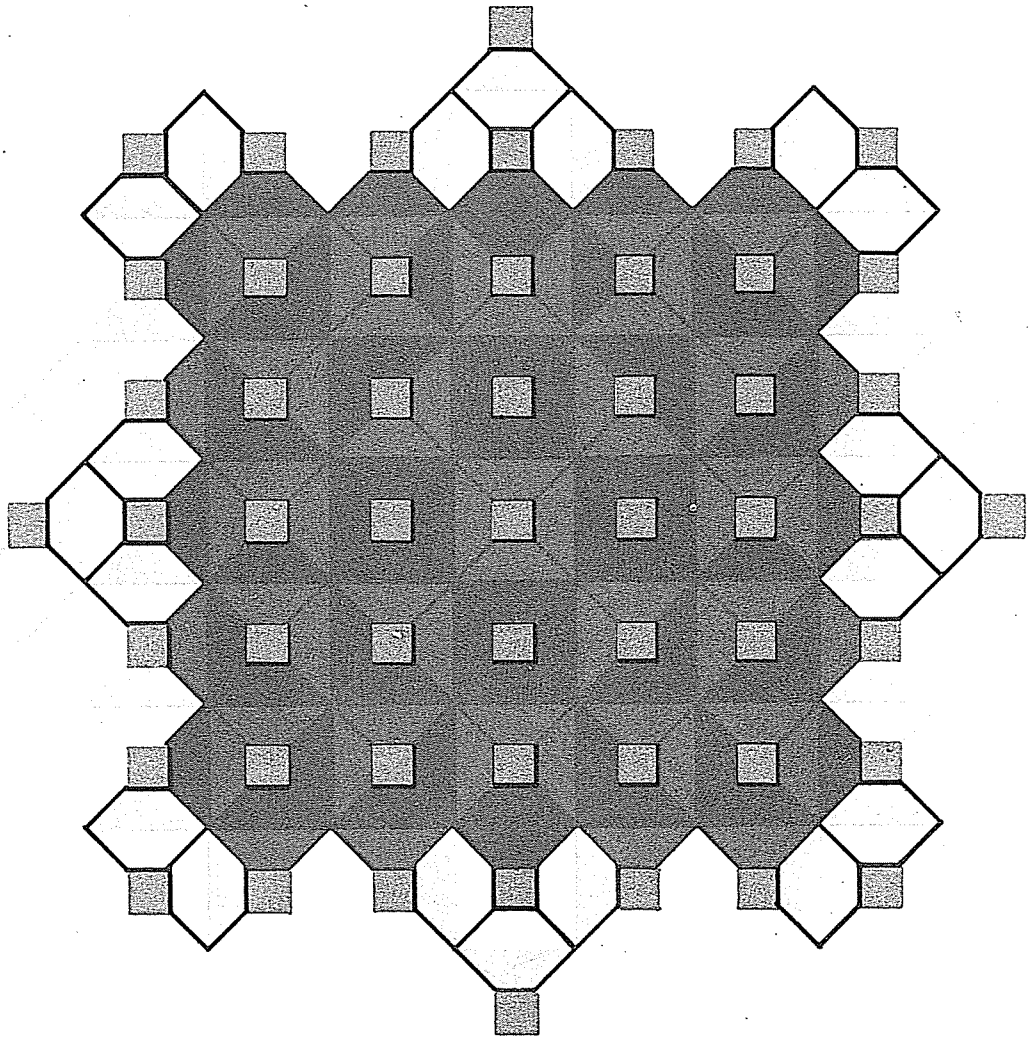
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EDITORIAL

This issue of the Gazette is dedicated to a giant of international mathematics education, Jean Piaget, who celebrated his eightieth birthday last August. Your editor was privileged to attend the 1972 conference of the International Congress on Mathematical Education, held in Exeter, England, in August, 1972. The conference paid special tributes to Piaget and George Polya, who between them have done much to clarify for teachers the dichotomy between (i) the abstract character of mathematics and the concrete way in which it is learned by children and (ii) the deductive nature of mathematics and the inductive manner in which most mathematics is created.

Polya attended the conference and made a big hit as a lively participant. Unfortunately, Piaget was not present because of illness. The paper which he sent to the conference is reproduced in this issue of the Gazette.

By a number of criteria Piaget is one of the world's greatest scholastic leaders of this century, perhaps of all time. The one thing in his career that impresses me most is the careful study of children as individuals by painstaking interviews, done by Piaget and his assistants over a span of forty or more years. He has identified the distinct stages through the cognitive processes of the child develop and the terminology he invented has become part of the language of developmental psychology.

Piaget is difficult to understand. I think, however, the effort is rewarding. He has his disciples and detractors; he cannot be ignored. The changes which have been made and which continue to take place in the Ontario curriculum owe much to Piagetian research and its interpretation (or misinterpretation). The rationale for **The Formative Years** cannot be fully understood without knowledge of Piaget's theories. When the so-called "new math" came in, the emphasis on "sets" in the early elementary grades was supported by the claim that Piaget had proved that cardination precedes ordination in the child's development. (See BOOK REVIEWS)

We shall soon have a new set of Ministry guidelines for the Intermediate Division (grades 7-10, ages 11-15). There is no doubt they are badly needed. The Ministry deserves credit at this time for correcting earlier mistakes and oversights with a number of improvements. One was the research on mathematics in the Intermediate Division, known as the Pogue Report, described in this issue. Another is the appointment of our president, Tom Griffiths, as O.A.M.E. representative on the curriculum revision committee. At last we mathematics teachers have an **official** voice in the decision making process. Thirdly, the recently announced return to a carefully spelled out compulsory provincial core is in the writer's view a welcome improvement.

Now for the bad news! Press reports indicate that the Minister of Education wants the curriculum committee to complete its work in two months. If true, that is ridiculous. I hope that common sense will prevail and the committee will be given **at least** six months to do justice to their enormous task. It is rumoured that the committee will have some members seconded fulltime. Even so they need ample time for study and consultation. Look at Piaget's example: a lifetime spent developing ideas of lasting worth, not a trendy pseudo-change brought out because of political pressures.

Your editor attended the November O.A.M.E. Council meeting as an observer. The organization is very much alive and growing. Many projects are under way. In the next issue we shall see Senior Division recommendations which will have been forwarded to the Ministry in the new year. Remember the survey reported in the September 1975 Gazette? By the way, have you renewed your membership?

Editorship brings many rewards to compensate for the hardships. This month I was surprised to receive a proof of Fermat's Last Theorem from overseas. Natural caution prevented me from recommending it for publication without careful scrutiny. If any reader would like a copy to study (and if possible refute) he or she is welcome to it. Just drop me a line.

Arn

PIAGET: A BRIEF BIOGRAPHY

Based on an article in **The Encyclopedia Britannica:** Jean Piaget is the world renowned psychologist and investigator of thought processes, especially among children. He was born August 9, 1896, in Neuchatel, Switzerland. At the age of 10 he wrote an article on an albino sparrow, and by the age of 15 his publications on mollusks made him known to international specialists. After he obtained his doctorate in science at the University of Neuchatel (1918), his biological training, together with an intense interest in logic and epistemology, or theory of knowledge, led him into psychology.



During two years of research at the Sorbonne, Piaget began to explore why children fail in tests of reasoning. This led to his appointment to the directorship of the Institut J. J. Rousseau, Geneva. Over his long career a vast number of his books and articles have been translated into English and published in Britain, U.S.A. and Canada. He was professor of psychology at the University of Geneva from 1929 to 1954 and in 1955 became director of the International Centre for Epistemology at the university. (See BOOK REVIEWS for a listing and discussion of Piagetian writings.)

"Piaget's description of the development of cognitive abilities is held in high professional repute. Adopting an earlier view that thinking arises in situations that cannot be handled by automatic, learned performances and reflexes, Piaget suggested that thinking is a refined, flexible, trial-and-error process. He traced four stages in the development of children's thinking. The first,

or sensorimotor stage, occurs in the first two years of life and is characterized by an empirical, largely nonverbal intelligence. Experimenting with objects and connecting newer experiences with older ones, the child can be said to be learning from experience. In the second, or pre-operational level, from two years to seven, objects of the child's perceptions come to be represented by words, which he now manipulates experimentally in his mind as he had previously experimented physically with concrete objects. In stage three, from 7 to 12, his first logical operations occur, and he classifies objects by their similarities and differences. In the last stage, from 12 and into adulthood, the individual begins experimenting with formal logical operations, and thinking becomes a more flexible kind of experimentation. Piaget's later work attempted to describe the interaction between cognitive and emotional factors in his postulated four stages of thought development."[1]

1. Encyclopaedia Britannica: 15th edition, 1974, Micropaedia, vol. 7, p. 982.

LETTERS TO THE EDITOR

Department of Mathematics
Queen's University
Kingston, Canada
September 9, 1976

Professor A. Harris
Editor, Ontario Mathematics Gazette

Dear Professor Harris:

Dale Cooper, in a couple of recent letters to the Gazette, has made some interesting comments about multiple roots of a function, and how they can be used to test whether critical points of a function are points of inflection or extreme points. Below is an alternate definition of multiple roots which provides an easy confirmation of the conjecture Cooper makes at the end of his letter in the September Gazette, and which also makes it easy to see why the "Generalized Pinnegar Conjecture" is true.

Definition. x_0 is a root of the function f with multiplicity m if there is a function g such that

$$f(x) = (x - x_0)^m g(x) \text{ and } g(x_0) \neq 0.$$

Cooper's definition says that x_0 is a root of f with multiplicity m if $0 = f(x_0) = f'(x_0) = \cdots = f^{(m-1)}(x_0)$ and $f^{(m)}(x_0) \neq 0$. The two definitions are seen to be equivalent by noting that Cooper's definition amounts to requiring that the first non-zero term in the Taylor series of f about x_0 is $(x - x_0)^m f^{(m)}(x_0)/m!$, and hence that m is the largest integer for which $(x - x_0)^m$ is a factor of all the terms in the series, (assuming, of course, that the functions we are dealing with are all sufficiently well-behaved to have Taylor series).

The following theorem verifying Cooper's conjecture about multiplicities follows immediately from the definition above:

Theorem 1. If x_0 is a root of f_1 and of f_2 with multiplicities m_1 and m_2 respectively, then x_0 is a root of the product f_1f_2 with multiplicity $m_1 + m_2$.

Proof. $f_1(x)f_2(x) = (x - x_0)^{m_1}g_1(x) \cdot (x - x_0)^{m_2}g_2(x)$

$$= (x - x_0)^{m_1+m_2}g_1(x)g_2(x) \text{ , and}$$

$$g_1(x_0)g_2(x_0) \neq 0 .$$

A straightforward proof of Cooper's "Generalized Pinnegar Conjecture" also follows:

Theorem 2. Let f be a function for which x_0 is a root of the derivative f' with multiplicity m. Then x_0 locates a point of inflection of f if m is even, and an extremum of f if m is odd.

Proof. Since $f'(x) = (x - x_0)^m g(x)$ with $g(x_0) \neq 0$, it follows that $f''(x) = m(x - x_0)^{m-1}g(x) + (x - x_0)^m g'(x)$

$$= (x - x_0)^{m-1}[mg(x) + (x - x_0)g'(x)],$$

with the second factor different from zero when $x = x_0$.

Now, whether x_0 locates a point of inflection or an extremum of f depends on whether or not the concavity of f changes at x_0, and hence depends on whether or not the sign of $f''(x)$ changes at x_0. But the equation above shows that $f''(x)$ changes sign at x_0 iff $(x - x_0)^{m-1}$ changes sign at x_0, which happens iff $(m - 1)$ is odd, i.e., iff m is even.

P.S. As a postscript to the letter I sent you yesterday, I should emphasize that the function g in the definition of multiple roots must at least be continuous at x_0 for the definition to make reasonable sense.

For example, $f(x) = \sin x$ has a root of order 1 at $x = 0$ since

$$\sin x = xg(x) \text{ where } g(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases},$$

this g being continuous at $x = 0$ since $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

This example also points out a third equivalent definition of multiple root, namely, that x_0 is a root of f with multiplicity m if $\frac{f(x)}{(x - x_0)^m}$ has a non-zero limit as x approaches x_0.

13 Fraser Avenue
Onaping, Ontario
August 28, 1976

The Editor
Ontario Math Gazette

Dear Arn:

The following may be worth printing:

Bode's Law

The following sequence is related to the distances of the planets from the sun, and is known as Bode's Law.

4, 7, 10, 16, 28, 52, 100, 196, 388 .....

I spoke on this sequence at the last OAME conference and commented that as far as I knew no one had produced a formula to generate each term. I now have one at hand produced by Dave Skoyles which reads thus:-

$$t_n = 4 + 3x2^{n-2} - \frac{3}{2}[(n-2)(n-3)(n-4) \ldots (n-r)]^{n-1}$$

$$n \leq r$$

This formula will produce all the terms up to and including the rth term.

Yours sincerely,
Peter Weygang

BACK (?) TO BASICS - RATIO AND RATE

Doug Henderson, Master Teacher of Mathematics, Sudbury Board of Education

Mathematics educators are at present faced with a surfeit of choice in terms of program content to which students might be exposed. Research is demonstrating that there are a multitude of worthwhile topics in mathematics which students **can** learn. There is, at the same time, advice from many quarters concerning topics which the pupils **should** learn. Neither of these two sources of pressure would cause much discomfort for teachers were it not for the third force acting on them.

The constraint of time and the place of mathematics in the total school program mean that we can certainly not teach every topic that we would like to teach nor even to teach every topic which others say that we should teach.

There are many criteria which can be listed in choosing the content of the mathematics program. To be included for all students a topic should:

(i) be applicable as widely as possible both outside of the school and in both the present and future school program.

(ii) be teachable. That is, it should have been demonstrated through our own experience that the majority of pupils of the age group with which we are dealing are able to grasp the topic in some real way.

(iii) lend itself to implanting such non-content objectives as working both independently and cooperatively, expressing ideas clearly and concisely, problem solving and pattern recognition.

If we are trying to move the curriculum "back to basics" then surely the twin topics of ratio and rate form a fundamental idea which runs throughout the school program. However it is doubtful whether we wish to go "back" to this topic as it was handled in the fifties. What is required is a careful analysis of the entire topic and a suitable teaching technique for its presentation.

Both the current psychological theories of learning and our own experience indicate that pupils will undergo considerable difficulty with the ideas and applications of rate and ratio if the presentation is made at a level which is too abstract. However students from age eleven or twelve on upwards can begin the work in this area so long as they are not confronted with an overly sophisticated symbolism and slick but ill-understood problem solving approaches.

The series of experimental situations found at the end of this article are meant to give the student contact with a concrete situation from which the ideas of rate and ratio will emerge. Each experiment involves observation, summary through a table, graph or otherwise, and an attempt to generalize in words and symbols the overall pattern or rule which emerges. After the pupils have worked their way through a number of these exercises, it is expected that the teacher will draw the class together, explaining the "official" notation which should be used in these situations.

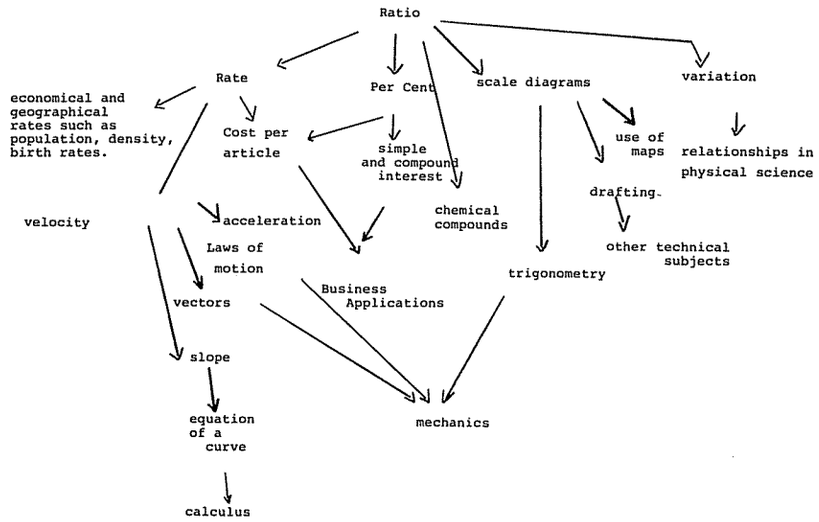
It is our advice that the pupils perform problem solving tasks in rate and ratio from purely fundamental ideas and that they are not shown slick, algebraic problem solving techniques before they reach secondary school.

These experiments have been used successfully in grade seven and grade eight. They do not constitute a complete list of the possibilities and if you wish to place pupils into additional experimental situations, it should be quite easy for you to formulate more.

The trick, as usual, is not to give the pupils too much or too little, and to sense the time at which the teacher should enter the discussion and help the pupils to draw all of their ideas together.

Good luck!

The Place of Ratio and Rate in the School Program. a Partial List



RATIO EXPERIMENT #1

Equipment: (i) Overhead projector and screen
(ii) Soda straws cut to different lengths
(iii) Rulers

Procedure: (i) Focus the overhead projector so that the image on the screen is rectangular in shape.

(ii) Place the shortest length of straw on the table of the projector. Measure the length of the straw and the length of its image on the screen.

(iii) Repeat this procedure with the other pieces of straw and complete the following table.

length of straw	length of image

(iv) Display the results of the table on a graph. Is there a pattern in the graph? If so, describe the pattern in words.

(v) Is there a pattern in the numbers in the table? Describe it in words. If the length of a piece of straw were 20 cm, could you find the length of its image

without measuring? Verify your answer by cutting a piece of straw 20 cm long and measuring its image. Describe in words your procedure for calculating the length of the image without measuring.

(vi) Stage your law in:

(a) ratio form

length of straw : length of image =

(b) fraction form

$$\frac{\text{length of straw}}{\text{length of image}} = \text{_____}$$

(c) equation form

$$\text{length of image} = \text{____} \times \text{lengtn of straw}$$

OR

$$\text{length of straw} = \text{____} \times \text{length of image}$$

(vii) Repeat part (vi) using algebraic notation. Let s represent the length of the straw and i represent the length of the image.

Problem

A piece of straw 10 cm long has an image 40 cm long.

(a) Complete the table:

straw (cm)	image (cm)
10	40
4	
2.5	
19	
	100
	50

(b) State the law in this case using algebraic symbols in the ratio, fraction and equation forms.

Teachers' Hints

(i) Cut the straws to lengths of 2.0 cm, 4.0 cm, etc.

(ii) Focus the overhead onto a wall or blackboard so that the image may be measured easily.

(iii) 'Cook' the experiment by positioning the overhead yourself. Set the longest piece of straw on the table and adjust the distance between the projector and the wall so that the image length is a simple multiple of the straw length. This minimizes errors and maximizes the pupil's chance of identifying the relationship.

RATIO EXPERIMENT #2

Equipment: (i) 3 slides of rectangles with different shapes

(ii) Slide projector.

Procedure: (i) Choose the slide marked 'A', insert it into the projector. Move the projector about 50 cm from the wall and focus the image. Measure the length and width and record your results. Move the projector farther from the wall and re-focus the image. Repeat several times and complete the table.

width (cm)	length (cm)

(ii) Display your results on a graph. Is there a pattern? Describe it.

(iii) Look at the numbers in the table. How would you find the length if the width were 7.3 metres?

(iv) State the relationship between length and width

(a) as a ratio

length : width =

(b) as equivalent fractions

$\frac{\text{length}}{\text{width}} = \frac{\quad}{\quad}$ or $\frac{\text{width}}{\text{length}} =$

(c) as an equation

length = ____ × width or

width = ____ × length.

(v) Repeat steps (i) - (iv) with each of slides B and C.

Problems I If slide D were projected on the wall the results would be as follows:

width (cm)	length (cm)
9	12
12	15
21	
48	
	100
10	

II If a rectangle measuring 15 mm by 25 mm is projected on a drive-in movie screen so that the width of the image is 18 metres, what would be the length of the image?

III A rectangle has width and length in the ratio 2:3. If the perimeter of the image is 70 cm, find the length and width of the image.

Teachers' Hints

(i) Have the AV office make slides of rectangles, the length of whose sides have ratio 2:1, 3:1 and 3:2.

(ii) It is tempting to try to find a relationship between the image size and the distance from the projector to the screen. The lens system of the projector means that pupils of grade seven or eight have very little chance of finding this relationship and looking for it would likely be unwise use of their time.

RATIO EXPERIMENT #3

Equipment: (i) trolley and platform
(ii) graph paper

Procedure: (i) Place the platform on top of the trolley on top of the graph paper. Align the pointers on the trolley and platform so that both have the same starting point.

(ii) Move the platform and trolley forward together. Be certain that the wheels roll freely and that they do not slide.

(iii) Complete the following table. 'P' stands for the distance moved by the platform; 't' for the distance moved by the trolley.

t	p
1 unit	
2	
3	
4	
5	

(iv) Draw a graph showing the relationship between t and p. Show t on the horizontal axis.

(v) State the relationship between t and p

(a) in words

(b) as a fraction

$\frac{t}{p} =$ ——— or $\frac{p}{t} =$

(c) as a ratio

t:p = or t:p =

(d) as an equation

$t = \frac{\quad}{\quad} \times p$ or $p = \frac{\quad}{\quad} \times t$

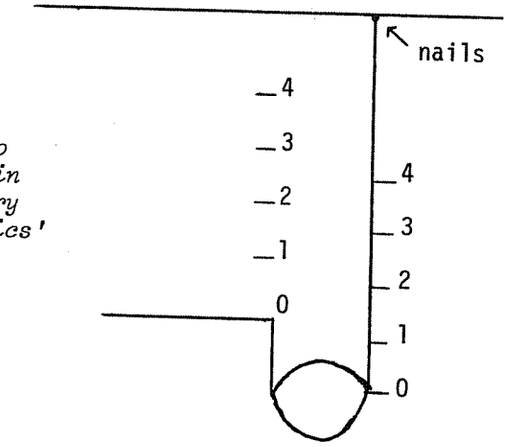
Teachers' Hint

The trolley is basically a platform on rollers. For a complete description of this experiment see 'Vision in Elementary Mathematics' by W.W. Sawyer.

RATIO EXPERIMENT #4

Equipment: (i) Pulley and string

Procedure: (i) Arrange the pulley and string as shown. Mark two scales on the wall — one for the position of the pulley and one for the position of the end of the string. The spaces on both scales must be the same.



Thanks to 'Vision in Elementary Mathematics' Sawyer.

(ii) As the end of the string is raised, the pulley will also rise. Observe the motion and determine a rule relating the distance which the pulley moves and the distance which the end of the string moves.

(iii) Write your rule

(a) in words

(b) as a ratio

distance string moves : distance pulley moves =

(c) as equivalent fractions

$\frac{\text{distance pulley moves}}{\text{distance string moves}} = \frac{\quad}{\quad}$

or

$$\frac{\text{distance string moves}}{\text{distance pulley moves}} = \text{____}$$

(d) as an equation

$$\text{distance string moves} = \text{____} \times \text{distance pulley moves}$$

or

$$\text{distance pulley moves} = \text{____} \times \text{distance string moves}$$

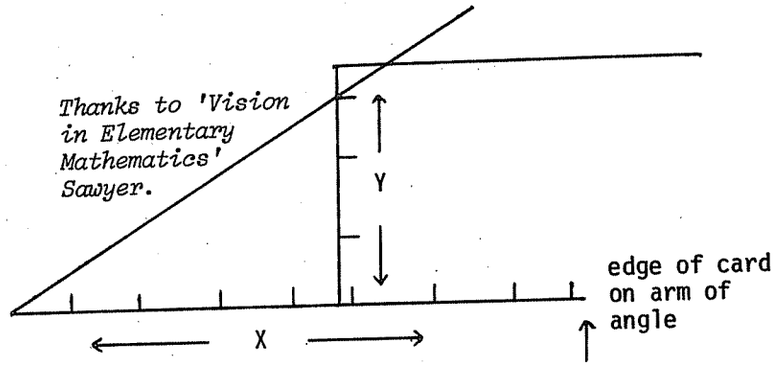
(e) Use algebraic symbols to re-write parts (a) - (d).

Let s represent the distance moved by the string and p represent the distance moved by the pulley.

RATIO EXPERIMENT #5

Equipment: (i) Three pieces of bristol board, each with an angle marked on it.
(ii) Another card with a scale marked on one edge.

Procedure: (i) Place the moveable card on the angle as shown.



Change the position of the card to obtain different pairs of values for x and y. Choose only those cases which give both x and y whole number values.

(ii) Complete the table for several values.

x	y

(iii) Draw a graph of your results. What pattern exists in the graph?

(iv) What pattern exists among the numbers in the table? State the pattern:

(a) in words

(b) as a ratio x:y = or y:x =

(c) as equivalent fractions $\frac{x}{y} = ____$ or $\frac{y}{x} = ____$

(d) as an equation $y = ____ \times x$ or $x = ____ \times y$

(v) Repeat steps (i) - (iv) for each of the other two cards.

Problems

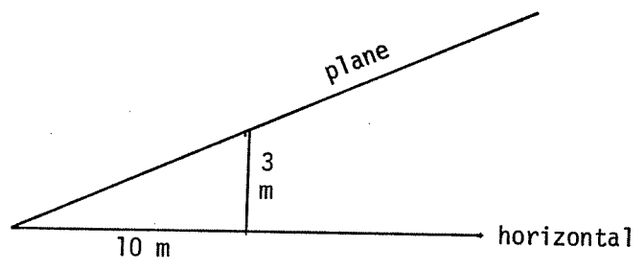
I (a) Complete each of these tables:

x	y
2	10
3	15
4	
5	
12	
6.3	
	75

x	y
4	5
8	10
12	
20	
60	
	60
	12

(b) State the patterns for each of these tables in words, as a ratio, as equivalent fractions, as an equation.

II A jet plane on taking off increases its height by 3 metres for every 10 metres which it travels horizontally.



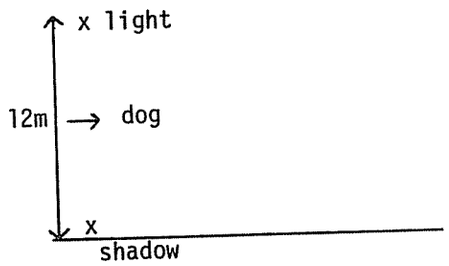
(a) How far must it travel horizontally in order to increase its height by 450m?

(b) When it travels 350m horizontally, by how much does its altitude increase?

RATIO EXPERIMENT #6

Equipment: Graph paper and a ruler

Procedure: (i) Use the graph paper to make a scale diagram of this situation. Use one square for one metre.



A dog is standing midway between a spotlight and a long black wall. The spotlight is directed at the dog and its shadow is projected on the wall. As the dog begins to walk parallel to the wall, its shadow also moves. When the dog moves one metre, how far does the shadow move?

(ii) Complete the table and draw a graph. Place the distance which the dog moves on the horizontal axis.

dog (m)	shadow (m)
1 m	
2 m	
3	
4	
5	
6	

(iii) What pattern exists on the graph?

(iv) What pattern exists in the numbers in the table? Write this pattern:

(a) in words

(b) as a ratio

distance dog moves : distance shadow moves =

(c) as equivalent fractions

$$\frac{\text{distance dog moves}}{\text{distance shadow moves}} = \frac{\quad}{\quad}$$

(d) as an equation

dist. dog moves = —— × dist. shadow moves

or

dist. shadow moves = —— × dist. dog moves.

(e) Using d to represent the distance the dog moves and s to represent the distance the shadow moves, re-write parts (b), (c), and (d) using algebraic symbols.

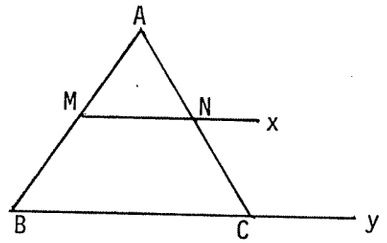
(v) Repeat the experiment if the dog begins at a point 9m from the wall and 3m from the light.

(vi) Repeat the experiment if the dog begins from a point 4m from the wall and 8m from the light.

RATIO EXPERIMENT #7

Equipment: A triangle made of plastic strips and held together with fasteners. An additional strip joining points on two sides of the triangle.

Procedure:

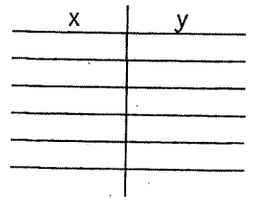


(i) Begin with triangle #1.

(ii) Fasten strip x first.

(iii) Fasten strip y so that it fits exactly.

(iv) Count the number of spaces between the fasteners of x and y and complete the table.



Draw a graph, placing x on the horizontal axis. What pattern do you notice on the graph?

(v) State the pattern between x and y

(a) as a ratio x:y = or y:x =

(b) in fraction form $\frac{x}{y} = __$ or $\frac{y}{x} = __$

(c) as an equation X = —— xY or Y = xX

(vi) Repeat the experiment with triangles #2 and #3.

Teachers' Hints

(i) In triangle #1, M and N are the midpoints of AB and AC. Δ's #2 and #3 they divide AB and AC in the ratios 1:2 and 2:1 respectively.

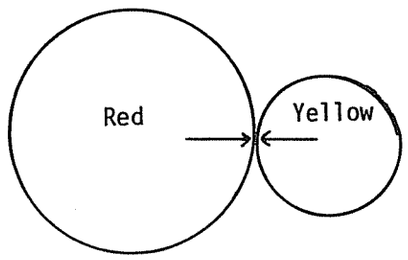
(ii) Strips x and y have holes 2 cm apart.

(iii) The strips may be of bristol board but something sturdier will last longer.

RATIO EXPERIMENT #8

Equipment: three plastic gears

Procedure: (i) Place the red and yellow gears so that the teeth interlock and the arrows point to each other.



(ii) Rotate the red gear and count the number of complete turns of the yellow gear.

(iii)

Red (turns)	Yellow (turns)
1	
2	
3	
4	
5	

Complete the table and draw a graph. What pattern do you notice on the graph?

(iv) State the relationship between the number of turns on the red and yellow gears:

(a) in words

(b) as a ratio

red : yellow = or yellow : red =

(c) in fraction form

$\frac{\text{red}}{\text{yellow}} = \frac{\quad}{\quad}$ or $\frac{\text{yellow}}{\text{red}} = \frac{\quad}{\quad}$

(d) in equation form

red = —— × yellow or yellow = —— × red.

(e) Re-write parts (b), (c), and (d) in algebraic notation. Use r to represent the number of turns of the red gear and y to represent the number of turns of the yellow gear.

(v) Repeat steps (i) - (iv) for the red and white gears.

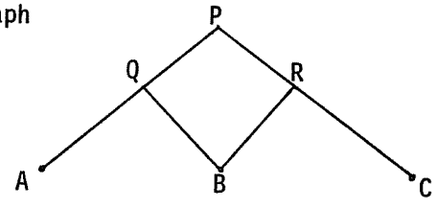
(vi) Repeat steps (i) - (iv) for the yellow and white gears.

Teachers' Hint

These gears are available in toy stores from 'Lego' Company.

RATIO EXPERIMENT #9

Equipment: (i) A pantograph



Procedure: (i) Draw a straight line 4 cm long on a piece of paper and lay the paper flat on the desk.

(ii) Place the pantograph so that A is not on the paper and B is on one end of the line. Through the hole at C make a penmark on the paper. This point is called the image of the point at B.

(iii) Holding point A fixed, move point B to the other end of the 4-cm line. Once again mark the point at C. This is the image of the other end of the 4-cm line.

(iv) Join the two pen marks and measure the length of the image line.

(v) Complete the table:

length of line	length of image line
4 cm	
6	
8	
10	
12	

Draw a graph showing the length of the original line on the horizontal axis. What pattern is in the graph?

(vi) What pattern exists in the numbers in the table?

(v) Write the relationship between the length of the origional and the length of the image.

(a) in words

(b) as a ratio

original : image = or image : original =

(c) in fraction form

$\frac{\text{original}}{\text{image}} = \frac{\quad}{\quad}$ $\frac{\text{image}}{\text{original}} = \frac{\quad}{\quad}$

(d) as an equation

image = —— × original or

original = —— × image

(e) Using o to represent the length of the original line and i to represent the length of the image line, rewrite parts (b), (c), and (d) in algebraic form.

Teachers' Hints

(i) The strips are free to pivot at P,Q,B,R.
(ii) The figure PQBR is a rhombus.
(iii) Try to buy one of these rather than making your own.

Editor's Note: This article is based on a presentation made by the author at the London conference of O.A.M.E. in May of 1975. A second set of experiments will be printed in the March issue of the Gazette.

q.e.d.o.a.m.e.

MOTIVATIONAL TECHNIQUES TO IMPROVE COMPUTATIONAL SKILLS

Rita C. Janes

Reprinted with permission from Teaching Mathematics, the journal of Newfoundland Mathematics Teachers, April 1976 issue

When designing an instructional activity to assist elementary students learn mathematics, the teacher should be constantly striving for methods that will motivate the student. The purpose of this paper is to offer a few suggestions to accomplish this while helping to improve the student's computational skills.

An effective technique is to start class with an interesting or exciting statement or question which will stimulate and gain the attention of your students. By way of example, you could start class by writing the number 15,873 on the board and advising the students that this is a very special number. Have one of the students suggest his favourite number. If it is 4, have him multiply the special number by 28 (answer 44,444). If 2, have him multiply the special number by 14 (answer 222,222). Repeat this process. Encourage the students to see the pattern. If you achieve success with this type of approach, you may wish, at a subsequent time, to suggest that they multiply 12,345,679 by 9, 18, 27, and 36 respectively; if they have chosen 1, 2, 3, or 4 for their favourite number. Don't you feel compelled to determine the answer? Again, what is the pattern?

A veritable gold mine of stimulating problems can be found in puzzle books awaiting exploration and development by the teacher for the enlightenment of the student. Here's a little gem!

Write a 2-digit number between 50 to 100. Add 76 to this number. Cross out the digit in the hundreds place of the answer and add this to the remaining 2-digit number. Next, subtract the result from the original number. The answer will always be 23.

For those students who are not engrossed in the computational skill involved, have them investigate **Why is the answer always 23?** This should produce a flurry of intellectual activity.

When you find yourself at the beginning of the mathematics period trying to make the transition from a geography period or confronted by Johnny without his homework assignment, have an interesting problem ready for the class. "Open your books to page 86", is such a poor substitute for a challenging problem. A problem in rhyme is always fascinating:

Sally's cat climbed up a tree
Which was 60′ and 3
Every day she climbed 11
Every night she slipped back 7
Tell me, if she did not drop
When her paws would touch the top.

Another problem that will keep children meaningfully **busy** is the following:

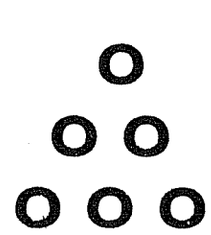
Using exactly four fours and using only the operations of addition, subtraction, multiplication and division, try to write an expression that names each of the numbers from 1 to 10. You do not have to use all the operations in any one expression, but you must use exactly four fours, e.g. $2=(4\div4)+(4\div4)$. While students are working on this problem, the teacher can have time to teach long division to David who has been absent for a week.

Are the answers to this problem unique? Is it possible to vary the conditions of this problem, such as using only three fours? As you discuss the student's answers to this problem, you will notice it not only gives meaningful drill on computational skills, but allows opportunity for divergent thinking. It also promotes positive attitudes in mathematics and gives students a chance to recognize and determine patterns in mathematics.

The teacher, as part of her instructional activity, can use games and activities designed to reinforce rules of computation with whole numbers, decimals, fractions and percents. These can be used with individual students, small groups, or assigned as class projects.

A problem to the entire class or two teams formed among the students could be given in the form of a challenge. How many different ways can you use all nine digits from 1 through 9 to represent 100? Answers could be posted on the bulletin board.

An activity with the entire class is always fun especially if students are challenged to use their problem solving skills. Here is an example. The teacher draws on the board seven circles in the form of a triangle as below:



Can you place each of the numbers 1 through 6 in the triangular chain of circles so that each side has the same sum? How many different ways can you place these numbers? Is there a pattern? Are there any restrictions on where the digits can be placed?

Elementary students are usually enthused about recreational problems, especially ones in which they can become actively involved. Here is one that students enjoy.

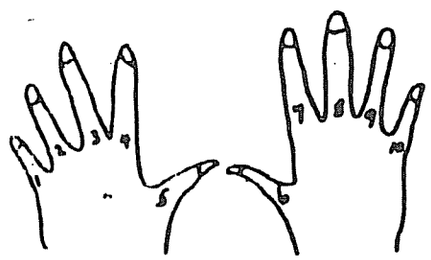
It is decided that whenever a hockey team meets, each member must shake hands with everyone else. If only two people meet, then only one handshake is necessary. If three people meet, then three handshakes are necessary (Have 3 students get up and shake hands to check this.)

a) How many handshakes if 4 people meet?
b) How many handshakes if 5 people meet?

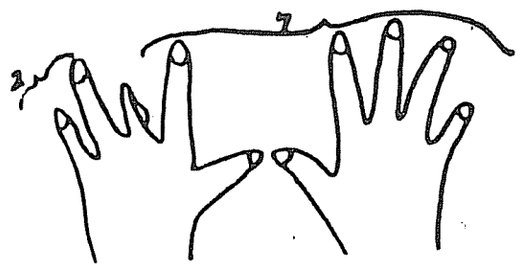
Have students shake hands and count. Then a table could be drawn on the board (as below) and have students complete it. Is there a pattern? The last line is a challenge for the better students. Others could check to see if the number of handshakes for **n** peoples works for say 5 handshakes.

PEOPLE	HANDSHAKES
2	1
3	3
4	6
5	10
6	—
7	—
—	—
—	—
—	—
n	$\frac{n(n-1)}{2}$

Students sometimes have problems in remembering the 9 time multiplication facts. Teachers can get interesting results in this area by showing students how to find the product of two numbers, one of which is nine, by using the fingers. the fingers on the two hands are numbers from 1-9 as shown in the picture below.



To find a product such as 3 x 9, we bend the finger that represents 3.



The two fingers to the left of the bent finger represent the tens digit in the product and the seven fingers to the right of the bent finger represent the units digits. Thus, from the diagram, 3 x 9 = 27.

You can also find products of two digit numbers and 9. Suppose we have to multiply 9 x 38. Bend the fingers that represent the ones digit in the 38 and make a separation after the finger that represents the tens digit as below.



The three fingers to the left of the separated fingers are the hundreds in our product. Between the separation and the bent finger, there are four fingers. This is the tens digit and the two fingers to the right of the bent down finger is the ones digit. Thus, from the diagram, 9 x 38 = 342.

These are but a few examples of the mathematical problems that are available to help motivate our students. As teachers, we must realize that we can never hope to use all the motivational devices available in any one classroom or in any one year. Also, we should realize that not all students will be motivated to the same degree. But it is our responsibility as teachers of mathematics to make mathematics enjoyable for students by using every possible device to create as good a learning climate as possible. Hopefully, the motivated student will show a change in behaviour observed by a greater achievement and interest in mathematics. Motivating the student is truly our job!

q.e.d.o.a.m.e.

COMMENTS ON MATHEMATICAL EDUCATION

Jean Piaget

A paper sent to the Second International Congress on Mathematical Education, Exeter, August 1972, and reprinted from the proceedings.

The orientation one would consider giving to mathematical education depends naturally on the interpretation adopted of psychological development or the acquiring of operations and logico-mathematical structures; this interpretation depends equally on the epistemological meaning given to those things, the two questions of their psychogenesis and their epistemological significance being very closely related. If Platonism is right and mathematical entities exist independently of the subject, or if logical positivism is correct in reducing them to a general syntax and semantic, in both cases it would be justifiable to put the emphasis on the simple transmission of the truth from teacher to pupil and to use, as soon as possible, the language of the teacher, that is, the axiomatic language, without worrying too much about the spontaneous ideas of the children.

We believe, on the contrary, that there exists, as a function of the development of intelligence as a whole, a spontaneous and gradual construction of elementary logico-mathematical structures and that these 'natural' ('natural' in the way one speaks of the 'natural' numbers) structures are much closer to those being used in 'modern' mathematics than to those being used in traditional mathematics. There is, therefore, a body of facts which are, in general, little known to the teacher, but which, once he has a better psychological knowledge, would be of considerable use to him and would help him rather than make things more complicated. This would also favour the realisation of creative vocations in pupils rather than treating them simply as conforming 'receiving' instruments.

However, in order to arrive at this stage it is necessary to revise our ideas about the relation between language and action. It would seem, in fact, psychologically clear that logic does not arise out of language but from a deeper source and this is to be found in the general coordination of actions. In fact, before all language, and at a purely sensori-motor level, actions are susceptible to repetition and then to generalisation thus building up what could be called assimilation schemes. These schemes organise themselves according to certain laws and it would seem impossible to deny the relationship between these and the laws of logic. Two schemes can be coordinated or dissociated (reunion), one can be partially nested in the other (inclusion), or only have a part in common with the other (intersection); the parts of a scheme or the coordination of two or more schemes can allow either an invariant order of succession or certain permutations (types or order), as well as one-to-one correspondences, one-to-many or many-to-one (bijections etc.), and once a scheme imposes a goal on an action it is contradictory for the subject to go in the opposite direction. Briefly, there is a whole logic of the action that leads to the construction of certain identities and these go beyond perception (for example the permanence of the hidden object) and to the elaboration of certain structures (the practical group of displacements already described by Poincare in his epistemological essays).

Therefore, it would be a great mistake, particularly in mathematical education, to neglect the role

of actions and always to remain on the level of language. Particularly with young pupils, activity with objects is indispensable to the comprehension of arithmetical as well as geometrical relations (as was the case with the empirical mathematics of the Egyptians). The mathematics teacher's aversion to activities involving material experimentation is quite comprehensible. They probably see a sort of reference to the physical properties of objects and might fear that empirical verifications will harm the development of the deductive and purely rational mind which characterises their discipline. But this is, in fact, a fundamental misunderstanding and psychological analysis allows us to dispel these fears and reassure mathematicians with regard to their essential demand that the deductive and formal aspect of the mind should be educated. There exist, in fact, two types of 'experience', one very different from the other, which are related to the subject's actions. In the first instance, there is what is known as 'physical experience' (in the broad sense) which consists in acting on objects in order to discover the properties of the objects themselves, for example, comparing weights or densities, etc. But there also exists, and this is generally not known, what could be called 'logico-mathematical experience'; this type of experience gathers its information, not from the physical properties of particular objects, but from the actual actions (or more precisely their coordinations) carried out by the child on objects - these two types of experience are not equivalent. A friend of mine and a well-known mathematician says that the beginnings of his interest in mathematics were triggered off by an experience of the second type which happened to him when he was about 4 or 5 years old. Seated in his garden, he started to amuse himself by placing some pebbles in a straight line and counting them, for example, one to ten from left to right. After this he counted them from right to left and to his great surprise he still found ten. He then put them in a circle and, with enthusiasm, counted them - again ten so he counted them in the opposite direction and he found there were ten in both directions. He went on arranging the pebbles in all sorts of ways and finished by convincing himself that the sum, ten, was independent of the order of the pebbles. It is evident that neither the sum nor the order are physical properties of the pebbles until such time as the child has actually arranged them or put them all together. In this instance the child has discovered that the action of uniting the pebbles gives results and these results are independent of the action of ordering the pebbles. He could have observed this with any solid objects as, in this action, the physical properties of the pebbles played no particular role (apart from the fact that they 'let themselves' be acted on; their nature, however, remains unaltered, that is, it is conserved, but conservation itself also gives rise to logico-mathematical experience).

Thus this initial role of actions and logico-mathematical experience, far from hindering the later development of deductive thought, constitutes, on the contrary, a necessary preparation and this for two reasons. This first is that mental or intellectual operations, which intervene in the subsequent deductive reasoning processes, themselves stem from actions: they are interiorised actions and once this interiorisation, with the coordinations it supposes, is sufficient, then logico-mathematical experience in the form of material actions is no longer necessary and interiorised deduction is sufficient. The second reason is that coordinations of actions and logico-mathematical experience, whilst interiorising themselves, give rise to the creation of a particular variety of abstraction which corresponds precisely to logical and mathematical abstraction: contrary to ordinary or Aristotelian abstraction which derives its sources from the physical properties of objects and the reason is called 'empirical abstraction', logico-mathematical abstraction would be referred to as 'reflective abstraction' and this for two related reasons. On the one hand, this abstraction 'reflects' (in the same way as a reflector or projector) everything that was on a lower plane (for example, that of action) and projects it to a higher plane, that of thought or mental representation. On the other hand, it is a 'reflective abstraction' in the sense of a reorganisation of mental activity, as it reconstructs at a higher level everything that was drawn from the coordinations of actions.

However, between the age where material actions and logico-mathematical experience are necessary (before 7/8 years old) and the age where abstract thought begins to be possible (towards 11/12 years old and through successive levels until about 14/15 years), there is an important stage whose characteristics are interesting to the psychologist and useful to know for the teacher. In fact between the age of 7 and 11/12 years an important spontaneous development of deductive operations with their characteristics of conservation, reversibility, etc. can be observed. This allows the elaboration of elementary logic of classes and relations, the operational construction of the

whole number series by the synthesis of the notions of inclusion and order, [1] the construction of the notion of measurement by the synthesis of the subdivision of a continuum and the ordered displacement of a chosen part which serves as a unit, etc. Although there is considerable progress in the child's logical thinking it is nonetheless still fairly limited. At this level the child cannot as yet reason on pure hypotheses, expressed verbally, and, in order to arrive at a coherent deduction, he needs to apply his reasoning to manipulable objects (in the real world or in his imagination). For these reasons, at this level we refer to 'concrete operations' as distinct from formal operations. These concrete operations are, in fact, intermediaries between the actions of the pre-operational stage and the stage of abstract thought which comes much later.

Thus, having established the continuity between the spontaneous actions of the child and his reflexive thought, it can be seen from this that the essential notions which characterise modern mathematics are much closer to the structures of 'natural' thought than are the concepts used in traditional mathematics. First, the importance should be pointed out of the spontaneous role of operations which allow the establishment of correspondences between sets and thus the construction of morphisms and in particular when these can be combined with recurring sequences. We have, for example, with B. Inhelder, asked children between 4/5 and 7/8 years old to put a bead from one hand into a transparent cylinder and simultaneously with the other hand put another bead into a second transparent cylinder which was, however, hidden behind a screen. The questions were designed to find out whether or not the child understood that the two sets, thus constituted, were equivalent and also to discover whether if this action were to be continued indefinitedly, this equality would be conserved. All the children questioned admitted the equality of the two sets whilst the action was going on, however the youngest children refused to generalise to the case where the action was continued indefinitely. From about 5 or 6 years onwards they admit this generalisation and one small boy of 5½ found the following very amusing formula: 'When one knows for one time, one knows for ever.' However this same child, after having seen a set of ten red counters in a one-to-one correspondence with a second set of ten blue counters, refused to admit the conservation of this equivalence once the elements of one of the sets had been spaced out a little and the correspondence between the two was no longer visible. This example demonstrates the constructive role of the establishment of a correspondence combined with the idea of recurrence.

An extremely striking example of convergence between theory and the spontaneous development of the child is that of geometric intuitions. Historically these intuitions appeared in Euclidean geometry, the structures of projective geometry were not discovered until much later and topology only in the nineteenth century. Psychologically children of 3 and 4 years old, who do not yet know how to draw squares and tend to compare them to circles - shapes such as rectangles and triangles etc. being assimilated to simple closed curves - are very careful, however, to make the distinction between closed and open figures, and they are able to draw with as much care a circle inside a figure, outside a figure, or on the frontier of a large figure. From these early topological intuitions arise, later and simultaneously, projective notions (with verification by 'taking aim' or 'sighting') and Euclidean notions according to a process which is nearer psychological theory than history.

From the level of concrete operations - at about 7/8 years - another interesting convergence can be found, that is the elementary equivalent of the three 'mother structures' discovered by Bourbaki, and this itself shows the 'natural' character of these structures. First of all there is the construction of structures of an algebraic nature, in as much as their laws of composition have an inverse and an identity element $+ A - A = 0$. This can be observed particularly in the system of logical classes (classifications, etc. with quantification of the inclusion $A < B$ if $B = A + A'$ - and neither are empty sets). Secondly, order structures can be found whose laws of composition are based on reciprocity and this characterises the system of relations (ordering). Finally, topological structures based on ideas of continuity, neighbourhood and separation can be observed. These elementary structures later combine with each other. In particular, inverses (or negations (— A)) and

[1] Several authors (Freudenthal, etc.) seem to have understood that I think the ordinal number is more primitive than the cardinal number, or the opposite. I have never made such a statement and have always considered these two aspects of finite numbers indissociable and psychologically reinforcing one another in a synthesis that goes beyond both the inclusion of classes and the order of asymmetrical transitive relations. If order is necessary it is because units which have become equivalent by the abstraction of their qualities can only be distinguished from one another by their ordered position. But the order of the elementary units is relative to the number (cardinal) of units which precede each of the units thus ordered.

reciprocities, which do not combine with each other at the concrete operational level, can be composed with one another from the 11/12 formal level onwards, in a four-group which renders possible such compositions: in this case the beginning of propositional logic with the combinatorial (set of all sub-sets) system, superposes itself on the elementary structures of logical classes and relations. The subject is then capable of handling systems that have four transformations. Let us take, for example, the propositional operation $p \supset q$ and define the four transformations:

1. (I) the identify of 'null' transformation $I(p \supset q) = p \supset q$,
2. (N) the inverse transformation $N(p \supset q) = p \cap \backsim q$,
3. (R) the reciprocal transformation $R(p \supset q) = q \supset p$,
4. (C) the correlative transformation $C(p \supset q) = \sim p \cap q$.

In this case $RC = N$, $RN = C$, $NC = R$, $NRC = I$ which ensures finally the coordination in a unique system of inverses and reciprocities.

Many other examples, in particular the construction of elementary and 'trivial' forms of categories, could be given. However, it is now the moment to describe how these convergences between the spontaneous thought of the child in his 'natural' development and certain fundamental theoretical notions can be of use to the teacher. It can, of course, happen that certain people will try to teach young children 'modern' mathematics with archaic teaching methods, based exclusively on verbal transmission from teacher to child with a premature use of formalisation. With such methods there are bound to be a certain number of failures and these help to explain the scepticism of certain great mathematicians such as J. Leray.[1] However, it is not the 'modern' character of the mathematics programmes that is at fault but the methodology and psychology used in such cases. In fact, it is often particularly difficult for the teacher of mathematics, who, because of his profession, has a very abstract type of thought, to place himself in the concrete perspective which is necessarily that of his young pupils. However, from the developmental point of view and in relation to the progressive assimilation of the structures already mentioned, there would seem to be no contradiction (as we have seen above) between the initial concrete phases of structures and the final stage when they become formal and abstract. The teacher can only be aware that there is no contradiction between these two levels of thought if he is fully acquainted with (and this is the difficulty for the teacher) the details and functioning of these successive spontaneous thought structures. Briefly, the practical problem that is difficult to solve is to graft these general types of notions which the teacher understands in his language on to particular cases of these same notions constructed and used spontaneously by the children, without these yet being for them objects of reflection or sources of generalisation.

In order to make this necessary conjunction between the logico-mathematical structures of the teacher and those of the pupil at different levels of his development, certain very general psychopedagogical principles should perhaps be mentioned. The first is that real comprehension of a notion or a theory implies the re-invention of this theory by the subject. Once the child is capable of repeating certain notions and using some applications of these in learning situations he often gives the impression of understanding: however, this does not fulfil the condition of re-invention. True understanding manifests itself by new spontaneous applications, in other words an active generalisation supposes a great deal more: it seems that the subject has been able to discover for himself the true reasons involved in the understanding of a situation and, therefore, has at least partially re-invented it for himself. Naturally, this does not mean that the teacher has no role any more, but that his role is less that of a person who gives 'lessons' and is rather that of someone who organises situations that will give rise to curiosity and solution-seeking in the child, and who will support such behaviour by means of appropriate arrangements. Should the child have difficulties in his attempts to grasp a certain idea, the procedure with an active methodology would not be directly to correct him, but to suggest such counterexamples that the child's new exploration will lead him to correct himself.

A second consideration should constantly be present in the teacher's mind: that is, at all levels, including adolescence and in a systematic manner at the more elementary levels, the pupil will be far more capable of 'doing' and 'understanding in actions' than of expressing himself verbally. In other words, a large part of the structures the child uses when he sets out actively to solve a problem remain unconscious. In fact, it is a very general psychological law that the child can do something in action long before he really becomes 'aware' of what is involved - 'awareness' occurs

[1] See the very critical report presented by Leray for Academie des Sciences de Paris (Report No. 276, p. 95, Session of 13 March 1972).

long after the action. In other words, the subject possesses far greater intellectual powers than he actually consciously uses.[1] Consequently, once the teacher has had the opportunity of becoming acquainted with the psychological research mentioned above, and knows the subjacent thought structures the child possesses, he can more easily help the child to become aware of these either by appropriate discussions between the child and himself, or by the organisation of the work in groups where partners of the same age or similar ages (an older child acting as leader of a small group) discuss between themselves, which in turn favourises verbalisation and 'awareness'.

A third remark would seem important: in traditional mathematics it was often necessary for children to solve quantities of problems, some of them quite absurd, and this would mean a huge number of numerical or metrical calculations. In this case, the only way to succeed with children who were not particularly talented in mathematics was to proceed in two stages (but this was often forgotten): the first stage was purely qualitative and dealt with the logical structure of the problem and only afterwards in a second step were numerical or metrical facts introduced with the additional difficulties this type of calculation would create. With modern mathematics programmes the problem is less acute as they are basically qualitative. However, in this case, the problem can be found at another level - the teacher is often tempted to present far too early notions and operations in a framework that is already very formal. In this case, the procedure that would seem indispensable would be to take as the starting point the qualitative concrete levels: in other words, the representations or models used should correspond to the natural logic of the levels of the pupils in question, and formalisation should be kept for a later moment as a type of systematisation of the notions already acquired. This certainly means the use of intuition before axiomatisation and the scorn of logicians for all intuitive of 'naive' thought is well known. However, once it is remembered that mathematical intuition is essentially operational and the nature of operational structures is to dissociate 'form' from 'content', then the final formalisation would seem to be prepared and becomes progressively necessary by the construction itself of these initial intuitive structures. We do not believe with Pasch that formalisation goes in the opposite direction to that taken by 'natural' thought, but so that there may be no conflict between the former and the latter, formalisation should be allowed to constitute itself in its own time and not because it is forced to by premature constraints.

[1] Euclid himself was not aware of all the operational structures he used in reality, for example, the group of isometries.

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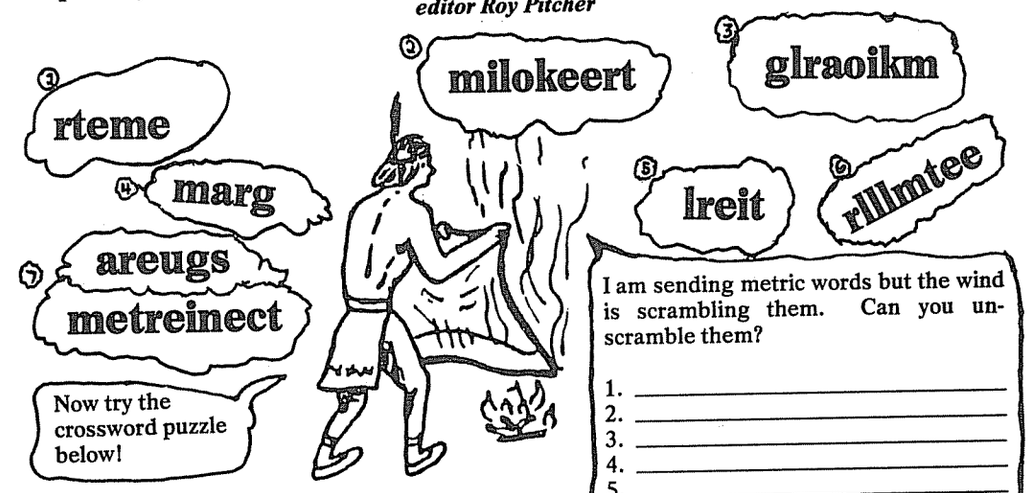
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TWO METRIC PUZZLES

Reprinted from the February, 1976 issue of Teaching Mathematics, St. John's Newfoundland, editor Roy Pitcher

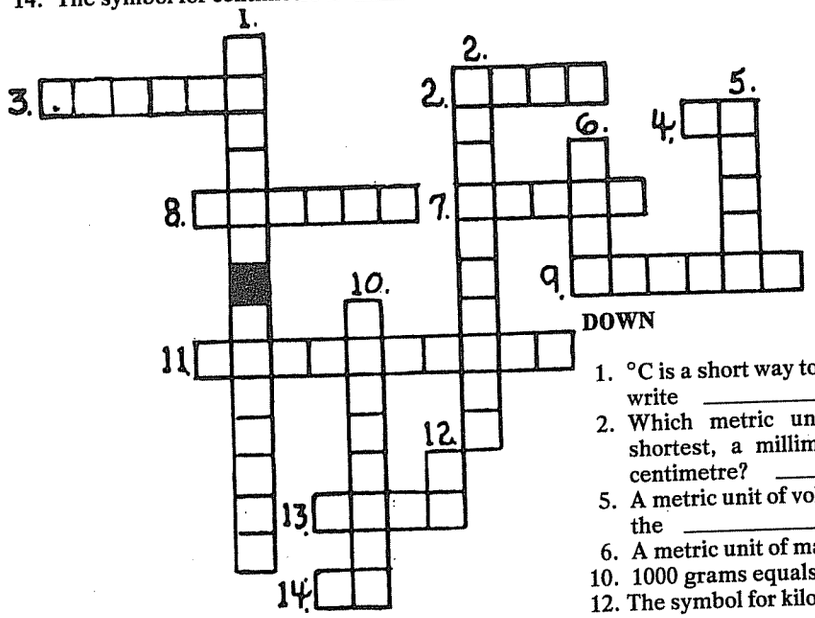


I am sending metric words but the wind is scrambling them. Can you unscramble them?

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

ACROSS

2. A kilogram is a unit of _____
3. A millilitre is a unit of _____
4. The symbol for millilitre is _____
7. Which metric unit has the largest volume, a millilitre, a litre, or a cubic centimetre? _____
8. The metre is a unit of _____
9. 1000 _____ is equal to 1 kilometre.
11. 1/100 of a metre is equal to 1 _____
13. Which metric unit has less mass, a gram or a kilogram? _____
14. The symbol for centimetre is _____



DOWN

1. °C is a short way to write _____
2. Which metric unit of length is the shortest, a millimetre, a metre, or a centimetre? _____
5. A metric unit of volume is the _____
6. A metric unit of mass is the _____
10. 1000 grams equals 1 _____
12. The symbol for kilogram is _____

WHY WAS MARY SO CONTRARY?

James R. MacLean, Education Officer,
Research and Evaluation Branch, Ministry of Education, Ontario

This article was originally published in the Journal of the Canadian Association for Young Children, May 1976

"Now, Mary, let me hear you count. Johnny, count backwards from ten." Do these commands strike responsive chords in your memory -- either as a student or teacher? Did you ever think that both these requests were utter nonsense? Confusing the recitation of number names with the ability to count has been a contributing factor to the dreadfully poor start we give children in learning mathematics. This has not been the only factor or even the major one. There are many practices cemented by tradition and prejudice that do not support or extend children's learning. It is not my intention to catalogue the mistakes that have been made but rather to present what I believe to be a reasoned and practical approach to early childhood mathematics.

It was not so long ago (and perhaps it still survives in some parts of the country) that principals and inspectors insisted that the day begin with mathematics because the children were fresh and could undertake such and exacting discipline. Happily, many schools have changed this pattern and mathematics is being learned at any time during the day that suits the children's and the teacher's needs. But how does the day begin in Kindergarten or even in a Nursery School? There, on most mornings, the children start their day with mathematics! Teachers of young children may be shocked at this, but the evidence is there for all to see -- a group of children building a wall of blocks, one child spinning a wheel on an overturned wagon, a pair are setting the table in the house corner, two or three are making patterns with coloured tiles, a girl is sorting out doll's clothes, a boy is crawling through a cardboard tunnel.

In defense, teachers claim that the children are playing. But what is play? In a recent publication, **Education in the Primary and Junior Divisions, (1)** the Ontario Ministry of Education states

> "Play is an essential part of learning. It is free from the restrictions of reality, external evaluations, and judgment. Children can try out different styles of action and communication without being required to make premature decisions or being penalized for errors. Play provides a context in which the teacher can observe children's handling of materials and social situations, assess their stage of development, and encourage experiences that further their growth."

This supports Susan Isaacs' earlier contention **(2)** that the key to full development is the child's activity. She argues that the best way to lead children out of the narrow circle of their own egotistical desires and naive assumptions about the world is through play with real things that can be seen and handled in meaningful situations. Not only must they be used in whatever way is deemed appropriate at the moment, but these experiences should be put into words through description, discussion and argument. "Children should not be expected to talk or listen instead of doing; they should be encouraged to talk about what they are doing." **(3)** This linking together of two of the child's most effective strategies for learning -- representation and language -- adds enormous power to his capacity to interpret and control his world.

What is his world and what does it have to do with mathematics? Mathematics is an invention of man that helps us to understand, analyze, and communicate both qualitative and quantitative ideas about our environment. Although it consists of the relations of space, the relations of quantity, and the relations of number, these are closely interwoven. Because young children learn through their own actions on the things of their environment and geometry is the study of spatial relations, it makes good sense to begin the study of mathematics by looking at geometry.

From the moment a child is born, he explores space. His first spatial knowledge is of an object being placed in his mouth. He begins to understand what shape it is, and that the shape that he sees corresponds to the shape he can feel. The child develops an awareness of the properties of objects by experimenting with them. He learns what they look and feel like and what he can do with them. He hits things to see if they will move, pushes them to discover if they will roll, shakes them to see if they will rattle, and drops them to find out how they fall and where they go. Through

these experiences he learns that objects have an identity and a permanence even though they may appear different when viewed from a variety of positions and distances. According to Jean Piaget, (4) experience for the young child falls into two categories, each markedly different from the other. Physical, or empirical, experience consists of acting upon objects in order to discover the properties of the objects. Logico-mathematical experience, gathers information, not from the physical properties of the objects, but from the actual actions carried out by the child on the objects. For example, watch how a child plays with blocks. The child who has physical experience only will understand that the objects are cuboids (right rectangular prisms) that have flat sides and can be placed on the floor or table without moving. The child who has gained logico-mathematical insights from his experience will not be content to just sort the blocks but will try to build a wall or tower by fitting the flat sides together. He has reasoned how he is able to use the physical properties to carry out his actions. Similar observations can be made as children interact with a variety of objects such as containers, tiles, hoops, lengths of rope, and so on.

What his unique experiences mean to the child can only be expressed to others by means of language. There is considerable doubt that this communication can be effected unless he has acquired the logical knowledge needed to apply the deductive reasoning necessary to formulate his actions. Some children, who have the power to reason logico-mathematically may not have the vocabulary to interpret and communicate even their physical experiences. Other children, able to imitate and use words fluently, may be much less skilled in thinking. These observations suggest that the ability to think is not identical with the ability to use language. Insofar as concepts in mathematics are concernced, it is essential that they be communicated in words and eventually in symbols. A new concept brings a new word. Without the concept, the child will not understand the word; without the word he cannot assimilate or accommodate the concept as easily into his existing schema because the word categorizes the concept with a mathematical interpretation. For this reason, language must be regarded as an integral part of the young child's mathematical experience.

This brings me to another cogent reason for beginning the study of mathematics through geometry. Children listen to language for five years and use it orally for about four years before we expect them to read or write it. Is it not reasonable to give them a comparable period hearing and using mathematical language before expecting them to comprehend the written and symbolic forms? In contrast with arithmetic, the child is not confronted with a host of symbols that require memorization not only of themselves but also of the rules governing their manipulation. Geometry provides the process of abstraction of the attribute by shape or size and illustrates the necessity of forming clear ideas about the characteristics and relationships. Almost every object has some geometric property, and as Vance (5) remarks, children have an innate curiosity about geometry. They see shape, size, and position as entities to explore, manipulate and control and as the means of organizing their world so that geometry becomes their natural vehicle for developing intuition, creativity, a spirit of enquiry and problem-solving.

The nature of its properties makes geometry the framework of number through emphasis on observation, collection, and organization of data. This leads to discrimination and generalization. The conscious adjusting of means to meet ends requires comparisons to be selected by asymmetric relationships such as big and small, tall and short, wide and narrow. The necessity of exact balance, or equivalence transforms vague qualitative differences into definite quantitative ideas of more or less. Powerful concepts such as those relating to symmetry, similarity and congruence can be developed through early and systematic experiences with geometry.

In defense of the title 'Why Was Mary So Contrary?', I would suggest that not only Mary, but also Tom, Dick and Harry found the mathematics they were offered uninteresting and irrelevant because it was unrelated to the problems they were facing as boys and girls trying to make sense of their world. However, Mary had a special problem. She was a girl. Sex stereotypes held by our society dictate to a large extent how children should play. Young girls have been encouraged to engage in homemaking tasks, while young boys build with blocks, push toy trucks, and take part in athletic activities. One outcome is that spatial visualization, involving the visual imagery of objects and movement or change in the objects themselves or in their properties, is less well developed in girls than in boys (6). Another problem arises because young girls tend to remember and use symbols much better than young boys, with the result that teachers are inclined to move girls rapidly from concrete to symbolic representation. Lacking the concrete experience needed to develop spatial visualization skills both in their school work and out-of-school activities, girls often

have difficulty with those aspects of mathematics that have spatial components. As an individual's level of achievement drops there is a corresponding change in self-esteem and in attitude toward the subject. Confidence in one's ability to learn and use mathematics is essential. Teachers have a vital role in reducing the effect of such stereotyping. They must be continually alert to ways of structuring the learning environment so that girls are not handicapped by a lack of appropriate experience; they can develop and maintain a positive belief in their own abilities to do mathematics, and that they appreciate the importance of mathematics.

It is my contention that the most fruitful approach to the mathematical development of young children is through geometry. For the youngest children, the natural environment, some manmade materials and play equipment will provide all the opportunities children require. New tools and materials should be gradually introduced and subsequently removed as the need for them declines. Their range of experiences should include large toys, boxes of various sizes and weight, containers, wet and dry sand, water, clay, living things, blocks, puzzles, home corner, dress-up corner, cooking, music, books and so on. They need to handle three-dimensional shapes -- spheres, cubes, cuboids, cones, cylinders -- and to experiment with shapes of all kinds on a flat surface. From these activities the children will acquire mathematical competence and be able to compare shape and size -- big and small, tall and short, wide and narrow; distinguish points and locations -- above and below, front and back, inside and outside; construct by discriminating shape and size; to recognize shapes in three-dimensions; to order by size in three-dimensions; to balance by weighing; and to compare capacities of containers. Eventually, by acting upon the objects, the children will develop an awareness of comparisions, matching, invariance and order, and intuitive notions of straight lines, parallels, right angles, distance, direction, perimeter, area, circumference and some of the properties of simple plane figures. The children's natural delight with pattern leads to an exploration of symmetry. While most natural creatures, including the children themselves, exhibit a high degree of symmetry in their form, it is through their investigations with mirrors and their recorded impressions -- self-portraits and portraits of parents, drawings of houses and landscapes -- that we see how influential symmetry is in their thinking. It is important that ideas of planes and axes of symmetry (although not in this language) be established in the real world of three-dimensions, otherwise work in two-dimensions will tend to be merely techniques designed to get a 'right' answer. Following from these early experiences with various symmetries come ideas of congruence and similarity. Young children should have this repertoire of powerful and practical concepts to apply to their study of arithmetic.

It would be foolish to expect that children could develop their competency in mathematics to the level outlined here all on their own. Within school, or outside of school, an understanding adult is the key to the children's attitude towards learning. They need

- a person who recognizes and understands their needs;
- a person who provides relevant materials and experiences that will meet those needs;
- a person who is prepared to patiently guide and sympathize with them when they are confronted with difficulties or in times of stress;
- a person who knows when to intervene by asking a question, adding some equipment or taking some away, so that individual children are neither frustrated nor bored;
- a person who is alert to those occasions when children indicate basic understandings of mathematical ideas such as order, symmetry, scale, etc. and can take them on from there.

These qualities may seem unrealistic when one considers all of the demands being made on teachers today. But are they really new and additional expectations or simply restatements of the skills that professionals in the field of education should have? In any event, it must be acknowledged that a true appreciation of young children's early experiences can be gauged only by an adult who, as a result of training and experience, knows young children and how they learn mathematics. Hopefully, the day is not far away when all Toms, Dicks, Harrys, and especially, the Marys, will benefit from new experiences orchestrated by knowledgeable and caring teachers.

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MATHEMATICAL APPLICATIONS AND MODELS FOR AGES 11 - 14 IN BRITISH SCHOOLS

Douglas H. Crawford, Queen's University, Kingston, Ontario

In Part I of this article, published in the September 1976 issue, the significance of mathematical modelling and its relation to elementary applications of mathematics was outlined and some examples of simple mathematical modelling used by the Fife Mathematics Project were given.

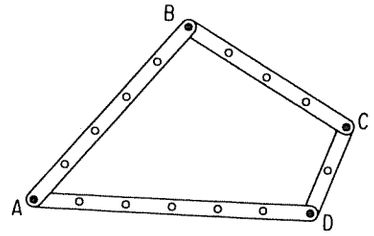
In this second part, the Mathematics for the Majority Project and its Continuation Project at Exeter University are used to illustrate common elementary applications of mathematics. The article concludes with an account of how mathematical modelling was used in an English secondary school to solve a common everyday problem.

PART II

1. The Mathematics for the Majority Project

In 1967, as a result of a study undertaken to assess the implications for school mathematics of the Newsom Report of 1963 which considered the education of pupils of average or lower ability, ages 13 to 16, a Project entitled **Mathematics for the Majority** was set up for 5 years at Exeter University.

Must such children "play out time until they are released from the burden of schooling? Or can they appreciate, profit by, cooperate in a curriculum, a way of learning, that enlists their interest and justifies itself to them by its relevance?". The project set out to show that they **can** by preparing a series of teaching guides such as **Space Travel on Mathematics, Luck and Judgement, Simple Functions** to advise teachers on how to structure their mathematics programmes with this range of pupils. Much excellent material was produced, of which the book **Machines, Mechanisms and Mathematics** is an excellent example. It deals with topics like Linkages, Rotary Motion, Shapes and Loci. To give a very brief flavour of the work, I have reproduced Work Cards No. 9 and 10 on quadrilateral linkages - two of a series of 22 Work Cards dealing with this topic (see Figure 6). This work approaches the study of polygons using real models, and many of their properties can be demonstrated much more effectively than by the usual 2-D approach.



Make a quadrilateral linkage with 6, 5, 3 and 7-hole strips as shown.

Hold AD and turn CD clockwise about D.

1. Draw the linkage when
   (a) C is at its greatest distance from A,
   (b) C is at its closest point to A.
2. Draw the linkage when
   (a) angle BAD is largest,
   (b) angle BAD is smallest.
3. Describe the motion of AB as C makes complete revolutions about D.
4. If CD is replaced by a 5-hole strip could C turn completely around D?

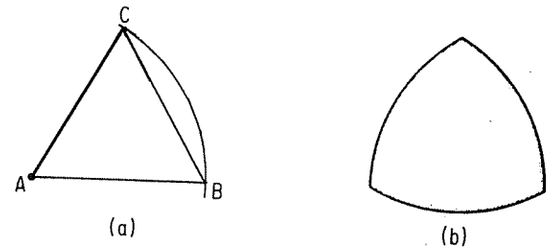
Figure 6

Is it possible for all the angles of a quadrilateral to get larger at the same time?

Make a number of quadrilaterals and investigate.

Another fascinating topic considered in Chapter 5 (Shapes for a Purpose) is "Curves of Constant Breadth". Such curves have the property that no matter how they are orientated or turned, they always have the same height. Work Card 46, which is shown in Figure 7, shows how to construct one. The seven-sided British 50p piece is a curve of constant breadth. One important application of this shape is as the cross-section of the rotor of the N.S.U. Wenkel rotary engine, which has the tremendous advantage of no reciprocating pistons.

You will need: a template of an equilateral triangle, paper, pencil, cardboard, scissors, ruler.



Draw around the triangle on the cardboard. Centre your compasses on A and draw an arc from C to B. Repeat for the other corners to obtain the shape in figure (b). Cut out this curved shape.

Figure 7

2. The Mathematics for the Majority Continuation Project (MMCP)

Excellent though the MM project was, evaluation of its work showed that the existence of teachers' guides was not a sufficient stimulus for change. Hence in 1971 a Continuation Project was set up to provide and try out classroom materials for these pupils. It is a unique project in that it is a communal effort by groups of teachers from 51 Local Education Authorities linked to the Project H.Q. staff by regional coordinators. A group of teachers write a draft set of materials on **one** environmental topic and its interaction with mathematical thinking e.g. Mathematics and Buildings. These are then tried out locally before publication. Twelve Packs are due to be published by 1976. Their titles are shown in Figure 8. Each pack contains about 15 wallets, each containing assignments on a sub-topic of the theme in the form of workcards and booklets. The sub-topics for Theme I - Buildings are also shown in Figure 8. The cost of each pack is £25. One of the assignments in B5 "A Roof Over Your Head" is a Survey on Housing. The first page of the work card is reproduced as Figure 9. It shows how extensive the survey is designed to be. A second assignment entitled "The Pitch of a Roof" occurs in B3. Two cards are involved, one for purposes of introducing the concept; the second is reproduced as Figure 10. It requires the student to use a model of the Holland Bungalow, which appears on another work card. Finally, in this very brief look at MMCP, Figure 11 depicts a work card on Television Programme from Theme 2 **Communications,** with accompanying comments. Only fairly simple calculations are involved, but the comments indicate the kinds of common sense conclusions which can be made. A similar approach in Ontario could lead to various assignments and comparisons in a Canadian context, e.g. the amount and type of advertising which is shown on different TV channels, and possibly a study of costs of advertising, and discussion of why particular types of programme are prevalent.

This project in my view is one of the most exciting programmes in middle school mathematics ever to appear.

Problem-Solving, Modelling and Applied Mathematics

Finally, I would like to outline a fascinating piece of work carried out by Alan Tammadge in an English Grammar School with a first year class of boys, aged 12 to 13.

Mathematics for the Majority Continuation Project

TITLES OF 12 PACKS PUBLISHED BY SCHOFIELD AND SIMS LTD.

1. Buildings
2. Communication
3. Travel
4. Physical Recreation
5. Seas and Rivers
6. Family
7. Motor Car
8. Countryside
9. Factories
10. Food, Shops and Advertising
11. Fashion and Design
12. Holidays, Entertainment and Recreation

Mathematics for the Majority Continuation Project

CONTENTS FOR PACK ON BUILDINGS

B1 Don't Collapse
B2 Going Straight
B3 The Roof
B4 Working from Plans
B5 A Roof over your Head
B6 Soyden School
B7 A Model Cathedral
B8 Usworth Comprehensive School
B9 Bricks
B10 Castles
B11 Build
B12 Electricity at Home
B13 Furnishings
B14 Saving Up
B15 Walls and Floors

Figure 8

Read this right through before you start.

You are going to make a survey to find out the sort of houses people live in and how they live in them.

First you must decide what questions you are going to ask.

Here are some suggestions:

(i) Do you live in a house or a bungalow or a flat?
(ii) How many bedrooms has it?
(iii) How many people live in it?
(iv) How is it heated: by electric heaters, or by coal/coke fires, or by gas?

Next you must decide whom you are going to ask. You could start by asking the children in your class. Perhaps later you could ask other children in the school.

Then you must find a good way of writing down the answers. One way is to make a large table like the one on the top of the next page.

Figure 9

THE PITCH OF A ROOF 2

The angle marked A is called the **PITCH** of the roof.

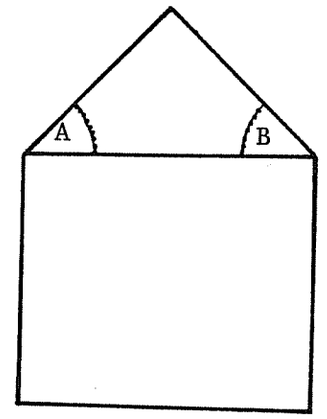
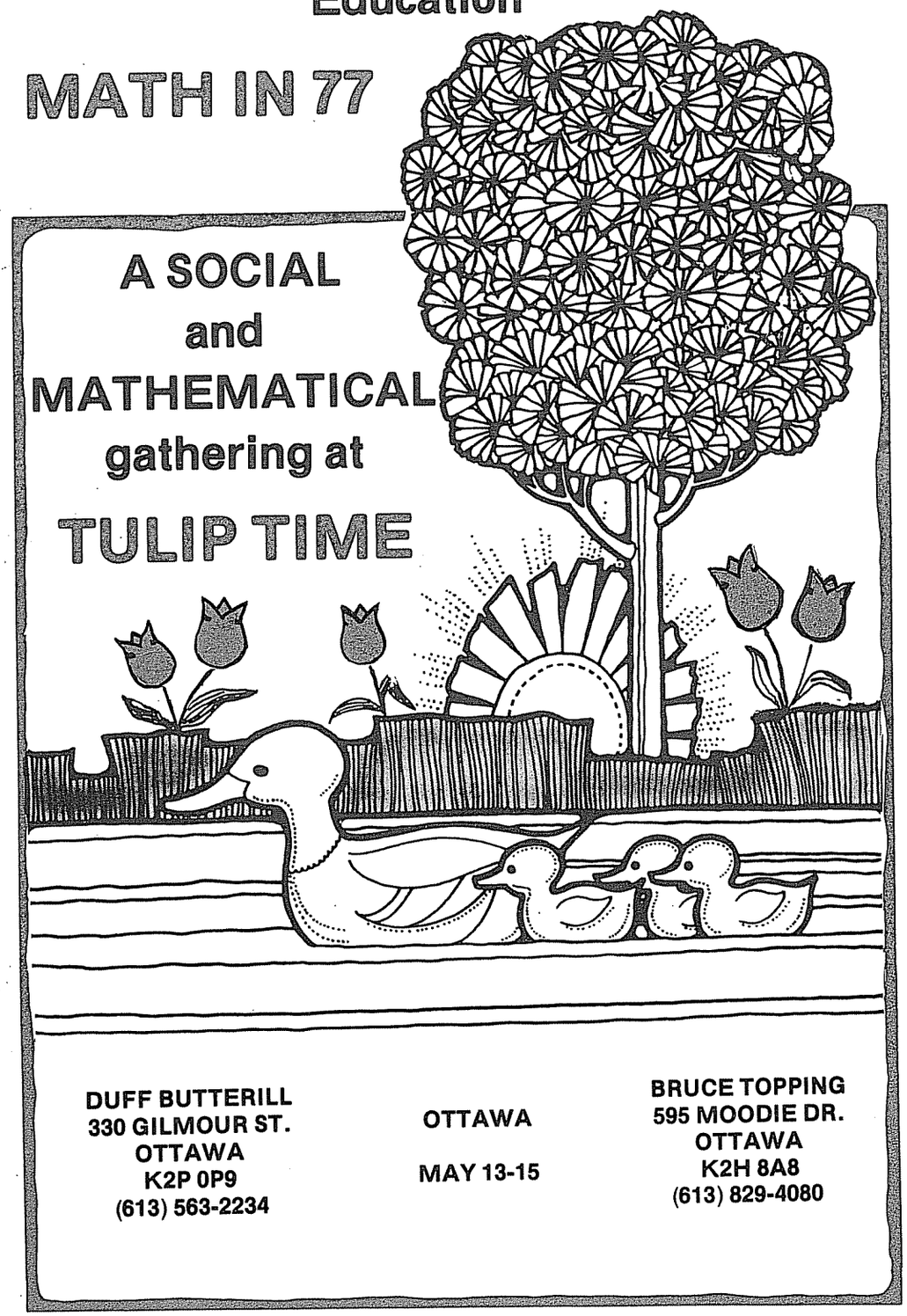


Figure 10

1. Is angle B also the pitch of the roof? Could you have a house in which A and B were different?
2. Use your protractor and the Wallchart to measure the pitch of the roof of the Holland Bungalow.
3. Is it true that the steeper the roof the greater the pitch?
4. How many degrees do you think is the pitch of the steepest roof you found when working through Card I? How many degrees is the shallowest?
5. Could you have a roof with a pitch of 80° or 90° or 10° or 0°? What about 100°?

Ontario Association for Mathematics Education

MATH IN 77



DUFF BUTTERILL
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OTTAWA

MAY 13-15

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OTTAWA
K2H 8A8
(613) 829-4080

OAME MATH IN 77
OTTAWA CONFERENCE MAY

A.) NOTES:

1.) The official conference is Friday night through Sunday noon - however, there is a full day Friday program with Thursday night accommodation facilities available.
2.) The accommodation included in the packages below, is in residences which are approximately 25 metres from all lectures, presentations, sports facilities, and meals. (no walking!)
3.) The University of Ottawa used for #2 above is in the downtown centre of Ottawa - therefore, several hotels, main shopping areas, many points of interest (Parliament Buildings etc.) and excellent restaurants are all within walking distance.
4.) Rooms will be available at convention rates (approx. $20 - $30 single, $25 - $36 double) in the following hotels, on a "book yourself" basis:
Lord Elgin Hotel, 100 Elgin Street, Ottawa;
Park Lane Hotel, 111 Cooper Street, Ottawa.
5.) Registration will be available from Thursday noon through Saturday noon til 11:30 P.M. each evening.
6.) Thursday evening (for those who arrive) will be your "night on the town".
7.) For those not opting for packages 2e offer, meals are available on a cash basis in the "close - at - hand" cafeteria.

B.) ALL INCLUSIVE OPTIONS: (Select appropriate package(s) below; AND CIRCLE ON REGISTRATION FORM.)

	MEMBERS		NON-MEMBERS	
	before Mar. 15/77	after Mar. 15/77	before Mar. 15/77	after Mar. 15/77
#1 REGISTRATION FEE only: includes: -all presentations and workshops Friday, Saturday and Sunday.	$10.00 person	$10.00 person	$10.00 person	$14.00 person
#2 SINGLE PACKAGE: includes: -registration as above, Friday and Saturday accommodation, sports passes, 2 days, Sat. & Sun breakfasts and lunches, parking 3 days, Fri., night welcoming party (see extras)	$50.00 person	$55.00 person	$60.00 person	$65.00 person
#3 DOUBLE PACKAGE -as above for #2 except 2 per room	$40.00 person	$45.00 person	$50.00 person	$55.00 person

C.) "SUPER-PRICED" EXTRAS: (Select appropriate package(s) below AND CIRCLE ON REGISTRATION FORM)

	MEMBERS	NON-MEMBERS
#a) SPOUSE/GUEST PACKAGE: includes: -Friday and Saturday accommodation, sports pass 2 days, Saturday and Sunday breakfasts and Saturday restaurant lunch, bus tour shopping, sightseeing, visits to points of interest, Friday night welcoming party (see #c this section)	$25.00 person	$25.00 person
#b) THURSDAY NIGHT PACKAGE: includes: -Fridays program, accommodation extra night, Friday breakfast and lunch, sports pass extra day, parking extra day, night on the town (your cost)	$15.00 person	$17.00 person
#c) FRIDAY NIGHT PARTY:* includes: -entertainment, hors' d'oeuvres, dancing, cash bar. * included in packages #2 and #3 part B	$1.50 person	$2.00 person
d) MEMBERSHIP IN O.A.M.E.: includes: -Membership until Aug. 31/77, 3 issues of math magazine "Gazette", 4 issues of "Abacus", reduced fees at conference, voting rights at annual meeting,	$10.00	$10.00

EGISTRATION APPLY NOW
3 - 15, 1977 INFORMATION

affiliated membership in available
local chapter-on request, and
opportunity to apply for funds to assist in math
projects under O.A.M.E., and for chapter
sponsorship and representation in your area.

***N.B.: If you include membership fee with
your registration, you are entitled
to all the above members' rates for
this conference now!!

e) SATURDAY NIGHT PRESIDENT'S
DINNER - DANCE PARTY:
includes:
-hip o' beef dinner, complimentary
wine, orchestra, disc jockey, casino,
''centre of activities'' lounge, bridge
tournament, cash bar, and transportation
from residence, to and from party site.

**President's dinner limited to 400

#f) SATURDAY NIGHT PRESIDENT'S DANCE
PARTY ONLY:
-as above #e except NO dinner

	$14.00 person	$15.00 person
	$8.00 person	$9.00 person

D.) NEEDED INFORMATION: (CIRCLE THE APPROPRIATE NUMBER FROM BELOW ON THE REGISTRATION FORM)

#1) I will be attending Friday's sessions.
#2) I am interested in mainly: (a) elementary/primary, (b) intermediate, (c) secondary (d) post-secondary.
#3) Send me a program booklet when available.
#4) I am now a member.
#5) I am including membership fees.
#6) I am not and do not wish to be a member.

DETACH AND RETURN WITH CHEQUE

O.A.M.E. 77 05 13 - 15 REGISTRATION FORM

PRINT ALL INFORMATION

NAME: _____
(FIRST) (INITIAL) (SURNAME)

ADDRESS: _____
(POSTAL CODE)

EMPLOYER: _____

NAME OF _____
(SPOUSE) / (GUEST) / (SHARED ACCOMODATION)

Circle Appropriate of the Following:

B.) #1 (10.00/10.00/10.00/14.00)
#2 (50.00/55.00/$60.00/$65.00)
#3 ($40.00/$45.00/$50.00/$55.00)

C.) #a ($25.00)
#b ($15.00/$17.00)
#c ($1.50/$2.00)
#d ($10.00)

D.) #1
#2 a, b, c, d.
#3
#4
#5
#6

***TOTAL AMOUNT ENCLOSED: $_____
***Make cheque payable to **O.A.M.E. 77**
***Send to W.J. McLeod, 330 Gilmour Street, Ottawa, Ont. K2P 0P9

O.A.M.E. 77 Ottawa May 13 - 15

Synopsis of Program

A.) Friday: An exciting program including: an address by Dr. Lola May (what to do on Tuesday after teaching Monday), workshops and presentations in French and English with speakers including Dr. Derek Woodrow from Manchester England, David Ireland of O.I.S.E., Yvette Ferland, Roberte Legris, Roger Lalonde, Roger Fischler, Dr. Eric McPherson, and many others with topics such as: interdisciplinary science studies, thinking games, problems and their solutions, anxieties, curriculum study K to 13, P J, ways of working with textbooks, mathematics activities, and many more for primary through post secondary.

B.) Saturday: a full day program with something for everyone from kindergarten teacher to University professor: Dr. Lola May (take note elementary teachers!) speaks on "What every mathematics teacher ought to know", also Peter Hilton, Jean Paul Collette, Paul Pogue et al (Pogue Report), and many others with a full range of workshops and panel discussions. All the exhibitors and publishers will be in attendance, complete with complimentary coffee and donuts.

The all-important annual election and meeting will be from 11:30 - 12:30 - members let's VOTE!!

C.) Sunday: a morning filled with multi "shorts" for everyone - 10 min. - 30 min. presentations on just about every topic you can imagine - something to use the very next day - Monday!! or - clear the head from the night before - Saturday!!

***N.B.: Program booklets will be finalized by March - register **NOW!!**

KEYNOTE SPEAKERS

Friday: **Dr. Eric McPherson;** "Present Day Anxieties in Mathematics."
Saturday: **Peter Hilton;** "Causes & Consequences of Anxieties in Teaching & Learning Mathematics."
Sunday: **David Robitaille;** "Identifying & Correcting Learning Difficulties in Mathematics."

Television Programmes

How much TV transmitted by ITV during week 30 Oct - 5th Nov 1971

	Start	Close	How Long? hr	min	Daily T hr	
Sat	1135	2410	12	35		
Sun	1100	2345	12	45		
Mon	1020 1340 1555	1200 1433 2401			9	43
Tues	1020 1345 1555		7	57 13	9	50
Wed		52 2404	1 1 8	35 02 07	10	44
	0 1340	1200 2400	1 10	40 20	12	00
	1020 1340	1215 2425	1 10	55 45	12	40
			Total for Week:			

Do not copy these figures. This page is merely a suggestion of the sort of thing you might do.

Comments

1. There is more television at the weekend (Friday, Saturday and Sunday), and least on Tuesday.
2. There are 24 hours in a day, and 24 x 7 or 168 hours in a week. Therefore the ITV programmes were transmitted for just under half the week.
3. The results for BBC might be quite different.

Figure 11

It was based on a question on Page 250 of the School Mathematics Project Book I in a Chapter entitled "Bread and Butter Arithmetic", in a section on "Real Arithmetic". The section contains a set of 15 real life problems, some with hints, the later ones without hints.

The one I shall discuss is number 8, entitled, "How much does it cost to keep a dog?" and is fully reported in Mathematics Teaching #57, Winter 1971. It took a full week of mathematics time and according to the author it was a real learning situation for him as well as the class.

The project began with a class discussion which centred entirely on the problem of feeding the dog. After some time, the teacher halted it, and asked if there was anything else to be talked about other than food. This shook the class as they suddenly realized that answering the question was not so simple as they had thought.

They were then given the homework assignment shown below.

> Think of your own dog, or one that you know, find out the facts that you need or make reasonable assumptions and then write your own answer to the question.

The results were chaotic. One boy had written:

Cost of food per week	85p
Cost of dog	£10.00
New basket	£ 1.25
Total cost of keeping a dog:	£12.10

This gave the teacher the opportunity to raise the distinction between capital costs, and running costs, and to introduce the idea of depreciation, which is spread over the life of an object. An example of this was the dog's basket, and so they were given a second homework assignment as follows:

> Bearing in mind that there are capital costs and running costs, find out the weekly cost of keeping your dog.

Reasonable answers resulted, almost all in tabular form, though mostly very messily presented. Thus the next stage was to find a systematic way of organising the data. The result was the standardised four-column table shown below. To allow for variations in which dog was chosen, the data shown was added.

Item	How often Bought	Cost in Pence	Weekly Cost in Pence
Food	daily		
Vitamins	daily		
Cod liver oil	weekly		
Licence	annually		
Holiday boarding	annually		
Cost of dog	occasionally		
Baskets (say 3)	occasionally		
Vaccination	occasionally		
etc.			

We had also to decide what to take as the number of years in the dog's life so we inserted:

Type of dog _____
Length of life _____
Any other assumptions _____

Discussion about the calculation, and wide variations in the estimates for individual item costs and the length of time they would last were encountered. This brought up the need for a flow plan, and hence the next homework assignment, H3:

Read through pages 250 and 251 of SMP Book 1 and sketch a flow plan for the dog problem. Do not get down to details of costs but distinguish between weekly, daily, annual costs, etc.

Results were creditable, but the problem of having only one piece of information in each box of the flow plan still had not been adequately solved, and other points such as the difference between a variable (years in a dog's life) and a constant (weeks in 1 year) had to be cleared up. On this basis, the outline flow plan shown in Figure 14 was built up.

Each boy was then given the final homework assignment shown below:

Use your outline flow plan to build up a detailed flow plan showing the individual items and how you will deal with them.

In the final lesson of the week, they fed their own figures into their own flow plans, and did hectic calculations, coming up with weekly costs, varying from about 80c to $2.50, with a mean of about $2.00.

Here then, to my mind, is a piece of school mathematics which has everything -- a real problem arising from daily life, and the built-in need to model situations mathematically. Notice that we have two models in this problem. Firstly, there is the table on page 12 in which the relevant abstract features of the situation have been identified and displayed. This is a modelling of the real world. However, another type of flowplan may be used, in that it is a blue-print, or algorithm or mathematical model of the process of calculation involved in using the tabular data.

Conclusion

Mathematical modelling, and applications of mathematics in a sense represent the wholeness of mathematics. We want to help the average student learn to use mathematics in his own life. To do so, we need to motivate him with interesting relevant problems, but unless we help him develop the mathematical tool of modelling which enables him to see the mathematical representation of the problems, and the computational and processing tools he needs to use this representation, we will not have taught him much mathematics.

So perhaps there is little new after all! We need motivation, computation and structure - as always!

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PEDOE'S FORMULATION OF URQUHART'S THEOREM

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Urquhart's theorem of Euclidean geometry states:

Let ℓ and ℓ' be two straight lines which intersect at A. Let B and C be points on ℓ with C between A and B. Let D and E be points on ℓ' with E between A and D. Suppose that BE and CD intersect at F. If AC + CF = AE + EF, then we have AB + BF = AD + DF.

In a recent article Pedoe [2] asserts that an equivalent version of Urquhart's theorem is the following:

If C and E are points on an ellipse with foci A and F, then $B = AC \cap EF$ and $D = AE \cap CF$ lie on a confocal ellipse.

Unfortunately, this is not quite correct as it stands. To make it correct, we must insert the requirement that C and E lie on opposite sides of AF. More precisely we have:

Let C and E be points on an ellipse ξ with foci A and F, and set $B = AC \cap EF$ and $D = AE \cap CF$. If C and E lie on opposite sides of the major axis of ξ, then B and D lie on a confocal ellipse; whereas, if C and E lie on the same side of the major axis, then B and D lie on a confocal hyperbola.

To prove this, we choose our co-ordinate system so that has the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $0 < b < a$. Writing e for the eccentricity of ξ, we have $A = (-ae, 0)$, $F = (ae, 0)$. Since C and E lie on ξ, we can suppose $C = (a\cos\theta, b\sin\theta)$, $E = (a\cos\phi, b\sin\phi)$. Setting $\alpha = \frac{1}{2}(\phi + \theta)$, $\beta = \frac{1}{2}(\phi - \theta)$, a straightforward calculation shows that

$$B = \left(ae\,\frac{\cos\alpha(\sin\alpha + e\sin\beta)}{\cos\beta(\sin\beta + e\sin\alpha)},\ be\,\frac{(\sin^2\alpha - \sin^2\alpha)}{\cos\beta(\sin\beta + e\sin\alpha)}\right)$$

and

$$D = \left(ae\,\frac{\cos\alpha(-\sin\alpha + e\sin\beta)}{\cos\beta(\sin\beta - e\sin\alpha)},\ -be\,\frac{(\sin^2\alpha - \sin^2\beta)}{\cos\beta(\sin\beta - e\sin\alpha)}\right)$$

It is easily verified (remembering that $b^2 = a^2(1 - e^2)$) that B and D lie on the conic

$$\frac{x^2}{L} + \frac{y^2}{M} = 1 , \tag{1}$$

where

$$L = a^2e^2 \frac{\cos^2\alpha}{\cos^2\beta}, \quad M = a^2e^2 \frac{(\cos^2\alpha - \cos^2\beta)}{\cos^2\beta}.$$

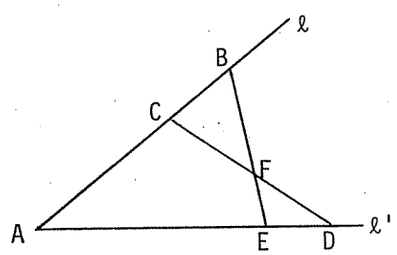
Since $L = a^2 + \lambda$, $M = b^2 + \lambda$, with

$$\lambda = a^2 \left(\frac{e^2\cos^2\alpha}{\cos^2\beta} - 1 \right).$$

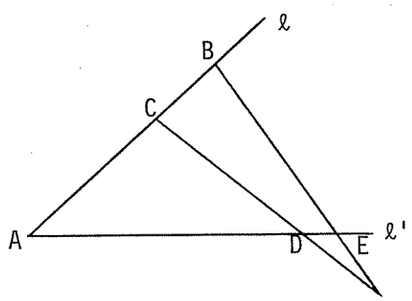
(1) is a conic confocal with ξ. Now, if C and E lie on opposite sides of AF, the chord joining them, viz., $\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = \cos\beta$, meets the x-axis between $x = -a$ and $x = a$, so we have $-1 \le \frac{\cos\beta}{\cos\alpha} \le 1$, that is $\cos^2\alpha - \cos^2\beta \ge 0$, i.e., $M \ge 0$ and (1) is an ellipse (possibly degenerate). On the other hand, if C and E lie on the same side of the major axis, then reasoning as above, we obtain $M \le 0$, so that (1) is a hyperbola. This completes the proof.

The case when C and E lie on the same side of AF leads to the following complement to Urquhart's theorem (see for example [1]):

Let ℓ and ℓ' be two straight lines which intersect at A. Let B and C be points on ℓ with C between A and B. Let D and E be points on ℓ' with D between A and E. Suppose that BE and CD intersect at F. If AC + CF = AE + EF, then we have AB - BF = AD - DF.



URQUHART'S THEOREM:
AC+CF = AE+EF => AB+BF = AD+DF



COMPLEMENT TO URQUHART'S THEOREM:
AC+CF = AE+EF => AB-BF = AD-df

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FACTS AND OPINIONS REGARDING THE TEACHING OF STATISTICS IN ONTARIO SECONDARY SCHOOLS: A SAMPLE SURVEY OF MATHEMATICS HEADS*

by
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INTRODUCTION

"Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write". (H. G. Wells)

The prediction embodied in this quotation, now a number of decades old, is probably somewhat extravagant. Nevertheless today it is widely recognized that some statistical and probabilistic reasoning is demanded of nearly everyone. How is the citizen to gain the competence to deal with the statistical arguments put to him through the mass media by business and government? Statistics education is part of the academic program for a substantial portion of those who attend university but most people, in this country and elsewhere, do not go on to university. This suggests that statistics should be seriously considered as a subject for study at the secondary school level. Certain experience along this line has been gained in several foreign educational jurisdictions. See, for example, Gani and Rising in reference (1) and Rade (2). The Secondary School Statistics Project (SSSP) of The University of Western Ontario has assessed the state of statistics education in Ontario high schools and is developing experimental programs. What statistics is presently being taught? What is the statistical background of the Ontario mathematics teachers? How valuable is statistics regarded as an addition to the mathematics program? What difficulties need to be surmounted before statistics can be widely taught at the secondary level? To obtain information necessary to answer questions such as these the SSSP conducted a sample survey of the mathematics heads of Ontario secondary schools.

DESCRIPTION OF THE SURVEY

A questionnaire pertaining to the teaching of statistics at the secondary level was sent to each mathematics head in a random sample of 202 schools. The sample, which included approximately one third of the secondary schools, was stratified with respect to region according to the regional structure present in Ontario during the 1973-74 academic year. A response rate of approximately two thirds was achieved. Table 1 contains the breakdown by region of the number of schools to which questionnaires were sent and the number of schools returning questionnaires.

*This research was funded by the Ministry of Education, Ontario through a grant-in-aid of research.

**Research supported in part by the Canada Council.

REGION	MAILED	RETURNED
Separate Schools	24	10
1 - Thunder Bay	8	3
2 - Sudbury	10	6
3 - North Bay	9	6
4 - London	21	13
5 - Waterloo	18	10
6 - Hamilton	22	14
7 - Toronto	36	19
8 - Toronto	43	25
9 - Kingston	14	13
10 - Ottawa	22	14
TOTAL	202	133

TABLE 1

Number of Questionnaires Mailed and Returned by Regions.

STATISTICAL BACKGROUND OF ONTARIO MATHEMATICS TEACHERS

The success or failure of a statistics program for the high schools must rest upon the ability of the teachers to teach the subject. Can one expect at least one teacher in a mathematics department to have sufficient statistical background to be able to deal effectively with a course at this level? The initial section of the questionnaire pertained to this problem.

The first question concerned the number of full time and part time teachers in the mathematics department. Question 2 inquired into the statistics background of the mathematics teachers by asking how many statistics courses had been taken at university. This information is tabulated by region in Table 2. Roughly 1/5 of the teachers were not classified by the department heads. The reasons for this are unknown but perhaps some heads did not consider the part time teachers when answering question 2. However, it is quite apparent from the data of Table 2 that teachers with zero or one course in statistics predominate. If one averages the number of statistics courses over those teachers that are classified one obtains the figure 1.08 as an estimate of the average number of statistics courses taken by Ontario mathematics teachers. Our information does not tell us anything about the content of this statistics course that the average mathematics teacher has taken. It could have been a course in mathematical statistics or it could have been a quantitative methods course with a heavy bias toward applications in one of the social or biological sciences.

	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL
Full Time	11	32	33	79	37	73	125	188	63	63	33	737
Part Time	14	20	12	40	20	40	44	64	38	39	27	358
0 Stat. Course	13	14	8	25	14	18	43	77	32	22	21	287
1 Stat. Course	8	17	15	43	16	31	52	98	33	32	28	273
2 Stat. Course	4	8	2	11	4	18	28	35	9	22	6	147
3 Stat. Course		3	4	5	6	4	15	9	6	9	4	65
4 Stat. Course		1				2	1	1	1			7
5 Stat. Course							3		1			4
More							3		6			9
Stat. Degree							1					5
Unclassified	0	9	16	35	17	36	3	31	13	17	1	198

TABLE 2

Number of University Statistics Courses Taken by Teachers of the Schools in the Sample

ATTITUDES OF MATHEMATICS HEADS TO THE TEACHING OF STATISTICS

Question 3 asked if statistics forms a part of the present mathematics program. The responses by region to this question are to be found in Table 3. Of those responding, ⅔ have a statistics component to their mathematics program. Question 4 asked this group to indicate the topics covered and the approximate total time allotted. The material covered tends to be descriptive statistics only and the amount of time devoted to statistics is short. The comments made indicate that some of those schools that do schedule some statistics may leave it out if there is some difficulty in completing core material.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL	%
YES		5	2	12	8	8	13	17	9	9	6	89	67
NO	3	1	4	1	2	6	6	8	4	5	4	44	33

TABLE 3

Number of Schools in the Sample Teaching Statistics - by Region

The math heads were asked whether or not they considered it important to teach statistics at the secondary school level. The answers to this question are given in Table 4. Overall, about 5/6 answered this question in the affirmative.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL	%
YES	3	6	6	10	7	10	18	19	12	12	7	110	83
NO	0	0	0	3	3	4	1	5	1	2	2	21	15
NO ANS.								1			11	2	2

TABLE 4

Mathematics Heads in the Sample Who Consider it Important to Teach Statistics - by Region

The mathematics heads were then asked if they would be willing to teach statistics in their program? The answers are overwhelmingly favourable as can be seen from Table 5. The 94% in favor can be compared with the 83% who said they thought statistics was important for secondary school students (Table 4) and the 67% who said that statistics is presently part of their program (Table 3). Comparing the results from Tables 4 and 5 indicates that there are a few mathematics heads who would be willing to teach statistics even though they do not regard it as a valuable addition to the program.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL	%
YES	3	6	5	11	10	13	18	23	13	14	9	125	94
NO				1		1		2			1	5	4
NO RESPONSE			1	1							1	3	2

TABLE 5

Mathematics Heads Willing to Teach Statistics

The ⅓ portion not teaching statistics were asked to indicate the reasons for not doing so. Reasons given include:

i) Not considered to be a proper topic
ii) Unsuitable or unavailable teaching materials
iii) Lack of pupil interest
iv) Lack of adequate staff
v) This topic has been overlooked
vi) Not considered to be a "core" topic
vii) Other

The numbers of schools giving these reasons for not teaching statistics are tabulated in Table 6. The fact that statistics is not a core topic is the most common reason for not teaching it.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL
i)												
ii)	1			1	1	1	2	1	1	3		11
iii)				1			1		1	1	1	5
iv)	1			1				1		1	2	6
v)	1				1		1	2		2	1	8
vi)	2			1	1	4	3	5	4	3	3	26
vii)	1		2		1	2	3	2	1	1	2	15

TABLE 6

Reasons Given by Sampled Schools for Not Presently Teaching Statistics - by Region

The fact that the totals of Table 4 add up to more than the numbers of schools not teaching statistics can be accounted for by individual heads giving more than one reason for leaving statistics out of the syllabus.

The second section of the questions asked mathematics heads to comment on the appropriateness of certain topics in statistics for the secondary school mathematics program. The statistics topics fall into four categories: 1) Data analysis and presentation; 2) Elements of probability theory; 3) Elements of statistical inference and 4) Correlation and regression analysis. The respondents were asked whether or not they felt the specific topics should be added to the mathematics program. Table 7 contains the percentages for and against adding certain topics to the mathematics programs of the Ontario secondary schools. The most acceptable topics appear under the headings of descriptive statistics, probability and statistical inference. However, the acceptability of the more advanced topics in probability tails off sharply. Correlation and regression analysis are not regarded as suitable for high school students. It is interesting to note that whenever the "no" rating of a topic is high so also is the "In no position to judge" rating. Perhaps the topics with high acceptability are well known to the respondents and those topics with low acceptability are not well understood.

TOPICS IN STATISTICS AND PROBABILITY	Yes	No	In no Position to Judge	No Response
A. Data Analysis and Presentation				
____ a. Tabulation of data, graphical representation of data, histograms, cumulative frequency polygons.	94%	3%	1%	2%
____ b. Measures of central tendancy for raw and grouped data. Arithmetic mean, median, mode.	94	3	1	2
____ c. Measures of variability for raw and grouped data. Variance and standard deviation, coefficient of variation, range, interquartile range.	75	16	6	3
____ d. Measures of skewness and kurtosis.	32	43	20	5
____ e. 'Quantiles, percentiles, deciles, quartiles:	69	16	7	5
B. Elements of Probability Theory. (Some of these topics are regularly taught in Grade XIII).				
____ a. Sample space, events and basic manipulations using union, intersection and complements.	93%	2%	2%	3%
____ b. Probability measures.	87	4	5	4
____ c. Equally likely events and the combinatorics associated with counting such events.	93	2	2	3
____ d. Conditional probabilities and Bayes' rule	59	21	17	3
____ e. Independent events.	93	2	3	2

____ f. Random variables - discrete and continuous. Probability distribution, mass and density functions.	39	32	27	2
____ g. Jointly distributed random variables: Joint, conditional and marginal probabilities.	15	50	31	4
____ h. Specific discrete random variables: binomial, Poisson, geometric, hypergeometric, multinomial.	20	45	31	4
____ i. Specific continuous random variable: uniform, exponential, gamma normal.	10	53	32	5
____ j. Mathematical expectation: mean, variance, higher order moments, covariance, correlation coefficient, moment generating functions.	21	45	31	3
____ k. Central limit theorem, law of large numbers.	24	43	28	5
C. Elements of Statistical Inference				
____ a. Notion of a population and a sample from a population.	81	3	14	2
____ b. Estimation of population parameter(s) by sample statistic(s).	71	7	18	4
____ c. Interval estimates. This section might include some of the following topics. Distribution of the sample mean from a normal distribution, and the standardized normal variable. Distribution of the sample variance from a normal population and the chi-square distribution. The t-distribution. Use of standard tables. Confidence intervals for the mean and variance of a normal distribution.	41	32	25	2
____ d. Hypothesis testing. This section might include some of the following topics. Null versus alternative hypothesis. Test statistics. Critical regions and the level of significance of tests. Type I and Type II errors.	30	35	32	3
D. Correlation and Regression Analysis.				
____ a. Correlation. This section might include some of the following topics. Population and sample correlation coefficients. Interpretation of correlation coefficients.	39	33	25	3
____ b. Simple linear regression model. This section might include some of the following topics. Least squares estimators for regression coefficients. Distribution theory for least squares estimators assuming a normal distribution for the observations. Confidence intervals and hypothesis tests for regression coefficients. Prediction using fitted lines.	20	45	31	4
____ c. Multiple regression	3	54	35	8

TABLE 7

Percentages For and Against Adding Certain Statistics Topics to the Mathematics Program

HELP REQUIRED TO INCREASE STATISTICS OFFERINGS

A final set of questions inquired as to the type of help that teachers would require in order that statistics offerings could be increased. Table 8 contains the responses to the question "Are more suitable teaching materials required?" 86% of the respondents felt this to be the case.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL	%
YES	3	5	4	12	10	12	17	18	13	12	9	115	86
NO		1	1			2	2	5		1	1	13	10
NO RESPONSE			1					2		1		5	4

TABLE 8

Are More Suitable Teaching Materials Required?

Table 9 contains the responses, by region, to the question, "Are in-service training programs required?" 75% of the respondents felt they would be helpful.

REGION	1	2	3	4	5	6	7	8	9	10	SEPARATE	TOTAL	%
YES	2	6	6	10	8	11	13	16	8	11	9	100	75
NO				3	2	3	3	7	3	3	1	25	19
NO RESPONSE	1						3	2	2			8	6

TABLE 9

Are Inservice Training Programs Required?

Finally, the teachers were asked "In order to help in teaching topics of statistics, do you think it is necessary to obtain additional help (describe briefly)?" A representative sample of the responses are included below.

Materials and resource personnel might be obtained from governmental agencies and industries which are involved in work of a highly statistical nature.

Better textbooks.

Outlines from schools presently teaching statistics.

Courses of study.

Directions as to which topics are essential, particularly to students going on to courses in the social sciences, psychology, etc.

Consultants.

Suitable projects - meaningful, but not too difficult.

Development of a good text.

More real data.

Summer course in statistics for teachers at Althouse since many have few courses and/or little understanding of terms.

With reference to teaching materials and in-service training for teachers, materials and texts must contain plenty of concrete and useful examples relating to things which the student can understand. Assistance is needed in deciding what topics can be presented to students of low,

average and high abilities.

Need a short course to familiarize teachers with the subject.

More detailed outline of topics would be helpful.

Teachers should be made more aware of the uses of statistics in further education and in life outside the academic environment.

Computer programs that allow the use of some of the standard statistical measures available to the student and computer time available to go more deeply into the subject.

Specific outlines of suggested course.

Need to relate the subject to the use of statistics, hence the need for statisticians to lend the occasional practical flavour to the course.

An outline of content for a 2 month optional topic at the Grade 13 level would be of great help.

Should we not offer a "fourth" math. in grade 13? You would have to agree if we were to encompass some of the aforementioned topics otherwise, how could we possibly find time under the current guidelines. There it is, March 6, and I still have to teach logical reasoning, math. of investments, and probability and statistics to my class. Most of these students want a "wide" exposure to mathematics, hence I suggest we create, at the Ministry level, the logical alternative. I am a bit tired of this "please submit" your requests for Ministry approval for any "new" or experimental courses jazz.

More textbooks. A "lab" oriented problems book.

Research carefully the degree of difficulty of the course that can be presented to our students keeping in mind that a lot of them will not be university material.

This department head has to take a statistics course soon.

Teachers have access to university courses. They have been given some incentive to do specified or approved courses. They need **formal courses.**

Good text or reference that even a high school student would be able to read and understand. i.e. Keep it simple and meaningful.

CONCLUSIONS

In summary one can say that i) mathematics heads are receptive to the idea of statistics in their programs; ii) there is room for improvement in the statistics background of mathematics teachers and iii) suitable teaching materials need to be developed for these topics of interest to the secondary schools.

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Mathematics is the science of doing necessary things the easiest way.
Bertrand Russell

PIAGETIAN RESEARCH - A NOTE ON AN AFRICAN STUDY

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A large number of studies have been done on the empirical verification of Piaget's developmental hypotheses. More often than not, these studies have concentrated on the development of conservation, and they have indicated that conservation is developed in young children at about age 7 with no marked differences between males and females. Further, a strong relationship between performance in conservation tasks and arithmetic achievement has been reported (e.g., Kaminsky, 1970), and this relationship has been shown to be stronger than the relationship between arithmetic achievement on the one hand, and IQ, sex, or socio-economic status on the other (Stommel, 1966).

The present study was primarily concerned with the cross-cultural validity of the Piagetian framework. The procedural details of the study have been described elsewhere (Omotoso & Shapiro, 1976), but its general purpose was to determine with respect to a sample of 120 Nigerian children, aged 4 - 8, (a) the age at which conservation, seriation, and classification abilities develop, and (b) the nature of the relationship between these abilities and achievement in mathematics.

The results of the study indicated that in terms of developmental levels, the proportion of children at the operational level in conservation, seriation, and classification increased steadily with age. A majority of the Nigerian children did not, however, reach the operational level until the age of eight - approximately 1 year later than for North American subjects (cf. Steffe, 1966). Further, at each age/grade level and for each cognitive task, there were no statistically significant differences between girls and boys. Thus, in general, the findings indicated that Nigerian children develop according to the developmental hypotheses of Piaget. That is, cognitive development is age related with little evidence for sex differentiation in development.

In addition, the results of this study indicated that there were positive relationships between achievement in mathematics and the abilities to conserve, seriate, and classify. The actual correlation coefficients ranged from $+.31$ to $+.80$ with a median value of approximately $+.55$. Thus, it appears that young Nigerian children who do relatively well in arithmetic are also at relatively higher levels of cognitive development in Piagetian terms.

Interesting, moreover, was the fact that within this sample, the association between cognitive level and arithmetic achievement was much stronger for seriation than for either conservation or classification. Previous research has concentrated on the conservation dimension; future studies might well focus on seriation. Whether or not future studies include this particular dimension, the present result suggests the need to move away from the standard paper-and-pencil tests used in such investigations. It is quite possible, for example, that the strong relationship between seriation and mathematics achievement is a result of the role of both the regular algorithms and the standard spatial arrangements used for solving simple computational problems. Thus, the ability to arrange mathematical symbols appropriately on paper may be the critical skill involved. That is, the results may simply reflect this relatively accidental aspect of achievement tests in mathematics rather than some underlying relationship between a cognitive skill and mathematical understanding. One possible alternative would be to work with orally administered tests although these would, of course, introduce other variables such as concentration, auditory memory, etc.

From the perspective of a classroom teacher, this study suggests that (a) the pattern of cognitive development in other cultures is not dissimilar to that of our own experience, and (b) there is an obligation to provide students with a variety of contexts in which their attainment can be observed.

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THE EFFECTS OF ELECTRONIC CALCULATORS ON MATHEMATICS AND SOME OTHER THINGS

N. L. Dain, Mathematics Department, Simcoe Composite School

"It is about five years since pocket-sized electronic calculators came on the mass market. By about 1970 it was possible to buy a good basic calculator for under $100. Now that the industry is some 100 million units further along the low end of the price range is well under $20.00."

The above is an extract from **Scientific American** of March 1976. I have on my desk a calculator which will do the four basic operations of airthmetic, and find square roots, inverses, deliver trigonometric functions and their inverses, logarithms, factorials, exponential values and also perform memory problems. This device is available at $40.00. Two years ago its price was over $100.00.

For under $200.00 it is possible to obtain a calculator which can be programmed for a fairly extensive set of operations. It is a small scale computer. What will be the price of this one two years from now?

The next type, presently in the range of $500.00 and up, accepts a small library of much more elaborate programmes, which are pre-printed on magnetized cards, and inserted into the calculator as required. This model seems to be a bit beyond the range of this present discussion, but there is some evidence that its influence may be felt, within the foreseeable future, even at the high school level.

It was with these facts in mind that I headed for Trent University on March 10 to inquire about the effects of calculators upon the teaching of mathematics in general, and also to ask about the success or otherwise of our courses in preparing students for college Maths. Trent is a small University. It does have some auditorium classes of up to 150 in first year general courses, but there are groups as small as four or five for students in the final year of Mathematics degree. The instructors are closer to the students than in most of our universities.

My first interview in the Trent Mathematics department was with Dr. Cyril Carter. Dr. Carter is an Englishman who took all his training in England and taught in England for some years before coming to Trent. He was not impressed by calculators nor much concerned about the effects which they might have upon mathematics. In the Science Department they are useful, he felt, but in pure mathematics they don't make a great deal of difference. Trent University has no faculty of engineering, so Dr. Carter had no views on calculators and engineering maths.

Our conversation turned to other topics. I asked Mr. Carter if he could compare the content of secondary level mathematics courses in the U.K. and also the state of preparation of students who started University maths courses there, with the students of Ontario. Regarding course content, he said that the U.K. schools were at least a year and a half further along than we are in Ontario. English students of the Grade 12 level finish their year with mathematics which is well into the first year work of Trent University.

Dr. Carter was quick to point out that this did not happen because U.K. students were born with larger brains. Academic specialization takes place earlier in the U.K. than it does here. Furthermore, a much smaller proportion of the high school population in the U.K. will continue on to University. He felt that he did not understand the educational set up in Ontario well enough to suggest what might be done to speed up the coverage of work in mathematics, in our high schools.

He found Canadian students somewhat more open minded than those in the U.K. and receptive to new ideas. He found a pronounced difference in backgrounds amongst Canadian students. The difficulty which he had with Canadian students stemmed from the fact that those who come to Trent to take the degree course in mathematics were, in the main, students who had been good at maths all through school.

Consequently, they were accustomed to solving the most difficult problems, even in Grade 13, in a matter of ten or fifteen minutes. Maths problems which might require an hour or an hour and a half of hard effort seemed to these students to be somewhat indecent or even downright immoral. He had had difficulty in persuading them that such problems should be expected, and indeed welcomed. He felt that this area was one in which our high schools were not really doing a good job.

The next interview was with Dr. George Hamilton, also of the Trent Mathematics Department. I mentioned Dr. Carter's opinion that calculators were not very significant in the Arts Maths courses which were offered at Trent. Dr. Hamilton began by saying that he was inclined to agree, and then stopped. He reached into a drawer and brought out his own calculator, a Hewlett-Packard '25' which can be programmed internally, and demonstrated the setting up of programmes for me.

Dr. Hamilton was an engineer before he became a mathematics instructor and explained that in his engineering days he had never been more than an arm's length away from his slide rule. Upon reflection he agreed with Dr. Carter to the extent that there were many parts of their courses in which calculators were not useful. Indeed, he could think of sections in which the calculators were "a damn nuisance", because students wanted to use calculators instead of other methods more suitable to theoretical work.

However, there were many situations in which the calculators were valuable time savers. If calculators are to be used by high school students he hopes that they will learn to use them properly. He had in mind types of problems which involved formulas which should be simplified algebraically to a final conclusion. That is, that the simplification should be completed before the calculator is brought into use. His students tend to reach for the calculator as soon as formulas were written down, even at the beginning of a problem.

We discussed briefly the programming of calculators as he had done with his Hewlett-Packard, which many college students are also doing.

Dr. Hamilton was under the impression that most high schools in Ontario were giving some instruction in computer work and programming. He was quite surprised when I explained that the computer boom, which had existed in maths departments six or seven years ago, died out when universities started to charge for the use of computer time. There may be some schools, close to computer facilities, which do carry on this work, but it certainly is not general.

We discussed the notion that there will be extra time available in high school math courses, when calculators are in general use and the use which might be made of this time. Some of the points urged by Dr. Hamilton, or Dr.Carter, were

a) that matrices could well be taught before Grade 13. If matrices are taught, starting in Grade 10, they should be used as the method for solving equations. This topic should be repeated in Grades 11 and 12 and used in Grade 13. In short, the topics should be treated thoroughly, and students should understand that they are receiving only an introduction to the study of matrices, that there is much more to come later.
b) a sound grasp of logic and order in mathematical operations is necessary for setting up computer programmes. At present, only a small number of people in colleges, business or industry need to use calculators which can be programmed. However, it seems certain that the prices of such calculators will drop and their capabilities increase during the near future. Their use could become much more general. It may become desirable to adapt our high school courses to provide more background for or training in computer programming. I do not think it

is possible to deal adequately with this problem at the present time.

There is much interest in formal logic among the instructors at Trent. However, they feel that formal logic is a good thing for some students, but they are skeptical about including formal logic in the programme for all students.

c) Common logarithms, as such, have relatively little use now in theoretical mathematics. Naperian logarithms are of considerable importance, especially in higher calculus. It would be great if the Naperian logarithms received more coverage in high schools.

After lunch, Dr. Hamilton arranged a meeting with Mrs. Morrison, who is instructor of remedial Maths and English at Trent. Mrs. Morrison is not a member of the Trent faculty in the ordinary sense. She is one of a group of six who are doing a research project on remedial work in (smaller) Canadian Universities.

Mrs. Morrison has found that many students enroll in courses such as Psychology, believing that they will not need Mathematics. Indeed, because of the options system, good numbers of them have taken practically no Maths while in high school.

She had given assistance to 50 students out of 180 enrolled in Psychology and 20 of the 50 receive help on a regular basis.

Even with students who have taken Math courses, such things as the parallel lines theorems of geometry, the meaning of the number $\sqrt{47}$, and trigonometric functions as continuing functions are not understood.

The question of what is the earliest level at which the use of calculators should be permitted was discussed briefly but no firm conclusions were reached.

Concerning this matter, my own view has been that the logical place to get information would be from Japan. It has been my belief, (not quite correctly) that Japanese children received instruction on the soroban (the Japanese version of the abacus) from the start of their schooling. Hence, the problem of teaching arithmetic while the students have calculators is an old one over there.

En route to Trent University, I had a most interesting visit at the home of Professor Nogami of the Physics Department of McMaster University in Hamilton. Professor Nogami has taught in Japan. Mrs. Nogami had all her schooling in Japan, and their oldest son, who is in Grade XIII at Hamilton C.I., has had six months of High School in Japan.

Mrs. Nogami herself started the soroban at the end of Grade III and continued through Grade VI, in formal class study. From then on in her school, only selected students continued the soroban in regular class periods. In some schools, classes in the soroban were held after hours. (Other special subjects such as music or art might also be held after hours. The organization of these things varied from one region to another.)

The soroban is, by nature, well suited to addition and subtraction. It can be used for multiplication and division but this involves learning of a system of complicated rules. Students who became skillful with the soroban, were top candidates for jobs in banks and accounting firms. The electric calculators (adding machines) which we have had for many years have had a limited acceptance in Japan.

Mrs. Nogami has recently received a letter from a relative in Japan, who stated that electronic calculators have been introduced in the schools of her home town, at the end of Grade VI.

There has been considerable controversy as a result. This does not mean that pupils are permitted to bring the calculators to school, as at S.C.S., but that the calculators are provided and that the pupils are taught how to use them.

It would seem that the Japanese educators are carrying on a tradition of training their students in the use of calculators, and also that the soroban is going to follow the slide rule into history.

Concerning Canadian university students, Professor Nogami said that he found them more free thinking and innovative, but not as well inured to sustained hard work as those in Japan.

I asked Professor Nogami's son if he could compare the course content of our Ontario math courses with those in Japan. The answer was that he felt that Japanese math students were more than a year, perhaps a year and a half ahead of those in Ontario.

I found these comments especially interesting in the light of Mr. Carter's comments at Trent University.

Upon leaving Trent University I went to Sir Sandford Fleming Community College in Lindsay to visit Mr. Gaile Fleming who teaches Maths and Surveying the the Natural Resources Division there.

Last year, when I talked with Mr. Fleming, about half his students owned calculators. Students who did not own a calculator could borrow from a stock kept in the school library. This year 80% of his students have their own calculators. The school's authorities have decided not to add to the stock of calculators in the library.

Mr. Fleming's view is that next year students will all have their own. They are being encouraged to buy ones which have trig functions and their inverses, logarithms and anti-logarithms. "The calculator is a tool which greatly implifies many mathematical tasks. Indeed, many problems which our students now do quite handily would be virtually impossible for them without a calculator. It's only reasonable that they develop a mastery of their calculators while here at the college, so that they can use them efficiently and productively on the job."

Mr. Fleming had one further comment to make about calculators. He spotted that many of his students were not using their calculators efficiently.

For example, students would do: $\frac{131 \times 47.2 \times 16}{1200 \times (-76) \times 42}$

by multiplying out numerator and denominator separately and writing down:

$= \frac{90,921.2}{3,830,400}$

then dividing = .02379.

Not having acquired a proper grasp of operations and order of operations, they wasted time.

When I suggested that the Japanese idea of giving training in the use of calculators might be helpful, Mr. Fleming was vehemently negative. "Not for our students--give them a formal course in using calculators and they'll promptly forget all about it." "If you can show them, in a problem, that your method of using the calculator is better, then they'll buy the new method. Otherwise, no. All you need is enough different practical problems, and they will learn to use the calculator as it should be used."

CONCLUSIONS

I'm not sure how many final conclusions can be drawn at present about the effects of calculators upon the teaching of mathematics, but certainly a number of interesting possibilities can be suggested.

The recent Scientific American article, referred to at

the beginning of this account, stated that the industry is producing electronic calculators at under $20.00. Attached is a circular received in last week's mail, which gives prices on calculators, starting from $7.95. As far as price is concerned, it's reasonable now, for any math student to have a calculator on his desk.

What type of calculator is best for high school students? We can safely say that one having the four basic operations of arithmetic, a percent button and a square-root button would be satisfactory through high school and useful after school days are over, for household calculations, income tax, and the like. Many of our senior students are now using more elaborate models, which saves them the trouble of looking up trigonometric, logarithmic or other tables. These are suitable also for technical courses at community colleges, and probably for first year science and engineering courses at universities.

What advice a teacher should give to a student who wishes to buy a calculator which will see him all through university in science or engineering or in business courses which have their own peculiar problems, is another matter. Probably it's best to suggest that the student should talk with someone who is at a university and well along in the particular course.

$49 + 9 = 51?$ $\quad \frac{1}{5} + \frac{1}{25} = \frac{1}{30}?$ $\quad 2 + 9 = 12?$

$8900 - 4000 = 8500?$

The items above, which are selected from two recent tests in Grade XIII maths, suggest one further observation about the use of calculators. Calculators do save a great deal of time, and they do cut down substantially on the number of mistakes, but there are many calculations, even in senior work, which are so simple that it's not worth while to pick up a calculator to do them. In matrix problems the calculator is simply clumsy.

Mistakes like those above do happen, but it's the number of them which is surprising. The trend which is said to be current in the elementary schools, to re-emphasize drill

in arithmetic, deserves encouragement. This drill does not need to be on the long elaborate problems of earlier days, but much simpler ones. The importance of accurate work in very simple arithmetic is, if anything, enhanced by existence of calculators.

In Maths 340 I have found the calculator to be a worthwhile indirect teaching aid. Students bring new calculators to class, quite sure that their troubles in mathematics are over, and are quite surprised that they still make mistakes. Soon, they realize that the calculator is not to blame. It deals accurately with the information given it. These students are then prepared to pay closer attention to the correct handling of formulas and equations, so that the calculator can do its job.

Also, when all people in the class have calculators, it will be possible to do more problems and more difficult problems than at present. In effect, more time will be available for study of theory and procedure. This becomes very obvious when teaching Income Tax, which involves a long succession of calculations, all of which must be correct.

Many people are concerned that students who are allowed to use calculators will not learn mathematics because they will come to rely solely upon the calculator. This matter has several facets which must be considered separately.

Mr. Fleming at the Lindsay Community College is quite definite in his stand. "A student who reads the value of sine 47° from a calculator does not know any less or any more than one who reads sine 47° from a book of tables, as we've always done." For him that is the beginning and the end of the matter.

Mrs. Morrison, the remedial Maths teacher at Trent University, had quite a different viewpoint. "I certainly don't object to them using calculators provided that they understand what they are doing." Mrs. Morrison has been very surprised at the number of students who understand, for example, that $\sqrt{64} = 8$, but are really mystified by what is meant by $\sqrt{63}$ or $\sqrt{65}$, even though they can read these values on their calculators. They have simply never mastered the basic concepts of irrational numbers.

When the "New Maths" arrived, **the** approved method for finding square-roots was to be Newton's averaging method. I don't know of any teacher who uses it now, or has for the last several years. It simply died, because classes got bogged down and bored by the calculations involved. Newton's method is by far the best for giving an understanding of the nature of square roots, and of irrational numbers. It is now quite practical to demonstrate Newton's method in class, even if only a few of the students have calculators.

In other words, the calculator can be used to teach many of the concepts which some people feel that it may obscure. What happens in this area will depend, I think, very much upon how well teachers are able to make full use of the potential of these new calculators.

POSTSCRIPT (22 July '76)

Since this P. D. Day report was written, I have received some further information on the subject of calculators that may be of interest.

One of our Grade XIII students was accepted in an engineering technology course in Mohawk College, Hamilton. Enclosed with his notice of acceptance was a sheet instructing him to bring a 'scientific calculator' with him, and specifying precisely the capabilities which the calculator should have. A copy of this sheet is attached.

Another student who graduated from S.C.S. last year visited us after finishing his first year in engineering at Queen's University. He reports that it is "not possible to stay alive" in the engineering course without an electronic calculator. The minimum capabilities of the calculators used by the students at Queen's were closely those specified in the Mohawk College instructions.

Concerning the use of electronic calculators in the Japanese educational system, I was able to talk, last week, with members of the staff of the Japan Trading Centre in Toronto. These people had all had their education in Japan and were in regular contact with friends and relatives in their homeland.

None of them had heard of electronic calculators being used in Japanese elementary schools, as mentioned by Mrs. Nogami, and all were certain that this must be a local arrangement. I got the impression from them that basic arithmetic, done mentally or with pencil and paper, is taken very seriously in Japan.

All of these people had studied the soroban in classes from the end of grade III through grade VI as Mrs. Nogami had. They pointed out that the use of the soroban requires a good knowledge of simple addition and subtraction. The soroban, as well as being a calculator, provides drill and review for basic arithmetic. They felt sure that electronic calculators would not be allowed (generally) in Japanese schools until the latter part of the secondary school program.

Finally, a conversation with Mr. J.T. Shook, District Sales Manager for the Calculator Products division of Hewlett-Packard Canada confirmed that the prices of calculators continue to drop. The company's latest catalogue will quote substantial price reductions on calculators, and will include new models with wider capabilities, also at lower prices.

When asked about a published statement that fully programmable calculators will be sold for less than $100, 'in the foreseeable future', Mr. Shook replied that he would not give the exact number of years in which this would happen, but he felt that the statement was quite reasonable.

MOHAWK COLLEGE OF APPLIED ARTS AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE AND TECHNOLOGY

SCIENTIFIC CALCULATORS **APRIL 1976**

A "Scientific Calculator" will be required for calculating purposes when you arrive at Mohawk or shortly thereafter in the fall semester of 1976. This requirement is for students enrolled in the Technician and Technology Programs.

WHEN? Since the prices of calculators are continually going down, it is highly recommended that you wait until September before purchasing one.

PRICE? The current price of calculators ranges from $25.00 to $125.00. It is expected that this range will be lower by September. A general statement which can be made is that you get what you pay for. There are exceptions to this statement!

TYPE? A good source of information regarding calculators is the Consumer Reports found in most libraries. You should consider a "scientific calculator" which can **at least** perform the following functions:

FUNCTION

1. Take a common log
2. Take an antilog
3. Take a natural log
4. Take an anti*ln*
6. Take a sin, cos, or tan
   (for angle greater than 90°)
7. Take an inverse of sin, cos, tan

POWER? In particular you will be faced with the question of purchasing a calculator which could:
a) operate only on batteries (not rechargeable)
b) operate only on batteries (not rechargeable) or plugged into a 110v outlet by "an adapter"
c) operate on **rechargeable** batteries or plugged into a 110 v outlet sometimes called a "charger"
The cost of (a) or (b) calculators, although cheaper initially, can be expensive in the long run because of the cost of batteries. This is especially true when you forget to turn it off every so often.

DISPLAY? A point to consider is that the cheaper models usually have an 8 digit display, while the more expensive models have a 10 digit and 2 digit exponent display. Although an 8 digit display is adequate, the latter display is more useful.

LOGIC? There are basically two types of calculators. One uses algebraic logic and the other uses Reverse Polish Notation (RPN). Each has its merits. It's up to you.

SERVICE? Most calculators have a 1 year warranty. Some stores have an immediate over the counter replacement policy. Others offer no replacement but send your calculator away sometimes for a prolonged period of time and you are without a calculator. While others expect you to deal with the manufacturer directly for any repair or replacement.

Mohawk College will not require a student to purchase a **particular** calculator model nor that it be purchased at a particular outlet. This is completely at the discretion of the student. For your information only, the Mohawk College Bookstore does sell calculators.

FORWARD TO BASICS

The National Council of Supervisors of Mathematics (U.S.A.)
Position Paper on Basic Mathematical Skills

Mathematics supervisors are concerned that as a result of the "back-to-the-basics" movement, today in many schools there is too much emphasis on computation and not enough stress on other important mathematical skills. To respond to this trend, the National Council of Supervisors of Mathematics (NCSM) set up a twelve-member task force to write a position paper on basic mathematical skills. The position was first written in July, 1976, and later revised on the basis of ideas from supervisors throughout the country.

The position paper urges that we move forward, not "back" to the basics. The skills of yesterday are not the ones that today's students will need when they are adults. They will face a world of change in which they must be able to solve many different kinds of problems. The NCSM position paper lists ten important skill areas that students will need.

Problem Solving: Students should be able to solve problems in situations which are new to them.
Applying Mathematics to Everyday Situations: Students should be able to use mathematics to deal with situations they face daily in an everchanging world.
Alertness to Reasonableness of Results: Students should learn to check to see that their answers to problems are "in the ball park."
Estimation and Approximation: Students should learn to estimate quantity, length, distance, weight, etc.
Appropriate Computational Skills: Students should be able to use the four basic operations with whole numbers and decimals and they should be able to do computations with simple fractions and percents.
Geometry: Students should know basic properties of simple geometric figures.
Measurement: Students should be able to measure in both the metric and customary systems.
Table, Charts and Graphs: Students should be able to read and make simple tables, charts and graphs.
Using Mathematics to Predict: Students should know how mathematics is used to find the likelihood of future events.
Computer Literacy: Students should know about the many uses of computers in society and they should be aware of what computers can do and what they cannot do.

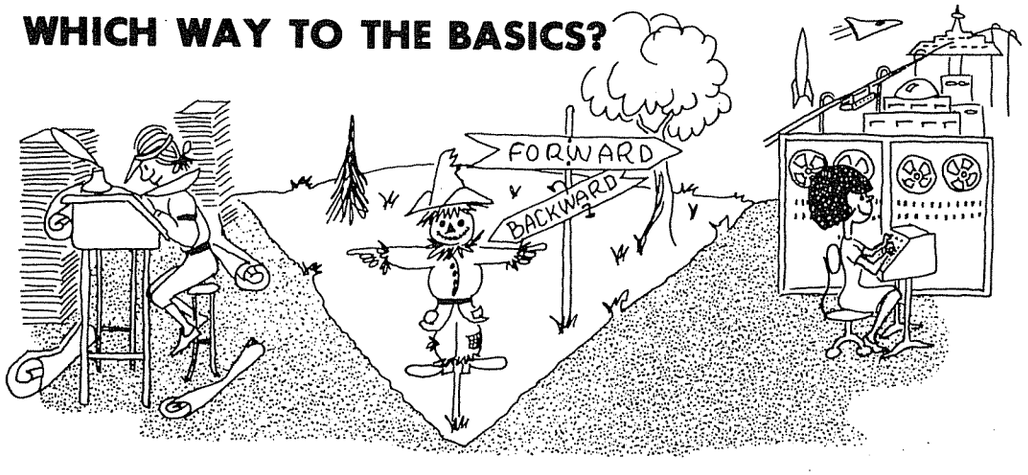
The role of computation is put into its proper place. Long computations will usually be done with a calculator, but computation is still important. Mental arithmetic is a valuable skill. Computational skills by themselves are of little use, but when used with other skill areas they give the learner basic mathematical ability. School systems which try to set the same requirements for all students should beware of requirements which either are too difficult or which stress only low-level skills.

Rather than using only a single method such as drill and practice for learning basic mathematical skills, many different methods should be used. Hands-on experiences with physical objects can provide a basis for learning basic mathematical skills. Standardized tests are usually not suitable for measuring individual student progress. Instead, the tests used should be made especially to measure the mathematical skills being taught.

The NCSM position paper sets forth a basis for identifying which basic mathematical skills are important and for determining if students have learned these skills.

TO OBTAIN THE NCSM POSITION PAPER ... send a stamped, self-addressed, business envelope to Ross Taylor, NCSM Basic Skills, Minneapolis Public Schools, 807 Broadway, N. E., Minneapolis, Minnesota 55413.

WHICH WAY TO THE BASICS?



Seen on a Math classroom door: "Free, every Monday through Friday, mathematical knowledge. Bring your own container."

BOOK REVIEWS

On Reading Piaget: Reflections and Recommendations for Teachers of Mathematics

Dr. William Higginson, Queen's University

"I shall go to my farmhouse in the Alps again this summer. There is no phone there. The villagers bring my mail up to me. I have instructed that any letter marked 'special delivery' be given to the slowest walker."

Jean Piaget, June 1976.

Quoted in memo from Murray Wood, Ontario Ministry of Education, October, 1976.

Jean Piaget is a unique and remarkable phenomenon. An active thinker and prolific author in 1976, this same man had published several reputable scientific papers before the outbreak of World War One. (He had to turn down the first professional post offered to him, the curatorship of a collection in the Geneva Museum of Natural History, because he still had two years of high school left to complete.) Although he had been publishing significant research results in child development since 1923, very few North American academics recognized the value of Piaget's work until some forty years later. An exception to this general rule was Harvard where a perspicacious faculty awarded him an honorary doctorate in 1936; "a deft inquirer into the growth of those mental processes which mark the gradual development of the normal child".

In the past decade, however, there has been a veritable deluge of 'Piagetiana' and he has been a seminal influence on the thinking of educators and psychologists in North America. Documents such as the recent Primary-Junior Guidelines in Ontario **(The Formative Years, 1975)** are firmly, albeit implicitly in places, based on Piagetian foundations. While a very strong case can be made for the contention that this is a desirable step for education, many difficulties arise.

The range and complexity of Piaget's publications are such that it is easy to misrepresent his views and it would seem that most North American 'interpretations' need careful scrutiny. One finds misunderstandings and misplaced emphasis in documents ranging from student papers; "In her work Jean Piaget ...", to the pages of **Scientific American.** In his frequently quoted work on 'The Origins of Number Concepts' **(Scientific American,** 1973, **228** (3), 101-109) Dr. Charles Brainerd of the University of Alberta claims that "the **conception of natural number** derives from a prior understanding of ordination and not from a prior understanding of cardination". Brainerd's contention is that 'new math', which he sees as proceeding from cardination to ordination, has things backwards and should be replaced. The implication is that Piaget has in some way been refuted. Even if this experiment of Brainerd's was acknowledged as being technically sound (which is not universally the case; see, for instance, David Fielker's 'Cardinal Sins' **Mathematics Teaching,** 75, June 1976, 10-13), the implied refutation of Piaget is not justified. Piaget's theory does not consider cardination to be more fundamental than ordination (or vice-versa; see the footnote in his article 'Comments on Mathematical Education' page 24 of this journal).

The major source of difficulty would seem to be the desire of North Americans to see Piaget primarily as an education psychologist. While there is no doubt that his work has major educational implications, Piaget's research is essentially biological and philosophical in nature. As a genetic epistemologist his concern is with the nature of knowledge and its relation to human development. Some of the fundamental premises on which Piaget's theoretical work is based differ radically from the assumptions underlying the traditional North American school of psychology, that of neo-behaviourism. Where neo-behaviourists see man as **reacting** to his environment, Piaget and his fellow cognitivists see man **interacting** with his environment. In their classroom manifestations these different views become visible in the emphasis on, respectively, environment (stimuli) and activity. This difference is so basic and so important that it calls into question all attempts at synthesizing the two approaches. In our immediate situation it means that thousands of 'person-hours' may be being wasted in attempts to write behavioural objectives for Piagetian tasks. Furth and Wachs (1975) comment on this in the preface to their book:

"When readers ask for a precise curriculum for the various activities and for their relations to

Piaget's stages, we have the impression that Piaget's concepts are being interpreted in a fashion that does violence both to the theory and the child. It cannot be repeated too often that for Piaget it is not the child that is in a stage, but the way a task is performed by the child. There is no question here of an overall state of the child, similar to an IQ level. And it is not only possible but entirely normal and expected that the same child's performance will vary, up **and** down, form occasion to occasion. Because of this, Piaget's theory is not consistent with a diagnostic-prescriptive approach to education where the weaknesses of the child are systematically explored, nor does it encourage a premature emphasis on achievement; rather it allows the teacher to accept and nourish at appropriate times what is positive and best in every child, namely his capacity for further intellectual development." (pp. ix-x).

It would seem important, therefore, to note carefully those few places where Piaget does address educational issues directly. In these cases, he makes strong, human statements.

"The principal goal of education is to create men who are capable of doing new things, not simply repeating what other generations have done - men who are creative, inventive, and discoverers. The second goal of education is to form minds which can be critical, can verify, and not accept everything they are offered. The great danger today is of slogans, collective opinions, ready-made trends of thought. We have to be able to resist individually, to criticize, to distinguish between what is proven and what is not. So we need pupils who are active, who learn early to find out by themselves, partly by their own spontaneous activity and partly through material we set up for them; who learn early to tell what is verifiable and what is simply the first idea to come to them."

(Conference at Cornell University, 1964,
see E. Duckworth, 'Piaget Rediscovered'
Journal of Research in Science Teaching.
2 (1964) 172-175.)

"Affirming the right of all human beings to education is to take on a far greater responsibility than simply to assure to each one reading, writing, and arithmetic capabilities; it is to guarantee fairly to each child the entire development of his mental faculties and the acquisition of knowledge and of ethical values corresponding to the exercise of these faculties until adaptation to actual social life. Moreover, it is to assume the obligation - keeping in mind the aptitudes and constitution that each person possesses - of not destroying or spoiling those possibilities that he may have that would benefit society first of all, or of allowing the loss of important abilities, or the smothering of others."

(Piaget, 1974(a), pp. 53-54)

With respect to mathematics education in particular, Piaget feels that very frequently new math is being taught ineffectively because old and inappropriate methods of instruction are being used.

"A lot of the mathematics which is being taught now is modern math, but it is often still taught with very archaic methods. And, at least in Europe, nothing at all is done to develop an experimental frame of mind. Experiments are performed in front of the child, but the child is not the experimenter ... it is important that teachers present children with materials and situations and occasions that allow them to move forward. It is not a matter of just allowing children to do anything. It is a matter of presenting to the children situations which offer new problems, problems that follow on from one another. You need a mixture of direction and freedom."

(Evans, 1973, p. 53)

It seems feasible to think that the sophisticated mathematics used in Piaget's theories makes understanding difficult for the average North American reader. It is possible that most psychologists are fully aware of the technical implications of phrases like "the INRC group". It seems likely, however, that statements such as the following would baffle even the most diligent social scientist:

"The logician must now formalize structures suitable to these successive stages, preoperatory structures (without reversibility, transitivity, or conservations but with qualitative identities and oriented functions, likewise qualitative with corresponding but quite elementary, trivial

MacLane type of 'categories') or operative structures (with 'group' or 'groupoid' characteristics)."

(Piaget, 1972(a), pp. 9-10)

Education is at present being extensively criticized. It is crucial that the lay public realize what it is that schools are attempting to do. We should not be castigated for "low scores" if the public, accustomed to basketball, sees the results of a hockey match. And, in a real sense, the rules of the game have been changed much more than many people realize. Perhaps the major influence in this change has been the work of Jean Piaget. Teachers have a professional obligation, therefore, to acquaint themselves fully with this work. If we succeed, we may be able to extend this research to answer some pressing questions. The Piagetian position needs, in almost all disciplines, extensive development of curriculum materials. The formal operations stage is badly in need of investigation as is the whole non-rational side of child development. If we fail, many children may, through exposure to repressive and reactionary methods, never have a real chance to develop emotionally and intellectually.

It is the intention of the following list to indicate some starting points in the massive Piagetian literature. It is hoped that it may assist a few educators in their attempts to understand this remarkable man and his most important theories.

A Brief Guide to the Piagetian Literature for Mathematics Educators

(Where appropriate, the International Standard Book Number and an **approximate** price have been included.)

Non-Technical Introductions:

Hall, Elizabeth. A Conversation with Jean Piaget, **Psychology Today,** 1970 (May), 25-32.
Elkind, David. Giant in the Nursery - Jean Piaget. **New York Times Magazine,** May 26, 1969.
Evans, Richard I. **Jean Piaget: The Man and His Ideas: A Dialogue with Piaget.** New York: Dutton, 1973, Paper. Bibliography. 0525473602 ($4.00).

General Overviews:

Ginsburg, Herbert and Opper, Sylvia. **Piaget's Theory of Intellectual Development: An Introduction.** Englewood Cliffs, New Jersey: Prentice Hall, 1969. paper. 013493017X ($6.00).
Piaget Jean. **The Child and Reality: Problems of Genetic Epistemology.** New York: Viking Compass, 1974. Paper. 0670004081 ($2.35).

Introduction to Children's Mathematical Thought:
Piaget, Jean, How Children form Mathematical Concepts. **Scientific American,** 1953 **189** (5), 74-79.
Isaacs, Nathan. **A Brief Introduction to Piaget.** New York: Schocken, 1974. Paper. 0805204563 ($3.50).
Copeland, Richard W. **How Children Learn Mathematics: Teaching Implications of Piaget's Research.** Second Edition. New York: Macmillan, 1974. 0023247509 ($11.50).

Empirical Studies:

Inhelder, Barbel and Piaget, Jean. **The Growth of Logical Thinking from Childhood to Adolescence: An Essay on the Construction of Formal Operational Structure.** New York: Basic, 1958. Paper. ($4.50).
Inhelder, Barbel and Chipman, Harold H. (Eds.) **Piaget and His School: A Reader in Developmental Psychology.** New York: Springer-Verlag, 1976. paper. Bibliography. 0387072489 ($15.00).

Piaget on Education:

Piaget, Jean. **To Understand is to Invent: The Future of Education.** New York: Viking Compass, 1974(a), 0670005770 ($2.60).

Piaget, Jean. **Science of Education and the Psychology of the Child.** New York: Viking Compass, 1971. paper. 0670003115 ($2.50).

Piagetian Curricula:

Furth, Hans, G., and Wachs, Harry. **Thinking Goes to School: Piaget's Theory in Practice.** New York: Oxford, 1975. paper. 0195019273 ($4.50).

Piagetian Theory and Mathematics:

Rosskopf, Myron F. **et al.** (Eds.). **Piagetian Cognitive-Development Research and Mathematical Education.** Washington: National Council of Teachers of Mathematics, 1971. Paper ($5.00).

Beth, Evert W. and Piaget, Jean. **Mathematical Epistemology and Psychology.** Dordrecht-Holland: Reidel, 1966, ($30.00).

Quintessential Piaget:

Piaget, Jean. **Biology and Knowledge: An Essay on the Relations between the Organic Regulations and Cognitive Processes.** Chicago: University of Chicago, 1971. Paper. 0226667758 ($3.95).

Piaget, Jean. **Psychology and Epistemology: Towards a Theory of Knowledge.** New York: Viking Compass, 1972(a). Paper. 067000362x ($2.50).

Interdisciplinary Piaget:

Piaget, Jean. **Structuralism.** London: Routledge & Kegan Paul, 1971. paper. 0710070446 ($3.00)

Biographical Piaget:

Piaget, Jean. **Insights and Illusions of Philosophy.** New York: World Meridan, 1971. Paper. ($3.95).

RECOMMENDATIONS FOR INTERMEDIATE DIVISION MATHEMATICS IN ONTARIO;
Based Upon a Study of Local and International Programs (Final Report);
PATRICK BABIN, Principal Investigaor; PAUL POGUE, Project Director, ROBERT KNOOP, Research Officer.
This research project was funded under contract by the Ministry of Education, Ontario, 1976. 180 pages, $4.00. Available from the University of Ottawa Press, 65 Hastey Ave., Ottawa, Ontario, K1N 6N5. (613-231-2270)

ABSTRACT (Prepared by the Authors)

Relevant educational theory and applicable local, national and international mathematics programs were reviewed and used as a basis for making recommendations for the mathematics program of the Intermediate Division.

A curriculum model, adapted to school mathematics, served as a rationale for this study. The model consisted of these five variables: goals and objectives, program content, learning experiences, approaches and methods, and evaluation procedures. The variables provided also the structure of the report.

It was recommended that the major goal of intermediate mathematics education be to provide the learner with mathematical concepts, facts, and skills, that develop his ability to think logically, through interdisciplinary problem solving, to enable him to function effectively in a technological society. Specific objectives were developed that stressed the cognitive, affective, and psychomotor domains.

Concerning program content, recommendations were made that (a) place a strong emphasis on statistics and probability, flow charts, computer awareness. estimating and approximating, transformations, and measurement; (b) advocate an earlier introduction of relations, functions, and trigonometry; and (c) advocate a deletion of division of polynomials, factoring, rational algebraic expressions, complicated word problems, and the heavy emphasis on structure. A series of

detailed content recommendations was given in the form of a content matrix.

A varied succession of learning experiences was recommended that stressed relevant, real world, interdisciplinary and application-oriented problem-solving activities and the use of mathematical models, games, constructions, flow charts, and calculators.

It was strongly recommended that the problem-solving approach be made the predominant approach and that varied and diverse teaching methods be used that emphasize an activity-centred learning process that is relevant and realistic, of interest to the student, and that encourages free inquiry and good thinking.

Evaluation procedures were advocated that applied not only to content but also to learning experiences, approaches and methods, and to the appropriateness and achievement of objectives. Specifically, it was recommended that evaluation of a mathematics program that stresses a problem-solving approach be a continuous, ongoing process employing a variety of methods which result in an evaluation profile that strongly emphasizes doing, experiencing, and participation.

To implement the recommendations, it was suggested that the Ministry use the vehicles of in-service training, teacher education, and selection of suitable textbooks.

Editors Note: This important report will be reviewed in detail in the March issue of the Gazette.

DIVERSIONS

BY T. G. Friday

Game HEX

The two player game of Hex can be played by children of all ages up to and including those of at least 92. It has the great advantage of being extremely simple to understand, and still be playable on many different intellectual levels.

The board consists of a tesselation of regular hexagons, with from 5 to 11 on a side, although a 5 sided board produces a trivial game. Two different coloured counters are required, one for each player. The object is to produce a chain of one coloured counters from one side to the opposite side of the board while preventing the opponent from building his chain in the opposite direction. The players construct their chains by each in turn placing a counter on the board, which counters remain in that position for the rest of the game.

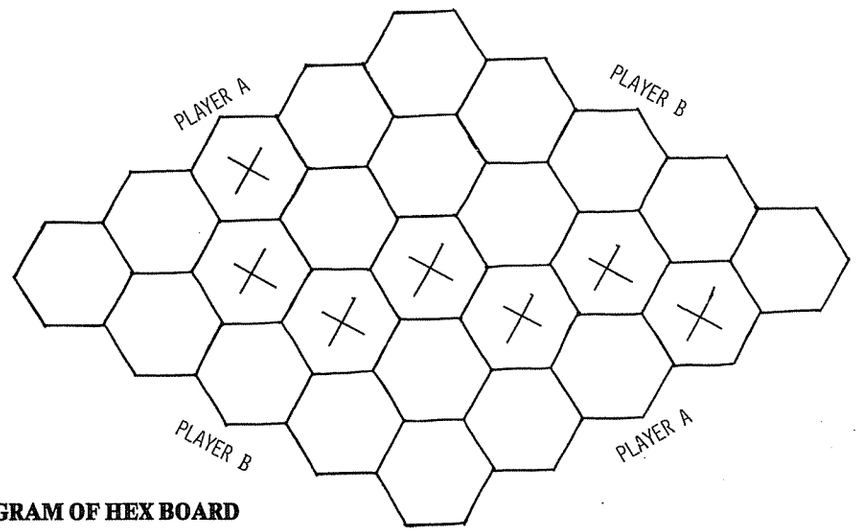


DIAGRAM OF HEX BOARD

The Four Colour Problem

The hypothesis of this problem is that no more than four colours are needed to shade any map so that no two adjoining countries are the same colour. This famous problem, first stated in 1853 by a London graduate student Francis Guthrie, has at last been solved.

Mathematicians Kenneth Appel and Wolfgang Haken, by reducing a map to a graph and considering all possible situations, have offered a proof of this long standing theory. The most interesting aspect of their proof is that they used 1200 hours of computer time. I really cannot see this proof becoming standard undergraduate bookwork.

Rene Descartes Medals

Congratulations to our fellow O.A.M.E. members Arnold Harris and Paul Sherk. Professor Harris is the editor of this journal, and Mr. Sherk is an O.A.M.E. councillor.

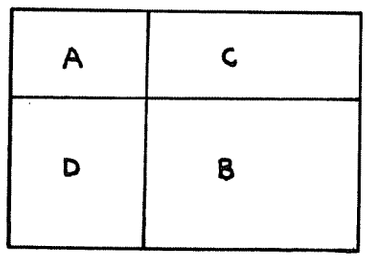
Rene Descartes medals are awarded each year by the University of Waterloo to school mathematicians who have exhibited excellence in classroom teaching and mathematical leadership.

PROBLEMS

15-2-1 Junior: - Up to grade 9.

A man must transport a fox, a goose and a sack of corn over a river in a boat which will hold only him and one of the fox, goose or sack of corn. He cannot leave the fox and goose, or the goose and corn alone together or one will eat the other. How does he accomplish his task?

15-2-2 Intermediate:-
If a rectangle is divided as shown in the diagram, prove that A.B=C.D where A, B, C and D are the areas of the sections in which they lie.



15-2-3 Senior:-Canadian Olympiad Paper 1970

Find all the ordered triples (x,y,z) such that when any one of these numbers is added to the product of the other two the result is 2.

THE FAIR TICKET

Mr. Cash plumped his money on the counter and received in exchange a ticket entitling him to enter the Fair Grounds. "Interesting," he muttered to himself as he examined the number stamped on his ticket; "it has five different digits, the first pair and the last pair are squares, so is the middle digit, and it isn't one either. Then too, the whole number is a square, and its root is a prime number and a palindrome at that. I am sure this is unique." "Yes, Mr. Cash, it certainly is unique." Can you find out what the number of the ticket was?

ONTARIO ASSOCIATION

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MATHEMATICS EDUCATION

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PRESIDENT'S MESSAGE



You, the individual member are the most important person in O.A.M.E. It is through you and only through you that the learning situation of our students can be improved, and that is our prime objective. It is also through you that other teachers of mathematics who are not members will become aware of our objectives and hence hopefully help us to improve mathematics education. Remember that you will have influence on a large number of future teachers during your career, and they in their turn will have influence on many more. Your excellent work will have wide repercussions.

Please let us assist you in this work. If there is any way we can provide information, speakers for conferences, or any other aid, write, phone or send a pidgeon to someone on the executive or council or contact your local chapter.

Futhermore you have good ideas. Let others hear about them. Write articles for our publications and pass them on to your colleagues by word of mouth. Become a councillor and have even greater influence. The opportunity is there for you to become involved and our organisation is involved with some important work. We are at the moment attempting to influence the faculties of education to offer even better mathematics courses for elementary school teachers. We have been requested by the ministry to give our recommendations for the Interface study, and to take an active part in preparing a new curriculum for grades 7 to 10. You can assist in this work and have your influence felt.

For myself, one of my main interests at this time is to provide a mathematics education for each student at his or her own ability level. This applies especially to the most able students, who I fear are in many cases not having extended to their full potential. These students are our ideas people of the future, and as a country we must have use of their full ability. Also for their own satisfaction they must feel that they are being given the opportunity to excel. Further I am convinced that if we improve the learning standards of the top students, this will in turn positively influence the whole system.

Our organisation is going from strength to strength, and our influence is at present increasing rapidly. To conclude I reiterate that this strength and influence comes from you the individual member.

Tom Griffiths
President

SEVEN CHAPTERS OF OAME

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Representative - L. Lefebvre, P. O. Box 4, Casselman, K0A 1MO

COMA

Chairman - Steve Thompson, 6 Donna Street, Ottawa, K2G 2V4
Representative - Nettie Sharkey, 26 Elliot Ave., Ottawa, K1S 0M4

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N.O.M.A.

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Representative - Andy Czempinski, Box 369, Iroquois Falls, P0K 1E0

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President - Dr. Hugh Allen, Fac. of Ed., Queen's University, Kingston
Representative - Jim Whiteley, Queen's University

SWOAME

President - Gary Peters, Chatham C.I., Chatham

CHAPTER ITEMS

AMEO/EOMA: An evening conference on mini-calculators is planned for April. Details will be announced later.

THE GOLDEN SECTION: **The Golden Section** is the newest chapter of O.A.M.E. The geographical area covered by this chapter includes the counties of Halton, Wentworth, Lincoln, Niagara, Brant, Welland, Haldimand, and Norfolk. The steering committee of this chapter has decided to stress the fraternal aspects of such an organization rather than the "professional development" aspect. We plan to meet three times a year. Each meeting will have a keynote speaker - not necessarily a mathematician. Much of the time at these meetings will be devoted to sharing ideas with other teachers. It is planned that one meeting a year will include spouses. We hope that this format meets with your approval and that you will support this new organization. More details regarding **The Golden Section** will be presented at our first meeting, 1976 11 23, in the Great Hall of the Faculty Club, McMaster University, Hamilton.

NOMA: A very successful conference was held on October 2. For details, contact Jeannette Bedard.

SWOAME: The annual conference has been rescheduled for Saturday, 1977 03 05 (also known as March 5, 1977) at University of Windsor. The principal speaker will be Dr. A. J. Coleman, Queen's, well known to the mathematical community as past president of the Canadian Mathematical Congress and O.M.C. author of text books and **The Mathematical Sciences in Canada.** For additional information, contact Dr. Alan Gold, Department of Mathematics, University of Windsor.

A TOTAL SUCCESS!
The 1976 Leadership Seminar

Bruce McFarlane, Principal, Bridgeport Public School, Kitchener, Ontario.

I have been asked to share with the readers of the Gazette, my experiences during this year's O.A.M.E. leadership seminar held at the Ascot Inn, Toronto, October 20-23.

This is an intrinsically subjective report. I offer no apologies whatsoever for the superlatives contained herein.

This seminar, the fifth of its kind, was a tremendous success. Co-ordinators Mr. Morley MacGregor of Sarnia and Mr. Bruce Jay of Niagara Falls are to be commended for their fine efforts in organizing and overseeing this worthwhile conference. These gentlemen met the needs of all who were in attendance.

Participants at the seminar were rewarded with a programme that was interesting, informative, creative, motivating and above all - practical.

It has often been stated that one can best demonstrate a leadership style by example. The group leaders responsible for the uniquely interesting and eye opening "Monday Morning Specials" did just that. The various topics included Geometric Reflections, Problem Solving Techniques, Probabilities, Fun with Numbers, The Mathematics Box and The Language of Mathematics. Mary Aggus (Hamilton), Audrey Baird (Mississauga), Bill Jeffers (Oshawa), Shirley McIntyre (East York), Bruce Schmitt (Waterloo) and Neil Williamson (North York) exemplified what this conference was all about - leadership.

A successful conference must have, in its repertoire, several motivating forces. I have already alluded to the group leaders. Mr. MacGregor and Mr. Jay also provided the participants with several exciting guest speakers who very capably held the interest of their audience - not necessarily an easy task on the morning following the night before!

Included as members of this team were: Art Dayman, Waterloo (Leadership Aspects and Skills), Bob Robinson, Hamilton (Resources in Mathematics), Leroy Callahan (Leadership in Mathematics) and Don Fraser, Toronto (How to Run a Workship). Did we really only have three in agreement Don?

The conference concluded with three excellent presentations on evaluation processes in mathematics. Larry Axcell (Peterborough), Shirley Robinson (Lincoln County) and Ed Baumgart (Waterloo County) represented their respective Boards admirably.

As I reflect back through the days October 21-23, I would attribute the success of this particular conference to many factors. It was extremely well organized. It was practical. It was a positive personal experience. It had a sense of direction. Leadership styles based on positive interpersonal relationships were in evidence throughout. Quite simply - it was a fun learning experience.

This article represents one person's view. A view, however, shared by all of those fortunate enough to have had the opportunity to attend. I would strongly suggest that all Boards in the Province continue to support the Ontario Association for Mathematics Education by sending as many representatives as financially possible to future conferences.

INTERNATIONAL MATHEMATICS EDUCATION

N.C.T.M. IN ONTARIO: For many years Canadian Mathematics teachers have been interested in the National Council of Teachers of Mathematics, subscribing to that organization's excellent magazines and even on occasion taking leadership roles in the organization. This has culminated in 1976 with our own John Egsgard as president of N.C.T.M. Over 2000 Canadians now belong to N.C.T.M., 700 of those being from Ontario. Also, Ontario teachers have contributed over $2300 to a fund for the new N.C.T.M. building in Reston, Virginia.

O.A.M.E. has a working relationship with N.C.T.M. We jealously guard our independence, as we should. In no sense is O.A.M.E. an affiliate of N.C.T.M. We are friends and partners in an excellent cause.

In order to assist our liaison with N.C.T.M., O.A.M.E. appoints representatives to that organization. These are listed below.

N.C.T.M. 55TH ANNUAL MEETING: Cincinnati, Ohio, 20-23 April, 1977. O.A.M.E. members wishing further information should contact their local N.C.T.M. representative.

NCTM Representatives Appointed by OAME '76-'77

Revised 1976 11 03

Jeannette Bedard, 35 Victoria Street, Swastika, Ontario, **NOMA**, Midnorthern Ontario (MN), Northeastern Ontario (NE)

Jean Bennie, 88 Leslie Avenue, Hamilton, Ontario, L9C 1M3, **Golden Section,** Niagara (NG)

Sharon McPhail, 1250 Wiltshire Drive, Sarnia, Ontario, N7S 3Z7, **SWOAME**, Western Ontario (WE).

Doug Neil, 276 Summit Drive, Wingham, Ontario, N0G 2W0, **GVMA,** Midwestern Ontario (MW)

Paul Pogue, 43 Cynthia Court, Barrie, Ontario, L4M 2X3, Central Ontario (CE)

Louis Rancourt, 75 Laurier East Street, Ottawa, Ontario, K1N 6K9, **AMEO/EOMA, COMA,** Ottawa Valley (OV)

Edwin Silcox, 8 Riverstone Drive, Weston, Ontario, M9P 2R6, Central Ontario (CE), (Toronto)

Robert Tuck, 247 Lee Drive, North Bay, Onatrio, P1A 3J2, **NOMA**, Midnorthern Ontario (MN), Northeastern Ontario (NE)

Dr. G. Vervoort, Lakehead University, Thunder Bay, Ontario, P7B 5E2, Northwestern Ontario (NW)

David Wing, 6 Crerar Boulevard, Apt. 1, Kingston, Ontario, L7M 3M6, **Quinte,** Eastern Ontario (EA)

Ontario Chairman - Bob Robinson, Mathematics Department, Hamilton Board of Education, 100 Main Street West, Hamilton, Ontario.

MEMBERSHIP NOTICE

There are still a number of 1975-76 members who have not renewed their membership. In anticipation of some late renewals a small number of December 1976 Gazette and Abacus were ordered from the printer. Please send in your renewal using the form below at once. The publications will be distributed on a first come basis.

If OAME Membership is going to increase we ask that if you are a member please encourage colleagues to use the renewal form and become a new member. The Association, as in past years offer any new member joining after Jan. 1, 1977 membership privileges and publications till August 31, 1978.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
MEMBERSHIP APPLICATION

DATE .....

Mr. ☐ Mrs. ☐ .....
Ms. ☐ Miss ☐ (Please print) NEW MEMBER ☐

APARTMENT No. ..... RENEWAL ☐

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Chapter Affiliation
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PLEASE ENCLOSE CHEQUE OR MONEY ORDER
FOR $10.00, PAYABLE TO OAME

Important Reminder

If members wish charter seals to attach to the Membership certificate. Please detach or copy the form below and send at once.

I desire an

OATMP seal_____ OATM seal_____ OAME seal _____

check on one line one seal per member.

_____
Signature

YOUR ASSISTANCE IS REQUIRED

At the Annual Meeting of OAME in Ottawa in May, 1977 we shall elect the following: 1 President-elect, 3 Executive Members (Directors), about 10 Councillors. If you would like to fill one of these positions or wish to nominate someone else please copy the following.

I nominate ..... (name)

..... (professional address)

for the position of ..... (president-elect, director, councillor)

Name of Nominator .....

Professional address .....

Note: 1. This nomination should be accompanied by information supporting the nomination.
2. A summary of the qualifications of those members who are allowing their names to stand in nomination will be published with the March Gazette.
3. Return to: Mrs. Joan Routledge,
Box 194, Aurora, Ontario
L4G 3H3
by March 1, 1977.

Using the facts given can you determine the names of the men playing the various positions on the baseball team.

Andy dislikes the catcher. Ed's sister is engaged to the second baseman. The centre fielder is taller than the right fielder. Harry and the third baseman live in the same building. Paul and Allen each won $20 from the pitcher at pinochle. Ed and the outfielders play poker during their free time. The pitcher's wife is the third baseman's sister. All the battery and infield, except Allan, Harry and Andy are shorter than Sam. Paul, Andy and the shortstop lost $50 each at the racetrack. Paul, Harry, Bill and the catcher took a trouncing from the second baseman at pool. Sam is undergoing a divorce suit. The catcher and the third baseman each have two children. Ed, Paul, Jerry, the right fielder and the centre fielder are bachelors. The others are married. The shortstop, the third baseman and Bill each cleaned up $100 betting on the fight. One of the outfielders is either Mike or Andy. Jerry is taller than Bill. Mike is shorter than Bill. Each of them is heavier than the third baseman.

Construct a chart to help you solve this puzzle in an orderly fashion.

Canadian Educational Showplace, Automotive Building, Exhibition Place, Toronto, February 22 - 24, 1977.

SUBSCRIPTION RATES

$5.00 FOR ONE YEAR'S SUBSCRIPTION
$2.50 FOR A SINGLE COPY

O.A.M.E. members receive the Gazette as an adjunct to their membership.

Invoices will be sent 30 days prior to expiry of a subscription, and 60 days is allowed for payment.

Orders for subscription and single copies should be sent to the Secretary-Treasurer.

M. B. MacGregor
247 Bright Street
SARNIA, Ontario
N7T 4F9

REVIEW OF SENIOR DIVISION CURRICULUM

The Council of OAME invites responses to the report of the Committee published in the September, 1975 Gazette. They should be directed to Neil Williamson, 1754 Walnut Lane, Pickering, Ontario. L1V 2X3.