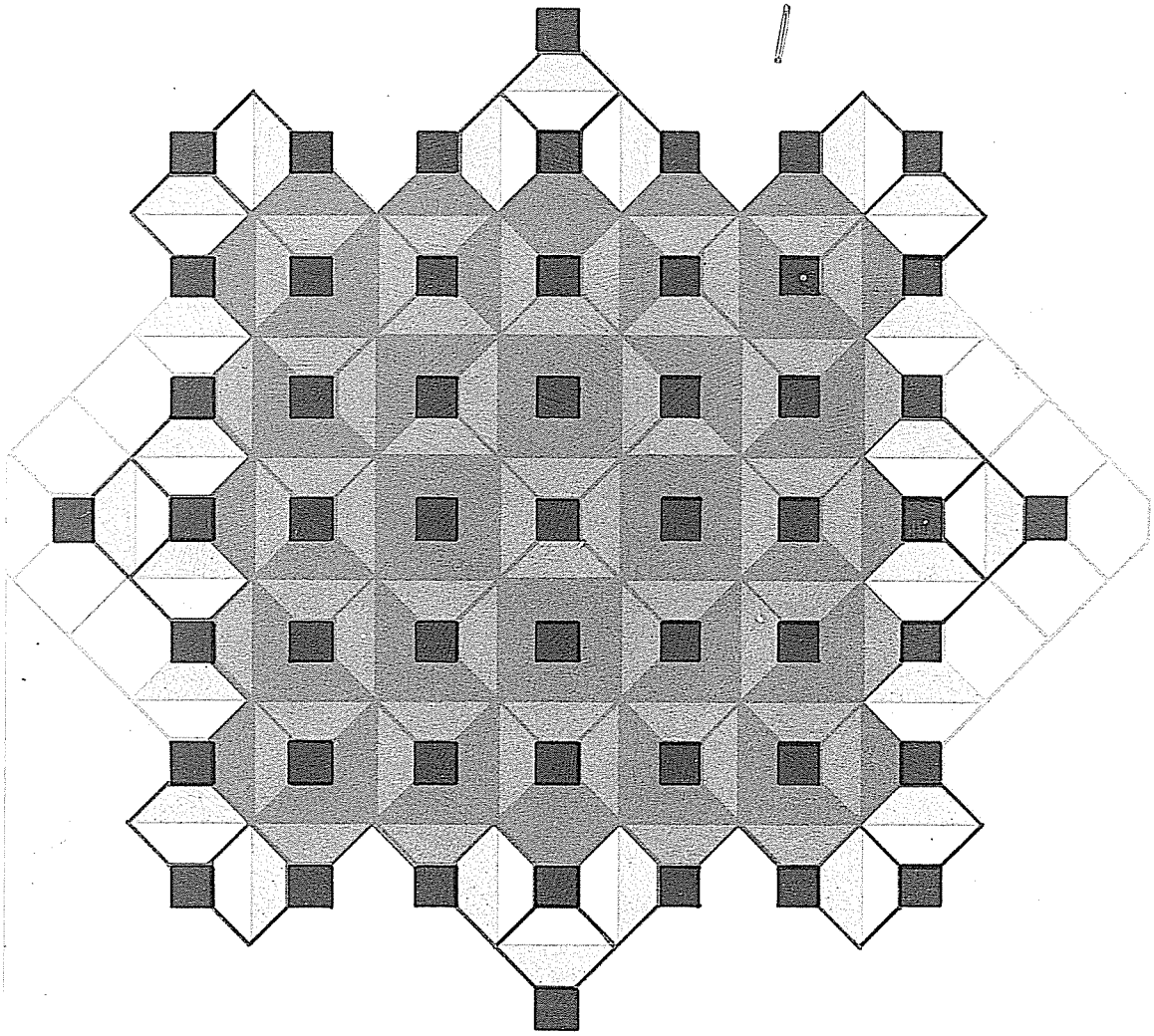
ontario mathematics gazette

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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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EDITORIAL

By the time this issue of the Gazette reaches your desk, another school year will be well under way. May it be a **good** year for you; here's hoping that those shining faces retain some of their lustre at least until Thanksgiving.

Perhaps we should dedicate the September issue to an invisible group who may never read this editorial: the large number of young mathematics teachers across the country who cannot find work. This waste of talent and resources makes a former faculty of education person very sad. What is the answer? For that matter, what is the question? Some of the lucky ones ought to take time to study the question of teacher supply and speak out, to O.A.M.E. Council, to the Federations and to the provincial government.

During 1977-78, the controversy over what and how to teach will continue, especially in the Intermediate Division (grades 7-10). These curriculum questions are legitimate and complex. However, they should not be allowed to divert attention away from the central role of the teacher. Your example of scholarship, your personality and character, your ability to organize effectively, your ability to get along well with your students, ("You can lead a horse to water, but you can't make him drink" is corny, but has some truth) and your stock of proven teaching techniques - all these things are vitally important to the students, who continue to be the reason why we are gathered together.

Some would say that we teach children; I claim we teach children mathematics. Now mathematics is a varied discipline; it is easy to misrepresent its nature by lack of care in selection of material. (Do you remember the fable of the blind men and the elephant?) It seems to me there are about six main facets of the subject which ought to be emphasized in every course, at a level appropriate to the needs and abilities of the student.

Mathematics is, in part, a language; its vocabulary has to be learned, even over-learned. This requires practice, even **drill.** The concept of proof is a legitimate concern - after all it was invented by geometricians. Bertrand Russell defined mathematics as the study of relationships; our curriculum ought to reflect this insight. We must remember that for many of our students mathematics is a tool - of the sciences, business, the home - and that the applications give the subject life. Problem solving is central; in a real sense the techniques of the discipline have been invented in the search for solutions to problems. Let's not forget heuristic. Finally, mathematics is a GAME and can provide FUN. The recreational aspects of the subject should never be forgotten.

Sounds just too complicated, doesn't it? Remember the old song: "I never promised you a rose garden!"

Arn

WHAT'S NEW AT THE GAZETTE

The Executive and Council of O.A.M.E./A.O.E.M. have appointed an Editor-Designate. He is Professor Robert S. (Bob) Smith of the Department of Mathematics and Science, Faculty of Education, University of Western Ontario. Professor Smith will be assuming the editorship in 1978, for a two-year period. His **Taped Talks** has been a regular feature for the past year.

Welcome aboard, Bob! The nautical jargon is a concession to his hobby of sailing. He was for several years a mathematics master at London Teacher's College, now Elborn College of U.W.O.

You will note that we have assumed a bilingual character, using the new name of the sponsoring organization. It will be our policy henceforth to have at least one article en français in each issue, commencing in December, 1977.

Our printer, J.B. Pole Printing, moved into fine new quarters in August. Congratulations to Jamie and his excellent staff. If you should have the occasion to visit Forest, why not drop in?

What do you think of our cover? The first person to discover the significance of the colour scheme will receive an old Syl Apps sweatshirt (or Russ Jackson) as an award.

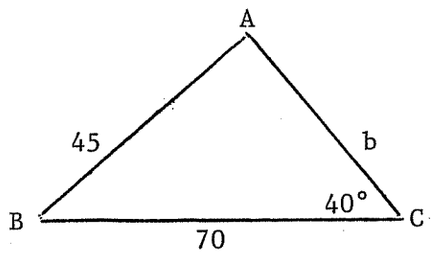
LETTERS TO THE EDITOR

Laurier S.S.,
London, Ont.

Dear Arn:

I thought the readers might be interested in this "solution" given by one of my students, which resulted in the correct answer. Does anyone else have a good original "boner"?

Barbara Palmer,
Mathematics Department



$$\cos 40^\circ = \frac{b}{70}$$

$$b = .70 \times \cos 40^\circ$$

$$b = 70 \times .7660 = 53.62$$

$$\therefore b \doteq 54$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - 2bc}{a^2}$$

$$= \frac{2916 + 2025 - 2(54)(45)}{4900}$$

$$= \frac{81}{4900}$$

$$= .0165$$

$$\therefore \angle A = 89^\circ$$

WESTMINSTER CLOCK

Readers will have learned recently that the famous "Big Ben" attempted to take his own life, without success we are happy to say. He is now restored to health and should tick along for at least a couple of centuries. The following problem is taken from **536 Puzzles and Curious Problems** by Henry Ernest Dudeney (Scribner 1967).

"A man crossed over Westminster Bridge one morning between eight and nine o'clock by the tower clock (often mistakenly called Big Ben, which is the name of the large bell only, but this by the way). On his return between four and five o'clock he noticed that the hands were exactly reversed. What were the exact times that he made the crossings?

The answer may be found elsewhere.

INTRODUCING MEASUREMENT TO PRIMARY CHILDREN

Susan Lakaski

This article was submitted by Ms. Susan Lakaski, previously with the Hamilton Board, who is now living in Mexico. The approach that Ms. Lakaski describes in the article is one that she used in Readiness through to level 3. It is to be understood that the length of the unit is subject to the class experience in measurement and their capabilities.

Sharon McPhail

SKILLS: - estimation of size;
introduction and use of a measuring tool;
using non standard units of measurement;
developing accuracy;
introduction and use of measurement terms - length, height, width, taller, shorter, wider, longer, etc.

To initiate the study of measurement with primary pupils they must be presented with a practical problem, "How long should we make the paper to be put on the wall if it is to go from the floor to the chalk trough?" Even little children respond with the idea, "Measure it!", and they usually suggest the use of a ruler, since this term is heard at home.

In preparation for the standard unit of measure, suggest to the children that if there were no rulers in the entire world "what could we use?" This will allow students to invent many non standard units of measure that are totally their own creation and are personal to them, hopefully to encourage the pupil to do his best with his unit.

The students are then given the task of measuring the length of the various items in question using some of the non standard units that they have created. It is at this time that the method of measuring is brought to the attention of the class. The correct form of stating results is also emphasized.

(The desk is six hands wide).

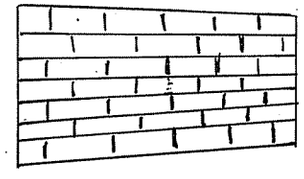


no fingers allowed

The children are then allowed to choose from a variety of measuring tools. (Hide all rulers!!) They work from activity cards and record their results in a record book. Through these activities, the method of measuring is reinforced.

How long is the wall?

The wall is .....long.



Continued use of the activity cards is refined to the extent that each child must use the same measuring tools. (crayons, straws, pencils, etc.) A record is kept by a teacher to illustrate the cards each child has completed. Results are checked for accuracy. It is now that we draw to the attention of the class the appropriateness of the tool for the job. (Using a toothpick for measuring the hall, for example, would hardly be realistic.)

The need for standard measure may now be present. This concept is easily introduced through the children themselves. Having children with varied sizes of hands measuring the same objects will yield many different answers. The children might then invent a single measuring tool for their class, giving it a name, and all pupils will use this device. The activity cards are then used again with their standard units this time and results are again reviewed.

(i.e.)

PIFFLE

POOF

The use of the piffle and poof will be adequate for the children within their own classroom; however we must bring the children to the realization of the need for a **universal standard.** This can be approached through sending the children to different teachers (pre-arranged please!) for a paper 3 poofs X 1 piffle. The following story might be used with the students when the results from the other classes is other than satisfactory. (Embellish to please! I don't remember how much, if any, is true . . . .)



THE METRIC KING

Once upon a long time ago, in a far away land, there lived a giant King. He lived in a castle and was quite happy there. One day he decided to have a big party. He decided he wanted a hundred people to come. But he had a problem -- all the rooms in his castle were too small. He thought and planned and pondered. "I know what to do! I'll have the Royal Builders make a big, big room!"

The King measured 400 footsteps long and 600 footsteps wide and decided that that was a good size for his new room. He called the Royal Builders in and told them his idea. "Make it 400 footsteps long and 600 footsteps wide," he commanded. So the Royal Builders started immediately.

They worked and measured and measured and worked. Pretty soon, they had finished the party room. They ran to get the King. The King hurried down to see his big new party room. When he saw it he was shocked! The room was far too small! He shouted! He stamped his feet! The poor Royal Builders were frightened.

When the King calmed down, he asked, "Why is this room so small?" "But, but!", said the Royal Builders. "You said 400 footsteps by 600 footsteps and it is! It is!" "Prove it!" roared the King. So the poor Royal Builders counted out loud -- 400 steps -- "See!"

"Oh, no!" said the King. "I see the problem now."

At this time show the children a picture of the **giant** King and the Royal Builders.

"Well from now on," said the King, "Everyone must use exactly the same measuring tool. Then this won't happen again."

So the wise King invented a METRE STICK.

And they all measured happily ever after.

At this point the metric measuring unit is introduced using the standard units of measure: metre, centimetre.

NOTES:

(1) It is to be noted that each step indicated is actually an introduction to the following step and should be extended and expanded to allow for mastery.
(2) I teach measurement the same way as always, except that now my universal standards are METRIC.
(3) WHAT CAN WE MEASURE?

walls	halls	rugs	bodies
shoes	gym	desks	snowsuits
floors	tables	chairs	teachers
doors	books	toys	principals
papers	shelves	gym mats	blackboards
etc.	etc.	etc.	etc.

(4) For Further Reading:

Bates, Clifford and Park. **Exploring Metric Measure.** McGraw Hill.

IDEAS FROM THE MAY O.A.M.E. CONFERENCE

Barbara Kaiser, Faculty of Education, University of Western Ontario

Are you looking for something different, as a quick addition drill, or a longer discovery lesson? Try ARITHMOGONS. In a session given by Dr. Derek Woodrow from Manchester, England, he shared his ideas on arithmogons which are suitable for use in almost any elementary grade. In an arithmogon the sum of the end 2 numbers (in circles) must equal the number in the middle of the side (in boxes). Thus in figure 1, $3 + 2 = 5$, $3 + 4 = 7$, and $2 + 4 = 6$.

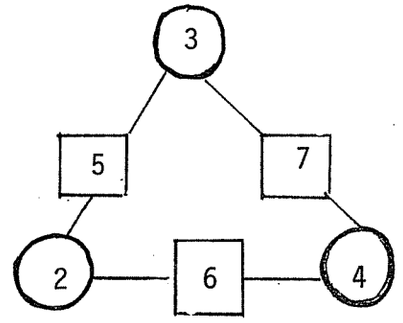


Fig. 1

For use in primary grades, try giving the end numbers and finding the sums (see figure 2). To make the arithmogon more challenging for junior grades, give the sums and try to work out the end numbers. In figure 3, and answers are 7, 9, 8 in a clockwise rotation beginning at the top.

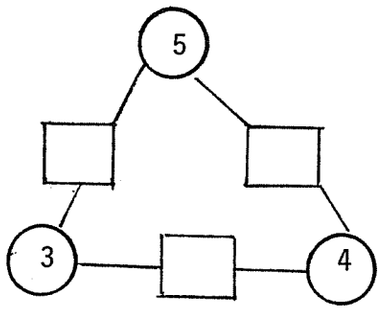


Fig. 2

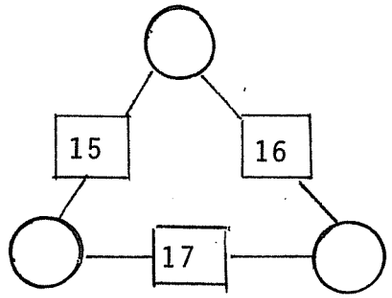


Fig. 3

In the senior grades your arithmogons can involve fractions and integers. In figure 4, the solution must involve fractions. Why? The 3 side numbers have an odd sum. In figure 5, the solution must involve integers. Why? One of the side numbers is greater than the sum of the other two side numbers.

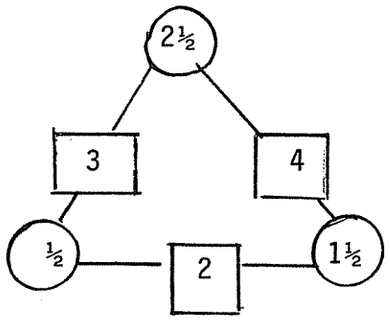


Fig. 4

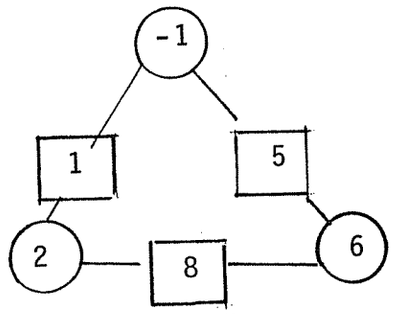


Fig. 5

Do you need a new way to drill basic operations? Dr. Woodrow would suggest HAPPY NUMBERS. 23 is a happy number (see figure 6.) By following the pattern of taking a number and squaring each digit, finding the sum of these squares, and repeating until your sum equals one, you arrive at a happy number. Can you find more? What is an unhappy number?

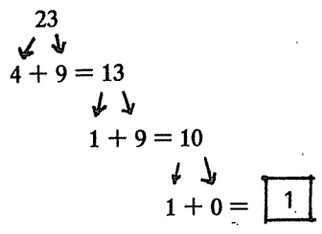


Fig. 6

q.e.d.o.a.m.e.

O.A.M.E.'S PRESIDENTS GET TOGETHER AT OTTAWA



It was 1973 when The Ontario Association of Teachers of Mathematics and the Ontario Mathematics Commission dissolved and The Ontario Association for Mathematics Education was formed. The strength of O.A.M.E. is in part due to the excellent leadership of the officers.

Pictured above are all of the presidents of O.A.M.E. and Don Attridge, the last president of O.A.T.M., who acted as past-president in the first year of the new organization.

Left to right: *Don Attridge, Dave Alexander 1973-74, John Egsgard 1974-75, Joan Routledge 1975-76, Tom Griffiths 1976-77, Neil Williamson 1977-78, Dave McPhail president-elect.*

BRAIN TEASERS

Six ears of corn are in a hollow stump. How long will it take a squirrel to carry them all out, if he takes out three ears a day?

The proprietor of a menagerie was asked how many birds and how many beasts it included. He replied, "Well the lot have 36 heads and 100 feet."

How many of each were there?

The answers may be found elsewhere.

These were obtained from Math Trek, Mathematics Consultants, Waterloo County Board of Education.

NUMBERS I - INVESTIGATING UNDERSTANDING OF OPERATIONS ON WHOLE NUMBERS

Robert Smith, Department of Mathematics, Faculty of Education, University of Western Ontario

This is the second part of an article which began in the March, 1977 issue.

Background

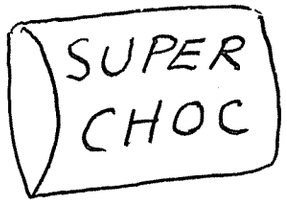
On September 11, 1964, the Nuffield Mathematics Teaching Project was formally announced. Professor Matthews was chosen director and accepted the challenge, after the age range was changed from 8-13 to 5-13. In 1965, Professor Matthews visited Geneva and began a liaison with Piaget which eventually resulted in one of Piaget's researchers coming to Britain to work with the Nuffield writing team. This link also resulted in one of the Nuffield Book series called Check Ups, in which some aspects of conservation are considered. Today, Professor Matthews is still at work, now at Chelsea College, University of London, where he holds the chair of Mathematics Education, and continues his interest in Piagetian theory. Under a grant from SSRC, the Concepts in Secondary Mathematics and Science project was set up in October of 1974, with the aim of investigating the levels of conceptual demand of various mathematical and scientific areas within the secondary curriculum. In the area of mathematics it was decided to look first at numbers and an understanding of the basic four operations among children in the first year in secondary school.

Development

The chief researcher, Margaret Brown and her colleagues, were interested not so much in computation with whole numbers as in whether children could see what operation should be applied in a given situation and conversely, whether, given an arithmetic expression like 9 x 3, they could make up a 'story' for which the expression was a model.

For part of the test, they wrote items like the three in figure 1, some involving small numbers (<10) and others, larger numbers (between 12 and 500).

Figure 1.



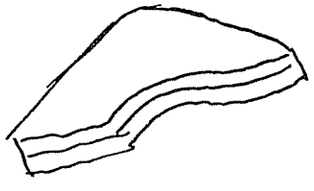
1.

A bar of chocolate can be broken into 12 squares
There are 3 squares in a row

How do you work out how many rows there are?

$12 + 3$ $\quad$ 3×4

12×3 $\quad$ $3 - 12$
$6 + 6$ $\quad$ $12 \div 3$
$12 - 3$ $\quad$ $3 \div 12$



7.

A shop makes sandwiches. You can choose from 3 sorts of bread and 6 sorts of fillings.

How do you work out how many different sandwiches you could choose?

3×6 $\quad$ $6 - 3$

$6 + 3$ $\quad$ $3 - 6$
$18 \div 3$ $\quad$ $6 \div 3$
6×3 $\quad$ $3 + 3$



8.

A gardener has 391 daffodils These are to be planted in 23 flowerbeds. Each flower-bed is to have the same number of daffodils.
How do you work out how many daffodils will be planted in each flowerbed?

$391 - 23$	$23 \div 391$	
$23 - 391$	391×23	
$391 + 23$	$23 + 23$	
23×17	$391 \div 23$	

This material is from a report to CSMS. The tests will probably be published by NFER.

The form of operation involved was varied. For example, the shop item is more like the Cartesian product form of multiplication than to the repeated addition form of multiplication. Again the daffodils item involves partition (sharing among a given number) whereas the chocolate bar item involves measurement (how many shares if they contain 3 each).

The research team then took these trial items out into classes where they interviewed children from all ability levels in their first and second year of secondary school. The children were asked how they would tackle the problem. Sometimes the pupils were asked to go so far as to carry out the steps if their strategy was not clear. Only then were they asked to choose an expression from the sets shown on the right in figure 1, which were on the reverse side of the problem cards.

The interviews were taped, later transcribed and classified. The replies to the 'problem' items seemed roughly to fall into five different levels that the team identified in the following manner.

Figure 2.

STAGE		
1.	Intuitive	idiosyncratic answers or guesses
2a:	early concrete	successful strategies but mostly based on addition and subtraction.
2a/b:	middle concrete	multiplication and division were sometimes recognized. The sandwich item likely to cause trouble.
2b:	late concrete	fairly clear identification of multiplication and division, although often the inverse was chosen. e.g. 3 x 4 for the chocolate item.
3a:	early formal	generally very quick answers and a recognition of non-commutativity of subtraction and division.

The following are examples from the tapes with I standing for the interviewer and the initials of the child as identification.

I. Stage 3a: **Early Formal**

L.D. How many 23's go into 391 ...
I. Which one of these would ...? (showing back of card)
L.D. This one (391 ÷ 23)

2. Stage 2a: **Early Concrete**

I. (reads question) What would you do with those two numbers to work it out?
S.B. 23 there they take away 23, 23, 23 ...
I. Keep on taking 23 away?
S.B. Yes.
I. Which one of these would be right? (Back)
S.B. That one - 23 take away 391
I. Any others?
S.B. That one - 391 take away 23

The interviewers were interested to find that almost all children could produce successful strategies for solving problems, even when they did not recognize the operations involved. Complementary addition (subtraction by "adding on" in the "shopkeeper's fashion") was successfully used mentally, even when large numbers were involved. Sometimes ingenious forms of repeated addition and subtraction were employed by many who, although recognizing the problem required multiplication or division, confessed that they could not remember the rules. A surprisingly large number did not distinguish between $12 \div 3$ and $3 \div 12$ or $12 - 3$ and $3 - 12$ stating that the answer is the same whatever the order. Occasionally, the child did not recognize the nature of the operation. The team wondered if there might not be an argument for considering whether the traditional algorithms are the most useful ones. "Could it be that a method more closely related to children's 'concrete' methods might not be more suitable, even if slightly less efficient?" they asked themselves.

II. At this point, various versions of the group tests were tried. Since the very straightforward items on addition and subtraction and those on the 'take away' form of subtraction were done correctly by almost all pupils, they were omitted. Part I of the class test consisted of three problem items on harder forms of addition and subtraction, three on multiplication and three on division. Part II consisted of the 'story' items on subtraction, multiplication and division.

As I have been advocating the idea that pupils might try constructing problems from number expressions for some years, I was most interested in discussing the results of Part II with members of the team. They felt it was hardly surprising that addition and subtraction were found to be the easiest. Nearly 80% of first year secondary students were successful in making up a story to illustrate subtraction and that almost all were of the 'take away' variety, e.g.

Story

9 - 3

If you had 9 pence but lost 3 how much would you have left

They were more surprised to find that division was consistently easier than multiplication, but felt that the clue to this was perhaps in the stories given, which were generally of the sharing type. e.g.

Figure 3.

Story:

84 ÷ 28

If 28 children shared 84 sweets how much would they each have

The word 'times' which is commonly associated with multiplication, appeared to have a much less concrete image than that of 'sharing', and children seemed to have a correspondingly harder time constructing a story. Concrete level children seemed either to use the word 'times' in their story, e.g.

John had 9 times the amount of Peters 3 marbles how many did he have?

9 x 3

OR used a repeated addition story e.g.

Julie bought 3 chews
Mary " 3 chews
Janet " 3 chews
Shelley " 3 chews
Jane " 3 chews
Andrew " 3 chews
Peter " 3 chews
Pam " 3 chews
Lee " 3 chews

How many chews altogether?

84 x 28

The best stories rarely used any of the 'cue' words e.g.

There are 84 skirts on each rack at a clothes shop. There are 28 racks. How many skirts altogether?

Conclusion

On balance, the researchers felt that there was evidence to support the following order of difficulty +, —, ÷, x. The 'Cartesian product' problem (item 7, figure 1) was found to be significantly harder than the "repeated addition" form of multiplication, and "about 35% of first year secondary students (11-12) were successful in constructing a story from multiplication."

The team then went on to build a model in 3-D lattice form which shows the results, using ordered triples. One mark was given for addition/subtraction (grouped together) if the pupil answered two out of three questions correctly. A second mark was given if he gave a correct subtraction 'story' so that the child could score 0, 1, 2. The same was true for multiplication and division. Thus the ordered triple 210 would identify a child successful in parts 1 and 2 of the test in addition and subtraction, successful in only one part for multiplication and in neither for division. This allowed levels to be set up and a suggestion of the probable lines of development of children's understanding. The work is well worth looking at in greater detail so I suggest you write for further information (address below).

To conclude this article about operations of whole numbers, allow me to quote from Margaret Brown's article in the Times Educational Supplement titled "Investigating Understanding." She wrote on 23.3.76 that "about eight per cent of pupils at 11-12 were completely successful in recognizing the operations involved in the problems (including getting the numbers in the right order), a further 25 per cent were generally competent, and another 25 per cent had at least some

idea of multiplication and division. These proportions correspond roughly to those in the large-scale survey by the science team in the CSMS project, who found at this age about 12 per cent of pupils to be at the early formal stage and 52 per cent at the late concrete stage, as defined by class version of Piagetian tasks''.

The research team plans to continue its work by looking at the child's basic understanding of such topics as graphs, ratio, proportionality, probability, vectors, matrices, geometrical ideas, simple algebraic formulae and measurement.

CSMS - Centre for Science Education
Chelsea College, University of London
88-90 Lillie Road
LONDON, England SW6 7SR

1. Brown, M., **Investigating Understanding, The Times Educational Supplement,** 23.4.76
2. Ibid.

INTRODUCING INTEGERS WITH A MIRROR

George Ropchan, Weston, Ontario

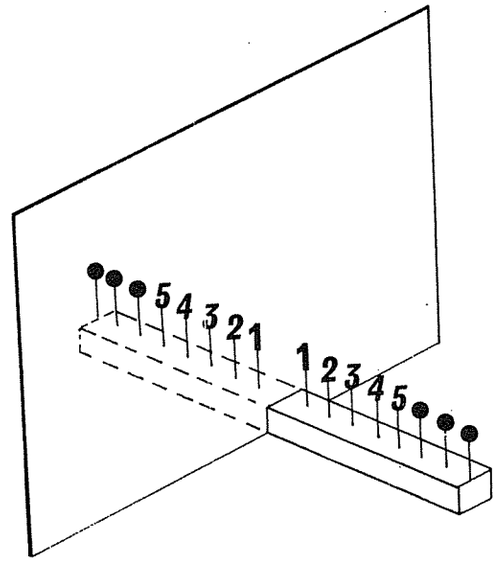
A student is introduced to the integers in Grade Nine, or, in some schools in Grade Eight. At this time the operations in arithmetic are taught in the extended system. Since the child does not have the ability to extract the properties of mathematical systems a lot of confusion exists. It may be bad enough to turn the child against any mathematics and indeed is a cause of dropouts and discipline problems.

Various methods have been used, one being the labelling of the integers with raised symbols that do indicate a qualitative difference. Directed line segments or vectors are used to indicate the properties, and as an aid to the computational process. When some degree of success is achieved it is either assumed that the child will discard the use of the raised symbols, or he or she is directed to drop them.

It is my claim that the raised symbols provide more problems than they cure. For students of better than average ability there is an awareness of the fact that a subtle difference exists between the two sets, an awareness that they cannot fathom. A situation such as this will tend to make them retain the raised symbols for much longer than necessary, in some cases refusing to dispense with them at the suggestion of a teacher. The student having more problems is going to balk at the need of a raised sign as there is no difference between 3 and $^{+}3$ in his eyes. Certainly in a lesson or two he will be told that they can be lowered and in the case of the positive integers dispensed with entirely. A situation such as this only enhances the uncertainty of the child as to what the symbol represents, an operation or a quality.

Some authors use a further refinement, that of absolute values. Rather than comment on this, I will quote a rule given and leave it to the proponent reader's imagination as to the effect of this on the mind of the child, ''In general, if ^{+}a represents a positive integer, there exists a negative integer ^{-}a, having the same absolute value such that $^{+}a + {}^{-}a = 0$''. This is one example culled from many in mathematics texts written for a general level class.

To avoid these problems I would like to set forth a method that I have developed. The apparatus consists of a mirror having dimensions of approximately two feet by five feet. It is mounted on the wall with an arm perpendicular to its plane. The arm has the natural numbers mounted on it with three balls to indicate the continuity of the numbers. They are colored black, while the side hidden from the student is red. The child can see red colored numbers in the mirrors while on the arm he can see the natural numbers as black colored numbers. A simple diagram of this is below.



Now the word "opposite" is used freely in place of minus or negative, though no attempt was made to exclude the other two words. No defense of this is made. It is there by design.

A distinction was made at the outset that the number to the right is greater than its predecessor. Using the mirror number-line, a step-by-step procedure was instituted to show that since 5 is greater than 4, that $5 - 4 = 1$. Of course the subtraction of the following in succession, $4 - 3$, $3 - 2$, $2 - 1$, $1 - 0$, $0 - -1$, $-1 - -2$, all led to the same result. The relative position of these numbers as viewed by the student reinforced this concept.

Now if $5 > 4$, and as the students could see the opposites as red coloured numbers in the mirror, it was easy for them to accept that the opposite of 5 was less than the opposite of 4. Further confirmation was obtained by appealing to the step-by-step process indicated above.

In developing the addition of the negative integers, the addition of the counting numbers was used to show that the addition of $3 + 2$ resulted in a number greater than either of the addends. Now if the opposite of 2 was added to the opposite of 3 the result would be a smaller number, in this case the opposite of 5. Very little time was spent on this as the idea was easily grasped.

Multiplication was developed from the notion that it can be treated as repetitive addition. Since $4 + 4 = 8$, then -4 added to -4 was -8. An appeal was made to the knowledge of $2 \times 4 = 8$, and then taking the opposite of this. The result was used to make the commutative idea unnecessary as one could consider that if $2 \times -4 = -8$, then $-2 \times 4 = -8$ as one could consider multiplying 2 by 4 and then taking the opposite. From here it was an easy step to consider the multiplica-

tion of a negative number by a negative for one could then interpret it as taking the opposite twice and so having the answer positive.

The operation of subtracting a negative integer from its opposite was developed by resorting to the concept of distance as viewed by the student in the mirror and to the fact that the greater number was to the right. It is easy to demonstrate with numbers whose absolute value is the same. Choosing 5 - -5, the pupil would intuitively use the reflecting property of the mirror to come up with the answer 10. No counting of spaces occurred although this was resorted to in the subtraction of integers whose absolute values were not the same. The operation of subtracting a positive number from a negative was carried out by appealing to the distance being in the opposite direction. It was strengthened by noting that subtracting 3 from a positive number diminished the number by three, and demonstrating the fact that $4 - 3 = 1$, $3 - 3 = 0$, $2 - 3 = -1$, $1 - 3 = -2$, ..., $-4 - 3 = -7$. Division was also operated on in this fashion, first by demonstration of their knowledge of division in the natural numbers, then division of a negative number by a natural number. The last case, dividing a negative number by a negative number was then tackled from the knowledge of multiplication.

It is my contention that the use of the mirror will provide better results than a line on the blackboard. A child at this age considers the numbers real in the same sense that mathematicians did until a century ago. The reflection in the mirror fits in with his knowledge of the properties of mirrors and can be put to good use to visualize the properties of numbers and their opposites. One would have to consider this from the child's likely reaction to what he sees on the blackboard and what he sees with the mirror and objects on an arm labelled as the numbers he knows. I reiterate that the raised symbols provide more problems than they solve. Both the bright student and the mediocre find it difficult; the first assumes that they are not the same (I hesitate to use the word isomorphic) and the other sees no difference and so is puzzled by what appears to be hairsplitting on the part of the teacher.

"A UNIQUE DEVELOPMENTAL OPPORTUNITY"

Alexander L. Norrie, Coordinator of Mathematics, Peel County Board of Education

" ... that metric conversion be accepted as an opportunity to reassess pedagogical approaches toward the topic of measurement as a whole; that students be provided with opportunities to acquire a practical and experiential knowledge of metric units through activities that involve comparing, estimating, measuring and related problem solving;"[1]

Recently, I had the most exciting opportunity to initiate metric training for almost one hundred top executives from a large nation-wide retail business. This experience positively reinforced my perceptions of how measurement is learned and also taught me many things about the people

who are involved in the metric changeover. One of the Vice-Presidents was asked how he would convince key employees to positively participate in the changeover. He replied, "You can't force the involvement or legislate participation; therefore you just tell them that they have the opportunity to participate in a unique developmental opportunity."[2]

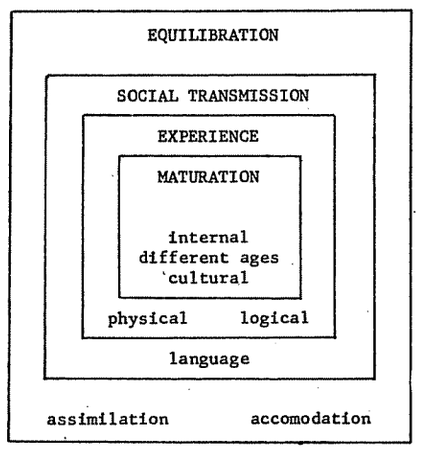
This approach certainly brought results!

The unique developmental opportunity that you can participate in is not only the changeover to metric measurement, but much more. As the Ontario Ministry of Education outlined in 1975, you have the opportunity to reassess pedagogical approaches toward the topic of measurement and this in the long run will prove more beneficial to education than the metric units themselves.

Historically, the opportunity is even more unique. You can now look to your northern neighbour for an educational model of a metric changeover. Canadian educators who looked to the United States and Britain for mathematics education leadership in the nineteen sixties are even now using the skills you taught them to develop revised measurement programs for young children. More than that, they are proving to be a most positive force in the changeover to SI.

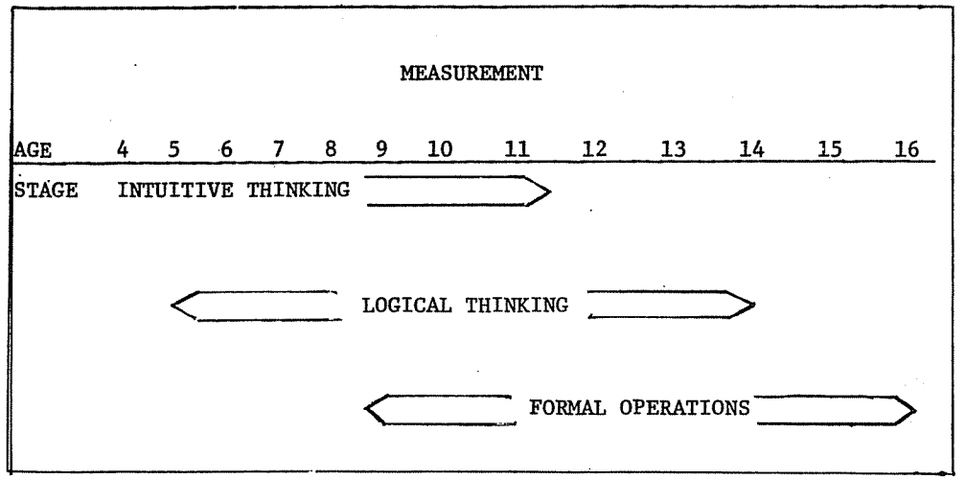
As I discuss the topic "How Children Learn to Measure" I will try to share my own unique developmental experiences with you. These experiences encompass the sphere of mathematics education radiating from American centres similar to Syracuse University, as well as direct involvement in the Canadian metric changeover.

A cognitive explanation seems to be most appropriate in dealing with complex behaviour such as measurement concepts. Ginsburg indicates that cognitive research conducted over the past 5 years seems to deal mainly with arithmetic. [3] However, as early as 1970, he and other cognitive psychologists were listing factors that affect the learning of measurement concepts. David Elkind sums it up nicely when he indicates that a basis in logic must be developed in the child.[4] This logic base is dependent on many factors some of which are maturation, experience, social transmission, and equilibration. One way of outlining these is in diagram form such as the following:



For purposes of this discussion, we need only to be aware that a multitude of factors influence the learning of measurement and that the factors apply in differing degrees throughout the learning continuum.

Various stages closely paralleling those suggested by Piaget and his students also categorize the learning of measurement. We must realize that these stages are not mutually exclusive; all children's thinking progresses in this manner, but many children reach certain stages at many different times; a child may be at the stage of logical thinking in terms of one type of measurement, let us say linear, and at the same time may very well be back at intuitive thinking in terms of area mass or volume. If we are flexible in our interpretation of the age scale we can summarize the stages as follows: [5]



You can see by this outline that a science colleague of mine should not have been surprised or amazed that some second and third year secondary school students have not yet reached the formal operations stage with some concepts. The concept of proportionality in the abstract domain is an example of a particularly difficult concept even for some adults.

Using the above mentioned stages as a general guideline, let us investigate how children learn to measure.

Use of language
longer, shorter,
big, little,
near, far

↓

Play and pre-measuring activities are a vital aspect of children's learning. Much of the work at this stage will involve refinement of vocabulary. Given blocks, hoops, boxes, glue, junk materials (every primary room should have a substantial junk centre), string, tape, paint..., children will make models that represent things in their world.

Our class went to the zoo!
The children made animals from junk materials.
Can you compare the giraffe to the snake?
I'll stand the snake on its tail.
It looks like the snake is longer!

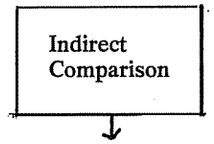
How could you check to find out?
Move the snake over beside the alligator.
Good, the snake is longer! 6

Direct
Comparison

↓

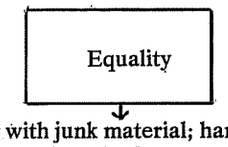
During the stage of **direct comparison** a child's interest in size is centred around gross differences and obvious contrasts. Sorting experiences based on size differences are excellent at this stage. By visual discrimination and direct matching children will use language such as big and little, tall and short, etc. Many experiences are necessary to refine the child's perception to include finer differences using language such as smaller, heavier, and so on. **Direct comparison** involves comparing two quantities and noting a relationship between them and the comparing is done by bringing the two things together.

After many experiences involving direct comparison, children will enjoy the use of parts of the body as an aid in measuring. This achieves several very important objectives. It personalizes the experience for the child. "I can use **my** hands." Personal involvement makes a very positive contribution to the learning process. Body units also encourage the development of the language of comparison. "The elephant is four hands wide". "But look!" "I need five hands for the giraffe." Again the use of models created from realistic child centred experiences (a trip to the zoo) make the measurement concepts very personal. Estimation and the need for counting units are informal by-products of this experience.



To compare two objects an intermediary measure is often necessary. As outlined above, the use of hands represents an example of an intermediary. Notice that children at this stage do not recognize the need to use only one hand repeatedly.

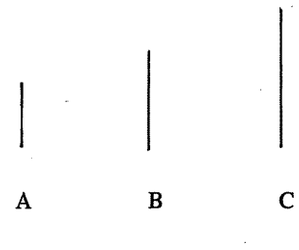
Distance and length should also be treated differently because children cannot fully understand the spatial perceptions involved. Copeland [7] explains distance as linear separation and length as a property of an object.



From experiences such as measuring with junk material; hand spans and pieces of string children begin to find things that are the same length; for more precision they use the same object repeated over and over. How about a crayon box that is about 10 cm long? Through a wide variety of experiences using body units (hands, feet) and non-body units (pie plates, crayon boxes, paper strips) the child may be ready to move along the measurement continuum.

But wait! Some critical developmental issues still have to be considered; the concept of order is crucial to all measurement and serial order (seriation) is particularly important. Can the child put three or more things in order from shortest to longest or longest to shortest? Many experiences such as this must precede any further measurement activity. **Order by size!**

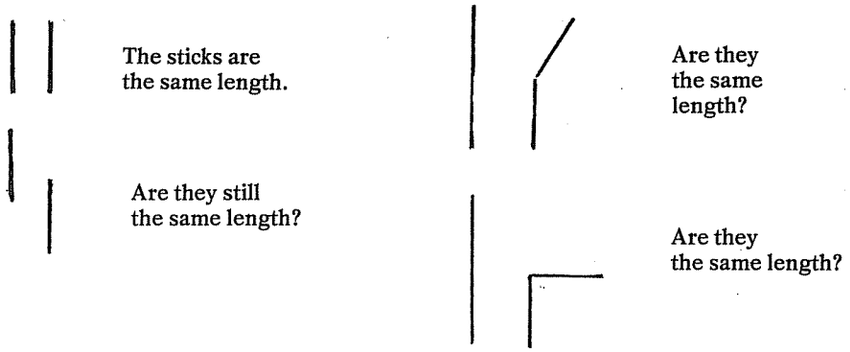
The idea of the transitive property involving relationships between three things is also a necessary experience for children; however, it is also extremely difficult.



Seriation
and transitivity

If stick A is shorter than stick B and stick C is longer than stick B then stick A must be shorter than stick C. This kind of experience can first be approached by using sticks, paper strips or pieces of string that are the same length as a particular item. Then we can find things shorter than, longer than and the same as the item; be sure to have children estimate first.

Does a child think that the distance from A to B is the same as the distance from B to A? This reversibility development also applied to measurement of length. Similarly, many children think that length changes as an object is moved about or distance changes when something is placed between two objects.



First, before attempting systematic measurement, the child must be able to conserve the idea of length of an object. The child arrives at the stage of conservation not by having explanations given but largely on his own. [8] Thus the operation of measurement involving parts and wholes is formed by children who have had **appropriate** experiences in the early primary years.

Experience with materials itself is not enough. To understand the need for more precise measurement and to communicate discoveries and relationships, children must be given the language to describe measurement experiences. Verbalization with adults as well as with their peer groups clarify the mathematical concepts as they develop.

More precise measurement is required if we are going to design a box for our giraffe. We may decide on a non-standard arbitrary unit such as a crayon box and everyone in the group will agree to use the same measurement unit.

Arbitrary Units

↓

New measurement involves giving the unit a name (a giggle, or a goggle) and repeating the unit over and over again by a change in position. Conservation of length is a necessity! Children also quantify measurement by assigning a counting number (1, 2, 3, ...).

Quantification

↓

If each mark is a giggle, how many giggles long is the desk? At this point many standard units are placed in the junk centre (i.e. sticks 1 m, string 1 m) but are not named.

Standard Units

↓

We must be very inventive and get children to recognize the need to be precise and use a standard measurement unit. My developmental sequence suggests that this should not occur until late level 1 or even level 2, about the age of 7 or 8. A suggested teaching sequence based upon ideas outlined above follows:

LINEAR MEASUREMENT

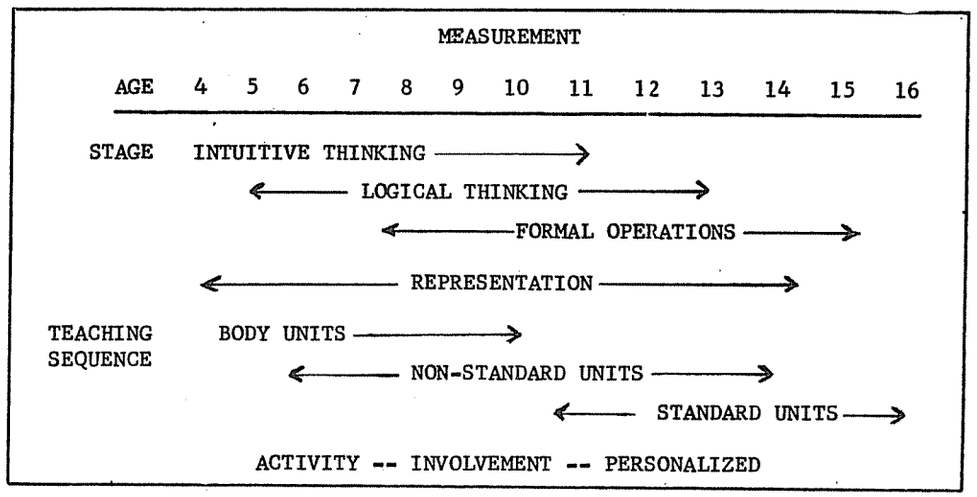
Suggested Grade Level	K	1	2	3	4	5	6
Length and Distances	Pre-measure Experience	→	metre	metre	m, cm, dm	mm, km	→
	Non-standard	→	centi-metre	centi-metre 1 m = 100 cm	relation-ships	→	→

Traditionally, measurement has been badly taught. We tended to introduce standard units too early and we did not develop an intuitive understanding of the units. Thus, measurement was reduced to a matching and quantifying abstract mathematical exercise. Now it should be recognized that children need numerous opportunities to see and feel measurable quantities, to estimate their sizes and then to check their guesses by actual measurement. [9] In standard measurement, metric units should be stressed and a continuous mixture of two systems should certainly be discouraged.

Similar stages of development as outlined above, apply to all measurement situations including linear, area, volume, mass and temperature relationships. This is outlined below:

Suggested Grade Level	LINEAR	AREA	VOLUME	MASS	TEMPERATURE
K	pre-measurement experiences non-standard units	pre-measurement experiences	pre-measurement experiences	pre-measurement experiences	pre-measurement experiences
1		non-standard units covering surfaces	non-standard units filling containers (liquid, cubes, counting)	non-standard units	sensory experiences (approximate temps)
2	metre centimetre	non-standard units counting squares with metric dimensions	litre		comparison indoor-outdoor temperatures
3	metre centimetre 100 cm = 1 m	counting squares		kilogram	read and record temperatures in degrees Celsius
4	m, cm, dm begin relationships among these units	multiplying length by width no formula cm^2 m^2	m counting cubes with metric dimensions	grams	
5	mm, km relationships	area of rectangles using ℓ x w dm^2	L, mL cm^3, dm^3 relationships	1000 g = 1 kg	
6			m^3	tonne	

Given the current changeover to metric we should recognize the **unique developmental opportunity** to reassess our teaching of measurement and to help children learn by recognizing and utilizing developmental stages. The teaching of measurement will be maximized by these considerations:



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WESTMINSTER CLOCK: The times were 8 hours 23 71/143 minutes, and 4 hours 41 137/143 minutes. We are always allowed to assume that these fractional times can be indicated in clock puzzles.

"No man may be so great that he can not be proven wrong."

Aristotle

LET'S REALLY 'GO METRIC' — TO METRIC TIME and METRIC MAP COORDINATES

Howard Munson, Professor of Education, Winona State University

We are today witnesses to a rapid and accelerating interest in conversion to the metric system. There are many reasons for the growing interest in an international system of weights and measures. I shall not recount them here. Suffice it to say that those reasons which support the adoption of metrics by the U.S. seem most logical. Opposition, for the most part, seems based upon economics and perhaps emotion — it's difficult to swap something you are familiar with, with something strange. My purpose in this article is to propose an extension of the metric system into several additional areas where dividing units into tenths and hundredths would not only be logical but would greatly facilitate both computation and communication. Whether or not this is regarded as a serious proposal is not important. Its real merit may lie in the stimulation it may provide teachers and students alike.

First, I propose that the circle be declared to contain 100 degrees (or give the unit a new name such as "centicircles" if you like) rather than 360. If one views this proposal without prejudice, one finds good reasons to promote the idea. After all, it is only by convention that we agree there are 360 degrees in a circle and that convention seems to trace back to Babylonian times and their base sixty. With a 100 "centicircle", a straight line would have 50 centicircles and a right angle would have 25 centicircles. Furthermore, we could divide each centicircle into ten smaller units and then again into ten units. Thus, we could have a circle with decicircle, centicircle, and millicircle as subunits. Think of the accuracy that could be achieved in geometric constructions! Think of the accuracy one could achieve in circle graphs where 33⅓% could equal 33.33 centicircles! Simplicity of graph construction would result from the avoidance of a need to convert percents to degrees.

A circle with 100 even units would have great advantages in describing locations on earth (although we can understand a reluctance to tamper with present systems of mapping). Instead of 180° east or west longitude, one could simply describe a place as being at 26.7558 centicircles west, or at 13.94 centicircles east. One would start at an agreed line of longitude such as our present prime meridian and measure west or east. While the accuracy may not be better than degrees and minutes, the system would be understood easily by anyone who can manipulate decimals.

Let's go a step further. There seems little sense in insisting that a year is 365 days, 5 hours, 48 minutes and 45.7 seconds in length. If we are going to use arbitrary values, let's be consistent. Why not use powers of 10 to describe the movement of the earth around the sun and the earth on its axis? Surely this has all been proposed before but somewhere along the line we have sacrificed convenience and logic for something not quite so defensible.

While I am proposing changes let's examine our peculiar 12 hour clock with 60 minutes each having 60 seconds. Would it not be just as well to adopt a clock on mod 100? Then we could record time beginning either at high noon or at midnight and divide the day into 100 units, each divisible by 10 and by 10 again. Thus, after giving the units a new name such as "da" we would have decidas, centidas, and millidas. At the equinox you would have five decidas or 50 centidas of daylight and an equal number of darkness. On December 21 (or a da by another designation if we go all the way) you would have 2.5 decidas of daylight (more or less depending on the latitude at which you live).

Per haps we have paid sufficient tribute to the Babylonians. Perhaps we are ready really to 'go metric'. Why not have your math students explore these propositions? They will enjoy the process of proposing and testing additional ideas. They may get involved in math history or with navigation, or with map reading, but the result will be enjoyment of learning.

Editor's Note: It is interesting to hear American views on metrication. By the way, is there not a metric unit for angle measure, 1/100 of a right angle?

HOCKEY DATA: STATISTICS FOR THE CLASSROOM

Bonnie H. Litwiller and David R. Duncan, Professors of Mathematics, University of Northern Iowa, Cedar Falls, Iowa

Mathematics teachers at all levels of instruction are searching for problems involving statistics and probability from real world situations. The sports world is a rich source from which to draw data for such problems. Such data is readily available in newspapers, magazines, and yearbooks.

Table I: Shots On Goal

W \ L	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
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Table II: Percent of Successful Shots on Goal

W \ L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0																									
1																									
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11				2	1	2		1						1	1										
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14	3			1		1		1		2									1						
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We considered the following questions:

I. What is the probability that a team ahead at the end of the first period in the NHL (National Hockey League) will win the game? Similarly, what is the probability that a team ahead at the end of the second period will win the game?

II. What is the probability that the team that scores first will win the game?

III. How many "shots on goal" are attempted by the winning and losing teams? What percent are successful?

IV. What is the relationship between winning and losing scores?

To gather data upon which to answer these questions, we considered the first 100 hockey games played in the 1976 calendar as reported in **The New York Times.**

I. What is the probability that a team ahead at the end of the first period will win? In 27 of the 100 games, the score was tied at the end of the first period. Of the 73 games in which a team was ahead at the end of the first period, the team that was ahead won 49 games, lost 12 games, and was tied in the final score 12 times. Consequently, the probability that a team which is ahead at the end of the first period will win the game is 49/73 or 67%. The probability that they will lose is 12/73 or 16%. The probability that they will be tied is 12/73 or 16%.

What is the probability that the team ahead at the end of the second period will win the ame? Of the 100 games played, the score was tied at the end of the second period 10 times. Of ne 90 games in which a team was ahead at the end of the second period, the team that was ahead von 75 (83% of the 90 games), lost 3 (3%) and was tied in the final score 12 times (13%).

II. What is the probability that the team which scores first will win the game? In the 100 games played, the team which scored first won 68, lost 18, and was tied in the final score 14 times.

III. How many "shots on goal" are attempted by the winning and losing teams? What percent are successful?

Table I reports the results for the 86 games in which there was a winning team.

In "row 44, column 22", the three dots indicate that there were three games in which the winning team had 44 shots on goal and the losing team had 22. In 23 of the 86 games, the losing team had more shots on goal than did the winning team.

The average (mean) number of shots on goal for the winners was 34.5 (with standard deviation 7.7) while the average number of shots on goal for the losers was 26.8 (with standard deviation of 6.6). The total number of shots on goal for an entire game averages 61.3 -- approximately one per minute.

Table II reports the percents of shots on goal which were successful.

In only four of the 86 games did the losing team have a higher percent than the winning team. It is striking to compare this with the shots on goal (successful or not) as described in the previous paragraph.

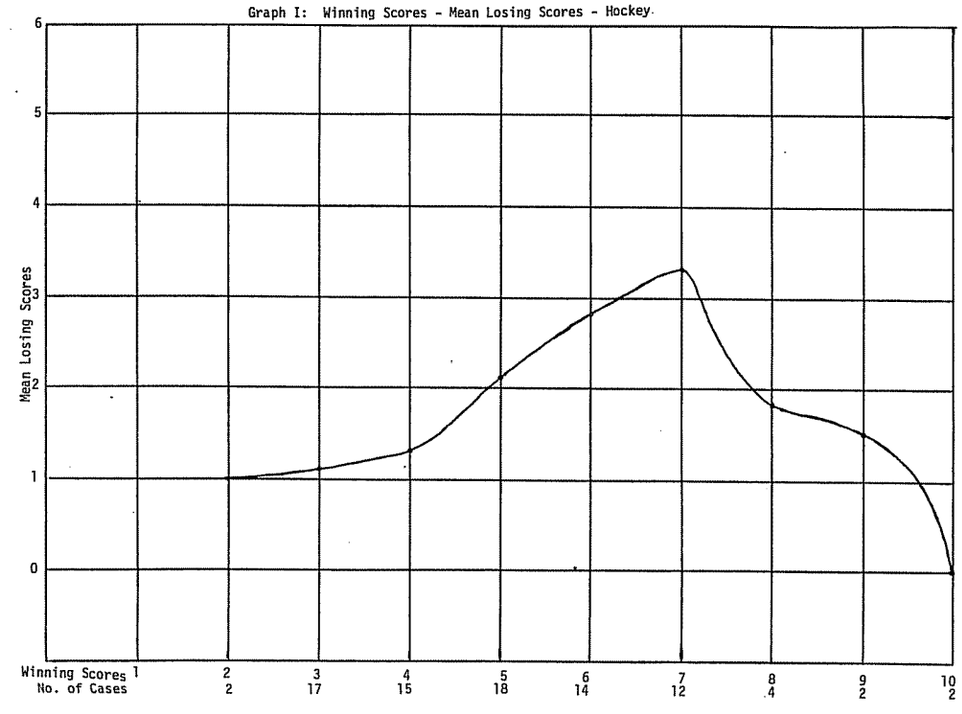
IV. What is the relationship between winning and losing scores?

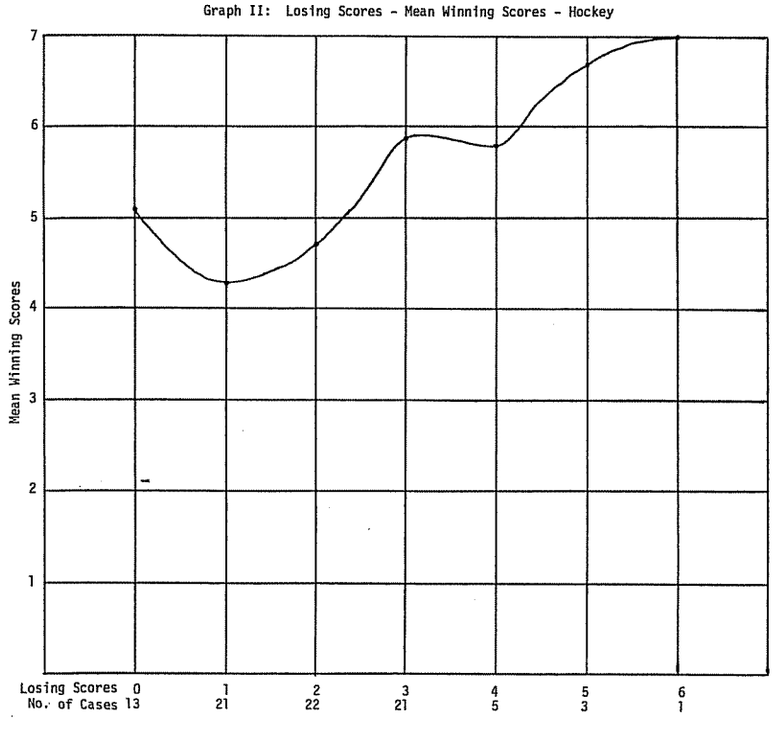
Table III depicts the winning and losing scores for the 86 games in which there was a victor.

Table III: Winning-Losing Scores

W \ L	0	1	2	3	4	5	6	Totals	Means
0									
1									
2		2						2	1.0
3	4	8	5					17	1.1
4	3	4	6	2				15	1.3
5	2	3	5	7	1			18	2.1
6	2	1	1	5	4	1		14	2.8
7			3	6		2	1	12	3.3
8		2	1	1				4	1.8
9		1	1					2	1.5
10	2							2	0
Totals	13	21	22	21	5	3	1	86	
Means	5.1	4.3	4.7	5.9	5.8	6.7	7		

In Table III, the column and row entries were totalled and means were found. For instance, there were 17 games in which the winner scores three goals. The mean losing score for these 17 games was 1.1. The data from Table III were used to construct Graphs I and II.*





* A special thanks to David Tschiggfrie for constructing the graphs.

Neither graph appears linear. How can this observation be interpreted?

The reader and his/her students are invited to ask similar questions for other sports of interest to them.

THE NEW YEAR GAME 1977

This is a postscript to a letter from Don Pickersgill, Port Dover, which appeared in the March, 1977 issue. Don's students produced at least one solution for every number from 1 to 100. The results for 75 - 95 are given below.

A similar problem appeared on the Colorado Mathematics Teacher recently, authored by Timothy D. Cavanagh of the University of Northern Colorado, Greeley. It involved producing the natural numbers using four 4's.

e.g. $21 = 4! - \sqrt{4} - (4/4)$

$74 = -1 \times \sqrt{9} + 77 = \bar{1} \times 9! \div 7! + [\sqrt{7}]$

$75 = 1 - \sqrt{9} + 77 = 1 + 9! \div 7! + [\sqrt{7}]$

$76 = -1^{9} + 77 = 19 \times [\sqrt{7}] \times [\sqrt{7}]$

$77 = [\frac{1}{9}] + 77 = 1^{9} \times 77 = (1+9) \times 7 + 7$

$78 = 1^{9} + 77 = -1 + 9! \div 7! + 7$

$79 = -1 + \sqrt{9} + 77 = +1 \times 9! \div 7! + 7$

$80 = 1 \times \sqrt{9} + 77 = 1 + 9! \div 7! + 7$

$81 = 1 + \sqrt{9} + 77 = 1 \times 9 \times (7 + [\sqrt{7}])$

$82 = -1 + \sqrt{9!} + 77 = 1 + 9 + [\sqrt{7}] + [\sqrt{7!}]$

$83 = 1 \times \sqrt{9!} + 77 = 1 \times \sqrt{9!} + 7 + [\sqrt{7!}]$

$84 = 1 + \sqrt{9!} + 77 = 1 + \sqrt{9!} + 7 + [\sqrt{7!}]$

$85 = -1 + 9 + 77$

$86 = 1 \times 9 + 77 = 1 \times (9+77)$

$87 = 1 + 9 + 77$

$88 = 1 \times 9^{[\sqrt{7}]} + 7$

$89 = -1 + 97 - 7$

$90 = 1 \times 97 - 7 = (1+9) \times (7+[\sqrt{7}])$

$91 = 1 + 97 - 7$

$92 = 1 + (\sqrt{9!} + 7) \times 7 = [\sqrt{\sqrt{\sqrt{\sqrt{19}}}} \times 77]$

$93 = (1+9)^{[\sqrt{7}]} - 7 = [19 \times 7 \times .7]$

$94 = -1 + 97 - [\sqrt{7}]$

$95 = 1 \times 97 - [\sqrt{7}] = 19 \times (7 - [\sqrt{7}])$

BACK(?) TO BASICS — RATIO AND RATE

(conclusion)

Doug Henderson, Master Teacher of Mathematics, Sudbury Board of Education

This is the third part of an article which appeared in December, 1976 and March 1977.

RATE EXPERIMENT #6

Equipment: (i) coiled spring on rubber band
(ii) set of weights
(iii) metre stick

Procedure: (i) Locate the zero point for the length of the spring before any weights have been added.

(ii) Add weights 25 grams at a time, and measure the increase in the length of the spring.

Complete the table.

weight (gm)	Increase in length (mm)
0	
25	
50	
75	
100	

Record the results on a graph.

(iii) Calculate the rate of increase in mm per g (one decimal place).

(iv) Write a formula connecting the increase in length of the spring with the weight.

(a) as a fraction: $\frac{\text{increase}}{\text{weight}} =$

(b) as an equation:

increase = —— × weight.

(c) Rewrite (a) and (b) using alphabetic notation.

(v) If 65 g were used what would be the increase in the length of the spring?

Problem

A piece of metal increases in length as the temperature increases. The length changes for 120 cm to 123.5 cm when the temperature increases from 0°C to 100°C.

(a) Find the rate of increase in cm per degree.
(b) If a metal bridge is 90 m long when the temperature is 10°C what will be its length at 24°C?

RATE EXPERIMENT #7

Equipment: (i) a heatable container of water
(ii) thermometer
(iii) wrist watch

Procedure: (i) Take the temperature of the water at different time intervals as it is heated. Be certain to stir the water so that all of it is at the same temperature.

Complete the table:

time (min)	temp (degrees Celsius)
0	
0.5	
1.0	
1.5	
2.0	
2.5	

Draw a graph to illustrate your results. Find the average rate of increase in degrees per minute.

(ii) Write a formula which connects the increase in temperature (I) with the time in minutes (t).

(iii) If this rate were maintained, how long would it require to heat the water from freezing (0°C) to boiling (100°C).

Problem

The temperature of the air decreases at a rate of 0.72 degrees Celsius for every 100 metres of height above the ground. If the temperature is 21°C in a valley what would be the temperature at the top of a mountain which was 1350 m higher?

RATE EXPERIMENT #8

Equipment: (i) a set of nuts and bolts
(ii) a ruler

Procedure: (i) Turn the moveable nut until it is touching the top of the threaded bolt.

(ii) Turn the nut and measure the distance between the nut and the top of the bolt.

(iii) Complete the table and draw a graph. Place the number of turns on the horizontal axis.

Number of turns	Dist (cm)
0	
5	
10	
15	

(iv) Write a formula which connects the distance (d) with the number of turns (T).

(v) Repeat parts (i)-(iv) for each of the other two bolts.

Problem

If the nut moves 1.4 cm in 10 turns, how far does it move in 3.5 turns?

Teachers Hint

It is best to get your secondary school machine shop to cut bolts of 3 different pitches.

RATE EXPERIMENT #9

Equipment: (i) an aquarium pump
(ii) a watch or clock with a sweep second hand
(iii) apparatus for collecting gas.

Procedure: (i) Set the pump at a moderate rate.

(ii) Collect the air delivered by the pump and complete the table.

time(min)	volume(ml)
0	0
1	
2	
3	
4	

(iii) Draw a graph, placing time on the horizontal axis. What pattern do you notice in the graph?

(iv) In 4 minutes the pump delivers ——— ml.

Therefore in 1 minute the pump delivers ——— ml.

Your answer is the rate at which the pump moves air in millilitres per minute

(v) How much air would this pump move in one day? Explain how you obtained your answer.

(vi) State the relationship between volume of air and time

(a) in words

(b) as a fraction $\frac{\text{volume}}{\text{time}}$ =

(c) as an equation volume = ——— × time

(d) Rewrite parts (b) and (c) using algebraic notation. Let V represent the volume and t represent the time.

Problems

1. A furnace fan can move 40 m^3 of air in 15 minutes. How much air does it move in a day if it runs for 20% of the time?
2. An irrigation pump can deliver 50 litres per minute. If a field of tomatoes requires 125000 litres per day, how many pumps are needed?

RATE EXPERIMENT #10

Equipment: (i) salt in shaker
(ii) an instrument for measuring volume, graduated in ml
(iii) a watch with a sweep second hand

Procedure: (i) Invert the shaker for 5 seconds. Measure the volume of the salt which is poured.

(ii) Repeat the procedure for 10, 15, 20 seconds. Refill the shaker each time. Complete the table. What pattern appears on the graph?

Time(s)	Volume(ml)
0	
5	
10	
15	
20	

(iii) In 20 seconds the volume is —— ml.
Therefore in 1 second the volume
is —— ml.

Your last answer is the rate of pouring in ml per second.

(iv) How much salt would run through this shaker in one minute? How do you get your answer?

(v) State the relationship between volume and time as

(a) a fraction $\frac{\text{volume}}{\text{time}}$ =

(b) an equation volume = —— × time

(c) Rewrite parts (a) and (b) using algebraic symbols. Let v represent the volume and t represent the time.

Problems

1. 40 ml of dry sand will pour through a small hole in two minutes.
   (a) What is the rate in ml/s?
   (b) How many ml must be used to time a three-minute egg?

2. A leaky faucet drips 25 ml in 2 minutes. How much water will be lost in one week?

"In order to reach the truth, it is necessary, once in ones life, to put everything in doubt — so far as possible."

Descartes

MY CHILD WANTS A CALCULATOR

Glenadine Givv, Past President, National Council of Teachers of Mathematics

Reprinted with permission from the* Teaching Mathematics, *February 1977, the journal of the Mathematics Council, Newfoundland Teachers' Association of Teachers of Mathematics.

Should I get a calculator for my child? Does it interfere with children's learning mathematics? We do want our children to know mathematics — the basic facts, how to add, subtract, multiply, and divide. What do you advise?''

Undoubtedly you have been asked these questions, as have I, by parents of your students or by other parents of school-aged children. They may also ask, "And which calculator should I buy?'' During the holiday season, we can expect even more of these questions. With prices for calculators declining, they are among possible gifts on those Christmas lists.

In the classroom. Many schools are already using minicalculators in a variety of ways, supported by the NCTM statement that ''the minicalculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics.'' Finding effective and creative ways is not necessarily an easy assignment, however. The NCTM Instructional Affairs Committee has suggested some uses in a report to be published in the January issue of the *Mathematics Teacher* and the *Arithmetic Teacher.* Other articles in those journals, as well as articles in the *Instructor, Today's Education,* and other professional journals and books can help you, as can the workshops and discussions at our professional meetings.

Suppose that we look ahead and make a checklist of the mathematical concepts and skills that we shall be teaching during the remainder of the year. How can the calculator be used as an instructional aid to enhance learning beyond what we might otherwise expect?

Obviously our lists will reflect the age of our students and the mathematical content of our program. One guide for all of us, however, is that we *not* use the calculator until our students have developed a concept of number, a system for naming numbers, and an understanding of the meaning and processes of the basic operations — that is, until our students understand what the calculator is doing for them.

Once students have this understanding, using the calculator to check answers to paper-and-pencil computations might stimulate more thought than checking by simply looking up the answers. Checking answers, however, seems a very narrow, limited use of the minicalculator. More thinking may be stimulated by using it to ''debug'' problems; to obtain insight into the operations — as, for example, finding the solution to a division problem with subtraction; to explore topics in number theory; to become more sensitive to the order of operations; to generate patterns; ... and our list continues. Not to be overlooked is the opportunity to develop skill in estimating and to have more experiences with decimals.

We need not be reminded of the difficulties that many students encounter in problem solving. The calculator can be an impetus for them to formulate more carefully the mathematical relations to be solved in order to provide the calculator with the necessary information. Naturally, it enables students to solve problems that can be fascinating to them but were previously avoided because of the tedious, laborious computations involved. Thus, with the hand-held calculator it may be possible to increase considerably our students' skills in problem solving!

Selecting your calculator. The selection of a calculator must depend on how it is to be used and how much money is to be spent. Minimal capabilities for beginning use should include —

- a minimum of six display digits (eight is better);
- easily accessible, springloaded click keys;
- four functions: +, —, ÷, x.
- a floating decimal;
- two clearing keys (clear entry and clear);
- no key having a dual purpose;
- a rechargeable unit or AC adapter;
- algebraic logic.

Also, the warranty, the reliability of the manufacturer, repair service should be considered.

Like any teaching device, the calculator can create more problems than it can solve if it is misused. It does not replace the need for understanding, skill, or good teaching in mathematics. It is not a substitute for pencil-and-paper skills, although it reduces their use. Creative use of minicalculators after students' mathematical understandings have been abstracted, however, can establish it as a valuable asset among the instructional devices already in today's mathematics classrooms.

Editor's Note: N.C.T.M. has available an excellent bibliography of minicalculator information sources. It may be obtained by writing

N.C.T.M., 1906 Association Drive, Reston, Virginia 22091

and asking for **Minicalculator Information Resources**

PUBLICATIONS AVAILABLE

TO O.A.M.E. MEMBERS

	Regular Price	O.A.M.E. Member Price
Handbook of Trigonometry (1975)	$2.00	N/C
A Preparedness Test for Grade Nine Students (1973)	2.00	N/C
Sample Grade 13 Examinations	1.00	N/C
Grade 11 (Level 5) Sample Examinations	1.00	N/C
Reprints of 1974 & 1975 Gauss Contest Papers (Elementary)	.10	N/C

To receive these please contact:

Mr. R. G. Scoins,
Faculty of Mathematics,
University of Waterloo,
WATERLOO, Ontario.

Boxes, Squares and Other Things (Elementary) A leather guide for a unit in informal geometry		1.80	N/C

Available from:

Mr. M. MacGregor,
Secretary-Treasurer O.A.M.E.
247 Bright Street,
SARNIA, Ontario
N7T 4E9

Proof: Define $f(z) = \cos z + i \sin z$ where $z \in R$ and $i^2 = -1$

$\therefore f'(z) = -\sin z + i \cos z$

$\therefore f'(z) = i f(z)$

$\therefore \frac{f'(z)}{f(z)} = i$ $\quad (f(z) \neq 0$ since $|f(z)| = 1)$

$\therefore \int_0^x \frac{f'(z)}{f(z)} iz = \int_0^x i \, dz$

$\therefore \ln f(x) - \ln f(0) = ix$

$\therefore \ln f(x) = ix$ $\quad$ (since $\ln f(0) = \ln 1 = 0$)

$f(x) = e^{ix}$

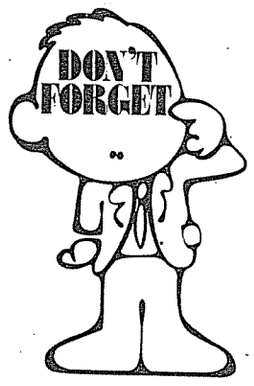
$e^{ix} = \cos x + i \sin x$.

The only potential complaint about this proof is that I assume that the logarithmic and exponential functions are defined in the complex numbers. While this assumption (which is correct) might offend those mathematicians who feel that a proof isn't rigorous enough if anyone can understand it, this proof is at least as rigorous as 99% of the mathematical proofs given in high school. In fact, if we were to make high school proofs as rigorous as some university proofs it would be self defeating, since most students would be totally lost.

Therefore, you now have a proof that you can use at the grade 13 level and thus expose your students to this mathematical masterpiece - Euler's formula - a thing of beauty that should not be restricted to those students who are studying mathematics at university.

q.e.d.o.a.m.e.

BRAIN TEASERS: (i) The "catch in this problem is the word "ears". He carries out two ears on his head (his own) and one ear of corn each day; hence it will take him six days. (ii) 22 birds and 14 beasts.



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For more information
see page 67.

AYIMESE

AN INVITATION TO TAKE YOUR LOGICAL SKILL OUT OF STORAGE AND INTO PROBLEM SOLVING

Dr. Mary Anne Ayim, Department of Philosophy, Faculty of Education, University of Western Ontario

Dr. Ayim has taught secondary school mathematics and now teaches Philosophy of Mathematics.

Foreword

The role of geometry in the mathematics programs of the elementary and secondary schools has been the subject of a great deal of discussion, oftimes very heated, in recent years. The proposed emphasis on transformations in the new Intermediate Division Guidelines, and the opposition this emphasis is meeting in certain quarters, is a recent example of these concerns.

Long-range goals associated with Geometry have been to teach students how to carry along a chain of deductive reasoning - how to think in the "if ... then ..." style and to appreciate the nature of logical reasoning and proofs. If young people are to learn that the validity of an argument can be determined from the structure of the argument then this aspect of geometry, in contrast to simply learning geometric properties, will require more emphasis.

The mini-axiomatic system presented in this article by Professor Ayim warrants consideration as a means to assist in the attainment of those goals mentioned in the previous paragraph. The time required to develop a reasonable understanding of the processes involved should be reasonable and the abstract nature of the exercise, although it may, at times, resemble "symbol pushing", could well overcome the difficulties students encounter when, in an interpreted system, their intuitive ideas about points, lines, and planes lead to contradictions. It is unlikely that such an exercise is suitable for the less-able student but it is worthy of consideration as material to be presented to the advanced level students or as an enrichment exercise.

Professor George Lawson, Faculty of Education, U.W.O.

Ayimese is an example, on a small scale, of an uninterpreted axiomatic system. Ayimese is uninterpreted because the statements and symbols occurring within it have no meaning. It is possible, of course, to give Ayimese an interpretation by assigning a meaning to its symbols, but that has not been done here. As an axiomatic system, Ayimese consists of four basic components: primitive symbols, formation rules, axioms, and rules of inference.

Primitive Symbols. The primitive symbols of an axiomatic system, roughly analogous to words in a natural language, are the most elementary units of expression possible. In Ayimese, there are three primitive symbols: '$', '/', and '#'. These are the only symbols which will occur in the system.

Formation Rules. The formation rules of an axiomatic system are roughly analogous to grammar rules in a natural language—they specify which combinations of symbols are permissible. A legitimate sequence of primitive symbols is called a well formed formula (wff, for short) in Ayimese and a grammatical sentence in English. Ayimese has only one formation rule: Any sequence of primitive symbols consisting of more than one primitive symbol is a wff. For example, ' $ $ $ $', '/ #',

and '# $ / / /' are all wffs in Ayimese. Neither '$', '#', nor '/', however, are wffs, as the violate the formation rule.

Axioms. The axioms are the starting points for deduction. No attempt is made to demonstrate their legitimacy. They are simply assumed to be acceptable. In Ayimese there are two axioms:

A1: $ / / #

A2: # # #

Rules of Inference. The rules of inference specify the legitimate moves that may be made to derive new wffs from axioms or other wffs. In Ayimese there are four rules of inference:

R1: Any wff containing an odd number of primitive symbols may be rewritten with the middle symbol removed. E.g., '/ # / $ $' may be rewritten as '/ # $ $ ' using R1.

R2: Any wff that begins with a '$' may be rewritten in reverse order. E.G., '$ # / / / #' may be rewritten as '# / / / # $' using R2.

R3: If a wff either begins with a '/' or ends with a '#', (or both) then it may be conjoined with any other wff in either order. In conjoining two wffs, only one need fulfill the specified condition, although both may fulfill it. E.g., '$ / / / #' and '$ $ $' may be rewritten as either '$ / / / # $ $ $' or as '$ $ $ $ / / / #' using R3. Similarly, '/ $ $' and '$ #' may be rewritten as either '/ $ $ $ #' or as '$ # / $ $' using R3.

R4: If a wff contains an even number of primitive symbols greater than two, it may be divided in half and either half may be rewritten separately. E.g., '/ # $ / / #' may be rewritten as either '/ # $' or '/ / #' using R4.

Deriving Theorems in Ayimese. Using the two axioms and four rules of inference, Theorem 1—'# # / $' will be derived. The usual pattern of numbering the steps consecutively to the left and justifying each step to the right will be followed. A step may be justified either because it is an axiom (in which case the number of the axiom is cited) or because it is the result of applying a rule of inference to a previously justified step in the proof (in which case the number of the rule and the step(s) to which it was applied are both cited).

T1: # # / $

1.	# # #	A2
2.	# #	1, R1
3.	$ / / #	A1
4.	$ /	3, R4
5.	/ $	4, R2
6.	# # / $	2, 5, R3

Notice that R3 is the only rule which must be applied to two previously justified steps of a proof. The other rules are always applied to single steps. There is no limit to the number of times an axiom or a rule or any particular step can be employed in any particular proof.

Theorems 2 and 3 will be derived and seven further theorems will be left for the reader to solve.

T2: # # # # # # # #

1.	# # #	A2
2.	# # #	A2
3.	# # # # # #	1, 2, R3
4.	# # # # # # # # #	3, 2, R3
5.	# # # # # # # #	4, R1

This system allows a variety of solutions, with the result that many different, but equally legitimate proofs of its theorems are possible. An alternative proof for Theorem 2, for example, is as follows:

1.	# # #	A2
2.	# # #	A2
3.	# # # # # #	1, 2, R3
4.	# #	1, R1
5.	# # # # # # # #	3, 4, R3

T3: / # /

1.	$ / / #	A1
2.	$ /	1, R4
3.	/ #	1, R4
4.	/ $	2, R2
5.	/ $ / #	4, 3, R3
6.	/ # / $ / #	3, 5, R3
7.	/ # /	6, R4

An alternative proof for Theorem 3 is as follows:

1.	$ / / #	A1
2.	/ #	1, R4
3.	/ #	1, R4
4.	/ # / #	2, 3, R3
5.	# # #	A2
6.	# #	5, R1
7.	/ # / # # #	4, 6, R3
8.	/ # /	7, R4

Can you formulate proofs different than those given for Theorems 1, 2, or 3?

You are invited to solve the following seven theorems in Ayimese:

T4: / # # #

T5: # / #

T6: $ / #

T7: $ # /

T8: $ # $

T9: / $ /

T10: $ $ / / # #

Are there any wffs which could not be proved in Ayimese? Can you think of any?

. The idea for such a system comes from the late Professor Douglas Morgan's system, entitled Morganese''.

. Professor George Lawson provided me with several helpful comments on this article. The lternative proofs for Theorems 2 and 3 are his.

MODELS OF MINIATURE GEOMETRY

Dr. St. Augustine Ball, Assistant Professor, Department of Biostatistics, Graduate School of the State University of New York at Buffalo

In ordinary Euclidean plane geometry, we start by supposing there are some things called points and certain sets of points called lines such that certain assumptions or axioms are satisfied. There are infinitely many points on every line and infinitely many lines.

There are geometries called ''miniature'' in the sense that they contain only a finite number of points. In **finite projective geometry,** a ''line'' has a finite number of points.

In setting up a set of axioms for a miniature geometry, we begin with certain ''undefined terms'' - undefined elements and undefined relations. For instance, for the undefined elements, we have ''point'' and ''line''; for undefined relation, we have ''on'', as ''point on line'' and ''line on point''.

Our axioms should be **consistent, independent,** and **complete.** An axiom system is considered **consistent** if contradictory statements are not implied by it. To say that the axioms of a set are **independent** means that no one of the axioms can be deduced from the others. An axiom system is **complete** if it is impossible to state another axiom in terms of the undefined elements such that the new axiom is **independent** of the system.

For example, let this be the set of axioms for ''Axiom System S'':

1. Each pair of lines is **on** at least one point.
2. Each pair of lines is on not more than one point.
3. Each point is on at least two lines.
4. Each point is on not more than two lines.
5. The total number of lines is four.

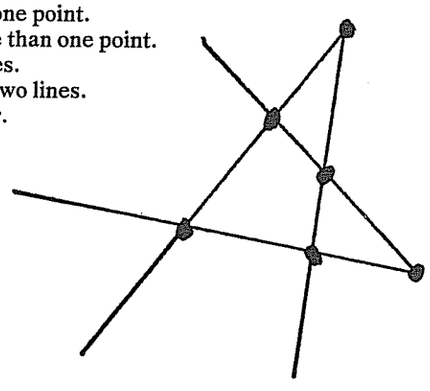


Figure 1

The interpretation is made by constructing in the Euclidean plane a **model** consisting of certain lines and points which will be interpreted as the "points" and "lines" of "Axiom System S". Our model might look like this:

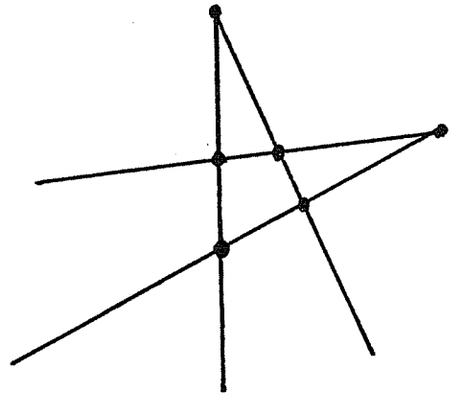


Figure 2

Another example of this same "Axiom System S" might be POSTULATES (or AXIOMS):

1. Each pair of wires has at least one bead in common.
2. Each pair of wires has not more than one bead in common.
3. Each bead is on two wires, at least.
4. Each bead is on two wires, at most.
5. Total number of wires is four.

From 1 and 2, each pair of wires has exactly one bead in common. From 3 and 4, every bead is on exactly two wires.

The **model** for this might be:

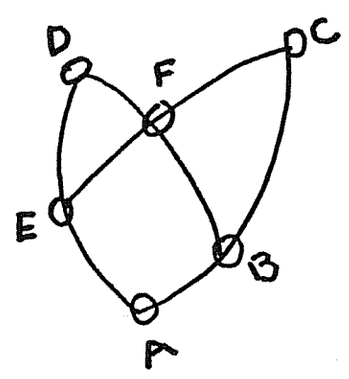


Figure 3

An example of a "finite projective geometry" might be:

Consider (in the ordinary Euclidean plane) a circle with a circumscribed equilateral triangle with its altitudes and let us make the following observations:

1. We consider only the numbered points as being **points** of our projective plane.
2. We agree that the **lines** of the projective plane will be just those triples of numbered points which lie together on a side or altitude of the triangle or on the circle.
3. The "drawn lines" are on the figure to help group the points together and aid the eye.
4. The line consists of only three points.
5. It is easy to check that all four axioms are satisfied, so even though this geometry may seem rather strange, it must be accepted as legitimate. This geometry has a much simpler set of axioms than ordinary Euclidean plane geometry. By definition, a projective plane consists of some things called points and certain sets of these points called lines, such that:

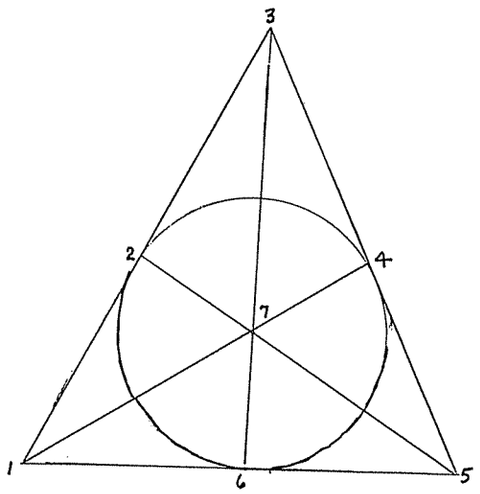


Figure 4

AXIOM I: Every two points are contained together in exactly one line.
AXIOM II: Every two lines contain in common exactly one point.
AXIOM III: Every line contains at least three points.
AXIOM IV: There are at least two lines.

The seven lines in figure 4 are easily identified in this matrix:

LINES \ POINTS	1	2	3	4	5	6	7
(1,2,3)	1	1	1	0	0	0	0
(3,4,5)	0	0	1	1	1	0	0
(1,5,6)	1	0	0	0	1	1	0
(1,4,7)	1	0	0	1	0	0	1
(3,6,7)	0	0	1	0	0	1	1
(2,5,7)	0	1	0	0	1	0	1
(2,4,6)	0	1	0	1	0	1	0

Figure 5

The above 7 by 7 square matrix describes a seven-point geometry. When a point is on a line, we enter a "1" in the column labeled by this point and in the row labeled by this line; if the point is not on the line, we enter a "0" in this row and column. The above matrix is called an **incidence matrix** for the given seven-point geometry because it shows which points lie on which lines and which lines contain which points. In our seven-point geometry, the following postulates hold:

1. Any two distinct points are contained in one and only one line.
2. Any two distinct lines contain one and only one point in common.
3. There exist four points such that no three are contained in one line.
4. Every line contains at least three points.
5. There exist four lines such that no three of them contain the same point.

We notice several things about this seven-point geometry. First, we have already seen that it has exactly seven lines. Second, each line has the same number of points on it, namely three; and each point is on the same number of lines, again three. (2, 4, 6) is considered a line by definition.

These observations are based on the following:

THEOREM: For every finite projective plane P, there is a positive integer n, called the order of P, such that each line of P contains exactly $n+1$ points, and each point of P is contained in exactly $n+1$ lines. Furthermore, P has exactly n^2+n+1 points and exactly n^2+n+1 lines altogether.

The reader is invited to work out a proof for the above theorem.

The seven-point projective plane is of order 2. There are $2+1$ or 3 points on each line; there are $2+1$ or 3 lines containing each point; and there are 2^2+2+1 or 7 points and 2^2+2+1 or 7 lines altogether.

Referring to figure 4 and reviewing the five postulates listed above, it is now possible to identify:
a) the four points such that no three are contained in one line, namely 1, 3, 5, and 7; also,
b) the four lines such that no three of them contain the same point. As a matter of interest, in the model now under consideration, there exist seven sets of these four lines. In each of these sets, no three lines contain the same point.

1.	2.	3.	7.
(2,4,6)	(1,6,5)	(1,6,5)	(1,2,3)
(2,7,5)	(1,7,4)	(1,7,4)	(1,6,5)
(3,4,5)	(3,4,5)	(2,4,6)	(2,4,6)
(3,7,6)	(3,7,6)	(2,7,5)	(3,4,5

4.	5.	6.
(1,2,3)	(1,2,3)	(1,2,3)
(1,6,5)	(1,7,4)	(1,7,4)
(2,7,5)	(2,4,6)	(2,7,5)
(3,7,6)	(3,7,6)	(3,4,5)

q.e.d.o.a.m.e.

I (LOVE)/(HATE) MATHS

J. E. Silcox, Weston, Ontario

It has always bothered me as a mathematics teacher, when I could not make mathematics as crystal clear to all my students after they have studied a topic as it is to me. Admittedly, there are forms of mathematics which I have not even considered since university days and which would be confusing to me now without a refresher course, but ... "how can those students of mine not see it? It is easy!"

I accept the fact that I am not a musician because, although I have been exposed to the art of creating music, I did not practise endlessly in order to become proficient; I was not a "born musician". Similarly, not being born an artist, I could not paint a picture which anyone would want to display anywhere. And so with all forms of sports or any other art form; I have no inborn desire to excel in them. However, mathematics, even though it is definitely an art, has always seemed to me to be quite different; I contend that it does not require being born a mathematician in order to be able to understand and correctly do any high school mathematics. To become proficient at mental arithmetic does require much practice and drill, but as humans we are all born logical to some extent, and mathematics, being symbolized logic, should be self evident to any human.

When I have commented on other people's skills in music, art, etc., they often have modestly made some comment to the effect that their skill was to them as mathematics was to me. I have never agreed! Consequently, last fall I handed copies of the following questionnaire to about one hundred of my acquaintances and others in various walks of life - parents of my students, senior citizens in my wife's Meals-on-Wheels, and so on - to try to ascertain if there was any pattern for adults who "can't do maths", for it had seemed to me that whenever I had mentioned in public that I was a mathematician, someone would say, "Mathematics! I never could do that stuff!"

	yes	no
1. Have you found any direct application of your secondary school maths (excluding arithmetic) in your post school life? If so, please specify: ----------	____	____
2. Do you believe that your study of mathematics prepared you for analyzing situations and making decisions based thereon?	____	____
3. Do you remember: (a) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	____	____
(b) Pythagorean Theorem: "the square on the hypotenuse equals the sum of the squares on the other two sides".	____	____
(c) SOHCAHTOA	____	____
(d) BODMAS	____	____
4. Do you like 'whodunits'?	____	____
5. Did you enjoy your formal study of history in school?	____	____

6. Please check only one of the following: do you prefer informal social gatherings ____, intellectual social gatherings ____, active sports in team

games ____, active sports in individual efforts ____, passive entertainment (theatre, opera, movies, TV, etc.) ____, quiet meditation and study ____?

7. Do you like puzzles? ____ ____
   mechanical ones (jig saw puzzles, SOMA, etc.) ____ ____
   logic (riddles, brain teasers) ____ ____

8. Did either of your parents pursue a math. oriented career or excel in maths.? ____ ____

9. Do you have any children who excel in maths.? ____ ____

10. Do you have any children who dislike maths.? ____ ____

11. Do you remember any math. teacher you liked very much? What do you particularly remember about him/her? ____ ____

Did he/she have a strong sence of humour? ____ ____

Were his/her tests challenging? ____ ____

12. Do you remember any math. teacher you disliked? What do you particularly remember about her/him? ____ ____

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13. Please check only one of the following: did you find high school maths.:

    (a) very enjoyable ____
    (b) satisfying ____
    (c) tolerable ____
    (d) difficult ____
    (e) abhorrent ____

(If you answered a, b, or c, you may skip questions 28 to 37; if you answered d or e, you may skip questions 14 to 27).

14. Do you or did you play any musical instrument? ____ ____

15. Did you ever study Latin successfully? ____ ____

16. Do you consider yourself a good organizer? ____ ____

17. Could you go <u>directly</u> to you copy of last year's income tax form? ____ ____

18. Is your career math. oriented? ____ ____

19. Which did you enjoy most (check one): algebra ____, Euclidean geometry ____, analytic geometry ____, trigonometry ____, calculus ____, statics ____, statistics ____?

20. Could you do some of the non-trivial geometric constructions (such as constructing a circle to touch a line and pass through two other points)? ____ ____

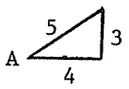
21. Can you multiply 13 × 17 in your head? ____ ____

22. What does $(x-y)^2$ equal? _____

23. What are the factors of $x^2 - y^2$? _____

24. Do you remember what πr^2 stood for? ____ ____

25. What is π "approximately equal" to? _____

26. 

In the given 3-4-5 triangle, what does sin A = ? _____

27. Are you all in favour of the switch to S.I. (metric)? ____ ____

28. Did maths. seem like a guessing game to you: you felt lucky if you got the right answer? ____ ____

29. Do you remember any of your math. teachers telling you that you had to do it their way to get full marks? ____ ____

30. Did any math. Teacher tell you, or imply that you were "stupid"? ____ ____

31. Did any of your teachers make math seem like army parade ground drill? ____ ____

32. Did your mother or father ever refuse to help you with math. because they "never could do it"? ____ ____

33. Did your spouse like high school maths.? ____ ____

34. Is the answer to 2 + 3 × 4 equal to 20? ____ ____

35. Is $\sqrt{x^2+y^2} = x + y$? ____ ____

36. If log 2 = .3, what does that mean that 2 equals? ____ ____

37. Answer only those of the following questions which you can do <u>completely</u> "in your head". (please do not write anything anywhere except the answers).

(a) 13 + 27 = ____ (b) 7 × 53 = ____ (c) 7 - 13 = ____

(d) 11 × 73 = ____ (e) 4910.4 ÷ 1200 = ____ (f) $3 + 7^2$ = ____

(g) $\sqrt{81}$ = ____ (h) 4.78 + 12.05 = ____ (i) 73.4-12.8 = ____

(j) 7.2 ÷ .12 = ____

Of the hundred questionnaires, thirty-four were returned completed, which perhaps is a good percentage response for a survey but not sufficient from which to draw ironclad conclusions. However, the following results were in some aspects very interesting and very revealing:

Only one in thirty-four claimed that math was horrendous and three claimed it to be difficult, although mathematics is always the epitomization in shows, plays, and even educational discussions for all that is difficult or challenging in school! Nine found it tolerable, nine thought it was satisfying and twelve (the mode!) found it very enjoyable. Surprisingly skew to the right! I believe the sample was fairly random, transcending class and culture; but perhaps, being a very academic oriented survey, the questionnaire whispered to the responders, "Answer as you think the teacher wants you to answer.", or perhaps the above surprising nature of the response indicates that if adults hate maths, they stand in awe of it and do not want to tell anyone else, but if they like it they are proud of the fact and want to tell the world.

The full response follows for the benefit of those who are very statistically inclined and wish to exhaust the survey of all possible implications. Following the table I have noted the most obvious observations. In the table, numbers indicate the percentage of positive response in each set, noting that a positive response for questions 34 and 35 is incorrect.

12	9	9	Number of responses in each set		3	1
Enjoyable	Satisfying	Tolerable	Question		Difficult	Horrendous
75	55	55	Direct application inpost school		33	0
90	66	44	Analysis and decision transfer		0	0
50	33	44	Quadratic formula		66	100
66	66	80	Pythagoras		66	100
0	0	0	Mnemonics (c) and (d)		0	0
66	80	66	Like 'whodunits'		33	100
50	44	44	Like history		100	100
75	66	44	Recreational preference:	social	66	
8		11		intellectual		
16				sports		100
25	11	11		movies/TV	33	
25	11	33		meditation		
66	80	44	Mechanical puzzles		33	100
75	55	55	Logical puzzles		0	0
20	0	0	Math parents		33	0
60	55	80	Math children		33	100
10	20	10	Non-math children		0	100
80	90	33	Liked math teacher (comments below)		16	0
66	33	10	Disliked teacher (comments below)		66	100
90	33	80	Musical	guessing game	66	100
66	33	55	Latin	"do it my way"	66	100
75	80	90	Organizer	"stupid"	33	100
100	90	90	Income tax	parade drill	66	100 plus
60	55	10	Math career	no parent help	0	100
66a	33a	33t	Most enjoyable: algebra-a	math	33	0
				spouse		
25t	33e	22a	Euclid-e, an. geom-g, trig-t			
16e	33s	11e	calc-c, statics-p, stats-s			
		11g				
		11s				
40	66	22	Construction-deductions	$2+3x4=20$	0	100
100	100	55	13 x 17	$\sqrt{x^2+y^2} = x+y$	0	100
40	33	33	$(x-y)^2$	log 2	0	0
25	33	11	x^2-y^2	arithmetic	80	60
90	55	55	πr^2			
75	55	55	π			
16	22	11	sin A			
90	55	55	S.I.			

Some general observations from the data:
The more you understand mathematics, the more you use it in life and the more you prefer abstract study.

Euclid was never popular, but everybody remembers Pythagoras.

Mathematics students and history students appear to be mainly disjoint sets (and yet both are obligatory subjects in the new curricula!)

A mathematics/music correlation is more evident than mathematics/latin; a transfer effect from mathematics to organizational ability is strongly indicated. There is no evident correlation between interest in mathematics and in any one form of recreational ability.

Mathematics is not hereditary, but non-math people avoid mathematicians for spouses.

Even math lovers do not remember high school mathematics: they claim they can multiply 13 x 17 but cannot factor $x^2 - y^2$! Incredible as it seems they do not all remember!

The common man in the streets is not eagerly awaiting the impact of complete S.I.

Good mathematics teachers are remembered for: their challenging tests, their strong sense of humour, their spirit of fun, their being fair, patient, superior, friendly, their carefully explaining difficulties.

Poor mathematics teachers are disliked for: being tedious, prejudiced, conceited, sarcastic, unprepared, unknowledgeable, inflexible, and unable to take charge.

The impact of poor math teachers on the set of math haters seems more powerful than the healing effect of any good teachers they must have had: in response to the questions concerning mathematics being a guessing game, or a teacher demanding it be done one way, or calling a student 'stupid', or acting a drill sargent, the math hater not only confirmed such notions and methods but pointedly emphasized their reaction. This might indicate that a poor math teacher early in the life of a student did irreparable damage before a good teacher had any opportunity to save the student. Of course, the questions were loaded, but the fact that responders **triple** checked 'yes' and underlined it cannot be ignored.

As a positive response to #34 and #35 is incorrect, it is interesting to note that all who found mathematics difficult avoided these traps; perhaps the very wording of the question begged a negative response.

It seems logs are soon forgotten, but I wonder how many of those who do not know what log 2 = .3 means could actually understand and use log curves.

The last point of note from the survey is that even those who as students found math difficult or horrendous, can as adults answer 60-80% of basic arithmetic questions correctly; I doubt if they did as well in school. Could it be that errors in arithmetic outside school cost money, instead of just marks?

Probably each mathematics teacher who reads this has complained at various times of students who will not answer a question as it is asked. For what consolation it is worth, adults are no different! Question 1 specified the exclusion of arithmetic but one responder's reply was just "percentages"; question 6 asked them to check only one, but four persons thought it meant "more than one", two or three adults could not remember any math teacher they liked or disliked, but then they remembered what they did or did not like about this unremembered person!

Since an overwhelming majority of what seems a random sample of adults seems fairly good at mathematics and/or likes it, I still claim that a mathematician is primarily a product of good mathematics teachers, that they are made, not born. Therefore, let us continue to press for the essential requirement for good mathematics education, namely only mathematically inclined adults to be allowed to teach mathematics, particularly in the formative years from grade 1 to 6.

BOOK REVIEWS AND RESEARCH

RESEARCH AND THE CLASSROOM TEACHER 1: THE CLASSICAL PARADIGM

Dr. W. Higginson, Queen's University

In a paper at a major educational conference in 1976 a speaker characterized much of educational research as "the science of the strange behaviour of children in strange situations with strange adults for the briefest possible periods of time". Had this paper been given by a radical critic of the system in one of a large number of sparsely attended sessions in a huge meeting, this statement would have attracted little attention. This, however, was not the case. The context for the quotation was none less than the Award Address to the American Educational Research Association (Bronfenbrenner, 1976). In some ways this speech can be seen as a 'my gosh I have no clothes on' statement from the emperor, for it is a long and scholarly document which implies, among other things, that the classic, statistically-oriented model of research has been a failure in the field of education.

For many mathematics teachers this is hardly news. The research literature in mathematics education has, for far too long, been largely an inaccessible jumble of narrow, unimaginative, poorly-designed and badly-executed experiments. In very many cases the naive but hopeful classroom teacher who has set out to find 'what research has to say' about a given topic, returned with a record of conflicting results or that old favourite of the 'ed research biz', "NSD" (no significant difference).

Traditionally there have been two major problems with research in mathematics education; accessibility and quality. In the last decade considerable progress has been made in making the results of research endeavors available to interested parties. The two institutions largely responsible for this progress are the National Council of Teachers of Mathematics (NCTM) and the Education Resources Information Center (ERIC). The combined efforts of the NCTM publication policy and the indexing/microfiche programme of ERIC have meant that masses of well-indexed and current information is easily available to teachers. (For a list of ERIC collections in Ontario and other details about publications see the end of this article).

The NCTM's **Journal for Research in Mathematics Education** (JRME) is now quite well established. At the elementary level there are three publications written specially for the teacher who wishes to have a convenient summary of research results (Callahan 1975, Suydam 1975, Suydam 1976). At the secondary level a recent teacher's handbook (Travers 1977) is the first to make research results an integral part of its material.

Unfortunately, one cannot report equal progress on the quality front. The results which the ERIC/NCTM people are doing such an admirable job of systematizing are in all too many cases not worth making public. (Perhaps the obscurity, oblivion and inaccessibility of the pre-computer days had its merits). The JRME is perhaps the publication which suffers most because of this. Page after page of formal, compact, symbol-filled articles almost defy the reader to attempt to follow the argument outlined. The newly appointed and very able editor, James Wilson, certainly has a 'sow's ear syndrome' challenge on his hands with this journal.

Why should it be that there is so little first-class work done in the field? One major reason, in the opinion of this reviewer, is that the structure of educational research is not conducive to the production of high quality studies. The great majority of mathematics education research studies are carried out by people in graduate programmes. For very many of these people this study will be the first and the last one of its type.

A second, and perhaps more important inherent weakness of the classical research paradigm lies in the inappropriate assumptions on which it is based. In an attempt to appear scientific, educational researchers have accepted an antiquated vision of simple cause and effect linked to primitive classifications and sophisticated statistical techniques. To compare Grade Three Class A, Teacher B, and Treatment C with Grade Three Class X, Teacher Y and Treatment Z (and all the combinations thereof) may be convenient and attractive from the viewpoint of research design; however it is not a basis for concluding anything about A, B, C, X, Y, or Z. Stated differently this criticism says that in a typical classroom one has not a 'two-body problem'with one

T (teacher) and one C (class), but a 'thirty-six body problem' with one T and thirty-five C (children). The latter is a very much more complex situation than that former; it has, however, the merit of reflecting the reality of the classroom.

Fortunately for our discipline some classical studies have transcended these difficulties and there are now a number of studies appearing based on a different paradigm which is much closer to the one Bronfenbrenner outlines. We shall turn to some of these studies in the second part of this article.

Bronfenbrenner, Urie. **The Experimental Ecology of Education.** Teachers College Record 78 (2), December 1976. (157-204).
ERIC/SMEAC Educational Resources Information Center. Information Analysis Center for Science, Mathematics and Environmental Education, 1200 Chambers Road, The Ohio State University, Suite 300, Columbus, Ohio, 43212.

Ontario Information Systems:

EISO - Educational Information Systems for Ontario,
EISO Search Analyst, OISE Library, S218
252 Bloor St. West, Toronto M5S 1V6
416-923-6641 *487.

ONTERIS - Ontario Educational Research Information System
Ministry of Education
Research and Evaluation Branch
Room 1215, Mowat Block
900 Bay Street, Toronto

ERIC Microfiche Collection in Ontario. In the university library system of: Brock, Guelph, O.I.S.E., Ottawa, Queen's and Western; in the board of education library of Toronto, Toronto Separate and North York and in the National Library of Canada.

Journal for Research in Mathematics Education. James W. Wilson, University of Georgia, Editor. A journal published by the National Council of Teachers of Mathematics 1906 Association Drive, Reston, Virginia USA 20091. Five numbers per annual volume. Volume seven, 1976, 336 pp. $8.00 p.a. to NCTM members.

Elementary School Mathematics: A Guide to Current Research. Fourth Edition. Callahan, Leroy G. and Glennon, Vincent J. Washington: Association for Supervision and Curriculum Development, 1975. 0871200767. 188pp. pb. $5.00 Available through NCTM.

Using Research: A Key to Elementary School Mathematics. Suydam, Marilyn N. and Weaver, J. Fred. Columbus, Ohio: ERIC, Centre for Science and Mathematics Education, OSU, 1975. 136 pp. $3.25. Loose bound. Available through NCTM.

Mathematics Teaching. Travers, Kenneth J.; Pikaart, Len; Suydam, Marilyn N. and Runion, Garth. New York: Harper and Row, 1977. 0060452331. 591 pp. $12.00

Classroom Ideas from Research on Computational Skills. Suydam, Marilyn N. and Dessart, Donald J., Reston Va.: National Council of Teachers of Mathematics, 1976. 58 pp. $3.00 pb.

BOOK REVIEW

Geometry Jacobs, Harold R. San Francisco: W.H. Freeman, 1974. xii + 701 pages. $9.00

This is an excellent book! The layout and style are similar to that of Jacobs' superb book **Mathematics: A Human Endeavour.** Motivation is achieved and interest is maintained through the use of entertaining and appropriate photographs, drawings and cartoons relating to geometry.

There are essentially two ways to approach the teaching of geometry in the school. One way is to treat geometry as the study of space and shape; the other is to treat geometry as a deductive system in which theorems are proved by deductive reasoning from axioms and postulates. It is not easy to blend these two approaches. The "space and shape" approach emphasizes the ways in which transformations and symmetry develop and relate geometrical phenomena in the environment and is not greatly concerned with proving results. The "deductive" approach emphasizes proof and the way in which the proof is to be written. The "deductive" approach considers applications to be secondary to the formal development of theorems.

Jacobs does a commendable job of the unenviable task of blending these two approaches together in a single book. The deductive thrust is most noticeable in the first four chapters. But even here, amid the definitions, postulates, and emphasis on proof and reasoning, there is a refreshing lightness achieved through the use of interesting and amusing examples and illustrations. Formal, yes - but not plodding or pedantic, and never dull. The applications and the relationship of geometry to the real world are always evident in the book. (Additional applications and ideas on how to motivate students to want to learn geometry are presented in the useful Teacher's Guide, available separately, ($5.00)).

The sixteen chapters deal with the nature of deductive reasoning, lines and angles, basic postulates and theorems, congruence, transformations, inequalities, parallel lines, quadrilaterals, area, similarity, right triangles (including some elementary trigonometry), circle geometry, concurrence theorems (including Ceva's Theorem), regular polygons, geometric solids, and non-Euclidean geometries.

A student who has mastered the material in this book would have a thorough grounding in high school geometry and no doubt would have found the study interesting.

This book is highly recommended to all persons interested in geometry. It is an extremely valuable source of ideas for teachers of geometry.

Hugh Allen, Faculty of Education, Queen's University

Network: A Mathematics Series The Leapfrogs Group, Coldharbour, Newton St. Cyres, Exeter, U.K.

An imaginative and energetic group of mathematics teachers in the United Kingdom have blended together under the name of 'The Leapfrogs Group' to produce an attractive and inexpensive set of material for children of ages 10 - 14. At the moment the book side of **Network** consists of three types of title: 'Action' books, 'Link' books, and 'With Few Words' workbooks.

Action books are sixteen-page A5 booklets which are intended to bridge the gap between workcards and textbook chapters. They invite the reader to explore the consequences of specific actions. The thirteen action books published to this point are: Animations, Bands, Chance, Codes, Cubes, Doodles, Dots, Folds, Moves, Orders, Pegboards, Prints, and Tiles. The cost of these booklets is 35p. which would probably work out to approximately $0.80 with postage.

Two of the three Link books, Links, and Leads (= $2.50) are subtitled: "A book of pictures and diagrams which have been selected with certain mathematical ideas in mind, not to illustrate these ideas but to show parts of the real world from which mathematics comes. Those who turn the pages of this book are invited to reflect on what they find herein and to pursue such actions as they think appropriate in drawing, writing, making models, talking, making their own collections of pictures etc.,etc."What else is there to say except that the pictures and diagrams selected are intriguing, whimsical and thought provoking. The third Links book is entitled 'Leaves' (= $3.00) and it is an eighty page "flicker book" on 12 mathematical themes.

There are three With Few Words workbooks (= $0.60). The Green book is a set of nets for various mathematical models and the Blue and Orange books consist of a range of non-verbal numeric and geometric tasks.

The Leapfrogs Group intends to keep increasing the number of **Network** titles. Upcoming booklets will be on Calculators and Fitting. Available are posters, film-loops, slide packs and materials on the history of mathematics.

The Leapfrogs writing arrangement is unique. Any teacher who 'subscribes' to the Network series (through the Exeter address) receives copies of the Newsletter which tells of upcoming plans. As a member of the Leapfrogs Group one can make suggestions and carry out classroom testing of materials in the development stage. Any profits from the sale of material are used as development funds for new projects. The Hutchinson Publishing Group (3 Fitzroy Square, London W.1, England; Canadian agent: GLC Publishers, 115 Nugget Ave., Agincourt M1S 3B1) have published the first titles and it appears that this relationship will continue.

It is unlikely that the Leapfrogs Publications can be used directly in very many classrooms in the province at the moment. They make excellent open-ended project starters, however, and very good 'ideas' sources for the math staffroom. One last word of warning: librarians and art teachers have a tendency to 'liberate' this sort of book for their own purposes, so be on guard.

Primary Mathematics Today. E.M. Williams and Hilard Shuard. New Metric Edition, London: Longman, 1976. 455pp. Limp Binding: 0582180422 $15.00. Case bound: 0582180430 $20.00.

The first edition of this book appeared in 1970. At that time it was suggested by some reviewers that this work might become the classic handbook for teachers of elementary school mathematics. The intervening years have produced no comparable publication to dispute this title and the changes made in this edition should serve only to strengthen PMT's claim.

The outstanding features of the book are its comprehensiveness and its smooth synthesis of mathematics, psychology and pedagogy. The forty chapters deal with ten major themes. These, with the number of chapters devoted to each, are: children's early mathematical development (2), children's activities using number (7), the development of the number system (9), the growth of notation and representation (3), children's ideas of space (6), the development of spatial concepts (3), children's experience of quantity and its measurement (5), the development of the idea of measurement (3), the use of mechanisms (1) and the unity of mathematical structure (1). Within this format we are taken smoothly from an intelligent discussion of the stages of intellectual development to an outline of groups of transformations.

Mrs. Williams and Miss Shuard are highly respected members of the British mathematics education community. Their sensitivity to the needs of classroom teachers and their familiarity with mathematics can be sensed on nearly every page. The book is attractively produced with a large number of diagrams and eight pages of colour photographs. The latter pages include many examples of children's work, a feature which should be part of more books of this type.

Most of the revisions for this edition have been in the direction of 'decimal system' and the introduction of metric terms at the expense of advanced work with fractions. It is unfortunate that more consideration was not given to electronic calculators and their role in the elementary school classroom. The 'cogs and levers' example of a "desk calculating machine" (p. 166) is one of the few instances where the validity of the title for 1977 is called into question.

Generally speaking, the language and terminology provide few problems for the North American reader - we can easily substitute for 'minis' and 'Rolls-Royces' and the relation 'is the father of' on the set of Tudors is not a critical one. Somewhat more confusing is the question of Canadian rights. Following the 1970 publication of the first edition in the U.K. by Longman, Addison-Wesley brought out an American edition in 1972. The contents stayed exactly as they were but the title was changed to **Elementary Mathematics Today: Grades 1-8** (0201086441, $15.00). The question of Canadian rights seems to be somewhat cloudy and at one time both publishers, and later neither, claimed to be handling the publication.

In any case, whoever the publisher may be, this is a very fine book. If your budget can afford only one teacher's handbook at this level, this probably should be it.

Math Is/3 Frank Ebos, Bob Tuck. Thomas Nelson and Sons Canada Limited, Don Mills, Ontario, 1976. 0-176-31584-5 $7.25.

This review of the book Math Is/3 will be specifically directed toward its use as a text book for a university bound stream (Ontario 5 year stream).

This is the third book of the **Math Is** mathematics series. The same printing features are evident again as in the earlier books - interesting photos, useful graphs and two colours of ink - all of which add to the over all impact of the book.

There are twelve sections in the book starting with "a basic foundation" and working through algebra including polynominal applications, equation solving and problems. The geometry section follows and includes the basic constructions, mensuration and the beginnings of co-ordinate geometry. The book ends with a brief look at "solving everyday problems" which is a beginning to basic statistics.

In working through **Math Is/3**, one of the most obvious features is the great number of exercises for the student, (answers are provided). Many of the exercises have an introductory statement which sets out the objective of the exercise. Other features of the book are the inclusion of recreational mathematics, math tips, and practice exercises which are liberally interspersed throughout the book.

In summation, the reviewer found the book very well written, easy to understand, very practical in its applications and the best book now available for use in this course.

Walter Schofield, Oakridge Secondary School, London, Ontario.

BRIEFLY NOTED

Mathematics teachers will find a number of experiments of interest in the most recently translated of Piaget's books. (Jean Piaget. **The Grasp of Consciousness: Action and Concept in the Young Child.** Cambridge, Mass.: Harvard University Press, 1976. 0674360338. 360pp. $15.00 Translated from the French edition of 1974.) The objective of the experiments is to see how children gain cognizance, or become conscious of their actions and knowledge. Consistent with his other work, Piaget finds that children are able to perform actions "correctly" and hence have "knowledge" on at least one level, long before they can articulate how or why they did these particular actions. The fifteen ingenious experiments include ones involving objects launched by a sling, pushing symmetrical and asymmetrical objects, and the Tower of Hanoi.

Another active octagenerian is the noted mathematician/problem solver George Polya. His most recent book (George Polya. **Mathematical Methods in Science.** Washington: Mathematical Association of America, 1977. 088385600X. 234pp. pb. $5.00) has just been published as Volume 26 of the **New Mathematical Library.** A somewhat unusual book in format (Chapter four is one page long, chapter five 74 pages long), this work nonetheless contains Polya's usual rich mixture of science, history and mathematics. Teachers who long for the 'good old days' of 'Trig and Statics' will find this a particularly attractive work with its extensive treatment of Statics and Dynamics. Chapter one on Measurement and Successive Approximation provides a number of attractive examples from the history of astronomy. Chapter five deals with differential equations and their use in science.

The Science Council of Canada is sponsoring a conference entitled 'Educating Teachers of Mathematics: The Universities' Responsibility' to be held at Queen's University, 31 August - 3 September.

Some 35 mathematicians and mathematics educators from across the country have been invited to participate in this follow-up of one aspect of the Science Council's Background Report #37, **Mathematical Sciences in Canada.**

INTERVIEW

Taped Talks with Bob Smith

• • • LAURIE BUXTON

Mr. Buxton is the Chief Inspector for Mathematics of Inner London Education Authority. It was my privilege to interview him on June 7, 1976 at County Hall, located on the south bank of the Thames River, London, England, and continue a discussion started the day before at Ladroke Maths Centre.

R.S. Laurie, you have been interested in mathematics all your life. You have been a part of the revolution in mathematics during the sixties, here in Britain. You are aware that when a pupil reaches adolescence, when he moves into the structure and the discipline of mathematics that some seem to catch and understand and move on to enjoy the subject. Unfortunately, many - perhaps a majority of the students do not. They seem to fail in that area that Richard Skemp talks about as "building concepts on concepts," and ever after, even in their adult life, even in such a place as the Houses of Parliament, they admit that they never did like mathematics, they never understood mathematics. Why?

L.B. It is very difficult to know why people are switched off. Before I speak of reasons, let me say a word about change. One of the things that made me least concerned about changes - after all change always concerns one, you don't want to practice on people and do something worse than they had before - was that when you talk to the adult population about mathematics you find that 90% of the women had a strong feeling against mathematics and that more than half of the men have the same view. Thus change cannot make the situation worse.

R.S. Why is there this strong dislike?

L.B. Perhaps it is related to the nature of the subject. People believe it to be an unemotional subject; yet it is found to create the strongest negative emotions of any subject in the curriculum - fear and anxiety. The standard defence mechanism is the boast that one is no good at it and so if enough pupils around you are saying the same thing, you feel happy. You are able to resolve your fears and anxieties.

R.S. There must be more basic reasons.

L.B. I started doing some tape recordings with intelligent adults who didn't get very far in mathematics, who did not get their '0' level, for example. In trying to track backwards with them into what happened, characteristically, they would say that (1) things got into a fog, were not understood or (2) they were told that they were no good at maths. The subject confronts one with a clear cut question of whether you can do it or not. In solving a problem, if no solution comes, you are faced with a blank piece of paper. This does not happen in any other subject. No one likes nor enjoys looking foolish. Time may well be another part of the problem. It varies enormously across the ability range, far more so than in any other subject. A piece of work which one student can mop up in a few minutes will take a second student an hour or so, even if he is successful. So there are lots of problems of this sort.

R.S. How would you release the pupils from these problems?

L.B. Well, various changes have been tried. The attack started with change in content, because it was felt that a lot of content was not seen as being very useful. Yet, there is a curious ambivalence here. Some of the content was seen as being of little use yet the mass of the people saw mathematics as being a very useful subject.

R.S. True. It is often the parent who insists that his child will take mathematics.

L.B. Change in content however may not alter attitudes at all. Some modern topics may have no more relevance to the everyday world than the traditional ones. They may merely suit the structure of mathematics better. I am not persuaded that kids bother too much about relevance.

Then there were moves toward making mathematics more interesting - Edith Biggs practical approach. This was basically a change in methodology where the Nuffield is the SMILE (1) scheme - a program to individualize both the pace of the work and its direction. It has taken the pressure off the kids in terms of competition with others. The pupil can work at a pace that is appropriate to them. This takes the pupil out of the whole-class situation where he might well fall behind, get into a fog and then must either battle his way out or give up.

But, having said all these things, I think what gets the kids more than anything, is the teacher's attitude. It is a fact that primary teachers generally do not have much mathematics. Let us not be critical for many are very good teachers. Not only do they not have much mathematics, but what is more important, they are anxious about it. And more than anything else, what a teacher does with a class, whatever mould she has used, she communicates attitudes. I am keen on this. I first try, when teaching a class, to show my enthusiasm, to create an atmosphere that shows my enjoyment of maths. So, one of my main concerns is that the atmosphere at the early stages of mathematics instruction is dominated by teachers who really do not enjoy the subject of mathematics and no matter how much they try to conceal it, that feeling will manifest itself. At the secondary level, regrettably, we have had to few mathematicians coming into the field, that there just aren't the people who understand what it is all about.

R.S. Well, I suppose both are relative and hopefully will improve. Before, I leave you, do explain what you mean by distancing.

L.B. Children, I believe, do not like going through a lot of prodecures, without knowing what it is all about. I have been concerned that we should teach or encourage children to learn from different distances. e.g. On the one hand pupils might well be solving a small special problem of some sort but at another time they should be asking the question, ''What is Geometry all about''.

R.S. Thank you, Laurie Buxton.



1. SMILE, Smith, R.S., **A Model for Individualizing Instruction,** p. 8-10, ''Ontario Mathematics Gazette'', Vol. 15, No. 3, March 1977.

DIVERSIONS

BY: T.G. Friday

Welcome back to the new year, for although I am writing this early in July in a small log cabin on Manitoulin Island, by the time you read it, it will be September, and the new school year will be under way.

It is quite a rare occurrence to come across a piece of Mathematics which is both neat, or to be prosaic, beautiful, and which is also directly applicable to the high school mathematics course. At the annual meeting of the O.A.M.E. in May at Ottawa University, I was fortunate enough to attend one of the sessions given by Dan Moalem, a teacher from New South Wales, Australia, from whom I heard the following two goodies. These added to my belief that transformation approaches have considerable value in high school mathematics.

Goodie number 1

To prove that the slopes of lines which are perpendicular to each other are negative reciprocals.

PROOF Consider any line with finite slope m.
Since angle is preserved under translation we can translate this line to pass through the origin without loss of generality. One point on this translated line will be P = (1,m).
We then have the diagram shown below.

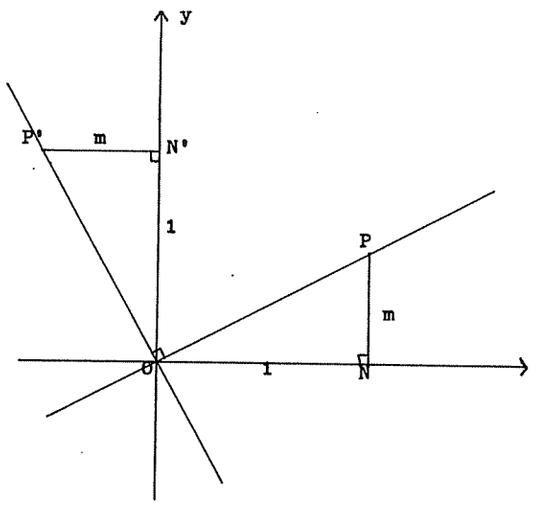


Diagram 1

Rotate Δ ONP through 90° about O. This is a congruence transformation. This will map N into N' on the y-axis.

. . the slope of OP', which is our required perpendicular line is

$$\frac{-ON'}{N'P'} = \frac{-ON}{NP} = \frac{-1}{m}$$ as required.

I should also like to give credit to Mrs. Roma Acheson from whom I had previously obtained an almost identical proof.

Goodie #2

A different and perhaps simpler method of obtaining the expansions of sin(A+B) and Cos(A+B).

Many transformations in the cartesian plane can be represented by a 2 × 2 matrix.

E.G., the point $x_1 = (3,2)$ under the transformation $T = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

gives $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ or $T \cdot x_1 = x_1'$

so $(3,2) \to (4,5)$.

Similarly $x_2 = (1,-3) \to (5,9) = x_2'$

or combining these $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 5 & 9 \end{pmatrix}$ or $T \cdot (x_1,x_2) = (x_1',x_2')$

Consider this same transformation on the points (1,0) and 0,1)

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, i.e., $(1,0) \to (2,3)$ and $(0,1) \to (-1,-2)$.

Note that this gives $T \cdot I = T$ where I is the identity matrix. Hence we have a method for obtaining very simply the matrix of a required transformation. E.G., for a rotation of 90° named for convenience $R_{\pi/2}$

$(1,0) \rightarrow (0,1)$ and $(0,1) \rightarrow (-1,0)$

hence $R_{\pi/2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

$$R_{\pi/2} \cdot I = R_{\pi/2} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

For reflection in the line y = x named T'

$(1,0) \rightarrow (0,1)$ and $(0,1) \rightarrow (1,0)$

so $T'\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$$T' \cdot I = T' = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

For a rotation of θ; transformation R_θ, consider the unit circle definitions of sin θ and cos θ,

$(1,0) \rightarrow (\cos\theta, \sin\theta)$

and $(0,1) \rightarrow (-\sin\theta, \cos\theta)$

hence $R_\theta \cdot \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$

$$R_\theta = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

This produces very simply the rotation mapping required in the conics section of the functions course.

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}x\cos\theta - y\sin\theta\\ x\sin\theta + y\cos\theta\end{pmatrix}$$

Consider the combination of mappings, a rotation of A followed by a rotation of B. This will give a total rotation of (A + B).

Hence $R_B \cdot R_A \cdot \bar{x} = R_{A+B} \cdot \bar{x}$

or $R_B \cdot R_A = R_{A+B}$

$$R_B \cdot R_A = \begin{pmatrix}\cos B & -\sin B\\ \sin B & \cos B\end{pmatrix} \cdot \begin{pmatrix}\cos A & -\sin A\\ \sin A & \cos A\end{pmatrix}$$

$$= \begin{pmatrix}\cos A\cdot\cos B - \sin A\cdot\sin B & -\sin A\cdot\cos B - \cos A\cdot\sin B\\ \sin A\cdot\cos B + \cos A\cdot\sin B & \cos A\cdot\cos B - \sin A\cdot\sin B\end{pmatrix}$$

But this equals $R_{A+B} = \begin{pmatrix}\cos(A+B) & -\sin(A+B)\\ \sin(A+B) & \cos(A+B)\end{pmatrix}$

Hence by equating equivalent terms

$$\cos (A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

and $$\sin (A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B .$$

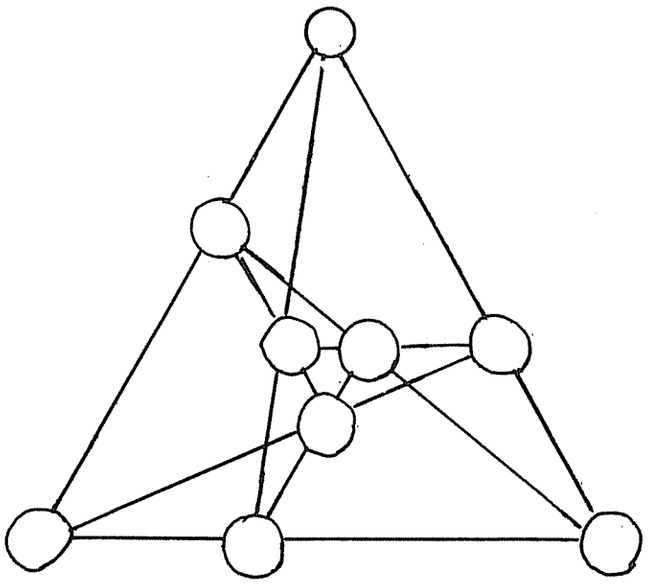
I have never seen these results demonstrated in a simpler way at a level which is applicable to high school students.

TRI-HEX

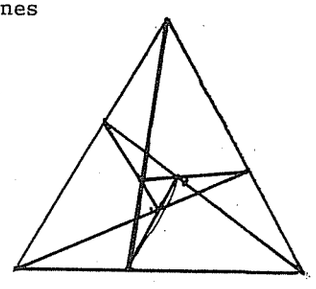
David Boyle, A.B. Lucas Secondary School, London

Tri-hex is a game similar to X's and O's but played on a special equilateral triangle. Each person takes a turn placing an X or an O, the first to have 3 in a row wins.

The different power plays for the first person to win or the second person to draw is a refreshing change from the standard X's and O's.

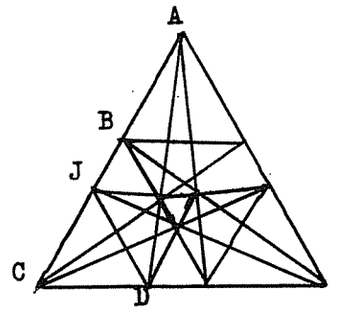


Problem. Construct the TRI-HEX board, diag. 1, or locate 9 points and 9 lines so that a line is defined by 3 collinear points and a point is defined by 3 concurrent lines.



diag. 1

Analysis. Symmetrical properties possessed by the diagram suggests that the TRI-HEX board is only part of a much more complex board (diag. 2).



diag. 2

Examining one triangle ACD suggests that AB = JC = CD = x

BJ = JH = y

JH ‖ CD

BH ‖ JD

DG ‖ AC

and ACD = 60°.

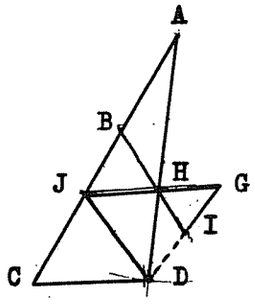
Thus AJ:JC = AH:HD = AB:BJ

or $(x+y) : x = x:y$

thus $x^2 = xy + y^2$

and $(\frac{x}{y})^2 - (\frac{x}{y}) - 1 = 0$

or $\frac{x}{y} = \frac{1 \pm \sqrt{5}}{2}$



diag. 3

Thus to construct triangle ACD requires the GOLDEN RATIO. Now if CD = JG = x = $1 + \sqrt{5}$ and JH = y = 2, then HG = $\sqrt{5} - 1$ and

$$\frac{JH}{HG} = \frac{2}{\sqrt{5}-1} = \frac{2(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{\sqrt{5}+1}{2}$$

again the golden ratio.

Construction. Construct triangle AJH so that AB = $1 + \sqrt{5}$, BJ = JH = 2 and angle AJH = 60°.

Join BH and construct JD parallel to BH meeting AH extended at D.

Construct CD parallel to JH, meeting AJ extended at C.

Thus AB : BJ = AH : HD = AJ : JC

or $(1+\sqrt{5}) : 2 = (3+\sqrt{5}) : JC$

therefore $JC = \frac{2(3+\sqrt{5})}{1+\sqrt{5}} = \frac{2(3+\sqrt{5})(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = 1+\sqrt{5}$

Construct rhombus JCDG so that HG = JG - JH

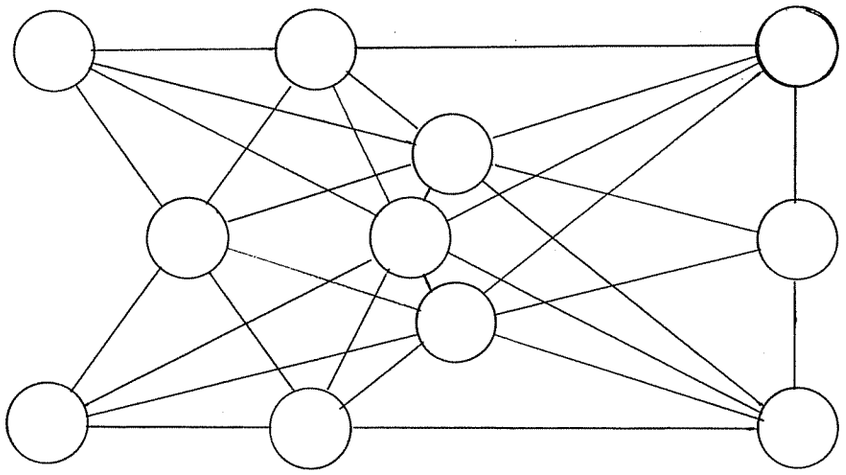
$= 1+\sqrt{5} - 2$

$= \sqrt{5} - 1$.

To complete the HEX-BOARD rotate the diagram 120°, and the 240° about the centre of the equilateral triangle HGI so that H → I, I → G,... A → C etc. Using the properties of rotation the remainder of the diagram can be established/proven.

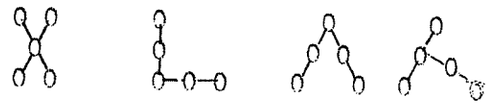
TWO - LINE TORMENT

George Lawson, Faculty of Education, U.W.O.



A game for two players. Each player has a set of five counters with each set being a different colour. The players decide who will make the first move which consists of placing a counter on an open circle on the board. The counters are placed on the open circles, one at a time, with the players alternating moves, and may not be moved until all counters are on the board. The first player to form two straight lines with three counters in each line is the winner.

Since each player has only five counters it is obvious that in a winning situation one counter will be common to both lines. Possible winning combinations are:



If, after all the counters have been placed on the board a winner has not been determined, the players continue to move in the following fashion. A counter may be moved **along** a **straight line** to an open circle, but counters may not be jumped or captured. Play continues in this fashion until a winner has been determined or both players agree to a draw.

"TOOROOT"

Larry Santa, Hammarskjold High School, Thunder Bay

Here is an entertaining game that my classes have enjoyed playing. The game provides a fun review of formal square root involving the Pythagorean formula. To play the game effectively, the students should be familiar with the formal square root and Pythagorean formula beforehand.

HOW TO PLAY

Divide the class into two sections. Let us call the two sections LIONS and ROBINS. Suppose the LIONS are on the left side and the ROBINS on the right side (when facing front).

On the board, in front of each section, is a right-angled triangle with letters h, a, b. On the teacher's word, each team begins to calculate values for h (hypotenuse) and a,b (sides) using formula $h = m^2 + n^2$, $a = m^2 - n^2$, $b = 2mn$ with specific values for m and n unknown to the opposing team.

As soon as the letter values are calculated, one team member (let's say from the LIONS) must rush up to the teacher and give the value of h. Immediately after this, some LION member must write the corresponding values for a and b on the triangle in front of the ROBINS. The ROBINS, in the meantime, have been attempting to do exactly the same thing but with different values for m and n.

A timer, at this point, informs the teacher on how much time has elapsed after the recording of a and b. (Say $L_1 = 50$ for the LIONS and $R_1 = 30$ seconds for the ROBINS). When the LIONS see the values for a and b on the triangle in front of them, they immediately proceed to calculate h using the Pythagorean formula and the formal square root. The ROBINS respond similarly to the values for a and b on the triangle facing them.

As soon as h has been calculated, some team member rushes up and writes down the value of h on the triangle in front of his section. The timer notes the total time that has elapsed from the start. (say $L_2 = 130$ sec for the LIONS and $R_2 = 140$ seconds for the ROBINS). Since $R_1 + R_2(170) < L_1 + L_2(180)$ the ROBINS would be declared the winners although they finished 10 seconds later (i.e., $R_2 - L_2 = 10$ seconds).

PENALTIES

Time penalties (perhaps 10-20 sec.) should be given for certain minor infractions such as: writing numbers in the wrong places, writing numbers illegibly, giving wrong value of h to the teacher.

An important penalty involves an error in calculating values of a and b which would cause the opposing team to waste time working on wrong values. Penalty suggested: TIME LOST + 30 seconds.

Occasionally, a team may generate too much confusion and should be disqualified. The opposing team would be declared the winner of that round.

CODE NUMBERS AND HOW TO START

Before starting the game, the teacher approaches each team and gives specific base number values for m and n unknown to the opposing team. Let us say the Lions' base numbers are $m_L = 9$, $n_L = 4$ and the Robins' base numbers are $m_R = 11$, $n_R = 7$. (The teacher can adjust

these values according to how heavily stacked one team is against the other).

At the starting signal from the timer, the teacher might tell the LIONS to adjust their base numbers by adding 2, so that $m_L = 11$, $n_L = 6$ and the ROBINS to adjust their base numbers by subtracting 1 so that $m_R = 10$, $n_R = 6$. Using these numbers, each team proceeds to play the game.

To play successive rounds, each team is merely assigned a new adjustment figure to their original base numbers. If necessary, the base numbers can be occasionally changed.

In the space of 20-30 minutes, several rounds can be played to determine a winner.

*NOTE: A team may think that they gain advantage by withholding values of a and b from the other team. However, the scoring procedure outlined prevents any advantage gain in this manner.

Example: The ROBINS quickly write down a and b values after 30 seconds (i.e., $R_1 = 30$). The LIONS hold back on a and b and immediately proceed to calculate their own value for h. The LIONS rush up to the board after 100 sec and put down the h value on their triangle and the a and b values on the ROBIN'S triangle. The time recorded is $L_1 = 100$ sec., $L_2 = 100$ sec.

The ROBINS complain because the LIONS finished first through unfair play. However, the ROBINS proceed to calculate and find the h value after 170 seconds have passed.

Result: $R_1 + R_2 = 200$, $L_1 + L_2 = 200$. Each team spent 70 seconds in calculating h. Since $R_1 < L_1$, the ROBINS could be judged the winner.

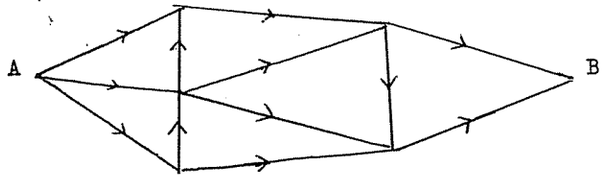
The game has generated a lot of excitement and fun for the students of my classes. I would appreciate hearing from you with your opinions.

PROBLEMS

For those of you who are involved in making up questions for contests or needing problems for your students, the following ten questions were given to the London senior mathematics teams. I would appreciate a copy of your contest questions for future issues.

In looking back over past issues there would seem to be a dearth of material suitable for elementary school students; this will be remedied in the next issue. This section of the Gazette will be devoted entirely to elementary material. Of course, the amount of material will depend on how much you send me, so please send in your party pieces and other favourites.

1. If movement is allowed only in the direction of the arrows find the number of paths from A to B.



2. A string is stretched tightly around the equator of a perfect sphere the size of the Earth, i.e., 6400 km. radius. 6 metres more string is added, and the whole circle of string is raised equally above the surface. What approximately will the height of the string above the surface be?

3. If a cable spool is 1 metre wide, has an inner radius of 1/2 metre and an outer radius of 1 metre, what is the maximum possible length of nylon rope of radius 1 cm. that can be wound on the spool?

4. Find an equation that would generate the following table of values.

n	1	2	3	4
s	0	2	6	12

5. A car depreciates at 20% per year for 3 years. What, at this time, is its percentage value of the original price?

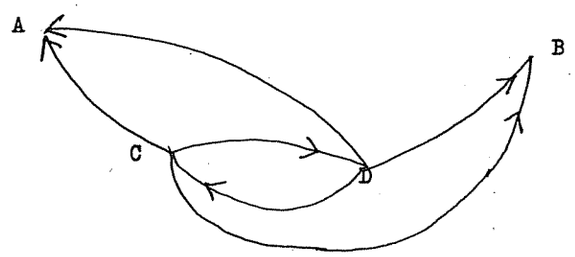
6. If each volume a twelve book encyclopedia is 3 cm. thick and the covers are 1 mm. thick, what is the distance from the first page of volume 1 to the last page of volume 12 when they are stacked in order on a shelf?

7. What is the sum of the first 30 odd natural numbers?

8. If a 3m. high statue takes 5 litres of paint to cover, how much will be needed to cover a 30 cm. high copy?

9. A hole of length 6 m. is drilled through a sphere of radius greater than 3 m. What is the volume of the remaining material? Yes - you are given enough information. Vol. of sphere is $4/3\,\pi r^3$.

10. If "→" means "__" is brother of "__", how is A related to B?
N.B. all arrows showing the given relationship are indicated.



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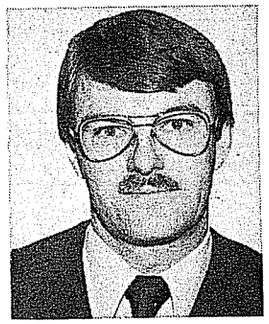
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PRESIDENT'S MESSAGE



I would like to welcome back all members of OAME (AOEM) and I trust that your summer break was an enjoyable one.

Congratulations to two of our members, Ross Leigh of Owen Sound and Don Attridge of Thornhill. Ross was recently awarded life membership in O.E.A. and also OAME for his many contributions to mathematics over the years. Don, a past president of OATM, is head at Thornlea Secondary School and Thornlea won the Waterloo Junior Math contest this year for Canada. The school will receive the "Father Faught Award" and $50.00 in books from the OAME.

Congratulations to Major Don Kaye of Deep River and Brock Rachar of London who received "Descartes Medals" this year from the University of Waterloo.

We welcome two chapters to the OAME this year. SEYMA, the Scarborough-East York Math Association, began with a very successful spring meeting with about 300 in attendance. Paul Sherk, recently retired from Cedarbrae Collegiate, is the founding president of this organization. The Pine Ridge Math Association has been formed in the Durham, Lindsay, Peterborough area and they have had two very successful meetings in the spring. Thanks to Bill Jeffers and Gord Roberts for their work in this area.

The ABACUS has two new editors this year. Shirley McIntyre of East York and Gareth Jones of Port Hope will share this responsibility. We wish to thank Andy Czempinski of Iroquois Falls for his efforts in launching the Abacus over the last two years.

On behalf of the membership and those in attendance at the OAME Conference in Ottawa this year I wish to thank our hosts for a most excellent conference. Both the sessions and the social activities were most enjoyable. Special thanks to Duff Butterill, Bruce Topping, Don Fox, Jacques Theoret, Joe Ryan, Edna and Weldon McLeod and Kathy and Peter Ullrich for chairing the various committees.

We look forward to the 1978 conference in Hamilton next year during the month of May.

At our first Council meeting in June, the committees for 1977-78 were formed and they will be examining Elementary Teacher Qualifications, Publications, Chapter activities, and Intermediate and Senior Division Curriculum,, Interface, Evaluation, French Language publications, future meeting sites and other projects. Thanks to all the members of the Council for volunteering to chair and serve on these committees.

We now have nine chapters of the OAME throughout Ontario and I urge all members to support their local organizations and get involved this year. For example, NOMA will be holding their meeting early in the fall, October 1, at Northern College in Kirkland Lake. We hope to see all northern members there.

If you have any concerns you would like to share with the Council, please write me direct and I will present your views to our executive for consideration.

Have a good year!

Neil Williamson,
President, OAME (AOEM)
1754 Walnut Lane,
PICKERING, Ont. L1V 2X3

NINE CHAPTERS OF OAME/AOEM

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Representative - Jim Whiteley, Queen's University

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President - Paul Sherk, 86 Banmoor Blvd. Scarborough. M1J 2Z6

SWOAME (Southwestern Ontario)

President - Barbara Palmer, Laurier S.S., London

Chapter secretaries are asked to update the above.

CHAPTER ITEMS

COMA: Congratulations to all members on the very successful annual conference. It was most profitable as well as enjoyable.

COMA's annual meeting was held on Mon., April 25 in Centennial School. The main topic of discussion was the Intermediate Division guidelines.

COMA publishes an excellent newsletter. Other chapters might consider following this example, "Lee - Jeff Bell, a 17 year old Grade 13 student at Fisher Park High School, won the 1977 Descartes Mathematics Competition." Congratulations. Upcoming — Mini Math Conference — Nov. 5, 1977.

GOLDEN SECTION: We understand that the plans for the 1978 Conference are proceeding well. See information elsewhere in this issue.

NOMA: Annual conference, Northern College, Kirkland Lake, October 1.

SEYMA: Congratulations to Paul Sherk et. al. on the founding of a new chapter in the Scarborough-East York area. The first conference was held on April 28. Speakers included: Grant Jones, Neil Williamson, Howard Russell, and Joan Routledge.

SWOAME: Plans are now being made for the annual conference to be held in Chatham on Saturday, Nov. 5.

SOCIALIZING IN OTTAWA

John improves relations with the Ministry, but will meet his Waterloo later. Is someone being transformed? Or is Euclid back?



left to right: Ken Fryer, Don Fox, John Delgrande.

FARMER HARRIS ASKS

Readers will be pleased to learn that this is **not** to be a regular feature. However, retirement gives one time to contemplate, if one does not take one's hobbies too seriously.

Let's look first at beef cattle. **The Farmer's Handbook** gives a formula for estimating the weight of a steer, based on its girth, measured just back of the shoulder with a snug tape, after the animal has not been fed for 12 hours. The authors state that this formula has been based on thousands of observations of choice quality (A1, A2) steers. It takes the form $y=mx+b$, where **x** is the number of inches in the girth and **y** is the weight in pounds (apologies to the Metric Commission). Its use is restricted to the domain 63-80. You might be interested to determine the formula: 63→715 and 80→1203. My question is, why is this a **linear** function?

My second contribution arose from several dozen rounds on a baler. This baler makes rectangular bales, small enough for one man (or woman) to throw onto a wagon. They are pushed out of the rear of the machine as they are tied, slide up a short shute, and in this case, fall to the ground a distance of about four feet. Most of them come to rest on a surface with the largest area. While baling hay in a large field I noticed that about 1 in 10 "stood on end", i.e. balanced on the surface with the smallest area. This ratio was determined for the whole field (15 acres and Lord knows how many hectares) and was about right for small samples as well. The other day I noticed that when baling oat straw, only 2 out of approximately 200 bales "stood on end". There must be some mathematics in this somewhere. Is it significant? Have you any theories?

This article is by way of a suggestion to you to write a letter to the editor. You have been silent over the summer. Is there anyone out there?

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O.A.M.E. COMMITTEES AT WORK

The effectiveness of our Association depends on the participation of the members in policy formation. The following committees will be at work during 1977-78: The Ad-hoc Committee on Teacher Qualifications, the Interface Committee, the French Language Committee and the Evaluation Committee. Submissions to these committees are welcomed and may be sent to the Secretary at the above address.