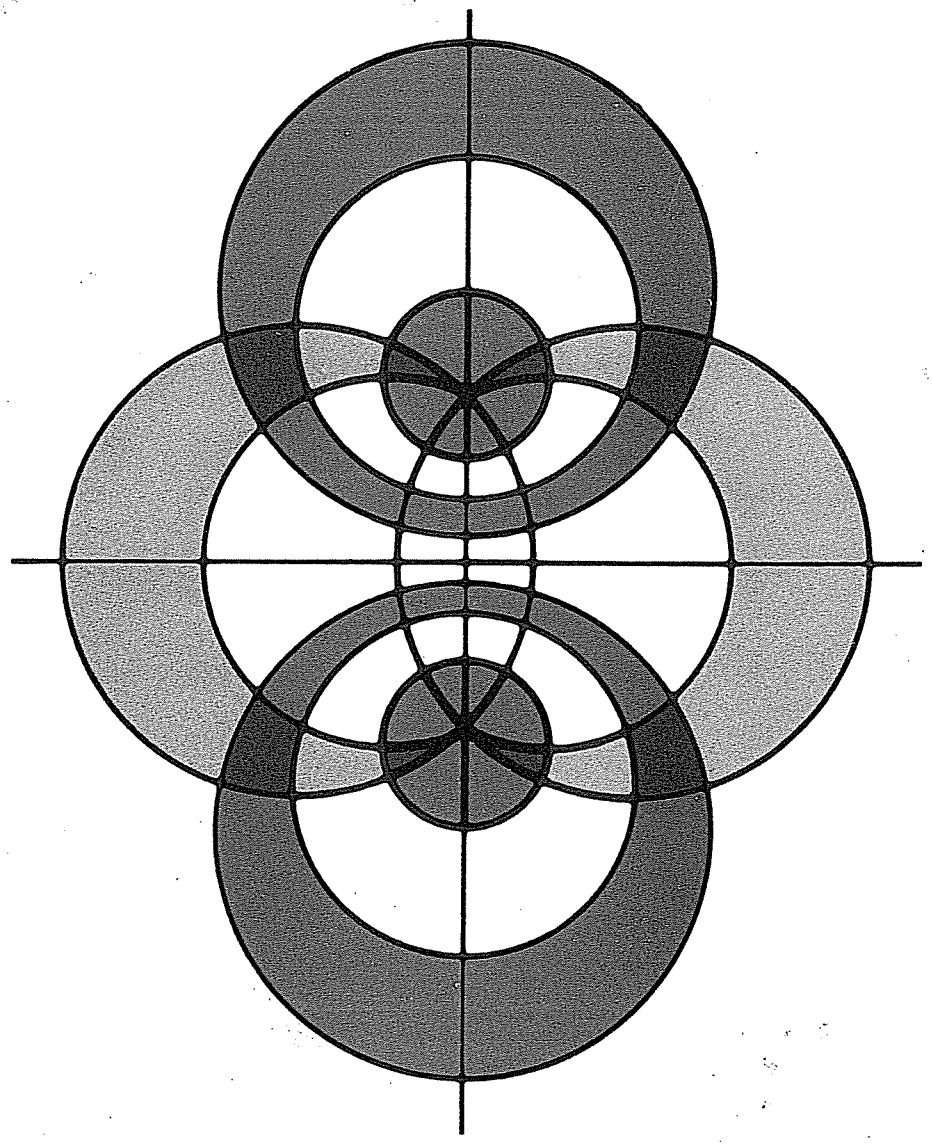
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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EVERYBODY PRAISES THE INCOMPARABLE POWER OF THE MATHEMATICAL METHOD, BUT SO IS EVERYBODY AWARE OF ITS INCOMPARABLE UNPOPULARITY.
J. Rosanes

PRESIDENT'S REMARKS

The Ontario Association for Mathematics Education represents a large number of math educators throughout Ontario. As such it is the responsibility of this organization to not only provide services to its members but to search out and express the concerns and ideas of its membership to the appropriate bodies of education: ie. Ministry of Education, Local School Boards, The Public Sector and other Subject Councils. To accomplish these tasks your council is attempting to clarify and establish its positions on current educational issues and directions.

How will computers enhance mathematics education?
Are there appropriate mathematics programs for the learning disabled and the gifted?
Do the senior guidelines meet the needs of the students?
How can education promote mathematics to the general public?

It is my hope as your president that OAME, in the future, through addressing such issues will be able to provide a service in co-operation with other organizations such as the Council for Exceptional Children and the Ontario Guidance Teachers to enhance mathematics in Ontario.

To be an effective and respected organization OAME must investigate means in which it can continually become a more proactive voice in Ontario education. Your elected members' attendance and input at council meetings and their contributions durlng the various committee gatherings is essential to accomplish this goal. Input from OAME members to your elected official therefore is not only encouraged but it is strongly recommended and essential to the life of this organization. For this year your council has formulated the following as its goals:

1. to determine and establish OAME positions on current issues such as:
   a) Senior Guidelines
   b) Computers in Mathematics
   c) Bill 82;
2. to review the constitution;
3. to initiate projects that will promote the name of OAME;
4. to provide services to our members as needs are identified;
5. to consider working co-operatively with other provincial educational organizations, for example OAIP; Federationa.

In an attempt to meet these goals the following committees have been established at the council level. As members of the organization your concerns or suggestions on any of these topics should be submitted to the Secretary Treasurer who will ensure that the information is then directed to the appropriate personnel.

Committee	Chairperson
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7. Bill 82 Committee	E. Harvey
8. Problem Solving Committee	C. Carlow
9. Constitution Committee	to be announced
10. Conference Liasion Committee	B. Robinson

Informations on the activities of these committees may also be obtained through contacting the OAME office.

The chapters of OAME are key components of the organization. It is hoped that in planning activities for the coming year they too will consider the general goals and aims of the the parent organization. Together OAME and its chapters must be proactive in dealing with current educational issues on behalf of its many members.

Your elected council is attempting to address its responsibilities to you its members. I now challenge you as members of this organization to provide input to your elected members to enable the council to truly represent you as THE VOICE for Mathematical Education.

Sharon McPhail
President, OAME

MATHEMATICS EDUCATION

"IF ALL THE SCHOOLS WERE APPLE PIE
AND UNIVERSITIES LIKE INK.
AND ALL THE TEACHERS BREAD AND CHEEZE
HOW WOULD WE EVER THINK?"

PERIODIC PICTURES

by
C. W. Zeller

Using periodic decimals as a starting point, it is possible to develop some intriquing geometric patterns through graphing. These in turn lead to a host of speculations, many of which are suitable for investigation by students.

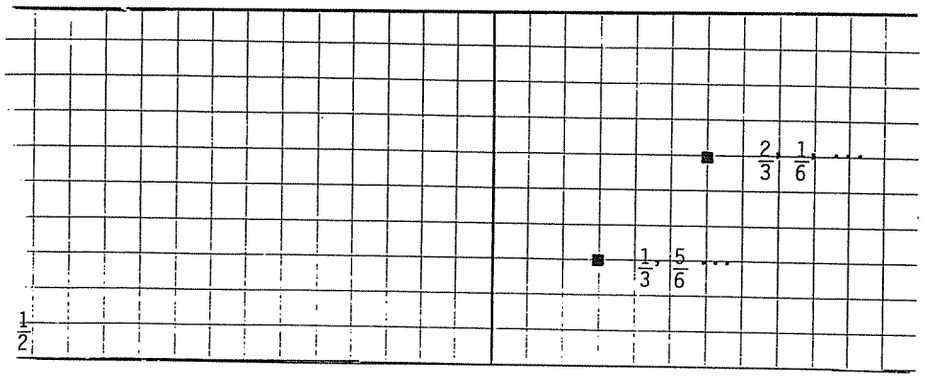
Let x/y be any rational number from (0,1). Represent the periodic decimal from of x/y by 0. a_1 a_2 ... a_n. Form ordered pairs, sequentially, as follows:

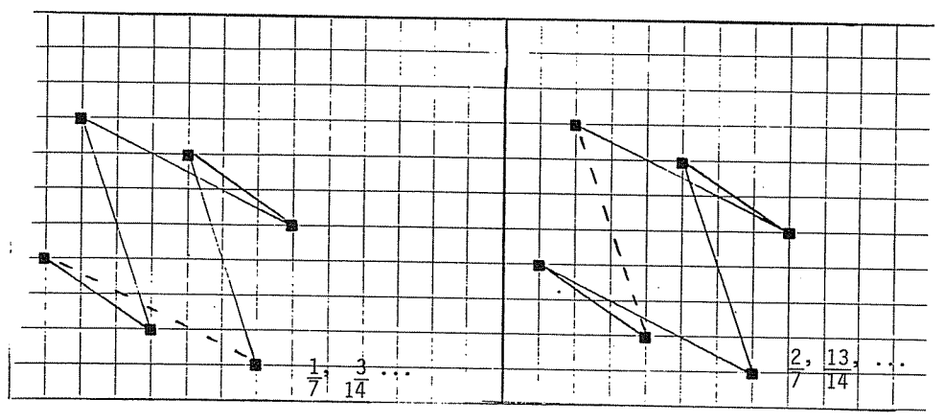
$(a_1, a_2), (a_2, a_3), \ldots (a_{n-1}, a_n), (a_n, a_1)$

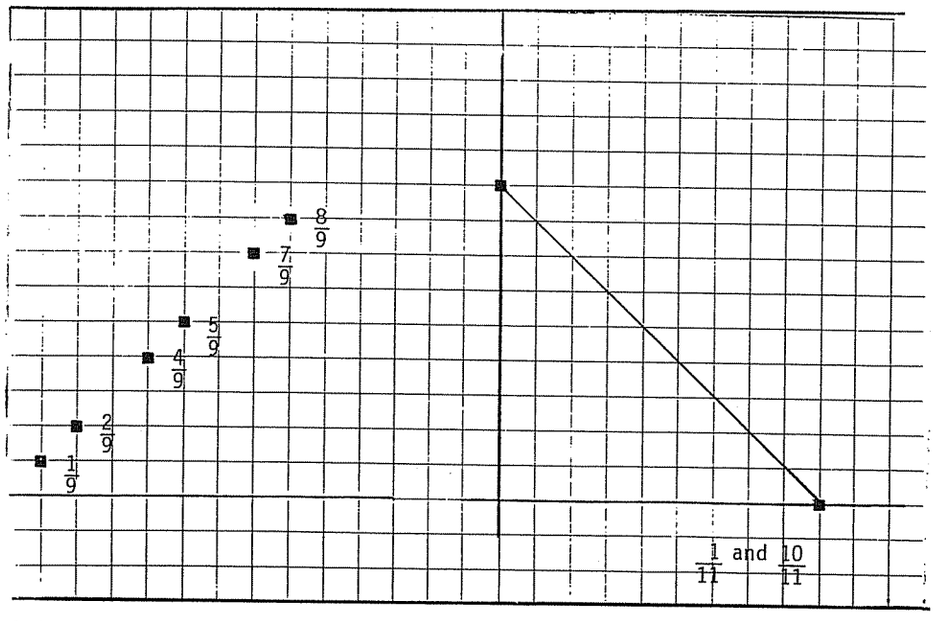
Plot each of these points and join them sequentially. (I've joined the last to the first with a dotted line to close the figure). Specifically, 1/7 = 0.142857. The points plotted would be:

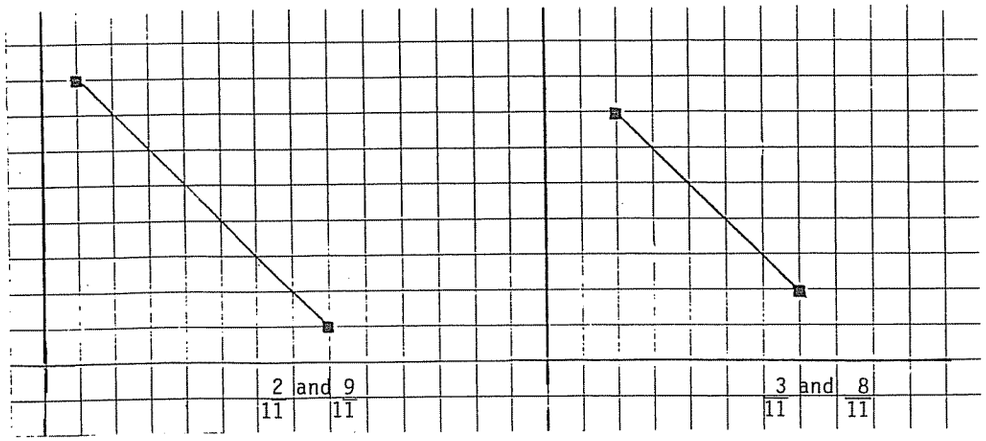
[(1, 4), (4, 2), (2, 8), (8, 5), (5, 7), (7, 1)] and they would be joined in the order presented.

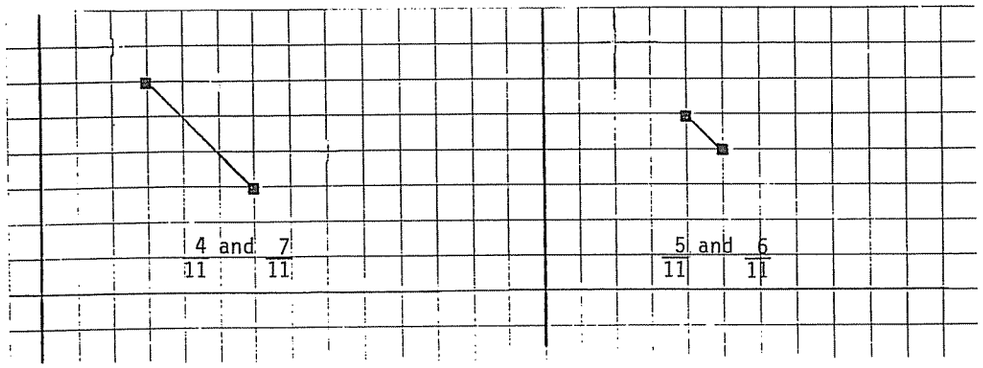
(7, 1) and (1, 4) are dotted to close the figure. all distinct periodic pictures for denominators up to 25 are sketched on the following pages.

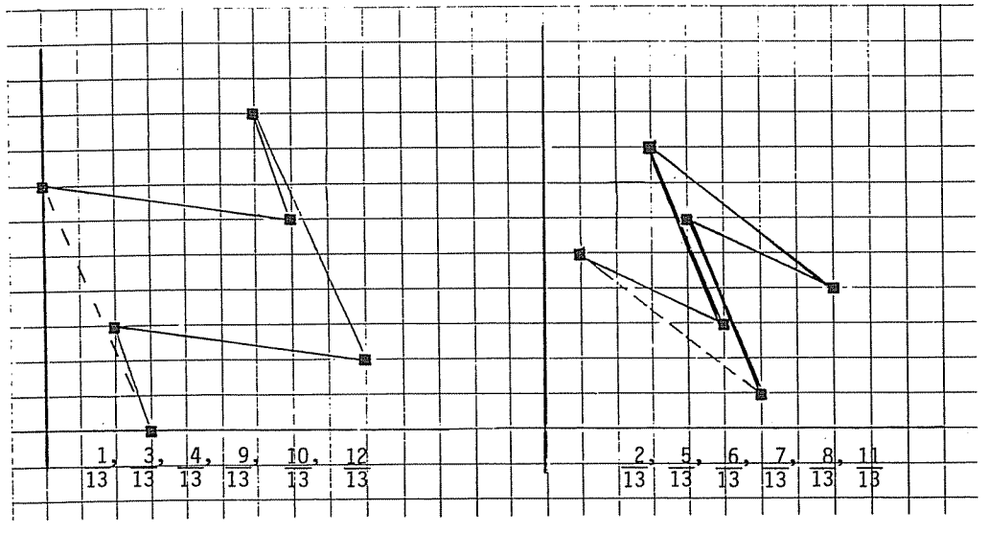

$\frac{2}{3}, \frac{1}{6}, \ldots$
$\frac{1}{3}, \frac{5}{6} \ldots$
$\frac{1}{2}$

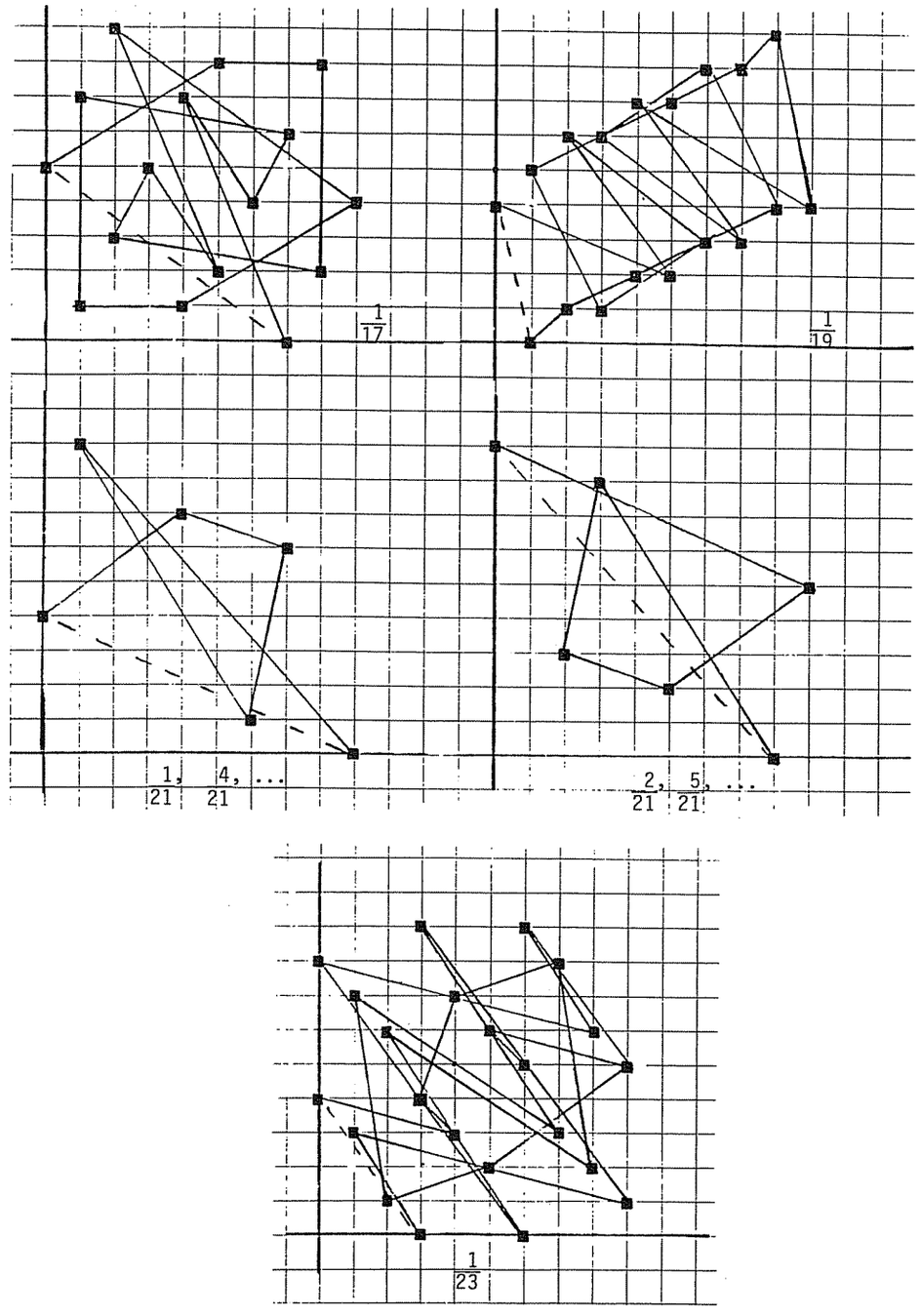

$\frac{1}{7}, \frac{3}{14} \ldots$
$\frac{2}{7}, \frac{13}{14}, \ldots$


$\frac{8}{9}$
$\frac{7}{9}$
$\frac{5}{9}$
$\frac{4}{9}$
$\frac{2}{9}$
$\frac{1}{9}$
$\frac{1}{11}$ and $\frac{10}{11}$


2 and 9
11 11
3 and 8
11 11


4 and 7
11 11
5 and 6
11 11


1, 3, 4, 9, 10, 12
13 13 13 13 13 13
2, 5, 6, 7, 8, 11
13 13 13 13 13 13



Perusal of these pictures does indeed suggest some patterns and symmetries in the individual pictures, as well as many additional questions. I have shared these findings with a grade 11 student who will try to develop a computer analysis for denominators larger than 25. I consider the following questions among those worth some exploration.

i) Are there other ways of plotting these periods which are more revealing?

ii) Are the pictures of composite denominators predictable from their prime factors' pictures? e.g., 7, 21)

iii) Do all pictures of prime denominators reflect internal symmetries?

iv) Is there some significance to the -Z- pattern which appears several times?

v) Are there picture characteristics which would characterize primeness?

The author is grateful for the suggestions of the editor and reviewer in the preparation of this paper.

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GREAT FLEAS HAVE LITTLE FLEAS UPON THEIR BACK, TO BITE 'EM
AND LITTLE FLEAS HAVE LESSER FLEAS, AND SO AD INFINITUM.

THE ESSENCE OF MATHEMATICS LIES IN ITS FREEDOM.

Cantor, George

"HOWEVER DIVERSE THEIR TALENTS, TEMPERAMENTS AND DIFFERENCES, ALL GREAT ACHIEVERS HAVE ONE TRAIT IN COMMON: THEY NEVER BOTHER TO COMPARE THEMSELVES WITH OTHER PERSONS, BUT ARE CONTENT TO RUN THEIR OWN RACE ON THEIR OWN TERMS."

S. J. Harris

THE ROLE OF MATHEMATICS IN SCIENCE EDUCATION

by
W. Higginson

I Introduction

The real problem is that, overall, as the world is getting more technologically complicated, the average American student -- the one who does not intend to major in mathematics or science -- is becoming less technologically sophisticated.

- National Science Foundation official
(Finney, 1980, p.1)

This paper is intended to be a skeletal survey of the common ground between science and mathematics at the school level. It is proposed that it provide the basis for preliminary discussions between the staff of the study and representatives of the Canadian mathematical community.

The structure of this paper is such that it is suggested that each of its six substantive sections could become the subject for detailed scholarly investigation and presentation at a workshop. The scope of the subject necessitates an impressionistic treatment in a paper of this length. The list of supplementry references included in the bibliography provides some indication of possible sources for further investigation of the issues raised in this preliminary partitioning of the area. An attempt has been made to stay cognizant of, and where possible be consistent with, the previously stated aims of the study and its various components, such as discussion papers, (Aikenhead (1980), George (1981), Page (1979)), policy and text analyses, case studies and surveys of science educators. Of particular influence, however, are three procedural components of the overall strategy of "deliberative inquiry", namely:

1. The stimulation and maintenance of a continuing commitment to deliberation (both by practitioners and by others having a stake in Canadian science education).

2. The development of a reliable knowledge base about Canadian science education (to inform deliberation).

3. The generation of a range of issues and alternative courses of action (to form the substance of deliberation).

II What is (1): Contemporary School Curricula in Science and Mathematics.

"We would be stupid as a country to permit the continued erosion of science and math training in our primary and secondary schools."

- Frank Press (Walsh, 1980, p. 17)

"Many of us who teach scientific subjects at all levels, have noticed a significant deterioration in the mathematical skills of our students over the past few years."

- K. J. Laidler (1975, p. 33)

In a paper published ten years ago, entitled 'Relating Mathematics and Science at the Secondary Level', Douglas Crawford (1971) wrote that his motivation for attempting such a challenging task was its critical importance and his conviction that, "for many pupils there is little realization of how mathematics and science are related" (p. 1). A decade later, there seems little reason for concluding that Crawford's judgement about this situation in Canada would be any different. An analysis of

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current ministerial guidelines would reveal that little consideration is given to the ways in which science and mathematics overlap. At the school level it most frequently happens that mathematics and science departments exist as independent entities. Where this is not the case, it is often because of admistrative or staffing convenience rather than because of a decision based on philosophical or pedagogic grounds.

In mathematics textbooks, which are by far the most important factors in determining the nature of day-to-day activities in a high percentage of classrooms, we find very few examples taken from science. In most science texts mathematics is seen strictly in an instrumentalist form; equations are to be solved or formulae manipulated and there is little sense of mathematics as a discipline in its own right.

It may be small consolation, particularly to the students caught in the middle of this rift, but the Canadian situation seems very typical seen in an international context. At the recent UNESCO conference on 'Co-operation between Science Teachers and Mathematics Teachers' (Steiner, 1979) speakers from France, Germany, the United States, Sweden, England, Japan, Brazil and Australia played variations on this theme.

The Canadian situation differs from some in that it lacks detailed documentation. In the United States the reports of the NSF project 'Case Studies in Science Education' (Stake and Easley, 1978; Weiss, 1978) gives a vivid picture of the reality of mathematics and science instruction in a number of schools. In England the recent survey (Department of Education and Science, 1979) of secondary education included a section on mathematics and science which might well have been written about Canada.

> There have long been problems in establishing a proper relation between the teaching of mathematics and science, and these merit separate discussion. Recently the problems have been accentuated by the efforts to teach mathematics (and not only arithmetic) and science to a broader range of pupils, and also by changes in syllabuses and in the classroom approaches to both disciplines. There is particular need for continuing liaison between mathematics and science. The successful application of mathematical skills in science is primarily a matter of consultation and joint action; it certainly is not an automatic consequence of any particular kind of syllabus. In one comprehensive school, where the long square root process was still taught and examined, where calculators were discouraged except in sixth form statistics courses, where there was much practice of techniques and where homework was regularly set and marked, the science department made the same complaints about the level of mathematical achievement as those which can be heard in some schools with 'modern' syllabuses -- complaints concerning difficulties in calculations, the manipulation of equations and formulae, proportion and the interpretation of graphs.
>
> It is necessary to coordinate the methods to be employed. A joint policy is needed for coping with awkward numbers. Practice in algebraic manipulation may not carry over into science lessons, and it can help if some practice is based on the formulae actually used by the science teacher. A better fit is often needed between the levels of difficulty and the timing of certain topics in science and mathematics. An extreme case was seen in one school where pupils were expected to cope with newtons, newton/cm^2, dynes and joules in the science lesson, but the same pupils were naively delighted with practical work in mathematics involving no more than metres and millimetres. Not infrequently science teachers attempted unilateral solutions by managing without mathematics as much as possible. This is understandable, and there is a positive side to this approach, but it cannot be the best solution for the pupil in the long term. Coordination was often made difficult by having different setting arrangements for the two subjects; there would not appear to be any simple solution to this problem which would be acceptable to both departments. (pp. 151-152).

Hart (1981) in a short report of a sub-committee of the Mathematical Association (U.K.) sees as a basic problem the fact that "mathematics and science teachers view mathematics in different ways, one as an entity in itself and the other as a tool to be used" (p. 12). With respect to particular content difficulty she comments:

> The mathematics problem areas as seen by the scientists were decimals, fractions and rearrangement of algebraic formulae; fractions and operations on them were seen to cause trouble more often than decimals. Ratio and graphs appeared often in science books and the problem of non-integer numbers arising in experiments, with the child's consequent inability to deal with them, was mentioned. When asked whether children had mathematical difficulties in science lessons because (a) they had not been taught the mathematics or (b) they found it difficult to apply

and transfer to other situations, most schools which completed the larger questionnaire opted to (b). There may be a case to be made that children should not be expected to recognize the applicability of their mathematics in science situations and that science teachers should see "applied mathematics" as an intermediate step, prior to using mathematical concepts in science. (p. 14)

When asked to account for the existence of these difficulties some commentators (the best known is probably Kline, 1974) point unequivocably to "the New Mathematics". Laidler (1975), found his university-level chemistry students wanting in mathematical skills with regard to:

(1) Calculations involving exponents, indices, logarithms and square roots; (2) Solving quadratic equations; (3) Plotting experimental data; (4) Relating a graph to a linear equation; (5) Use of logarithm tables; (6) Understanding significant figures; (7) Addition and multiplication of fractions, and (8) Understanding the concept of proportionality. (p. 33)

Laidler saw several reasons for this "disastrous situation", of which the three most important in his mind were:

(1) The decline in intellectual discipline in our school and universities; (2) the related decline in general educational standards at all levels and (3) the introduction of the "new mathematics". (p. 33)

Be this as it may, and it is at least debatable, (see, for instance, the foregoing comment in the DES study), the last few years have seen a reaction against the formalism of the so-called new math and a trend toward problem-solving and applications (Hilton, 1979; NCTM 1980 (a), 1980 (b), as the major emphases for school mathematics curricula. (At least on the part of mathematics educators; on the part of laymen there has, of course, been very strong pressure from the "back to basics" movement, to emphasize computation.) It is by no means clear how these new emphases will be implemented in schools, but there is little indication in the early materials that science-based problems or applications will be prominent (Krulik and Reys, 1980).

III What Is (2): The Wider Context

Yet, despite such indicators of the need for scientific literacy, no public clamor for increased emphasis on science and mathematics is apparent. The cries of "back to basics" rarely include science, and stress computation, not mathematics. Thus we find ourselves in 1977 confronted with this paradox: we, the American public, need to know more science in order to live in and run our society but we appear unwilling to invest the time, energy, and the activities of our teachers and students to the pursuit of excellence and understanding in the science and mathematics fields.

- L. Shulman and P. Tamir (1978, p. 2)

For one wishing to understand the current relationship between mathematics and science in Canadian schools, and possible avenues for change, it would be a mistake to look only at the interactions between the two disciplines, for there are wider issues which are of considerable significance. Of immediate importance are the serious criticisms which exist of science and mathematics education programs considered independently. In the 1976 Background Study, **Mathematical Science in Canada**, we read that, "No subject was discussed at greater length by contributors to the study than mathematics teaching. No one was satisfied with the current situation in Canadian schools." (Coleman et al, 1976, p. 113). At the 1980 International Congress on Mathematical Education the well-known Brazilian mathematics educator, Ubiratan D'Amrosio stated:

The system itself will liquidate mathematics education as such. Everyone (at this Congress) is claiming that the level of mathematics education is dropping. Students are knowing less and less -- everybody claims this. What is developing is a feeling, a correct feeling I think, that mathematics education is a useless appendix in the educational system. It's really something that does not fit. What's happening is that it is being incorporated with other disciplines and taught by others.

- (Steen and Albers, 1981, p. 126)

The problem of student disaffection with mathematics has become so widespread that 'math anxiety' is a household phrase in North America (Tobias, 1978; Kogelman and Warren, 1978) and math anxiety clinics are commonplace on college campuses. In the United Kingdom a government

Committee of Inquiry into Mathematics Teaching is close to reporting after three years of deliberation.

Nor is science free of these criticisms. A recent publication is entitled **Science Anxiety: Fear of Science and How to Overcome It**, (Mallow, 1981). Even before the submission of the report of the Mathematics Committee of Inquiry in Britain we have the announcement of a major review of the science curriculum in secondary schools. This review, which will take five years to complete is being described as the "most ambitious analysis of the secondary science curriculum yet undertaken" (**New Scientist**, 9 April 1981, p. 71).

Mathematics and science are not alone as disciplines receiving largely negative reviews from their clients. The whole edifice of public education as we have come to know it in North America in this century has been called into question by a number of critics, both 'romantic' and 'conservative.' Central to the enterprise of public education is the classroom teacher. Observers of many curriculum projects in the past fifteen years have commented on how critical it is to the success of the project implementation that the teacher understand and support the aims and methodology of the project. Often this is not the case and the new curricula fail badly.

It is not uncommon to find instructors of mathematics and science who have a weak academic background and inadequate preparation in teacher education (the OECD **Review of National Policies for Education** (1976) emphasized the pressing need for improved teacher training in this country (p. 101)). When combined with weak material and leadership support at the school level, the effect can be quite powerful. The organizational structure of most schools and professional organizations does not lend itself to interdisciplinary cooperation. Any proposals for change which do not consider these factors will not succeed.

Mathematics and science education programmes have direct links to several important societal issues of the moment. The highly technical nature of almost all research in science and mathematics makes it particularly difficult for the media to report on developments without misrepresentation. The role that mathematics and science play in limiting the employment possibilities of women is clear (Tobias, 1978). What is more difficult to account fully for is the set of reasons which leads to the small enrolment of female students in senior secondary courses in these subjects (Ferguson, 1981; Fox, Brody and Tobin, 1980; Shelanski, 1981).

Some children show their precocity in early math and science classes and these early experiences can be very influential on their later lives. Consider the case of the young female biologist, Anna, observed by Goodfield (1981):

> One experience stands out from these early years. She was six or seven years old and the whole class was struggling with the problem of multiplying by zero. The teacher, Donna Fausta, a lovely old woman, set what appeared to be an enormous challenge. "Try to find out what happens when you multiply things by zero." For most children, the very idea is silly. "Nothing" happens, but why should the answer be "nothing?" You have one thing in your hand, and you are multiplying it by nothing and still you have the thing in your hand. How can zero and "one thing" be made to fit? Anna was fascinated. She remembers sitting there, multiplying away like mad, and what quickly came across was the notion of abstraction. She describes this as a fantastic moment. So now the problem was: How could one multiply a "thing" by an abstraction? She became entranced by the idea of a pure, abstract, transparent thought -- and has remained entranced. (p. 8)

Much of the societal concern for the quality of mathematics and science programmes has in it a strong element of self-interest. These programmes are seen as being important because without a cadre of highly trained technical specialists no nation will be able to compete internationally be it in the economic or military arenas (Stockton, 1981; National Science Foundation, 1980 (b)).

IV What Has Been (1): The Relation Between Mathematics and Science

> Mathematics, as the word is customarily used, consists of at least two distinct subjects, and I propose to call them "mathology" and "mathophysics."
>
> - P. Halmos (1968 (a), p. 376)

> It made no sense to learn complicated mathematical methods to follow baloney.
>
> - J. Watson (1969, p. 27)

Any conception of the interrelation of school science and mathematics must be set in a more general scheme which contains the two disciplines. In acknowledging this one has to accept that one is opening the door to some major philosophical problems of very long standing. Kac and Ulam (1968) write, "the relations between mathematics and the empirical disciplines have puzzled philosophers and historians of science for centuries" (p. 145), and Wigner (1970) notes, "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and there is no rational explanation for it" (p. 223).

It is clear, however, that whatever the precise nature of the relationship may be, it is seen by many prominent mathematicians as a particularly important one. Von Neumann (1956) writes: "the most vitally characteristic fact about mathematics is, in my opinion, its quite peculiar relationship to the natural sciences, or more generally, to any science which interprets experience on a higher plane than purely descriptive level" (p. 2053). (For a dissenting view on aspects of this issue see Schwartz's (1962) 'The Pernicious Influence of Mathematics on Science.'). The interaction between physics and mathematics has been a specially rich one at least since the time of Galileo (Polya, 1977) and particularly in the first half of this century (Feuer, 1974). The last thirty years has seen the emergence of what Eilenberg has called "the algebraization of mathematics" (COSRIMS, 1969, p. 153) and a concomitant loss of influence for the classical tradition of applied mathematics (Halmos, 1968 (b)).

This move toward pure mathematics has been regarded with dismay by some observers. Kline (1977), for instance, writes, "today most mathematicians no longer know any science or even care whether their work will have any bearing on real problems" (p. 51). An examination of current popular reviews of mathematical research (COSRIMS, 1969; Lighthill, 1978; Steen, 1978; Saaty and Weyl, 1969) make Kline's view seem rather extreme. What is happening is that fields other than physics are becoming the scene for mathematical investigation. Biology, in particular, seems to have become a rich source for problems (Judson, 1979, Smith, 1968; Thom, 1975; Zeeman, 1977).

The whole question of the nature of applied mathematics is, of course, another very difficult one (Coleman et al., 1976; Lin, 1978). Historically one can find a number of instances of one decade's pure math becoming precisely what is needed to solve another decade's difficulties in applied math. Richard Feynman (1967) acknowledges this in one of his Messenger lectures entitled 'The Relation of Mathematics to Physics', "When the problems in physics become difficult we may often look to the mathematicians who may already have prepared a line of reasoning for us to follow." (p. 45).

The fact that even at the highest levels we encounter the same dichotomous views (what Bochner (1966, p. 47) calls the "handmaiden" and "dictatorial mistress" positions) can be seen in Peter Hilton's (1967) reaction to Feynman's lecture:

> "the fact that we have now isolated a criterion for distinguishing mathematics from science should not delude us into believing that this distinguishing feature represents the essence of mathematical activity. This was precisely the error made by Professor Feynman in his fascinating and exciting Messenger Lectures at Cornell University in the winter of 1964, in which he discussed the nature of physics. He devoted one lecture to an attempt to express the distinction between mathematics and physics; and I found myself in violent disagreement with his conclusions, since his picture of mathematical activity was so unfamiliar and unacceptable. Indeed, his description of the procedures of a physicist was perfectly appropriate to the case of a mathematician, provided one replaced the physicist's 'hardware' by that of the mathematician. He portrayed the mathematician, however, as starting from a rigidly prescribed set of axioms (and, presumably, rules of inference) and then slavishly grinding out all the consequences of those axioms; and contrasted this servile attitude with that of the physicist, stepping lightly from context to context with delicate imaginative leaps, and every now and again penetrating the unknown with bright exciting shafts of light. It transpired that, in fact, Feynman, who is not only an outstanding physicist but also a highly skilled user of mathematical tools, did not realize that there are questions to which mathematicians want to know answers; he imagined that all was now uninspired system-building. He had surely been misled by this mistake into confusing a formal description of mathematics as a deductive system with the living, creative activity of mathematicians.

The criterion for distinguishing mathematics from science to which Hilton refers is the distinction between mathematical theorems which can be proven and scientific theories which cannot. In any educational system which purports to encourage the development of the rational abilities of young people, this distinction is an important one.

V What Has Been (2): Previous Attempts at Curriculum Integration

> Incidentally I am only coming to Princeton to do research, not to teach. There is too much education altogether, especially in American schools. The only rational way of educating is to be an example -- if one can't help it, a warning example.
>
> - Albert Einstein (1973, P. 65)

It it is the case that relatively few teachers consciously work toward the integration of science and mathematics topics in their classes, it is not because materials to support such a programme do not exist. Over the past fifteen years up to a dozen curriculum project in different parts of the world have taken up this challenge. In many cases the quality of their material is high. Their failure comes at the stage of implementation, not of content development.

At the elementary level two American programmes supported by the National Science Foundation deserve attention. The MINNEMAST (Minnesota Math and Science) programme consists of a series of twenty-nine coordinated mathematics and science units for kingergarten through grade three. The goals of this project are to provide both mathematical and scientific knowledge and to emphasize the development of tools for learning by stressing methods of acquiring knowledge rather than memorization of facts. Eight clusters of threads are woven through the curriculum; three of these are related to processes (observation, generalization, and linking of observations with generalizations), and five to subjects (real numbers, geometry, formal concepts of science, scientific topics and the links between science and mathematics). Units seven and fourteen of the MINNEMAST series deal, for instance, with the concept of symmetry. Unit seven is intended for the kindergarten level and has four sections: (1) rotational symmetry, (2) repeating patterns, (3) bilateral symmetry and (4) symmetry in sound and movement. The five sections of unit fourteen, intended for grade one pupils who have worked through unit seven, are: (1) rotational symmetry, (2) repeating patterns and translational symmetry, (3) bilateral symmetry, (4) symmetry in sound and movement, and (5) other interesting patterns.

USMES, Unified Science and Mathematics for Elementary Schools is a curricular programme designed to develop the problem solving abilities of pupils attending grades one through eight. The USMES emphasis is on student-initiated investigation of situations which arise in their school and community environment. Titles of USMES units include, 'Pedestrian Crossings', 'School Rules', 'School Supplies', 'Traffic Flow', 'Lunch Line', 'Advertising', and 'Dice Design'. The AAAS supported project 'Science: A Process Approach', had considerable mathematical content. In continental Europe, only the Dutch IOWO group would seem to have given a high priority to math/science cooperation at the elementary level.

In the United Kingdom elementary school curriculum materials prepared by the very influential Nuffield Project shared many of the characteristics of the USMES and MINNEMAST series. The 'Mark II' version of Nuffield elementary mathematics which is currently being developed has, however, abandoned the attempt to work almost exclusively with teacher handbooks, and is instead developing pupil materials which seem much more like traditional approaches to the subject at this level.

At the secondary level there is a long tradition of nonrigorous applied mathematics in British schools. In the late sixties and early seventies several curriculum projects produced materials which extended this tradition. As its name would imply, the **Mathematics for the Majority** project focused on materials for the general student. Among these were the two books, **Mathematics from Outdoors** and **Machines, Mechanisms and Mathematics**. The major aim of another project, Mathematics in Education and Industry, is "to promote the links between Education, Industry and Mathematics at the Secondary School level." The Mathematics Applicable Group is a spinoff from the Schools Council Sixth Form Project, **Mathematics Applicable**, which was housed at Reading University for many years. One book in the Mathematics Curriculum Project at the University of Nottingham dealt specifically with **Mathematics Across the Curriculum** (Ling, 1977).

In the United States the major new math programme, the School Mathematics Study Group, considered math/science integration only in some of its peripheral materials as did its British counterpart, the School Mathematics Project. A very fine book, entitled **The Man-made World**, was, however, produced by the Engineering Concepts Curriculum Project (1971) based at the Polytechnic Institute of Brooklyn. Other American packages include the Mathematics Resource Project at the Univesity of Oregon (Hoffer, 1977), and the recent School Science and Mathematics Association monograph called, **Interactions of Science and Mathematics**, (House, 1980).

Lacking as we do in Canada any equivalent national funding body to the American National Science Foundation or the British Schools Council any math/science materials which have been produced in this country have achieved only local circulation.

A university-level project which might provide a good model for possible future materials development is UMAP, the Undergraduate Mathematics and its Applications Project. Its module approach seems to yield both high quality materials and an effective means of distribution.

VI What Might Be (1): The Promise of Technology

School will disappear, education chief says: Home computers will replace teacher: minister
Kingston Whig-Standard

(Headline), 27 November 1980

Any serious fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution.

- A. N. Whitehead (1949, p.81)

We have, according to some commentators (Dertouzos and Moses, 1979; Toffler, 1980), embarked on a cultural transformation from the "industrial" to the "information" age. The effects of this transformation on individuals and institutions will, it is claimed, be as great as was the case in the last similar transformation, that of the agricultural to the industrial age. Whether or not all the predictions of these thinkers eventually came to be, it is already clear that the impact of technological innovation, particulary in the form of computers, on our culture is very great.

Schools are institutions which are almost invariably pointed to when examples of likely areas of structural change are discussed. As one indicator of the rapidly increasing interest in this area we note that fewer than 200 people attended the 1980 conference of the Educational Computing Organization of Ontario; in 1981 there were 800 participants and 400 more had to be turned away. We have here a situation which is at once a problem and an opportunity for mathematics and science teachers. Within the next five years it seems likely that most school curricula will have to be modified in some way to account for the existence of computers. One issue which might well receive consideration in such modifications is the integration of math and science topics.

Despite the very rapid increase of interest in school computing in the last two years (largely attributable to the development of microcomputers) we have, in fact, over a decade of experience on which to reflect. Of the many different types of approaches tried in those years perhaps the area of simulation with its strong links to scientific problems and elementary mathematical modelling (Friedland and Frishman, 1973; Lancaster, 1976) offers the most promise.. Our experience with the use of calculators in schools over the past five or six years (Burt, 1979; Suydam, 1980) provides us with something of a 'pilot-study' in this area.

It seems possible that some problems which are now fairly significant, such as memory size and language compatibility, will diminish with time. The language APL has many strong supporters and would seem to be particularly suited for work in mathematics and science at the secondary level. The work of Papert (1980) and his colleagues on the language LOGO represents the only major attempt to consider the wider social and educational implications of computing in schools. Initially, LOGO was seen as a tool for use at the elementary level, however, with the publication of Abelson and diSessa's work (1981) on **"Turtle Geometry"** it can be seen how naturally the ideas of LOGO extend to the consideration of questions in secondary and university level physics.

The real difficulties with the use of Technology in schools will come not with problems of hardware, languages or even finances. As was the case with the curriculum reforms of the sixties, the critical point will come with the classroom teacher and his or her understanding of, preparation for and support of the reforms. Unless definite steps are taken soon the mistakes of the 'curriculum-reform sixties' will be relived in the 'computer-reform eighties'.

VII What Might Be (2): A Broader Vision of Mathematics/Science Integration

The solution which I am urging is to eradicate the fatal disconnection of subjects which kills the vitality of our modern curriculum.

- A. N. Whitehead (1949, p. 18)

Althusser and his followers have made the important point that 'science' does not exist; only particular sciences do. To talk about science is to fall prey to an illusion or to perpetuate the same on someone else.

- J. C. Guēdon (Hooker, 1980, p. 89)

Among the many attractive features of mathematics and science are their clear and tangible results. As practitioners in those fields it is tempting for us to stay within their confines, even when considering questions like the ones here. This is a temptation we must resist. As one critic says of his colleagues who are suggesting computers as a panacea for educational ills:

No fix, technological or otherwise, of the American education system that does not recognize that American schools are rapidly becoming America's principal juvenile minimum security prisons can be expected to have socially therapeutic effects. Giving children computers to play with, while not necessarily bad in itself, cannot touch this or any other real problem.

(Weizenbaum, 1979, p. 440)

As described in a previous section, the educational institutions in which our mathematics and science classes are taking place are under very considerable pressure. Of even more importance, however, is the global context. Our world at present faces a complex of major problems such as food scarcity, pollution, over-population, resource depletion and the threat of nuclear annihilation, which have come to be known as "the world problematique" (Waddington, 1977 p. 13). Any educational endeavour worthy of the name cannot ignore these problems, and since some of them (like the nuclear threat) are a direct consequence of scientific and mathematical research, we perhaps have a special responsibility to work toward some ameliorization of the human prospect wherever we can.

The question of the nature of science and its influence on contemporary western culture has been a contentious one in the past two decades (Bohm, 1981; Bronowski, 1973; Capra, 1975; Feyerabend 1975, 1978; Kuhn, 1962; Lakatos, 1970; Lovins 1980; Nagel, 1961; Pirsig, 1975; Polanyi, 1964; Popper, 1968; Snow, 1959; Suzuki, 1980). There has been less discussion of the parallel questions in mathematics (Barrett, 1978; Davis and Hersh, 1980; Kline 1980; Lakatos, 1978). It seems generally agreed that it is important to make the general public aware of the human aspect of science and technology (Hooker, 1980).

One aspect of the human face of science and mathematics is the strong commitment made to aesthetic considerations (Weschler, 1978). As Henri Poincarē (1958) put it:

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living. (p. 8)

This stress on the aesthetic in science and mathematics is not something of which the general public is very aware. It could perhaps be used as a springboard to help in the generation of curricula which not only ameliorate some of our immediate math/science problems, but also make some progress in solving more general educational difficulties.

The gap which exists between science and mathematics at the school level is just one of many which riddle our academic landscape. Seen in Snow's (1959) terms it is only a minor tiff in the more powerful of the two cultures; a frog-mouse affair. If physicists find it difficult to communicate with mathematicians these days (although there may well be problems with biologists and chemists too), it is as nothing compared with the differences which separate him from historians or students of literature. At the root of the problem is specialization. As a discipline evolves it develops a language and a set of procedures of its own (the notorious Kuhnian paradigms) and xenophobia becomes the order of the day.

One way to try to counteract this in a school curriculum is to focus on the concepts which are common to several disciplines, rather than on the disciplines of themselves. Symmetry, for example, is a concept which plays an important role in art and music as well as in physics and mathematics. Other 'big ideas' might include proportion, boundary, polarity, continuity, system, equilibrium and isomorphism. Whitehead was a firm believer in the importance of big ideas. In the **Aims of Education** (1949) we find, "the really useful training yields a comprehension of a few general principles with a thorough grounding in the way they apply to a variety of concrete details" (p. 37)

and "the art of reasoning consists in getting hold of the subject at the right end, of seizing on the few general ideas which illuminate the whole, and of persistently marshalling all subsiduary facts round them. Nobody can be a good reasoner unless by constant practice he has realized the importance of getting hold of the big ideas and of hanging on to them like grim death." (p. 87).

Perhaps not suprisingly for one of his background, Whitehead saw a special role for mathematics in this approach, "mathematics, well taught, should be the most powerful instrument in gradually implanting this generality of idea" (1949, p. 60). He was also aware, though, that teaching through such a method would not be easy. In an essay entitled, 'Mathematics and Liberal Education' he wrote, "Now, nothing is more difficult than to transmit to our pupils real general ideas, as distinct from pretentious phrases. Nobody with any sense can confront a class for long without discovering that all sound teaching is concerned with definite, accurate achievements on the part of the pupils." (1948, p. 186).

Under the name 'integrative education' a group of thinkers under the leadership of Henry Margenau (1972) worked for many years on the possibility of a 'big idea' approach to curriculum. More recently the attractive book and sophisticated television and film support materials for Judson's **Search for Solutions** (1980) has shown the power of a conceptual approach to mathematics and scientific ideas.

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MATHEMATICS IN GENERAL IS FUNDAMENTALLY THE SCIENCE OF SELF-EVIDENT THINGS.

Felix Klein

NUMBER PATTERNS ON THE EXENDED ADDITION TABLE

by

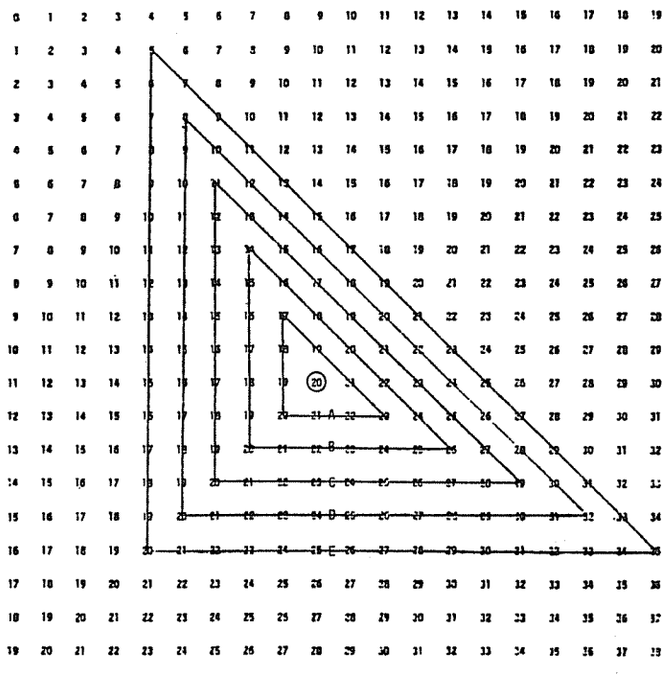
David R. Duncan and Bonnie H. Litwiller

The extended addition table is a numerical array that is rich in number patterns. To discover these patterns, students can use their calculator skills and they must be able to make generalizations. Both of these activities are important in the study of mathematics.

We shall present several activities for pattern development. All of these activities are on the extended addition table.

1. Figure I displays the extended addition table with "nested isosceles right triangles" drawn upon it. The innermost triangle contains an interior number. The legs of the isosceles right triangles are horizontal and vertical.

Figure I



Have your students:

1. Find the sum of the entries that lie on the perimeter of each of the nested triangles; call this sum P.
2. Call the interior number which lies in the interior of the innermost triangle I.
3. Find the ratios of P to I.
4. Count the number of entries that lie on the perimeter of each triangle.
5. What pattern do you observe?

Table I show the results of the computations related to the triangles which lie in Figure I.

Table I

Triangle	P	I	$\frac{P}{I}$	# of Entries Which Lie on each Δ	# of Entries Which Lie in the Interior of the Innermost Δ
A	180	20	$\frac{180}{20} = \frac{9}{1}$	9	1
B	360	20	$\frac{360}{20} = \frac{18}{1}$	18	1
C	540	20	$\frac{540}{20} = \frac{27}{1}$	27	1
D	720	20	$\frac{720}{20} = \frac{36}{1}$	36	1
E	900	20	$\frac{900}{20} = \frac{45}{1}$	45	1

Observe that the numerator of the ratio $\frac{P}{I}$ is the same as the number of entries which lie on the perimeter of each triangle. The denominator reflects the fact that there is one number in the interior of the innermost triangle.

Have your students draw several sets of nested isosceles triangles with one interior number. Does the pattern hold?

II. Figure II displays a set of nested isosceles triangles. In this case, the innermost triangle contains 3 interior entries.

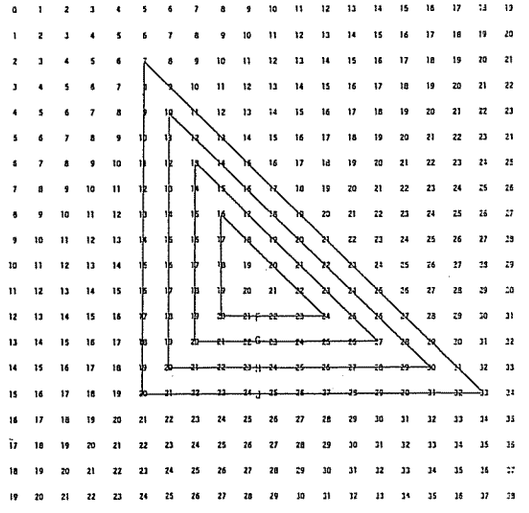


Figure II

Have your students:

1. Find the sum of the entries that lie on the perimeter of each of the nested triangles. Again, call this sum, P.

2. Find the sum of the entries that lie in the interior of the innermost triangle. Call this sum, I.

3. Find the ratio of P to I. Since this ratio can be expressed in many equivalent ways, select the representation that will be most useful in finding a pattern which involves the number of entries in triangles and in the innermost triangle.

4. Count the number of entries that lie on each triangle.

5. What pattern do you see?

Table II shows the results of these calculations.

Table II

Triangle	P	I	$\frac{P}{I}$	# of Entries Which Lie on each Δ	# of Entries Which Lie in the Interior of the Innermost Δ
F	240	60	$\frac{240}{60} = \frac{4}{1} = \frac{12}{3}$	12	3
G	420	60	$\frac{420}{60} = \frac{7}{1} = \frac{21}{3}$	21	3
H	600	60	$\frac{600}{60} = \frac{10}{1} = \frac{30}{3}$	30	3
J	780	60	$\frac{780}{60} = \frac{13}{1} = \frac{39}{3}$	39	3

Again the numerator of $\frac{P}{I}$ represents the number of entries which lie on the perimeter of each triangle. The denominator reflects the number of entries which lie in the interior of the innermost triangle. Have your students draw other sets of nested triangles and check the pattern.

III. Using the nested triangles of Figures III, IV and V, do the same steps as in Activity II. Observe that there are 6 entries which lie in the interior of the innermost triangle of Figure III, 10 in the interior of the innermost triangle of Figure IV, and 15 in the interior of the innermost triangle of Figure V.

Figure III

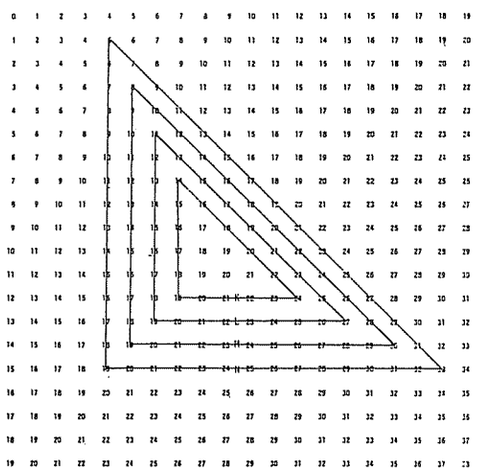


Figure IV

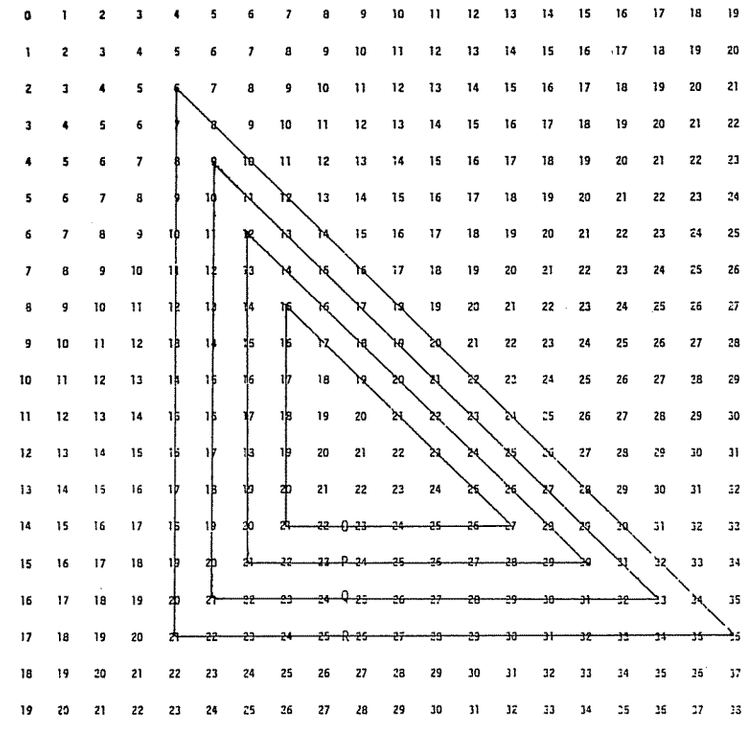
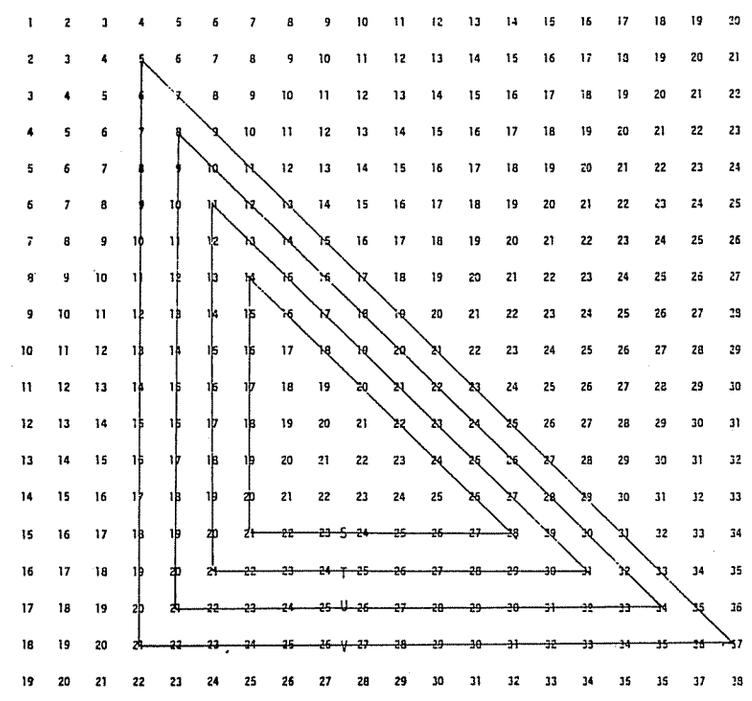


Figure V



Tables III, IV and V show our computations.

Table III

Triangle	P	I	$\frac{P}{I}$	# of Entries on the Perimeter	# of Entries Which Lie in the Interior of the Innermost Δ
K	285	114	$\frac{285}{114} = \frac{15}{6}$	15	6
L	456	114	$\frac{456}{114} = \frac{24}{6}$	24	6
M	627	114	$\frac{627}{114} = \frac{33}{6}$	33	6
N	798	114	$\frac{798}{114} = \frac{42}{6}$	42	6

Again, the numerator $\frac{P}{I}$ is the number of entries which lie on the perimeter and the denominator is the number of entries which lie in the interior of the innermost triangle.

Table IV

Triangle	P	I	$\frac{P}{I}$	# of Entries on the Perimeter	# of Entries Which Lie in the Interior of the Innermost Δ
O	378	210	$\frac{378}{210} = \frac{18}{10}$	18	10
P	567	210	$\frac{567}{210} = \frac{27}{10}$	27	10
Q	756	210	$\frac{756}{210} = \frac{36}{10}$	36	10
R	945	210	$\frac{945}{210} = \frac{45}{10}$	45	10

Table V

Triangle	P	I	$\frac{P}{I}$	# of Entries on the Perimeter	# of Entries Which Lie in the Interior of the Innermost Δ
S	441	315	$\frac{441}{315} = \frac{21}{15}$	21	15
T	630	315	$\frac{630}{315} = \frac{30}{15}$	30	15
U	819	315	$\frac{819}{315} = \frac{39}{15}$	39	15
V	1008	315	$\frac{1008}{315} = \frac{48}{15}$	48	15

Observe that the pattern holds. Have your students look for other number patterns on the extended addition table. Does this pattern also hold for an extended subtraction table and an extended multiplication table?

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MATHEMATICAL EXPOSITIONS

A STATISTICIAN IS SOMEONE WHO, IF HE PUTS HIS HEAD IN A HOT OVEN AND HIS FEET IN A BUCKET OF ICE WATER, WOULD REMARK, 'ON THE AVERAGE I FEEL FINE'.

J. Simons

OMAR KHAYYĀM, MATHEMATICIAN

by

Cherie Gates

Here with a Loaf of Bread beneath the Bough, A Flask of Wine, a Book of Verse - and Thou Beside me singing in the Wilderness - and Wilderness is Paradise enow.[1]

In 1859, Edward Fitzgerald brought this now well-known verse to the attention of the English-speaking world in his free translations of the **Rubāiyāt** of Omar Khayyām. Omar Khayyām, the eleventh century Persian, has achieved general renown as a poet, but few are acquainted with the fact that he was "the greatestmathematician and astronomer of his time, and the most original of all the Saracen mathematicians."[2]

Abu-l-Fath Omar ibn Ibrahim Khayyami was born near Nishapur in north eastern Persia and lived between the years 1040 and 1123 A.D.[3] He received his early education in Nishapur. Later, the vizier of the Seljuk Sultan and a childhood friend, Nizam al-Mulk awarded Omar with an annual stipend for the pursuit of his mathematical studies. (It is reported that Omar Khayyām refused an office in the Sultan's court in favour of his study.)

If I may be content with simple living, Such as my hand may gain by its own efforts, I shall be safe 'midst all the turns of Fate.[4]

Omar Khayyām was held in great respect by his contemporaries - an authority on philosophy, astronomy and mathematics. In 1074, Sultan Malik-Shah invited Omar to undertake large scale astronomical observations, revise the astronomical tables and reform the calendar. His series of astronomical tables was published as **Ziji Malikshahi.** The calendar he composed resulted in the Jalalian era, which began on March 15, 1079, and proved to be more accurate than that later proposed by Pope Gregory XIII and now adopted by most people. Among his other writings, still preserved, are a number of philosophical essays as well as a work on algebra, **On Demonstrations of problems of Algebra and Almucabala** (1079) and **Commentaries on the Difficulties in the Postulates of Euclid's Elements.**

As a mathematician, Omar Khayyām placed very little stress on the practical aspects of mathematics. He was an "algebraist in his choice of material but a geometer in method".[5] Omar showed some Diophantine influence in his subject material, i.e. the exact solution of equations, and yet he preferred the earlier techniques of the Greeks to the more convenient symbolism used by Diophantus of Alexandria.

Omar Khayyām's treatise on algebra is considered to be the "most notable contribution of his country and age to the study of mathematics".[6] In his **Algebra,** Omar classified equations according to degree. He was most interested in the general solution of the third degree, or cubic, equations. He extended Euclid's method for the solution of the quadratic equation, explained in the **Elements,** to the solution of cubic equations. Omar gave only geometric solutions to the cubic equation because he did not believe that arithmetic solutions were possible.[7]

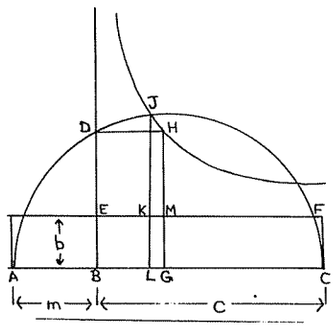
For Euclid, the quadratic equation was interpreted in the form $x^2 + b = ax$ where each term represented an area and a and b were both positive, and the unknown segment x could be found using straightedge and compass. To solve the cubic equation (in modern notation, $x^3 + ax^2 + b^2x + c^3 = 0$) Omar Khayyam had to consider the intersection of conics. (This method had been used earlier to solve cubics but had not previously been generalized to include all cubic equations). Since only positive roots held meaning for Omar, each problem had to be separated into all cases according to the parameters (a, b, c) being positive, negative, or zero. He was also obligated to specifically identify his conic sections since the concept of a general parameter was not known to him.[8]

The cubic equation: $x^3 + b^2x + a^3 = cx^2$ is the type of problem described by Omar as "a cube, some sides, and some numbers are equal to some squares."[9] The solution of this cubic equation by the method of Omar Khayyám can be accomplished in the following manner:[10]

By the basic construction of finding the fourth proportional to three given line segments, find the line segment z such that b:a = a:z and then again by the same construction, find the line segment m such that b:z = a:m. Then $m = a^3/b^2$.

Now construct a line AC such that $AB = m = a^3/b^2$ and BC = c. Draw a semicircle on the diameter AC and a perpendicular to AC at B, intersecting the semicircle at D. On BD, mark off BE = b and draw EF parallel to AC. By the basic construction find a G on BC such that ED:BE = AB:BG and complete the rectangle DBGH. Draw through H, the rectangular hyperbola having asymptotes EF and ED. Let the hyperbola cut the semicircle in J and let the parallel to DE through J cut EF in K and BC in L. Let GH cut EF in M.

Figure 1



1. Since J and H are on the hyperbola,
   (EK) (KJ) = (EM) (MH)

2. Since ED:BE = AB:BG, we have
   (BG) (ED) = (BE) (AB)

3. From 1. and 2.,
   (EK) (KJ) = (EM) (MH) = (BG) (ED) = (BE) (AB)

4. Now (BL) (LJ) = (EK) (BE + KJ)
   = (EK) (BE) + (EK) (KJ)
   = (EK) (BE) + (AB) (BE) by 3.
   = (BE) (EK + AB)
   = (BE) (AL)
   Thus $(BL)^2 (LJ)^2 = (BE)^2 (AL)^2$

5. But from elementary geometry,
   $(LJ)^2 = (AL) (LC)$.

6. Therefore, from 4. and 5.
   $(BE)^2 (AL) = (BL)^2 (LC)$,
   or $(BE)^2 (BL + AB) = (BL)^2 (BC - BL)$

7. Substituting BE = b, $AB = a^3/b^2$, BC = c, in the above, we have
   $b^2(BL + a^3/b^2) = (BL)^2 (c - BL)$
   and thus
   $BL^3 + b^2BL + a^3 = (BL)^2c$

   It follows that BL = x is a root of the given cubic equation.

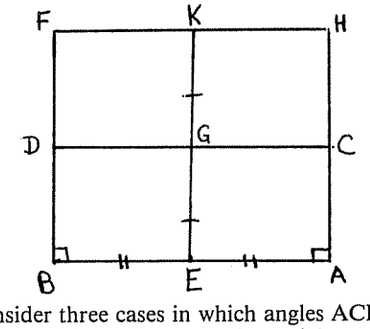
Because Omar Khayyám did not accept negative roots and all of the intersections of the conic sections were not considered, his method, while a great achievement, did not yield all of the roots of a given cubic equation.

Omar's assertion in his **Algebra** that he had discovered a law, independent of geometric figures, by which he could find higher powers of numbers indicates that he also must have known the formula for the expansion of the binomial (a + b)m, where n is a positive integer.

Omar Khayyam's second work, **Commentaries on the Difficulties in the Postulates of Euclid's Elements,** deals with such topics as the parallel axiom, and theory of proportions.

The parallel axiom (stated in slightly different form as Euclid's Fifth Postulate) states that through a point not on a line one and only one parallel can be drawn to the line. Omar tried to bridge the gap between Euclid's Fourth and Fifth Postulates. In his proof, "Omar Khayyam began with a quadrilateral, the two sides of which are equal and are both perpendicular to the base, and then considered the other (upper) angles of the quadrilateral, which are necessarily equal to each other."[11]

Figure 2



He found that he had to consider three cases in which angles ACD and BDC (Figure 2) are each less than a right angle; larger than a right angle, or equal to a right angle. Omar rejected the first two possibilities as they contradicted the principles of Aristotle. "We find here, apparently for the first time in history, the three situations later known as the hypothesis of the acute angle, that of the obtuse angle, and that of the right angle. These three situations are now known to lead respectively to the geometry of Bolyai-Lobačevskii, to that of Riemann, and to Euclidean geometry."[12]

In his discussion of the theory of proportions, Omar Khayyam defines ratios to be equal when they can be expressed by the ratio of integer numbers with as great a degree as we like.[13]. Now we see by this that Omar Khayyam was nearing a definition of the irrational number.

Thus Omar Khayyam emerges not only as a poet and a philosopher, but also as "a mathematician of considerable penetration, who contributed to the theory of equations, to the understanding of the parallel axiom, the the development of the concept of real number, and probably to the generalization of the binomial theorem."[14]

With them the seed of Wisdom did I sow,
And with mine own hand wrought to make it grow;
And this was all the Harvest that I reap'd-
'I came like Water, and like Wind I go.[15]

FOOTNOTES

1. Edward Fitzgerald, "Selections from the Rubāiyāt of Omar Khayyam" in Roberta A. Charlesworth, and Dennis Lee, **An Anthology of Verse,** (Toronto: Oxford University Press, 1964, p. 291.

2. Edna E. Kramer, **The Main-Stream of Mathematics,** (New York: Oxford University Press, 1963), p. 100.

3. This is an estimated date since no accurate dates are recorded.

4. Ali Dashti, **In Search of Omar Khayyam,** (London: George Allen & Unwin Ltd., 1971) p. 48.

5. Edna E. Kramer, op, cit., p. 101.

6. **Encyclopedia Britannia,** Volume 16, 1971.

7. Carl B. Boyer, **A History of Mathematics,** (New York: John Wiley & Sons Inc., 1968), p. 264.

8. Ibid., p. 265.

9. Julian Lowell Coolidge, **The Mathematics of Great Amateurs,** (London: Oxford University Press, 1950), p. 27.

10. Howard Eves, "Omar Khayyam's Solution of Cubic Equations" in **The Mathematics Teacher,** Volume 56, April 1958, pp. 285-286.

11. Carl B. Boyer, op. cit., p. 266.

12. D. J. Struik, "Omar Khayyam, Mathematician" the **The Mathematics Teacher,** Volume 51, April 1958, p. 283.

13. Ibid., p. 284.

14. Ibid., p. 284.

15. Edward Fitzgerald, op. cit., p. 242.

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IT IS EASIER TO SQUARE THE CIRCLE THAN TO GET ROUND A MATHEMATICIAN.
A. DeMorgan

THE DOUBLES DILEMMA

by

Todd Cardno

Hockey card collecting has long been popular among Canadian youngsters. Earlier this winter, my two younger brothers collected a set put out by a certain chewing gum company. The set, somewhat different from the traditional offering, which is a pack of 8 or 10 cards, consisted of 'stickers' which could be purchased four at a time in an envelope for 15c. The stickers, 269 in all, were to be entered into an album, which held a corresponding place for each member of the set. The album costs a modest 25c.

A large part of the joy that comes from collecting hockey cards is not knowing what cards you will receive in the pack or packs you have chosen. The cards, or stickers in this case, are packaged at random. The buyer cannot dictate what he receives. The packaging of cards is not a random process, only to the degree that, (i) each card of a pack is distinct, or at least as Mr. Ben. E. Houston of the Donruss Company informs me, "the machinery is such that it will not happen" that two of the same cards appear in a pack (Donruss packages baseball cards), (ii) the same number is produced of each card.

Every card collector experiences, what I call, the 'doubles dilemma'. For example, as the number of stickers in my brothers' album increased so did the average amount of 'doubles' in each pack. (When I say double, I mean a card appearing of which they have previously collected.) Now, the goal of most card collectors is a complete collection. However, he finds that as he comes closer and closer to this point, the ultimate end seems longer and longer away. For example, when two brothers have obtained 250 of the 269 size collection, it was not uncommon to purchase 4 packs and receive only one necessary entry. The cost of the collection begins to escalate.

Those last cards which the buyer is to receive, or may not receive at all are often termed 'hard to get'. Kids generalize that all collectors have just as much trouble obtaining these cards. In turn the pessimistic parent accuses the card producers of holding back certain cards. This is an all too common conclusion. Some thought shows that this practice is not necessary on the part of the gum companies. Suppose, for example that 250 of the 269 stickers have been collected, then 93% of the collection is complete. In which case if one chooses 20 stickers at random (5 packs) chances predict he will already have $20 \times .93 \sim 19$ of these stickers, and it is easily seen that as the size of a collection increases so does the probability of obtaining a needed card decrease. Finally, we propose the question, how many envelopes should be purchased to complete a collection? We rephrase and generalize the question to:

How many distinct cards does one have after the purchase of n packs, where each pack contains K cards and the size of a complete set is S?

Solution: Realize that we can only approach the amount, nevertheless we develop a sufficiently good approximation and denote it by S_n. Easily $S_1 = K$ (recall there is n two the same in a pack), then we give $S_2 = S_1 + K(1 - S_n/S)$, because generally $S_{n+1} = S_n + K(1 - S_n/S)$.

To see this, observe that in adding an n th pack to a collection one has S_n distinct cards previously, and so a fraction S_n/S of the collection is complete in which case the chance of obtaining some needed cards is $1 - S_n/S$. From a pack of K random cards, one receives $(1 - S_n/S)K$ necessary cards. We rewrite this recursion as $S_{n+1} = S_n(1 - K/S) + K$. We can continue this recursion backwards as follows, since $S_n = S_{n-1}(1 - K/S) + K$, we have

$$S_{n+1} = [S_{n-1}(1 - K/S) + K](1 - K/S) + K$$

$$= S_{n-1}(1 - K/S)^2 + K(1 - K/S) + K$$

we could continue, substituting for S_{n-1}, then S_{n-2}, ... so that we ultimately obtain

$$S_{n+1} = K(1 - K/S)^n + K(1 - K/S)^{n-1} + \cdots + K,$$

using the formula for the sum of a geometric progression

$$S_{n+1} = K\frac{(1 - K/S)^n - 1}{(1 - K/S) - 1} = K\frac{(1 - K/S)^n - 1}{-K/S} = S[1 - (1 - K/S)^{n+1}]$$

or more generally

$$S_n = S[1 - (1 - K/S)^n]$$

Thus we have an approximation. Notice that the graph of S_n, with respect to n is increasing but concave downward, a result that explains some of the observances we discussed earlier. Notice as n grows larger S_n approaches but the approximation never actually reaches it.

To answer our original question, how many packs of stickers should be purchased to complete the 269 sized collection, we begin here $K = 4$ and $S = 269$. We wish to find n such that $S_n = S$, but this is never so, however the smallest n such that $S_n > 268$ is a good approximation to the number we desire. We solve

$$269(1 - (1 - 4/269)^n) > 268$$

$$n = 374.$$

This precludes spending more than \$56 to complete the collection, an amount substantially larger than the simple \$10.87 which many persons might figure since it is 269 multiplied by the average cost per sticker. Approximately \$23 must be spent to fill just 90% of the album.

Now, not many youngsters will spend \$56, or even \$23. There are ways to avoid this. One is of course 'trading'. It is possible to take account of this way, in an exact sort of way, but calculations become sufficiently less simpler than the ordinary way in which we obtained S_n. Otherwise, the gum company offered to sell at 10c per piece, through the mail, specified stickers. Using this strategy, let us determine the approximate minimum amount of money necessary to complete a collection.

Let's say my brothers purchase n packs. They then have approximately $269 - S_n$ stickers to obtain. In this way we wish to minimize the expression

$$g(n) = .15n + .10(269 - S_n)$$

$$g(n) = .15n + .10(269 - 269(1 - (1 - 4/269)^n)$$

then

$$= .15n + 26.9(1 - 4/269)^n$$

$$g'(n) = .15 + 26.9(1 - 4/269)^n \times \ell n(1 - 4/269)$$

$$= .15 - 0.430\cdot(.9851)^n,$$

but for a minimum here, $g'(n) = 0$, then we must solve for n where $0.430\,(.9851)^n = .15$ we obtain $n = 65.83$. Since we desire a natural number we take $n = 66$. Thus to minimize costs one should purchase 66 packs after which one has obtained close to 170 stickers. The remaining should be purchased at a cost of 10c per entry. All in all, this minimal cost is, and again we say approximately, \$21.80.

This is a suprising result, but perhaps not all too surprising to the parents of young card collectors who wonder how all their change is being eaten up. In concluding, I emphasize it is not at all financially wise to purchase those last 'hard to gets' by purchasing more packs. To save considerable time and money they should be obtained through other means.

110-151 Gateshead Crescent
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DEPARTMENTS

THE GOOD OLD DAYS, THE GOOD OLD DAYS,
WE ALL SO FONDLY SPEAK OF;
WHICH IF THEY EVER SHOULD COME BACK,
NO ONE COULD STAND A WEEK OF.

PROBLEMS? DID YOU SAY

by
Pat Yip

Notice:
Problems and solutions are solicited from readers. Came across a good one lately? SEND IT IN! (to P. Yip, Department of Mathematical Science, McMaster University, Hamilton, Ontario.)

PROBLEMS:

21.1.1 Three is a wonder.

One can determine whether a number is divisible by 3, by 'casting out' 3's. For example, 1,231,572 is divisible by 3 since the groupings: (12) (3) (15) (72) all have zero remainder by 'casting out' 3's. In other words, n is divisible by 3 if the sum of its digits is. Show why this procedure works.

21.1.2 It seems rather odd.

The sum of 3 consecutive natural numbers is divisible by 3. It is also true for 5, 7, 9...

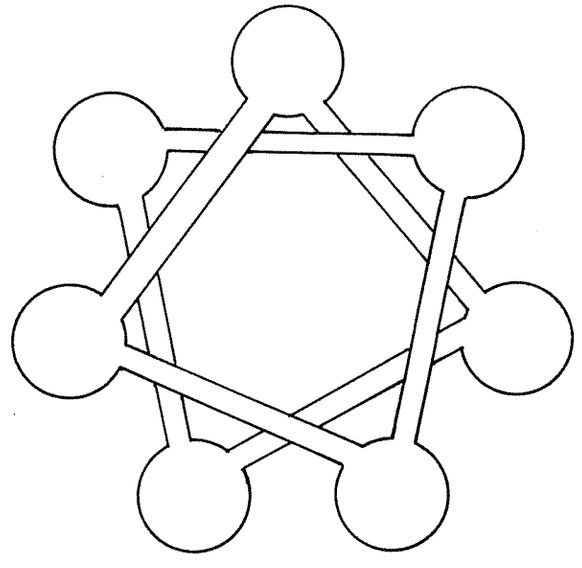
Explain. Is it true for all odd numbers?

What about even numbers?

21.1.3 Out there in space.

A space station consists of seven ports, interconnected by de-contamination tunnels as shown:

Diagram 21.1.3



Six different space ships are arriving, one after the other, at the station. If the crew of each ship can dock at any vacant port, but must go through a de-contamination tunnel to settle in the next vacant port, how should the six crews dock in order to settle eventually in the space station, each to a port?

SOLUTIONS: (for problems in volume 20.3 and problem 20.2.1)

20.2.1 Straight-edge and compass.

(Proof by proposer and alternate solution sent in by J. Byrnes of Ottawa)

Given: Two intersecting straight lines OK and OL and a point P.

To construct: Circle or circles passing through P and having OK and OL as tangents.

Construction: See vol. 20, no. 3.

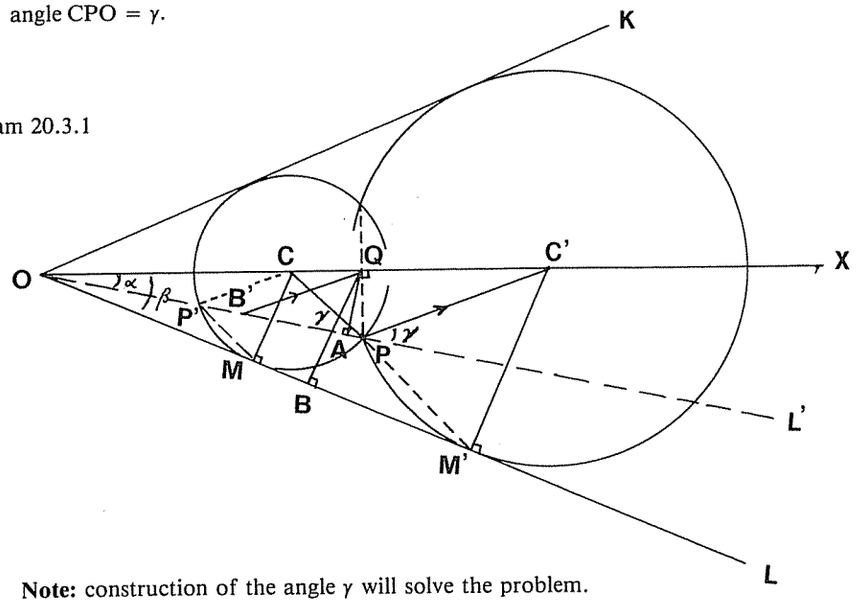
Proof:

Let C and C' be the centers of the required circles

Let angle XOL = β, angle XOL' = L,

angle CPO = γ.

Diagram 20.3.1



Note: construction of the angle γ will solve the problem.

1). Angle CPO = angle C'PL', since triangles CP'M and C'PM' are similar and triangle CPP' is isoscles.

2). sin γ/OC = sin L/CP = sin L/CM,

therefore: (sin γ)/(sin L) = OC/CM = 1/(sin β)

therefore: sin γ = (sin L)/(sin β)

3). In the construction, sin L = QA/OQ and

sin β = QB/OQ, thus, sin γ = QA/QB

4). QB'A is a right triangle with QB' = QB, therefore,

angle QB'A = γ. C' is therefore located.

5). The location of center C is similarly constructed.

20.3.1 Circle and radii.

2n equally spaced radii separate circle into 2n segments. A secant, not parallel to any of the radii, will, at most, intersect n radii, dividing the traversed $(n+1)$ segments into $2(n+1)$ segments. Therefore, the total number of non-intersecting segments is $2(n+1)+(n-1) = 3n+1$.

20.3.2 Fractions and function.

$f_1(x) = (2x-1)/(x+1)$ and $f_{n+1}(x) = f_1(f_n(x))$

Find $f_{28}(x)$.

It is easily seen that $f_m(x) = f_p(f_q(x))$

where $p + q = m$.

1). $f_1(x) = 2 - 3/(x+1)$ and $f_2(x) = 1 - 1/x$;

2). $f_4(x) = f_2(f_2(x)) = 1/(1-x)$

Therefore: $f_8(x) = f_4(f_4(x)) = 1 - 1/x = f_2$

3). $f_{28}(x) = f_4(f_4.....(f_4(x)))))))$

$= f_2(f_2(f_2(f_4(x))))$

$= f_2(f_4(f_4(x)))$

$= f_2(f_2(x)) = f_4(x) = 1/(1-x)$

20.3.3 Such is life.

Let x be the father's age

then, the son lived x/2 years,

and:

$x/2 = (1 - 1/6 - 1/12 - 1/7)x - 9$

Therefore: $x = 84$.

...WHAT IS PHYSICAL IS SUBJECT TO THE LAWS OF MATHEMATICS, AND WHAT IS SPIRITUAL TO THE LAWS OF GOD, AND THE LAWS OF MATHEMATICS ARE BUT THE EXPRESSION OF THE THOUGHT OF GOD.

Thomas Hill

THE OBJECT OF PURE PHYSIC IS THE UNFOLDING OF THE LAWS OF THE INTELLIGIBLE WORLD: THE OBJECT OF PURE MATHEMATIC THAT OF UNFOLDING THE LAWS OF HUMAN INTELLIGENCE.

J.J. Sylvester

BOOK REVIEWS

THE ORIGINS OF CAUCHY'S RIGOROUS CALCULUS

Judith V. Grabiner, The Mit. Press, Cambridge, Massachusetts, 1981

Although the beginnings of differential calculus date from the 1630's (with Fermat) and integral calculus is thousands of years old, it was not till the nineteenth century that mathematicians gave serious thought to the foundations of calculus. The chief figure in the program of putting calculus on a rigorous basis was the French mathematician Augustin Cauchy (1789-1857) but, as the book under review points out, the inspiration for his work lies in the writings of certain eighteenth-century mathematicians.

Before the 1660's, the problems of finding areas and volumes were extremely difficult, but then Newton and Leibniz made calculus into a systematic method by exploiting the Fundamental Theorem of Calculus. So it was only natural that during the latter part of the seventeenth century and the entire eighteenth century mathematicians plunged headlong into the task of using this powerful new method of calculus to make exciting discoveries in all aspects of infinite processes (integrals, infinite series, infinite products, areas, volumes, lengths of curves) and to use it in solving difficult problems in physics. It was assumed without a second thought that what was true for ordinary arithmetic and algebra must also be true for infinite processes. It was not appropriate during this period to stop and take stock of the foundations of the subject. After all, as Horace Lamb once put it, "A traveller who refuses to pass over a bridge until he has personally tested the soundness of every part of it is not likely to go far; something must be risked, even in mathematics."

Nevertheless there were a few voices crying in the wilderness. The most famous of these was Bishop Berkeley who objected to both the differentials of Leibniz and the "evanescent increments" of Newton. The latter, Berkeley said, "are neither finite quantities, nor quantities infinitely small, not yet nothing. May we not call them ghosts of departed quantities?"

The author makes the interesting point that the eventual concern for the foundations of the subject arose when mathematicians were asked to **teach** calculus. Many scientists of the eighteenth century were supported by royal patronage and did not teach. But when the revolutionary government established the Ecole Polytechnique in Paris in 1795, the major scientists and mathematicians taught there. "Having students forces a teacher to expound the first principles of a subject clearly and to think those principles through anew. This helps to explain why the contributions to the foundations of the calculus of Lagrange, Cauchy, Weierstrass, and Dedekind were all stimulated by their teaching."

The author identifies Lagrange as a transitional figure between the eighteenth and nineteenth centuries. Lagrange thought it was important to establish rigorous foundations for calculus but his own attempts to do so were flawed. (He defined f'(x) as the coefficient of the linear term in the Taylor series expansion of $f(x+h)$.) Cauchy was affected by Lagrange's concern for rigour but he decided to take d'Alembert's advice and base calculus on Newton's idea of a limit. (Most continental mathematicians had been following Leibniz in talking about differentials or infinitesimals.)

The two main works in which Cauchy carried out his program were **Cours d'Analyse** (1821) and **Calcul Infinitesimal** (1823). In the former we read Cauchy's definition of a limit: "When the successively attributed values of the same variable indefinitely approach a fixed value, so that finally they differ from it by as little as desired, the last is called the **limit** of all the others." This is not so very different from Newton's original formulation of a limit, but the important thing is that when Cauchy used this definition in a proof of a theorem he often translated it into the algebra of inequalities. The first occurrence in history of the famous $\varepsilon - \delta$ formulation of a limit occurs in Cauchy's proof of a theorem which is related to the Mean Value Theorem. The first line of the proof reads as follows:

"Designate by δ and ε two very small numbers; the first being chosen in such a way that, for numerical values of i less than δ, and for any value of x between x_0 and X, the ratio $(f(x+i)-f(x))/i$ always remains greater than $f'(x)-\varepsilon$ and less than $f'(x)+\varepsilon$."

(Notice that Gauchy thought that the same value of δ would work for all values of x. At this stage of his career, Cauchy did not distinguish between convergence and uniform convergence.)

Cauchy's works also contain a more general and more careful treatment of continuity, integrals, and convergence of series than was done by his predecessors. The author finds the roots of Cauchy's treatment in the use of inequalities and error estimates by d'Alembert, Lagrange, and Ampère and even further back in the work of Newton and Euler on approximations of solutions of algebraic equations. But that is not to deny the originality of Cauchy's achievement.

Ivor Grattan-Guinness (1) had accused Cauchy of plagiarizing the work of Bernhard Bolzano. A paper of Bolzano dating from 1817 contains a careful definition of continuity, a proof of the Intermediate Value Theorem, and a statement of the so-called Cauchy criterion for the convergence of series, four years before Cauchy published similar material. But Bolzano was isolated in Prague and his papers did not become generally known till much later. The book under review refutes Grattan-Guinness by showing that both Cauchy and Bolzano were influenced by Lagrange.

There were some gaps in Cauchy's account of the foundations of caluclus. But it was his lectures and writings that inspired others such as Abel, Riemann, Weierstrass, Heine, Dedekind, and Cantor to finish the task by filling in the gaps and developing the modern theory of real numbers. "Augustin-Louis Cauchy neither began nor completed the rigorization of analysis. But more than any other mathematician, he was responsible for the first great revolution in mathematical rigor since the time of the ancient Greeks". Judith Grabiner deserves our thanks for painstakingly researching the story of this revolution and presenting the results of her scholarship in a manner which engages our interest.

REFERENCE

1. I. Grattan-Guinness, The Development of the Foundations of Mathematical Analysis from Euler to Riemann, The MIT Press, Cambridge, Massachusetts, 1970.

James Stewart
Department of Mathematical Sciences
McMaster University, Hamilton, Ontario

THE HANDBOOK OF ARTIFICIAL INTELLIGENCE, VOLUME I

Edited by Avron Barr and Edward A. Feigenbaum, William Kaufman, Inc., Los Altos, California, 1981.

The field of artificial intelligence deals with the construction of programs exhibiting intelligent behaviour. Examples of such programs include theorem provers, game playing programs, speech understanding programs, and programs capable of diagnosing bacterial infections. Even though this field is barely 25 years old it has accumulated a large number of terms, ideas and techniques. The history of artificial intelligence is marked by many controversies and fads making it hard to separate the important ideas from the irrelevant details. The Handbook of Artificial Intelligence is one of the first attempts to present an unbiased and relatively complete survey of this field.

This three volume handbook is divided into sixteen chapters each addressing one area of artificial intelligence. The chapters are divided into articles that have been produced by some of the leading experts in this field. The first article in each chapter gives a general overview of the area with subsequent articles covering some of the important ideas in the area and several applications of these ideas. By using a wide range of authors the editors have avoided the bias and uneven coverage that often characterizes books in this field.

The four topics that are covered in volume I are search, knowledge representation, understanding natural language, and understanding spoken language. Many artificial intelligence programs use a large knowledge base to store the information they require for solving problems. In order to find particular pieces of information these programs must use some searching techniques. The techniques used for encoding the information in the knowledge base are called knowledge representation techniques. One of the most glamorous applications of artificial intelligence is the processing of natural language. The computer understanding of written English has been one of the most active research areas in this field. Related to this research problem is the more difficult task of understanding spoken language. This research area must deal with the processing of auditory information and ungrammatical utterances.

While not intended as a textbook the clarity and completeness of this book equals and surpasses most if not all of the textbooks in this field. The editors have produced a book that is relatively free of jargon allowing the reader to cover the chapters in any order he pleases without excessive references to other chapters. Most of the discussions in this book can be followed by a reader with only a basic understanding of computer science. While the three volume set will be rather long I recommend this book to anyone wanting a thorough introduction to the field of artificial intelligence.

Mark Green
Unit for Computer Science
McMaster University
Hamilton, Ontario

ERROR PATTERNS IN C)N PUTATION

Robert B. Asblock, Charles E. Merrill Publishing Company, Ohio, 1982

The purpose of this book is to provide teachers with the skills needed to assist students to overcome recurring problems in computation. Individualized, remedial assistance which helps to build the students' self-concept is recommended, and step-by-step guidelines for instruction to mastery are provided.

The author has included a large section in which teachers can participate in the indentification of error patterns using simulated student work. This is followed by an analysis section which develops the teacher's ability to search for the cause of the erroneous computation. A series of corrective procedures concludes the main body of the text. These include several remedial strategies designed to help the child to eliminate the error patterns which have been demonstrated.

A lengthy list of References is included for readers who wish to further pursue the topic of the book, and the Appendices provide additional information about error patterns in both computation and other areas of mathematics.

Eileen Collins
Hamilton Board of Education
Hamilton, Ontario

IF A MAN'S WIT BE WANDERING, LET HIM STUDY MATHEMATICS; FOR IN DEMONSTRATIONS, IF HIS WIT BE CALLED AWAY NEVER SO LITTLE, HE MUST BEGIN AGAIN.

Lord Bacon

NEWS ITEMS

1.
The National Council of Teachers of Mathematics (NCTM) announced the election of **Bob Robinson** to a three-year term on its Board of Directors at its 60th Annual Meeting in Toronto, Ontario (14-17 April 1982).

A resident of Dundas, Ontario, Robinson is an elementary mathematics consultant and metric coordinator for the Hamilton Board of Education. He received the B.A. degree from Wilfred Laurier University and the M.Ed. from the University of Toronto. He received an elementary teacher's certificate from Hamilton Teachers' College and a Specialist Certificate in Special Education and an Elementary Mathematics Certificate from the Ontario Ministry of Education.

Before assuming his present position, Robinson was an elementary teacher and mathematics team leader in elementary school.

In addition to his membership in many professional organizations, the new director has been a director for the Ontario Association for Mathematics Education (OAME), served on numerous OAME committees as well as the editorial panel of the ONTARIO MATHEMATICS GAZETTE, and been principal or instructor of several Ontario Ministry of Education courses in elementary mathematics education.

He has been a representative for the Ontario Association for Teachers of Mathematics; Councillor for OAME; chairman for an OAME Annual Meeting Program Committee; speaker at numerous regional, provincial, and state meetings; and chairman of several elementary mathematics curriculum committees. His NCTM activities include serving on the Committee on Affiliated Groups, the program committee for the 55th Annual Meeting and the Toronto regional meeting, and the meeting rooms committee at the 60th Annual Meeting. He has also been a reporter and referee for the ARITHMETIC TEACHER a reviewer for the Educational Materials Committee, and delegate to delegate assemblies and speaker at annual and regional meetings.

Robinson is coauthor of the mathematics texts, MATH IS...1, MATH IS...2 published by Nelson Canada, and Mathematics Workbooks, Grades 1-8. He has been a contributor and reviewer of the METRIC STYLE GUIDE published by the Council of Ministers of Education, Canada.

2.

A Hamilton born Mathematician and Russian Mathematician have been named the first winners of an international prize in mathematics that is administered, like the Nobel Prize in physics and chemistry by the Royal Academy of Sweden.

Louis Nirenberg, born in Hamilton, Ontario in 1925, educated at McGill University and New York University, and presently professor of mathematics at the Courant Institute of Mathematical Sciences, and **Vladimir Igorvich Arnol'd** of the University of Moskow, were each designated to receive $30,000.00 and a gold medal at a ceremony in Stockholm on September 29. They were honoured for their research contributions to the theory of non-linear differential equations, an area of mathematics of intense interest.

Professor Nirenberg was director of the Courant Institute, a Sloan and Fulbright fellow and is a member of the National Academy of Science and recipient of the M. Bôcher prize. Last October he visited McMaster University in Hamilton to deliver the annual Britton Lectures; he spoke on Variational and Topological Methods in Non-linear Problems.

The new prize is named for the Swedish industrialist **Holger Crafoord,** who died earlier this year, and his widow, **Anna Greta Crafoord.** Crafoord made a fortune in pulp and paper products and was also a producer of artificial kidney equipment. The Crafoord mathematics prize will be followed by awards in Astronomy, Biological science, Geological science, and Arthiritis research.

3.

Canadian wins the **Cole Prize** in Number theory. **Robert P. Langlands,** of the Institute for Advanced Study in Princeton, N.J., was presented with the prestigious Cole Prize in Number Theory by the American Mathematics Society. Professor Langlands received his award for pioneering work on automorphic forms, Eisenstein series, and product formulas. The Cole Prize is awarded at five year intervals, and the recipients are selected on the basis of an important research memoir within this five year period.

4.

Nova Math - a Journal for Teachers of Mathematics has been introduced to serve mathematics educators in the Atlantic region. It is produced jointly by the Department of Mathematics, Statistics and Computer Science and the Department of Education at Dalhousie University, Halifax.

Best wishes and success to its editors, Mary Cowley, Ken Dunn, Glane Gorveatt, Jules Gribble and Phil Reid.

IN MOST SCIENCES ONE GENERATION TEARS DOWN WHAT ANOTHER HAS BUILT AND WHAT ONE HAS ESTABLISHED ANOTHER UNDOES. IN MATHEMATICS ALONE EACH GENERATION BUILDS A NEW STORY TO THE OLD STRUCTURE.

Herman Hankel

COUNCIL COMMENT

by
Bob Robinson

Ed. Note:
Council Comment appears in an issue of the Gazette following O.A.M.E. Council Meetings. The Editorial Panel hopes that you find this section of interest. Your comments are welcomed and should be addressed to:

The Editor
Ontario Mathematics Gazette
c/o Department of Mathematical Science
McMaster University
Hamilton, Ontario
L8S 4K1

The June Council meeting was held on Saturday 1982 06 12 at the Ramada Inn (Airport West)

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COMMITTEE CHAIRPERSONS AND EXECUTIVE LIAISON

NOTE: All Councillors are members of one of the following committees

Nominations
G. Knill (Executive liaison)
B. Robinson
E. Collins

Problem Solving
C. Carlow (Chair person)

Publications
L. Wiggam (Chair person)
R. Connelly (Executive liaison)

Bill 82
E. Harvey (Chair person)
B. Lambframboise (Executive liaison)

Senior Guidelines
T. Griffiths (Chair person)
D. Davidson

Computers
A. Jones (Chair person)
G. Knill (Executive liaison)

Public Relations
J. Symington (Chair person)
S. McPhail (Executive liaison)

HE WHO SEEKS FOR METHODS WITHOUT HAVING A DEFINITE PROBLEM IN MIND SEEKS FOR THE MOST PART IN VAIN.

D. Hilbert

OAIP JUNIOR

The geometry instruments were tested and marked. A random selection of boards and schools was used. Results will be brought to the October Council Meeting. Sharon McPhail, Ralph Connelly and Ron Sauer are on the Advisory Committee.

OAIP INTERMEDIATE

A lengthy survey has been conducted and a chief examiner may be appointed.

SERP

At the time of the June Council meeting, we were waiting for the Minister to announce the direction in which the ministry is to proceed.

LEADERSHIP SEMINAR

The 11th Annual Leadership seminar is to be held 1982 10 21-23 at the Ramada Inn. See Calendar of Events for further details. Registration must be in by September 24. Each school board should have people attend this most valuable seminar.

NCTM ACTIVITIES

Ralph Connelly is OAME's representative to the NCTM. If any person wishes to order NCTM materials, they should prepay or indicate prepayment on the requisition and instruct the board or school to send this to Ralph, c/o Faculty of Education.at Brock University. The cheque should be made payable to NCTM and Ralph will forward this on to NCTM. In this way OAME receives a commission.

WATERLOO ANNUAL MEETING

The next Annual Meeting of OAME will be held in Waterloo on 1983 05 13-15. The Conference Chairperson is Ron Scoins at the University of Waterloo and the Program Chairpersons are Bill Proctor and Gary Flewelling. An exciting weekend is being planned.

SENIOR GUIDELINES

The actions of the Senior Guidelines committee will be different if there is a major revision and compression of curriculum due to the Minister's actions pertaining to the SERP report.

BILL 82

The council recommended that the ministry, faculties of education, and other institutions offering special education courses place much greater emphases on Mathematics in such courses.

PUBLIC RELATIONS

The Public Relations Committee will focus on assembling published materials relating to (a) Careers in Mathematics and (b) Awards in Mathematics.

COMPUTERS

Technology is changing at a rapidly increasing rate. It is felt that all students have the right to use computers throughout their education, K to 13 and that we must overcome obstacles such as, the strained economic times, resistance to change and lack of adequate teacher training. Contact with ECOO will be established and support and assistance from the Computer Committee to the 1983 OAME Conference committee will be offered.

The Ottawa Board of Education is presenting an educational computer conference on 1982 10 29-30 entitled "The Silicon School". See Calendar of Events Section.

CHAPTER REPORTS

COMA continutes to be a very active association. Ann Jones, the chair person distributed copies of the COMA newsletter for February-March, April, and May, 1982. The Canadian Mathematical

Society, The Mathematics Interface Project Steering Committee and information pertaining to the publication **Crux Mathematicorum.** Copies of the newsletter are available from Ann Jones (see council listing).

NOMA is planning their Annual Meeting for Friday, 1982 10 01 (evenings) and Saturday, 1982 10 02. Further information is available from Mickey Sandblom (see council listing).

Pine Ridge consists mainly of the Durham Region at present. They planned a day of workshops for Monday, 1982 06 28. Sessions on Bill 82, Programmes for the Gifted and Computer Sessions were planned

QSLMA is electing a new executive and is planning another year of interesting events.

GUMA had a successful spring meeting with about 150 attending. They are planning the OAME '83 Annual mentioned earlier in these Comments.

SAME is progressing well. Their first meeting is planned for Thursday, 1982 10 14 evening.

TORNY held three successful meetings last school year and three more are being planned again this year.

NEXT MEETING OF COUNCIL -- SATURDAY, 1982 10 16

CALENDAR OF EVENTS

The following conferences are sponsored by the OAME.

11TH ANNUAL LEADERSHIP SEMINAR

Another exciting Leadership Seminar is being held at the Ramada Inn on Dixie Road in Mississauga. The dates are 1982 10 21-23 with the registration taking place on the evening of 1982 10 20. Preregistration is required to attend this valuable seminar.

Some topics to be presented are Enrichment Mathematics, Geometry Potpourri, Creative Problem Solving, Resources for the 80's, Individualizing Independent Individuals, and Motivation -- Organization Strategies -- Practice.

Another important program component has been added this year. Each participant will receive 1-1.5 h of instruction and interaction with a computer sometime during the seminar. This will be hands on and one-to-one (machine-participant-instructor).

Although primarily for elementary teachers, this seminar is of value to principals, secondary school mathematics department heads and supervisory personnel.

As in the past the number of participants is limited. Letters and applications were sent out to Boards of Education in Ontario in February.

There are some places left ... so

(a) Inquire to see if your Board is sending anyone

(b) For more information contact the following by September 24

Morley MacGregor
Leadership Seminar Administrator
OAME

247 Bright Street
Sarnia, Ontario
N7T 4E9

NOMA

Dates: Friday evening 1982 10 01
Saturday 1982 10 02

For futher information contact Mickey Sandblom (see council listing).

SAME

Date: Thursday evening, 1982 10 14

For further information contact Iris Brown (see council listing).

ANNUAL MEETING

Date: 1983 05 13-15
Place: Waterloo

Contact Ron Scoins at the University of Waterloo for more information.

OTHER CONFERENCES OF INTEREST

SILICON SCHOOL

An educational computer conference for educators, administrators, trustees and parents is being held on 1982 10 29-30 at Woodroffe High School in Ottawa. Sixty workshops and presentations are planned along with exhibits, displays and a software exchange. Program, registration forms and details can be obtained by contacting:

The Silicon School
c/o Ottawa Board of Education
330 Gilmour Street
Ottawa, Ontario
K2P OP9

ONTARIO ASSOCIATION FOR CURRICULUM DEVELOPMENT

The 31st Annual Conference is being held on 1982 11 11-13 at the Westbury Hotel in Toronto. The theme is Curriculum: The Stationary, Moving Target. For further information and registration forms write to:

OACD
c/o Edgewood Junior P. S.
230 Birkdale Road
Room 29
Scarborough, Ontario
M1P 3S4

FWTAO COMPUTER/TECHNOLOGY CONFERENCE

To be held 1982 11 26-27, at the Airport Holiday Inn in Toronto. For more information contact, FWTAO, 1260 Bay Street, Toronto, M5R 2B8.

INDIANAPOLIS CONFERENCE

To be held 1982 11 18-20. Contact NCTM, 1906 Association Drive, Reston, Virginia, 22091.

BOSTON CONFERENCE

To be held 1982 12 2-4. Contact NCTM.

61st ANNUAL MEETING - NCTM

To be held 1983 04 13-16 in Detroit. Contact NCTM.

October 1-2, 1982.

The 10th Annual Mathematics and Statistics Conference, Mathematics and Computing will be held in Miami.

Information: Don Koehler or Fred Schuurmann,
Department of Math. and Stat.
Miami University, Oxford, OH45056

December 10-12, 1982.

Canadian Mathematics Society,
Annual Winter Meeting,
University of Toronto.

January 7-9, 1983

The 66th Annual meeting of the **Mathematical Association of America** is held in Denver, Colorado.

August 24-30, 1984

The Fifth International Congress on Mathematical Education (ICME) will be held at the University of Adelaide, Australia. It is expected that the program will span all levels of education and discuss problems of different interest while recognizing different cultural perspectives. The closing dates for applications for support to visit Australian Institutions in 1984 is October 1982.

For further details to: Dr. John Mack
Department of Pure Mathematics
Sydney University
NSW 2006, Australia

THE SECOND INTERNATIONAL MATHEMATICS STUDY - ONTARIO

Ontario has joined with twenty-four other "nations" in the International Association for the Evaluation of Educational Achievement's (IEA) Second Study of Mathematics. The Study involves the assessment of students' attitudes towards and achievement in mathematics at two levels: grades 8 and 12/13. Teachers are also queried on their teaching approaches as well as their attitudes towards mathematics and mathematics learning. Principals provide information on the size and structure of their schools.

For further information write to:

Dr. Les McLean, Head
Education Evaluation Centre, OISE,
252 Bloor Street West,
Toronto, Ont. M5S 1V6

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION ASSOCIATION ONTARIENNE POUR L'ENSEIGE DES MATHEMATIQUES

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- regional chapters
- affiliation with the National Council of Teachers of Mathematics

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Membership Application

Dr. ☐ Miss ☐
Mr. ☐ Mrs. ☐

_____

Address _____

_____ Postal Code _____

Educational Affiliation _____

Chapter Affiliation (if known) _____

Please detach and mail, with a cheque or money order, to: OAME, 247 Bright Street, Sarnia, Ontario N7T 4E9.

CALL FOR EDITOR

The Ontario Association for Mathematics Education (O.A.M.E.) invites applications for the position of editor of its journal, the Ontario Mathematics Gazette. The appointed editor will be responsible for publishing Volume 22 (September 1983, December 1983 and March 1984) and Volume 23 (September 1984, December 1984 and March 1985) of the Gazette. Additional information regarding this position may be obtained through the O.A.M.E. office.

Applications should be **clearly identified** as application for editor and submitted by November 1, 1982 to the following address:

O.A.M.E.
247 Bright Street
Sarnia, Ontario
N7T 4E9