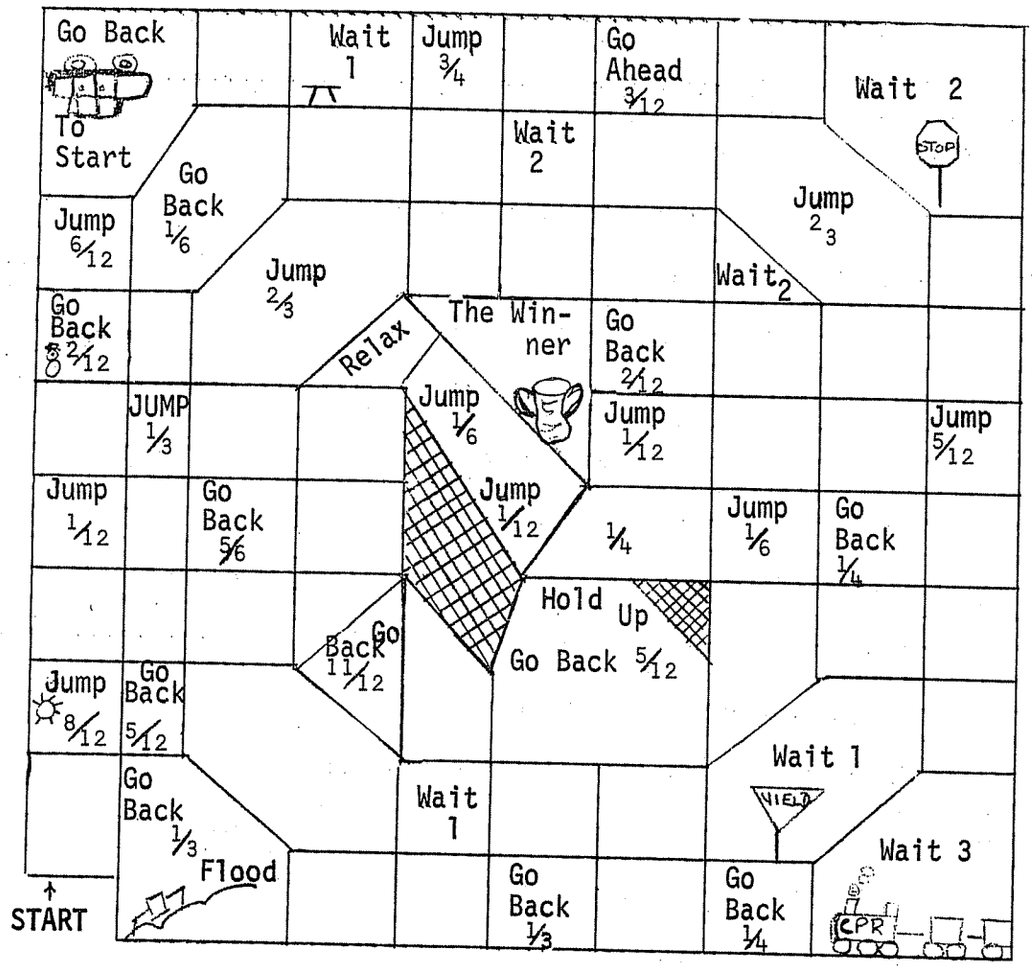
A playing board as shown on the next page.

RULES:

In this game, each step on the way to the prize, represents one-twelfth of a unit. The player rolls the dice and advances a number of units equal to the sum of the up-faces on the dice. For example, if one die shows $\frac{1}{2}$ and the other shows $\frac{1}{4}$, the player would advance 9 space.

The winner is the first person to reach the prize (or the person farthest along the way if time has been called). In attempting to reach the prize, you must throw the <u>exact</u> total required.

The other players must not help the player with the dice. If the player with the dice calculates his total correctly and advances to the proper square, he remains there until his next turn. If the player with the dice makes a mistake in calculating his total and/or advances to the wrong square, he either loses his turn or must play it correctly WHICHEVER IS LESS FAVOURABLE.


Go Back
To Start
Wait 1
Jump 3/4
Go Ahead 3/12
Wait 2
Wait 2
Go Back 1/6
Jump 6/12
Jump 2/3
Jump 2/3
Wait 2
Go Back 2/12
Relax
The Winner
Go Back 2/12
JUMP 1/3
Jump 1/6
Jump 1/12
Jump 5/12
Jump 1/12
Go Back 5/6
Jump 1/12
1/4
Jump 1/6
Go Back 1/4
Hold Up
Go Back 5/12
Go Back 11/12
Jump 8/12
Go Back 5/12
Wait 1
Go Back 1/3
Wait 1
Flood
Go Back 1/3
Go Back 1/4
Wait 3
START

BASEBALL

D. R. Campbell, Althouse College of Education

AIM: To provide practice and/or testing for any section in mathematics in an interesting manner.

THE GAME

1. Each team should be made up of 5 players for best results. However, any number of players could be used on each team.

2. Each player is paired off with another player of the opposing team.

3. The team at bat gets the chance to select a SKILL TESTING CARD (all cards are face down). If he answers the question correctly, he has the right to draw a card from the BASEBALL CARDS which are also face down. (See example below.) These cards give the offensive action, and the player moves his marker according to instructions on the card, e.g. advance one base, out, etc. Runs and outs are recorded on the score sheet (see Playing board).

4. If the offensive player does not answer the Skill Testing Card correctly, his opponent with whom he is paired has a chance to answer. If the defensive player answers it correctly, then it counts as an OUT against the offensive team. However, if the defensive player fails to answer it correctly the question goes to the next offensive player and the cycle is repeated.

5. Each team gets 3 outs. After 3 outs, the offensive team becomes the defensive team.

6. There is a pre-arranged time limit to answer the question.

7. The game is finished by a pre-determined number of innings or at a certain time.

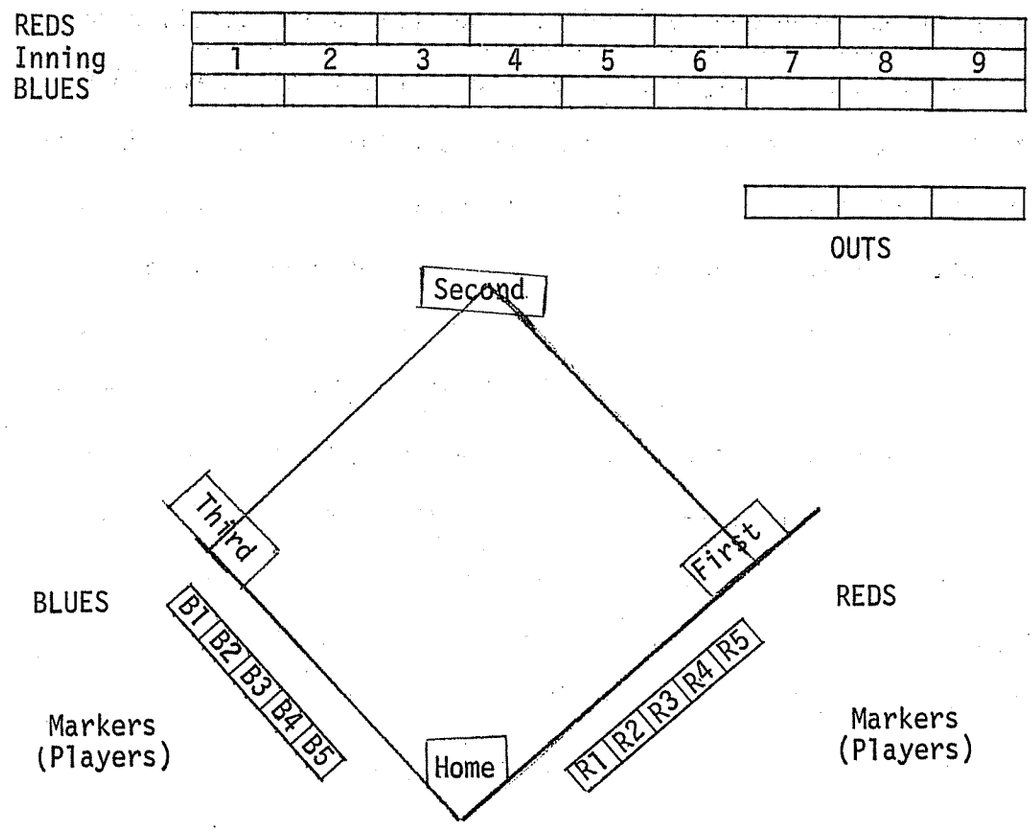
8. Teachers may wish to revise the rules to suit their situation.

9. Students can take turns acting as umpire.

THE PLAYING BOARD

There should be a large playing board approximately 1 foot square, somewhat as shown below. There should be markers for each player, preferably with a colour for each team.

SCORE



A WORKING SET OF BASEBALL CARDS

Below are a set of instructions which can be put on cards to provide sufficient variety for a game. Others can be added if desired.

1. Strike three, you're out!
2. Single to centre field. (Runners advance one base. If a man is on second, he scores.)
3. Deep fly to centre field. Caught, and batter is out.
4. Strike three, you're out!
5. Single to centre field. (Runners advance one base.)
6. Double to deep centre field. (All runners advance two bases.)
7. Home run into the left field stands setting a new home run record.
8. Double off the left field wall. (All runners advance two bases.)
9. Batter walks. (Others advance one base if forced.)
10. Pop out to short stop. (Man doubled out on second. Runner on third scores, if the play does not make three out.)
11. Double to right off the wall. (All runners advance tow bases.)
12. Strike three, you're out!
13. Runner picked off third and is out. (Take another card.)
14. Foul ball (take another card).
15. Batter safe on first because of a fielder's choice. (If runner is going to second he is out. All others advance one base.)

16. Strike three, you're out!

17. Strike three, you're out! (Runner on first takes second safely.)

18. Grounded out second to first. (All runners advance one base.)

19. Single to right field. (Runners advance one base.)

20. Batter walks. (Other runners advance one base if forced.)

21. Single to left field. (Runners advance one base.)

22. Double play: short to second to first ... (Two men out; if no runner is on first only the batter is out. Runner on third scores if not three out.)

23. Home run into the right field stands.

24. Batter walks. (Other runners advance if forced.)

25. Ground out short to first. (Runners advance one base.)

26. Home run into the bleachers.

27. Triple to deep centre field. (Three base hit; all base runners score.)

28. Bunt to third and batter is safe on first. (Runner on second makes third safely. Runner on third out at home.)

29. Strike three, you're out!

30. Runner picked off first and is out. (Take another card.)

31. Single to left field. (Runner advance one base.)

32. Double to left field. (All runners advance two bases.)

33. Single to right field. (Runners advance one base.)

34. Strike three, you're out!

35. Stolen base for all base runners. (If there is a man stealing from third to home, he is out. Take another card.)

36. Popped out to short stop. (Runner doubled out on third. All others hold their bases.)

37. Double off the right field wall. (Runner on first goes to third. Other base runners score.)

38. Runner picked off second, and is out. (Take another card.)

SAMPLE SKILL TESTING QUESTIONS

These can be made up to test or practice any desired skill. Some samples are given below. One question is put on each card.

1. $3\sqrt{2} \times 5\sqrt{7}$

2. $h \underset{6}{\triangle} 8$ Find h.

3. $\frac{-5}{6} \times \frac{-18}{25-}$

4. Solve for x: $\frac{2}{3}x = 12$

5. $-6(7-9) =$

6. $3x^7 \times 5x^3 =$

HOCKEY GAME

The hockey game is made up of Skill Testing Cards, Defensive Cards and Offensive Cards.

- There are five players on each team (2 teams). The players are paired off on both teams.

- The game starts with the offensive player selecting a Skill Testing Card which is face down. If he answers the card correctly, he then selects an offensive card. The offensive card could read "He shoots, he scores" or "He shoots and misses the net". In other words a correct answer does not always mean a goal. This way or manner causes suspense and excitement between the 2 teams. If the offensive card does not denote a goal the defensive team picks up a card from the defensive cards which could read "The defenseman gets the puck and makes a rush down the ice" or "The defensive player misses the puck and the offensive player gets it". Depending upon the situation any team could instantly be offensive or defensive and have the chance to answer the skill testing question.

- If the offensive player does not answer correctly his opponent with whom he is paired gets the chance to answer. If the opponent answers it correctly he becomes offensive and gets the chance to select the offensive cards and the possibility to score.

This game creates suspense, and most of all mental speed.

N.B. (1) There is a time limit for the answers.

(2) This game can be used for mental drills for K-13.

(3) The game creates excitement and competition for rapid answers.

CHALLENGE

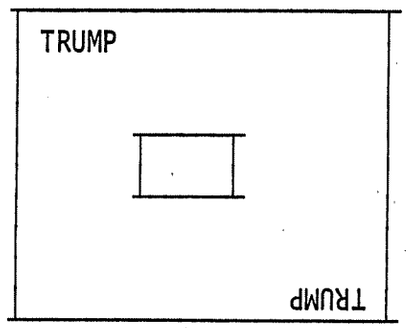
PURPOSE

This is a team game which can be used to provide review of the basic number facts by solving simple equations, evaluating expressions, or similar activities in a new and intriguing manner.

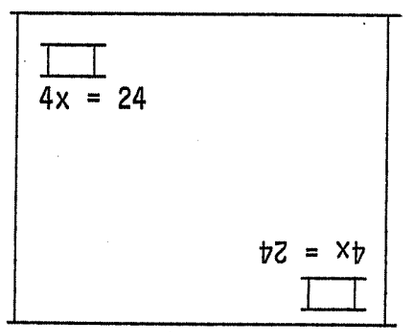
METHOD

The deck of cards consists of 12 TRUMP cards -- 3 of each suit, and 40 PLAYING cards -- 10 of each suit. The symbols (suits) and equations should be placed on the cards as indicated below. A nylon tip marker works well for putting the suit design on the card. The equations can be typed or hand-written as desired.

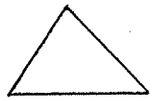
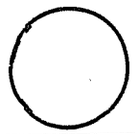
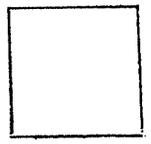
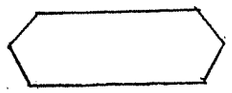
SAMPLE TRUMP CARD



SAMPLE PLAYING CARD



The following chart gives a suggested set of simple equations for the playing cards. Playing cards could be made to provide practice for any desired skill.

			
$4x = 24$	$\frac{x}{3} = 4$	$3x = 27$	$\frac{x}{5} = 1$
$\frac{x}{5} = 3$	$9 + x = 16$	$x - 5 = 3$	$x - 7 = 2$
$5x = 25$	$x - 7 = 1$	$2x = 16$	$\frac{6}{x} = 1$
$\frac{20}{x} = 2$	$x + 18 = 26$	$7x = 28$	$x + 1 = 4$
$x - 0 = 7$	$\frac{x}{6} = 2$	$\frac{x}{4} = 2$	$4x = 16$
$8 - x = 6$	$6x = 36$	$14 + x = 19$	$\frac{x}{5} = 2$
$x + 6 = 15$	$x + 3 = 11$	$\frac{x}{2} = 4$	$x + 3 = 8$
$\frac{x}{3} = 3$	$2x = 14$	$x + 10 = 13$	$6x = 18$
$x - 6 = 3$	$x - 0 = 3$	$x - 3 = 3$	$2 - x = 0$
$x + 12 = 16$	$8x = 64$	$x + 4 = 10$	$x - 3 = 1$

OBJECT OF THE GAME

First player to reach a pre-arranged total OR player with highest score at end of a certain time wins the game.

NUMBER OF PLAYERS

3, 4, or 5. However, if three players, then 1 card of each suit is to be taken from the 40 PLAYING cards.

RULES OF THE GAME

1. Shuffle the 40 PLAYING cards and 12 TRUMP cards separately.
2. Place TRUMP cards face down in center of table.

3. Deal PLAYING cards, one at a time, to the players.

4. Each player sorts cards into like-suits.

5. To begin play, dealer turns up TRUMP card to determine suit.

6. Starting at dealer's left, each player plays card of that suit, calling out the root of the equation as he plays.

7. The player with the largest value card wins the trick.

8. Each player must play a card on every trick even if he cannot follow suit. However, he cannot win the trick if he cannot follow suit.

9. In case of a tie in the value of the card played, the winner of the trick is the first player to play one of the tieing cards.

10. Winner of the trick turns up a new TRUMP card to determine suit and the game continues.

SCORING

Each player receives 1 point for each trick taken during the hand. Several hands may be played before winner of the game is determined. Bonus points can be received by challenging or by predicting.

Challenging - At any time during the game, an opponent can challenge the value of a card played by saying, "I challenge". Any other means of slowing the play of the game does not constitute a legal challenge. (e.g. statements like "Hold it", "Wait a minute" or "Are you sure" do not constitute a legal challenge.)

A. If the wrong value of a card is called, the challenger wins

all tricks the challenged has accumulated that hand.

B. If the correct value of a card is called, the challenged player takes all of the tricks the challenger has accumulated that hand.

C. If an incorrect call is not challenged it can win the trick, if it has the largest value.

Predicting

A. After PLAYING cards are dealt and each player has arranged cards, each player predicts how many tricks he will take during that hand. If his prediction is correct, he receives 1 point for each trick plus an additional 5 points.

B. There is no penalty for a wrong prediction.

C. It is suggested that the predictions be recorded on paper.

FRACTION WAR

D. B. Wing, Queen's University Faculty of Education

EQUIPMENT

A deck of 66 cards on each of which is written one of the fractions $\frac{a}{b}$ where $1 \leq b \leq 12$ and $1 \leq a < b$.

i.e. $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \ldots, \frac{1}{3}, \frac{2}{3}, \frac{1}{2}$

The deck is shuffled and dealt face down to all the players in turn, until all the cards have been distributed. Each player keeps his cards face down in front of him. Each player turns his top card face up. The player whose card shows the greatest fraction, wins all the cards that were turned up, and places them face down at the bottom of his pile. In the event of a tie for the greatest fraction, a "war" is declared. Each of the players involved in the war, places the next three cards from his pack face down. He turns over the fourth card. The person with the highest fraction now showing takes all the cards. The winner is the player with the most cards at the end of the time period.

S'MAD 1

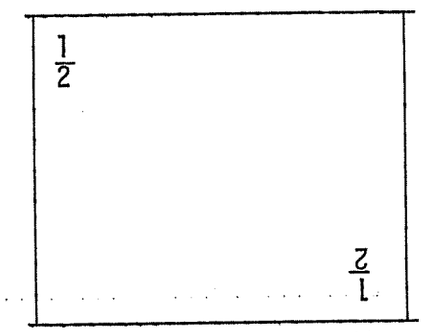
R. Wigle, Department of Mathematics, North York

PURPOSE

This is a card game which may be used to review the four basic operations with fractions.

METHOD

The deck consists of 52 cards in the form shown below:



The deck should contain:

Five cards, 1 each of the following: $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{7}{6}, \frac{13}{12}$.

Thirty-eight cards, 2 each of the following: $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{6}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}$.

Nine cards, 3 each of the following: $\frac{5}{12}, \frac{7}{12}, \frac{11}{12}$.

OBJECT OF THE GAME

The first player to reach 25 points wins the game.

Other goals for determining a winner might be:

1. A numerical score less than 25 points.
2. A specific period of time (e.g., 15 minutes for a game; the

player with the highest score wins).

3. A specific number of hands (the player with the highest score wins).

It is important that the goal of the game be clearly stated before play begins. With a classroom set of cards S'MAD 1 can be extended into a round-robin tournament (single or double elimination).

NUMBER OF PLAYERS

Two to five.

BASIC RULES

1. Shuffle the deck and deal 7 cards face down to each player.
2. Put the remaining cards in the center of the table and turn the top card face up beside the deck.
3. Each player looks at his cards to see if he can combine them into books that will equal 1, using any operation or combination of operations (addition, subtraction, multiplication, or division).
4. Multiplication by zero is not permitted.
5. Play starts with the person to the left of the dealer and moves clockwise.
6. A player may pick up the top card from either pile, and after drawing, must discard one card from his hand face up on the discard pile.
7. Play continues until someone is able to lay down all 7 cards in combinations of books and discard one card on the discard pile.

8. A person may not lay down any of his cards until he has all seven in books of 2 or more cards. When one person lays down his seven cards in books, he has won the hand and all drawing and discarding must stop. After the winner has laid down his hand, the other players may lay down what books they have in their hands. After all books have been laid down, scoring begins.

9. Score 1 point for each card played in a book. Deduct 1 point for each card left in the hand.

10. If the entire deck is used, play stops and each player lays down his books. No one may draw from the discard pile.

11. After all the books are laid down face up, they may be challenged. The person who challenges correctly takes all of the challenged player's books. If the person who challenges is incorrect, he loses all of his books to the person he challenged.

12. A person may also be merely asked to explain the operations and groupings used. This does not constitute a challenge.

SCORING VARIATIONS

1. The person who lays down all of his cards gets a 3 point bonus.

2. Play by variation 1 above, except use a one minute or one-half minute egg timer to limit the time opponents have to lay down their books and/or challenge the player who lays down his cards first. Other timing devices could be substituted.

NOTE:

Blank playing cards are available at $25.00 for 3000

cards from: The Arrco-Saxon Company, 3195 Airway Drive, Malton, Ontario.

ADVENTURES IN DIGITLAND - A TRUE STORY

Keith McLean, Downsview Secondary School, Downsview, Ontario

A boy lives in the strange work of Digitland, a land where there are no tens or hundreds ... only digits 0, 1, 2, 3, 4, 5, ..., 9.

When we enter his world at Digitland Customs and Immigration, we must discard all our tens and hundreds. Thus, our 100 becomes his 0, our 15 his 5, our 1738 his 8. To indicate this equivalence (exchange rate, as the DCI refer to it), we use ≡ - an almost equal sign.

As Digitlanders are optimists, they take a very dim view of negative numbers, prefering to convert them to positive numbers by the following rule: Reduce a negative number to digit form, and add 10. Thus -278 becomes -8 which in turn becomes 2. Negative numbers have a very specialized use in Digitland which we will come to in a moment.

Exercise 1: Convert the following numbers to Digitland numbers, using ≡. Thus -172065 ≡ 5 ≡ 5.

7065	-508	6043	33
8001	-99	356	-2
550	1706	-15	-53
-6	-706	78	109
133	-11	67	13

The boy had no trouble multiplying. For instance his 5 times table consists of only 0's and 5's. Complete the following

multiplication table neatly - it will be useful for you to refer to it later. Thus $3 * 7 \equiv 21 \equiv 1$. The $*$ is used to denote multiplication in Digitland.

*	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

Exercise 2: After completing the multiplication table, find all numbers which multiply to get 1, 7, 8. (Since 1 appears in the 3 row's 7th column, $3 * 7 \equiv 1$.)

Multiply the numbers in the first column of exercise 1 with their adjacent numbers in column 2. ($7065 * (-508) \equiv 5 * (-8) \equiv 5 * 2 \equiv 0$) Similarly for columns three and four.

Factoring in Digitland is easier and harder than in our world. Easier, because there are only a limited list of possibilities, and harder, because sometimes things don't work out the way they should - 'should' meaning the way that we're used to having

them work out.

The solution of $x^2 + 8x + 2 = 0$ presents no problem: he simply wants two numbers which, when multiplied give 2 and when added yield 8. The boy simply checks his multiplication table for those numbers which multiply to give 2: $1 * 2$ $2 * 1$, $2 * 6$, $3 * 4$, $4 * 8$, etc. Checking their sum - 8 he may write $x^2 + \dot{8}x + 2 \equiv (x+2)(x+6)$.

Factoring $x^2 + 8x + 1$, he notes $9 * 9 \equiv 1$ and $9 + 9 \equiv 8$, thus $x^2 + 8x + 1$ is a perfect square.

To factor $x^2 - 2x + 2$, Digitlanders replace $-2x$ with $8x$ and continue as before. Common factors are strange: $3x^2 + 2x + 8 \equiv 3(x^2 + 4x + 6)$, checking the multiplication tables.

Exercise 3: Common factor so that the term with highest degree has coefficient 1:

$3x^2 + 2x + 7$ $\qquad$ $6x^3 + 2x^2 - 4$

$4x^2 + 2x + 6$ $\qquad$ $7x^2 + 9x + 6$

Show that even Digitlanders cannot factor $x^2 + 8x + 3$.

Find all m such that $x^2 + 8x + m$ factors, and demonstrate these factors.

Show $(x+6)(x+7) \equiv (x+2)(x+1)$.

Find another pair of linear factors equivalent to $(x+8)(x+9)$.

Find all n such that $x^2 + ny + 8$ factors.

Find all trinomials that are perfect squares (expand)

$(x+k)^2$ for all $k = 1, 2, \ldots, 9$).

Find all difference of squares: $x^2 - a^2$.

The Factor theorem is still true but with some surprises. The theorem states that $f(a) \equiv 0$ iff $x - a$ is a factor of $f(x)$. Since Digitlanders prefer all numbers in the form that is positive, the factor theorem becomes $f(a) \equiv 0$ iff $x - a + 10$ is a factor of $f(x)$.

To factor any polynomial, we look at the factors of its constant term. In Digitland, this turns out to be somewhat a larger number of factors than our normal world. For instance, if we were to factor $x^3 + 2x^2 + 5x$, instinctively, most of us would take out the common factor (x) leaving:

$$f(x) \equiv x^3 + 4x^2 + 5x \equiv x(x^2 + 4x + 5)$$
$$\equiv x(x+5)(x+9)$$

However, if we were to apply the factor theorem, we would note that the factors of 0, in addition to 0 itself are 2 and 5, 4 and 5, 6 and 5 and 8 and 5.

$f(2) = 3 + 6 + 0 \not\equiv 0$

$f(4) = 4 + 4 + 0 \not\equiv 0$

$f(6) = 6 + 4 + 0 \not\equiv 0$ Thus $x - 6 \equiv x + 4$ is a factor

$f(8) = 2 + 6 + 0 \not\equiv 0$

$f(5) = 5 + 0 + 5 \equiv 0$ Thus $x - 5 \equiv x + 5$ is a factor

Dividing $x + 4$ into $x^3 + 4x^2 + 5x$:

$$
\begin{array}{r|l}
x+4 & x^2 + 5 \\
\hline
 & x^3 + 4x^2 + 5x \\
 & \underline{x^3 + 4x^2} \\
 & \quad 0 + 5x \\
 & \quad \underline{5x + 0} \\
 & \quad\quad 0
\end{array}
$$

But $x^2 + 5$ is a perfect square so,

$f(x) \equiv (x+4)(x^2+5)$

$\equiv (x+4)(x+5)(x+5)$

Similarly with $(x+5)$:

$$
\begin{array}{r|l}
x+5 & x^2 + 9x \\
\hline
 & x^3 + 4x^2 + 5x \\
 & \underline{x^3 + 5x^2} \\
 & \quad 9x^2 + 5x \\
 & \quad \underline{9x^2 + 5x} \\
 & \quad\quad 0
\end{array}
$$

Thus,

$f(x) \equiv (x+5)(x+9x)$

$\equiv (x+5)x(x+9)$

our original factors.

Thus, $(x+4)(x+5)*x+5) \equiv x(x+5)(x+9)$ if we divide both sides by $(x+5)$, we see that $(x+4)(x+5) \equiv (x+9)x$.

Exercise 4: Using the factor theorem, factor $x^4 + 3x^3 + 5x + 7$. Be very careful to divide the linear factors into the original polynomial, not whatever resulting quotient may arise. (Note in the above example that $(x+4)$ is not a factor of x or $x+9$.)

MATHEMATICS FROM KINDERGARTEN ON

W. W. Sawyer, University of Toronto

In November 1971 I spent two days in an elementary school with the object of making recommendations that would ensure every child going through the school with a steadily increasing enjoyment and understanding of mathematics. This objective is a perfectly feasible one; all the evidence indicates that young children enjoy arithmetic but that this interest is progressively destroyed by the adult population.

How should one go about such a task? I decided to spend most of my time talking to individual children who were not getting on too well, and trying to find the point where things went wrong. This procedure implies concentration on the negative aspects of the school; it concentrates on the symptoms of disease, and gives no measure of the number of children who are working enthusiastically and successfully. In case anyone is in a position to identify the school in question, I would place on record that it was a particularly good one in regard to the mathematical qualifications and concern of the teachers, the attitude of the administration and the enlightenment of the coordinators and consultants involved. A weaker school would never have allowed me in on such a project.

Some lessons that emerged from this experience will now be listed.

RECORDS OF INDIVIDUAL SKILLS AND UNDERSTANDING

It is very desirable that every child should have a record of his mathematical progress. For instance, one boy was having difficulty with subtractions of the type 315-189. In conversation with him it soom emerged that he knew subtraction facts such as 9-5=4 quite well but that he had no idea of 11-4 or any other subtraction involving a number larger than 10. Clearly his class work was presenting him with a completely impossible problem, and tackling impossible problems leads to fear and frustration. A continuing record of skills and concepts understood would have prevented this situation*. (I heard of a school where such records were kept, but the teachers were highly competitive and all the time seeking to prove how quickly their students were learning. It is essential that teachers making such records should realize that some children learn slowly; the competition, if any, should be as to who best maintains the confidence and interest of the students.)

CONCRETE MATERIALS

It emerged very clearly that, for many children continuing use of concrete materials is essential. Semi-concrete materials, pictures, are not enough. In one textbook there were quite sensible questions; a student would see 0000000000 0000000000 0 0 0

* A possible record chart is attached.

and be expected to recognize 2 tens and 3 units and write 23. Some children, who totally failed to cope with this, responded readily when a bundle of 10 sticks was laid over each of the ten groups in the picture, and a single stick was put on each single object. The same effect was seen at a much later stage with a student who was completely baffled by long division. We considered the question of dividing 72 sticks fairly among 3 persons. The 72 sticks were set out, 7 bundles of ten and 2 single sticks. The boy was then asked to make a handout. He gave a bundle of 10 sticks to each recipient. This was then recorded in writing

$$\begin{array}{r} 72 \\ \underline{30} \\ 42 \end{array} \qquad 10$$

and the process continued, the dramatization all the time preceding the written record. With a slow student, this procedure might have to be followed for many days for full understanding to be achieved.

In Grade 1 it is not a profitable use of time for the children to sit around the blackboard while the teacher chats about the null set. There is an old and effective device for teaching the meaning of numbers, including zero. A number of compartments carry labels 0, 1, 2, ... Each day a different child has the responsibility of placing 1 toy in the 1 compartment, 2 toys in the 2 compartment, and so on, including no toy in the 0 compartment. If the teacher feels that sets must be mentioned,

the word can be used, - "I see you have the set of 2 toys here, a set of 1 toy here, and the empty set in the end space". Personally, I feel that emphasis on sets has been somewhat overdone.

AVOIDANCE OF COUNTING-ON

In the Grade 1 textbook there were groups of questions such as 7+4= ..., 4+7= ..., 11-4= ..., 11-7= ... . This questions has a very sensible object, to give the idea of a "fact team". But some children were dealing with it in such a way that they never saw the point of the question grouping. They would answer 7+4 by counting-on with the fingers 8, 9, 10, 11 and writing 11. For 4+7 they counted 5, 6, 7, 8, 9, 10, 11 and wrote 11, but by this time they had forgotten all about the first answer. To get the point of the work, the answers must come quickly and be seen in relation to each other. The students had a number line marked out in inches. I cut strips 4 inches and 7 inches long; by holding them against the number line it could be seen that they covered the same distance, 11 inches, whichever order they were in, and by removing them one at a time, the children could see that 4 was left when 7 was taken away from 11, and 7 left when 4 was taken away.

Also, as Catherine Stern has so rightly emphasized in her excellent book, Children Discover Arithmetic, counting-on is a disastrous habit, and the teacher should always see that some material (strips, or commercially produced rods) is provided to

ensure that children never get into this habit. Once formed it is extremely difficult to break.

MATHEMATICS AND THE REAL WORLD

I was asked to teach a class of 10-year-olds any subject I liked. I decided on fractions because in every country I have been in, I have found children "doing" fractions without any understanding of the meaning of fractions. Once the meaning is clearly understood, work with fractions is not difficult. I found the students had quite a clear idea of simple fractions such as 1/2, 1/4, or 2/3. I then gave each child a strip of paper ten inches long and asked them to mark this strip out in fifths. I wandered around the class and watched them. Several children seemed to take an extremely long time to do this. I am fairly sure that if I had asked, "What is 10 divided by 5?" hands would have gone up all over the room. The reaction to the actual situation was quite different. Part of the difficulty may have been because 10/5 had been explained mainly as "How many fives in ten?" with little emphasis on the aspect, "If 10 things are shared fairly between 5 persons, how much does each person receive?" A deeper reason, I believe, was that mathematics was thought of as a paper-and-pencil exercise, something you did from a book. It had not been linked to everyday activities. This very often happens and it is a pity. It both prevents children applying their common sense and everyday experience to mathematical problems, and also

makes it unlikely that they will apply mathematical knowledge to any problem that arises in actual living - and there is no point in teaching mathematics if it is never going to be used.

Some children in the class coped easily with the task, and went ahead with much harder work.

AVOIDANCE OF DOGMATISM

The activity approach is rather fashionable at present, and there is a danger that schools may encourage activities without any clear goal in mind. It should therefore be stressed that we do not start with activity. We start with a mathematical concpet we wish to impart. We then look for an activity, or series of activities, that will help children to see what this concept means.

The same applies to the use of concrete materials. These are intended to help children who have not yet arrived at the stage where they can imagine a situation. Years ago the Montessori teachers found that at a certain stage a child would spontaneously put their apparatus aside and say, "I do not need that any more." It is entirely wrong to force a child, who has understood a concept or a process, to work with apparatus he does not need. But on the other hand, the idea that concrete materials are only for kindergarten and Grade 1 is entirely incorrect. Whenever a student of any age (I do not exclude university graduates) is finding particular difficulty with an idea, the use of concrete

aids is worth considering.

FLEXIBILITY AND ORGANIZATION

The main job of a teacher is not to talk but to listen, to discover just what difficulty is holding a child back. Often a teacher needs to listen to a single student, or at most a small group of students. Also it is a fact of life that children advance at very different rates. This clearly creates a problem of organization. One has to create something of the atmosphere of the old one-room school, where children went on with their own work when the teacher was helping other children. In the school I visited, a great asset was a very well organized kindergarten, in which the teacher would perhaps say, "I am going to read a story for those who are interested; the rest of you can get on with your own work", and children would immediately go to their various tasks. If self-reliance is encouraged early, it is not too difficult to maintain it through the grades. Teachers accustomed to the look-step class organization anticipate tremendous confusion with individual and group work; they should realize that discipline difficulties inevitably arise when one-quarter of the class has not understood anything in arithmetic for the last two or three years, and at the other extreme one-quarter of the class are capable of much more advanced work and are thoroughly bored. If one can achieve a situation in which every child understands the work and is advancing successfully, quickly or slowly, the

problem of organization is very much eased. Children usually like a subject at which they are succeeding.

PROGRESS CHART FOR CONCEPTS AND SKILLS

If children are working, each at his own pace, they will spread out widely. It is essential to have a record for each child, showing what skills he has and, above all, what ideas he understands.

On such a chart there would be spaces, each for a certain skill or insight. If a child has been exposed to some experience designed to form a certain concept, the appropriate space might be marked /. When the child displays some grasp of the concept, but with some uncertainty, the mark might become X. Finally, when the child has mastered the concept, the space could be blakced in completely.

A child may acquire an idea and later forget it. The chart should make provision for checking that this has not happened. For instance, concepts could be arranged one on each line of the paper, with successive years as columns, e.g.

	Kindergarten	Year 1	Year 2
Recognizes division situation	/	X	▓
Recognizes even and odd	X	▓	▓

Such a chart needs to be developed with the co-operation of teachers. Below appear some suggestions for items it might

contain, and for ways of testing whether a child has a concept or not. I do not envisage formal examinations or multiple choice tests. For instance, a child in kindergarten, trying to fit a block into a space may say spontaneously, "No. That's too big". This indicates that the child understands what "too big" means.

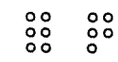
LIST OF SUGGESTED ITEMS

1. Qualitative understanding. Uses correctly such terms as bigger, smaller; longer, shorter; heavier, lighter; more, fewer.

2. Basic Number Sense. Recognizes numbers up to ten in patterns such as 

(See Catherine Stern, Children Discover Arithmetic for the use of such patterns with children 3 years old, and the importance of giving number sense before teaching counting. The most disastrous habit a child can acquire is that of solving problems by "counting on".)

3. Conservation of number. Realizes that rearrangement of objects does not change their number.

4. Associates "even" and "odd" with arrangements such as 

5. Knows the order of numbers. (5a) to 10, (5b) to 99, (5c) to 999. e.g. can answer "What is the number before 7?"; rapidly arranges number cards in correct order; can find a page in a book quickly and systematically.

6. Addition. Recognizes an addition situation (6a) with concrete material, (6b) in a story. (6c) Associates + with addition.

7. Subtraction. 7a, 7b, 7c

8. Multiplication. 8a, 8b, 8c. On analogy of 6a, 6b, 6c.

9. Division. 9a, 9b, 9c.

10. System of Numeration. Can interpret and demonstrate (10a) numbers 11 to 99, (10b) number 100 to 999.

For example, single objects bundles of ten and bundles of a hundred might be provided. Shown a bundle of ten and 3 single objects, the child should say 13 without counting from 1 to 13; similarly, asked to set out 23 objects, should pick 2 bundles of ten and 3 single objects, not count from 1 to 23. Similarly for HTO.

11. Can demonstrate additions and subtractions, such as 23 + 45 or 57 - 32, (no carrying) with concrete materials.

12. Regrouping. Can discover or demonstrate (12a) result such as 7 + 6 = 13 with suitable apparatus (Stern, Cuisenaire, bundles, board of 100 nails). (12b) can demonstrate 17 + 6 = 23, 27 + 6 = 33, 37 + 6 = 43, 47 + 6 = 53, etc. (12c) has recognized pattern in 12b.

According to Edith Biggs, 12c is a crucial readiness test. Children should not go on to work such as 23 + 18 if they have not understood this idea.

13. Addition Facts. (13a) can think out any result from 1 + 1, 1 + 2, ... to 9 + 8, 9 + 9. (13b) has done this so often that the results have been memorized.

14. Multiplication Facts. (14a) can obtain any result from 1 × 1, 1 × 2 ... to 9 × 8, 9 × 9 with concrete material, (14b) can think out these results without concrete materials, by a mixture of memory and reasoning, (14c) has memorized all these results.

15. Multiplication by 10. Can demonstrate multiplication by 10 and explain why it leads to the routine "put 0 at the end of the numeral".

16. Multiplication by 2-digit numbers. (16a) has discovered or can demonstrate how to multiply by (say) 13 with the help of item (15) above. (16b) understands how and why multiplication by 20, 30, 40 ... works. (16c) understands the ideas involved in multiplication by (say) 23, 33, 43 ... .

17. Extension of principles of (15) and (16) to multiplication by 100, by 234, etc.

18. Can demonstrate or explain any procedure for carrying out computations involving division of one whole number by another.

The central theme in mathematics is thinking about ideas; terminology and conventions occupy a minor position and should never be allowed to usurp the centre of the stage. Such things should be mentioned, casually and incidentally, on occasions when they arise naturally. With this caution, we include the following items related to language and symbolism.

19. Is familiar with questions such as "Find the factors of 15". (This question can be posed concretely; arrange 15 objects in the

form of a rectangle.)

20. Knows the meaning of "prime number". (This arises naturally in connection with a question "Express 12 by prime factors;" $12 = 2 \times 2 \times 3$.)

21. In prime factors $16 = 2 \times 2 \times 2 \times 2$; somewhat wearisome to write. Does the student know the abbreviation 2^4?

22. Does he know $5^2 = 5 \times 5$ = the square of 5?

23. And $5^3 = 5 \times 5 \times 5$ = the cube of 5?

24. Brackets. Understands the distinction between $7 \times 2 + 3$ and $7 \times (2+3)$.

(21) to (24) are of little significance in arithmetic itself, but it is very helpful to a student beginning algebra if he is familiar with these. He can concentrate on the meaning of $\times$, and not be distracted by having to become accustomed to squares, cubes, x^2, x^3, x^4 as well. (24) is highly relevant to algebra; if a student does not understand $7 \times 2 + 3$ he has little chance of realizing what $7y + 2$ means.

The teaching of conventions should be limited to such essential ones as those above; I am against the BODMAS business. Expressions such as $60/4\times5$ should never be used. If an author means $60/(4\times5)$ or $(60/4)\times5$, he should take the trouble to use a clear notation and put his meaning beyond doubt.

Factors and prime factors are natural and good exercises on the multiplication tables, and thus lead to exponents. Brackets

may arise in discussing, say, 27 × 13 as 27 × (10+3), in a discussion of the multiplication procedure.

25. Fractions. (25a) has met fractions informally in activities involving such things as half a yard of ribbon, a quarter of a pound of butter, experiments with half gallon and gallon containers of milk, etc. (25b) can indicate correctly (say) two-thirds or three-fifths of any physical object (length of string, area of paper, volume of liquid in cylindrical jar, etc.). (25c) associates symbols 2/3, 4/6, 8/12. (25e) can demonstrate physically how to find the solution to such questions as "What is two-and-a-half times one-and-a-half inches?" or "How many times does an inch and a half go into a foot?" (25f) can answer any question, involving reasonably small numbers, about adding, subtracting, multiplying or dividing fractions, by common sense with a minimum reliance on formalisms.

26. Fractions. Understands and can explain or demonstrate the formal procedure for multiplying fractions, e.g.

$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$. This formalism may be needed for discussions of decimals, e.g. 0.2 × 0.3 as (2/10) × (3/10), or later for algebra.

27. Percentage. (27a) knows that e.g. 73 per cent is simply another way of writing 73/100.

(27b) by working examples has become familiar with such facts as 50 per cent = 1/2, 75 per cent = 3/4, 33 per cent = approximately 1/3.

28. Decimals. (28a) has experience of measuring lengths such as 2.3 inches. (Where rulers marked in tenths of an inch are not available, graph paper with 10 divisions to the inch can be used.) (28b) can demonstrate meaning of e.g. 2.34 with money or squared paper. (28c) realizes 2.34 means the same thing as 2 + (3/10) + (4/100). (28d) can explain multiplication by 10 or by 100 by demonstrating (say) the relative magnitudes of \$0.12, \$1.20, and \$12.00. (28e) appreciates that the processes for addition and subtraction of decimals are essentially the same as for whole numbers, since e.g. \$2.34 + \$5.67 is the same as 234 cents + 567 cents. (28f) appreciates that $2.3 \times 4.5 = \frac{23 \times 45}{10 \times 10}$ so that, apart from the question of locating the decimal point, the calculation is identical with that for 23 45. (28g) can estimate approximate value of 2.3×4.5, and so locate decimal point correctly in the answer. (28h) realizes that division also shows strong analogy since e.g. \$3.72/\$1.20 is the same as 372 cents/120 cents. (28i) division, unlike the otheroperations, has a special feature in that the quotient may have figures after the decimal point, as in 2.34 ... or may even begin beyond the decimal point as in 0.0037 ... . The student is able to adapt the explanation of long division (item 18) to give a rational explanation of the process he uses.

29. Graphs. Understands graphs. (29a) made of actual objects or persons. (29b) made with pictures. (29c) of block type. (29d)

of column type. (29e) with line or curve, as in secondary school graphs. (29f) in relation to a simple scientific law or mathematical formula (e.g. graph of 2 times table).

NOTE ON TEACHING AND TESTING

The mystery and anxiety surrounding arithmetic would disappear if teachers and students realized that any question in arithmetic can be settled by experiment with actual objects. For instance, many children do not know 8 × 7. To find it out, they need an understanding only of items (8) and (10). What does 8 × 7 mean? On the board of 100 nails the child sets out 7 rows with 8 rings in each. What number gives the answer? The child moves the rings up, to fill as many complete rows as possible, and obtains 5 complete rows of ten each, and a row with 6 rings. Answer, 56. Later, the child may discover ways of thinking this result out. It is not hard to see that 8 × 5 is 40. Two more eights are needed to make seven eights. So 40 + 16 gives the answer.

Understanding of operations may be texted/by asking for a demonstration; "Show me what three fours means" (3 groups of 4, or 3 rows of 4) or by asking "Tell me a story about three fours" (3 boys each had 4 marbles, etc.). Both tests should be used.

A SOLUTION OF PROBLEM #8 (ONTARIO MATHEMATICS GAZETTE, DEC. 1968)

Edward Evans, Guelph, Ontario

The produc of two linear transformations S and T of the form

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} \text{ or the matrix form } \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad (ad - bc \neq 0)$$

although not in general commutative, will, when acting on the circle $x^2 + y^2 = r^2$, produce ellipses of the same area. The necessary and sufficient condition that the ellipses also have the same eccentricity, is that $a^2 + b^2 + c^2 + d^2$ be the same for both the products ST and TS.

This condition is satisfied by any of the following Types;

Type I: If S is of the form $\begin{vmatrix} a & -b \\ b & a \end{vmatrix}$ for any T, a simple rotation.

Type II: A transpose, such as $S = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$ $T = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or the inverse $\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$.

A more interesting possibility is

Type III: Given any $T = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$, a transformation S of the form $\begin{vmatrix} a & b \\ 1 & a \end{vmatrix}$ can be found by applying the sum of squares condition, and we get the required result by letting $b + 1 = 0$, or

$$b - 1 = \frac{2a(r-8)}{p + s}, \quad (p^2 - s^2 \neq 0)$$

(Note, b will be an integer if $a = p + s$.)

If $b = -1$, we get Type I.

If $p - s = 0$, we get Type II.

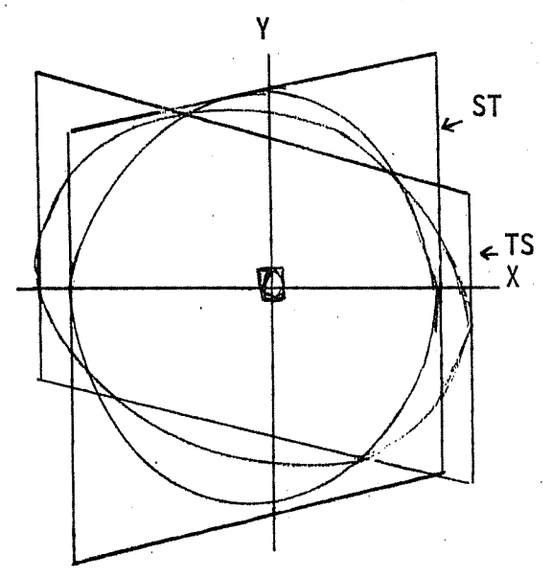
If $p + s = 0$, we require the condition $a(b-1)(r-q) = 0$.

A good example to show Type III, by a graph, is to let $T = \begin{vmatrix} 2 & 8 \\ -1 & 3 \end{vmatrix}$ and let $a = (p+s), = 5$, and let $b - 1 = 2(r-q), = -18$.

You will notice that the sides of the parallelograms are tangents at the ends of conjugate diameters.

TYPE III

$$ST = \begin{vmatrix} 18 & -2 \\ 6 & 32 \end{vmatrix} \qquad TS = \begin{vmatrix} 27 & -3 \\ -11 & 23 \end{vmatrix}$$



AN EFFECTIVE WAY OF KEEPING STUDENTS IN TOUCH WITH THE WHOLE YEAR'S WORK, RATHER THAN JUST THE PRESENT TERM

Neil MacEwan, Head of Mathematics, Patterson Collegiate Institute

This problem is common to many subject areas and the following is what is done by some of the teachers at this school.

Throughout the school year the class is kept supplied with Review Problems (see attached examples) and each Friday the class is given a test on those 5 problems.

Since, as teachers, we want to minimize our marking time, a scheme had to be devised to do this as well as force the student to prepare all 5 problems.

As a result, a deck of cards was constructed (52 in all) as follows:

6 cards having the numeral 1 on each
6 cards having the numeral 2 on each
6 cards having the numeral 3 on each
6 cards having the numeral 4 on each
6 cards having the numeral 5 on each
2 cards having the numerals 1, 2 on each
2 cards having the numerals 1, 3 on each
2 cards having the numerals 1, 4 on each
2 cards having the numerals 1, 5 on each
2 cards having the numerals 2, 3 on each
2 cards having the numerals 2, 4 on each
2 cards having the numerals 2, 5 on each
2 cards having the numerals 3, 4 on each
2 cards having the numerals 3, 5 on each
2 cards having the numerals 4, 5 on each

And just to keep them honest

2 cards with 1, 2, 3, 4, 5 on them

This deck is shuffled by the teacher and one of the

students cuts the deck for the whole class. If the cut card has a 2, 3, on it, say, then the whole class does problems 2, 3 in a reasonable amount of time (determined by the teacher) and hands it in to be marked.

Each test is given 10 marks and is marked closely for form, misuse of symbols, etc. The students are told beforehand how the test will be marked, and to prepare accordingly.

These review tests constitute 20% of the mark, chapter tests 30% and exams 50% but this will probably changed to 30:30:40.

It has been found that the class review test can be marked and recorded in less than 30 minutes, but you must be prepared to give them the whole period in the event they cut the 1, 2, 3, 4, 5 card. This has happened only 3 times this year in about 80 attempts.

It has also been found that there is a fairly close correlation between a student's performance on review tests and chapter tests and this can be used as a selling feature of the program.

ADVANTAGES:

1. See subject.
2. Students who are absent the day of the test make it up by coming in and cutting the deck for themselves.
3. Definitely helps those hard-working students who are not strong in Mathematics.

4. It's possible to emphasize FORM, ORGANIZATION, PRESENTATION by the manner in which the test is marked.

5. More important type-problems can be emphasized.

6. General strategies can be emphasized (see ex. 13.1, 15.1, 15.5).

7. Where a problem on a chapter test has been poorly handled it can be placed on a future review test.

8. General problems can be given more emphasis (i.e. state and prove the Law of Sines, Law of Cosines, etc.).

9. Cutting the cards on test-day lends itself to many student comments in class, which makes the spirit of the class better.

10. In better classes a broader range of problems can be assigned. The reverse is true in weaker classes.

11. It forces the lazy student in to some work.

12. On those days where you might not be properly prepared to teach you have a source of problems for them to do.

13. Gives many students a chance to work independently.

14. Students tend to help each other, but are not anxious to help the lazy ones.

15. Other assignments or projects could be marked and included in the total for review tests.

DISADVANTAGES:

1. Test-day can be a heavy day of marking, especially with 6 classes.

2. Keeping students supplied with review problems keeps the

teacher hopping.

3. You must be prepared to give students help with these problems either before or after school.

4. There is a certain amount of "lost time" during test day, but this is questionable.

CONCLUSION

We have been generally pleased with the student-response to this idea and we are looking for ways to improve it. We feel that the basic idea is sound that the other teachers could modify it in many ways to suit their own situation.

REVIEW

(1.1) Solve and classify the following equations

(a) $2(3x+1)(5x-2)-3(2x-1)(x+2) = 2(3x+1)(4x-3)+x,\ x \in I$

(b) $|2-3x| + x = -7,\ x \in R$

(1.2) Name all the sets (in the structure of the real number system) to which each of the following belong. Where possible write the given number as a ratio of integers.

(i) .5101520253035 ... (ii) .237373737... (iii) .75

(iv) -812 (v) $\sqrt{16}$ (vi) $\sqrt{13}$

(1.3) Simplify: (1) $5\sqrt{75} - 6\sqrt{72} + 3\sqrt{98} - 8\sqrt{27}$

(2) $\dfrac{6\sqrt{3} + 5\sqrt{6}}{3\sqrt{3} - 2\sqrt{6}}$ (3) $\dfrac{5x + 3}{x - 1} + \dfrac{x}{x - 2}$

(1.4) Find the domain and range of

$A = \{(x,y) | xy+y = 7\}$ and $B = \{(x,y) | y = \sqrt{2x-7}\}$

(1.5) If $F = \{(x,y) | 3x-2y = 7\}$ find k such that

$$f(k+2) + 2f^{-1}(k-3) = 3f(7) + 2k$$

.....

(2.1) Solve and graph

(i) $A = \{x | 3(2x-1)-7(2x-4)<5\}$ (ii) $B = \{x | -13 \leq 2(x+1)-3x<5\}$

(2.2) Find

(i) the range of $C = \{(x,y) | x = \sqrt{3y-6}\}$

(ii) the domain of C^{-1}

(iii) the graph of C

(2.3) If $D = \{(x,y) | 5x+7y = 6\}$ find m such that

$$d(m-1) + 2d^{-1}(3m+2) = d(3m) + 5$$

(2.4) Draw the graph of $E = \{(x,y) | y = \frac{2}{x-3}\}$ by first discussing its domain & range.

(2.5) Using equations, prove that the altitudes of ΔABC are concurrent. A is at (-2,-2) B is at (2,6) and C is at (6,2).

.....

(6.2) Given the following functions:

$f(x) = 3x+5$ find $f^{-1}(5)$; $g(x) = 8x-2$ find $g^{-1}(7)$; $k(x) = -5+7$ find $k^{-1}(3)$. Find K such that $f^{-1}(K) + g^{-1}(K+1) = k(5)$

(6.3) Solve for x and classify the equation

(i) $9^{3x-4} = 27^{5x-3}$ (ii) $16^{3x+2} = \frac{1}{4^{2x-1}}$

(6.4) Simplify:

(i) $\frac{3}{5^{-1}+2^{-2}}$ (ii) $\frac{3^{-1}+3^{-2}}{3^{-1}-3^{-2}}$ (iii) $\frac{64^{5/6}-27^{-2/3}}{12^{0}+125^{-4/3}}$

(6.5) Find the linear function with $m = 7$ such that $2f(a-1) - f(a+2) = 4$. Evaluate $f^{-1}(3)$.

.....

(7.1) Find the angles of quadrilateral ABCD where $A = (6,1)$, $B = (8,-2)$, $C = (-3,1)$, $D = (-1,-2)$. Use tables.

(7.2) Find the domain and range of:

(i) $A = \{(x,y) | 3x+2y = 7\}$ (ii) $B = \{(x,y) | xy+y = 7\}$

(iii) $C = \{(x,y) | x\sqrt{1-y} = 7\}$

(7.3) Solve & classify:

(i) $\frac{1}{3}x + 6 = \frac{1}{4}x - 1$ (ii) $\sqrt{3x-1} = -5$ (iii) $2(x+1)(x-1) = -x(x+12)+13$

(7.4) If $D = \{(x,y) | 4x-y = 7\}$ find K such that $d(K) + d^{-1}(K-2) = 5$.

(7.5) (i) Write the equation of F_1, the family of lines that pass through $A(3,1)$.

(ii) Select the one line from F_1 that is perpendicular to $L_1 = \{(x,y) | 3x+7y = 5\}$

(iii) Select the other line from F_1 which has the same x-intercept as $L_2 = \{(x,y) | 5x-2y = 10\}$

.....

(10.1) Evaluate using logarithms:

(i) 25.61×732.5 (ii) $.5354 \div .002713$

(10.2) Simplify:

(i) $\log_2 56 - \log_2 7 + \log_3 54 - \log_3 2$

(ii) $\log_{10} 40 + \log_{10} 2.5 - \log_4 \frac{2}{3} - \log_4 96$

(iii) $\log_7 14 + \log_3 108 + \log_7 3.5 + \log_3 \frac{3}{4}$

(10.3) Solve: (i) $\log_{10}(x+1) + \log_{10}(x-2) = 1$

(ii) $\log_9(x-6) + \log_9(x+2) = 1$

(10.4) Simplify, using the graph of $y = 2^x$

(i) 1.3×2.9 (ii) $(1.9)^3$ (iii) $\sqrt{7.2}$ (iv) $\sqrt[3]{7.4}$

(10.5) Solve (i) $3^{5-x} = \frac{1}{3}$ (ii) $2^{3x+5} = 0.5$ (iii) $x^2+2x-7 = 0$

.....

(11.1) Simplify: (i) $\frac{2^n \times 4^{n-1} \times 16^{n+2}}{(4^3)^{n+1}}$ (ii) $\frac{(3^a)^3 \times (3^{a+2})^{a-2}}{(3^{a+1})^{a-4}}$

(11.2) Graph A A^{-1} where $A = \{(x,y) | 3x-4y \leq 12\}$

(11.3) A(2,1) B(4,3) C(x,y) D(3,7) are the vertices of a parallelogram in the first quadrant. Find (x,y).

(11.4) $F = \{(x,y) | 5x+7y = 2\}$; find K such that $f(K+1) + f^{-1}(K-1) = 3$.

(11.5) Evaluate using logs. $\sqrt{3571 \times .00008123}$

.....

(12.1) Evaluate using a slide-rule and check using logarithms

(i) $\frac{\sqrt{357 \times (29.3)^2}}{6.12}$ (ii) $\frac{\sqrt{391 \times .00391}}{5.76 \times 631}$

(iii) $\frac{\sqrt{53.7 \times 71.2 \times 8.14}}{7.93 \times .00612}$ (iv) $\frac{\sqrt{357 \times 93.1 \times 16.2}}{112 \times .231}$

(v) $\frac{80.3 \times 57.6 \times \sqrt{3840}}{61.2 \times (23.2)^2}$

.....

(13.1) Evaluate using a slide rule $\frac{a \times b \times c}{d \times \sqrt{e}}$ where a, b, c, d, e will be given on the day of the test.

(13.2) Given the parabola $y = ax^2 + bx + c$ find the coordinates of the vertex. Under what conditions will this parabola open up?

(13.3) Evaluate $\frac{a \times b \times c}{d \times e}$ using a slide rule where a, b, c, d, e will be given on the day of the test.

(13.4) An object projected vertically upwards is h feet above the ground after t seconds according to $h = -16t^2 + 64t + 16$. Find the maximum height reached and the time of flight!

(13.5) Solve for x without using tables $\log(x+1) + \log(x-3) = \log 9 + \log 5$.

.....

(14.1) Graph A B where $A = \{(x,y) | y \text{ x } -2x + 4\}$ and $B = \{(x,y) | y -2x +4x-7\}$.

(14.2) Graph C D where $C = \{(x,y) | y -3x -5x+1\}$ and $D = \{(x,y) | y -3x -5x-1\}$.

(14.3) Find x without using tables given $\log(2x+1)+\log(x+18 = 2$.

(14.4) If $G = \{(x,y) | 3x-2y-4 = 0\}$ find K such that $g(2K+1)+ g^{-1}(3K-2) = 4$.

(14.5) Prove that the altitudes of Δ A(5,3) B(-3,1) C(1,-4) are concurrent.

.....

(15.1) You will be given a class set of marks on a test out of 37.

Convert these marks to a mark out of 50 using a slide rule. Each mark /50 is to be estimated to the nearest 1/5 of a mark.

(15.2) Find the vertex, y-intercept, x-intercepts, domain and range of $A = \{(x,y) | y = -1/3x^2+4/3x+17/3\}$. Illustrate this information on a sketch of the graph f A.

(15.3) Solve $2x^2 - 5x + 3 = 0$

(i) by factoring (ii) by completing the square (iii) by formula.

(15.4) Graph $A \cap B$ where $A = \{(x,y) | y>x^2+4x+3\}$ and $B = \{(x,y) | 2x-y-6>0\}$.

(15.5) A problem on evaluating using logarithms, like 12.1.

.....

(16.1) A picture 20 inches wide and 10 inches high is to be centrally mounted on a rectangular frame with total area three times the area of the picture. Assuming equal margins on all four sides, find the width of the margin.

(16.2) A man travels 120 miles by bus and 180 miles by train. The train's average speed is 5 mph more than the bus. The combined trips take 7 hours. Find the average speed of the bus.

(16.3) The time required for a train to make a 175 mile trip is $1\frac{1}{2}$ hours less than the time required by a bus. The average rate of the bus is 15 mph less then the average speed of the train for the same trip. Find the average speed of each.

(16.4) Two boys agree to mow a rectangular lawn 80 feet by 60

feet so that each cuts one half of the lawn. How wide a strip around the outside must the first boy cut to do his share.

(16.5) A dealer bought a certain number of melons for \$1.50. After throwing 5 bad ones away he sold the others at 8 cents a piece more than he paid for them. If his profit was 30 cents, how many melons did he buy?

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(17.1) Graph $A \cap B$ where $A = \{(x,y) | y<2x^2-4x+5\}$ and $B = \{(x,y) | y>2x^2-4x+3\}$.

(17.2) (i) $2x^2 + 3x + k = 0$, $x \in R$, has 2 equal roots. Find k.

(ii) $kx^2 + 7x - 1 = 0$ has 2 complex roots. Find k.

(iii) $x^2 + kx + 9 = 0$ has 2 real roots. Find k.

(17.3) Use the factor theorem to factor $f(x) = x^3 - 7x + 6$ and as a result find the x-intercepts of $f(x)$.

(17.4) Are the following lines concurrent? Defend your answer.

$L_1 = \{(x,y) | 2x+y-7=0\}$ $L_2 = \{(x,y) | 3x+4y+6=0\}$ $L_3 = \{(x,y) | 2x-y-7=0$

(17.5) Simplify using logarithms

$$\frac{\sqrt[3]{.0053} \quad (7.56)^2}{392 \times .00731}$$

.....

(18.1) Evaluate each of the following.

(i) $8^{2/3} - 5^0 + \left(\frac{4}{9}\right)^{-3/2}$ (ii) $\frac{2^{-1} - \left(\frac{1}{2}\right)^{-2}}{2^{-2}}$

(iii) $\frac{2^{-2}}{2^{-2}+2^{-3}}$

(18.2) Do problem (15.5).

(18.3) Find the vertex, the x and y-interecepts, the axis of symmetry, the domain and range of $A = \{(x,y) | y = \frac{1}{2}x^2 - x - \frac{15}{2}\}$. Sketch the graph of A.

(18.4) (i) The parabola $y = 2x^2 + 3x + k$ doesn't cross the x axis. Find the possible values of k.

(ii) The curve $y = 5kx^2 + 2x - 7$ just touches the x axis. Find the possible values of k.

(18.5) An object projected vertically upwards is h feet above the ground after t seconds according to $h = -16t^2 + 64t + 80$. Find (i) the initial height (ii) the maximum height reached (iii) the time of flight of the object.

.....

(19.1) A jet-airliner travels 100 mph faster than a turbo prop airliner. For a trans-Atlantic flight of 3000 miles the jet takes an hour less time than the turbo prop. Find the speeds of both types of aircraft.

(19.2) Solve (i) $x^3 - 5x^2 + 3x + 9 = 0$, $x \in C$

(ii) $\frac{1}{3}x^2 - 2x - 7 = 0$

(19.3) Solve (i) $9^{x-5} = 27^2$ (ii) $9^{3x+2} = (9^x)^2 (27)^{x+1}$

(19.4) Graph A B where $A = \{(x,Y | 3x-2y<6\}$ and $B = \{(x,y) | y<x^2-2x+3\}$.

(19.5) (i) In any ΔABC prove that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(ii) Solve $\triangle ABC$ given $a = 23.4$ $*B^a = 53°^b$ and $*C^c = 41°$.

.....

(20.1) (i) In any $\triangle ABC$ prove that $a^2 = b^2 + c^2 - 2bc \cos A$

(ii) Solve $\triangle ABC$ given $a = 43.2$ $b = 54.3$ $*C = 55°$

(20.2) G is a coastguard station located 9.6 miles due north of C, a second coastguard station. From G the bearing of a ship at sea is S34°E. From C the bearing of the same ship is N62°E. How far is the ship from the nearest coastguard station?

(20.3) Two roads intersect at A at an angle of 37°. Smithtown is 4.7 miles from A on one road and Johnsonville is 7.2 miles from A on the other road. If a straight road was built connecting Smithtown and Johnsonville how much would it cost if it costs $10,000 per mile to build a road over that type of terrain?

(20.4) (i) Calculate the length of the arc ABC and the area of the sector ABCD given the radius is 6 feet.

(ii) An angle θ in standard position has the point $P(-3,-2)$ on its terminal arm. Calculate

(i) $\sin \theta$ (ii) $\cos \theta$ (iii) $\tan \theta$

(20.5) Graph $A \cap B \cap C$ where $A = \{(x,y) | y > 2^x\}$ $B = \{(x,y) | x > 0\}$ and $C = \{(x,y) | y < -x^2+2x+2\}$.

.....

(21.1) Two boys agree to mow a rectangular lawn. The first boy cuts a 5 foot border, leaving a rectangular area 4 feet longer than the width, having an area of 940 square feet. If the job

will pay \$6.20 how much should each boy get for his part of the job?

(21.2) Without using tables, evaluate

(i) cos 120° (ii) tan 225° (iii) sin 330° (iv) sec 150°

(v) tan 210° (vi) csc 315° (vii) cot(-300°)

(21.3) Solve ΔABC given $\angle B = 21°$, $\angle C = 37°$, $a = 25.7$

(21.4) Solve ΔABC given $a = 31.2"$, $b = 15.7"$, $c = 25.3"$

(21.5) Solve ΔABC given $c = 23.4"$, $b = 15.8"$, $A = 110°$

.....

(22.1) In any ΔABC prove $a^2 = b^2 + c^2 - 2bc \cos A$

(22.2) Solve completely $x \in C$

(i) $x^3 + x^2 + 4x - 6 = 0$ (ii) $x^4 + 35x^2 - 36 = 0$

(22.3) A numerical calculation using logarithms, the question to be given on the day of the test.

(22.4) Simplify:

(i) $\frac{3^{-1}\times 27^4\times 81}{27^2\times 9^{-3}}$ (ii) $\frac{2^{-1}+3^{-1}}{2^{-1}-3^{-1}}$ (iii) $\frac{10^0+4^{-1}}{5\times 2^{-3}-5}$

(iv) $6\sqrt{18}-5\sqrt{12}-3\sqrt{50}+2\sqrt{27}$

(22.5) In any ΔPQR prove that $\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$

.....

(23.1) Find the domain of $A = \{(x,y) | y^2 = x^2 - x - 20\}$.

(23.2) Solve: (i) $x^4 - 4x^2 + 3 = 0$ (4 answers) (ii) $(x^2-2x)^2 - 4(x^2-2x) + 3 = 0$ (4 answers)

(23.3) If a car increases its speed by 20 mph it will reduce the travel time for a 120 mile journey by one hour. What is the

increased speed?

(23.4) Prove that:

(i) $\frac{1}{1+i} = \frac{1-i}{2}$ (ii) $\frac{1}{2-3i} = \frac{2+3i}{13}$

(23.5) Evaluate using logarithms $\left\{\frac{345\times(.00831)^2}{.671}\right\}^{2/3}$

BOOK REVIEWS

Two years ago when I first came to Lambton College, the first semester course in mathematics offered to Liberal arts students consisted of a combination of topics that came from several branches of mathematics. It introduced mathematics to students who "have no interest in a conventional mathematics course". The topics taught were: Elementary Set Theory, Combinatoric Topics, Probability, Statistics, Boolean Algebra, Matrices, Number Systems & Bases, Topics from Number Theory, History of Mathematicians, Graph Theory, Seminars were delivered by students, An Essay was asked for on Mathematics in Western Culture using Kline as a reference.

Each of the topics mentioned was skimmed very lightly and attempts were made to bring out only the simplest concepts which it was felt the student could understand. The course was mathematical in content at all times but the emphasis was to try to keep the student's motivation high by making the mathematics as interesting as possible. This consideration of the student's power of concentration prevented me from doing any serious mathematics.

There was no textbook available to cover the topics I wanted with the degree of difficulty - or rather the degree of simplicity I needed - hence the course was given largely in the form of lecture notes. The course was well received and the

attendance was next to excellent. I did not make attendance compulsory but I do not exclude the possibility that attendance was excellent because the course was compulsory. Class results for final grades were from average to good.

Students would benefit greatly from a textbook. I've looked at several available texts written especially for liberal arts students and none fit the standards I required. All were too difficult or did not contain the variety of material at the level of achievement I wanted. Not ignoring the fact that "there is no royal road to mathematics" I was still looking for one.

I finally did find a book I was looking for called *Mathematics: A Human Endeavor*, written by Harold R. Jacobs and published by Freeman 1970. It contains topics similar to the ones I have been giving and then some. It does a far better job of making the subject matter interesting than any other text I have reviewed. Martin Gardner, Editor of the Mathematical Games Department of Scientific American says "Above all, he knows how to present mathematics in a way most likely to provide an average student with what is fashionable to call 'motivation'." Other people comment: "I can't believe it - it seems just too good to be true. I have examined more than a dozen books within the past weeks in an effort to find a text for my spring course in 'mathematical ideas'". "An excellent text. It is just the kind of book I have been looking for to use in our College Mathematics class.

Presentation is lively and to the point. The topics are appropriate for liberal arts students and the exercises are at about the right level of difficulty". "Overall I am quite excited about the book. It is certainly the most visually attractive, stimulating, and provocative book of this sort I have seen: when I first opened it I was compelled to leaf through, looking at the cartoons and pictures. I expect that many students might have the same reaction -- this may well be the first mathematics book they will really want to read. Passing to the subject material, I was brought to earth by the table of contents, which suggests that the book really treats the same old stuff -- numbers, functions, graphs, probability. And indeed it does, for the author's originality lies not in novelty of subject but in novelty of approach". "This is one of the most intriguing books I have seen in orver 38 years of the teaching of mathematics, from the kindergarten to the university".

The text contains the following topics: Inductive-Deductive Reasoning, Arithmetic and Geometry sequences, Number Patterns, Idea of a Function, Coordination Geometry, Graphing, Extrapolating and Interpolating, Logarithms, Slide Rule, Polygon and Polyhedra Models, Symmetry, Conic Sections, Sine Curves, Spirals, Cycloids, Combinatorics, Probability, Statistics, Topology.

This text does not cover the particular topics I want but this difference in taste is offset by its method of presentation,

which I like.

There is a Teacher's Guide accompanying the text which contains complete lesson development and procedure for each and every chapter of the text. Unquestionably it is a remarkable addition to the main text for it contains little gems of interesting anecdotes that you can relate during the presentation of your lesson: for example; in the chapter on Measures of Central Tendency, lesson 3, page 95, the author writes:

> "One of the most extraordinary books ever written was published in Los Angeles in 1939. It is probably safe to say that it is unique, that there is no other book like it in the English language. The book was a novel written by Ernest Wright and is titled *Gadsby, A Story of Over 50,000 Words Without Using the Letter E.* You know that E is the most frequently occurring letter in written English, yet it does not occur even once in the entire novel. Think of how many common words contain E: "the", "he", "she", "they", "be", "are", "were", "have", "there", "then", and "when", to name a few.
>
> As he wrote the book, the author had a difficult time keeping words containing the letter E from slipping in. In the preface, he says: "As I wrote along, in long-hand at first, a whole army of little "e's" gathered around my desk, all eagerly expecting to be called upon. But gradually as they saw me writing on and on, without even noticing them, they grew uneasy; and, with excited whisperings amongst themselves, began hopping up and riding on my pen, looking down constantly for a chance to drop off into some word; for all the world like

seabirds perched, watching for a passing fish! But when they saw that I had covered 138 pages of typewriter size paper, they slid off onto the floor, walking sadly away, arm in arm; but shouting back: 'You certainly must have a hodge-podge of a yarn there without us! Why, may! We are in every story ever written, hundreds of thousands of times! This is the first time we were shut out.'" (this is related on pages 739-740 of Kahn's The Code-breakers.)"

You will find several references to other books in the textbook as well as the Teacher's Guide and it's no doubt that such books should be on your library shelves to be used by your math students.

In the second semester we offered a course called "Introduction to Computers" consisting of the following topics: History of Calculating and Computer Devices, Computer Components, Binary Arithmetic, Manipulation of Numbers and Symbols within the Computer, Machine Language (numerical and mnemonic), The Compiler, Supervisor, etc., and Simply Fortran and Algorithms, Writing and Running of Fortran Programmes on Punched Cards.

Attendance to this second semester course began falling and steadily grew worse until a peak was reached when I began to teach programming languages. I felt the liberal arts student had no desire nor interest to learn the exactness of a computing language, and for example, when his programme is rejected by the computer for his having left out a comma in one instance, or his

having put in a comma when he should have left it out in another, is the type of thing that "turns him off" and away from the subject. If he has to sustain a thought of three or four things simultaneously, as he must when dealing with numerical machine language for example, the student seems to give up easily and he loses interest. Of course, as he gets behind in such material and he quickly does, there is almost no chance for recovery. The minute details of syntax and semantics together with the mastery of algorithms needed to be successful, in a section of the course which covers 40% of the semester's work, overtaxes the average liberal arts student's mind. Most of my students confessed that they hated math most of their lives.

I could see, at that time, that if first year math was not compulsory Lambton College would not have any students at all in the second semester course. I felt a revision in the second semester course was necessary and so I began the arduous task of looking for the right text for the course I had in mind. First of all, the course had to have little or no mathematical content. Originally, I thought I might teach a course similar to the material contained in the History of Mathematics by Howard Eves, except more time would be spent on the History of the Mathematicians and the historical events of that time rather than any in-depth Study of Mathem-tics itself. Such things as the Pythagorean Theorem, origin of numbers, the quest for solutions to

equations that led to discovery of certain numbers, the application and invention of trigonometry to determine the diameter of the earth and the distance to the moon, trisection of an angle and Morley's Theorem, the long history of π etc., would provide the incentive for such an approach.

I recalled that a credit course was given to a television audience by Professor Morris Kline of N.Y.U. several years ago and so wrote to his publisher for copies of some of his texts. They were: *Mathematics In Western Culture, Mathematics For The Liberal Arts Student, Mathematics: A Cultural Approach.* I found the material was different from what I had in mind but it was the kind of course suited for liberal arts students. *Math in Western Culture* contained little to no mathematics and was my choice for the considered course. At any time where mathematical theory was applied, which was in very few instances, I told students to read the material on their own if they were interested. *Mathematics: A Cultural Approach* is more mathematically and historically enriched. The treatment of Math in Western Culture is more indepth. It is a good reference for both teachers and students. These latter texts also have exercises from which assignments can be drawn. Morris Kline states that these two texts each contain more material than can be handled in a one semester course.

Mathematics In Western Culture begins by describing

mathematics as an art rather than a series of techniques and claims that the pursuit of mathematical discoveries in a given civilization promotes its culture. Some facts are presented concerning mathemtics in Egypt and Babylonia, thus presenting the origins of mathematics and its application in relatively primitive civilizations. At this point in time the emphasis is on the empirical nature of pre-Greek mathematics and inductive reasoing (a reference to 1st semester math can be made at this time).

The Classical Greek period (600 to 300 BC) is discussed and here the change in the approach to mathematical activity is emphasized. The success in mathematics, which was the result of rigorous treatment of abstract ideas - deductive proofs from explicitly stated axioms - influenced the Greeks to apply rigor in literature, philosophy, etc.

The Alexandrian period (300 BC to 600 AD) comes next. The mathematics of this era favoured science and measurement. Trigonometry arose at this time as a step toward quantitative astronomy.

The final topic in the Greek period centres about the Greek doctrine that nature is designed in accordance with mathematical law and that man can fathom that design through the mastery of mathematics. Euclidean geometry is looked upon as a perfect pearl in man's quest for absolute truth. The importance of Euclidean geometry and the supreme Greek achievement in

astronomy, (the Ptolemaic Theory for example) as evidence for the rationality of nature is discussed.

All the while the cultural influence of mathematics is developed. A brief look is taken at the structure of Greek society and its centres of learning. A mathematician-philosopher is looked upon as a member of the upper class. The priestly class, of which many were mathematicians, looked upon mathematics as a source for developing knowledge about man and his universe. These people worked together fairly well, and no conflict is noted between this culture's religious beliefs and the discoveries in mathematics or science. Emphasis is made at this time to the many gods people in this pre-Roman culture worshipped.

At this point a brief discussion ensues concerning the Roman Empire and its decline, the rise of Christianity, the dark ages, and the power of Christianity.

The Renaissance period is sketched. The creation and introduction of the heliocentric theory by Copernicus and Kepler is discussed. Its affect on religious dogmas, the conflict between Church and Science, the inquisitions and persecution of learned men is raised. In a sense the Church is viewed as a barrier to scientific progress. "The Greeks had many gods and no theology and the mathematical spirit flourished whereas the Medieval period had but one God and a great deal of theology. Here the mathematicial spirit was at its lowest ebb." At this time a code

existed called the Roman Code of Mathematics & Evil Deeds which dammed and forbade the art of mathematics. Mathematical logic and reason was in constant conflict with the logic and reason of the Church and one risked the fate of Giordano Bruno who was put to death by the Inquisition. He declared that "man is no more than an ant in the presence of the infinite". We all know the fate of Galileo. When Galileo said to his circle of mathematicians, and scientists, priests and royal members, "if you don't believe my word then look through the telescope and see for yourself". And when they did look, they claimed they saw nothing, much to Galileo's demise. How many books of historical significance have been destroyed by both writer and state? No one will ever know.

Religious freedom was a rarity during the 17th century and the Church had effective ways to exert its influence as evidence by such historical events as: The Spanish, Roman, and Mexican Inquisitions, The St. Bartholemew's Day Massacre in France, The Piedmont Massacre in Italy and The Thirty Years War in Germany. These are but a few of the inspired efforts to educate mankind which are mentioned.

Two chapters of the book are devoted to perspective in painting and it discusses how these techniques led to the development of projective geometry.

In the 18th century a rationalistic movement was

inspired by the development of the Universal Laws. "Imbued with the conviction that reason, personified by mathematics, could solve all of man's problems, the great minds of the 18th century undertook a sweeping reorganization of science, philosophy, religion, ethics, literature, and the social sciences". It explains how science became more dependent upon mathematics, that doctrines such as materialism and determinism were built upon mathematical and scientific foundations, that the source and nature of truths were re-examined, that new religious movements culminating in Deism resulted, and that poetry was depreciated in favour of prose.

The rationalistic spirit of the Age of Reason had enforced the Social Scientist with the desire to discover the universal laws of their domains. This desire to interpret results mathematically lead to the development of statistics and probability.

Finally the advancement of non-Eucliden geometry is presented as the destroyer of the belief that Eucliden geometry represents the absolute truth: The ideas in Relativity gives the liberal arts student a surprising impression of the power of mathematics especially when it is learned that it predicted the now famous equation $E = mc^2$ and that it still predicts contraction of a moving object when the object approaches the speed of light. Mathematics is shown as an art which continually creates changes

in old ideas by presenting new and improved ones. Take for example the cyclotron which is capable of smashing atoms which, according to Dalton in Dalton's Atomic Theory, indivisible particles. We show that mathematics is a viable and moving discipline, not stilted nor worn and that mathematics to a mathematician is an exciting, rewarding field of study.

Since the course makes contact with painting, music, philosophy, literature, the social sciences, and so on, a course such as this consequently can blend well with other courses the student takes in the pursuit of his general arts education. It is hoped that when a student completes this course he will have a greater understanding and appreciation of the role mathematics plays in the development of a culture in civilizations. When he leaves this course although he does not understand mathematics as such, he will leave with a good feeling for it, and when he speaks of mathematics to others he will have the background to articulate in favour of it. One does not have to write prose or poetry, or to paint or sculpture to appreciate these arts and I contend one does not have to do mathematics to appreciate mathmeatics. There is little doubt in my mind that a course in mathematics appreciation is a sensible one for the arts student and it gives us as mathematicians an opportunity to sell our subject matter to non-mathematicians. Besides, these people whom we teach may someday be in the position to determine the fate of

mathematics in some areas.

A most important addition to such a course would be to have a good library. Titles to references are supplied at the back of the text. It is mainly through these sources that essays will be written. Also, on occasion, when the text makes reference to a Bibliography the student will want to read it first hand. Teachers are busy people and it is not always possible for them to read every book available for such a course in spite of the wish to do so, hence it is possible to enrich this course for your benefit as well as the student's by asking students to conduct seminars in areas of: significant historical events, geographical locations of ancient centers of culture, personal histories of certain mathematicians and scientists such as Copernicus, Kepler, and Galileo, to mention a few.

As I see it, mathematics for the first semester liberal arts students should be mathematical in nature whereas in the 2nd semester he should have some choice. The choice could be a course in *Introduction to Computers* as originally designed with perhaps less emphasis on computing languages, *History of Mathematics and Mathematicians* as described earlier using, perhaps, E. T. Bell's *Men of Mathematics* as a main text, *Mathematics in Western Culture*, and a course strictly on mathematics, continuing in the same vein as my first semester course - perhaps the same topics but with more depth. Such a student might also take the math 103

course which we offer to students who want a conventional treatment of mathematics plus a little calculus. I see these students coming into the 2nd and third year of the Liberal Arts School to continue to develop their mathematics skills until a point of proficiency is reached equal to the Math 20 course of the 1st year arts and science course at the Universit of Western Ontario. The Math 20 texts used to be *Introduction To Finite Mathematics* by Kemeny and Snell and *Introduction to Analytic Geometry and Calculus* by Burdette. The course mentioned earlier would represent a terminal point in the study of mathematics for these students.

Through these courses we have an opportunity to get students interested in our subject matter which in the past they may have found uninteresting.

A most unfortunate situation occurs in our primary systems which is to find teachers who hate mathematics and who have never done well in mathematics teaching mathematics to our children. It often happens that this dislike of a subject is transmitted from the teacher to the child in various ways: spending less than the allotted time in the subject to spend more time on the teacher's favorite subject; presenting mathematics as a dull and boring subject, and in general, teaching the subject matter incorrectly because of the lack of the lack of academic background. It is now possible for a grade 13 student to go to

Teacher's College without having studied more than grade 10 mathematics and yet in his teaching environment he or she will teach the same class all day in all subjects, which of course includes mathematics.

Not only our students, but many other students, are the product of 12 to 13 years of this type of exposure and from year to year the situation gets progressively worse for them instead of better. When people are asked what is mathematics for? people usually respond by saying its use is to build bridges and to keep track of financial accounts and so on. But to a student in Mathematics in Western Culture he is taught that it is far more. He learns to see mathematics as a flourishing branch of our culture, the application of some of it, still unknown, requiring imagination, insight and creative ability. Mathematics to him is an art and he realizes that mathematical work must also satisfy aesthetic requirements.

These are the things I do with my students in the mathematics course for liberal arts students.

W. Lovsin, Lambton College

OMC NEWS

The executive of the Ontario Mathematics Commission wishes to make the following progress report to readers of the Gazette.

A Single Organization in Mathematics is proposed in a constitution for the "Ontario Association for Mathematics Education" which has been drawn up by a joint negotiating team of OMC and OATM representatives. At the May, 1972 meeting of the Ontario Association of Teachers of Mathematics the constitution was presented as a notice of motion to be voted on at its May, 1973 meeting.

The OMC executive has approved the proposed constitution and will recommend it to the Annual Meeting of the Ontario Mathematics Commission next September for implementation in June, 1973.

A French Edition of the Handbook for Teachers of Mathematics in the Intermediate Grades has been under active preparation during recent months by the Scholarship, Research and Publications Committee, chaired by Dr. Girhiny. To assist with the financing of this edition a grant was obtained by OMC from the Ontario Ministry of Education. This will be the third printing of this popular Handbook and it is expected to be available by the end of September. An announcement will be mailed to schools having French speaking teachers.

Planning of Leadership Seminars for October and November, 1972 has been completed, one of these being a follow-up Phase II Seminar for October 19-22 to be attended by those who were enrolled in the Phase I Seminar of last year. The purpose is to develop classroom leaders in elementary school mathematics who will aid their associates in local schools and communities.

A second Phase I Seminar, similar to that of October, 1971, has been planned for November 9 to 12 in Niagara Falls (tentative dates). Applications already exceed the 120 places available. Dr. Girhiny is continuing to serve as Director of these seminars and is supported by experienced capable staffs.

Many teachers in schools offering three units in Grade 13 Mathematics have been aided by class sets of booklets on Polar Coordinates and on Complex Numbers which have been made available by OMC with the kind permission of the authors. Requests have been filled for approximately 2700 copies of each booklet.

Publications available at OMC office, 1260 Bay Street, Toronto, 185 include:

Polar Coordinates by D. T. Faught	$ .50
Complex Numbers by J. E. Hogarth and B. J. Kirby	$ .50
Handbook for Teachers of Mathematics in the Intermediate Grades (English Edition)	$2.00

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