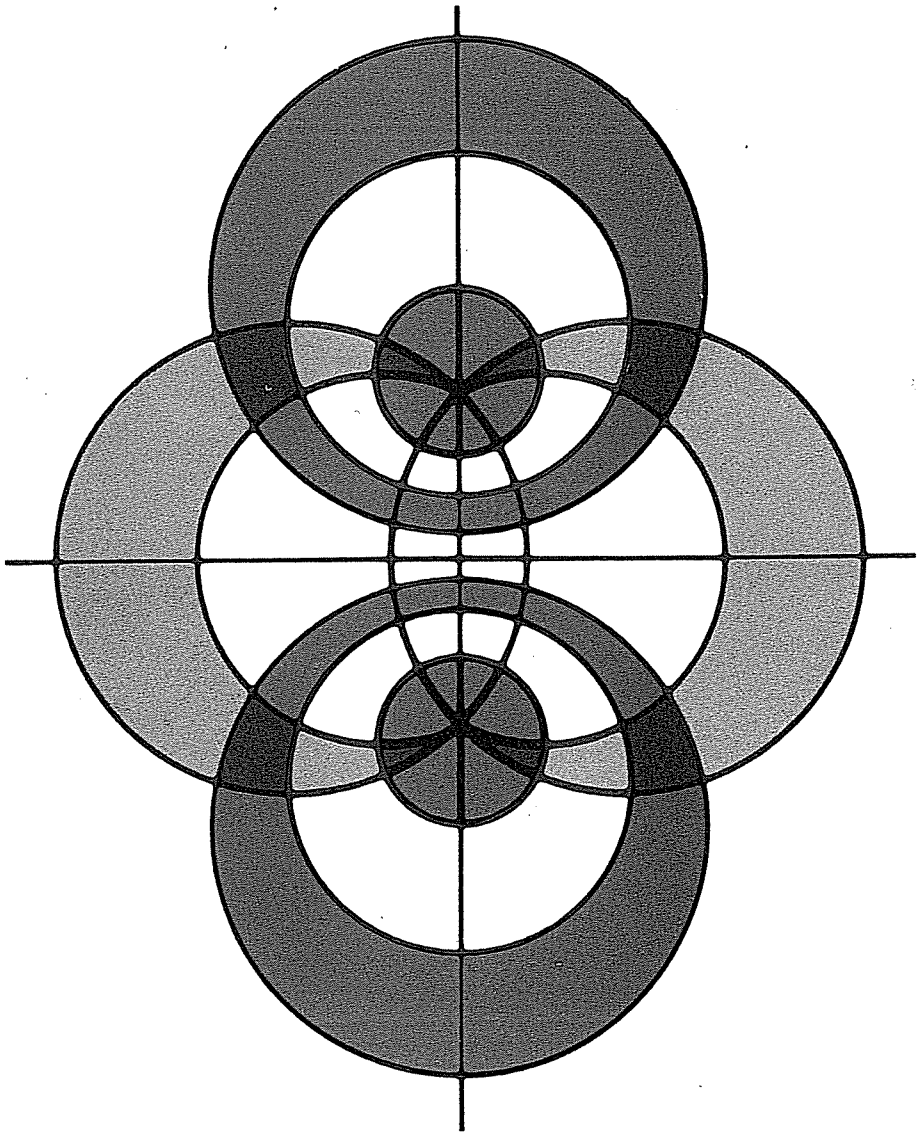
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EDITORIAL

After two years of involvement with the production of the *Ontario Mathematics Gazette* this is the last issue that I will bring you. It is time for a change and perhaps new ideas. Therefore, a new editor and maybe a new editorial staff will provide you with Volume 22 beginning in September.

Like most new endeavours, I found the job as editor exciting, challenging and sometimes even frustrating. There were deadlines to meet, proofs to be read and letters to be answered. If you were pleased with our format and content, much of the credit is due to the work of my associate editors. Their careful and constructive refereeing improved many articles which were submitted. Their reviews and regular columns provided information, enjoyment and insight to the readers. I very much appreciated their work and am grateful to them. Special thanks go also to Tom Davison, Mark Green and my daughter Nina who gave help when I needed it. I also wish to acknowledge gratefully the support of McMaster University, particularly the Department of Mathematical Sciences' secretarial staff.

The quality of a journal such as the Gazette depends to a considerable extent on the quality of the articles submitted for publication. I was most fortunate that many educators in the United States and Canada submitted excellent articles of high scholarly standards. Their contributions were most appreciated and it is my hope that the new editor receives the same excellent contributions. In addition, the editor needs the support of the community of mathematics educators in Ontario if he or she is to function effectively. The Gazette is a vital and effective instrument of communication for teachers of mathematics and their association. It is my hope that the new editor will receive the support from both the Publications Committee and the O.A.M.E. Executive.

Good luck,
H.P. Heinig

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MATHEMATICS EDUCATION

APICS MATHEMATICS, SCIENCE AND ENGINEERING EDUCATION COMMITTEE BRIEF ON EDUCATIONAL POLICIES

by

F. W. Dobson, BIO

1. We are convinced that in Canada, and particularly in Atlantic Canada, the educational system at all levels underemphasizes mathematics, science and technology in the teaching of non-mathematics and non-science students. We are also deeply concerned that the education of mathematics and science students in the schools is not being supported as it should be. The support which we feel is needed is outlined in detail later in this brief; it involves an attempt to provide a cost-effective use of human and material resources within the existing system, and an enhancement of public understanding. We base our assertions principally on two studies: the Report on Science Education by the Science Council of Canada, and the report entitled "Barriers to Science Education Improvement in Canada: A Case in Point", by Charles P. McFadden of the Atlantic Institute of Education.

These reports and our Committee, clearly indicate the following conclusions:

(a) Curriculum development departments, at least in the Atlantic Provinces, are understaffed. This leads, through lack of adequate human resources, to the adoption of "prepackaged" science programmes, with little or no local emphasis. The intent of curriculum development should be to enhance the relevance of the course material by making it realistic.

(b) Many practicing mathematics and science teachers, particularly at the elementary and junior high levels, in general lack education and experience in their field, especially in laboratory work. This lack will be further exacerbated as the number of available teachers falls with falling enrollment in the schools. Each teacher will be required to instruct in more and more subject areas.

(c) Many schools lack useful facilities, such as relevant, stimulating textbooks (see (a) above) and intelligently equipped laboratories at the elementary and junior high school levels.

(d) Much of the media fall woefully short in their general understanding of mathematics and science, and this has led to a lack of public awareness. We recognize the excellent work of the CBC in "The Nature of Things", "Quirks and Quarks", and, locally, the work of Dr. Robert Fournier of Dalhousie. There must be more, much more; there should be no "mystique" associated with science and technology.

2. We believe the situation as it exists needs rectification, since mathematics and science - both pure and applied - are in fact a part of every citizen's life. Several cogent reasons can be given to clarify this.

(a) Technological development is today proceeding at a rapid pace, and many political decisions are called for to allow society to respond. Such decisions must be based on understanding.

(b) An explosion of information available to the individual is being brought about by television, satellite communications, and the computer. This explosion is producing many changes in the role an individual plays in society. The everyday consumer, for instance, must understand an ever-more-complex technology to buy wisely. The society without a proper background in mathematics and science, will rapidly become out of touch with reality in all aspects of its cultural development.

(c) Both mathematics and the scientific method are central to cultural advancement. Logic is an integral part of both philosophy and mathematics, and "natural philosophy" stands for our curiosity about Nature herself. The ethic, used in science, of progress in understanding through communication and criticism, is a central one in our political and moral systems.

(d) An infusion of qualified, enthusiastic scientists into society leads to enhanced debate and questioning on a broad range of intellectual topics. This in turn leads to a stronger stimulus of bright students in all fields, and a higher quality of life in the future. We should never lose sight of the lessons we learned from the Renaissance and the Sputnik era.

3. In view of the above, we believe the following actions are called for. We point out that all the proposals we mention are for improvements to the existing system, as much as possible within present levels of support.

(a) The structure of the degree programmes compulsory for mathematics and science teachers at all levels must be reoriented to provide them with more direct mathematics and science training. In particular, we strongly feel that science teachers at elementary and junior high school levels should have high-level courses in mathematics and all principal areas of science.

(b) Recruitment and placement policies for mathematics and science teachers should be restructured. The overall goal should be an increase in the absolute number and quality of mathematics and science courses at senior high and university levels required by qualified incoming teachers at the elementary and junior high school levels.

We feel that provincial Education departments, in trying to fulfill preconceived commitments from decreasing budgets, and teachers' unions, in trying to protect and expand the rights of their members, have increasingly over the last decade given their own ends a higher priority than that of educating students. It is of crucial importance to our entire mathematics and science education system that the education of the student be returned to the top priority, if necessary at the expense of educational programmes peripheral to the core of mathematics and basic science and at the expense of some (admittedly hard-won) privileges now enjoyed by the teachers. It is the effectiveness of the teacher in the classroom and laboratory which really matter.

(c) The type of on-the-job training available now to mathematics and science teachers needs reassessment with a view to making the science taught more stimulating (i.e., locally relevant, and up-to-date) from the point of view of the student, and increasing the equivalent number of formal higher-level courses for which the teachers are qualified and for which they should **remain** qualified. The teacher supervision now provided for in science and mathematics is considered inadequate to ensure that high standards are maintained throughout the Atlantic region.

(d) A cost-benefit oriented study of the classroom and laboratory facilities required for effective mathematics and science teaching at the elementary and junior high school levels is needed. In particular, direct student involvement in the scientific method, including the design and construction of theories, experiments and instruments, must be a central aim in developing the facilities for optimum use. Examples from everyday local life must be stessed, and used whenever possible as elegant student experiments, and demonstrations of mathematical and scientific principles. The possibility of integrating into the science program other school facilities, such as computers, photography laboratories, and metal and woodworking shops, should be fully investigated.

(e) Mathematics and science material should be integrated into other courses within the educational system. Mathematics, science and technology have played significant roles in shaping our history, our geography, our social structure and our culture. There is also a strong requirement, for instance, for mathematicians and scientists to be literate, articulate individuals; they need special help in developing their ability to communicate with others.

Mathematicians and scientists need to understand the implications of their discoveries. Scientists and non-scientists alike are in today's world continually required to interpret political statements and journalistic reporting relating to science and technology. In times when financial realities demand a minimization of the teacher/pupil ratio in all school systems, teachers must be qualified in a broad range of subjects to be useful in the system. The careful motivation and training of such teachers in the basic precepts of mathematics and science must be given the highest priority by their educators in the university system.

(f) The importance of guidance for careers in mathematics, science, and technology, particularly at the Junior High Level, cannot be overstressed. We feel existing guidance practices should be carefully and continually reviewed. Communication among the various levels of the educational system, particularly between the university and professional schools on the one side and guidance counsellors in the secondary schools on the other, must be made as efficient as possible. The engineering schools are particularly isolated from the high schools. If the guidance system is not found to be working on an informal basis, then the lines of communication should be formalized.

Lack of guidance at the junior level often leads to a general lack of preparedness in the general level of academic proficiency required at university. For example, students who do not take their English or Mathematics seriously due to lack of guidance, find it difficult or impossible when at university to become effective at communication or to master the mathematics which inevitably forms part of their science. High school university stream math and science students are easily sidetracked from core courses into the "more exciting" specialized electives, such as computer science, oceanography and ecology (such electives have their uses, of course, as general introductions for non-science students).

(g) Friends of Science, Technology and Engineering Research, or FOSTER, is a network of extracurricular science clubs run by volunteer professionals for junior and senior high school level students who want more than that offered in school. It is well-established now in Ontario and should soon become a nation-wide organization. The FOSTER programme (or something like it) should be adopted on as wide a geographical scale as possible, as an integral part of school extra-curricular activities. Science departments or teachers at all levels stand to gain. The FOSTER clubs and the communication facilities they contain help to strengthen contacts with professional scientists and their institutions. Community assistance becomes available to the teacher, ideally permitting an augmented general science program in the school and enhancing student involvement. Science fairs and olympics can also become self-starting yearly activities, and can be used as "media events" to inform the school community, and hence the public, on science activities both locally and in the country as a whole.

QUIET PLEASE! - I'M LEARNING MATH!

by
Richard Lucas

During the past score of years many changes have occurred in the teaching methods and techniques employed in the classrooms of our schools.

We have witnessed the introduction of regimented rows of pupils for uniform instruction. We have seen groups formed for appropriate level teaching. Team-teaching, open-concept and group strategies have come to give us new interesting approaches to the classroom scene.

Not many years ago my Grade Six classroom climbed onto the electronic bandwagon by making, editing, and using Math audio cassette tapes for instruction, drill, and remediation purposes. The pupils were engrossed with the gadgetry - and also learned their tables, equivalent percentages, etc.

Modern consumer education is such that newspapers offering comparative pricing with its focus on problem solving is another practical tool. And of course, we now have the availability of video cassette tapes with the added dimensions of colour, teen-age interests and lingo for impact. More recently, calculators, and now computers have appeared on the present classroom math scene and are as vital a tool as the compass and the textbook.

Throughout these changes, one must remember the reasons for teaching mathematics at all! - Yes it is the **pupils**. Mathematics is not simply the process of adding, subtracting, or solving problems, but is instead a way to communicate a method of notation designed to express thoughts efficiently. Just imagine the near impossibility modern technological life would be if we had never heard of decimals and we were still using only Roman Numerals.

Remember that pupils to-day don't have much patience - they seem to demand instant gratification - hence the popularity of video games.

Whether it is individually, collectively, or in associated pairs, let us not forget the centre of our mathematical deliberations - the pupils.

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CONCRETE REFERENCES

by

Dr. Charles D. Gallant

The purpose of this article is to give some illustrations of how concretion can be used in teaching secondary-level mathematics. Concretion is taken simply to mean giving concrete references, i.e., references to things with which students are familiar.

For convenience, the examples which follow are grouped under the headings of concepts, principles, and skills.

CONCEPTS

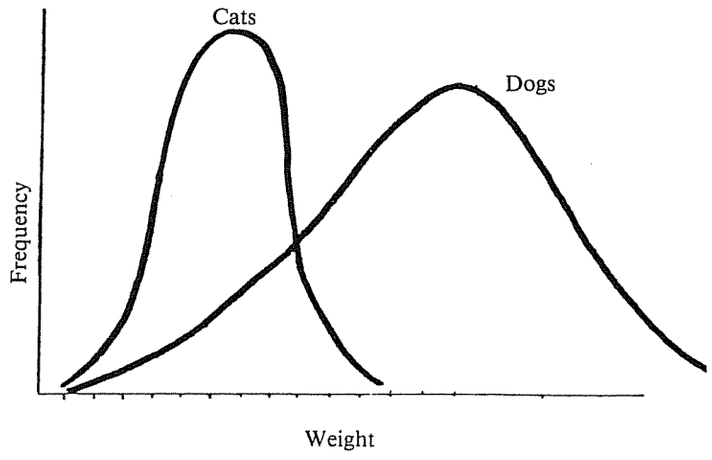
Teaching a concept can involve teaching the meaning of a term. Hence teachers use etymology to advantage; e.g., in teaching the concept of *percent* by referring to the derivative meaning "out of a hundred." Similarly the word *scalene* from the Greek means limping. A man with unequal legs would limp, hence a triangle with unequal legs is a scalene triangle. The word *focus* means hearth or burning place, particularly well suited when one thinks of the reflective property of a parabola. *Chord* from the Latin means bow-string, and *echelon* is derived from the French word échelle meaning ladder.

Exemplification is essential to concept formation, hence teachers seek good examples or embodiments. A menu is an example of a *function* and furthermore we often hear the accurate comment "the lobster is outside my *range*," the *mode* is a statistic of particular importance to a shoe salesman. Why? The *greatest integer function* has been appropriately dubbed the "How old am I?" function. The *intersection* of two streets is what the two streets have in common! Branching in *flowcharts* is reminiscent of the familiar snakes and ladders game. Photo enlargement illustrates *similarity*.

Some examples are particularly visual and may assist concept retention. *An angle of depression* is formed between the horizontal and the beam of a car headlight. A STOP-sign is an *octagon*. The profile of organ pipes is *an exponential curve*. For aerodynamic reasons the flight pattern of migrating birds is *parabolic*. A familiar picture postcard is often in the shape of a *golden rectangle*. An egg carton illustrates a 2 x 6 *matrix*. The sign rule for *cofactor determinants* is referred to as the checkerboard rule. Is a coat hanger in the shape of an *isosceles triangle?* Students refer to the *rectangular hyperbola* as the boomerang curve, and similarly the *supply/demand* configuration is known as Marchall's scissors. If a histogram were cut from sheet metal and balanced on a knife edge, then would the knife edge determine the *mean* or *median* of the distribution? Compare the terms quinella/exactor with *combination/permutation.* Does the symmetry of a circle not illustrate nicely why a bath-tub plug is round!

As a prelude to the concept of *variance* have your students discuss the diagram below and see what comments emerge.

Figure 1

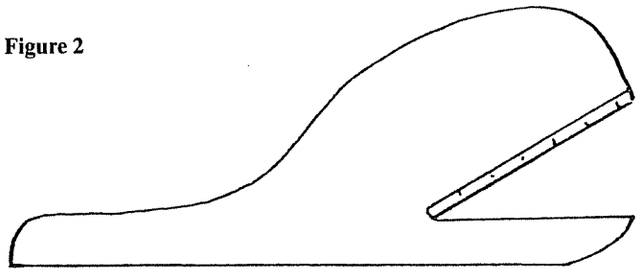


PRINCIPLES

An important move in teaching principles is application and many real-life applications are striking physical embodiments of seconday-level principles.

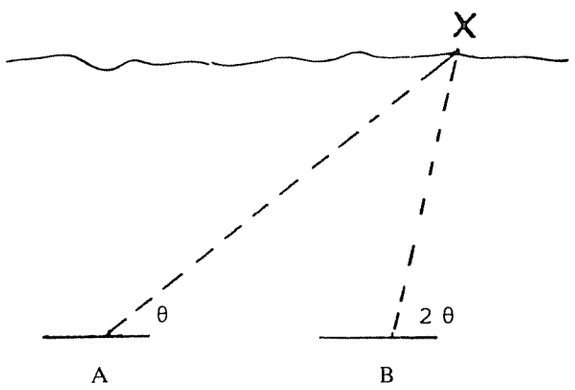
The diagram below shows a device used to measure thickness. The thickness of an object inserted into the "jaws" is one-half the reading on the scale along the upper jaw. The principle involved is simply $\sin 30° = ½$.

Figure 2



The technique of "doubling the angle at the bow" is used to estimate one's distance from shore.

Figure 3

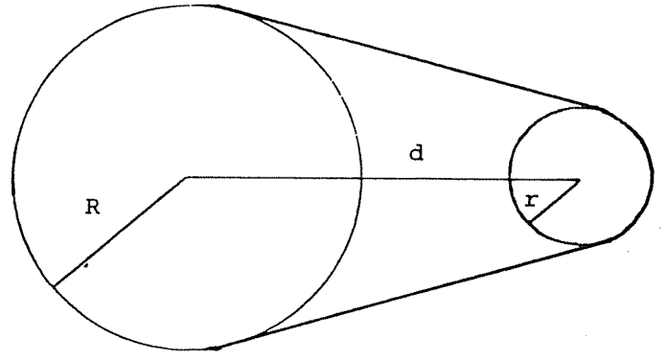


The object at X is sighted initially at an angle θ, and again at an angle 2θ. This distance from shore is approximately $\overline{AB}$. What principle is embodied here? What value would you choose for θ?

Three points determine a plane, hence a three-legged stool is likely to be more stable than a four-legged stool. How would you put a rope handle on a cylindrical pail? To find the opposite ends of a diameter place a carpenter's square so that the corner touches the circular top, the body and tongue of the carpenter's square then touch the circular top at the desired points. What principle is illustrated?

Try to devise a formula which will give the number of bricks needed for a straight wall, of length L feet, height H feet. Compare with the Bricklayer's estimate, N = 7LH. The figure below shows a

Figure 4

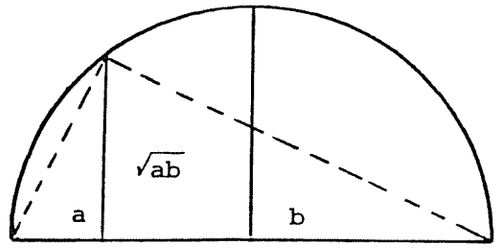


belt fitted about two pulleys. The Machinist's estimate for the length of that belt is 3.26 (r + R) + 2d. Find the length of a common external tangent and proceed to justify this estimate.

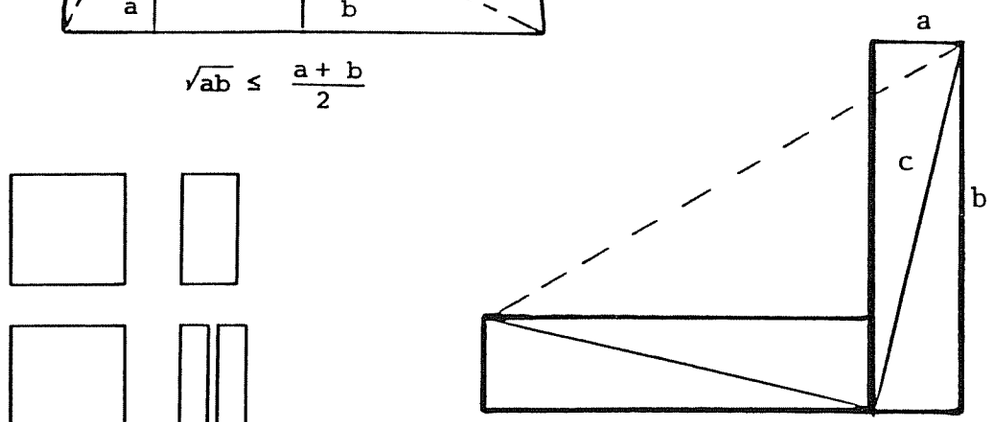
The old Sailor's formula ($d = \sqrt{3h/2}$) for finding the distance to the horizon is an application of the *Two-Chord power theorem*. The Bookkeeper's check uses the *Rule of Nines* to detect "reversal" entries.

The "behold" type proofs are also helpful in understanding and remembering principles as the following diagrams illustrate.

Figure 5



$$\sqrt{ab} \leq \frac{a+b}{2}$$



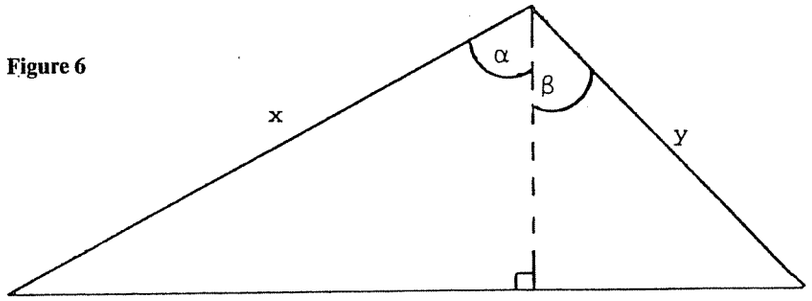
$$(a+b)\left(\frac{a+b}{2}\right) = \frac{c^2}{2} + ab.$$

Completing the square.

$$X^2 + AX = (X + A/2)^2 - (A/2)^2$$

Given the figure below, using only x, y, α, β, find the area of the given triangle in two ways, equate the answers, and discover a familiar principle.

Figure 6



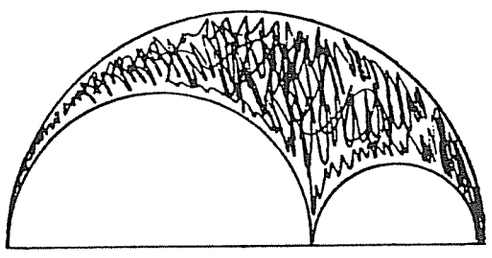
SKILLS

Teaching skills invariably requires that we provide interesting and challenging exercises for practice. The birthday problem gives an interesting (and contraintuitive) application of *probability*; the Capture-Recapture technique for estimating the size of wildlife populations is a nice exercise using *ratio*. Complete the following table where letters of the alphabet are arranged according to their *symmetry*.

Vertical - A
Horizontal - B
Both - H
None - F

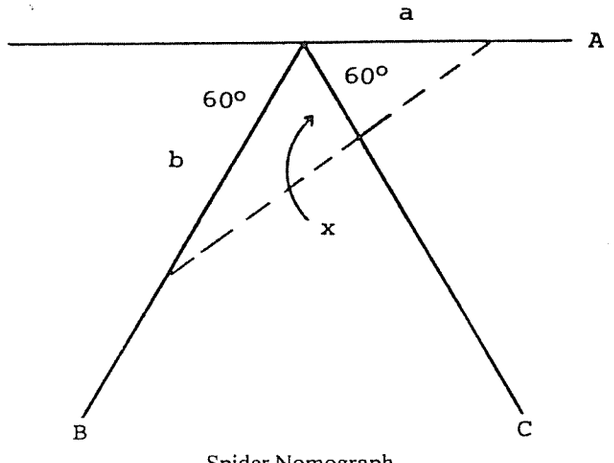
Use the familiar alphabet also to illustrate closed curves, simple curves, etc.

Consider the following diagrams:



Shoemaker's Knife

Figure 7



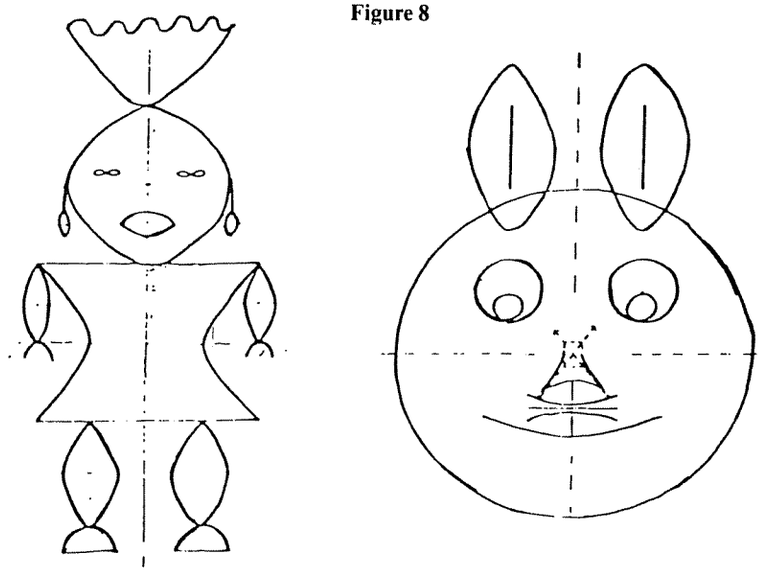
Spider Nomograph

Find the area and perimeter of the shaded area, i.e., the *Shoemaker's Knife*. The *Spider Nomograph* is an alignment chart which solves the equation $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$. A straight edge placed at *a* and *b* will determine *x* on the C-scale. Explain how it works. Hint -- Find the area of triangles!

Explain how the *Boy Scout Method*, for finding the height of an object, works. First, take 12 paces from the object marking the 11th pace, B, and the 12th pace, A. Place an upright stake at B, and sighting the top of the object from a prone position at A, mark the intersection on the stake. The height of this intersection in inches gives the height of the object in feet.

Routine practice can sometimes be replaced with innovative assignments which capitalize on student creativity. Graphing is a good example of such activity, e.g., producing a Christmas tree or a boat using line segments. The following figures were produced using conics.

Figure 8



The analysis of physical models such as the ellipsograph, sphereometer, hypsometer, pantograph, golden triangle compass, etc., provide good exercises and the motivational bonus is that students enjoy constructing them!

As a final note on teaching skills, consider the maxim "Relate new skills to old skills". This maxim represents, again, reference to the concrete. The student who wishes to simplify $\frac{1}{x} + \frac{1}{x+3}$ benefits from a reminder of how to add fractions or the student who knows how to sketch, using transformations, the curve $y = a(x - b)^2 + c$ can move easily to sketching other curves, e.g., $y = \frac{a}{x - b} + c$.

The following example (Goff & McKellips, 1974) relates the multiplication of a matrix by a vector to the familiar multiplication of polynomials. Consider the correspondence:

$$x + 3 \quad -- \quad (1, 3)$$

$$x - 2 \quad -- \quad \begin{matrix} 1 & -2 & 0 \\ 0 & 1 & -2 \end{matrix}$$

Note the matrix must have as many rows as the vector has columns!

Now, we have,

$(x+3)(x-2)$ and $(1, 3)$ $\begin{matrix} 1 & -2 & 0 \\ 0 & 1 & -2 \end{matrix}$

$= x^2 + x - 6$ $= (1, 1, -6)$,

and, again the correspondence,

$x^2 + x - 6$ -- $(1, 1, -6)$.

Try another example, say: $(x^2 + 3)(x^2 - 3)$.

CONCLUSION

For teachers who wish to gather more examples of concretion, the bibliography below provides some starting points. Of course, every textbook and every journal related to secondary-level mathematics is a potential source. Try browsing; it cannot fail to produce a gem!

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MATHEMATICS EXPERIENCES IN THE CURRICULUM

by
Marlow Ediger

Mathematics as a curriculum area is experiencing numerous changes. Society also experiences many changes. Which curricular changes in mathematics have significant implication in society?

Use of Technology

Manual labor continually decreases in terms of societal demands. Increased use of technology abounds. Calculators and computers seemingly are becoming less expensive and being used more and more in business and home settings. Thus, the mathematics curriculum must emphasize objectives, learning activities, and evaluation procedures involving the use of technology.

Utilization of hand held calculators may guide pupils to achieve the following:

1. Skill in estimation. For example, when adding a given set of numbers, the pupil must be able to estimate the sum. Otherwise, a sum may dogmatically be accepted as being accurate when incorrect values were punched on the hand held calculator.

2. Enjoy mathematics. Much drudgery can be taken out of mathematics if pupils enjoy ongoing experiences. No longer do pupils need to spend much time in checking answers to multiplication and long division problems. The same would be true in checking answers to complex addition and subtraction problems.

3. Drill and practice in mastering basic facts. For example, the pupil may silently or orally state the product of 9 x 6 = □, before pressing the "equal" key to check the accuracy of his/her response.

4. Make discoveries. Numerous discoveries may be made by pupils in utilizing hand held calculators. Thus, pupils may find the square of two, or the square root of nine. Learners need to attach meaning to and understand the concepts of "square" and "square root". Rote learning in mathematics is not recommended unless learners individually experience meaning in ongoing units of study.

5. Use in problem solving. Mathematics textbooks emphasize pupils solving story or word problems. This is recommended if the content presents realistic situations. Absurdity of content in word or story problems is to be frowned upon. In society, pupils also need to solve mathematics problems in the economic world. Pupils receive or earn allowances. They buy goods and services in society. What pupils have learned in the school curriculum needs to be applicable in society. The calculator is a valuable piece of equipment to utilize in the solving of problems in the economic world.

Programmed Learning

Programmed materials in the mathematics curriculum have become significant in use. In the use of programmed mathematics textbooks, the pupil may read a sentence or more, view an illustration, respond to a completion item, and check his/her response with that given by the programmer.

If a learner's response was correct, he or she is ready for the next sequential item. Should a pupil have responded incorrectly, the correct answer as given by the programmer has been noted. The learner is still ready then for the next linear sequential programmed item.

A similar or same procedure may be emphasized again and again in programmed learning -- read, view an illustation, respond, and check. The sequential steps involved in learning have generally been tested in pilot studies. Thus, pupils individually may make few errors indeed in progressing from one programmed item to the next.

The programmer selects objectives and learning activities for pupils in programmed mathematics. The sequence or order of experiences has been field tested and determined by the programmer. Basically, there are no opportunities for pupils to identify and solve problems when programmed materials are being utilized in the mathematics curriculum.

Which advantages may well be given in utilizing programmed materials?

1. Pupils individually might progress at their own individual optimal rates of achievement.

2. Few errors are made by learners. Thus, positive self-concepts may be developed in the mathematics curriculum.

3. Pupils need not compare personal achievement with that of other learners. Each will be at a different place in time when utilizing programmed materials.

Pretest and Post-test Methods of Teaching

Writers of numerous mathematics textbooks advocate a pretest and post-test emphasis in the teaching of mathematics. Thus, a selected series of mathematics textbooks might contain a pretest for each new unit of study in mathematics. If a learner achieves at a satisfactory level in the pretest, he/she may then take the pretest for the next more complex unit. Test results might reveal that this is the proper unit for the involved learner to work in. Sequential learnings are then being completed by the learner. Feedback from each day's experiences are provided to the pupil in terms of quality of work being performed. At the end of the unit, a post-test is taken to notice if an adequate number of goals have been attained within the unit. If the achievement level on the post-test is not adequate, remedial and additional related work is in the offing. If the level of achievement was satisfactory, the involved pupil is then ready for the next sequential unit within the confines of the adopted mathematics textbook.

If a basal mathematics text has no pre and post-tests inherent, the teacher may write the tests.

Advantages given in emphasizing a pretest and post-test method of teaching include the following:

1. Pupils may test out of any unit of study if pretest results indicate an adequate level of mastery learning. Duplication and repetition in learning might then be omitted.

2. Each learner may achieve as rapidly as possible on an individual basis. Test results indicate if a learner has truly mastered an adequate number of learnings before progressing to the next sequential unit.

Behavioural Objectives in the Mathematics Curriculum

Behaviorists recommend that teachers write measurable objectives for each pupil to attain. The teacher after instruction might then be certain if a learner has indeed attained an objective.

The following are examples of measurable objectives:

1. The pupil will add correctly ten problems, each with three four digit addends involving renaming in the tens column.

2. The pupil will solve correctly nine of ten story or word problems.

After instruction, the teacher may ascertain if a learner has/has not attained an objective. If an objective has been achieved, the pupil may work in the direction of attaining the next sequential goal. If a pupil was unsuccessful in goal achievement, the teacher needs to diagnose reasons for the occurance. Additional learning activities should then be provided to guide the pupil to realize the stated goal.

Advantages given for utilizing measurable objectives in teaching pupils include:

1. The teacher is certain as to what each pupil is to learn. Clarity in statements of objectives is vital.

2. The teacher may then select learning activities which assist pupils to attain the stated goals only. All other stimuli are eliminated in teaching pupils.

3. The teacher may measure to determine if a pupil has achieved the stated goal. The concept of diagnosis is important if a pupil does not achieve a measurable goal.

Humanism in the Curriculum

Humanists believe pupils need to be involved in decision-making procedures. Thus, learners need to choose alternative goals, learning activities, and evaluation procedures. A learning centers approach to teaching and learning might then be emphasized. To emphasize a humane mathematics curriculum, each pupil needs ample opportunity to select and choose. The following titles for learning centers, among others, may then be emphasized in an ongoing mathematics units of study:

1. A geometry center
2. An addition and subtraction center
3. A multiplication and division center
4. A mathematics laboratory center
5. A model making center
6. An audio-visual center

Each center must have an adequate number of tasks for pupils to pursue. A pupil then might choose which learning center and which tasks to complete sequentially. An adequate number of tasks needs to be in evidence so that each learner might truly select and also omit selected learning activities. The teacher does not assign tasks for learners to pursue. Rather, the teacher stimulates, encourages, and guides pupil learning.

Advantages given in emphasizing humanism in the mathematics curriculum include the following:

1. What is relevant to study can only be determined by the involved learner. Each pupil must choose what is to be learned.

2. Sequence is in the mind of individual pupils. The personal self orders learnings. The teacher, supervisor, or programmer cannot sequence experiences for others.

3. The learner individually is in the best position to appraise personal achievement. Evaluation is meaningful if the self perceives needs to improve in mathematics achievement.

Other Innovations in the Mathematics Curriculum

There are numerous other innovative ideas which need exploration and evaluation by teachers and supervisors. These include:

1. The contract system. An agreement in contract form is developed through teacher-pupil planning. The due date and signature of both the learner and the teacher need to accompany the contract.

2. Unit teaching. Subject matter in mathematics is divided into meaningful units of study. General objectives state the direction in which change of behavior within pupils is to be emphasized. A variety of learning activities need to be available to assist each pupil to achieve optimally. These activities include the utilization of textbooks, workbooks, transparencies and the overhead projector, illustrations, films, filmstrips, and slides. Activities chosen in teaching-learning situations must harmonize with the learning style of the individual pupil.

In Conclusion

There are numerous innovative procedures available in the teaching of mathematics. Teachers and supervisors need to appraise and select those strategies which guide pupils to perceive interest, purpose, and meaning in the mathematics curriculum.

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KNOWLEDGE IS CAPABLE OF BEING ITS OWN END ..., IT IS ITS OWN REWARD.
Henry John Newman

MATHEMATICAL EXPOSITIONS

COMPUTER GENERATED PERIODIC PICTURES

by

R. S. Nowak, J. E. Friedrich, C. W. Zeller

I. Introduction:

The Cartesian pictures of C. W. Zeller[1], that correspond to the periods of rational numbers, present an interesting interplay between geometry and algebra.

Zeller's article called "Periodic Pictures" explains how to produce these graphs:

Let $\frac{x}{y}$ be any rational number from (0,1). Represent the periodic decimal form of

$\frac{x}{y}$ by $0.b_1b_2\ldots b_m\ \overline{a_1a_2\ldots a_n}$.

Form ordered pairs, sequentially, as follows:

(a_1,a_2) , (a_2,a_3) , (a_3,a_4) , ...

(a_{n-1}, a_n), (a_n,a_1)

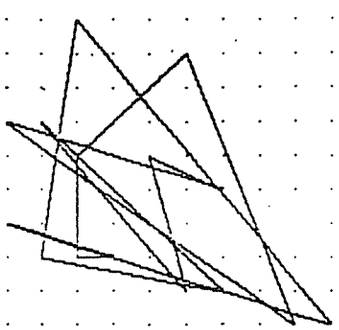
Plot each of these points and join them sequentially.

The investigation of the patterns and symmetries in these pictures for larger denominators creates an ideal situation in which the assistance of our Apple II Plus microcomputer could yield some interesting results.

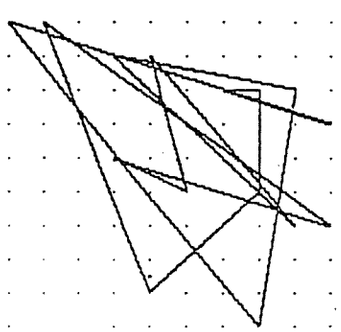
This article presents a BASIC Applesoft program which will plot the periodic picture for any positive fraction specified by a given numerator and an appropriate denominator up to 128.

(Extensions to higher denominators may be possible with our equipment, but are incomplete at present).

II. Sample Pictures:

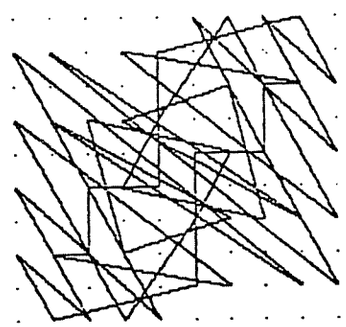


JFRACTION: 1/31 PERIOD LENGTH: 15

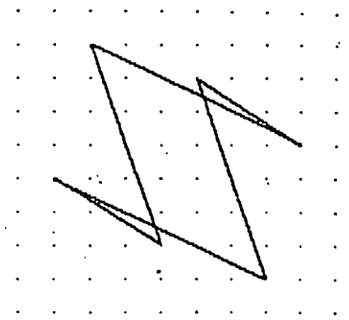


JFRACTION: 3/31 PERIOD LENGTH: 15

These are the only pictures generated by denominator, 31.

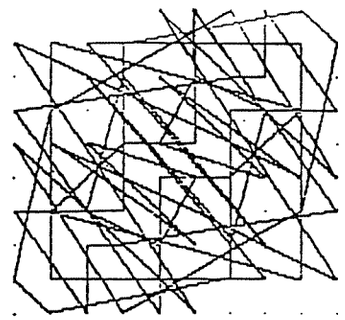


JFRACTION: 1/49 PERIOD LENGTH: 42

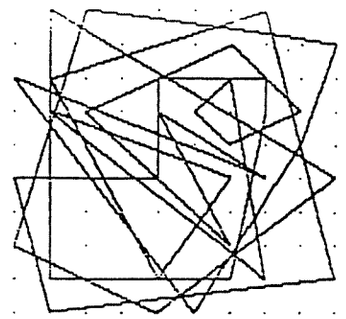


JFRACTION: 7/49 PERIOD LENGTH: 6

These are the only pictures generated by denominator, 49.
The right picture for numerators which are multiples of 7, and the left picture otherwise.



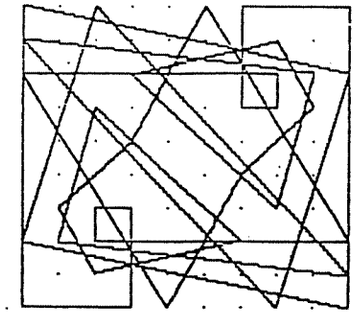
JFRACTION: 1/59 PERIOD LENGTH: 58



JFRACTION: 1/67 PERIOD LENGTH: 33

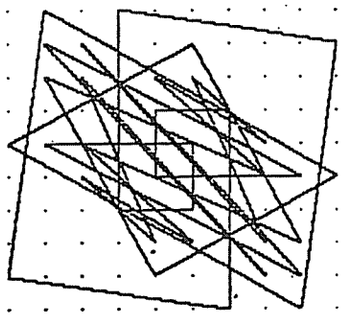
Unique for denominator, 59.

One of two TRANSFORMATION PAIRS for denominator, 67

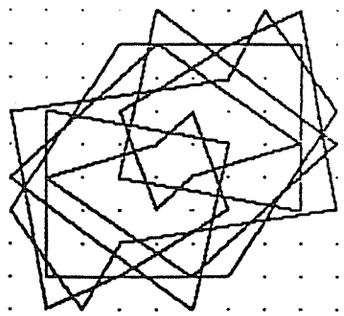


JFRACTION: 1/103 PERIOD LENGTH: 34

For denominator, 103, there are three distinct pictures. They are also original to 103 as they had not been drawn for an earlier denominator.



JFRACTION: 2/103 PERIOD LENGTH: 34



JFRACTION: 4/103 PERIOD LENGTH: 34

YET ALL EXPERIENCE IS AN ARCH WHERETHROUGH
GLEAMES THAT UNTRAVELLED WORLD WHOSE MARGIN FADES
FOREVER AND FOREVER WHEN I MOVE.

Tennyson

III. Summary of Program Output

DENOM-INATOR	PERIOD LENGTH	DISTINCT PICTURES	DENOM-INATOR	PERIOD LENGTH	DISTINCT PICTURES	DENOM-INATOR	PERIOD LENGTH	DISTINCT PICTURES	DENOM-INATOR	PERIOD LENGTH	DISTINCT PICTURES
2	1	1	40	1	1	71	35	2	101	4	25
3	1	2	41	5	8	72	1	9	102	1	3
4	1	1	42	1	3	73	8	9		16	3
5	1	1		6	3	74	1	1	103	34	3
6	1	3	43	21	2		3	12	104	1	1
7	6	1	44	1	1	75	1	3		6	2
8	1	1		2	5	76	1	1	105	1	3
9	1	8	45	1	9		18	1		6	3
10	1	1	46	1	1	77	2	5	106	1	1
11	2	5		22	1		6	11		13	4
12	1	3	47	46	1	78	1	3	107	53	2
13	6	2	48	1	3		6	6	108	1	9
14	1	1	49	6	1	79	13	6		3	6
	6	1		42	1	80	1	1	109	108	1
15	1	3	50	1	1	81	1	8	110	1	1
16	1	1	51	1	2		3	6		2	5
17	16	1		16	3		9	6	111	1	2
18	1	9	52	1	1	82	1	1		3	36
19	18	1		6	2		5	8	112	1	1
20	1	1	53	13	4	83	41	2		6	1
21	1	2	54	1	9	84	1	3	113	112	1
	6	3		3	6		6	3	114	1	3
22	1	1	55	1	1	85	1	1		18	3
	2	5		2	5		16	1	115	1	1
23	22	1	56	1	1	86	1	1		22	1
24	1	3		6	1		21	2	116	1	1
25	1	1	57	1	2	87	1	2		28	1
26	1	1		18	3		28	3	117	1	8
	6	2	58	1	1	88	1	1		6	18
27	1	8		28	1		2	5	118	1	1
	3	6	59	58	1	89	44	2		58	1
28	1	1	60	1	3	90	1	9	119	6	1
	6	1	61	60	1	91	6	15		16	1
29	28	1	62	1	1	92	1	1		48	2
30	1	3		15	2		22	1	120	1	3
31	15	2	63	1	8	93	1	2	121	2	5
32	1	1		6	9		15	6		22	5
33	1	2	64	1	1	94	1	1	122	1	1
	2	15	65	1	1		46	1		60	1
34	1	1		6	2	95	1	1	123	1	2
	16	1	66	1	3		18	1		5	24
35	1	1		2	15	96	1	3	124	1	1
	6	1	67	33	2	97	96	1		15	2
36	1	9	68	1	1	98	1	1	125	1	1
37	3	12		16	1		6	1	126	1	9
38	1	1	69	1	2		42	1		6	9
	18	1		22	3	99	1	8	127	42	3
39	1	2	70	1	1		2	45	128	1	1
	6	6		6	1	100	1	1			

IV. The Program

```
10  CLEAR : TEXT : HOME : VTAB 2
20  HIMEM: 8192
30  PRINT  TAB( 4)"***PROGRAM BY
    NOWAK-FRIEDRICH***": PRINT :
     PRINT
40  PRINT "THIS PROGRAM PLOTS PER
    IODIC PICTURES"
50  PRINT "CREATED BY CLAIRE ZELL
    ER IN THE"
60  PRINT "FOLLOWING WAY:"
70  PRINT : PRINT "LET 'X/Y' BE A
    NY RATIONAL NUMBER FROM"
80  PRINT "(0,1). REPRESENT THE P
    ERIODIC DECIMAL "
90  PRINT "FORM OF 'X/Y' BY '0.A
    A  ... A '."
100  PRINT "
    1 2       N"
110  PRINT : PRINT "FORM ORDERED
    PAIRS, SEQUENTIALLY, AS "
120  PRINT "FOLLOWS:"
130  PRINT : PRINT "(A ,A ),(A ,A
     ), ... (A   ,A ),(A ,A )"
140  PRINT "  1  2    2  3
      N-1  N    N  1"
150  PRINT : PRINT "PLOT EACH OF
    THESE POINTS AND JOIN "
160  PRINT "THEM SEQUENTIALLY."
170  VTAB 24: PRINT "HIT ANY KEY
    TO ENTER FRACTION 'X/Y' ";: GET
    Z$
180  TEXT : HOME : CLEAR
190  PRINT : PRINT
200  PRINT "REMEMBER THE FRACTION
     'X/Y' IS LESS"
210  PRINT "THAN ONE. FOR THIS PR
    OGRAM THE MAXIMUM"
220  PRINT "VALUE FOR 'Y' IS 128.
    "
230  PRINT : PRINT
240  INPUT "ENTER X ";X
250  PRINT
260  INPUT "ENTER Y ";Y
270 R = X
280 N = 2 * (Y - 1)
290  IF Y = 2 THEN N = 4
300  FOR I = 1 TO N
310 R = 10 * R
320 Z =  INT (R / Y)
330 A$ = A$ +  STR$ (Z)
340 R = R - Z * Y
350  NEXT I
360 N = Y - 1
370  IF Y = 2 THEN N = 2
380  FOR I = 1 TO N - 1
390  IF  MID$ (A$,N,1) <  > MID$
    (A$,N + 1,1) AND  MID$ (A$,N
     - 1,2) <  > MID$ (A$,N + 1
    ,2) THEN 440
400 L$ =  LEFT$ (A$,1)
410 R$ =  RIGHT$ (A$,2 * N - 1)
420 A$ = R$ + L$
430  NEXT I
440 L$ =  LEFT$ (A$,N)
450 R$ =  RIGHT$ (A$,N)
460  FOR I = N TO 1 STEP  - 1
470 L$ =  RIGHT$ (L$,I)
480 R$ =  LEFT$ (R$,I)
490  IF L$ = R$ THEN P$ = L$
500  NEXT I
510  PRINT : PRINT
520  PRINT "THE DIGITS IN THE PER
    IOD OF ";X;"/";Y;" ARE:"
530  PRINT : PRINT P$: PRINT
540 LP =  LEN (P$)
550  PRINT : PRINT "THE LENGTH OF
     THE PERIOD IS: ";LP
560  VTAB 24: PRINT "HIT ANY KEY
    TO PLOT PERIODIC PICTURE ";:
     GET Z$
570  DIM D(LP + 2)
580  FOR I = 1 TO LP
590 D$ =  MID$ (P$,I,1)
600 D(I) =  VAL (D$)
610  NEXT I
620 D(LP + 1) = D(1)
630 D(LP + 2) = D(2)
640  HGR
650  HOME : VTAB 23
660  PRINT "DO YOU WANT 'X' AND '
    Y' AXES?"
670  PRINT "('YES' OR 'NO') ";: GET
    Z$
680  IF  LEFT$ (Z$,1) = "Y" THEN
    710
690  IF  LEFT$ (Z$,1) = "N" THEN
    740
700  GOTO 650
710  HCOLOR= 7
720  HPLOT 0,8 TO 0,152
730  HPLOT 0,152 TO 252,152
740  HOME : VTAB 23
750  PRINT "DO YOU WANT GRID POIN
    TS?"
760  PRINT "('YES' OR 'NO') ";
770  GET Z$
780  HOME
790  IF  LEFT$ (Z$,1) = "N" THEN
    870
800  IF  LEFT$ (Z$,1) = "Y" THEN
    820
810  GOTO 740
820  FOR GY = 8 TO 152 STEP 16
830  FOR GX = 0 TO 252 STEP 28
840  HPLOT GX,GY
850  NEXT GX
860  NEXT GY
870  FOR I = 1 TO LP
880  HPLOT 28 * D(I),152 - 16 * D
    (I + 1) TO 28 * D(I + 1),152
     - 16 * D(I + 2)
890  NEXT I
900  VTAB 23: PRINT "WHEN READY T
    O PLOT ANOTHER PERIODIC"
910  PRINT "PICTURE, HIT ANY KEY
    ";: GET Z$: GOTO 180
```

V. Program Logic.

(An explanation of the major sections of the program)

The first part of the program briefly describes the way in which the periodic pictures can be created.
(Lines 40-160)

The program then asks for the numerator, 'x', and the denominator, 'y', of the fraction to be plotted, such that $x < y$ and $y \leqslant 128$.
(Lines 170-260)

Since the maximum period length possible is one less than the denominator of the fraction, or 'y-1' digits, the program long divides the denominator into the numerator (one digit at a time) until the quotient, A$, consists of '2X(y-1)' digits.
(Lines 300-350)

For example, the quotient generated for fraction, $\frac{1}{14}$ would be:

.071 428 571 428 571 428 571 428 57

13 digits 13 digits

Now, the program starts to "look" for the digits in the period by splitting the string, A$, and then comparing the left group of digits, L$, with the right group of digits, R$. Next, the first and last digits of A$ are discarded and the remaining left group of digits is compared with the remaining right group. This process repeats until all possible comparisons are made. The smallest half-group of digits in which the left sequence equals the right sequence constitues the period, P$.
(Lines 460-500)

A sample of the comparison process for the fraction, $\frac{1}{14}$ is shown below:

```
071 428 571 428 5 714 285 714 285 7
 71 428 571 428 5 714 285 714 285
  1 428 571 428 5 714 285 714 28
    428 571 428 5 714 285 714 2
     28 571 428 5 714 285 714
      8 571 428 5 714 285 71
        571 428 5 714 285 7
        (71 428 5 714 285)
          1 428 5 714 28
            428 5 714 2
             28 5 714
              8 5 71
                5 7
```

Note: Since 714 285 is the least left and right sequence to be equal, this becomes the period. This process may not be the most efficient, but does identify the period reasonably quickly without major constraints.

The comparison process, however, does identify an incorrect period for a few fractions in which the first and last digits of the period are the same, ($\frac{14}{29}$ is an example) or the first and last pairs of digits are the same ($\frac{9}{113}$ is an example). The reader might explore this situation with the fraction, $\frac{14}{29}$, which has period: 482 758 620 689 655 172 413 793 103 4.

The program handles these fractions by "rotating" the digits of the orginal quotient, A$, until the situation is resolved. It should be observed that the "rotating" does not change the picture to be plotted.
(Lines 380-430)

The program prints the sequence of digits in the period, and gives the length of the period, LP.
(Lines 520-550)

Next, the program uses the period to form the LP + 2 points to be plotted and connected.
(Lines 580-630)

If required, the X-Y axes and grid points can be plotted on the screen.
(Lines 660-860)

Finally, the actual periodic picture is plotted.
(Line 880)

VI. Problem Status:

We had hoped that further investigation might reveal answers to some of the questions posed in the Zeller article. Thus far, we've been consumed with printing the entire folio of pictures. (It is very time consuming on our equipment). It would seem that when we have a complete collection, certain comparisons and patterns will suggest at least some tentative conclusions. We shall try to report these as we develop them. Moreover, we shall gladly share a complete folio for the cost of duplication, when it is ready. Pursuit of these ideas in other contexts and with other methods would be welcomed by the authors and probably the Gazette.

Post Script:

Following the submission of this paper for publication, a somewhat more direct program strategy emerged for getting to the period. We share the details below:

Delete from the program statements 280 through 630 inclusive, and substitute the following new statements:

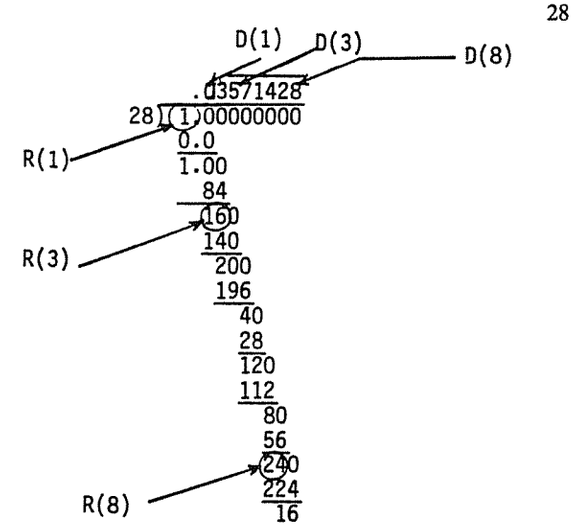
```
270 R = X
280 N = Y - 1
290  DIM R(N),D(N + 2)
300  FOR I = 1 TO N
310  IF R = 0 THEN LP = 1:D(1) =
     0:D(2) = 0: GOTO 430
320 R(I) = R
330 R = 10 * R
340 Z =  INT (R / Y)
350 D(I) = Z
360 R = R - Z * Y
370  FOR J = 1 TO I
380  IF R = R(J) THEN 420
390  NEXT J
400  NEXT I
410  GOTO 180
420 LP = I - J + 1
430  PRINT : PRINT
440  PRINT "THE DIGITS IN THE PER
     IOD OF ";X;"/";Y;" ARE:"
450  PRINT
460  FOR K = J TO I
470  PRINT D(K);
480  NEXT K
490  PRINT : PRINT
500  PRINT "THE LENGTH OF THE PER
     IOD IS: ";LP
510  VTAB 24: PRINT "HIT ANY KEY
     TO PLOT PERIODIC PICTURE ";:
      GET Z$
520 D(I + 1) = D(J)
530 D(I + 2) = D(J + 1)
```

Explanation:

Since the maximum period length possible is one less than the denominator of the fraction, or 'y-1' digits, the program long divides the denominator into the numerator (one digit at a time) up to a maximum of 'y-1' ('N') times. It stores each remainder, R, and each digit of the quotient, Z, in the respective numeric arrays R and D.

The division process is complete whenever any remainder repeats. Thus, the program checks each new remainder, R, with all previous remainders stored in R(N) to determine the location, J, of the repeating remainder in the array. The period consists of the digits from locations J to I in the array, D.

For example, the complete division process is shown here for the fraction $\frac{1}{28}$:



Note: the repeating remainder 16 is stored in array element R(3). Therefore, the period 571428 is stored in array elements D(3) to D(8) inclusive.

This new program allows exploration of positive fractions (less than 1) up to denominator 424 with the Apple II (48K).

**The authors wish to express their thanks to the reviewer and the editor for their helpful suggestions.

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(1) Zeller, C.W., "Periodic Pictures" The Ontario Mathematics Gazette Vol. 21, No. 1, September, 1982.

THE RATIONALS AND IRRATIONALS AS INFINITE DECIMALS

by

Gordon Barry

Operating with infinite decimals provides an excellent opportunity for our students to get a good feeling for the real numbers. Traditionally rational numbers have been explored extensively, both as fractions and as repeating infinite decimals, but the irrational numbers have been studied only fleetingly and as radicals.

But both the rationals and the irrationals are infinite decimals. Even the terminating decimals may be expressed as infinite decimals, for example, $1 = 0.\overline{9}$ and $2.34 = 2.33\overline{9}$.

In this article algorithms will be presented for adding, subtracting, and multiplying **infinite decimals** so the rationals and irrationals may be considered together, as a system.

The algorithms will be simple, teachable to any mathematics class, the perceptions magnificent and the mathematics beautiful in its simplicity.

ADDITION OF INFINITE DECIMALS

The reasons why the algorithm for the addition of the natural numbers is not appropriate for the addition of the infinite decimals are:

(a) we add natural numbers from the RIGHT.

(b) the carrying of digits LINKS COLUMNS from right to left. So the addition algorithm must be different.

The difference? Just one.

The carried digits are placed in a special row.

Two examples.

1.
```
 237    This is the carried digits row.
  97
  98
  94
 ---
 206
  32
 ---
 526
```

2.
```
 438
 158
 304
 ---
 880
   2
 ---
 800
   1
 ---
 900
```

With this one process change there are two major effects.

1. The sum of each column is now independent of the others, not depending on previously carried digits.

2. The partial sums may be done from left to right.

So the algorithm for adding infinite decimals is .

1. Add the columns from the left.
2. Place the carried digits in that special row.
3. Place the decimal point accurately via estimation.

Here are some examples:

1. Find the sum of $0.\overline{5}$ and $0.\overline{8}$

$$\begin{array}{r} 0.55555\ldots \\ +\ 0.88888\ldots \\ \hline 33333\ldots \\ 1\ 1111\ldots. \\ \hline 1\ 44444\ldots \end{array}$$

$\therefore\ 0.\overline{5} + 0.\overline{8} = 1.\overline{4}$

2. Add the irrational 0.272772777... to the rational 0.4545454545...

$$\begin{array}{r} 0.272772777\ldots \\ +\ 0.454545454\ldots \\ \hline 626217121\ldots \\ 1\ 11\ 111\ \ldots \\ \hline 0.72731823\ldots \end{array}$$

$\therefore$ 0.272772777 ... + 0.454545...
= 0.727318231823227323...

SUBTRACTION OF INFINITE DECIMALS

This may be done in at least two ways.

Method 1. By borrowing from the digit 10 times larger.

Example:

$$\begin{array}{r} 0.428571428571\ldots \\ -\ 0.142857142857\ldots \end{array} \qquad \begin{array}{r} 0.\ 4\ 2\ 8\ 5\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ 1\ \ldots \\ -\ 0.\ 1\ 4\ 2\ 8\ 5\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ \ldots \\ \hline 0.\ 2\ 8\ 5\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ 1\ 4\ \ldots \end{array}$$

Therefore, showing, with decimals, that $\frac{3}{7} - \frac{1}{7} = \frac{2}{7}$.

$\therefore\ 0.\overline{428571} - 0.\overline{142857} = 0.\overline{285714}$

Method 2. Exchange a terminating for an appropriate infinite decimal.

$1 = 0.999999999\ldots$

$2.34 = 2.339999999999\ldots$

Examples:

1. $$\begin{array}{r} 4 \\ -\ 1.6666666666\ldots \end{array} \quad \rightarrow \quad \begin{array}{r} 3.99999999999\ldots \\ -\ 1.66666666666\ldots \\ \hline 2.3333333333333\ldots \end{array}$$

2. $$\begin{array}{r} 1.93333333333333\ldots \\ -\ 0.777777777777\ldots \end{array} \quad \rightarrow \quad \begin{array}{r} 1.\ 9\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ \ldots \\ -\ 0.\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ \ldots \\ \hline 1.\ 1\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ \ldots \end{array}$$

MULTIPLICATION OF INFINITE DECIMALS

The algorithm.

1. Multiply from the left.
2. All carried digits go in the special carried digits row.
3. Place the decimal point correctly as a last step.

Examples.

1.

```
          0.123456789123456789...
        x                3
          ------------------------
Row A --> 369258147369258147
Row B --->   111222   111222   <-- Carried digits row
          ------------------------
            360360367360360 36...
              1 1     1 1      <-- Another carried digits row when adding rows A and B.
          ------------------------
            .3703703673703703...
```

2. Although this next problem is messy, I am going to include it, because it was, for me, the motivation for working with infinite decimals this way.

I wish to show that if $\sqrt{2} = 1.414213562373...$ that
$1.414213562373... \times 1.414213562373... = 2$ <-> Such a simple result.

```
  1.4142135...
x 1.4142135...
  ------------
   14142135...
    4646842...
    1 1  1
     141421...
      46468...
      1 1
       2828...
        141...
         32...
          1
           5...
           2...
  ------------
    1988776...
     11223...
  ------------
  1.9999999...
```

$\therefore (1.4142135...)^2$
$= 1.99999...$
$= 2$

DIVISION OF INFINITE DECIMALS

Division of the natural numbers is normally done from the left, so there will not be any changes to the algorithm.

Be careful though, since the multiplications and subtractions should be the new ones when infinite decimals are involved.

Example:

```
     .66...
  ------------
2)1.3333333333...
  12xx
  ----
   13
   12
   --
    13
    :
```

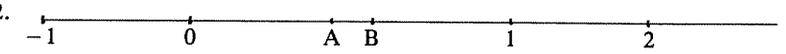
$\therefore \frac{1.333333...}{2} = 0.666...$

Now, can you do these?

1. Show that (a) $1.\overline{3} - 0.\overline{8} = 0.\overline{4}$

   (b) $0.\overline{6} + 0.\overline{9} = 1.\overline{6}$

   (c) $4 \times 3.434434443... = 1.373773777...$

2. 

On the number line, A and B are the two irrationals 9.434434443... and 0.565565556... Find the mid-point of $\overline{AB}$.

3. The mid-point of $\overline{CD}$ is $\overline{0.4}$. If C = 0.161161116... , find D.

4. Find two irrationals who average is 0.2.

5. Find an example of two irrationals which multiply to give a rational number answer.

6. Find an example of two irrationals which divide to give a rational answer.

7. Describe the rational number line.

8. Describe the irrational number line.

9. Evaluate 3.4567823 × 24.78 to three significant figures.

10. Is the average of two rational numbers always rational?

11. Is the product of a rational and an irrational always irrational?

The rational numbers are closed for addition and multiplication, are dense, but not complete.

The irrational numbers are dense but not closed for addition and multiplication.

The union of the two sets is dense, complete, and has not only closure for addition and multiplication, but the rest of the field properties. Voila, the real numbers.

I would emphasize that working with these algorithms allows the students some time to assimilate the concept of real numbers, correctly emphasizes that the real numbers are the infinite decimals, and allows the teacher to teach the radicals and exponentials for what they are - notations.

Carry on in this topic. You will be rewarded.

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EDUCATION IS THAT WHICH REMAINS WHEN ONE HAS FORGOTTEN EVERYTHING LEARNED IN SCHOOL.

A. Einstein

SOLVED AND UNSOLVED PROBLEMS IN MATHEMATICS

by

Hans P. Heinig

Problems, guesses, inferences and conjectures are the lifeblood of mathematics. The ingenuity by which their solutions are arrived often yield new mathematical methods and tools, initiate new theories and indeed new conjectures. Most problems or conjectures are solved or settled with considerable speed and affirm or reject mathematical or physical theory, while other defy solution for several decades or even centuries. Because of continuous attempts by many workers, these problems attain a certain glamour even if their mathematical significance has somewhat diminished. One of the best known conjectures, initiated by Euclid, is to deduce the "parallel axiom" in geometry from simpler principles. Any attempt to solve this problem affirmatively failed, but the study of this problem led to the development of the various non-Euclidean geometries in the nineteenth century. It further raised questions about the consistency of mathematics, which, in spite of Gödel's work, reverberate to the present.

The object of this article is to state and discuss a number of other well known conjectures and problems from various branches of mathematics ranging through the centuries. Some of these have been partially or completely solved while others still resist solution.

I. **Geometric Construction Problems.** Many problems from antiquity fall into the category of geometric construction problems. These are the problems of drawing a - usually plane - figure satisfying given conditions and using certain prescribed tools, such as a ruler and compass, only a finite number of times. If the problem is solvable it is called a possible construction problem, otherwise it is called an impossible construction problem.

One of the most famous problems is called **Apollonius' Problem,** after the great Greek mathematician (c.262-190 BC) of the classical period and is the following: *Given any three points, lines, or circles, or any combination of three of these, construct - using ruler and compass - a circle passing through the points and tangent to the given lines and circles.* This is a possible construction problem and many mathematicians, most notably Francois Vieta (1540 - 1603) and Isaac Newton (1642 - 1727) gave solutions.

Other famous construction problems of Greek origin and using a ruler and compass are the following:

(i) **The Trisection of an Angle:** *Divide a given angle into three equal parts.*

(ii) **The Delos Problem:** *Construct a cube whose volume is double that of a given cube.*

(iii) **The Quadrature of a Circle:** *Construct a square whose area is that of a given circle.*

Galois Theory may be applied to solve the first two problems, although not the third. However, Pierre L. Wantzel in 1837 proved that without using Galois Theory (i) and (ii) are impossible construction problems. C.L.F. Lindemann proved the impossibility of (iii) in 1882 by proving that the number pi (π) is transcendental. (A number is called transcendental if it is not algebraic, and an algebraic number is any real or complex root of the polynomial equation

$$a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0,$$

where a_i, $i = 0,1, \ldots, n$ are rational numbers).

It is interesting to note that one mystery about a fundamental constant still remains, namely whether the Euler constant defined by

$$C = \lim_{n \to \infty} \left(1 + \frac{1}{2} + \ldots + \frac{1}{n} - \log n\right)$$

is transcendental. At this time even the irrationality of C remains unproved although C = 0.57721566490153286060651209008 2... has been computed to more than seven thousand decimal places.

We conclude with two rather move recent construction problems.

(iv) **The Cramer-Castillon Problem (~1750):** *Given a circle 0 and three points P,Q,R, not lying on 0. Draw a triangle ABC inscribed in 0 in such a way that the sides BC, CA, AB pass through P,Q,R.*

(v) **The Steiner Problem (1848):** *Suppose we are given three straight lines l,m,n and three points P,Q,R. Draw a triangle ABC in such a way that the vertices A,B,C, lie on l,m,n and sides BC, CA, AB pass through P,Q,R.*

II. **Goldbach's Conjecture.** *In 1742, Goldbach, in a letter to the eminent Swiss mathematician L. Euler, stated that every positive integer can be expressed as the sum of two primes.* More precisely, he conjectured *that any even integer not smaller than six can be expressed as the sum of two odd primes* and that *any odd integer not smaller than nine can be expressed as the sum of three odd primes.* Very little if any progress was made towards a solution of this conjecture for nearly two centuries until 1937 when the Russian mathematician I.M. Vinogradov proved that every "sufficiently large" odd integer can be expressed as the sum of three primes. In proving this, he used the prime number theorem for arithmetic progressions and invented ingenious methods to estimate sums of the form $\sum_{p} e^{2\pi i p\alpha}$. In the case of even integers the complete solution is not known, however a number of mathematicians e.g. I.G. Van Der Corput, T. Esterman and N.G. Cudakov proved in 1938 simultaneously that "almost all" even integers (ie. except of a set of density zero) can be expressed as a sum of two primes. Their method was essentially suggested by G.H. Hardy and J.E. Littlewood in 1919, however they assumed that the Riemann hypothesis (see below) holds.

Various generalizations and abstractions of Goldbach's conjecture are known. T. Mitsui and O. Korner in 1960-61 generalized the Goldbach conjecture to algebraic number fields and obtained results in this setting. A related problem, which dates back to 1770 is **Waring's Problem:** *Is it possible that an arbitrary positive integer can be expressed as the sum of at most nine cubes?* Although much research has been generated by Waring's problem, a complete solution has not been found.

There are, of course, many other problems and conjectures in number theory. For example, the **Bertrand Conjecture** *asserts the existence of at least one prime number between n and 2n, where n is a natural number greater than 2.* This conjecture was proved by the Russian P.L. Chebycheff in 1848.

The last conjecture that we are going to list in this section is still unsolved.

It is well known (Euclid) that there are infinitely many prime numbers. If we denote the n_{th} prime number by p_n, then we call a pair of primes (p_n , p_{n+1}) **twin primes** if $p_{n+1} - p_n = 2$. For example (3,5), (5,7), (11,13), (17,19), (29,31) are twin primes. It is conjectured, (I think by P. Erdös), that *there are infinitely many twin primes.* Although there is some probabilistic evidence that the conjecture is true, no one has been able to give a proof todate. Incidentally, the largest pair of twin primes seems to be $(76 \times 3^{139} - 1, 76 \times 3^{139} + 1)$ (H.C. Williams and C .R. Zarnke, 1972).

III. **The Riemann Hypothesis.** One of the great achievements in mathematics during the last two centuries was the proof of the prime number theorem. If $\pi(x)$ denotes the number of primes less than or equal to x, then the prime number theorem asserts that $\lim_{x/} \pi(x) \log x/x = 1.$ The theorem was conjectured by Euler, Legendre, Gauss and others and many of the most powerful mathematicians of the 18th and 19th centuries have tried to prove it. B. Riemann, a German Mathematician in 1859, gave a conditional proof of the prime number theorem, the condition being that -what is now called - the Riemann Hypothesis holds. Although the prime number theorem was finally proved by Jacques Hadamard in 1898 and subsequently by several twentieth century mathematicians, the Riemann Hypothesis was never proved. To explain the conjecture we must introduce the so-called Riemann-zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + 2^{-s} + 3^{-s} + \ldots + n^{-s} + \ldots$$

where $s = x + iy$, $i^2 = -1$ is a complex number. For real s the function has already been studied in 1737 by Euler, who found the value of $\zeta(s)$ when s is an even integer. For example, he found that $\zeta(2) = \pi^2/8$, $\zeta(4) = \pi^4/96$, $\zeta(6) = \pi^6/960$ etc. Moreover, he established the connection of this function with the prime numbers. Namely, if p_n is the nth prime then for $s > 1$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_{n=1}^{\infty} \left(1 - p_n^{-s}\right)^{-1}$$

where || stands for the product of the argument e.g. $\prod_{n=1}^{\infty} a_n = a_1 \cdot a_2 \cdot a_3 \cdots$;

Now for complex s, say $x + iy = s$, then the series defining $\zeta(s)$ fails to converge absolutely for $x < 1$, but it is possible to define the function for $x < 1$ by analytic continuation. It is known that $\zeta(s)$ has infinitely many roots or zeros in the strip $0 < x < 1$ (We will not consider the case $x < 0$). *The Riemann Hypothesis asserts that all zeros or roots of $\zeta(s)$ lie on the line $x = 1/2$.* Thus, as noted above, if this conjecture is true, then Riemann's proof of the prime number theorem is correct. The conjecture is important in various areas of mathematical analysis and analytic number theory and the importance of its solution cannot be overestimated.

Here are some of the results regarding the conjecture. R.S. Lehman in 1966 showed that there are exactly 2.5 million zeros of $\zeta(1/2 + iy)$ for $0 < y \leq 170571.35$ and in 1968 this computation was extended to 3.5 million zeros. The great English mathematician G.H. Hardy proved in 1914 that there are infinitely many zeros of $\zeta(s)$ on the line $x = 1/2$ but he could not show that these are all. In 1942 A. Selberg, a Scandinavian, proved that if $N_0(T)$ is the number of zeros of $\zeta(s)$ on the line $x = 1/2$ and $0 < y < T$, then for some positive constant A, $N_0(T) > AT \log T$. Other sharper results were established in 1974 by the American, N. Levinson.

In spite of these facts and other positive evidence that the Riemann Hypothesis is likely true, there are many who are skeptical while others withhold judgement.

There is also a generalized Riemann Hypothesis which pertains to the zeros or roots of a class of functions (L-functions). This conjecture has also yet to be settled. However, as in the case of the Riemann Hypothesis there are several theorems which can be proved if the Generalized Riemann Hypothesis is true e.g. Artin's Conjecture in Group Theory.

IV. **The Four Colour Problem.** *Are four colours necessary and sufficient to colour a given geographical map on the plane or sphere?* This is the four colour problem. However, to be more precise we must insist that (i) every country or map is a connected domain - which is a most natural condition; (ii) a connected part of the sea is considered a country; (iii) two countries sharing boundary lines must be coloured differently, however, if two countries share only a finite number of boundary points, then they may have the same colour.

The four colour conjecture was made by Francis Guthrie and communicated to the logician August De Morgan in 1852. In spite of considerable effort the problem remained unsolved for 124 years. The foremost English mathematician at the time, Arthur Cayley, called attention to this problem in 1878 and in 1890 P.J. Heawood proved that five colours are sufficient for colouring any map (the necessity of the problem is fairly obvious). In 1968 the four colour problem for the case where the number of countries is not more than forty was solved by O. Ore and J. Stemple and in 1976 the general case was solved affirmatively by the American mathematicians W. Haken and K. Appel (Bull. Amer. Math. Soc. 82 (1976)). The interesting aspect of their solution is that it was solved largely by using a computer that considered many thousands of different subcases. The solution of this problem solves several other equivalent problems in graph theory, e.g. *every planar graph is four colourable.*

V. **The Bieberbach Conjecture.** This conjecture was formulated by the German mathematician Ludwig Bieberbach in 1916 and is a problem in complex function theory. Before we formulate this conjecture we need a definition: *A one to one analytic function defined on the unit disk* $D = (z: |z| < 1)$ *of the complex plane which satisfies* $f(0) = 0$ *and* $f'(0) = 1$ *(derivative) is called a Schlicht function.* (Schlicht in German means simple). Such a function has a power series expansion

$$f(z) = \sum_{n=1}^{\infty} a_n z^n , \quad \text{where} \quad a_1 = 1.$$

The Bieberbach conjecture asserts that for any Schlicht function $|a_n| \leq n, \; n \geq 2.$

Although a complete solution of this conjecture is still not known, it may be of interest to trace some of the work on this problem from its formulation to the present.

1916 Bieberbach proves $|a_2| \leq 2$;

1923 Loewner proves $|a_3| \leq 3$;

1955 Garabedian and Schiffer prove $|a_4| \leq 4$;

1968 Ozawa and (independently) Pederson prove $|a_6| \leq 6$;

1972 Pederson and Schiffer prove $|a_5| \leq 5$;

For exact bounds this is all and the conjecture is open for $n \geq 7$. For larger bounds of the coefficients the following results were obtained:

1925 Littlewood proves $|a_n| < en$, $n \geq 2$ (Here e is the base of the natural logarithm $e \cong 2.71828182845\ldots$)

1974 A. Baernstein II proves $|a_n| < \frac{en}{2}$, $n \geq 2$;

1965 I.M. Milin proves $|a_n| < 1.243n$, $n \geq 2$;

1972 C. Fitzgerald proves $|a_n| < 1.081n$, $n \geq 2$

1976 D. Horowitz proves $|a_n| < \left(\frac{209}{140}\right)^{1/6} n < 1.061n$, $n \geq 2.$

This last result is the best general constant to-date.

If one considers certain subclasses of Schlicht functions, e.g. those functions who have real coefficients then;

1931 J. Dieudonnē proves $|a_n| \leq n$ for all $n \geq 2$.

Thus, for this subclass the conjecture is true.

Finally, we mention a result of D. Bishouty in 1976 who proved that for **any** Schlicht function the following holds: If $|a_2| < 1.55$ then $|a_n| < n$ for all n.

For these and other results see [2] and the bibliography cited there.

VI. **The van der Waerden Conjecture** was posed by Dutch mathematician B.L. van der Waerden some fifty years ago. Given an n x n matrix A, one associates with it a scalar quantity, called the determinant of A or det A. The determinant of a matrix is obtained by summing certain signed elementary products. e.g. if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *is a 2 x 2 matrix, then det A* = ad - bc. Another scalar

quantity associated with an n x n matrix is called the permanent of the matrix. It is obtained by summing the elementary products disregarding the sign. Thus in the above example per (A) = ad + cd. Now if A is a n x n doubly stochastic matrix, that is, A is a matrix with nonnegative entries and both rows and columns add up to one, then the van der Waerden conjecture asserts that

$$\text{per}(A) \geq n!/n^n$$

and equality holds if and only if all the entries of A are 1/n. The conjecture was proved in 1980 by the Russian mathematician G.P. Yegorychev. The conjecture (now a theorem) attracted considerable attention in the last 25 years, since its solution has important applications in some branches of combinatorics, probability theory as well as physics and chemistry.

Another recently solved problem is **the Littlewood conjecture** formulated by the English mathematician J.E. Littlewood in 1930. *It asserts that*

$$\int_{-\pi}^{\pi} \left| \sum_{k=1}^{N} e^{in_k x} \right| dx \geq K \log N$$

where (N_k) are distinct integers and K, a constant, is independent of N. Although several smaller lower bounds were obtained by a number of mathematicians most notably by P. Cohen (U.S.A.), J. Fournier (Canada) and S.K. Pichorides (Greece). The full result was given by three American mathematicians, O.C. McGehee, Brent Smith and Louis Pigno in 1981. The result is of importance in harmonic analysis and seems to have applications in Banach space theory.

There are numerous other conjectures in mathematics most of which cannot be stated in simple terms. Thus, in group theory, there is the **Burnside conjecture,** that was solved in 1963: *Is a finite group of odd order solvable?* In algebraic geometry one has the famous and still unsolved **Pointcaré conjecture** which asserts that *a simply connected three dimensional closed manifold is homeomorphic to a three-sphere,* and in topology the "Hauptvermutung" or "fundamental conjecture".

These conjectures, their partial or complete solution, reflect the stubbornness of the mathematical community to concede defeat and the brilliance and ingenuity of its members. It shows that the study of mathematics is vital and alive.

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DEPARTMENTS

PROBLEMS? DID YOU SAY

by
Pat Yip

Notice:

Problems and solutions are solicited from readers. Come across a good one lately? SEND IT IN! (to P. Yip, Dept. of Math. Sciences, Master University, Hamilton, Ontario.)

PROBLEMS:

All letters denote positive integers.

21.3.1

If $(n + 1)(n + 2).....(n + k)$ is a multiple of k,

show that $n \leq k!$

21.3.2

Let $[1, 2, 3,....n]$ denote the least common multiple of the numbers $1, 2, 3...n$.
Prove that

$$\sum_{n=1}^{\infty} 1/[1, 2,....n] \text{ is not a rational number.}$$

21.3.3

Prove that the infinite sum

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

is finite, if a_n runs through all the integers which do not contain the digit 9 in their decimal expansion.

SOLUTIONS: (from problems in Vol. 21.2)

21.2.1 Let $1 \leq a_1 < a_2 .... < a_{n+1} \leq 2n$, where a_i's are positive integers,

(i) two of the numbers $a_1, a_2, \ldots, a_{n+1}$ must be consecutive and therefore have no common factor except 1;

(ii) if the equation $a_i + a_j = a_k$ is not soluble, then the $2n+1$ numbers:

$$a_1, a_2, \ldots, a_{n+1}, a_2 - a_1, a_3 - a_1, \ldots, a_{n+1} - a_1$$

are distinct. This is not possible since all these numbers are between 1 and 2n.

21.2.2 p is a prime and $k < p - 1$.

By binomial expansion:-

$$r^{k+1} - (r-1)^{k+1} = \binom{k+1}{1} r^k - \binom{k+1}{2} r^{k-1} + \ldots + (-1)^k r^0$$

Since $p^{k+1} = \sum_{r=1}^{p} [r^{k+1} - (r-1)^{k+1}]$,

therefore $p^{k+1} = \binom{k+1}{1} S_k - \binom{k+1}{2} S_{k-1} + \ldots, + (-1)^k S_0$ (1)

where $S_n = 1^n + 2^n + \ldots + p^n$.

Let $k+1<p$ and suppose $S_0, S_1, \ldots, S_{k-1}$ are all multiples of p, then by equation (1) $(k-1) S_k$ is also a multiple of p, i.e. p divides S_k since p is a prime. Since $S_0 = p$, the result follows by induction on k.

21.2.3 To show that $\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}$

Differentiate $(1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$ with respect to x and set x=1.

Without using the calculus, we argue as follows:

$n2^{n-1}$ are the number of ways of filling n-1 of n cups with either tea or coffee, since the empty cup can be any one of n, and there are 2 ways of filling the (n-1) cups.

If exactly p cups are filled with tea, there are

$\binom{n}{p}$ possible ways of choosing these p cups,

and of the remaining (n-p) cups, one is left empty.

Therefore: $n2^{n-1} = \sum_{p=0}^{n-1} \binom{n}{p} (n-p) = \sum_{k=0}^{n} k \binom{n}{p}$

21.2.4 Show that n(n+1) (n+2) (n+3) is not a perfect square.

Since

$$n(n+1)\ (n+2)\ (n+3) = (n^2 + 3n+1)^2 - 1$$

and consecutive positive integers cannot both be squares.

BOOK REVIEWS

WHY EXPERIMENTS? WHY STATISTICS?

An Introduction to Sound Experimental Practices, Hines, Lachmansingh and Miller. 1982, 97 pages, $4.50 softcover. Available through the Campus Bookstore, University of Guelph, Guelph, Ontario N1G 2W1.

This booklet was originally conceived as a guide to students entering Science Fair exhibits. It was hoped that it would enable them to avoid some common pitfalls. In the process of responding to this concern the authors have come up with much more.

At first glance, the reader will see this as a guide suitable to the science classroom (in particular, the Biology classroom) and so it is. However, closer inspection will show that while Biology is the vehicle it is **not** the message of this book.

A 26-page section of the book entitled 'Summarizing Your Observations' shows examples of the use of tables and graphs to summarize experimental results. The 5 experiments which are presented in this section are typical of what students might see in their science classes and thus perhaps, in some ways, 'better' than the examples that mathematics teachers often use.

The statistics presented do not go much beyond this, however, there is reflected in the writing an enthusiasm which is refreshing.

While this book is primarily directed to those entering Science Fairs and to students in biology classes, it does provide examples for the tables and graphs portion of the Statistics section of our courses in the Intermediate Division.

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Hamilton Board of Education
100 Main St. W.
Hamilton, Ont.

THE SAMUEL BEATTY ESSAYS, 1981 - 1982

Edited by E.J. Barbeau, published at the University of Toronto with a grant from The Samuel Beatty Fund, 1982 (92 pages, $3.50 paperback).

This volume of The Samuel Beatty Essays contains six of the best essays submitted for the 1981 and 1982 Samuel Beatty Essay Contests along with a few biographical comments about each of the six authors. Five of the authors were in their final year of secondary school when they wrote their essays and one was in grade eight. The topics of the essays are "A study of Pythagorean triplets", "Six numbers such that the sum of any two is a perfect square",

"$720 = 27^2 - 3^2 = 28^2 - 8^2 = 29^2 - 11^2$", "Establishing the sum $\sum_{k=0}^{n} k^r$",

"Investigation of factorials and logarithms of negative numbers" and "The mathematics of a 3x3x3 Rubik's cube applied to a 4x4x4 cube".

The annual contests have been discontinued but essays for future volumes of The Samuel Beatty Essays will be considered by its editor. Other information about The Samuel Beatty Essays appears in the September, 1981 and March, 1982 issues of the Ontario Mathematics Gazette.

E. Mead
Department of Mathematical Sciences
McMaster University
Hamilton, Ontario

THE HANDBOOK OF ARTIFICIAL INTELLIGENCE, VOLUME II

Edited by Avron Barr and Edward A. Feigenbaum, William Kaufman, Inc. Los Altos, California, 1981.

This is the second volume in a three volume work covering the major areas of artificial intelligence (AI). The first volume (reviewed in the September 1982 issue of the Ontario Mathematics Gazette) covers a number of basic AI techniques including search, knowledge representation, natural language understanding and understanding spoken language. The second volume concentrates on some of the major applications of artificial intelligence. While this volume has many references to the first volume a detailed understanding of the material in the previous volume is not necessary in order to follow most of the articles in this volume.

Like the first volume, this volume consists of a number of articles written by leading experts in the field. Each chapter addresses a particular area and consists of a general introduction to the area followed by short articles covering additional details and how the ideas have been used in major programs. The last section of each article contains suggested references where the reader can find more information on the particular topic. These references range from general articles to Phd thesis. This wide range of coverage accommodates both the casual reader and the active researcher.

The five chapters in this volume cover AI programming languages, scientific, medical and educational applications of AI and automatic programming. The chapter on AI programming languages contains a survey of the major AI programming languages, an introduction to LISP (the most popular AI programming language) and an introduction to the major issues in AI programming language design.

The next three chapters survey a number of applications of artificial intelligence. These chapters show how the basic techniques described in the other volumes can be applied to real problems. The first applications chapter deals with the scientific applications of AI. Most of this chapter is devoted to applications in chemistry with some material on applications in mathematics and geology. Chemistry was the first field where AI techniques were successfully employed in research projects.

Several programs in this area are essentially coauthors of published scientific papers. The second applications chapter reviews some of the applications of AI in medicine. Most of these applications deal with the diagnosis and treatment of diseases. The third applications chapter is devoted to educational applications. In this area AI techniques have been used to plan the presentation of material and evaluate how well the student understands the material.

The last chapter in this volume deals with automatic programming. This chapter surveys some of the techniques that have been used to automate the production of programs. These techniques range from generating a program from a description or example of what it should do to programs that attempt to understand programming and assist the programmer in routine tasks.

This volume makes quite interesting reading. It makes a good source of background material for a course on computer applications or computer literacy.

Mark Green
Unit for Computer Science
McMaster University
Hamilton, Ontario

TVO CHANGES TRADITIONAL APPROACH TO TRANSFORMATIONAL GEOMETRY

by
Susan O'Connor

Students' enthusiasm for transformational geometry will get a boost when teachers use TVOntario's new television series, **Landscape of Geometry,** in their classes.

The eight 15-minute programs in this unique series illustrate the relationships between geometry and industry, art, architecture, and nature. Hosted by the affable David Stringer (of TVO's **Fast Forward** and **Half-a-Handy Hour**), the programs combine animation with live action and film footage from around the world to put tranformational geometry into practical, everyday perspective. Although designed for students in grades 7 and 8, **Landscape of Geometry** is also suitable for students in grades 9 and 10, as well as advanced grade 6 students.

Written by freelance educational writer Alan Ritchie, **Landscape of Geometry** was produced and directed by TVOntario's David Chamberlain, who says the series is "the most complicated and sophisticated one I have ever done. But I know it's going to make students feel that it's both worthwhile and fun to learn geometry."

As with all TVOntario series produced for schools, a project committee of experts in a subject -- in this case math -- was formed to offer guidance for the writing and production of the series. The project consultants for **Landscape of Geometry** were: Neil Williamson, project leader, curriculum mathematics, for the North York Board of Education; Peter Ullrich, a mathematics consultant with the Ottawa Board of Education; and George Knill, an education officer working at the central Ontario regional office of the Ministry of Education.

In addition to the input from the project committee, four of the **Landscape of Geometry** programs were tested in Ontario classrooms by TVO's Office of Project Research to measure their appeal to the target audience. Results indicated that boys and girls in grades 7 and 8 rated the appeal of the programs very highly. The next phase of research will measure the target audience's comprehension of the programs -- the amount they learn from the programs and their ability to apply their knowledge to real world problems.

Landcape of Geometry has won an international award for educational excellence. Like many other TVOntario series for schools, it is supplemented by a teacher's guide filled with ideas for activities that go beyond the programs to offer students real-life applications of geometry.

Landscape of Geometry will be broadcast in two one-hour time blocks this spring -- Friday, 29 April from 3:00 to 4:00 p.m. EDT, and Wednesday, 4 May, from 3:00 to 4:00 p.m. The series may be taped off air for future classroom use, or purchased on videotape from TVOntario.

Teachers wishing to purchase videotapes or copies of the teacher's guide ($4 each) should check with their audiovisual representative or television resource teacher for ordering information. Broadcast and ordering information is given in the current copy of **School Broadcasts,** a magazine distributed by TVOntario to every teacher in the province.

For further information, contact:
Susan O'Connor (416) 484-2600

EDUCATION MUST ACCEPT AS A KIND OF LAW THAT EVEN THE REDISCOVERY OF THE PAST MUST HAVE RELEVANCE FOR THE PRESENT.

George N. Shuster

INTERVIEW

with

Bob Smith

This interview took place at the 1982 conference of R.C.D.P.M., held at S.U.N.Y., in Buffalo. The council seeks to stimulate, generate, co-ordinate and disseminate research efforts designed to understand and/or overcome factors that inhibit mathematics learning. Many teachers, as well as special education programs, are concerned with pupils at the low end of the scale. The concern of these two interviews is with students at the high end of the scale.

Kevin Bartkovich was a Phi Beta Kappa graduate in 1980, receiving B.E.S. and M.S.E. degrees in electrical engineering, who decided in his senior year to enter education. He is currently a graduate student in education at Duke University and a Project Associate with the Talent Identification Program in North Carolina.

Dipendra N. Bhattacharaya comes from Bengal, India, where he earned a bachelor's degree, but, to his Brahmin parents' chagrin, he joined a group of magicians. This proved unsatisfactory, so he came to America to continue his education. At S.U.N.Y. he earned a Master's degree in Applied Mathematics, and now his doctorate with a case study of Gifted Children in Mathematics.

R.S.: Please take a moment, Dip, to describe the study.

Dip: It is a sociological study of four students who were admitted to the gifted program at the University of Buffalo. Their scores were much higher than the norm for the program; the gifted among the gifted. For a twelve-week period I tutored them in the content of a one-year program for gifted mathematics students. In return, they agreed to take part in my investigation.

R.S.: Did they challenge you?

Dip: Not really. My personality helped them accept me like an older brother. I became a participant-observer, discussing the math with them and at the same time making notes on their feelings and behaviour. Finally, questionnaires were sent to the parents & interviews with the parents and students were held, so that a match could be made between parents and students for each of the sixty items.

R.S.: What were some of the things you found out?

Dip: 1) The children all had very strong memories. All said that they have an excellent memory. After we had proved the famous proof about infinite prime numbers, one of the kids, who was only twelve years old, said that he just could not erase it from his memory. 2) Each of the students showed very early signs of reading ability. One started reading the bedtime stories to his parents when he was eighteen months old. The father thought he was memorizing the story, so he changed to a different version of the story to find that the boy was indeed reading. Another parent was typing and found that her child was doing the same thing, and in fact understood what she was composing. 3) They do not perceive things the way average children do. Here is student K's experience when in Kindergarten. A set consisting of a horse, pig, chicken and cat were shown on a ditto along with the question "Which doesn't belong to this set?" The only answer accepted by the teacher was the chicken, because it has only two legs and the rest have four legs. K talked to his mother at home about his answer because he felt he was right. He had crossed out the cat because he had included chicken, horse, and pig together as farm animals, whereas the cat he identified as a pet. Mother then went to the school to support K's stand.

R.S.: Parental support was strong and did help their children. Yet they do not always praise their bright child. Why?

Dip: The student who stated he was never praised was aware of the reason, accepted it and recognized the parental concern and love shown in other ways. His father, in the interview, pointed out that his other son was of average ability. Gifted children are aware of their ability and are very generous, recognizing all students are doing the best they can.

R.S.: Why is reading so crucial?

Dip: In the middle grades, grades 7 and 8, these children find they can master the content of courses by reading the textbooks. Yet often, they are not allowed to take secondary school courses because it will create logistical problems. Text books for most children contain exercises only. For the gifted student, it is a source of food. They learn what the book has to tell them. They have learned to feed themselves. My personal feeling is that they have learned early to wonder about what the text is telling them. In India, we see holy cows eat grass quickly, then sit down to chew the cud; we call it in Sanskrit, "rummonton". I believe your equivalent word is "ruminate", meaning to ponder upon.

R.S.: Kevin, you have illustrated the classroom teacher's problem nicely with a picture of the educational continuum. Tell us about it.

K.B.: Well, you have a problem any time you have a student at the extreme in a classroom. Many of the teachers at this conference are concerned with the extreme at the low end and bringing those sutdents up to a minimal competency level. My concern has been, What about the students on the high end -- students at that extreme? With nothing special done for them, they'll end up falling back into the main stream, which is far below what they are capable of doing, and the ramifications of that are tremendous because they become bored, they become less productive, and they may never reach their full potential -- so I feel it is important to do something in the way of help to advance them further along, at a level they are truly capable of learning at, and with material that is at the level of difficulty that is consistent with their ability level.

R.S.: How did you identify the children you worked with?

K.B.: Our principal means of identification was the scholastic aptitude test. My main concern is with the mathematically talented, and when I say that, I mean those who reason very well mathematically, not those who may be able to do advanced addition in their heads. As in anything else, there is going to be great variability with just about any other characteristic you name. It might be there, it might not be there. You have the athletes, you have the non-athletes, you have the ones who like to read novels, those who don't like to read novels, the very extroverted, and the very shy. One other thing that tends to be true, is that they are very interested in science, and a lot of times, in computer science, but just as a follow-up to their mathematics.

R.S.: And, indeed, you also found those who liked to work and those who didn't like to work.

K.B.: Well, very often you'll find students, because they've been in the typical classroom and have not had to put out the effort that we expect from them in our program, that they have a period of adjustment there. Invariably they end up loving it because they get the satisfaction of knowing they've truly done what they are capable of doing, for some, maybe for the first time in their lives.

R.S.: There are several particular programs that you mentioned which are indeed a help for the teacher who is faced with one or more particularly gifted students in mathematics. Name some.

K.B.: The ones I would refer to as models to look at would be the program at Duke and the program at Johns Hopkins, in terms of the kinds of things that can be done using diagnostic testing and prescriptive instruction. Now there are two notable curricula that are under the heading of unified mathematics. One, called the Secondary School Mathematics Curriculum Improvement Study, or SSMCIS for short, out of Columbia University, and the one developed by CEMREL, Central Mid-Western Regional Educational Laboratory, and that's out of St. Louis, and those are for students in the upper 15% of their age group. They incorporate advanced mathematics with pre-calculus and unify so that it is no longer the traditional subject taking its place. I think each of those is a six-year program. There is a problem if you want to leave in the middle, because it does not correspond to the typical mathematics curriculum.

R.S.: We should look a moment at this mixture of math and science. Would you take a minute to describe the North Carolina program?

K.B.: Two years ago, North Carolina established the North Carolina School of Science and Mathematics, that is, a residential school for gifted students who have a special aptitude and interest in mathematics and science. Students come from all over the state of North Carolina.

R.S.: They've even been written up in **Time** Magazine and **Newsweek**.

K.B.: Yes. It's a very significant development because it's the only school of its kind. New York City has its specialized schools but none of those are residential. There is something special about the residential situation -- what they call the living and learning atmosphere. It's a two-year school for juniors and seniors, and that doesn't mean that every student there is typically aged junior and senior. Also, the restriction about in-state students is something that's planned to be relaxed so that eventually there will be students from outside the state and also from outside the country, I would hope.

R.S.: When you say junior and senior, what age level are you speaking of? Well, the typical senior is just turned eighteen and the typical junior is sixteen-seventeen, but they are also younger. I have one student in particular, she was fourteen when she entered there and she is just now turned sixteen.

R.S.: So again, age is not the necessary entrance requirement?

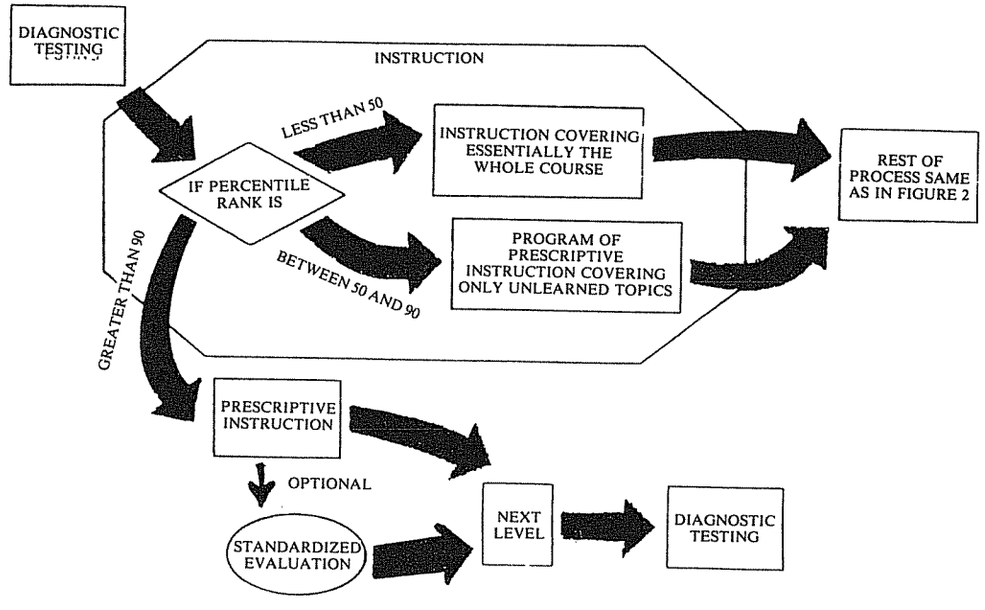
K.B.: The necessary entrance requirement is that they, upon entering, be able to gain a high school diploma in two years. In other words, they would have had to fulfill the requirements of a tenth grader in the school system in order to apply.

R.S.: In fact, you find some quite able in particular areas to be in this program, but deficient in others, so I understand through diagnosis and prescription they are brought to an acceptable standard. Please describe the process.

K.B.: The process starts by giving each of the students a diagnostic test in a pre-calculus subject. What we find is that a bright student will often know many topics, for instance, from Algebra II, without ever having taken the course. So those topics are not covered. We spend time only on those things that the student needs to study, and what happens is that they end up working together in class in small groups, usually progressing through the material very quickly. They take quizzes at periodic times so we're sure they're actually learn-material and progressing at their own speed -- not what they think their own speed is, but what the teacher thinks their own speed is, which is different for each student. If you leave a student on his or her own, they won't work up to what they are capable of doing. The teacher works with the individuals in small groups, for instance, at the blackboard maybe, or acting as a stimulator by setting objectives, such as "You should have this chapter done by tomorrow", or "You should be able to take this quiz tomorrow".

HUMAN BEINGS SHOULD ALWAYS BE TREATED AS ENDS AND NEVER AS MEANS.
E. Kant

R.S.: The following chart is a good model.



R.S.: I noticed, too, however, your serious purpose behind this instruction was that they might move from your doing the lecture or presentation to the English systems concept of "reading mathematics". How did you accomplish that?

K.B.: Well, right from day one, it's understood that when I go into class, that, look, here are your textbooks, you're to start working through this on your own. Now, typically, what happens is that they're on very shaky ground at first and right away twenty hands go up in the air: "I don't understand this, I don't understand that", but they come to realize that if they wait for the teacher to come around to answer their question, they might have to wait ten minutes. Sometimes we suggest, "Joe, you're working on the same thing as Bill; why don't you go sit over there next to him and work together and help each other out?" And it becomes a game, it becomes fun and the prime motivator there is that they know that as soon as they master a topic, they can go on to the next, they don't have to wait for everyone else in the class to master it.

K.B.: Now in terms of what I see is the long-term significance here, we work primarily with ones who are extremely capable.

R.S.: The 1% of the population.

K.B.: Maybe the upper 1% of the population. Now, it seems to me illogical that the ones who may be only in the upper 5% ought to do more than they're doing, because I don't think that when you get to the upper 1%, you all of a sudden jump out into a new species of human who is capable of things that no one else can do. There is a continuum there, so that there is a middle ground between what goes on in the average classroom and what goes on in our program. What I would like to see is more schools using something like this with maybe the one or two bright students, the one or two who are far more capable than the others and allowing them to progress through more advanced material, and allow them to learn at a rate that is consistent with their ability level.

R.S.: Now, we have talked then in terms of the brighter student doing more and at a quicker rate. There is a school of thought that thinks that perhaps rather than more and quicker, they should be doing things that are in other directions -- divergents as opposed to pre-calculus, calculus then advanced calculus. Any indication from the group you worked with that you'd like to investigate that aspect of it for the bright student?

K.B.: Well, our philosophy at Duke has been to allow a student to go as far in depth and as fast as he or she wants to in the subject of interest. For instance, someone really interesed in mathematics to be allowed the opportunity to go very far in that area; at the same time we encourage them to develop their other areas, not to neglect them. I think that both things are possible. I don't think it's an either/or case. I think it's possible to gain that diversity, and I believe that diversity is stimulated; that there's sort of an umbrella effect here; that the mathematics start to overtake these other areas and stimulate advancement in those areas also.

R.S.: True enough. Perhaps I didn't phrase my question well. I was thinking, though, of a math student working with you and in this direction towards the calculus courses, all of a sudden gets a real interest in the game and in the game theory. Do you let him go in that direction as opposed to trying to keep him channeled on this pre-planned course?

K.B.: My response to that would be that in terms of what we do in our classroom, there is a specific focus on moving through the pre-calculus. We encourage them in the evenings and study halls and things like that to investigate these other topics. Our philosophy is, and my feeling is, that it is important to keep moving through the traditional curriculum because it opens more doors and more interesting topics are available. In other words, once you know calculus, gaming theory becomes much more understandable, and you can see exactly why these things work once you understand the probability and statistics. So I think there is a certain amount of acceleration that has to take place if one wants to be able to enrich the curriculum at a significant level. Otherwise, I feel that things like you're suggesting sometimes regress into the pseudo-intellectual games, and are not really hard-core academic material.

R.S.: Fair enough. I accept what you're saying and I liked your analogy that my making sure that there are no gaps, you are laying a basis that opens more doors and that certainly has a real advantage, not only for the student himself -- but, in the long run, for the country, which has advanced so far because of basic knowledge and research in mathematics.

Thanks very much, Kevin & Dip, we've enjoyed talking with you both.

Mathematics - Science Department
Faculty of Education
The University of Western Ontario
London, Ontario

THE CHARACTER OF NECESSITY ASCRIBED TO THE TRUTH OF MATHEMATICS AND EVEN THE PECULIAR CERTAINTY ATTRIBUTED TO THEM IS AN ILLUSION.
John Stuart Will

AN INFORMAL NEWSLETTER

by
D.W. Alexander

MAinfo Two (Mathematics Information Newsletter #2) 82 11 1

The second meeting of the Project Committee took place on October 26, 27, 1982.

A major portion of the meeting was devoted to revising the Grade 7 Guideline. Most of the changes were directed toward making clearer the intent of the guideline as published in 1980. Some changes are suggested of a more major kind to which the committee invites reaction:

1) Numerical Methods Section 6 Integers was deleted since it is core in Grade 8.

2) Numerical Methods Section 7 Other Arithmetics was deleted since other optional topics and core topics appear to deserve greater attention.

3) Algebra Section 3 Relations was deleted since it is also in Grade 8 Guideline.

4) Numerical Methods Section 5 Factors was deleted with the intent of transferring to Grade 8 when the topic is more appropriately applied to fractions.

5.) Numerical Methods Section 3 (c), (d), (e) Ratios and rates and related references in Geometry Section 5 were deleted since

(i) many Grade 7 students have difficulty with the concepts and tend to learn the techniques by rote.

(ii) majority of Grade 8 teachers tend to teach the topic as a new topic.

(iii) deletion of a core topic should allow more time to be spent on problem solving, measurement, and geometry.

6) Geometry Section 3 Patterns with Tiles is changed from optional to core since it provides for experiences in informal geometry which should be the essence of the Grade 7 experience.

7) A new **optional** section is suggested: Mathematics and the Computer.

a) Using the computer in simple arithmetic applications.
b) Investigating the rules a specific computer uses to simplify numerical expressions.
c) Writing simple programs.
d) Discussing uses of the computer and its impact on individuals and society.

8) Numerical Methods and Algebra; Measurement and Geometry are identified as two major strands rather than leaving Algebra as a separate strand and linking Measurement with Numerical Methods.

Anyone who wishes to see the details of the DRAFT revision and does not receive it with this copy of MAinfo may obtain the DRAFT by contacting the source of their MAinfo copy (coordinator or education officer, etc.).

Other major issues on which the committee invites expressions of opinion are: Should there be a separate general level course focussed on the Mathematics for Business or would such a course be the only choice for general level students not taking the proposed Mathematics for Technology?

Whither geometry? What is the role of deductive geometry in the Senior Division? How much time should be devoted to it? At what grade levels (11, 12, 13)?

Before revisions of the Grade 11, 12 Advanced and the Grade 13 courses are undertaken some consensus must be clear as to the emphasis to be placed on deductive geometry.

Please send your opinions to: Dr. D.W. Alexander, Education Officer, Ministry of Education. Senior and Continuing Education, 17th Floor - Mowat Block, Queen's Park, Toronto, Ontario M7A 1L2

AN INFORMAL NEWSLETTER

by

D. W. Alexander

MAinfo Three (Mathematics Information Newsletter #3) 82 12 28

1. The O.T.F. representatives on the Advisory Committee are: Joyce Cornell, Brandon Gate P.S., Peel; Robert Lalibertē, Ecole Secondaire de la Salle, Ottawa; George Sachs, W. L. Mackenzie D.I., North York; Bruce Steffler, Marymount College, Sudbury; David Thorsell, Carleton Board of Education.

This group met with the Project Committee on November 24, 1982 to discuss general directions the revisions are taking and the specific revisions indicated in the Grade 7 DRAFT circulated with MAinfo #2. They will meet again on January 25, 1983 to consider reactions received regarding the Grade 7 DRAFT and to give their reactions to the Grade 8 DRAFT.

2. The Project Committee met November 24, 25, 1982 and again on December 7, 8, 1983.

The major portions of both meetings was devoted to revising the Grade 8 Guideline. The only major changes suggested were:

(a) Addition of some work with concept of Factors (transferred from Grade 7).
(b) Addition of **introduction** to concept of Ratio (transferred from Grade 7).
(c) Addition of an **optional** section

Mathematics and the Computer

(a) Using prepared programs related to numerical methods, algebra, measurement, or geometry and their applications; simulations.
(b) Writing simple programs.
(c) Discussing uses and misuses of computers and their impact on individuals and society.

The Committee invites your comments on these changes and other details of the revisions in the Grade 8 DRAFT by March 1, if possible. (The Grade 8 DRAFT, if not attached, may be obtained from your source for MAinfo coordinator, education officer).

3. The Committee is particularly anxious to receive input from you on:
(a) The Mathematics and the Computer sections (Grade 7 DRAFT and Grade 8 DRAFT).
(b) The question, - "What is a reasonable distinction (if any) to be made between the concepts of **'ratio'** and **'rate'** for Grade 8 students?"

4. The Grade 7 DRAFT and Grade 8 DRAFT should be read with the understanding that the intent is to retain the focus of the Intermediate Mathematics Guideline 1980 on Experiential Approaches, Mathematical Models, Applications, and Problem Solving. The expectation is that, where appropriate, these will be used as **procedures** in developing the items in the **content** list.

5. (a) The current planning is for the following courses to be offered at the general level:
Grade 9 Mathematics
Grade 10 Mathematics
Grade 11 Mathematics for Technology
Grade 12 Mathematics for Technology
Grade 11 Mathematics for Business
Grade 12 Mathematics for Business

(b) The Committee is considering that the advanced courses will include Grade 9, Grade 10, Grade 11, Grade 12 and three Ontario Academic Courses which would probably be Calculus, Algebra, and a course on applications related to Business and Commerce and the Social Sciences (replacing Relations and Functions).

6. The Committee is inclining toward the view that work in deductive geometry should be limited to essentially "numerical" problems and that the "formal deductive proof" aspects should be removed from the courses.

AS INDICATED IN MAinfo #1 THE INFORMATION CONVEYED IN THIS NEWSLETTER IS NOT POLICY BUT SIMPLY AN ATTEMPT TO SHARE THE CURRENT THINKING OF THE COMMITTEE. WE INVITE AND NEED YOUR REACTION IN ORDER TO MAKE FINAL DECISIONS.

Send your opinions to:

Dr. D. W. Alexander
Senior and Continuing Education
Ministry of Education
17th Floor, Mowat Block
Queen's Park
Toronto, Ontario
M7A 1L2

Members of the Project Committee (see MAinfo #1) and the Advisory Committee would appreciate the opportunity to react with groups of teachers concerning the revisions. Contact members directly or Dave Alexander.

Grade 8 Mathematics

Numerical Methods and Algebra

Section 1. Whole Numbers and Decimals

(a) Identifying whole number place values to millions.
(b) Identifying decimal place values to thousandths.
(c) Expressing a number in expanded form employing 10^6, ---10^2, 10, 1, 1/10, 1/10^2, 1/10^3.
(d) Consolidating and applying addition, subtraction, multiplication, and division of whole numbers and decimals with and without calculators.
(e) Multiplying and dividing a number mentally by 10, 100, 1000.
(f) Evaiuating numerical expressions using rules for the order of operations.
(g) Applying estimation, rounding, reasonableness of results to calculations and to problem-solving.
Optional (h) Identifying rounding errors resulting from successive calculations, in particular with a calculator.

Section 2. Factors, Powers, Square Root

(a) Divisibility tests for 2, 3, 5, 9, 10.
(b) Concept of a factor, a prime number and a composite number.
(c) Determining factors of a composite number up to 100, without a calculator.
(d) Writing repeated factors as powers; writing powers as repeated factors.
(e) Writing large numbers in scientific notation.
(f) Concept of square, square root (See MG 3(e)).
(g) Identifying the squares of whole numbers 1 to 10; identifying the square roots of perfect squares 1 to 100.
(h) Finding an approximation of the sqaure root of whole numbers using a calculator.

Section 3. Fractions and Decimals

(a) Addition, subtraction, multiplication, division involving fractions and mixed numbers with simple denominators (up to three operands per question); applications.
(b) Converting fractions to decimals with and without a calculator.
(c) Demonstrating that the decimal form of a fraction either terminates or repeats.
(d) Converting terminating decimals to fractions.

Optional (d) Investigating the patterns of repeating decimals with a calculator.

Section 4. Ratio and Rate

(a) Concept of the comparison of two or more quantities as a ratio.
(b) Expressing two-termed ratios in fractional form; distinguishing between fractions and two-termed ratios in fractional form.
(c) Concept of equivalent ratios; simple applications of equivalent ratios.
(d) Concept of rate as comparison of two quantities in different units; simple applications of rates in problem-solving.

Section 5. Percent

(a) Expressing percents (including percents greater than 100% or less than 1%) as decimals, as fractions, or as ratios.
(b) Applying percents to practical problems such as sales tax, simple interest, commission and discount.

Section 6. Integers

(a) Concept of integers from practical examples.
(b) Locating integers on the number line; ordering integers.
(c) Locating ordered pairs of integers in the plane.
(d) Addition and subtraction of one and two digit integers without a calculator.

Optional (e) Multiplication and division of one and two digit integers without a calculator.

Section 7. Organizing and Representing Data

(a) Collecting data; organizing data.
(b) Representing data by bar and line graphs.
(c) Reading and interpreting bar and line graphs.
(d) Reading and interpreting pictographs in which one element represents more than one object.
(e) Representing data by circle graphs.
(f) Reading and interpreting circle graphs.

Section 8. Variables, Formulae and Equations

(a) Concept of placeholder (variable).
(b) Translating word phrases and statements to algebraic expressions or equations.
(c) Translating algebraic expressions and equations related to applications to word phrases or statements.
(d) Evaluating algebraic expressions by substituting whole numbers and decimals with and without calculators.

Optional (e) Evaluating algebraic expressions by substituting integers.
(f) Developing simple formulae by generalizing from numerical cases.
Optional (g) Representing sequential activities by flow charts; representing algorithms for calculators by flow charts.
(h) Solving linear equations in one variable by methods such as inspection, systematic trial, replacement, or inverse flow charts.
(i) Substituting in formulae and solving the resulting linear equations with or without a calculator.

Section 9. Relations

Optional (a) Representing non-numerical and numerical relationships by arrow diagrams, by mappings, and by ordered pairs.
Optional (b) Representing simple relations using ordered pairs; graphing these relations.

Section 10. Mathematics and the Computer

Optional (a) Using prepared programs related to numerical methods, algebra, measurement, or geometry and their applications; simulations.
Optional (b) Writing simple programs.
Optional (c) Discussing uses and misuses of computers and their impact on individuals and society.

Section 11. Probability

Optional (a) Listing the possible outcomes of simple experiments.
Optional (b) Identifying the favourable outcomes among the possible outcomes; determining the associated probability.
Optional (c) Comparing predicted and experimental results.
Optional (d) Using sets and Venn diagrams to represent and analyse different outcomes of an experiment.

Measurement and Geometry

Section 1. Measurement

(a) Consolidating conversions among commonly used units within the metric system.
(b) Calculating in numerical cases the perimeter of figures bounded by line segments.
(c) Calculating in numerical cases the area of figures which can be decomposed into rectangles.
(d) Calculating the perimeter and area of rectangles, triangles, and parallelograms using appropriate formulae; applications.
(e) Investigating circumference/diameter for circular objects to determine an approximation of n.
(f) Investigating area/(radius)^2 for circular objects.
(g) Formulae for circumference and area of a circle; applications.
(h) Calculating the surface area of rectangular and triangular prisms by relating to faces or nets.
(i) Formula for volume of a rectangular prism; applications.
(j) Estimating and calculating the volume of non-rectangular prisms; generalization of volume of a prism as the product of height and area of the base; applications.
Optional (k) Determining the surface area and volume of a cylinder.

Section 2. Constructions

(a) Consolidation of constructions of congruent segments and angles, perpendicular lines, parallel lines, perpendicular bisector, angle bisector using geometric properties and a variety of techniques (ruler, protractor, compasses, paper folding, tracing paper, transparent mirror).
(b) Testing for perpendicular lines (paper folding, set square, protractor, grid paper, transparent mirror).
(c) Testing for parallel lines (parallel ruler, grid paper, ruler and set square, paper folding, ruler, transparent mirror, alternate or corresponding angles, protractor).
(d) Constructing 90°, 45°, 60°, 30° angles without a protractor.

(e) Constructing triangles and quadrilaterals from given information.
(f) Constructing a triangle congruent to a given triangle; investigating the conditions for congruent triangles.
(g) Constucting a circle given its centre and radius, centre and diameter, or centre and point of the circle.
(h) Locating the centre of a circular arc by paper folding or transparent mirror.

Section 3. Properties of Plane Figures

(a) Identifying the equal and supplementary angle properties of intersecting lines and parallel lines.
(b) Identifying the equal angle properties of isosceles and equilateral triangles.
(c) Identifying the sum of the angles property of a triangle.
(d) Applying the above properties in numerical cases.
(e) Investigating the Pythagorean Theorem; numerical applications.
Optional (f) Investigating the rigidity or non-rigidity of polygonal figures.
Optional (g) Identifying symmetries (line-symmetry, rotational symmetry) and angle properties of regular polygons.

Section 4. Three-dimensional Geometry

(a) Constructing skeletons of pyramids and prisms; classifying pyramids and prisms by their bases.
(b) Recognizing 3-D objects from sketches (perspective; front, side, top views).
(c) Sketching simple 3-D objects such as cuboid, pyramid, cylinder, cone).
Optional (d) Identifying the symmetries (line, plane, rotational, point) of 3-D objects.
Optional (e) Constructing regular polyhedra from their nets; stacking solids in space.
Optional (f) Investigating the rigidity or non-rigidity of polyhedral skeletons.

Section 5. Transformations

(a) Identifying the image of a figure under translation, reflection, and rotation using the basic property of each transformation; drawing such images.
(b) Identifying (using tracing paper, transparent mirrors) the transformation relating two congruent figures; drawing the translation arrow, reflection line, or rotation centre and angle.
(c) Drawing dilatation images; making and using scale drawings; concept of scale factor (scale ratio); applications such as maps and floor plans.
(d) Identifying the properties of similar figures (shape, sides, angles, area).
Optional (e) Determining the areas of similar figures in tiling patterns.
Optional (f) Observing and predicting reflections in parallel mirrors, intersecting mirrors; applications to art and special effects in films.

AND THEREFORE, WHETHER THE MATHEMATICIANS OF THE PRESENT AGE ACT LIKE MEN OF SCIENCE, IN TAKING SO MUCH MORE PAINS TO APPLY THEIR PRINCIPLES THAN TO UNDERSTAND THEM.

Bishop Berkeley

COUNCIL COMMENT

by

Bob Robinson

EDITOR'S NOTE:

Council Comment appears in an issue of the Gazette following OAME/AOEM Council Meetings. The Editorial Panel hopes that you find this section of interest. Your comments are welcomed and should be addressed to:

The Editor, Ontario Mathematics Gazette, c/o Department of Mathematics, McMaster University, Hamilton, Ontario L8S 4K1

The February Council Meeting was held on Saturday, 1983 02 12 at the Ramada Inn (Airport West). The names of council members are found on pages 41-43 in September, 1982 issue of the Gazette.

CHAPTER REPORTS

COMA

The main activity of COMA is the Ministry Interface Project. A one day seminar is planned for 1983 06 13. These other projects are well under way: a Lecture Series in Mathematics; the Differences Project; S.E.R.P. and Mathematics. Linkage II is being formulated as well.

GOLDEN SECTION

This chapter is presenting their third annual area mathematics conference at McMaster University on Friday evening 1983 03 11 and Saturday morning 1983 03 12.

GVMA

A successful general fall meeting was conducted and they have produced two new publications dealing with applied calculus problems and physics. This active chapter has been busily preparing for the O.A.M.E. Annual Conference to be held in Waterloo, 1983 13-15. This is always an exciting conference especially when it is hosted by GVMA and held at the University of Waterloo. There will be something for everyone at all levels of mathematics education. You should have seen a registration form and skeletal program by now. If you haven't, contact Ken Fryer or Ron Scoins at the University of Waterloo or the O.A.M.E. Office in Sarnia.

NOMA

Plans are well under way for holding their fall conference in Sudbury. They are hoping it will be as successful as last year.

PINE RIDGE

A mini conference is planned for 1983 04 05 at Vanier Collegiate in Oshawa. A full day session is planned to be held towards the end of June. Pine Ridge is continuing to gain new members.

QSLMA

The next general meeting will be held Wednesday, 1983 04 06 at the Royal Military College from 4:30 p.m. to 8:00 p.m.

SAME

The mini conference held on 1982 10 28 was relatively successful and the support of the East York and Durham teachers was appreciated. The Scarborough mathematics professional development program organized by SAME was very successful with 170 teachers attending the 22 different workshops scheduled. Another activity is to be planned in the late spring.

TORNY

A most interesting tour of the University of Toronto's Institute For Aerospace Studies was conducted on Thursday evening 1983 01 27. TORNY's next miniconference is planned for Saturday, 1983 02 26 from 08:30 to 12:30. The theme of this conference is "Positive Matheamtics In The Classroom".

GAZETTE EDITOR

The new editors of the Gazette are Ralph Connelly and Sam Pravica. The President and Council expressed their appreciation for the fine work of Hans Heinig and his Editorial Panel.

ABACUS EDITOR

Applications for the position of editor for the Abacus are requested in the current issue of the Abacus.

OAIP JUNIOR

Operations on whole numbers, decimals, and fractions will go out for screening trials later this spring or in the early fall.

OAIP INTERMEDIATE

This project is on hold until they see what curriculum recommendations come from the Intermediate/Senior Mathematics Curriculum Committee.

LEADERSHIP RESEARCH SEMINAR COMMITTEE

This committee was formed to investigate the direction of future Leadership Seminars. Interested persons should present their views to the chairman - J. Symington.

LEADERSHIP SEMINAR - 1982

George Knill reported that once again the Leadership was very successful.

LEADERSHIP SEMINAR - 1983

The next Leadership is being planned for 1983 10 19-22. More details will be available later.

NCTM

The Annual Meeting is to be held in Detroit from 1983 04 13-16. The O.A.M.E. delegate to this convention is Ralph Connelly and the alternate is Sharon McPhail.

MATHEMATICS TEACHER SHORTAGE

A motion was passed which asked the Executive of O.A.M.E. to examine the feasibility of conducting a study to gather information pertaining to the availability of qualified mathematics teachers at the Secondary levels in Ontario and throughout Canada. Furthermore, if such a study is undertaken that these findings be made available to O.A.M.E. Council for discussion and possible distribution later to other interested parties.

ANNUAL MEETING - 1983

Registration forms have already been sent out. The program looks exciting and all the members of GVMA have worked so hard to make this another outstanding O.A.M.E. Conference on the campus of the University of Waterloo.

Ed. Note: This writer regrets the inconvenience caused the organizers of the Tenth Annual O.A.M.E. Conference in that he inadvertently omitted this event from the Calendar of Events feature in the December issue of the Gazette.

OECA

Paul Pogue showed one of the 8 video cassettes available from OECA. These excellent tapes are entitled the Landscape of Geometry Series and are incorrectly placed in the Senior Division of the OECA Catalogue. They are designed for the Junior and Intermediate Division.

ANNUAL MEETING - 1984

The site of the 1984 Annual Meeting will be in Kingston and sponsored by the QSLMA. The chairperson is Elaine Harvey who will be assisted by Brian Gallagher (Program), Norman Rice (Local Arrangements), and David Handley (Publicity).

NOMINATIONS COMMITTEE

The council is elected by votes by mail. Every member is urged to return their completed ballots to the Sarnia office as soon as possible.

PUBLICATIONS COMMITTEE

The O.A.M.E. will participate on a trial basis in the publication of the Canadian Mathematics Teacher for the membership year of 1983-84. An order form for the CMT will appear in the September issue of the Gazette.

INTERMEDIATE/SENIOR MATHEMATICS CURRICULUM

A copy of this report is included with this issue of the Gazette. Tom Griffiths also distributed copies of a skeletal outline for Grade 13 Mathematics Courses. Copies may be obtained by writing Tom Griffiths. Members are encouraged to read MAINFO.

MEMBERSHIP COMMITTEE

There has been an increase in membership. Keep up the good work and let's see if we can obtain more members at our Annual Conference in Waterloo.

PUBLIC RELATIONS

J. Symington reports that posters will be drafted and that ideas should be sent to him.

COMPUTER COMMITTEE

O.A.M.E. is going to approach ECOO to form with ECOO an Interface relationship. The purpose of this interface would be to exchange information that relates mathematics to computers and computers to mathematics in education.

BILL 82 COMMITTEE

Elaine Harvey and her committee are busy preparing the "Position Paper on Bill '82".

PROBLEM SOLVING COMMITTEE

There are 6 members of this steering committee with Chester Carlow as Chairman. They have presented 11 general topics on problem solving for further study. It was recommended that teachers throughout Ontario who are presently involved in a successful problem solving program should develop a description of it and sent it to Bill Bisset, who will collate the information and make a report. Bill's address is: 41 Blythwood Road, Toronto, Ontario M4N 1A2.

LOGO

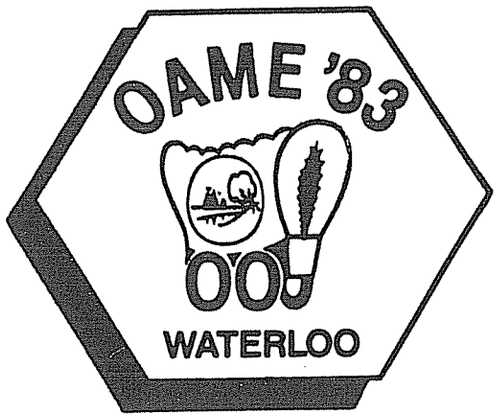
A new O.A.M.E. logo will soon be seen. The designer of the new logo is George Cumyn from Ottawa. Watch for it!

CLOSING REMARKS

Paul Pogue, speaking on behalf of council, thanked Sharon McPhail for her leadership during the 82/83 Council year. Members can thank Sharon personally at Waterloo in May.

CALENDAR OF EVENTS

The following conferences are sponsored by the O.A.M.E.



Tenth Annual Conference
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION
at University of Waterloo
May 13, 14, 15, 1983

Contact: • Ken Fryer or Ron Scoins at University of Waterloo.
• Secretary O.A.M.E./A.O.E.M. - Sarnia Office

COMA:
Date: 1983 06 13
Contact: Ann Jones

GOLDEN SECTION:
Dates: 1983 03 11-12
Place: McMaster University
Contact: Paul Lessard

PINE RIDGE:
Date: 1983 04 05
Place: Vanier Collegiate
Contact: Kumar Pasricha

QSLMA:
Date: 1983 04 06 - 20:00
Place: Royal Military College
Contact: Elaine Harvey

12TH ANNUAL MATHEMATICS LEADERSHIP SEMINAR:

Dates: 1983 10 19-22
Place: Ramada Inn (Airport West), Toronto
Contact: Morley MacGregor, Sarnia Office

OTHER CONFERENCES:

N.C.T.M.: 61st Annual Meeting
Date: 1983 04 13-16
Place: Detroit, Michigan
Contact: Ralph Connelly, Brock University
Bob Robinson, Hamilton Board

N.C.S.M.: Annual Meeting
Date: 1983 04 12-13
Place: Detroit, Michigan
Contact: Nancy Biggs
2597 Avery Avenue
Room 254
Memphis, TN
38112

R.C.D.P.M.: 10th Annual Conference on Diagnostic and Prescriptive Mathematics
Date: 1983 04 9-11
Place: Bowling Green State University
Bowling Green, Ohio
Contact: William R. Speer
EDCI, Bowling Green State University
Bowling Green, OH
43404

International Congress of Mathematicians
Date: 1983 08 16-24
Place: Warsaw, Poland
Contact: Congress Bureau ORBIS
P.O. Box 146
00-950 Warsaw, Poland
Deadline for Registration May 31, 1983

ANNOUNCEMENT

A SUMMER SEMINAR ON THE HISTORY OF MATHEMATICS FOR TEACHERS OF MATHEMATICS WILL BE CONDUCTED AT THE UNIVERSITY OF TORONTO, JULY 25TH TO AUGUST 2ND, 1983. THE SEMINAR WILL BE SPONSORED BY THE INTERNATIONAL COMMISSION ON THE HISTORY OF MATHEMATICS AND THE INSTITUTE FOR THE HISTORY AND PHILOSOPHY OF SCIENCE AND TECHNOLOGY OF THE UNIVERSITY OF TORONTO. THROUGH PLENARY SESSIONS AND SMALLER WORKING SEMINAR SESSIONS, PARTICIPANTS WILL EXPLORE WAYS IN WHICH THE HISTORY OF MATHEMATICS CAN ENRICH BOTH THE UNDERGRADUATE AND UPPER-LEVEL SECONDARY MATHEMATICS CURRICULUM. PRESENTATIONS BY LEADING SCHOLARS ON TOPICS RANGING FROM GREEK TO TWENTIETH CENTURY MATHEMATICS. FOR MORE INFORMATION WRITE HISTORY OF MATHEMATICS SUMMER SEMINAR, INSTITUTE FOR THE HISTORY AND PHILOSOPHY OF SCIENCE AND TECHNOLOGY, UNIVERSITY OF TORONTO, TORONTO, CANADA M5S 1K7.
PHONE: (416) 978-7391

ALGEBRA IS THE INTELLECTUAL INSTRUMENT FOR RENDERING CLEAR THE QUALITIVE ASPECT OF THE WORLD.

Alfred North Whitehead