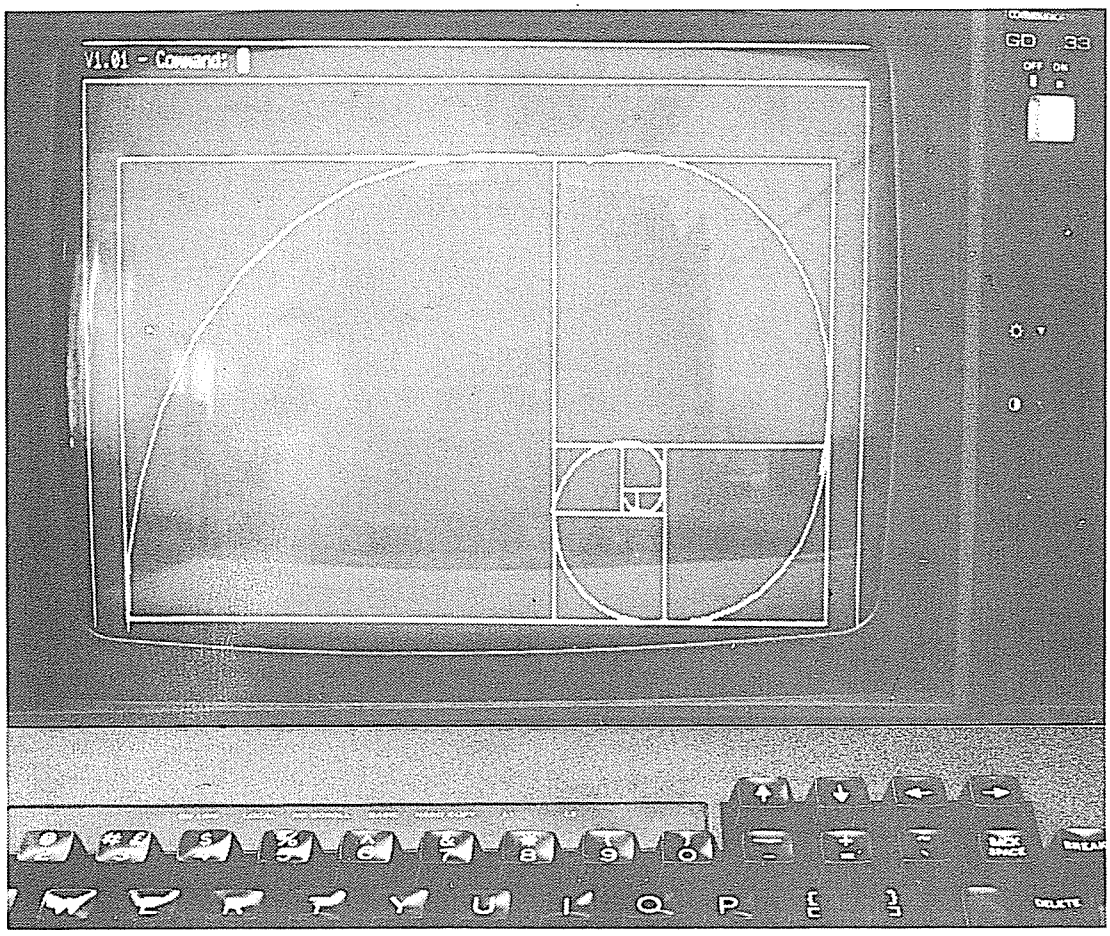
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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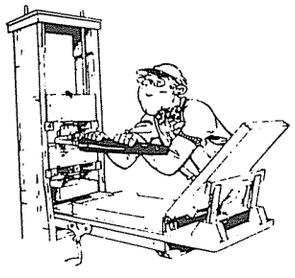
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Well, another deadline is upon us already. We're beginning to appreciate how appropriate the term **dead**line really is!

Our plea in the September issue with regard to the lack of articles aimed at the Primary/Junior levels bear repeating in this issue. Our faithful Primary/Junior followers will, we're sure, feel again that this issue is heavily slanted towards the Intermediate/Senior levels. We **need** articles at the K-6 level - please help us get them.

In past years, the **OMG** has tried to include an article in French. We're delighted that André Ladouceur has contributed an excellent article regarding the teaching of fractions for this issue.

This month's issue contains a wealth of information for those interested in what OAME Council has been doing. In addition to the regular "Council Comments" feature by Bob Robinson, this issue contains the Position paper on Bill 82 evolving from the work of Council's Bill 82 Committee, and a response from the Ministry's Computers in Education Committee to questions raised by Council's Computer Committee. Both should be of interest to readers and, again, comments are welcome. In fact they're necessary if Council is to continue to be OAME's voice on these important issues.

In our last issue, an "Open Letter to the Minister" section was included. We thank Dave Alexander, Chairman of the Int./Senior Division Mathematics Revision Committee for responding in this issue. This month's "letter" comes from Doug Henderson regarding his thoughts on **OAIP**. We welcome responses. It is nice to see this section becoming the forum we hoped it would be and hope it continues in this manner.

The Problem-Solving section this issue contains some interesting problems from antiquity (as well as the answers to last month's "Alice" problems). We hope others will see fit to submit their "favorite" problems. Question - would readers rather have the answers in the same issue, or would you rather have until the next issue to "mull" them over? Let us know.

Remember, we need your input to guide us. Solicit (and/or **submit**) articles; write a letter to us if you have ideas, suggestions, questions, or opinions. The **OMG** is **your** journal.

Ralph Connelly
Sam Pravica
Editors, **OMG**

THE UNIVERSITY OF GUELPH - WELLINGTON COUNTY SUPERMATH CLUB

by

Gary Flewelling and Jack Weiner

Most university mathematics departments generate a list of talks which professors are willing to give to high school audiences.

We tried something more ambitious. We wanted an ongoing program of enrichment for the top math students in the area high schools. This program would

- let professors, teachers, and consultants present "neat stuff" to "sharp kids"
- allow math students from different high schools to exchange ideas
- let students, teachers and profs rub elbows and (we hoped) gray matter
- expose math students to topics they wouldn't ordinarily see and
- show dramatically that their high school mathematics actually (surprise, surprise!) leads somewhere.

As a result the University of Guelph - Wellington County

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**************************
*  SUPERMATH CLUB    *
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was formed.

Five times a year, the students meet for an afternoon at the University of Guelph. To prepare, they must complete assignments designed to facilitate their understanding of the day's topics. This material is

1) prepared by the session leader
2) completed by the students
3) checked by the high school "contact" teachers
and 4) becomes the price of admission to the session.

The preliminary work is a fundamental part of any meeting. It both screens and prepares participants.

The session itself centres on an interesting topic from an area of mathematics outside the regular curricula. This year, we had talks on "Powerful Formulae" (number theory), "Instant Insanity" (graph theory), "REDDing" (creative thinking), "River Blindness in West Africa" (statistics), and "Fractal Geometry" (Fractal Geometry!).[1]

A common problem with presentations is the tendency for speakers to lecture. This is inappropriate for our club! Here, students are actively drawn into the discussion with the session leader. Furthermore, the session incorporates intervals for "hands-on" work. Students are NOT passive observers!

As with preliminary work, follow-through is fundamental to each session. The students take home a carefully prepared problem set which, ideally, will consolidate and extend their understanding of the day's topic. In some cases, research problems are posed, usually with references to articles in various math journals. Students are encouraged to work on the these either individually, or in groups. At later meetings, time is reserved for students to present their discoveries.

We feel that while each afternoon session has been valuable, it represents only one-third of the Supermath Club experience. The "pre" and "post" activities ensure that each meeting is much more than a half-day out of school.

The contact teacher in the high school plays a vital role with the "pre-post" activities. Student enthusiasm, preparation, and follow-through is largely determined by this individual. Without good people in the schools, the club simply would not work.

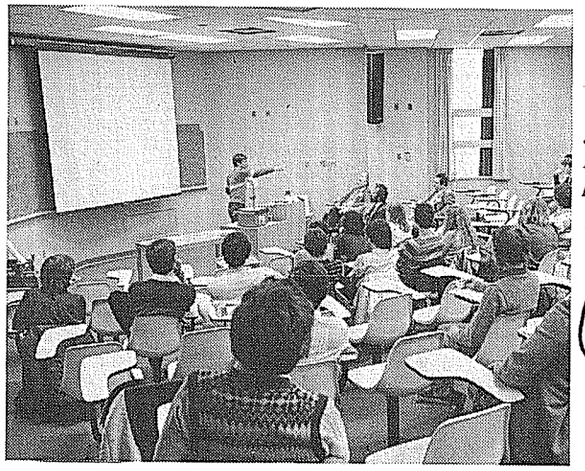
This year was experimental. The evaluations from students, teachers and faculty are in and are very positive. We predict the club will become a tradition.

We invite the participation of boards neighboring the Wellington County area. If interested, please contact either author. Alternatively, set up a similar program in your own area. We will be glad to assist you in any way we can.

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1 "*For interesting reading on this exciting new area of mathematics see Discover (June, 1982) or Omni (February, 1983). For a more complete presentation, read "The Fractal Nature of Geometry", by Benoit B. Mandebrot, published by W.H. Freeman and Company, c. 1982."



"At the first Supermath Club meeting, Jack Weiner discussed "Powerful Formulae". He issued this challenge problem:

"Justify geometrically the identity

$$\left(\sum_{i=1}^{n} i\right)^2 = \sum_{i=1}^{n} i^3 \quad "$$



At a subsequent meeting, Ida Seto, a student at Guelph Collegiate and Vocational Institute, presented an excellent solution to this problem.

INTENTIONS AND ATTAINMENTS IN THE TEACHING AND LEARNING OF ONTARIO GRADE 8 MATHEMATICS:

RESULTS FROM THE SECOND INTERNATIONAL MATHEMATICS STUDY

by
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Editor's Note:

We realize this is quite a long article, but found it difficult to find an effective way of condensing it. We feel the information presented is sufficiently important to readers of the ***Gazette*** *to warrant the inclusion of the full report.*

During the school year 1981-1982, as part of Ontario's participation in the Second International Study of Mathematics (SIMS) several thousand Grade 8, 12 and 13 students, along with their teachers and school principals contributed information concerned with the teaching and learning of mathematics in the Province of Ontario. Similar data were collected in over twenty nations.

Australia	France	Netherlands
Belgium (Flemish)	Hong Kong	New Zealand
Belgium (French)	Hungary	Nigeria
Canada (British Columbia)	Irish Republic	Scotland
Canada (Ontario)	Israel	Swaziland
Chile	Ivory Coast	Sweden
England	Japan	Thailand
Finland	Luxembourg	United States

The Ontario study was funded by the Ontario Ministry of Education and conducted by the Educational Evaluation Centre (EEC) of the Ontario Institute for Studies in Education. SIMS was a project of the Inernational Association for the Evaluation of Eductional Achievement (IEA), an association of educational research organizations and ministries of education whose primary goals are to conduct educational research on an international level and to assist member states in undertaking co-operative research projects. IEA has conducted international surveys in the past, among them the first mathematics study in 1964 involving twelve countries. Canada was not among these participants.

Present Report

The present report summarizes the Ontario grade 8 achievement findings analyzing the correspondence between the Intended, Implemented and Achieved curriculum in the Province. General overviews of the Ontario grade 8 and grade 12/13 findings are available elsewhere (1,2) as is an examination of the teaching and learning of geometry in Ontario grade 8 classrooms (3). A subsequent report will summarize the Ontario grade 12/13 achievement results.

Design of the Study

The design of SIMS has three components, diagrammed in Figure 1. A curriculum analysis examined the context in which teaching and learning was taking place within each participating nation. Prime focus was upon the mathematics content expected to be conveyed to the students enrolled in mathematics classes, the "Intended Curriculum".

Second, teachers supplied many details of their concepts of mathematics, their priorities and their teaching methods--the "Implemented Curriculum". The unit of study was one of the teacher's classrooms within the school, the "target class".

Finally, students answered questions designed to measure their mastery of the mathematics content and their attitudes towards mathematics and the learning of mathematics--the "Attained Curriculum".

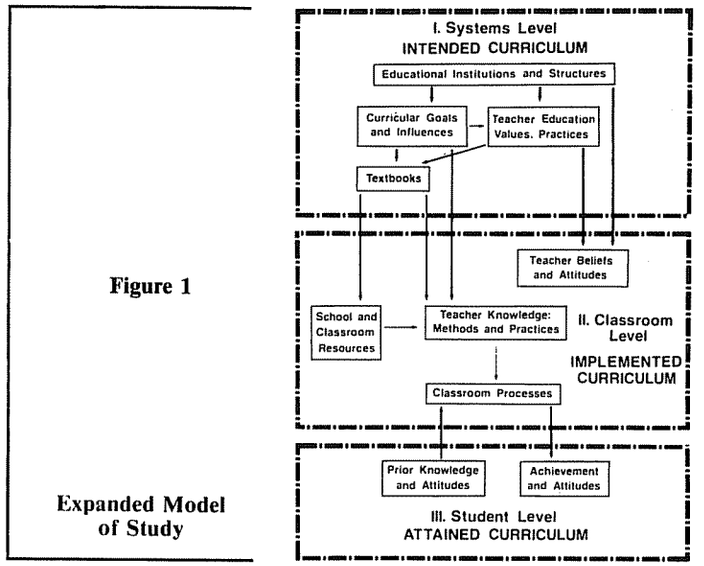


Figure 1

Expanded Model of Study

Two age populations were studied in SIMS. Population A was the grade where the majority of students had reached the age of 13. In Ontario, British Columbia and the United States this corresponded to Grade 8. Population B was those students in the normally accepted terminal year of secondary school who were studying mathematics as a substantial part (approximately five hours a week or more) of their academic program. In Ontario the group best fitting this definition were those Grade 13 students enrolled in two or more Grade 13 mathematics courses, while in British Columbia and the USA, the Population B students were in Grade 12. In Ontario, data was collected from grade 12 classrooms to allow comparisons to grade 12 students in other countries.

Participating nations had the option of conducting either one-time measurement (cross-sectional) or beginning and year-end measurement (longitudinal) versions of the Population A and B studies. Ontario participated in the longitudinal study at Population A and the cross-sectional version at Population B collecting data at the beginning and end of grade 8 and at the end only of the grade 12/13 courses.

SIMS in Ontario

An Ontario Steering Committee, consisting of mathematics educators at all levels: Ed Barbeau, University of Toronto; Jerry Morgan, Dufferin-Peel Roman Catholic Separate School Board; Lorna Morrow, North York Board of Eduction; Maurice Poirier, Carleton Board of Education (seconded to the Ministry of Education for a time during the study); Norman Sharp, Etobicoke Board of Education; Michael Silbert, Hamilton Board of Education; Robert Tuck, Nippising Board of Education and Howard Russell of OISE (Chairman) oversaw the study and advised its staff.

The committee examined the IEA instruments for their relevance to Ontario, reviewed materials prepared as part of Ontario's curriculum analysis and helped in the planning and location of the study and the interpretation of results.

Validating the SIMS Achievement Items for Ontario

There were 180 achievement items developed for the population A study: forty items were in a "core form" administered to all students at both pre and post-test and the other 140 items were placed in four forms rotated through each class at pre and post-test. Each student therefore, completed a core form and one rotated form at each time of testing. Steering Committee Members examined each achievement item to determine whether each was, in their opinion, (a) likely to be covered in a typical Ontario Grade 8 mathematics classroom; and (b) likely to relate to content covered in a widely used Grade 8 textbook.

Overall, 83 per cent of the Grade 8 items were judged to be covered either during or prior to the target school year. Since items had to cover content in over 20 countries, the match with the Ontario curriculum was amazingly good for Grade 8.

Analysis of commonly used texts at the Grade 8 level found one covering 72 per cent of achievement item content with an additional 20 per cent covered by content assumed to be prerequisite to the topics in the textbook. Another text covered 88 per cent with an additional 4 per cent of items assumed to be prerequisite to textbook topics.

The IEA items were therefore suitable for use in the Ontario study.

The Population A Samples

Schools containing Grade 8 classrooms were partitioned into 26 strata on the basis of (a) school size, (b) type of school (private, French ...), (c) rural or urban community and (d) geographical region of the Province. Five schools were selected at random from each stratum for the longitudinal study and one further school from each stratum for testing at the end of the year only. The supplementary sample provided a control group to determine whether the beginning of year testing and numerous teacher questionnaires were themselves strongly affecting the results.

Within each school, up to two Grade 8 classes were chosen for inclusion in the study. Thus, the longitudinal design sample involved 130 schools, 200 teachers and approximately 7,000 students. The care with which the sample was chosen allows the findings of the study to not only describe the sample of schools, teachers, and students studied but can describe practices and performances in the Province as a whole.

The Intended, Implemented and Attained Grade 8 Curriculum

The 180 SIMS population A achievement items were organized by IEA into a number of subtests of relatively homogeneous content. Eight of these subtests were particularly relevant to the Ontario grade 8 curriculum and will be the focus of this report. These subtests are: Whole Numbers, Common Fractions, Percent, Equations and Inequalities, Plane Figures, Informal Transformations, Representation of Data and Measurement.

The Intended Curriculum in Ontario

The Intended Curriculum in Ontario was the Ministry of Education Intermediate Mathematics Guidelines. The content of these guidelines is well known and will not be dwelled upon here. Members of the Ontario Steering Committee (OSC), based upon their knowledge of these guidelines, rated each achievement item as either appropriate, that is, the mathematics required to successfully answer the item should have been taught during grade 7 or 8, or inappropriate, covered either later than grade 8 or not at all. The OSC estimates of appropriateness of items constitutes the Intended Curriculum in this report.

The Implemented Curriculum in Ontario

Teachers of each target class were asked to indicate whether the mathematics required to correctly answer each achievement item had been taught during the school year. If it had not been taught, teachers indicated whether it was due to a) the content having been taught prior to grade 8; b) is taught subsequent to grade 8; c) not in the curriculum at all or d) other reasons. In all 190 teachers contributed this data and their reports constitute the Implemented Curriculum in Ontario.

In this report, the expression "items were covered" should be read as "the mathematics required to correctly answer the item had been covered." Figures reported are the average percentage of items in a subtest covered by the 190 grade 8 teachers.

The Attained Curriculum in Ontario

The Attained Curriculum was the average percentage of students correctly answering the items in a SIMS subtest. For example, the statement "49% correct for the Percent subtest" would indicate that on the average the Percent items were answered correctly by 49% of students. The data presented in this report was contributed by students in the 105 schools who contributed both pre and post-test information.

Whole Numbers

- of ten items in this subtest, the OSC rated 50% as appropriate for grade 7 and 50% appropriate for grade 8. All items therefore should have been covered or assumed as taught prior to grade 8.

- on average, teachers reported that 65% of the items should have been covered with an additional 17% assumed as taught prior. Total coverage was therefore 82%.

- on average, items were correctly answered by 60% of students at pretest and 61% at post-test. Four items showed improvement of three or more points; two items showed decline of four or more points.

Common Fractions

- of 12 items, the OSC rated 75% as appropriate for grade 7 and 25% for grade 8. All items should therefore, have been covered or assumed taught prior.

- on average, 88% of items were reported by teachers as covered during grade 8 with an additional seven percent assumed as taught prior for a total coverage percentage of 96.

- common fractions items were, on average, answered correctly by 51% of students at pretest and 57% at post-test. All 12 items showed improvement over the year with one item showing an increase of 15%.

Percent

- of the eight items the OSC rated 25% at appropriate for grade 7 and 75% for grade 8.

- on average, 91% of the items were covered during grade 8. Two percent were assumed to have been taught prior.

- items were answered correctly, on average, 36% of the time at pretest and 49% at post-test. All eight items showed improvement and seven items improved over 10%.

Plane Figures

- of the 20 items concerned with plane figures, 20% were rated by the OSC as appropriate for grade 7 and 75% for grade 8. Only 5% (one item) was viewed as involving mathematics taught subsequent to grade 8.

- on average, 64% of the items were seen by teachers as covered during grade 8 with only an additional 3% seen as taught prior. A full 21% of items were seen as involving mathematics taught subsequent to grade 8, four percent as not in the curriculum at all; eight percent of items were not covered for other reasons.

- items were on the average answered correctly 39% of the time at pretest and 49% of the time at post-test, an average improvement of 8%. Eighteen items improved over two percentage points of which 10 improved over 10%. One item declined over the year and one remained unchanged.

Informal Transformations

- of the eight items, 12% were rated by the OSC as appropriate for grade 7, 75% for grade 8 and 12% (one item) as involving material taught subsequent to grade 8.

- only 36% of the items, on average, were reported taught during grade 8 with only an additional six percent covered prior to grade 8. A full 36% was seen as taught subsequently, 7% as not in the curriculum at all, and 15% not covered for other reasons. Total coverage, taught or assumed was 42%.

- items were answred correctly, on average, 40% of the time at pretest and 51% at post-test. All items showed improvement and four showed change of 10% or higher.

Equations and Inequalities

- of the nine items, 22% were rated by the OSC as appropriate for grade 7, and 67% for grade 8. Eleven percent (one item) were seen as involving mathematics taught subsequent to grade 8.

- 78% of items were covered during grade 8 and only 3% assumed taught prior. On average, 9% of items were seen as involving mathematics taught subsequent to grade 8.

- pretest levels of 40% improved to 46% at post-test. All items showed improvement. Only one item improved by 10%.

Representation of Data

- of seven items the OSC rated 28% as taught during grade 7 and 57% during grade 8. Fourteen percent (one item) was seen as involving mathematics taught subsequent to grade 8.

- only 46% of the items were reported by teachers as taught during grade 8 and an additional 10% assumed to have been taught prior. Twenty percent of items were seen as requiring mathematics taught subsequent to grade 8, eight percent as not in the curriculum at all, and 16% of items were not covered for other reasons. Coverage therefore was only 56%.

- items, on average, were correctly answered by 55% of students at pretest and 58% at post-test. Four items improved (4-8%), one declined (2%) and two were unchanged.

Measurement

- of seven items, 57% were rated by the OSC as appropriate for grade 7 and the remaining as appropriate for grade 8.

- 78% of items were reported covered during grade 8 and an additional 11% assumed to have been taught prior. Seven percent of items were seen as involving mathematics taught subsequent to grade 8. Coverage, taught or assumed taught prior was 89%.

- pretest scores of 46% improved to 55% at post-test. Six items improved and one declined. Two items improved 15%.

Figures 2 and 3 summarize study findings on the Intended, Implemented and Attained Curriculum in Ontario.

Figure 2

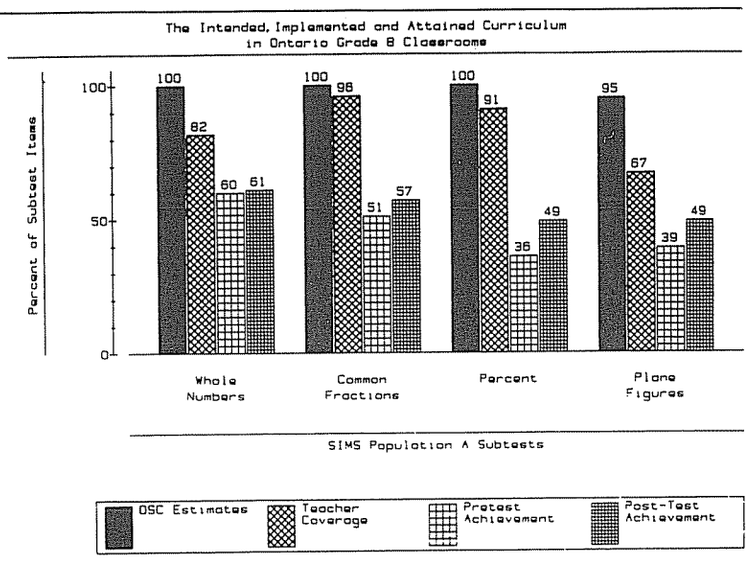
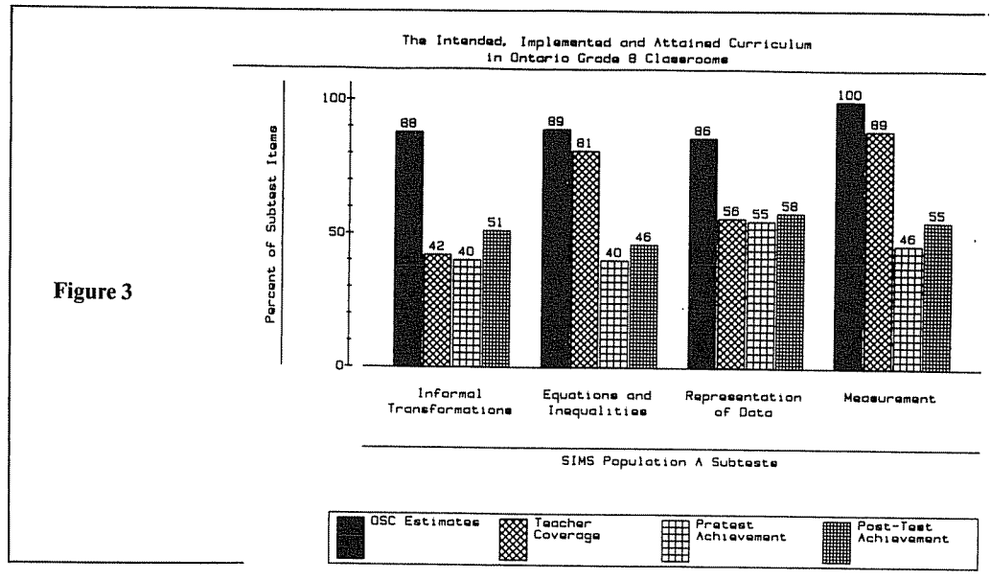


Figure 3



Analysis: The Intended, Implemented and Attained Curriculum in Ontario

As noted earlier, the main purpose of SIMS was the examination of the congruence between what students are supposed to be taught, what they are being taught and what they are learning.

The Intended Curriculum

According to the OSC virtually all of the SIMS achievement items measure mathematics knowledge appropriate for grade 8 students in Ontario. In the cases of Whole Numbers, Common Fractions, Percent and Measurement, the appropriateness of the items is 100%; in other subtests slightly less so: Plane Figures (95%), Informal Transformations (88%), Equations and Inequalities (89%) and Representation of Data (86%).

The Implemented Curriculum

To what extent do teachers report covering the mathematics required to answer these items? Subtest item coverage ranges widely. For some subtests, teacher coverage, taught during grade 8 or assumed taught prior, is relatively high: Whole Numbers (82%), Common Fractions (96%), Percent (93%), Equations and Inequalities (81%) and Measurement (89%). For others it is lower: Plane Figures (67%), Informal Transformations (42%) and Representation of Data (56%). The coverage for these latter subtests is especially interesting when contrasted with the OSC estimates of appropriateness (95%, 88%, 86% respectively). The failure of a significant proportion of teachers to cover selected topics in geometry is further documented elsewhere (3) and should be an area of concern considering the recent increased emphasis of the area in the curriculum. Similarly, the relatively low emphasis on Representation of Data merits attention.

The reasons for these findings could involve the relatively recent introduction of these latter areas into the curriculum at this level, teacher unfamiliarity with the topics or teacher perceptions of the greater importance of the traditional areas of arithmetic, measurement, and algebra. What is clear is that significant portions of the core curriculum are not being covered in many cases.

The Attained Curriculum

Rather significant gains in average achievement (10% or more) occurs for the Percent, Plane Figures and Informal Transformations subtests. Negligible change occurs (<4%) for the Whole Numbers and Representation of Data items. Fractions, Equations and Inequalities and Measurement items show modest (6-9%) gain over the year.

What is of interest is the correspondence between the Implemented and Attained curricula. It might be expected that topics with higher teacher coverage whould show the greatest change over the school year. To examine this question, the eight subtests were assigned ranks on each of the following: pretest and post-test achievement levels, improvement between pre and post-test, OSC estimates of appropriateness for grade 7 and 8, and teacher coverage of the topics during grade 8 as well as assumptions of the items having been taught prior.

The only significant predictors of the subtest rank ordering of growth over the year were pretest achievement (rho = −.90, $p<.01$), that is, subtests with relatively higher achievement performance showed the least growth over the year, and OSC estimates of item appropriateness for grade 8 (rho = .66, $p<.05$) that is, subtests with more items seen as introduced in grade 8 showed greater change over the year. Teachers report of subtest coverage during grade 8 was unrelated to both change over the year (rho = .03, ns) as well as to post-test achievement levels (rho = .13, ns).

These results suggest that subtests which contain novel content for grade 8 are likely to show the greatest growth over the year. Teacher coverage is not related to change and post-test levels since their coverage includes both grade 7 material which already show relatively high achievement levels as well as novel material introduced in grade 8. Therefore, their coverage of topics does not discriminate between those showing varying levels of growth over the year.

It is important to note that these novel areas, such as Plane Figures and Informal Transformations are ones in which coverage is reported as being relatively low. Had all teachers covered these topics it could be expected that achievement would show even greater gains.

In summary, teachers are reporting very high coverage of topics seen as appropriate by the OSC for grade 7 and which show relatively high pretest achievement levels and therefore relatively little growth over grade 8. Of course an important question may be to what extent deficits may have been observed had the material not been repeated? The issue of the repeating of material or the "spiral curriculum" is an important one. A recent paper has suggested that one main characteristic of the Japanese curriculum at the elementary level is the lack of repetition from year to year (4) in contrast to the North American curriculum at this level.

A final important question concerns the meanings of the absolute levels of achievement seen on the SIMS items in Ontario. The SIMS achievement items were designed to maximize variance between coutries, schools and classrooms and are therefore relatively difficult. It will be up to each math educator to evaluate what are acceptable levels of achievement.

To aid educators in the interpretation of SIMS findings and use of SIMS resources, evaluation materials have been developed at the Evaluation Centre at OISE which provide copies of the test items, information as to their mathematical content and skill level, and suggestions as to their use in evaluation of achievement in mathematics.

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REFLEXIONS SUR L'ENSEIGNEMENT DES FRACTIONS

by
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Les fractions demeurent, pour les élèves, un des sujets des mathématiques scolaires les plus frustrants et les moins réussis. Or, de plus en plus, les recherches indiquent que cette situation est le résultat de l'enseignement et qu'il est possible de l'améliorer. Cet article analyse les faiblesses de l'enseignement traditionnel des fractions au cycle intermédiaire, illustre la complexité de la tâche à accomplir et suggère des approches prometteuses.

Un examen des manuels de 7e et de 8e année nous montre que les fractions sont enseignées d'une façon abstraite. Comme au secondaire, cet enseignement est souvent magistral. Essentiellement, l'enseignant explique un algorithme, pour une des quatre opérations, et les élèves le pratiquent jusqu'à ce qu'ils le maîtrisée, on donnera des problèmes d'envergure où les élèves devront réfléchir et faire preuve d'initiative. Le plus souvent, les manuels offrent, comme application, des petits problèmes bêtes dans lesquels il suffit d'utiliser l'opération étudiée. par exemple, dans une section sur la multiplication, on trouvera l'"application" suivante:

> Michelle a fait une enquête auprès des
> 36 élèves de la classe et a découvert que
> les 3/4 des élèves économisent de l'argent.
> Combien d'élèves font des économies?

Ce genre de problème ne stimule pas la réflexion. Il encourage plutôt l'élève à chercher une règle pour chaque genre de problème.

On peut remarquer, aussi, que les manuels expliquent souvent les algorithmes d'addition et de soustraction à l'aide d'illustrations de rectangles dont les parties sont hachurées et que les algorithmes de ultiplication et de division sont expliqués de façon abstraite.

> Un manuel fait observer que
>
> $12 \div 3 = 4$ et $12 \times \frac{1}{3} = 4$,
>
> $16 \div 3 = 4$ et $16 \times \frac{1}{4} = 4$,
>
> $18 \div 9 = 2$ et $18 \times 1/9 = 2$ et donc,
>
> pour diviser par a/b, on multiplie par b/a.

Et pourtant le programme-cadre de mathématiques au cycle intermédiaire est clair. "La mise en oeuvre des cours proposés dans ce programme-cadre exige l'adoption d'une méthode expériencielle. ... (Le programme) doit faire usage de matériel concret, de recherches et d'expériences palpables, puisant souvent dans le quotidien de l'élève des éléments qu'il peut transformer en de stimulantes situations pédagogiques."

De nombreuses études, par exemple (1), (4) et (6), suggèrent que la faiblesse des élèves dans l'apprentissage des fractions est due aux caractéristiques que nous venons de décrire, soit l'utilisation précoce de l'abstraction, une attention à peu près exclusive aux algorithmes et un manque d'occasions où l'élève doit réfléchir. Cela encourage un grand nombre d'élèves à s'inventer des règles, souvent fausses, qui gouverneront leur travail (voir TABLEAU 1). Ces élèves ne sont pas confus. Ils "comprennent" à leur façon. Les règles qu'ils ont inventées leur permettent de produire des réponses. Et comme l'enseignement traditionnel leur apprend que les mathématiques consistent à choisir la bonne règle pour trouver la bonne réponse, leur réflexion s'arrête là. Ils ne cherchent plus à comprendre mais à trouver la règle.

TABLEAU 1

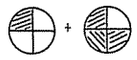
1. "On peut renverser des fractions. Par exemple, $\frac{1}{2}$, c'est la même chose que $\frac{2}{1}$. On peut le vérifier:

   $\frac{1}{2} + \frac{1}{2} = 1$ (deux moitiés, ça donne 1)

   Aussi, $\frac{1}{2} + \frac{2}{1} = \frac{3}{3} = 1$."

2. $\begin{array}{r} \frac{2}{7} \\ +\frac{3}{7} \\ \hline \frac{6}{4} \end{array}$ "7 + 7 donne 14. Je place le 4 et je retiens 1. Ensuite, 1 + 2 + 3 font 6. Donc, 6 sur 4."

3. " $\frac{1}{4} + \frac{3}{4}$ peut donner 1 ou $\frac{1}{2}$, selon la méthode employée.

   Car  donnent 1 entier.

   Mais $\frac{1}{4} + \frac{3}{4} = \frac{4}{8} = \frac{1}{2}$. Il faut employer la méthode qu'on nous dit de prendre.

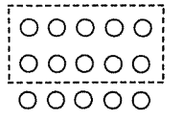
TABLEAU 1. Règles inventées par des élèves

Complexité du concept de la fraction

En enseignant les fractions comme une série d'algorthmes que l'on doit maîtriser pour ensuite les utiliser dans diverses applications, on augmente chez l'élève les difficultés inhérentes à l'apprentissage des fractions. Car la fraction n'est pas un concept simple; elle est plutôt un amalgame de concepts psychologiquement différents. Par exemple, la nouvelle ébauche du programme-cadre suggère qu'en 7e année, on enseigne trois concepts différents de la fraction: comme partie d'une unité, comme partie d'ensemble et comme division.

TABLEAU 2

1. Partie d'un tout: $\frac{2}{3}$ d'une tarte 

2. Partie d'un ensemble: $\frac{2}{3}$ d'un ensemble de 15 objets 

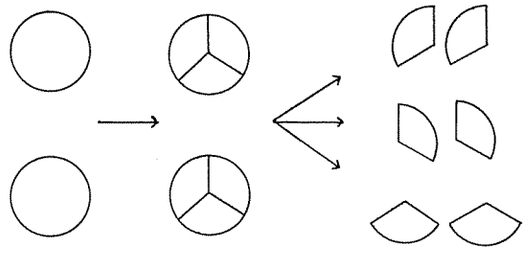
3. Division: on partage 2 pizzas parmi 3 personnes 

TABLEAU 2. Exemples de 3 concepts de la fraction

Le tableau 2 illustre bien qu'il s'agit de trois concepts différents. Dans le premier cas, on a un objet que l'on doit couper en 3 parties égales pour ensuite prendre 2 parties. Dans le deuxième cas, on doit considérer 15 objets comme un tout. Ce tout n'est pas aussi évident que dans le cas précédent, où il s'agissait d'**un** objet. la réponse, 10, n'est plus une partie d'une unité mais un groupe d'objets formant une partie d'un tout plutôt abstrait. Le troisième cas est encore plus différent. Pour partager 2 pizzas parmi 3 personnes, la solution la plus simple est de couper chaque pizza en 3 morceaux et de distribuer les 3 morceaux de chaque pizza. On voit, ici aussi, un concept psychologiquement différent des deux autres. Or les trois exemples illustrent tous la fraction ⅔.

L'enseignement traditionnel utilise le premier modèle pour en extraire une abstraction, la simagrée ⅔, qui soudainement, dans des applications perd sa signification première selon le problème rencontré. En effet, on se sert souvent du premier modèle, une partie d'un tout, pour que le concept soit concret. Cependant on passe très vite au concept abstrait, ⅔, qui se manipulera selon certaines règles, pour l'appliquer à des concepts qui sont psychologiquement différents. Et on voudrait que l'enfant soit logique. Il ne lui reste plus, pour survivre, qu'à apprendre les différentes situations concrètes ou pratiques dans lesquelles il devra utiliser cette abstraction uq'est devenue la fraction ⅔.

Le tableua 3 illustre des relations dont on doit tenir compte dans l'enseignement des fractions en 7e année si on veut que les fractions touchent à la réalité.

TABLEAU 3

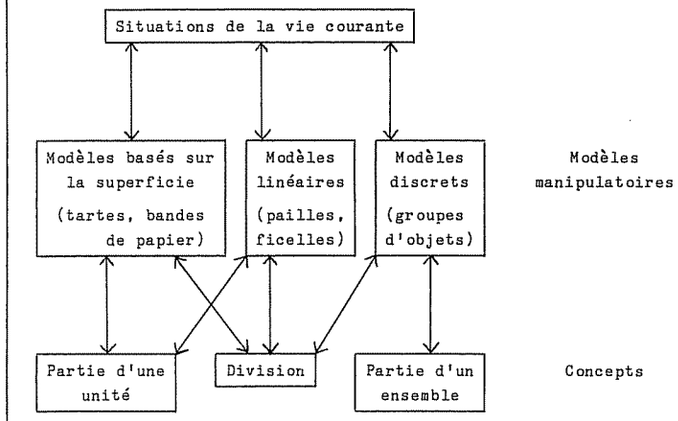


TABLEAU 3. Relations dans l'enseignement des fractions en 7e année

Il semblerait que les élèves montrent un certain degré de compréhension lorsqu'ils peuvent établir une relation entre une situation de la vie courante, un modèle manipulatoire approprié et le concept à utiliser. Il semble assez important qu'ils puissent maîtriser plusieurs modèles correspondant à un même concpet. Ce n'est qu'une fois que les élèves auront appris à établir ces relations et à manipuler les divers modèles qu'ils seront prêts à apprendre les règles abstraites.

Approche pédagogique saine

Piaget, dans une de ses rares suggestions pédagogiques, propose que chaque concept à apprendre doit subir les processus d'accommodation et d'assimilation dans l'ordre séquentiel des divers stades. Essentiellement, l'enseignant devrait suivre la séquence des phases de développement nécessaire à l'acquisition du concept en laissant d'abord l'élève manipuler librement les divers modèles concrets, ensuite en lui permettant d'effectuer les opérations nécessaires avec le matériel, puis en l'encourageant à résoudre des problèmes avec ce matériel. Ensuite l'enseignant devrait encourager l'élève à réfléchir

et à anticiper en pensée des opérations possibles jusqu'à ce qu'elles soient suffisamment intériorisées pour les reproduire sans dépendre de l'entourage (2). Bruner suggère que le symbolisme abstrait ne devrait être employé que pour représenter, d'une façon plus efficace, des opérations que l'apprenant faisant déjà concrètement ou à l'aide de dessins.

TABLEAU 4

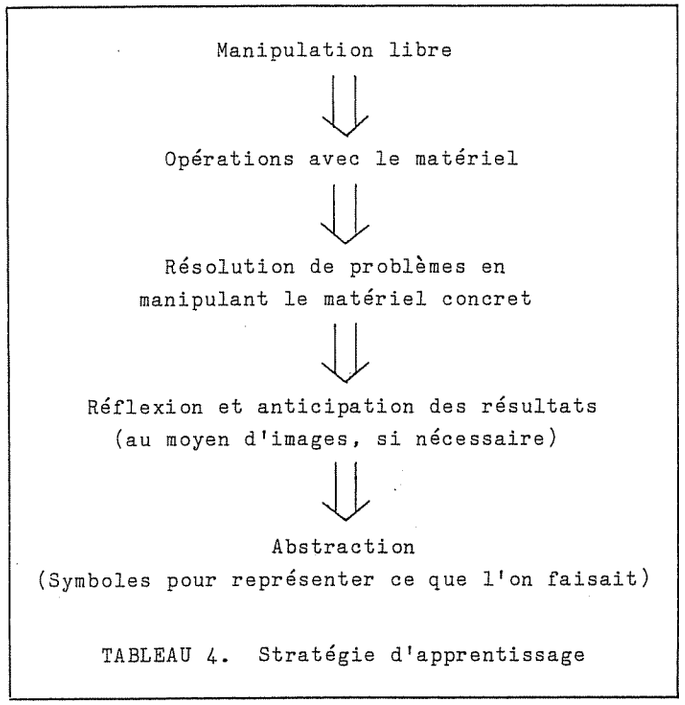


TABLEAU 4. Stratégie d'apprentissage

Tout en suivant ces étapes, on devrait aussi choisir des situations de la vie courante des élèves pour situer les exercices que l'on donne. Il faudrait aussi mettre divers modèles manipulatoires à la dispositions des élèves et voir à ce qu'ils puissent s'en servir pour représenter la situation présentée (partie d'une unité, partie d'un ensemble, division).

Les suggestions qui précèdent ne sont pas une recette. Elles ne forment qu'une charpente qui doit être complétée et étoffée. Les modèles manipulatoires ne sont pas tous aussi bons les uns que les autres. Il ne suffit pas de manipuler un objet pour que l'élève comprenne, comme l'indiquent les suggestions de Piaget. Il reste beaucoup de questions sans réponse. Cependant, il existe des recherches qui ont montré l'efficacité de certaines approches. Le National Council of Teachers of Mathematics s'efforce, dans ses publications, de publier et de rendre ces recherches pratiues pour l'enseignant. On trouvera des résultats de ces recherches et des suggestions pédagogiques pertinentes dans (3), (8), (10) et (11).

L'approche suggérée n'effacera pas tous les problèmes actuels. Cependant elle tient compte de la complexité du concept de la fraction et elle permet à l'enseignant de respecter le développement cognitif de l'élève. Ce sont deux aspects négligés par la pédagogie actuelle qui s'appuie trop sur les manuels.

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THE 24TH INTERNATIONAL MATHEMATICAL OLYMPIAD

by
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University of Toronto

The 1983 International Mathematical Olympiad was held in Paris, France from July 4 to July 12. In all, 186 school students from 32 countries participated, with Canada sending a team of six students. The four students, three from West Germany and one from the U.S.S.R., who received perfect scores of 42 on their examinations received gold medals along with five others. 27 silver and 57 bronze medals were also awarded. The following countries gained at least 100 points:

1. West Germany	212	6. Vietnam	148	11. Great Britain	121
2. United States	171	7. Netherlands	143	12. East Germany	117
3. Hungary	170	8. Czechoslovakia	142	13. Finland	103
4. Soviet Union	169	9. Bulgaria	137	14. Canada	102
5. Romania	161	10. France	123	15. Poland	101

The members of the Canadian team scored as follows:

Martin Piotte (Montreal, Quebec)	25
Michael Clase (St. John's Nfld.)	22
Neale Ginsburg (Loyalist C & VS Kingston)	18
William Rucklidge (Toronto French School)	17
Mike Molloy (Osgoode, near Ottawa)	14
Bruce Sutherland (Lorne Park S.S., Mississauga)	6

The first four students received bronze medals.

The coaches, G.J. Butler from the University of Alberta and E.J. Barbeau from the University of Toronto, wish to expess their gratitude to a number of organizations and individuals for assisting with the travel costs:

The Government of Canada
The Ministry of Education of Ontario
The Francis F. Reeve Foundation, Calgary
The Samuel Beatty Fund
The Royal Society of Canada
The McLean Foundation
Citibank of Canada
The Ontario Association for Mathematics Education
Norman S. Robertson, Q.C.
Jessie Current

Assistance was also provided by the boards of education and schools of the participating students.

There are three problems which I would like to take up with members of O.A.M.E., namely, funding, recruiting and training. We are still looking for organizations who might be willing to provide regular financing of some of the travel over an extended period. Unless this can be done, the Canadian Mathematical Society faces an annual round of applications and it may well happen that more of the burden will fall upon the boards or schools of the team members or the members themselves. Consequently, I would like to hear from any readers with useful information or contacts.

Recruitment is also a bit of a problem. Our main sources of information about potential competitors are the Waterloo competitions, the Annual High School Mathematics Examination and Canadian Mathematical Olympiad. There are two problems with these. First, they occur quite late in the academic year and none of them is completely reliable. There inevitably appear to be some very good students who are missed. Last year, we tried to mitigate this by obtaining a list of possibilities in the fall from Ron Dunkley, the executive director of the Canadian mathematics Competition at Waterloo. This left us having to choose a team in part from students we had corresponded with throughout the year and in part from those who somehow emerged as dark horses from one or other competition. It is clear that our identification of students is not as exhaustive as it should be.

Teachers can be enormously valuable in our quest for a Canadian team. If you have a suitable student in your class, please tell us about him or her. What I would suggest is that you have your students write up solutions to some problems which you can include with your letter. A good source of problems is **Crux Mathematicorum** (available from F.G.B. Maskell, Algonquin College, 200 Lees Avenue, Ottawa L1S OC5) which has a problems section and an "Olympiad Corner" department.

Training also presents difficulties. The small European countries can bring their teams together frequently for sessions. The cost of doing this in Canada would be prohibitive, and in the past three years, we have tried to arrange a few days just before the team leaves for the competition. What this has meant for the students is an extended period away from home, which appears to take some toll; many of the team will have spent a week at the Waterloo seminar, followed by a seminar for Canadian Olympiad winners before we get down with some regional meetings. Our intention is to identify potential team members early in the academic year, receive solutions from them in the mail and try to arrange for them to have a face-to-face session with someone who can gauge their abilities.

Once again, teachers can be of great assistance. Two aims of training are to enable students to transfer their ideas to paper in a coherent and logical way, and to be careful not to make silly errors or leave out important details. In general, their high school career does not equip them well for either of these, and we will be interested in finding out which schools are prepared to assign their bright students many problems for full written solutions and to mark them in a rigorous way. We will also be contacting teachers who might be willing to organize local training sessions and provide them with whatever assistance in the form of problems and workshops is appropriate.

Having participated in three International Olympiads, I am convinced that it is within our capacity to regularly place a team in the top ten. However, this will require a tighter operation and a better utilization of resources than we have been able to achieve in the past.

Information about potential students, requests for lists of Olympiad problems and other communications can be sent to E.J. Barbeau, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 1A1. (416) 978-5164.

Readers should be aware that the headquarters for the Canadian mathematical Olympiad will be moving from the University of Alberta to the University of Western Ontario; the new director will be Prof. Clive M. Reis, Department of Mathematics, University of Western Ontario, London, Ontario N6A 2B7. The provincial secretary continues to be Prof. Ken Fryer at the University of Waterloo (N2L 3G1). Teachers of bright students who are in their 13th school year or who are not chosen among the quota, should see that their students are nominated by their school principals to write the Canadian Olympiad, normally held on the first Wednesday of May.

OLYMPIADE INTERNATIONALE DE MATHEMATIQUE
PARIS

First day: Wednesday, July 6
Time: 4h 30
Each problem is worth 7 points

1. Find all functions f defined on the set of positive real numbers which take positive real values and satisfy the conditions:
   (i) $f(xf(y)) = yf(x)$ for all positive x, y ;
   (ii) $f(x) \to 0$ as $x \to +\infty$.

2. Let A be one of the two distinct points of intersection of two unequal coplanar circles C_1 and C_2 with centres O_1 and O_2, respectively. One of the common tangents to the circles touches C_1 at Q_1 and C_2 at Q_2. Let M_1 be the midpoint of P_1Q_1 and M_2 the midpoint of P_2Q_2. Prove that the angles $O_1A\,O_2$ and $M_1A\,M_2$ are equal.

3. Let a, b, c be positive integers, no two of which have a common divisor greater than 1. Show that
   $2abc - ab - bc - ca$
   is the largest integer which cannot be expressed in the form
   $xbc + yca + zab$
   where x, y, z are non-negative integers.

Second day: Thursday, July 7
Time: 4h 30
Each problem is worth 7 points

4. Let ABC be an equilateral triangle, and E be the set of all points contained in the three segments AB, BC and CA (including A, B and C). Determine whether for every partition of E into two disjoint subsets, at least one of the two subsets contains the vertices of a right-angled triangle. Justify your answer.

5. Is it possible to choose 1983 distinct positive integers, all less than or equal to 10^5, no three of which are consecutive terms of an arithmetic progression? Justify your answer.

6. Let a, b, and c be the lengths of the sides of a triangle. Prove that
$$a^2b(a - b) + b^2c(b - c) + c^2a(c - a) \geqslant 0.$$
Determine when equality occurs.

Premier jour: mercredi 6 Juillet 1983
Durée: 4h 30
Chaque exercice est noté sur 7 points

1. Déterminer toutes les fonctions de l'ensemble des nombres réels strictement positifs dans lui-même, qui vérifient les conditions suivantes:
(1) pour tous réels strictement positifs x, y: $f(xf(y)) = yf(x)$;
(2) $f(x) \to 0$ lorsque $x \to +\infty$

2. Dans un plan, on se donne deux cercles sécants C_1 et C_2, de rayons inégaux et de centres respectifs 0_1 et 0_2. Soit A un de leurs points communs. Les points de contact des tangentes communes à C_1 et C_2 sont P_1 (sur C_1) et P_2 (sur C_2) sur l'une des tangentes, Q_1(sur C_1) et Q_2(sur C_2) sur l'autre. Soient M_1 et M_2 les milieux respectifs de P_1Q_1 et P_2Q_2. Montrer que les angles $0_1A\ O_2$ et $M_1A\ M_2$ sont égaux.

3. Soient a, b, c des entiers strictement positifs et premiers entre eux deux à deux.
Montrer que
$$2abc - ab - bc - ca$$
est le plus grand entier qui ne peut pas s'écrire sous la forme
$$xbc + yca + zab$$
avec x, y, z entiers positifs ou nuls.

Second jour: jeudi 7 juillet 1983
Durée: 4h 30
Chaque exercice est noté sur 7 points

4. Soit ABC un triangle équilatéral. Soit E l'ensemble des points des segments fermés AB, BC, CA. Est-ce que, pour toute partition de E en deux sous-ensembles disjoints, il existe au moins un triangle rectangle dont les trois sommets appartiennent au même sous-ensemble? Justifier la réponse.

5. Peut-on trouver 1983 nombres entiers strictement positifs et distincts, inférier ou égaux à 10^5 et tels que trois quelconques d'entre eux ne soient pas les termes consécutifs d'une progression arithmétique? Justifier votre réponse.

6. Soient a, b, c, les longueurs des trois côtés d'un triangle.
Démontrer que
$$a^2b(a - b) + b^2c(b - c) + c^2a(c - a) \geqslant 0$$
Déterminer les cas d'égalité.

SUMMING SEQUENCES OF CUBES

by
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Teachers of mathematics frequently wish to use non-routine practice activities in their classrooms to review and maintain computational or calculator skills. It is a serendipitous occurrence if students also discover number patterns while performing this practice.

Consider Figure I, a sequence of cubes and their differences. Observe that the 3rd differences are all 6.

Figure I

$1^3 = 1$
$\quad > 7$
$2^3 = 8 \quad > 12$
$\quad > 19 \quad > 6$
$3^3 = 27 \quad > 18$
$\quad > 37 \quad > 6$
$4^3 = 64 \quad > 24$
$\quad > 61 \quad > 6 \qquad (6 = 6 \cdot 1)$
$5^3 = 125 \quad > 30$
$\quad > 91 \quad > 6$
$6^3 = 216 \quad > 36$
$\quad > 127 \quad > 6$
$7^3 = 343 \quad > 42$
$\quad > 169 \quad > 6$
$8^3 = 512 \quad > 48$
$\quad > 217$
$9^3 = 729$
. .
. .
. .

We shall modify the activity of Figure I in several ways.

Modification I

Figure II reports sequences of non-consecutive cubes.

Figure II

$1^3 = 1$
$\quad > 26$
$3^3 = 27 \quad > 72$
$\quad > 98 \quad > 48$
$5^3 = 125 \quad > 120$
$\quad > 218 \quad > 48 \qquad (48 = 6 \cdot 8)$
$7^3 = 343 \quad > 168$
$\quad > 386 \quad > 48$
$9^3 = 729 \quad > 216$
$\quad > 602$
$11^3 = 1331$
. .
. .
. .

Figure II continued on next page

Figure II continued

$1^3 = 1$
> 63
$4^3 = 64$ > 216
> 279 > 162
$7^3 = 343$ > 378
> 657 > 162 $(162 = 6 \cdot 27)$
$10^3 = 1000$ > 540
> 1197 > 162
$13^3 = 2197$ > 702
> 1899
$16^3 = 4096$
. .
. .
. .

$1^3 = 1$
> 124
$5^3 = 125$ > 480
> 604 > 384
$9^3 = 729$ > 864
> 1468 > 384 $(384 = 6 \cdot 64)$
$13^3 = 2197$ > 1248
> 2716 > 384
$17^3 = 4913$ > 1632
> 4348
$21^3 = 9261$

We conjecture resulting from Figures I and II that if consecutive differences are formed from the cubes 1^3, $(1 + n)^3$, $(1 + 2n)^3$, $(1 + 3n)^3$, ..., the 3rd differences are all $6(n)^3$.

Modification II

Figure III reports the results of summing **pairs** of cubes.

Figure III

$1^3 + 2^3 = 9$
> 26
$2^3 + 3^3 = 35$ > 30
> 56 > 12
$3^3 + 4^3 = 91$ > 42
> 98 > 12 $(12 = 12 \cdot 1)$
$4^3 + 5^3 = 189$ > 54
> 152 > 12
$5^3 + 6^3 = 341$ > 66
> 218
$6^3 + 7^3 = 559$
. .
. .
. .

$1^3 + 2^3 = 9$
> 82
$3^3 + 4^3 = 91$ > 168
> 250 > 96
$5^3 + 6^3 = 341$ > 264
> 514 > 96 $(96 = 12 \cdot 8)$
$7^3 + 8^3 = 855$ > 360
> 874 > 96
$9^3 + 10^3 = 1729$ > 456
> 1330
$11^3 + 12^3 = 3059$
. .
. .
. .

Figure III continued on next page

Figure III continued

$1^3 + 2^3 = 9$

> 180

$4^3 + 5^3 = 189$ $\quad > 486$

> 666 $\quad > 324$

$7^3 + 8^3 = 855$ $\quad > 810$

>1476 $\quad > 324$

$10^3 + 11^3 = 2331$ $\quad >1134$

>2610 $\quad > 324$

$13^3 + 14^3 = 4941$ $\quad >1458$

> 4068

$16^3 + 17^3 = 9009$

. . .

. . .

. . .

(324 = 12 • 27)

We conjecture that if consecutive differences are formed from the sums $(1^3 + 2^3)$; $(n + 1)^3 + (n + 2)^3$; $(2n + 1)^3 + (2n + 2)^3$; $(3n + 1)^3 + (3n + 2)^3$, ..., the 3rd differences are all $12(n)^3$.

Modification III

Figure IV reports the results of summing **triples** of cubes.

Figure IV

$1^3 + 2^3 + 3^3 = 36$

> 63

$2^3 + 3^3 + 4^3 = 99$ $\quad > 54$

> 117 $\quad > 18$

$3^3 + 4^3 + 5^3 = 216$ $\quad > 72$

> 189 $\quad > 18$

$4^3 + 5^3 + 6^3 = 405$ $\quad > 90$

> 279 $\quad > 18$

$5^3 + 6^3 + 7^3 = 684$ $\quad > 108$

> 387

$6^3 + 7^3 + 8^3 = 1071$

. . .

. . .

. . .

(18 = 18 • 1)

$1^3 + 2^3 + 3^3 = 36$

> 180

$3^3 + 4^3 + 5^3 = 216$ $\quad > 288$

> 468 $\quad > 144$

$5^3 + 6^3 + 7^3 = 684$ $\quad > 432$

> 900 $\quad > 144$

$7^3 + 8^3 + 9^3 = 1584$ $\quad > 576$

>1476 $\quad > 144$

$9^3 + 10^3 + 11^3 = 3060$ $\quad > 720$

> 2196

$11^3 + 12^3 + 13^3 = 5256$

. . .

. . .

. . .

(144 = 18 • 8)

$1^3 + 2^3 + 3^3 = 36$

> 369

$4^3 + 5^3 + 6^3 = 405$ $\quad > 810$

>1179 $\quad > 486$

$7^3 + 8^3 + 9^3 = 1584$ $\quad >1296$

>2475 $\quad > 486$

$10^3 + 11^3 + 12^3 = 4059$ $\quad >1782$

>4257 $\quad > 486$

$13^3 + 14^3 + 15^3 = 8316$ $\quad >2268$

>6525

$16^3 + 17^3 + 18^3 = 14841$

. . .

. . .

. . .

(486 = 18 • 27)

Figure IV continued on next page

Figure IV continued

$1^3 + 2^3 + 3^3 = 36$

> 648

$5^3 + 6^3 + 7^3 = 684 \quad > 1728$

$> 2376 \quad > 1152$

$9^3 + 10^3 + 11^3 = 3060 \quad > 2880$

$> 5256 \quad > 1152 \quad (1152 = 18 \cdot 64)$

$13^3 + 14^3 + 15^3 = 8316 \quad > 4032$

$> 9288 \quad > 1152$

$17^3 + 18^3 + 19^3 = 17604 \quad > 5184$

> 14472

$21^3 + 22^3 + 23^3 = 32076$

Again a pattern emerges. If consecutive differences are formed from the sums $(1^3 + 2^3 + 3^3)$; $(n + 1)^3 + (n + 2)^3 + (n + 3)^3$; $(2n + 1)^3 + (2n + 2)^3 + (2n + 3)^3$; $(3n + 1)^3 + (3n + 2)^3 + (3n + 3)^3$; ..., the 3rd differences are all $18(n)^3$.

Modifications II, III, and IV themselves suggest a pattern. That is, if consecutive differences are formed from the sums $(1^3 + 2^3 + 3^3 + \ldots m^3)$; $((n + 1)^3 + (n + 2)^3 + (n + 3)^3 + \ldots + (n + m)^3)$; $((2n + 1)^3 + (2n + 2)^3 + (2n + 3)^3 + \ldots + (2n + m)^3)$; ..., the 3rd differences are all $6m(n)^3$. For example, let $m = 4$ and $n = 3$. Consider summing quadruples of squares as in Figure V.

Figure V

$1^3 + 2^3 + 3^3 + 4^3 = 100$

> 648

$4^3 + 5^3 + 6^3 + 7^3 = 748 \quad > 1188$

$> 1836 \quad > 648$

$7^3 + 8^3 + 9^3 + 10^3 = 2584 \quad > 1836$

$> 3672 \quad > 648$

$10^3 + 11^3 + 12^3 + 13^3 = 6256 \quad > 2484$

> 6156

$13^3 + 14^3 + 15^3 + 16^3 = 12412$

The 3rd differences all are 648, which is $6 \cdot 4 \cdot (3)^3$ as conjectured.

Challenges for the reader:

1. If sums of other powers (2nd, 4th, 5th, ...) are used, what patterns result?
2. Is there a pattern in the following:
   $1^3 + 3^3 + 5^3$; $2^3 + 4^3 + 6^3$; $3^3 + 5^3 + 7^3$; $4^3 + 6^3 + 8^3$; ... ?
3. Prove all the conjectures using mathematical induction.

PRACTICAL METHODS FOR GEOMETRY

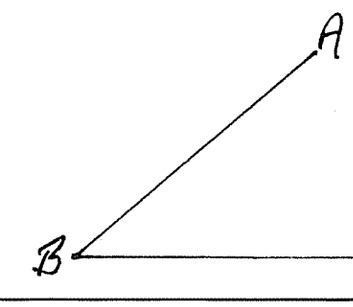
by
R. S. Smith
Faculty of Education
The University of Western Ontario

There are two types of objectives within geometry and measurement which are universally taught yet are seldom tested. The first of these are objectives requiring the actual manipulation of apparatus such as a ruler, protractor, mirror or MIRA. The second objectives involve the actual drawing or constructing of geometric figures or images. The concepts, the variety of methods used, the consolidation of properties of the figures, as well as the applications learned, are most worthwhile.

Activities such as using the protractor to bisect an angle, flipping an object about a line of symmetry, etc., as well as constructing a triangle and folding a paper to use the pin test to prove lines parallel, etc., however, do not lend themselves to pencil-paper computer-marked tests. Much better that an interview situation be used. That approach was used by the author and student teachers hired to act as interviewers. This article is a continuation of work begun in 1977 and reported in this journal in two previous articles. For details of the method and earlier results, I would refer you to them. (1)

The following items were designed to measure a goal or learning outcome implied by some Grade 8 objectives set forth in the London Board of Education Mathematics Guidelines. Each tester had a shoe box of materials. The pupils were encouraged to verbalize their thinking. Interviews were flexible. 'I' stands for interviewer and 'R' stands for pupil response.

Diagram 1



Interviewer places on the table a Mira, a pair of compasses, a ruler, a protractor and a sheet of paper with angle ABC on it.

I: Please bisect angle ABC. You may use any of the materials to divide it into two equal parts. Tell me what you will do.

R: Divide it into half?

I: Yes. How are you going to divide it in half?

R: Find the halfway point.

I: How?

R: Measure it. (He proceeds to measure the length of line BC).

I: You're going to divide the line into two equal parts for me?

R: Yeah.

I: Is that going to divide the angle?

R: Oh! I know how to do it now. You divide the angle (46°) by 2. That gives 23°. (He then marks off 23° and draws the angle bisector).

I: You're sure?

R: I think so.

I: How could you use the Mira to prove you are correct? (empirical proof)

R: Place the Mira on BD (the angle bisector) and you see BA on top of BC.

I: Suppose we pick up the paper and fold it along BD. What happens when you fold the paper?

R: You decrease the size of the paper.

I: Yes, and likewise the size of the angle decreases. Hold it up to the light and tell me what you observe.

R: BA is over top of BC.

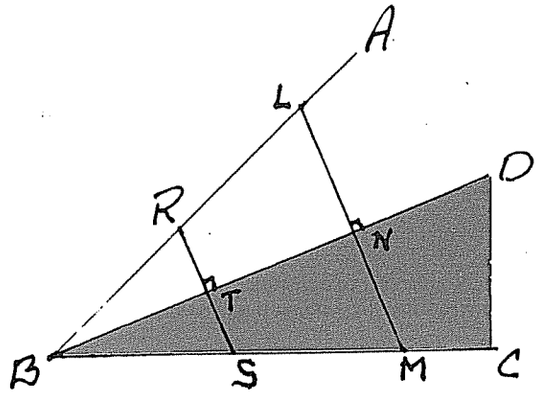
I: What do you think would happen if you stuck a pin through the paper at a point on line BC?

R: It should come out on the other side on line BA.

I: Try it.

In our testing more students used the protractor to measure the angle and then mark the mid-point then used the compasses to construct the angle bisector (the traditional construction technique). No one used the Mira nor the simple idea of folding the paper. One lad tried an interesting idea. He combined measuring with the ruler after estimating where the bisector would be. He was fairly accurate and his method was novel.

Diagram 2



For if the line RS is at right angles to the bisector at T and the line LM is at right angles at N then BD will be the bisector of the two lines RS and LM. He must have had an estimate of where the bisector would fall as I observed him move the ruler about until the distances RT and TS where 1 cm and the distance LN and NM were 2 cm to produce the points T and N. Joining TN produced the bisector BD. (None of these points were labelled in his work).

Parallel Lines

Interviewer places a sheet of dotty paper containing four sets of lines labelled a, b, c, and d, a ruler, Mira, protractor and pair of compasses on the table.

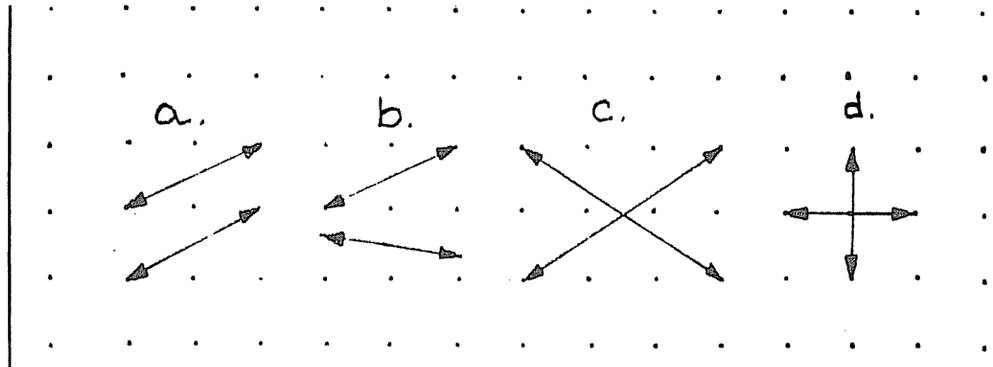


Diagram 3

I: Which of the lines are parallel?

R: Set a.

I: Suppose I say I do not believe that the lines in the set are parallel. How would you prove to me that they are parallel?

R: Well, parallel lines, these parallel lines are straight underneath each other.

I: I see. But I don't think that statement is good enough to convince me.

R: Is it that they are the same distance apart?

I: But you only think that is true. How will you convince me?

R: Measure them.

I: Do so and tell me what you find.

R: (Pupil uses a ruler and measures at several places). The lines are always 1 cm. apart.

I: If you used compasses set to a distance of 1 cm. perhaps it would have been easier to check to see if the distance between the lines in set a was constant?

Some pupils suggested that if they produced the lines in set a they would never meet. Mary drew a line perpendicular to the lines in set a, dug the protractor out of the shoe box and measured all four angles declaring them to be right angles. John joined dots above and below the lines in set a to make a vertical line across the rows of dots, then measured the alternate angles. After finding the measurement of the alternative angles formed by the transversal to be the same he declared the lines to be parallel. As in the case of bisecting an angle, no one suggested folding the paper to see if one line would fall on top of the other, nor placing the Mira between the lines to observe whether the image of one line could be made to fall on the object line. Both of these methods are simple applications of the symmetry concept.

One student, Peter, had an interesting idea which I encouraged. He suggested that the two lines do go at the same angle. On further questioning, it was found that he meant the angle formed by the row of horizontal dots and the parallel lines. He was encouraged to draw in the lines to form the angle and measure the two angles. These three cases of proving lines parallel by empirical proof are shown in diagram 4.

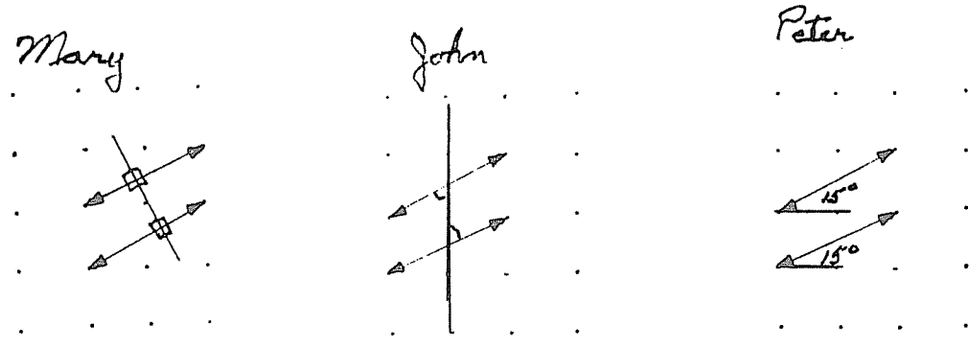


Diagram 4

Gerry was encouraged to try the Mira. He placed it diagonally across set a, turned the Mira slowly, then held it stationary and turned the paper. He was surprised to see the images of the two lines merge with the original lines. In fact, I remember his words vividly: "They're turning into straight lines!" We then folded the paper along the line of symmetry and proceeded to stick a pin through the line at various points to find that the pin did reappear on the line on the other side. In a last experiment I extended the line on one side of the fold only. After sticking the pin through the extended line I asked why the pin did not come through on the line on the other side of the fold. By this time he was with me and promptly stated that his line was only a line segment and, if extended, it would indeed pass through the pin hole. I requested him to do so to prove his statement correct.

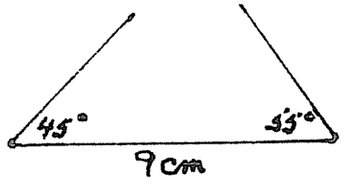
Constructions

Interviewer places a paper, pencil, ruler, pair of compasses, and protractor before the pupil.

I: Construct a triangle with base 9 cm and base angles of 45 and 55.

Some pupils had trouble using the protractor scales: turning the protractor over to make the second angle; trying to decide where to place the centre point of the protractor; and even using the base of the instrument rather than the base line scribed on the instrument. Several students did not bother to extend the arms - so drew three lines but not a triangle as requested.

Diagram 5



Interviewer places a paper, pencil, ruler, pair of compasses, and protractor before the pupil.

I: Construct a circle with a diameter of 6 cm.

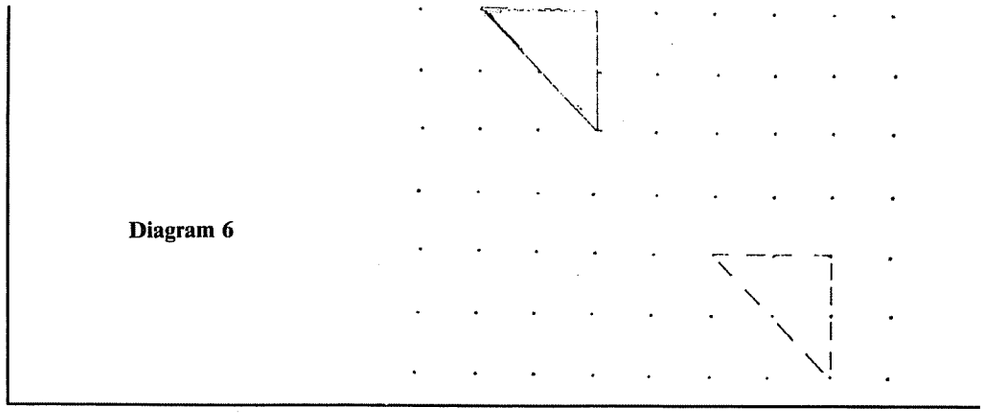
Fewer pupils were able to draw a circle of the correct size than could construct the triangle correctly. Most common error was incorrect radius even though the space allowed for this question was insufficient for a circle of 6 cm radius. Most children who drew the circle correctly found the radius mentally although a few drew the full diameter, measured to find the half-way point and then used this point as the centre and the distance to the end point as the radius. Perhaps the use of the English safety compass, an adaptation of the ruler with a plastic point at one end and a moveable slider with a hole for the pencil point, would help pupils recognize a radius. You could also have

some circles drawn where a string is used and kept taut while the circle is drawn.

Transformations (a kind of construction)

Interviewer places a sheet of dotty paper, a Mira, a ruler and a pencil on the table.

Diagram 6



I: On the sheet of dotty paper, the object is outlined in a solid black line and its image is outlined with a broken line. Draw in the flip line or the line of reflection, and explain how you do it.

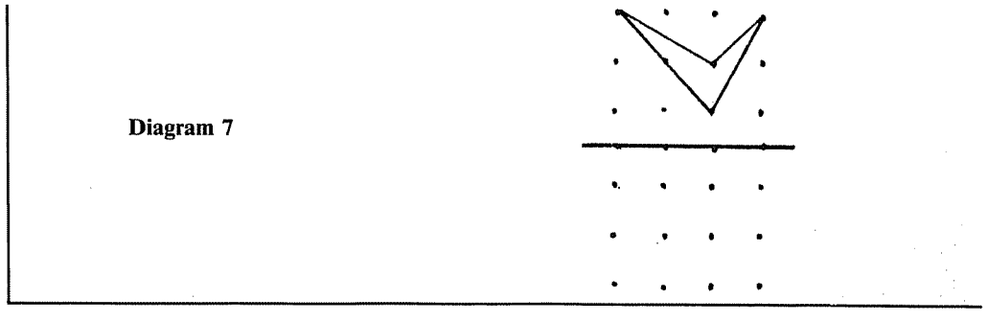
R: (Mary draws in the diagonal line of reflection). I turn the Mira around until I get the outline of the shape to fall back on the object which I can see through the Mira.

(John did not use the Mira but counted unit distances diagonally from the flip line. You take this point here, which is one space away from the line I will draw, because it is like the point in the image which is also one space away from the line. The other corners are three spaces and two spaces away from the reflection line.

The above question was used with Grade 8 students but the Mira was not available for use. The facility value dropped one-half when pupils were forced to use the method John used, i.e. to find end points and construct the figure by joining points. The next question required the pupils to construct an image of the original figure flipped about a horizontal line of reflection. Again, the facility value dropped from 91% in Gr. 7 to 51% in Gr. 8, when the Mira was not available for use. For pupils with difficulty, the overhead projector and transparencies are most useful tools, particularly if you colour in the original figure and label the corners. A second dotty transparency allows the pupil to trace in an image, lift the new dotty transparency complete with the image and flip it over the line of reflection. The pupil may now record this image on his work sheet.

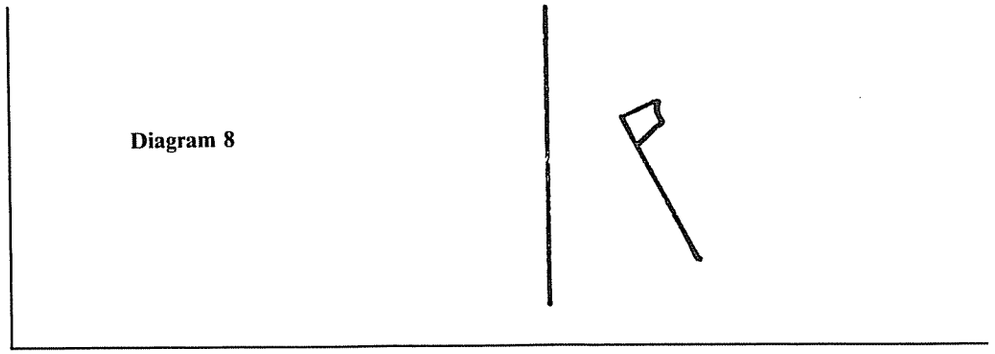
A second helpful device is to make a traced model of the arrow head, attach this to the clip on the ball point pen and flip it over the line of reflection keeping your thumb on the line and assuring the pen is at right angles. In this case, the pupil either traces about the figure or labels the new points and constructs the image figure.

Diagram 7



The last question asked the pupils to draw an image on plain paper.

Diagram 8



I: I have given you a vertical reflection line and a flag as the original figure. Draw the image. (Mira allowed).

A few students tried to draw the flag freehand, not using the Mira. One student attempted measurement. No student used the Mira to mark the end points of the flag pole and the flag before using a ruler to join them - a method which facilitates constructing an image.

The above described situations explored alternative strategies. These might well allow pupils to reach some of the goals suggested in the Intermediate Mathematics Guidelines such as "It is important that students be able to construct plane figures with reasonable accuracy. A variety of methods using different materials is suggested. In this way the properties of the constructed figures will be consolidated and students will be able to choose for themselves the most efficient construction methods... . These construction techniques are used when investigating transformation images and to determine their properties."(2) More important experimentation with various apparatus and strategies, as well as the discussion arising out of the situations cannot help but further develop concepts in geometry. We do become more aware of a pupil's development when observing pupils at work in practical situations.

REFERENCES

1. "Achievement in Measurement and Geometry in the Junior Division", *Ontario Mathematics Gazette*, vol. 17, no. 3, March 1979.
   "Practical Testing for Mathematics", *Ontario Mathematics Gazette,* vol. 20, no. 2, December 1981.

2. *Mathematics: Curriculum Guidelines for the Intermediate Division,* p. 14.

DEPARTMENTS

BILL 82

The Ontario Association of Mathematics Education at a recent meeting of Council adopted a Position Paper on Bill 82. This Bill deals with identifying students who have exceptionalities and the planning of programs that will meet their individual needs. The Position Paper is presented in this issue of the Gazette. Future issues will give some of the background and rationale that the Bill 82 Committee considered when writing the Position Paper.

O.A.M.E. POSITION PAPER ON BILL 82

1. Teachers responsible for mathematics instruction must accept responsibility for the mathematics education of **all** (including exceptional) students.

2. The mathematics curriculum of exceptional students in a secondary school should be the responsibility of the mathematics department in consultation with the special education advisor.

3. Mathematics courses for exceptional students in secondary school should be taught by teachers qualified in both mathematics and special education. Where this is not possible or practical, such courses should be taught to these students by mathematics teachers, and their programs supervised by a special education advisor.

4. Mathematics teachers should be encouraged to broaden their knowledge in the field of special education.

5. Mathematics for exceptional students should foster the development of creative thinking, divergent thinking, convergent thinking, and problem solving skills through activities involving patterns, puzzles, codes, geometry, research, etc.

6. The holistic nature of elementary gifted programs is recognized and endorsed, and the adoption of an holistic component at the secondary level is encouraged.

7. OAME should give leadership in providing programs that will suit the needs of the mathematically gifted.

8. The use of calculators and computers should be an integral part of the special education program.

9. Those students who are identified as mathematically gifted and/or talented should not be placed at a disadvantage with respect to university entrance and scholarships because of the method by which they are evaluated or the interpretation that is given to that evaluation.

10. The chief emphasis of the mathematics program for gifted students should be on enrichment rather than acceleration.

ICON QUESTIONNAIRE RESPONSE

Editor's Note:

The Computer Committee of the OAME Council contacted Mr. Lorne Smith, Chairman of the Computers in Education Committee of the Ministry of Education with questions related to the **CEMCORP ICON** microcomputer (the term "Bionic Beaver" never came up!). The following is the Ministry Committee's response. Thanks to both OAME Council's Computer Committee and the Ministry of Education Computers in Education Committee for enabling us to receive this information in time for this issue of the **Gazette**.

1. **Production:**

(A) Current Status:
The ICON is in phase 1 of the prototype field testing (at the University of Waterloo, OISE, and North York Bd. of Ed.). Units of the prototype are being distributed to those twenty boards that submitted successful proposals for the field trials in phase 2 (95 submissions were made). For more information on these phases, consult the Ministry's Policy/Program Memorandum #78.

(B) Production Schedule:
The assembly plant in Brockville will gear its production to the orders received. Should the demand for units exceed the capacity of the Brockville plant (estimated at 2 000 units per month), CEMCORP states that they would be able to set up another assembly plant in a very short time.

(C) Anticipated Requests:
The Ministry's contract with CEMCORP is expected to yield between 2,000 and 3,000 student microcomputers (the exact number is dependent on the kind of display screen selected, whether or not a built-in disk drive is selected, etc.). These units will be available at 25% of the stated contract price as identified in Memorandum #68, Appendix two. Contract prices:

ICON with monochrome monitor	
- no built-in disk drive	$2 500
- built-in disk drive	$3 100
ICON with colour monitor	
- no built-in disk drive	$2 900
- built-in disk drive	$3 500
Fileserver	$7 400

These units will all be produced during the 1984 calendar year. Other purchases will qualify for grants at the board's rate of "extraordinary expenditure". The number of machines that boards will order is unkown at this time.

(D) Can Demand be Met?
CEMCORP is quite confident of filling all orders. The company is flexible enough to gear production to various levels (even if it requires additional assembly plants, as indicated in section B). Delivery is expected to take 8 to 12 weeks from the receipt of an authorized purchase order.

(E) Standalone ICONS:
The ICON is designed for a network environment in a classroom, but it may be used also as a standalone microcomputer. The standalone unit (with a built-in disk drive) is available for purchase beginning in January of 1984.

(F) Purchase Dates?
Commercial sales will begin in January, 1984. Anyone may purchase ICON equipment at this time.

2. **Other Hardware Issues:**

(A) Current Hardware Use:

Boards of education, hopefully, have acquired their microcomputer hardware to satisfy some definite educational objectives: their current equipment should continue to satisfy those objectives. As new machinery, such as the ICON, comes into a school system, the board should develop an appropriate plan for the effective use of all computers. Areas of use include:

- Computer Studies courses
- word processing applications
- teaching/learning devices
- technological applications in music, art, drafting, etc.
- digital electronics or Computer Technology courses
- administrative functions

(B) What Should Boards Purchase?

The ICON will be available in quantity beginning in 1984. Boards of education are free to purchase **any** brand of micro that they wish; however, funding is available only for those machines that meet the Ministry's functional specifications for an educational microcomputer.

3. **Software:**

(A) Exemplary Software Projects:

These projects, which began in April of 1982, are to be completed by March, 1984. At present, about 15% of the projects have been completed, while about 60% of them are in the final stages of coding and field testing.

(B) Other ICON Software?

A software conversion project at O.I.S.E., under the leadership of Bob McLean, will have converted 30 public domain programs presently available for the Apple and PET into Waterloo BASIC. These programs will be available to boards now involved in the prototype field testing. In addition, the Ministry of Education has allocated $5.4 million in funds from BILD to stimulate educational software development in 1984-85.

The following languages and software products will be available for the ICON in 1984:

Waterloo microBASIC
Waterloo microPASCAL
LOGO
C
a word processor

Other languages soon to be supported include COBOL, FORTRAN, and APL.

(C) Waterloo BASIC Question:

Any software developed in Waterloo microBASIC for the ICON will run on any other micro that meets the Ministry's functional specifications. This BASIC (and the other programming languages called for in the specifications) must be implemented in such a way as to facilitate transfer to new or different computers (a concept known as "portability"). Existing versions of Waterloo microBASIC, on the Commodore SuperPET and IBM PC, are very close to the ICON version.

(D) Other Machines?

ICON softwre, written entirely in Waterloo microBASIC, will require very little, if any, modification to run on the other machines mentioned in (C) above. The reverse is also true (ie: programs written entirely in Waterloo BASIC on Commodore or IBM machines should also require little, if any, modification to run on the ICON). Of course, such programs could not include PEEKs and POKEs to memory locations of a particular machine.

(E) Cassette Recorder?

Software will be distributed in diskette medium. No cassette I/O is included.

4. **Distribution of Software:**

Boards will be notified as Lessonware projects are completed and ready for distribution. Boards will be able to order the programs on a cost-recovery basis. They may then reproduce and duplicate the materials according to their needs. Further information will be forthcoming about the specific

ordering and distribution method. It may be possible for boards of education to place an order for a large quantity of a particular software package, or packages, with the Ministry (or the developer) arranging for the reproduction.

Each Exemplary Lessonware Project will be available in a form compatible with one existing microcomputer (ie: PET, Apple, etc.). These materials will be converted to run on the ICON. Additional information about distribution is available from Luella Egerton, Education Officer in the Computers in Education Project.

5. **Compatibility with Other Micros:**

Software developed for the ICON (or any other Ministry-approved educational microcomputer) will not, in general, be portable to existing commercial microcomputers. There are two points to consider, however:

1. Programs written in Waterloo microBASIC will be portable (as described previously).

2. The floppy diskettes for the ICON are 5 ¼ inch, double sided, double density, 96 tracks per side, IBM format. This means that the ICON disk drives will be able to read programs stored on an IBM diskette. This fact, combined with (1), means that developers can use IBM PCs and Waterloo BASIC to create programs which can be read (and run) by the ICON. The reverse will also be true.

6. **Contacts:**

For more information about specific topics, contact:

a. ICON ..... CEMCORP
(416) 736-4550

CEMCORP
951 Alness Street
Downsview, Ontario
M3J 2J1

b. ICON Project Administration ..... Gary Morton

c. Software Development ..... Luella Egerton

d. Functional Requirements ..... Lorne Smith

The last three individuals may be reached at (416) 965-0692

Computers in Education Project
Ontario Ministry of Education
9th floor, Mowat Block
Queen's Park
Toronto, Ontario
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INTERVIEW WITH BROCK RACHAR

by
R. S. Smith

Brock Rachar is the Mathematics Coordinator for the London Board of Education. Brock is well known through his work as a text book author, speaker and his work with O.A.M.E./A.E.O.M., e.g. he was the first editor of ABACUS with its motto 'You Can Count On It'. I worked with Brock during the 70's on Curriculum, then Measurement and Evaluation Projects and have come to respect his thoroughness, his judgement as well as his concern for Mathematics Education. In this interview, we look at Problem Solving.

RS: Canadian Test of Basic Skills is perhaps the most used, standardized test for problem solving. What concerns do you have with respect to the CTBS?

BR: In terms of problem solving skills they lack the breadth of problem solving as it is viewed today. The CTBS tends to report a problem solving area in terms of the computational aspects of it. For example, a teacher might receive back information that a student is having difficulty with a problem that involves multiplying of decimals. This is not one of the problem solving skills, per se. It is certainly part of the whole picture but just one part.

RS: Yes, I agree. It is only a mechanical aspect then and not what you might think of as being problem solving. In fact, how would you define problem solving?

BR: I guess one of the neatest little definitions I've heard is that "problem solving is what you do when you don't know what to do".

RS: That sounds like a conundrum to me.

BR: It could be. I guess what is a problem for one is not necessarily a problem for another student. That's one of the difficulties in trying to devise a test for problem solving. Problem solving involves a number of skills. Sorting of information, finding if you have sufficient information, being able to represent your information in different ways, determining what is wanted, but not necessarily just following a pattern (as has been the practice in days gone by) doing a series of computations, say multiplying of two two-digit numbers followed by a series of 'problems' that incorporate multiplying of two two-digit numbers. And the question is, was this really a problem or was this simply a word exercise that follows up a computational skill?

RS: You were asked to do some work in setting up a new testing program that would look not so much at problem solving in terms of worded problems but to look at the problem solving skills. What have you done in that area?

BR: Most of the current texts and materials are trying to address other problem solving skills but the testing so far has not reflected this very well. We are attempting to develop a problem solving skills test to investigate and also to highlight the fact that these are skills that we are trying to give students experience with.

LONDON MATH PROBLEMS - 7

APPLICATIONS ↓ / SKILLS →	1. DECIDING ON OPERATION(S)	2. COMPUTING	3. CREATING	4. TOO LITTLE INFO. IDENTIFYING MISSING DATA	5. TOO MUCH INFO. IDENTIFY FACTS THAT ARE NOT NEEDED	6. INTERPRETING/ REPRESENTING	7. SYSTEMATIC TRIAL	8. SIMPLIFY USING PARALLEL, SIMPLER PROBLEM	9. RECOGNIZING ACCEPTABLE ALTERNATIVE SOLUTIONS	10. FINDING PATTERNS	11. IDENTIFYING REASONABLE ANSWER
A. MEASUREMENT	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓✓
B. GEOMETRY			?	✓	✓	✓			✓	✓	
C. ARITHMETIC (money)	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓✓
D. CATALOGUES CHARTS GRAPHS TABLES	✓	✓	✓	✓	✓	✓✓✓	✓	✓	✓		✓ ✓

RS: Please explain the chart that we have reproduced here.

BR: Across the top we have listed 11 skills which we see as relevant to problem solving, and then we have attempted to represent those skills in at least two or three of the measurement, geometry, arithmetic, or graphs/charts/tables/catalogues setting. We are trying to get at the skill through different contexts representing the main areas of math study.

RS: Illustrate your ideas using the content area of measurement and the skill of systematic trial.

Problem

John has a mass of 3 kg. more than Dave.
Sam's Mass is twice as much as Dave's.
All three together have a total mass of 163 kg.

		MASS IN KG.			
Here Are Solution Attempts	Dave's	John's	Sam's	Total	Decision
Trial 1	20	23	40	83	too small
Trial 2	50	53	100	203	too big
Trial 3					

THE BEST CHOICE FOR DAVE'S MASS ON TRIAL 3 IS:

a. 65 b. 35 c. 55 d. 15

BR: We want students to eventually become proficient problem solvers, to come up with accurate answers and refined solutions but we must also keep in mind that there is a process leading to this. We would like to give students experience in wrestling with the skills that are necessary and the sorts of decision making skills that are involved in problem solving. The example that you suggested here could be handled algebraically but, if it were, we would lose some of the significance of what is going on. Through systematic trial, we can start by making a guess (there's nothing wrong with making a guess in problem solving) that Dave has a mass of 20 kg. When we look at John's mass then, we relate it to that and say that's 23, Sam's mass is double Dave's mass, so that's 40 and we can easily calculate a total of 83. The important step is the next one. To make a comment on where we are in the problem. The student must decide here whether his guess was too big, too small or just right. He makes the decision that it's too small. It's this decision-making aspect that we should provide experiences in. We have avoided calling this 'trial and error' because it is not 'hit or miss', at least if his next step is to a larger number.

RS: And if, in fact, he has made a decision to continue the process.

BR: Yes. His next choice then is to try a bigger number. He's chosen 50, which gives him 53 for John and 100 for Sam, 203 kg. total which again requires a decision from him that his choice was too large. His next choice then must lie between these two and if it does, and probably he can even make a decision that it is closer to 50 than it is to 20 on the basis of the numbers he has come up with, then he can continue in the steps toward the answer.

RS: So by giving him an opportunity to do a number of these kinds of questions you are really helping him to build up a problem solving strategy which will be useful in many different kinds of problems.

BR: Yes, in many kinds because it has that element that it's being checked as he proceeds. He's getting a pattern, he's developing it, he's proceeding toward the answer.

RS: Good. I noticed also that in actually working through the problem yourself you talked about

the relationships in the numbers and it reminds me of a key idea. Reading in problem solving is a problem. The National Assessment of Educational Progress has tried to get around this by having a tape of the problems so that children not only see them in writing but also hear them read orally. Any comments?

BR: I think that reading has been a difficulty certainly, but I think it has been overstated. I think often it is that we require students to handle problems in ways that are unnatural to them. They are given certain clues that are more of the teacher's making, rather than letting them get involved right away in the problem. It's one of the reasons I favour systematic trial because once you've placed a guessed number in the context of the problem, it tends to clear the issue somewhat. I don't think it's that they can't read the individual words so much as failure to get the sense of what's going on.

RS: So it's the interpretation. It seems to me, Brock, that you had some other ideas about oral work when doing problem solving.

BR: Yes, the communication between students. For example, the possibility of working in pairs could address this difficulty. Often if students hear a problem read aloud they can internalize it better. Perhaps if a partner read the question aloud this would address part of that problem and I think your suggestion of the tape so that students receive the question in an oral form as well as in a written form might get around some of these difficulties.

RS: You have mentioned children speaking to themselves as they were doing problem solving, speaking out loud, working with a pencil, all of this as opposed to our thoughts of the past when children worked by themselves, in isolation, with perfect silence required.

BR: Yes, that's a good point you've suggested there. We have tended in the past to say "alright, you have ten minutes to do the following four problems. Go to work". Yet in experience with adults attempting to solve what might be classed as real problems (something that you don't just say, oh yes, it's one of those and jot it down), there is a need for 'muddling' time. Drawing, making sketches, talking about the problem, are all parts that are important to getting an internal concept of what's going on.

RS: I'm sure the best teachers are using these strategies in the classroom now. Standardized tests have shown that introduction of a new factor, for example, a fraction, or a decimal, causes the proficiency level to drop considerably. Why?

BR: I guess we have to be clear on what it is that we are trying to test. Perhaps we should try to avoid the complications of intricate number patterns, we should try to zero in on the skill that we are after, we should come at it from different directions so that we are seeing if this skill, in fact, is evident as opposed to trapping kids in issues that aren't directly problem solving ones.

RS: You are saying let's keep problem solving as our main direction and not the solving of the mechanics of fractions or decimals!

BR: Exactly. I think if I get a report back that says my children are having difficulty solving problems that involve multiplying two decimals I should be wondering 'what is that telling me about their problem solving abilities?'.

RS: Good. On the other hand, the complications of two step or multiple step problems you would still see as problem solving and, again, results in standardized tests show that two-step problems cause significantly more difficulty than one-step problems.

BR: Yes. I guess I'm not entirely happy with the arbitrary definition of difficulty in terms of how many steps the problem has. I think it may mislead us somewhat. No doubt some of the skills we have listed here may involve a number of steps but they approach the solution as exampled in the systematic trial model. While there are many steps in there, the approach, getting right down to numbers that generate themselves readily from the context of the problem gets around quite a bit of that.

RS: Surely, in terms of difficulty of mechanics, this is a place where the calculator could be used.

BR: Why not. If we are dealing with problem solving, then we re not after catching them on mechanical skills.

RS: Perhaps this is the place to consider the neatness of work, the completeness of work, the format in which the problem statement is written.

BR: I presume you meant the format in which the answer to the problem is written?

RS: Correct

BR: I guess my feeling is, or my philosophy is, that if a problem is posed and the problem is addressed or answered in a sentence that clearly states the answer and work accompanies that to show the steps that were used to arrive at the answer, this should be acceptable. A stereotyped format of answer that an individual teacher might expect during a specific training session would seem to be an unfair expectation in general. A test for grade 7 in problem solving must be based on the arithmetic skills, the experiences with geometry, measurement and graphs, etc., of the grades preceding it. We have to be careful to keep those aspects to a point where they don't obscure the problem solving skills that we are looking at.

RS: Let's turn in another direction. We have been talking so far about a printed, probably computer marked kind of test situation. But surely, there are other ways of obtaining information about the children's problem solving skills. I'm thinking about some of the work I did myself in geometry where it was done through an interview. What sorts of ways do you see?

BR: Well, first let me say that I think the testing of problem solving is extremely difficult and I'm not sure that it addresses what problem solving is all about. It's an attempt to sample from a number of skills and also to say, for a Board of Eduation where we do this system wide, that these are important aspects of problem solving. If we stayed with the old format that tested really only a small sampling of problem solving, we give a distorted view of what we mean when we talk about problem solving.

RS: So, in fact, we are broadening the whole understanding of problem solving with the teacher and examining this business of problem solving. It isn't simply a textbook, printed-word type of situation.

BR: Far from it. The best problem solving would be the situation in the class where kids solve problems: they talk about problems, they verbalize their thought processes, they share the techniques and ideas they have come up with, they build on their own native problem solving skills and teachers help them to see other ways.

RS: Sounds like the ideal classroom. Doesn't sound like the type of thing that is systems oriented which you can measure.

BR: No, we can only sample from it.

RS: Very good. Then it seems to me attitude is most important. How would you suggest we go about changing attitudes?

BR: Fortunately, there have been for years teachers who provided the atmosphere in a classroom which was non-threatening, which allowed students to fumble, if you like, to improve, to question, to try out different things. And that's really what we need if we are going to develop creative problem solving among students. It's alright not to get the right answer all the time. It's alright to explore some other strategies. It's alright to use the ways that they would have come to problem solving without an imposed format from the school. We don't want to discourage children from solving problems and we are not all equally good at solving problems. There should be no put down because a student is unable to solve problems well. On the other hand, many children enjoy the challenge. Beyond that, and in proper context, they can have this experience. In fact, we are preparing for a math contest this afternoon. These contests have involved some 350 Grade 7-8 students who come after school because of their desire to have fun in solving problems. Today, eight of the top school teams are competing. We look forward to a very exciting time.

RS: Having been in some of the classrooms where these teams are picked, I can assure you that there is great excitement in the classrooms as well. No doubt, that aspect rubs off on other students in the clssroom and in the school.

BR: Yes, and there are trophies that go back to give them a chance to talk up mathematics and

problem solving in the schools.

RS: Most worthwhile. We all would agree on the importance of drawing diagrams, and yet last year on a Gr. 8 perimeter question few pupils drew a diagram. I wondered why?

BR: Well, it's strange, but even those of us who have solved problems for years in various ways tend to avoid drawing sketches often, assuming that we can do it without. Here's a little example. Try this problem yourself.

The Problem:

Each book in a four-volume set is 6 cm. thick.
This includes the covers, which are 5 mm. thick.
As the books sat neatly on the library shelf a bookworm started at page 1 of volume 1 and bored his way straight through the set to the last page of volume 4. How far did he travel?

Choose the correct answer:

1 cm. 28 cm. 24 cm. 23 cm. 13 cm.

Tell why each of the other answers misses the mark.

BR: Finally, if as teachers we wish to have some input on the problem solving skills of our students, we have to provide lots of experience in conducive settings. We need to provide encouragement while levelling with the kids.

The following is a good message I believe.

YOU CAN SOLVE PROBLEMS

Many problems are solved outside of school.

Some problems take a long time to solve.

Some problems have not been solved by anyone yet.

Some people are particularly good at problem solving, but most of us have to work at it.

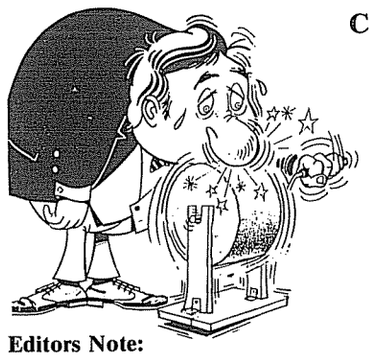
Most people can solve many more problems than they think they can, if they don't give up.

Thomas Edison, the inventor of the electric light bulb, carried out 100 experiments trying to find a battery that worked. All 100 experiments failed. One man said, "What a waste of effort!" But Edison replied, "Not at all. I now know 100 things I will not have to try again."

Like Edison, you should keep good records of the tries that don't work.

YOU CAN SOLVE PROBLEMS

RS: Thank you very much, Brock, for an interesting dialogue about problem solving.



COUNCIL COMMENT

by
Bob Robinson

Editors Note:

Council Comment appears in an issue of the Gazette following OAME/AOEM Council Meetings. Your comments are welcomed. The last meeting of Council was held 1983 10 15 at the Ramada Inn (Airport West).

CHAPTER REPORTS

Note: For the name and address of the chapter representative in your area consult the September, 1983 of the **Ontario Mathematics Gazette**.

• COMA

The September, 1983 COMA Newsletter outlines and summarizes the **Mathematics Interface Project** and offers some recommendations. Other topics discussed are "Grade 13 Relations and Functions - Student Perceptions"; "Reading Skills and Test Type for Grade 13 Students"; "The COMA Examining Board"; "Recommendations to the Provincial Mathematics Curriculum Committee re Calculus in Grade 13"; "Mathematics Curricula at the Interface" and "First-Year University Mathematics Courses in Other Schools". For further information contact the COMA Representative or Don Fox at the Ontario Ministry of Education regional office in Ottawa.

• GOLDEN SECTION

A meeting was planned to be held 1983 10 19 from 16:30 - 21:00 at Brock University with Dave Alexander being the guest speaker.

• GVMA

A fall meeting/conference was planned for 1983 11 10 at the University of Waterloo.

There are three new publications available: A Survey Test In Mathematics (Grade 9-12 Advanced); Mathematics For Life; and a related Problems Supplement.

• NOMA

A very successful conference was held in Sudbury on 1983 09 30 and 1983 10 01.

• PINE RIDGE (PRMA)

This chapter is going to be involved in organizing mathematics contests in the region, producing the Durham Mathematics Newsletter, and assisting with the planning of a Careers Night for Durham students and parents.

• QSLMA

The main thrust of the chapter this year is the planning and operation of OAME '84 to be held in Kingston, 1984 05 10-13.

A General Meeting was planned for 1983 11 03 at the Queen's Faculty Club with four sessions encompassng the kindergarten through post secondary levels.

• **SAME**

A fall mini-conference was conducted on Tuesday, 1983 10 18 at Maplewood Vocational School. Math workshops are planned for Tuesday, 1983 11 19, 1984 01 17 and 1984 04 03. These will be held at Howard Johnson's Hotel in Scarborough from 16:00 to 18:00. A Professional Development Day is planned for Friday, 1984 02 10 with the keynote speaker being Lorna Wiggin.

A special contest has been launched to have students in grades 5 through 13 design a new logo for SAME.

• **TORNY**

TORNY had four successful mini-workshops last year. They are planning to have three (perhaps four) activities during 1983-84. The first activity was held on 1983 10 19 at the Ontario Science Centre where J. A. H. Hunter, the creator of Fun With Figures in the Globe and Mail, was the guest speaker.

The second event will be a Computer Activity Workshop at the Ontario Science Centre in November. The third activity will be a multicultural workshop on various mathematical topics such as tangrams (China) and origami (Japan), etc. A possible fourth seminar may be offered in April on financial planning with emphasis on investment and taxes.

• **LEADERSHIP SEMINAR**

Approximately 60 people attended the seminar held once again this year at the Ramada Inn (Airport West). Thanks go to Sharon and David McPhail as well as the always dependable MacGregors.

• **ANNUAL MEETING - 1984** (1984 05 10-13, Kingston)

Elaine Harvey, the Conference Chairperson, promises several innovations this year. Many of you hve already seen the hockey stick promotion items. Quantity supplies of these for promotional purposes are available from David Handley at Belleveille Collegiate Institute. Registration forms will be available shortly and the deadline for early registration is 1984 02 29. The organizers hope to provide a list of registrants for all participants. From all accounts it looks like an exciting program with many worthwhile social activities thrown in as well.

• **ANNUAL MEETING - 1985**

Plans are underway for the '85 OAME Anual to be held in Sudbury. They are planning to make it a gala occasion.

• **MEMBERSHIP DUES**

Council approved to increase the membership dues for the association to $18.00 efective 1984 01 01.

• **SUBSCRIPTION RATES FOR THE GAZETTE**

Council approved the following subscription rates for the Gazette: Canadian $12; American $14; outside continental North America $16.

• **EDUCATIONAL TELEVISION**

The new geometry series of 8-15 min video tapes entitled "Landscapes of Geometry" are now available from OECA. Their excellent programs are suited for the intermediate grades and were written for the most part by OAME members. A teacher's guidebook is also available. Have a look!

• **NCTM ACTIVITIES**

New position papers pertaining to Research and Mathematics Education, Vertical Acceleration, The Mathematics Teacher Shortage and Retraining Programs, are available from NCTM. There are

several new publications available as well. Winnipeg is hosting a regional meeting of NCTM in October, 1984. The NCTM representative is Ralph Connelly at Brock University.

• SCHOLARSHIPS AND AWARDS

The McMaster University Scholarship was won by Tracey M. Gardiner, a student at Waterford District High School.

The McCartney Scholarship was awarded to Neale Ginsburg from Loyalist Collegiate in Kingston.

The Father Faught Award was won by Woburn Collegiate in Scarborough.

• CANADIAN MATHEMATICS OLYMPIAD

Tom Griffiths is serving on this committee and invites input from the membership.

• INTERNATIONAL MATHEMATICS OLYMPIAD

Disappointment was expressed regarding the press coverage of this olympiad.

• MEMBERSHIP

It was suggesed that each chapter of OAME elect or appoint a membership chairperson and a chapter plan be devised to find new members. Letters have been sent to members of NCTM who are not members of OAME in hopes that these mathematics educators will also join the provincial mathematics association. All mathematics educators should seriously consider membership in both of these fine associations - OAME and NCTM.

• SENIOR GUIDELINES

OAME will recommend to the Ministry of Education that the teaching of Business Mathematics courses be the responsibility of the Mathematics Department of each school.

• COMPUTERS

Plans are underway to gather information from the membership regarding what software we would like to see come out when the new contracts are granted.

Getting practical updated information to the teachers and administrators on the status of the Icon and its software was discussed.

• BILL 82

OAME has approved a position paper on Bil 82 which is published elsewhere in this issue of the Gazette. Elaine Harvey is commended for her hard work in heading up this project.

• PROBLEM SOLVING COMMITTEE

This committee is planning to meet once a month. A systematic review of recent interesting publications on problem solving has been undertaken and the results will be made available to the membership at a later date.

• NOMINATIONS

All nominations for positions of Councillor, Director and President-elect should be submitted to Sharon McPhail no later than 1984 01 01.

• NEW STANDING COMMITTEE

For the next council year (1984-85) there will be a new standing committee which will focus on math anxiety and issues involving women and mathematics.

NEXT MEETING OF COUNCIL - SATURDAY, 1984 02 11

September 28, 1983

Dear Mr. Pravica:

Your "Letter to the Minister" in the September issue of the Gazette provides an excellent opportunity for addressing issues specific to the mathematics curriculum. Since curriculum development in our era of rapid change must be considered as a continuous process, it is important that all those responsbile for implementation understand not only the substance of a specific change but also the intent of that change. As Project Leader for the renewal of the mathematics curriculum for the Intermediate and Senior Divisions, I should like to deal with the questions you have raised.

Your first question refers to a model for curriculum development and implementation. The Ministry has such a model which is described in **A Guide to Implementation, Suggested Planning and Operating Procedures for the Ministry of Education**, October 1982. This document was introduced to a broad spectrum of educators at a program seminar on November 29, 30, 1982, and is available through the Regional Offices of the Ministry.

With reference to your other questions:

1. How does the Minister propose to ensure that these two components (the establishment of stable fundamental foundations and the provision of carefully connected procsses by which curriculum change and renewal can occur) are part and parcel of the mathematics curriculum development model?

I believe that "the fundamental foundations on which secondary education can be based" were clearly stated in the Goals of Education **(Issues and Directions,** 1980). These goals are reemphasized in **Ontario Schools, Intermediate and Senior Divisions** and provide a basis for all curriculum development. The planning and operating procedures of the Ministry as described in **A Guide to Implementation** make clear the cyclic nature of review, development, and implementation. In the case of mathematics, the Senior Division Review, 1980, and the Secondary Education Review Project provided direction for the mathematics curriculum development project. The Curriculum Project Management Team for the Intermediate/Senior Mathematics Guideline has drafted an implementation plan which, when it has been approved, will provide a basis for regional offices and school boards to plan for the "curriculum change and renewal" initiated by the current curriculum development project. This plan will include a review of the effectiveness of the mathematics guideline in 1990 or shortly thereafter which may initiate further revisions in the mathematics curriculum.

2. How will the mathematics guideline address this concern (micro-technology and our understanding of what learning is, as well as how it must occur)?

It is the responsibility of the Project Committee to respond to this concern. I would note that the Ministry has funded a number of research efforts related to micro-technology and learning and it may be premature for curriculum guidelines to be overly specific on this topic untl we have a clearer picture of the nature of the change in our understanding of learning and in how learning must occur. The Computer Studies Guideline, June 1983, provides this guidance:

"Since the use of computers in the classroom is relatively new, their potential in the learning process is not fully known. Teachers are encouraged, therefore, to explore the capabilities and applications of computers and to share their experiences and understandings with their fellow teachers."

3. What are the Ministry's plans for ensuring that these notions are successfully translated into classroom practice?

Until the "changes in instructional strategies, materials and standards" have been established through the curriculum development project, it is premature to plan for their implementation except as they form part of the total implementation plan. The successful translation of any change in

classroom practice depends primarily on the commitment of teachers to undertake such a change. This commitment can ónly be achieved through the goodwill and cooperation of organizations such as the Ontario Association for Mathematics Eduction, board officials responsible for curriculum implementation, regional office personnel, and most importantly, teachers. The Ministry alone cannot "ensure" change in classroom practices and does not attempt to do so.

4. Does the Ministry intend to continue its traditional implementation policy?

A Guide to Implementation identifies the responsibility of school boards to work cooperatively with the regional offices of the Ministry of Education to implement curriculum policy and the responsibility of the Regional Services Secretariat with the assistance of the Curriculum Project Management Team to monitor the implementation activities. The Ministry will establish a detailed plan of action for its role in the implementation. Teachers who are concerned about a detailed plan of action for their school board might volunteer to assist board officials in drawing up such a plan.

5. How will the Minister monitor the procedures to ensure that both the policies and the spirit of the new guidelines will be reflected in programs and courses offered to prospective teachers?

As you are aware each Faculty of Education undergoes a periodic review of its programs. I am confident that this review process and the commitment of the staffs of the faculties to providing quality teacher education ensures tht the spirit of new guidelines is reflected in courses for prospective teachers. With respect to the Intermediate/Senior Mathematics Guideline the proposed changes are unlikely to require much accommodation on the part of the Faculties of Education.

I was disappointed that you feel that the Faculties of Education have not been sufficiently involved in the current revision since I have welcomed the opportunity to exchange views with all interested groups and, as you are aware, have met with representatives of the mathematics professors of the Faculties of Education and responded to their written submissions. I believe my secondment from a Faculty to lead this project reflects a full appreciation of the Ministry of the important role the Faculties can play in curriculum development. As regional implementation plans are created, the role to be performed by the Faculties of Education will be clarified.

Thank you again for the opportunity to discuss some of the issues of implementation. Your help in the development of excellence in education in Ontario is most welcome.

Yours truly,

David W. Alexander
Education Officer
Senior and Continuing
Education Branch

Son of OAIP arrived in the mail yesterday. When teachers asked me whether they should take the time to read the first version, I advised them to wait until it was made into a movie.

I have been eagerly awaiting this latest version, therefore, not because the first was so enjoyable, but because my colleagues who teach in other subject areas have been assuring me that the OAIP material which they have received is eminently useful both in planning and evaluating their programs. Alas, I find the sequel as disappointing as the original and have decided to write this letter before we receive OAIP three-D.

There have been definite improvements in the way in which test-items have been field tested and the results reported and explained. It is obvious that a great deal of thought, planning and effort was invested in the testing and reporting of the suitability of the various test items for classroom use. The problem is that the items themselves render the entire document useless.

Classroom tachers do not need assistance in writing questions to test low level mathematics skills. If OAIP is to be of any use, it should provide creative test items which can aid the teacher in assisting student understanding of important ideas in the subject. Finding such questions is the real problem in creating a test to determine the progress of a student in learning mathematics.

We do not really need questions like "express 2/37 as a decimal correct to three places" (Page 58) or "solve $2(57 + 6s)/3 = 2$ (page 138).

Furthermore, the language or notation of some items is of low quality or of such ambiguity that they should never have appeared in print. What self-respecting grade VIII teacher would ask "75 out what number is 105%?"

How many teachers write ¾/¼ to test division of fractions? (page 48).

What teacher would use #5, page 112, to test the concept of perimeter? "What is the perimeter of an isosceles triangle with two sides 1 cm. and the third side 2 cm.?" I may be wrong; but a look at the results indicating that this was the easiest of the six items testing perimeter, has led me to conclude that the answer 4 cm. was accepted as correct, rather than the answer that this figure is not a triangle at all.

Consider #4, page 128, "Using the variable "n", write an expression which means "a number divided by 44, then diminished by 28". Dividing a number by 44 and diminishing it by 28 are two separate operations. Each time you start with the number. I cannot be certain, but I believe the author meant "a number divided by 44 and the result diminished by 28".

Leaving the quality of the test items aside, why was so much time and money spent on assessing student performance on skills which are not stressed in the classroom? Time after time test interpreters tell us that poor performance is not surprising, since the skill being tested is not important [Page 31, factors and exponents; P. 43, "performance might have been better if the item were 9.9Γ28.6 rather than 28.6/9.9", P. 47, 3 4/5 x 3 2/3, and more].

Spending resources working on less important portions of the program would be no great sin if the important ideas had been thoroughly covered. There is nothing in the document which looks at student understanding of place value; simple applications of per cent like discount and taxes; simple applications of perimeter, area and volume; and more.

Anyone who has read this far might be saying "So what? So you don't like the latest edition of OAIP? So why don't you just put it on the shelf, keep your mouth shut and ignore it, the way you did the first one?"

There are three important reasons.

(i) The document as it exists is not only useless but potentially dangerous. Picture the teacher who has spent twelve years teaching grades three and four, only to be bumped by the relentless advance of French Immersion to a position which includes the teaching of mathematics in grade seven.

If she looks to OAIP for guidance in program planning, she will get the wrong idea of what the mathematics is all about. She will implement a program emphasizing one-step skill tasks, (some of which are not even important) with little time spent on problem solving or developing concepts.

If she looks to OAIP for guidance on what standard of work can be expected from grade seven students, she will be satisfied, and allow her students to be satisfied, with tasks which do not demand the best of their abilities.

(ii) Statements by the Minister and her advisors suggest that the OAIP Pool will be used to measure the effectiveness of the programs of the various academic disciplines in the schools of Ontario. An assessment of mathematics education based on this OAIP document would be a disaster. It would

not provide accurate information of genuine achievement. It could lead to a distortion of the program, an overemphasis of low-level skills at the expense of developing problem solving.

(iii) Perhaps I am wrong. Perhaps while for me the latest instalment of OAIP has been a source of frustration and anger, for others it has been embraced, devoured, cuddled and admired. Perhaps while I was moved to write a letter to the Gazette, others have been moved to write to their area Ministry office for more copies.

Either the Gazette or some other medium should become a forum through which the community of mathematics teachers in Ontario can reach a consensus on the important issues of the day.

If there are teachers out there who disagree in whole or in part with my thoughts on this matter, let them state their ideas so that everyone may consider all sides of the argument. If there are teachers out there who agree with me, let them express their ideas so that the community, through informed opinions and collective activity, can achieve some influence over the direction of policies which have such an effect on our professional lives.

In either case we can do better.

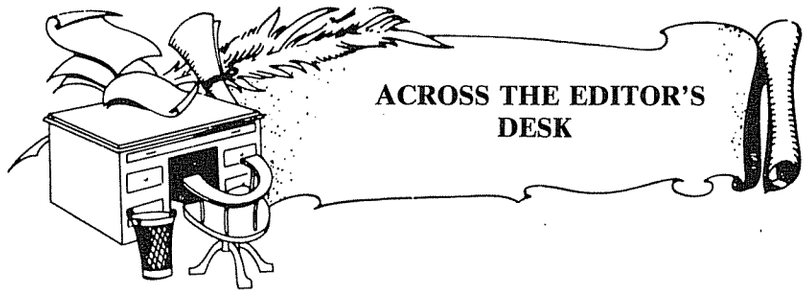
Doug Henderson,
Master Teacher of Mathematics,
The Sudbury Board of Education,
200 Brady Street,
Civic Square,
Sudbury, Ontario P3E 5K3

P.S. I don't believe that any good which might come out of OAIP could possibly be commensurate with the money and time which must be invested. Perhaps someone would like to express an opinion on whether the entire project should be aborted.

Editor's Note:

We should strongly consider Doug's request for comments because at a recent meeting re the implementation of OSIS it was strongly suggested that the Ministry fully intends to expand OAIP to be an instrument for the evaluation of the new OAC's. Could it be the Ministry's intent to introduce a new notion for evaluation ... The Provincials?





FURTHER ON THE IRRATIONALITY OF $\sqrt{2}$

A note on the Irrationality of $\sqrt{2}$ by Claire Zeller, The Ontario Mathematics Gazette, Vol. 22, No. 1, Sept. 83, prompts me to add the following proof, which is a departure from the usual proof displayed in text books.

A rational number can be expressed as

(A) a terminating decimal, or
(B) a repeating decimal.

(A) Let $\sqrt{2}$ be a terminating decimal with three decimal places
$2 = 1.a_1a_2a_3 \neq 0$

Write this in fractional form

$$\sqrt{2} = 1 + a_1a_2a_3/1000$$

$$1000\sqrt{2} = 1000 + a_1a_2a_3$$

Squaring $2\,000\,000 = (1000 + a_1a_2a_3)^2$

This is a contradiction, since the unit's digit on the left hand side is zero and on the right hand side is not zero. $a_3 \neq 0$

The same contradiction exists for
$\sqrt{2} = 1.a_1a_2a_3\text{-----}a_n$

(B) Let $\sqrt{2}$ be a repeating decimal with three decimal places being repeated.

$$\sqrt{2} = 1.a_1a_2a_3\ a_3 \neq 0$$

Write this in fractional form

$$\sqrt{2} = 1 + a_1a_2a_3/10^3 + a_1a_2a_3/10^9 + \text{---}$$

$$= 1 + a_1a_2a_3/10^3\,(1 + 1/10^3 + 1/10^6 + \text{---})$$

Using the formula $S\infty = \dfrac{a}{1 - r}$

$$\sqrt{2} = 1 + a_1a_2a_3 . \frac{1}{10^3} \Big/ \frac{}{1 - 1/10^3}$$

$$= 1 + \frac{a_1a_2a_3}{(10^3 - 1)}$$

$$\sqrt{2}\,(10^3 - 1) = (10^3 - 1) + a_1a_2a_3$$

Squaring

$2\,(10^3 - 1)^2 = [(10^3 - 1) + a_1a_2a_3]^2$

This is a contradiction since the unit's digit on the left hand side is 2, and on the right hand side is not 2 since the unit's digit os a square is 0, 1, 4, 5, 6 or 9.

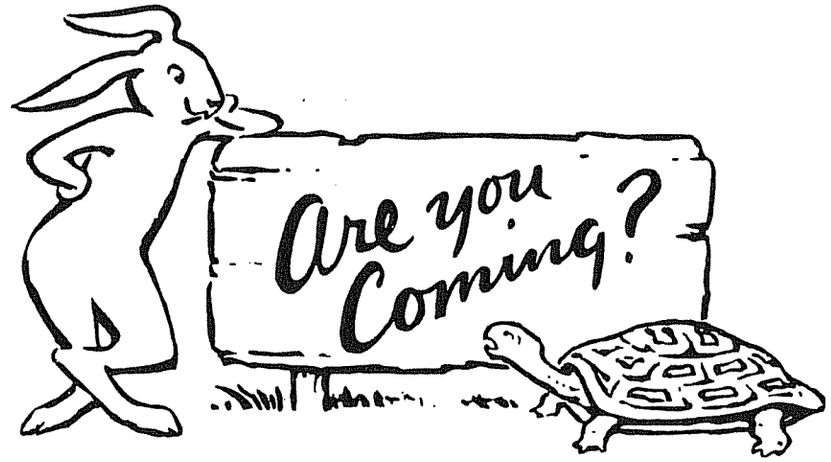
The same contradiction exists for $\sqrt{2} = 1.a_1a_2a_3\text{-----}a_n$

Hence $\sqrt{2}$ is not a rational number in the form of (A) or (B). It is irrational.

Garnet J. Greenbury, 123 Waverley Rd., Taringa. 4068. Brisbane. Qld. Aust.

Correction to A Note on the Irrationality of $\sqrt{2}$ in September, 1983 Issue, Page 29.

$\sqrt{2} \varepsilon Q$ should be $\sqrt{2} \varepsilon Q$ and $b \varepsilon Q$ should be $\sqrt{2} \varepsilon Q$.



OAME '84

Next May, mathematics educators from across Ontario, along with some of their colleagues from Quebec and New York State will descend upon the City of Kingston for the eleventh annual conference of OAME (Ontario Association for Mathematics Education).

Sessions, lectures and workshops begin on the afternoon of Thursday, May 10th and will run until noon on Saturday May 12th. Over 80 persons will be speaking on topics ranging from problem solving, enrichment, women in mathematics, the new Intermediate/Senior guidelines to geometric constructions using only a ruler!

On the social side: there will be a wine & cheese party after Thurday evening's Keynote Speaker; a Banquet & Dance on Friday, May 11th; and a tremendous choice of activities for Saturday and Sunday - including Dinner and/or Lunch at the General Wolfe Hotel on Wolfe Island; Dinner aboard the "Canadian Empress" while cruising on the St. Lawrence; Tours of Kingston & Environs; and much, much more!!

Participants are encouraged to bring their families - child care facilities will be provided, and, if there is sufficient demand, special programmes for children will be arranged.

Advance Registration is $35 (for OAME members) before February 29th. LEAP IN BY LEAP YEAR'S DAY!! Accommodation (complete package - 2 nights room and board at Queen's, inc. Banquet) - less than $80. Family Accomodation will be available for $35 - $60 per night at local motels. For more information, or to receive a registration form, WRITE: Dave Handley, Publicity Coordinator OAME '84, c/o BCIVS, 230 John St., Belleveille, Ontario K8N 3G1. Phone: 613-962-9581



Problems

Editor's Note:

December's problems come from an article in mu alpha Theta's newsletter, The Mathematical Fog. The article, by Don Allen (Truro, Nova Scotia) gives 12 examples of Victorian-era story problems. They are fascinating - unfortunately space restrictions allow for only three of them to be present here, but you'll get the "flavor" of the times from them. Answers will appear in the March issue.

1. On the Road to London

A and B travelled on the same road, and at the same rate, from Huntingdon to London. At the 50th mile stone from London A overtook a drove of geese, which were proceeding at the rate of three miles in two hours; and two hours afterwards he met a stage waggon which was proceeding at the rate of nine miles in four hours. B overtook the same drove of geese at the 45th mile stone, and met the same waggon exactly 40 minutes before he came to the 31st mile stone. Where was B when A reached London?

2. A Packet from Dover

A packet sailing from Dover with a fair wind arrives at Calais in two hours; and on its return, the wind being contrary, it proceeds six miles an hour slower than it went. Now, when it is half way over, the wind changing, it sails two miles an hour faster, and reaches Dover sooner than it would have done had the wind not changed, in the proportion of 6:7. Required the rates of sailing, and the distance between Dover and Calais.

3. Two Sportsmen Went Shooting

Two sportsmen A and B went out shooting and brought home 10 birds. The sum of the squares of the number of shots was 2880, and the product of the numbers of shots fired by each was 48 times the product of the numbers of birds killed by each. If A had fired as often as B and B as often as A, then B would have killed 5 more birds than A: find the number of birds killed by each.

ANSWERS TO SEPTEMBER PROBLEMS

1. If the Cheshire Cat ate the salt, then all three are lying, so this possibility is out. If Bill ate the salt, then all three of them are telling the truth, so this possibility is out. Therefore, the Caterpillar must have eaten the salt (hence, also, the first two statements are lies and the third is the truth).

2. There are four boys and three girls in the family. Tony has three brothers and three sisters; Alice has four brothers and two sisters.

3. Exactly three right is the same thing as exactly one wrong, so the choice is between exactly three right and exactly two right. Now, it is impossible to get exactly three right, because if three are right, the fourth must also be right. Therefore, she got exactly two right.

4. Between the first stroke and the sixth stroke there are 5 intervals of time, and it takes 30 seconds to cover those 5 intervals; so the interval between any 2 consecutive strokes is 6 seconds. Now, between the first stroke and the twelfth, there are 11 time intervals; so it takes the clock 66 seconds.

5. If the speaker were telling the truth, then he would be Tweedledum and he would be carrying a black card, but he cannot be speaking the truth and also carry a black card. Therefore, he must be lying. This means that his card really is black, and since his statement was false, he is not really Tweedeldum carrying a black card; so he is Tweedledee carrying a black card.

****** NOTICE ******

OAME ANNUAL MEMBERSHIP FEE

$18.00

Price effective January 1, 1984

FATHER FAUGHT AWARD

Congratulations to Woburn Collegiate in Scarborough for an outstanding achievement in winning the Fermat Contest.

Team members: Alexander Pak (score 145), Jovanni Sy (124.50), and Moshin Lee (116.50) are all students in the school's gifted program and wish to pursue careers in Mathematics or computer science.

BOOK REVIEW

MATHEMATICS FOR LIFE

Jack Weiner. Published by the Grand Valley Mathematics Association Paper, 125 pages, $6.00 for members of GVMA, $8.00 for others.

As usual, my desk is cluttered. But, like the screen during the running of a computer program stuck in an endless loop, the clutter never leaves; it just moves around. At the moment, the papers, files and books on my desk deal with new Ministry guidelines, falling enrolment, examinations, meetings, memos and notes to myself. Next time through the loop, the pattern will change, but the essential components will remain the same.

"Mathematics for Life" doesn't fit any of the stuff that is normally on my desk. Without jargon, without embellishment and without apology, it recounts the efforts of one teacher, in his attempt to bring into being for his general level students, a program which is at the same time demanding and achieveable, practical and mathematical.

A list of chapter headings would not do justice to the author's intent. Briefly, it explains his approach to a unit in mathematics of finance, including annuities, mortgages, bonds, stocks and life insurance.

For the copyable material alone, it would be a worthwhile purchase in preparation for the units in mathematics of finance contained in the coming Ministry guidelines at both the general and advanced levels.

But, the author and publishers are not trying to lighten our load by giving us material from which we can run off thirty-five copies and then take it easy. The book seeks to foster a spirit of creation rather than emulation. In the words of Jack Weiner "mathematics for life is an attitude and not a program".

I have no hesitation in recommending it as useful reading for all teachers of mathematics, regardless of the ability level of their students.

Jack ends the book with some excerpts of students' year-end evaluation of the course.

"The course was great. I learned a lot of practical information and a more positive attitude towards maths. Thanks a lot, Mr. Weiner, and God bless."

To which I add my own.

"The book is great. I learned a lot of practical information. But, better than that, I was reminded again that, when you cut through the morass of guidelines, memos, meetings, policies and regulations which surround and distract us each day, teaching consists of helping the young people with whom we work to find and bring out what is best in them. Thank you, Mr. Weiner, and God bless.

Reviewed by

Doug Henderson,
Master Teacher of Mathematics,
The Sudbury Board of Education.