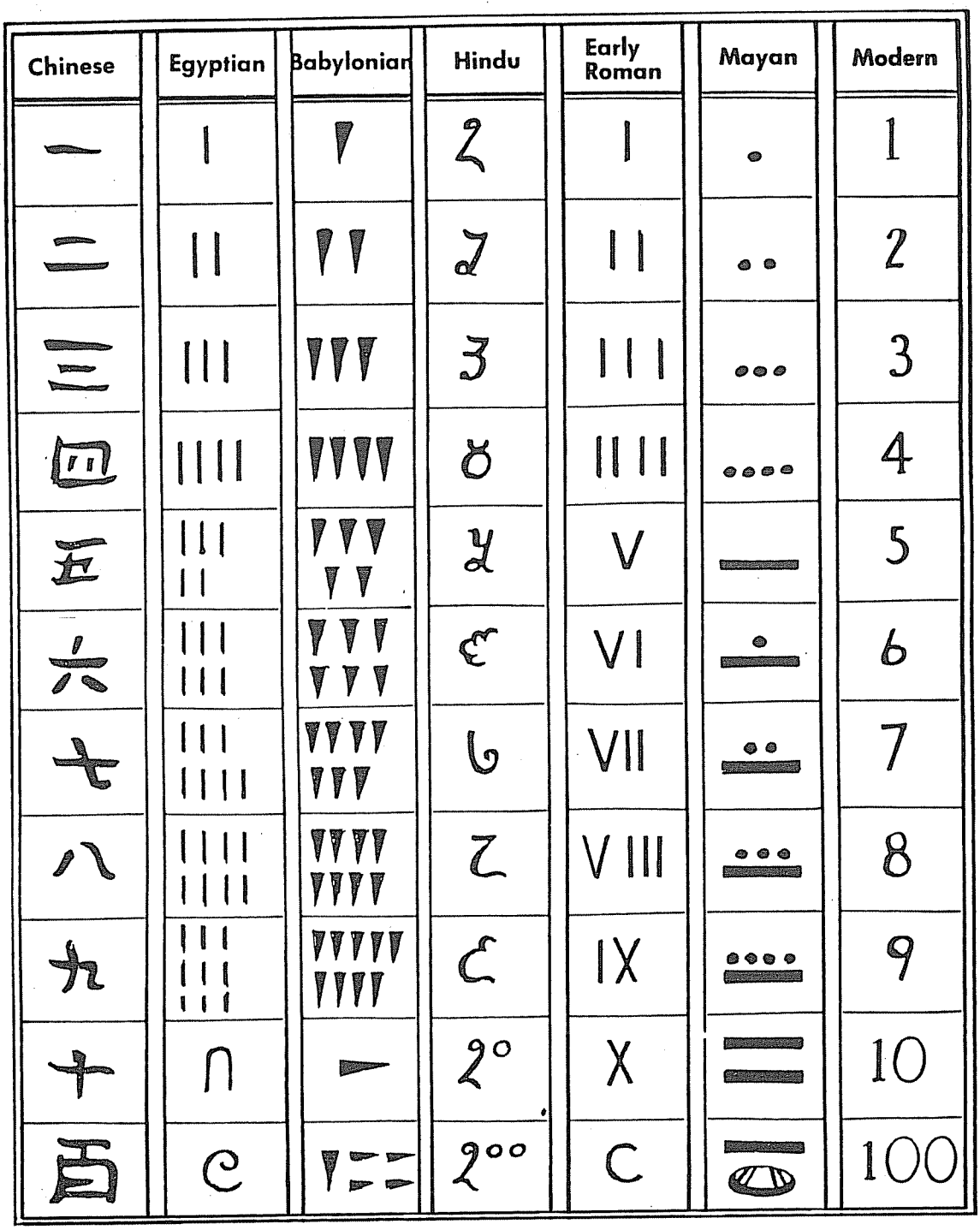
# ontario mathematics gazette

VOLUME 26 NUMBER 1 SEPTEMBER 1987



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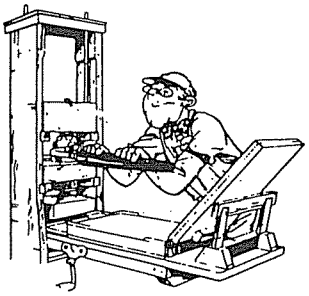
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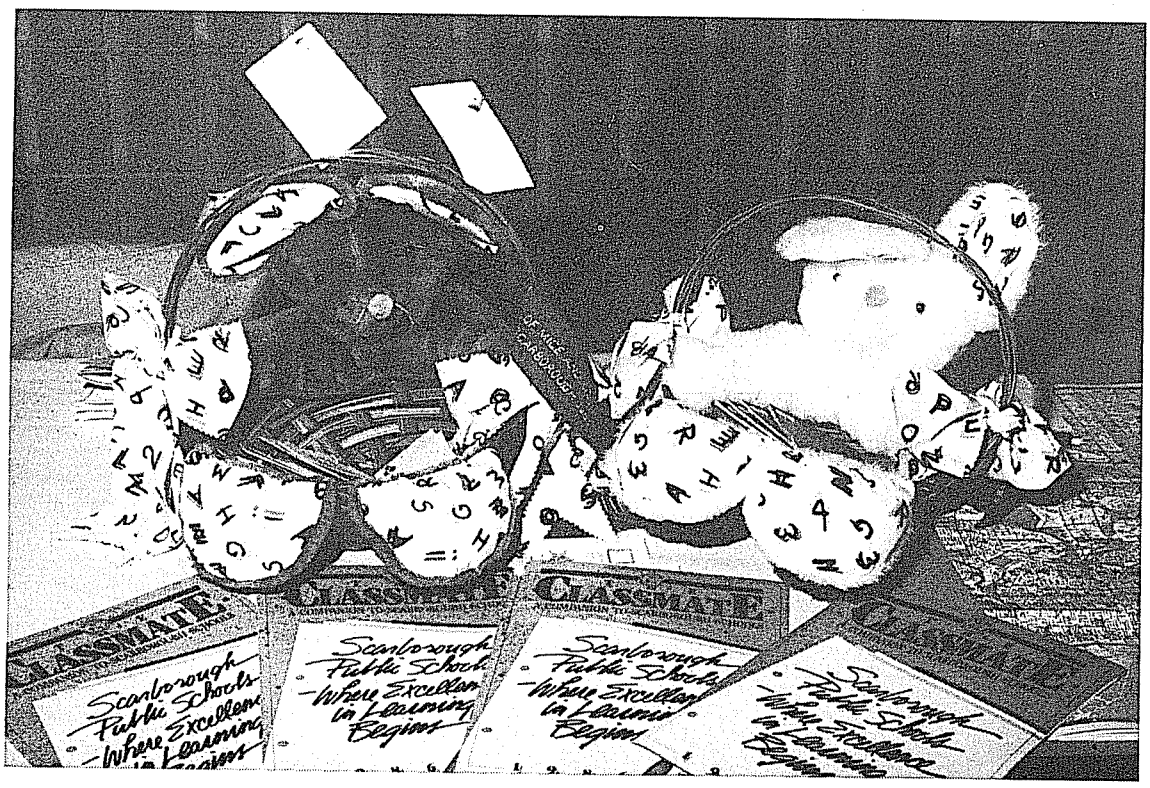



It is a pleasure to welcome OAME members to this September issue of Gazette. As one can tell from our articles this is an historic time of evaluation and evolution for OAME. This year marks a balance point, a fulcrum for possible adaption and change, on which we pause to recollect the past and marshall our energies for the future.

At such a time as this, it is important that all members contribute their voices and visions to the process of consolidating OAME's future focus; and what better means to facilitate this process than the OMG. There is no more efficient way to communicate your ideas to other members across the province or to express them to the Executive. So dear readers, sit down at your typewriters or personal computers today and let us know what you are thinking about the Gazette, the OAME, or mathematics and the great world beyond.

Clifford J. Gravelle

# SAME: OF MICE AND MATH
# 1987





## PRESIDENT'S MESSAGE

*by*
*Ralph Connelly*

"Good Grief He's back again!"

This might well be the reaction of those who thought (hoped?) they had heard the last from me with the end of my term as **Gazette** editor. I'd like to think of it as evidence that "you can't keep a good man (person) down," while others might observe, more crudely but equally accurately, that "cow chips keep floating to the top."

At any rate, I'm looking forward to my year as OAME President and hope I'll be able to carry on the excellent work of my predecessor, Dino Dottori.

The Committee structure of OAME Council will remain as it was last year. Indeed at the first Council meeting for the year in June, all the Committees busily set about following up initiatives from last year and establishing new goals for the coming year; it was without a doubt one of the most productive June Council meetings we've ever had. My thanks and congratulations to all the councillors for their efforts, and for their patience in "breaking in" the new President.

Congratulations are also in order for the organizers of OAME '87 in Scarborough. The Conference was an overwhelming success: good organization, good attendance, good sessions, good parties, .... what more can be said? We all look forward now to OAME '88 in the Niagara Region.

Dino already has the plans for the Leadership Seminar (Oct. 22-24, 1987) well under way and it promises to be an outstanding one.

This issue will mark the debut of Ron Ripley as editor of the ABACUS. Welcome aboard, Ron. On behalf of the membership I'd like to once again extend out thanks to Trevor Brown and Lorna Morrow for the outstanding job they've done as ABACUS editors over the last few years.

1987/1988 is shaping up to be a very busy year for OAME. We've given ourselves much to accomplish, but working together; the membership, chapters, Council and the Executive, we'll make it!

# MAGIC SQUARES AND ART

*by*
*Todd Romiens*
*Windsor Board of Education*
*Windsor, Ontario N9A 6K1*

A **MAGIC SQUARE** is an array of numbers such that each row, column and diagonal add to the same number. The number of rows and columns indicate the dimensions of the square. If the square has 3 rows and 3 columns it is said to be a 3x3 (read 3 by 3) magic square. The sum of each row, column and diagonal is known as the **MAGIC NUMBER.** In this article only basic magic squares will be considered. That is each square will consist of whole numbers, in sequence, starting with the number 1.

The basic 3x3 magic square is as follows:

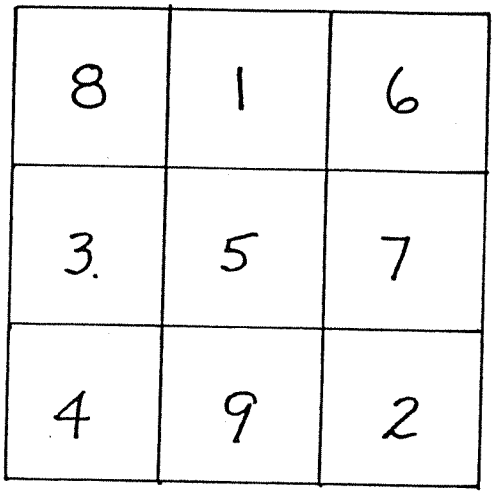



*FIGURE 1*

The magic number for this square is 15. There is only one basic magic square with these dimensions. (reflections, rotations, etc. of the square do no count as being different).

There are techniques beyond trial and error to generate magic squares of different dimensions. For example, consider the following method for determining magic squares with odd numbered dimensions. (3x3, 5x5, 7x7 . . ., etc.).

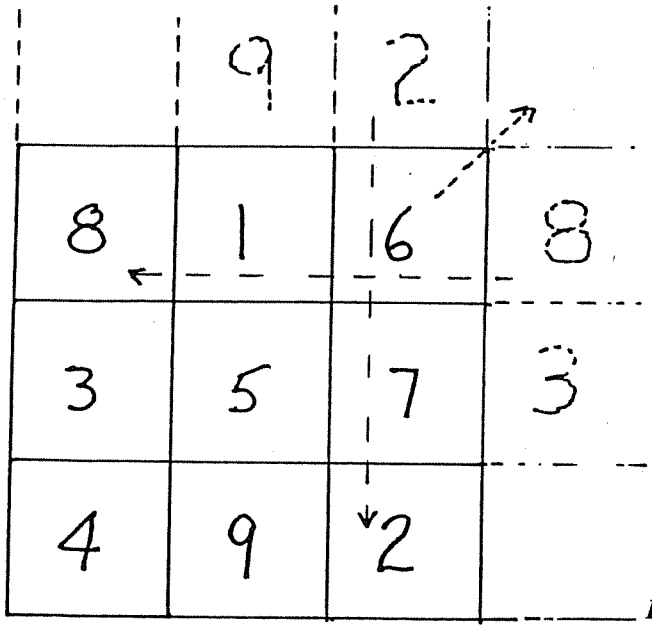



*FIGURE 2*

Begin by placing a 1 in the middle position of the top row. The rule now is always more diagonally to the right to fill in the next number (2, 3 - - -9). If there is no space on the magic square diagonally to the right, envisage a sqaure

above and to the right of the one you are filling in and place the number in the corresponding position in the square you are completing (see figure 2). When the position above and to the right is filled by another number (like 3 to 4 in figure 2) or if you would be off all squares (like 6 to 7 in figure 2), then "drop down" and continue.

Creating a 5x5 magic square:

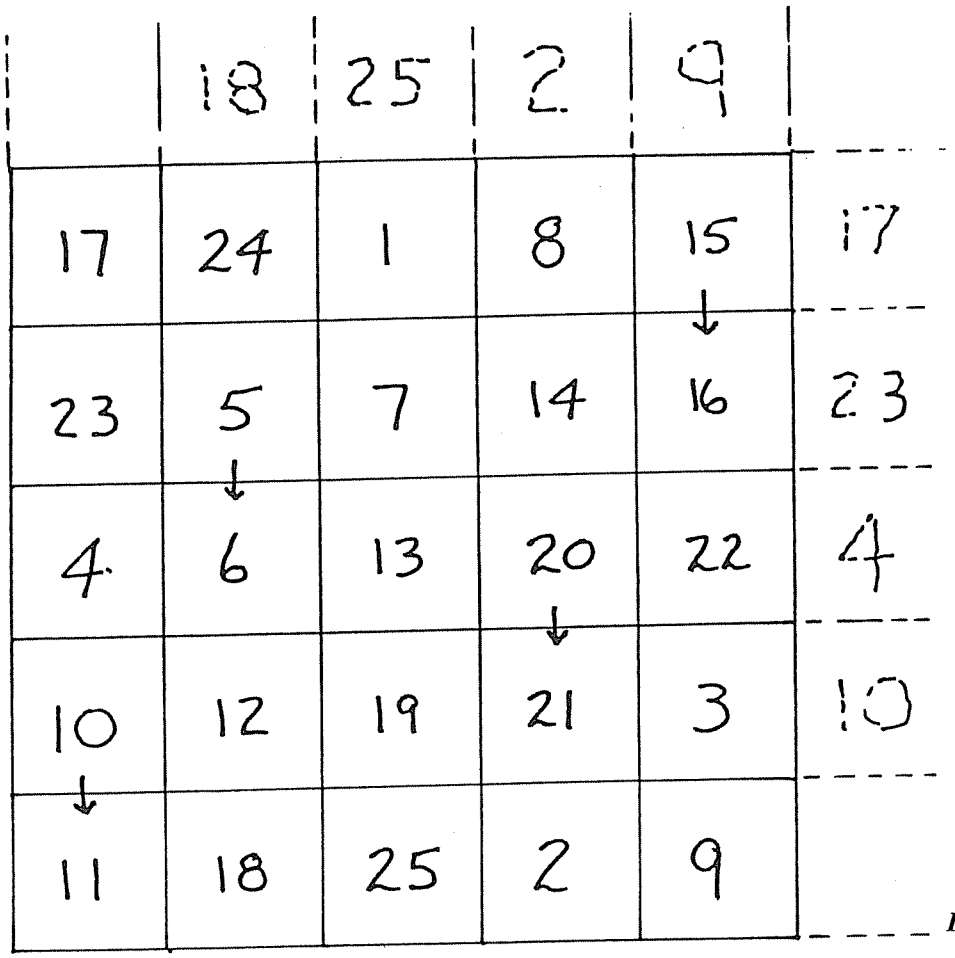



*FIGURE 3*

Another technique can be usd to generate a 4x4 magic square:

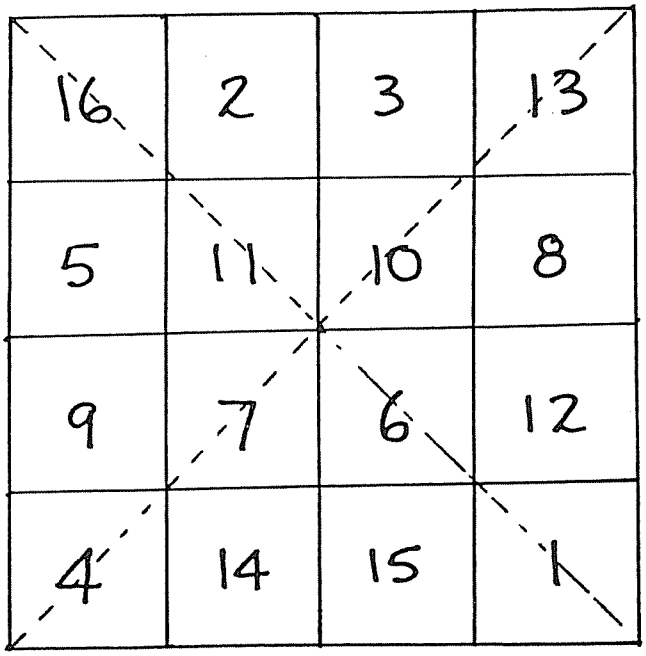



*FIGURE 4*

Draw dotted lines through the diagonals. Start in the top left hand corner and put in the numbers 1-16 in order.

You cannot put a number in a spot if there is a line through it. Therefore you cannot put 1 in the first spot, but you can put in 2 and 3. You cannot put in 4, but you can put in 5 etc. Once you have gone through the square and put in the numbers you were allowed to, start at the top left hand spot and begin putting the remaining numbers in by starting with 16 and working back to 1. The same rules apply.

Go back and calculate the magic numbers. Make the following table:

| DIMENSIONS OF SQUARE | MAGIC NUMBER |
|---|---|
| 3 x 3 | 15 |
| 4 x 4 | 34 |
| 5 x 5 | 65 |
| . | . |
| . | . |
| . | . |
| n x n | ? |

Can you conjecture the magic number for square which has dimensions nxn? (This is a difficult question).

To get artistic with magic squares, go to the 4x4 magic square generated previously (There are 880 different magic squares with these dimensions, perhaps you can find some more). Take each number in the magic square and represent it with a dot.

| | | | |
|---|---|---|---|
| 16 | 2 | 3 | 13 |
| . | . | . | . |
| 5 | 11 | 10 | 8 |
| . | . | . | . |
| 9 | 7 | 6 | 12 |
| . | . | . | . |
| 4 | 14 | 15 | 1 |
| . | . | . | . |

*FIGURE 5*

Join the dots in sequence starting with number 1 to 2 to 3 etc. to form a design. Join all the dots around the outside.

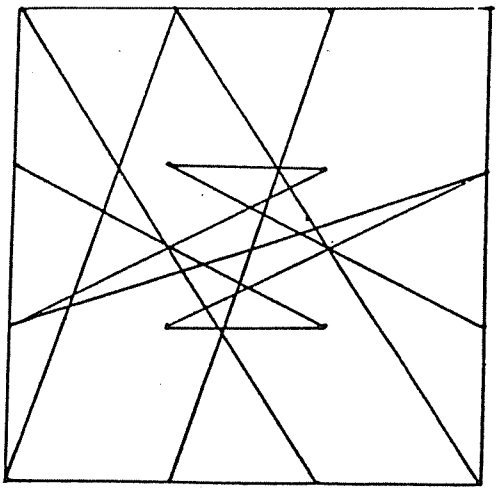


*FIGURE 6*

Colour the square to your liking.

A couple of different 4x4 magic squares are included here so that different designs can be made.

| | | | |
|---|---|---|---|
| 6 | 7 | 9 | 12 |
| 10 | 11 | 5 | 8 |
| 3 | 2 | 16 | 13 |
| 15 | 14 | 4 | 1 |

*FIGURE 7*

| | | | |
|---|---|---|---|
| 7 | 12 | 14 | 1 |
| 2 | 13 | 11 | 8 |
| 9 | 6 | 4 | 15 |
| 16 | 3 | 5 | 10 |

*FIGURE 8*

The following diagram shows the design for Figure 7.

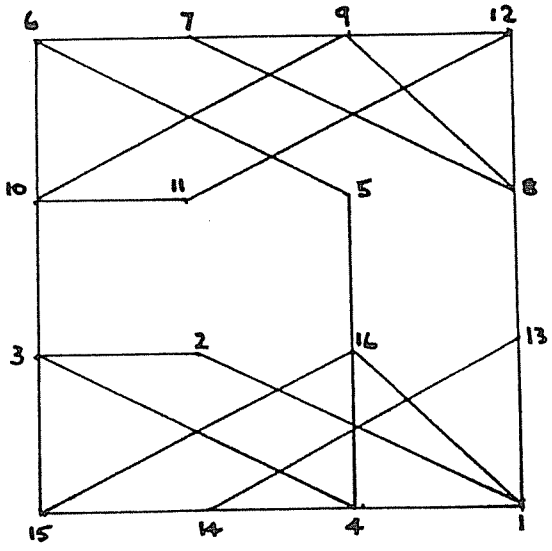



*FIGURE 9*

**Distorting the Basic Square:**

To generate new designs, variations of the basic ones can be made. Charging the distances between the numbers in the squares will distort the square and provide a different look.

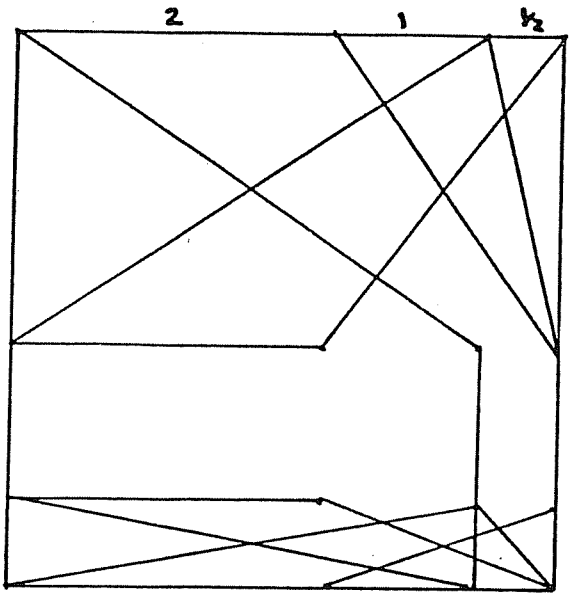



*FIGURE 10*

Do some experimenting with the following magic squares:

| | | | |
|---|---|---|---|
| 16 | 1 | 12 | 5 |
| 2 | 11 | 6 | 15 |
| 7 | 14 | 3 | 10 |
| 9 | 8 | 13 | 4 |

*FIGURE 11*

| | | | |
|---|---|---|---|
| 1 | 15 | 14 | 4 |
| 12 | 6 | 7 | 9 |
| 8 | 10 | 11 | 5 |
| 13 | 3 | 2 | 16 |

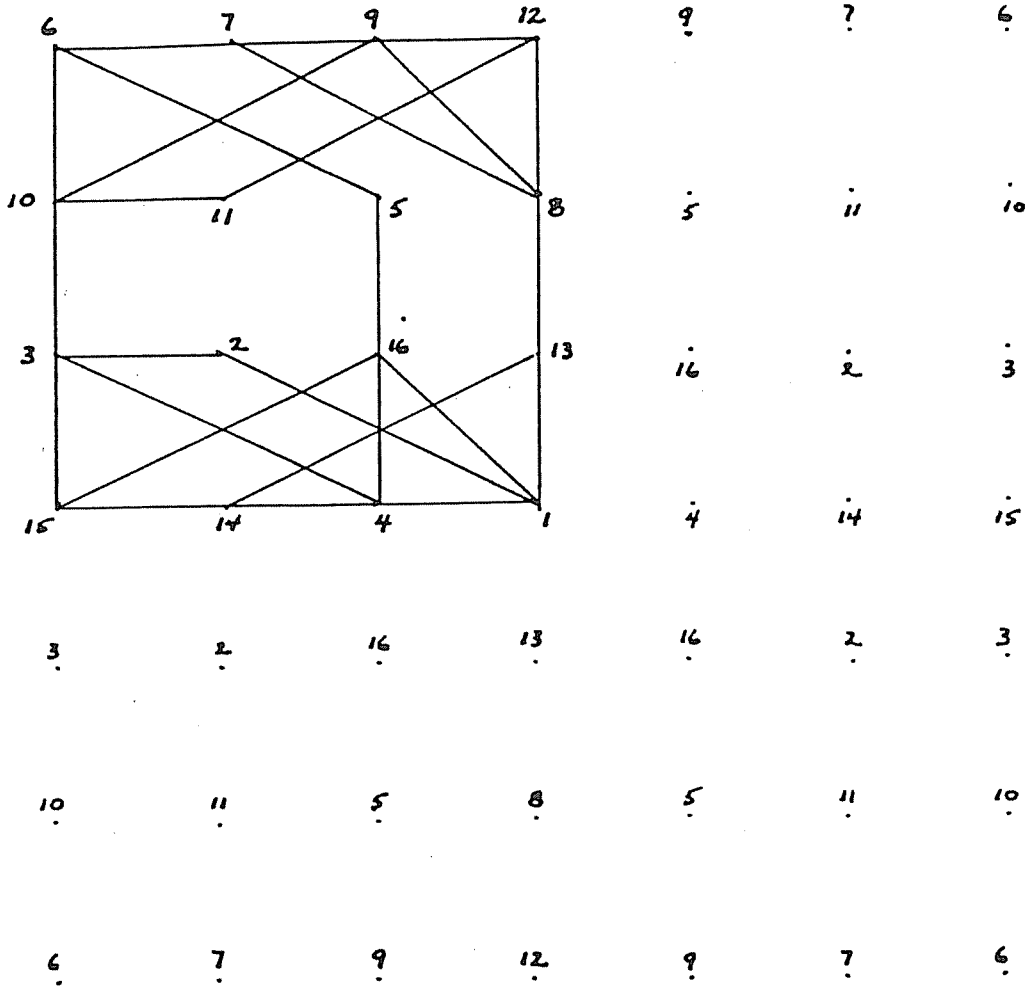
*FIGURE 12*

**Other Design Ideas:**

a) Start with the basic design in square 1 of the diagram, reflect or rotate the design in squares 2, 3, 4.




*FIGURE 13*



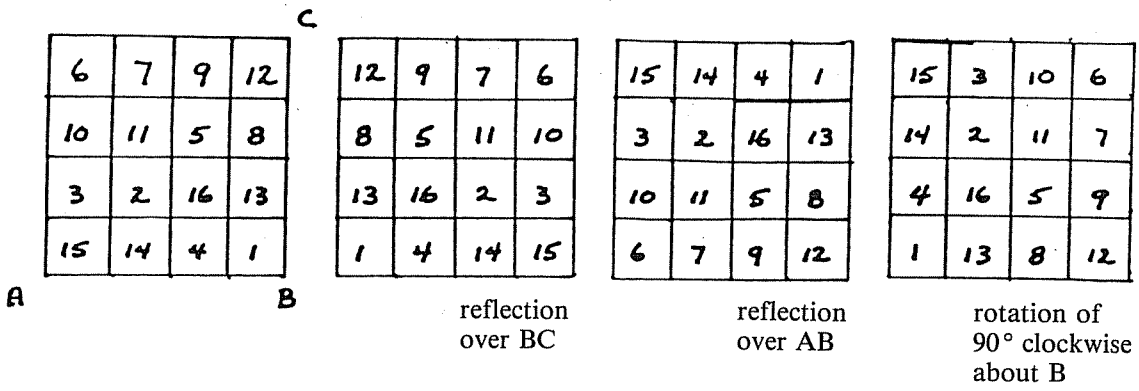



*FIGURE 14*

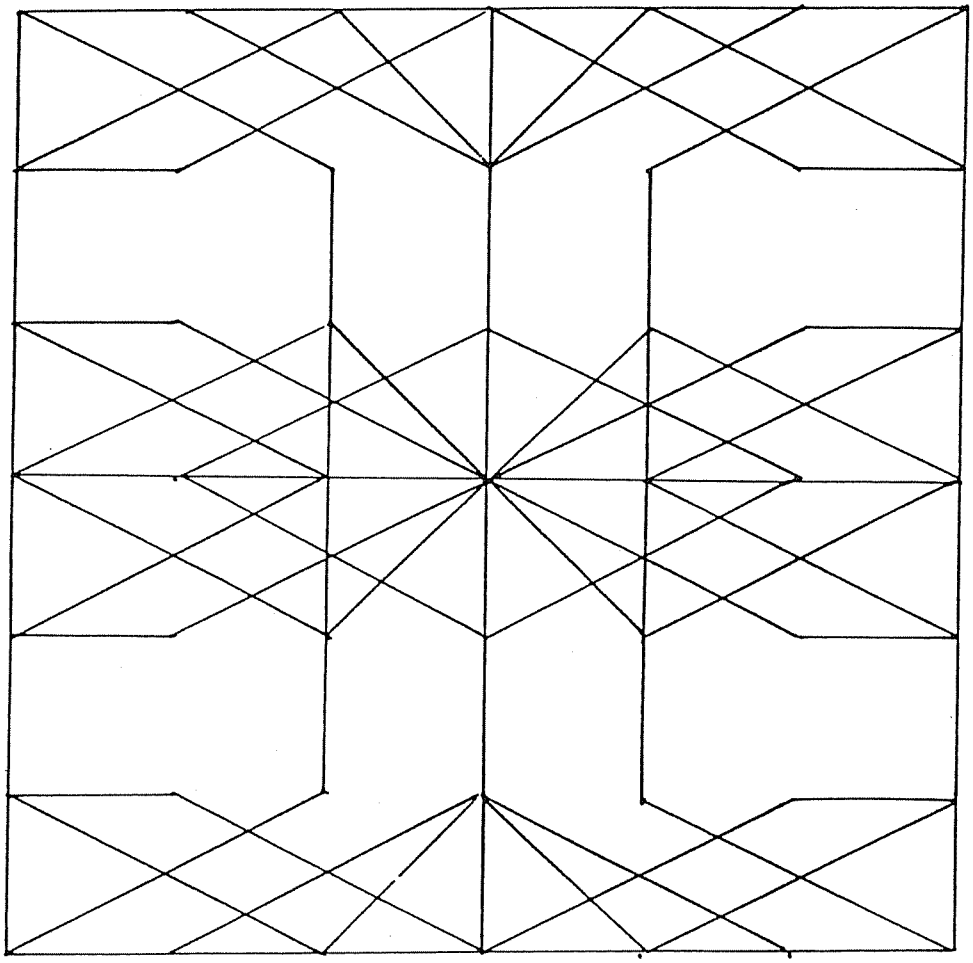


*FIGURE 15*

b) Try joining the even numbers, then the odd.

c) Try magic squares of large dimensions.

d) Consider different colour schemes.

e) Distort the arrays differently.

# PrIME: AN INTERVIEW WITH ANGELA WALSH

*by*
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*London, Ontario N6G 1G7*

**Editor's Note:** *The following is an edited version of a taped interview held by Robert S. Smith at Homerton College, Cambridge, England with Angela Walsh, a former teacher at the primary and secondary levels who later was an instructor in teacher education at Avery Hill College. The topic of this interview is the project known at PrIME (a mnemonic for Primary Initiatives in Mathematics Education).*

RS: Angela what is your title in this project?

AW: My title is Deputy Director of PrIME.

RS: Who has funded this project and in what amount?

AW: The project is funded by the Schools Curriculum Development Committee (50% from the Government and 50% from the local education authorities). The amount of funding that the project has received is £600,000.

RS: Hilary Shuard, the Deputy Principal here at Homerton, is a person most interested in mathematics. I am sure she had something to do with the setting up of PrIME. What was her role?

AW: Hilary really had everything to do with the setting up of PrIME. The origin of this project stems from the Cockcroft Report and from an article that Hilary wrote some time after the publication of the Cockcroft Report. In this article Hilary questioned the fact that not a lot had been done to look at the Primary Mathematics Curriculum. As a result, a feasibility study was set up. The feasibility study lasted for six months. During this time Hilary and the project officer reviewed what was already happening not only in England but also in other countries and published a report. The results of that study, really set in motion what is now known as the PrIME Project.

RS: PrIME has many aims. As you are six months into the project, you've probably sorted out main aims for the near future. What would you say those main aims are?

AW: I think the project's main aim is to help teachers develop their thinking about teaching and learning in relation to mathematics, with particular reference to the primary years (ages 5-11). In doing this we want to help teachers focus on their use of different teaching styles as spelled out in the Cockcroft Report. We also feel that it is vitally important that we tackle fully the issue of new technology regarding the contribution that it could make, the changes that it might bring about and the resultant reassessment of the primary mathematics curriculum for future years. All this has to be carried out within a broader context, namely that of society, not only for today but for the children of tomorrow.

RS: That suggests to me that pencil and paper arithmetic is gone.

AW: Well, I wouldn't like to say necessarily that it is **gone,** but certainly one of the major thrusts of PrIME will be looking at how we can use new technology, particularly the calculator, to help young children begin arithmetic by doing away with the teaching of the standard algorithms as we know them.

RS: You propose to develop a number curriculum using calculator awareness. Have you started that part of the project yet?

AW: Well, we have started developing ideas about it and we have already held two conferences with the people who will be invloved. We have also had detailed talks with the schools and their respective staffs and discussed openly their questions regarding the children and the mathematical approach to be adopted. Nearly 800 children at this first stage will be involved this September. We have written two draft papers on our ideas for a Calculator Aware Curriculum and offered these as discussion documents to the teachers, and we are presently engaged in producing some staring activities for the six and seven year olds who will begin work with their teachers in September on this aspect of the project.

RS: So during this trial year you will be testing materials and trying to revise and set out better strategies in the work sheets. At the end of the year will there be a monitoring?

AW: Yes. We have asked the teachers to write something for us, each term and each teacher involved will be keeping a daily diary in which we have asked them to record pertinent happenings. We want them to be totally honest about the way that they see it developing — dismal failures as well as success stories. We need as much information as possible on which to build a new way of looking at the number curriculum. We hope that we are going to find out a great deal more about the way mathematical thinking develops in the young learner and that this will lead us to a better understanding of learning and teaching. We will modify materials in light of the teachers' experiences and we will monitor the whole exercise through the use of both internal and external evaluation.

RS: The pilot then becomes more efficient and used by more schools in the second, third, fourth and fifth years. That means also, I take it, that children who start at say age 5 will also have a chance at age 6 and 7 to grow in the use of calculators in a number curriculum.

AW: Yes. In fact we won't actually have any 5 year olds because they are starting with 6 and 7 year olds. Children who start in September will continue with this curriculum right through the whole 5 years they remain in the primary school. What we obviously hope, and what we all believe will happen is that the mathematics will become more integrated within the curriculum and the use of the calculator will contribute to this.

RS: Interesting. Obviously working with teachers in schools would be helped if you had persons particularly interested in mathematics. I understand that the government is giving you help in this direction. Could you tell me a bit about its help?

AW: Again as a result of the Cockcroft Report the government has funded a program whereby special support teachers for mathematics education can be employed by each local education authority. Across the country 350 of these special education support grant persons for mathematics education have been appointed, many for primary work. Some of these from the LEA's where the project is operating will work along with the teachers that will be involved in the project. So we have a very strong team at work, ourselves, advisors, teacher educators, special maths support people and the teachers themselves.

RS: When I was in London a few years ago I heard Dr. Jeffrey Matthews say that the one thing that he wanted on his tombstone was that teachers' centres were started by Dr. Matthews. I take it you have taken a leaf from his book in demanding that anyone becoming involved in PrIME will also have a Maths Co-ordinator.

AW: Yes, in the sense that what Dr. Matthews was talking about was what came in the wake of the previous math project called the Nuffied Project. Obviously that project in itself was successful but there were problems in implementation and I think we have taken a leaf out of Jeffrey Matthews' book. We recognize that the implementation can only take place if there are a sufficient number of people experienced in the primary sector who are able to take teachers with them in implementing the changes that are required.

RS: So, in your project you definitely have in mind a model for a curriculum development.

AW: Yes we do. Our model is what is called a periphery centre model. What that means is that, although we have a PrIME team, the emphasis on the actual work development will come from the teachers in the schools themselves, who will be working in teacher groups across the country and who will be supported by the various persons that have outlines. The development work will actually be carried out by these teachers. They will write the material (in the first stage) which will then come to PrIME and will be edited and put together in suitable forms so that eventually it can be published. All the material produced will be INSET packs for use with other teachers.

RS: Two questions come out of what you have said. The first, could you explain what INSET is?

AW: INSET is a word that we use for inservice training for teachers.

RS: Secondly. Teachers being very busy people, will be looking for some kind of reward. What is their reward?

AW: Well the teachers don't get any monetary reward. Teachers in England do want to get a feeling of being involved and take opportunities even though it ofen demands large amounts of time. They find this rewarding in itself. It is very strange phenomenon for other countries to understand but it is one

very much present in English education. I thinks we would also find that some of them **will want to** register for higher qualifications as a result of some of the work they do. We will also **encourage** the teachers to write articles for journals such as ***Mathematics Teaching*** and ***Mathematics in School*** which allows them to reflect on their experiences and to express their views.

RS: You are quite right. That is the phenomena of the English teacher and I would be very interested in knowing how you encourage it.

AW: I don't know how. I started teaching as a primary school teacher quite some time ago and found that the teachers are very willing to give up their time and to extend themselves. The LEA's are also supportive. We know that quite a number of the LEA's involved in the project have offered to give teachers time off during school hours. We are making a strong point of trying to ensure that the teachers do get something in exchange for all the work and effort.

RS: The Cockcroft Report also stressed teaching styles and I know Hilary Shuard, herself, was interested in children **reading** mathematics. How does PrIME intend to help teachers promote mathematical thinking?

AW: One of the things that I think PrIME will do is direct teachers to materials that have already been published. There is quite a wealth of research material available now. A lot of new books have been written that have begun to make anyone involved in maths education really start to reflect on their practice. I think our main focus will be to try to provide the sorts of information, and workshops which will actually get teachers to reflect on what they do at this first stage. Once they become aware of what they are actually doing, then the next stage will be thinking how they might move forward. We believe very strongly that the model we are using stresses that the teachers must start from where they are.

RS: I have just finished visiting with Dr. Alan Bell at the University of Nottingham and he has been doing some very interesting work in remedial teaching. He feels that the setting up of conflicts for children to help them clarify their views is worthwhile. Do you see any of your work going in this direction?

AW: Yes, I think we do. This question of conflict is one area that is being explored in terms of math education. The first stage for many of our teachers in PrIME is listening to children. We will be asking teachers to make an effort to listen to children's language, to the whole aspect of language, then to compose suitable questions to respond in relation to the comments the child makes. When teachers listen carefully they realize that many children know a lot more than they ever imagined about math. Just keeping quiet for maybe two or three seconds, teachers find out a great deal more about themselves as teachers and also about the children as learners.

RS: At Ambleside in the Lake District, you are running a retreat for teacher educators, on a topic called "Images". Where does the conflict come within the teacher educator?

AW: There is a tremendous amount of conflict within teacher education and bringing up this question of Images in mathematics is wonderful. The great difficulty that teacher educators face is in terms of knowing where they are in their own thinking. I don't believe that any one of us ever reaches the point of thinking, "I know", because we are always learning, changing and reflecting. But the main difficulty is to actually change what we believe about mathematics and its teaching. The overwhelming problem concerning the type of thinking experiences that many student teachers have had is so strong that even after the completion of a teacher education program they revert back to previously held beliefs. This major difficulty is not just an English one or British one. I am sure it is a problem that is world wide. We are looking for new models for thinking about the learner and teacher. Also, if we want to encourage teachers to use a variety of teaching styles then we, as teacher educators, have to use a variety of teaching styles. We have to enable the students to actually experience them themselves if we are expecting them to be able to practise them in their own classrooms.

RS: Obviously **you** have done some model teaching.

AW: I have tried. I don't know how successful I have been.

RS: Thank you very much Angela. It has been a pleasure being with you and to again be challenged in thinking about mathematics, its work in the world we live in and the way in which we may change programs to meet future needs. It has been a pleasure.

AW: Thank you Bob. I hope that we can keep you in touch with the progress of PrIME and that between

now and 1989 we may be able to speak again about the developments that have taken place since our last talk.

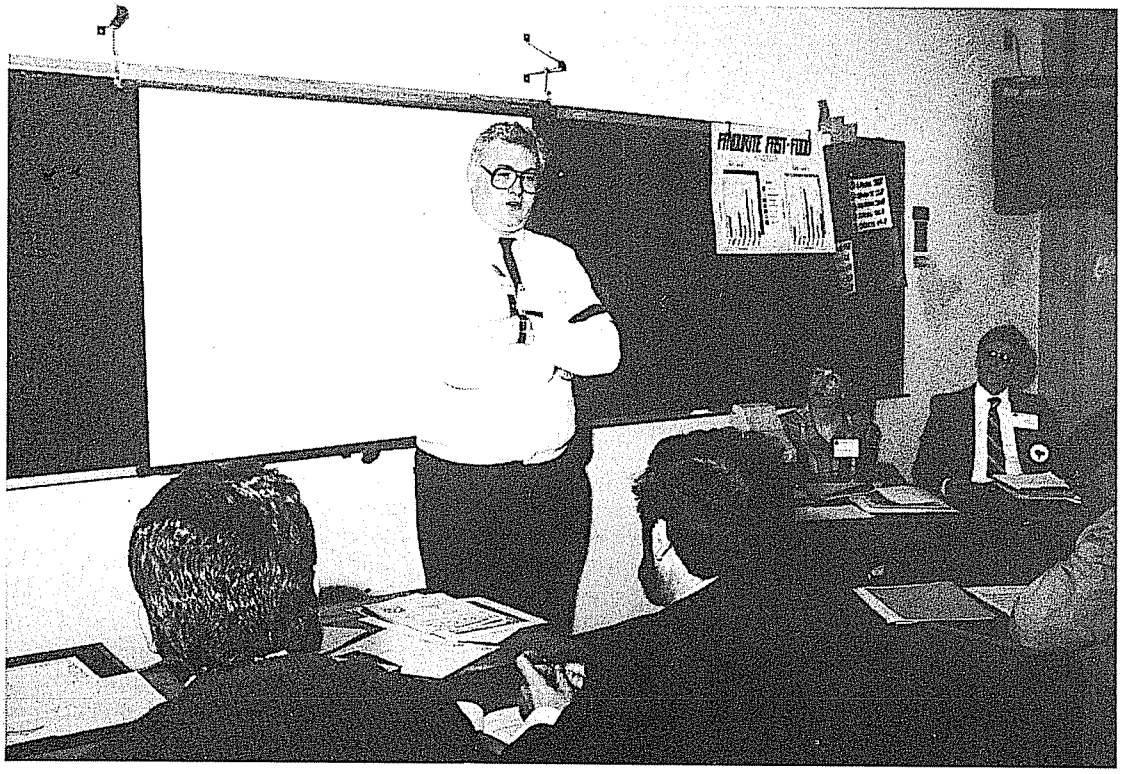
RS: Good.

For additional information you may wish to write to:

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Homerton College
Cambridge CB2 2PH
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**Attention**
Deputy Director
Angela Walsh

# SAME: OF MICE AND MATH
# 1987



**Arnie Carefoote**

# COMPUTERS CAN CHANGE OUR APPROACHES TO ALGEBRA AND THE CALCULUS

*by*
*Bill Newton*
*Education Officer (Seconded)*
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*Toronto, Ontario M2J 1W4*

*Calculators, microcomputers and, more recently, computer-based communications, have an undisputed place in your school's mathematics program. Powerful calculating and simulation techniques are available with microcomputers to plot and display functions, solve any type of algebraic equation through a process of successive approximation, model physical situations and provide dynamic views of physical phenomena, challenge (and replace?) the innumerable algorithms for differentiation and integration in the calculus. Alternative methods using the new technology ought to be explored now.*

**Computers are changing the nature of mathematics**

Recognize this?

```
?   SOLVE (X↑2 + 4*X + 1 = 0), X);
@   (X = -2 + 3↑(½),
     X = -2 - 3↑(½)
```

Once the blur of new symbolism fades away you might see the familiar quadratic equation, $x^2 + 4x + 1 = 0$, and its two roots, $-2 \pm \sqrt{3}$. The example is taken from MuMath, an interactive computer software package used for symbol manipulation. This package is now available for microcomputer users.[1] MuMath chomps up algebra. It can simplify algebraic expressions, solve equations, perform operations on matrices, logarithms, exponents and manipulate trigonometric expressions. What the calculator does for manipulating numbers, MuMath does for symbols.

Mathematics education throughout the Western world is now engaged in a process of evolutionary reform (Cockcroft, 1982; NCTM, 1980, 1985; Romberg, 1984; Hunting, 1985). The case for reform in mathematics however, could just as easily be built on the implications of the technological revolution for classroom instruction in mathematics. Is there a practising mathematician, actuary of statistician today who **doesn't** use a spreadsheet for their analyses? Is there a math classroom today that does? 'It seems clear that mathematics must adapt to this changing scientific workplace or lose its vitality' (Fey, 1984, p. 91).

Perhaps we should make a start by examining how the computer might be used to supplement existing approaches to traditional topics. Of course, as we start to examine some of these possibilities, many traditional sections of our mathematics courses will start to look very dated indeed. Let's look at a few examples on algebra and the calculus to see where we might start.

**1. Successive Approximation can be used to solve equations**

One of the things that a computer does best is repetitive calculation. A calculator is fine for a once-off calculation, but if you want to find, say, all the integer values of the function $y = x^2 - 4x - 6$ in the range from $x = -4$ to $x = 4$, then a computer can very easily and speedily perform all the necessary repetitive calculations. This facility suggests a powerful, but simple, equation solving strategy: successive approximation.

We teach factorising trinomial expressions like $2x^2 + 5x - 3$ primarily as a procedure for solving quadratic equations. Of course, factoring doesn't work for expressions like $x^2 - 4x - 6$; in fact, it works for only a limited number of quadratic expressions - those which have carefully chosen coefficients. The method of successive approximation enables us to progressively improve our first approximation to the zeros or roots of the equation. This is aided by simple BASIC program.

```
10  INPUT A, B, C
20  FOR X = A TO BE STEP C
30  PRINT X, (X-4) * X - 6
40  NEXT X
```

(After Fey, 1984, p. 14)

[1] *Symbol manipulating calculators have just arrived on the market. The Hewlett Packard 28C. It does differentiation and integration as well!*

A first RUN in the range -6 to 6 gives:

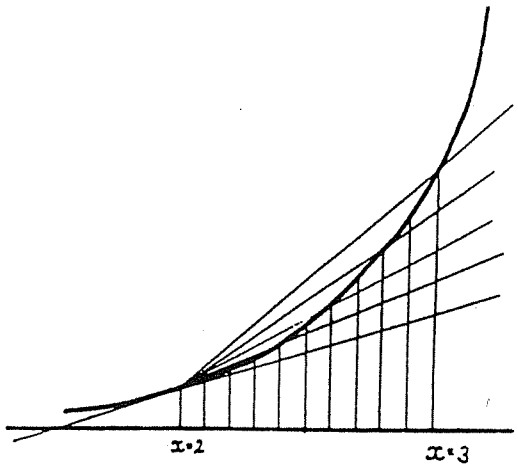
```
? -6, 6, 1
 -6    54
 -5    39
 -4    26
 -3    15
 -2     6
 -1    -1      First Zero
  0    -6
  1    -9
  2   -10
  3    -9
  4    -6
  5    -1
  6     6      Second Zero
```

From inspection, the quadratic has zeros in the intervals (-2, -1) and (5, 6). We can find a better approximation to the first zero by RUNning the program again in the interval (-2, -1) in steps of 0.1.

The advantage of this numerical approach is that it can be applied in the same way, not only to quadratics, but to cubics and other higher order polynomials, as well as trigonometric, logarithmic, exponential and simultaneous equations.

**2. Limits and Derivatives**

Simple programs can illustrate the nature of a limit by displaying successive values of a function as a particular value of the function is approached. This is best illustrated in the case where it is required to find the gradient of the tangent to a curve at a particular point. For example, find the gradient of the tangent to the curve, $y = x^2$, at the point $x = 2$. We can illustrate this geometrically by successively drawing the chords joining the points on the graph at $x = 2$ and $x = 3$, then $x = 2$ and 2.9; $x = 2$ and 2.8, 2.7, 2.6 down to the limiting value of 2 at which point students can see intuitively that the chord 'becomes' the tangent.




The gradient can be calculated at each point on the graph using the familiar difference quotient for gradient: $(f(x + h) - f(x))/h$. For example, the gradient of the chord joining the points $x = 2$ and $x = 2.5$ gives $(2.5^2 - 2^2)/(2.5 - 2)$ which is 4.5. However the gradient **at** $x = 2$ (the limiting value) is not able to be calculated since $(2^2 - 2^2)/(2 - 2)$ is indeterminate. It is here that the simple program below can be used to successively calculate the gradient over a selected range (A, B) in incremental steps (C) which approach the point $x = 2$.

```
10  INPUT A, B, C
20  FOR X = A TO B STEP C
30  GRAD = (X ** 2 - 4)/(X - 2)
40  PRINT X, GRAD
50  NEXT X
```

3. **Estimation of Definite Integrals (for finding the area under a curve)**
Just as a numerical approach can be used to estimate the **gradient** of a curve at a given point, so too can it be used to find the area under a curve. For example, to find the area under the curve of the function $y = x^2$ between the ordinates $x = 0$ and $x = 1$, the following (unrefined) program successively adds up the area of each of the columns to give an (over) estimate of the required areas.

```
INPUT A, B, C          A and B are the x ordinates, C is the width of the rectangles.
AREA = 0
FOR X = A TO B STEP C
AREA = AREA + X↑2 * C
PRINT X, AREA
NEXT X
PRINT AREA
```

4. **Spreadsheet uses algebraic formulae that students invent to explore their own data**
Spreadsheet originated in the commercial world and was designed to display and compute costs, profits, sales, ...It is simply an electronic balance sheet with rows and columns in which data is stored and manipulated. The example below records students measurements on the length and width of their classroom. Column 4 (C4) gives the area of the room and C5 gives the perimeter. The students enter their data, devise column formulae (C4 = C2 * C3, C5 = 2 * (C2 + C3)) and perform calculations on their data.

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| R1 | Student | Length | Width | Area | Perimeter | |
| R2 | Name | | | | | |
| R3 | John | 5.95 | 4.38 | 26.06 | 20.66 | |
| R4 | Marie | 5.91 | 4.42 | 26.12 | 20.66 | |
| R5 | | | | | | |
| | | | | | | |

This simple example utilises column formulae only: more complex examples would involve row formulae as well, for example, to calculate average values for the data. The algebra here is inherently sensible to students as it is linked to real data.

**These examples are just a beginning: you can explore some others.**
A wide range of additional examples from the traditional middle and senior mathematics courses remain to be exploited by the repetitive calculating power of the computer. Simple programs could be written now for:

- **sums of infinite series,** e.g. $1 + 1/2 + 1/4 + 1/8 + \ldots$,
  $e^x = 1 + x + x^2/2! + \ldots$

- **limits** e.g. $\sin x/x$, $(1 + 1/n)^n$

° **growth and decay problems,** e.g. calculate each successive year's capital (or population) growth when an initial principal (or population), $P_0$, grows at r% per annum.

° **Newton Raphson method** for estimating the root(s) of an equation (algebraic, trigonometric or otherwise).

**Numerical approaches challenge existing courses**

Numerical methods offer a powerful, and quite general, alternative approach to many of the algebraic and continuous methods of the calculus used in schools today. For the moment it is sufficient that numerical methods simply be explored in our classrooms. As Fey (1984, p. 63) comments: 'experimentation with such programs is likely to yield that satisfying corroboration of intuition denied to those who complete only the usual collection of required exercises'. Simple programs like these can enhance our traditional approaches to algebra, and the continuous methods of the calculus, and potentially, perhaps further down the track, might even suggest that our concentration on these approaches is somewhat misplaced given the availability and capacity of modern computing equipment.

*REFERENCES*


*Cockcroft, W.H. et al (1982).* ***Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in School.*** *London: H.M.S.O.*

*Fey, James T. (1984).* ***Computing and Mathematics: The Impact on Secondary School Curricula.*** *Report of a conference sponsored by the National Science Foundation. Maryland: N.C.T.M.*

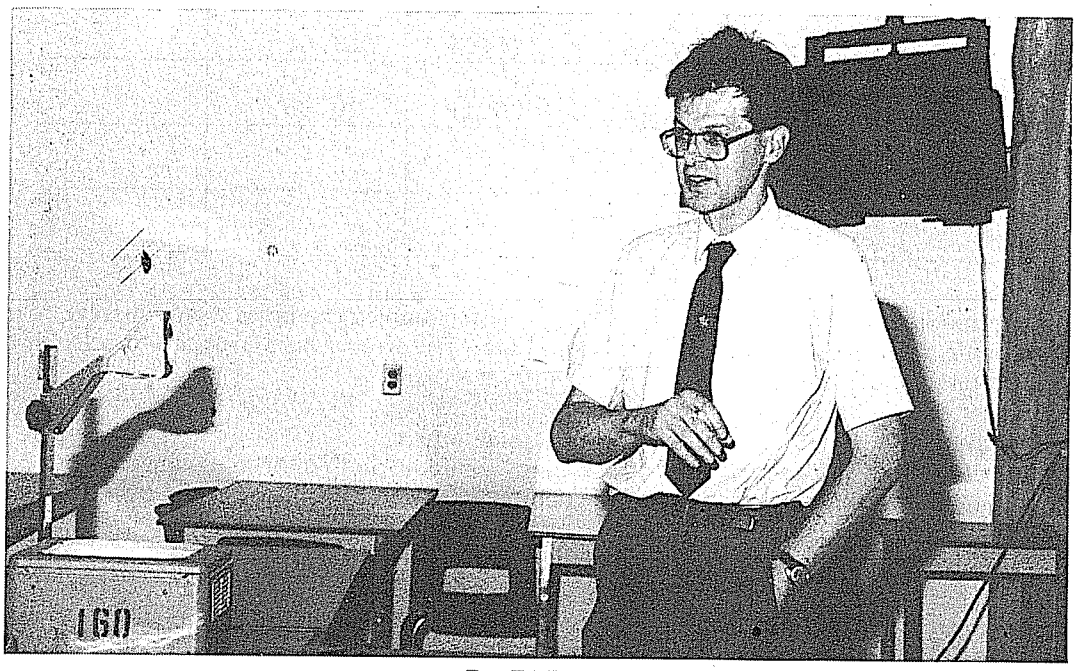
*Hunting, Robert (1985).* ***Perspectives on P-12 Mathematics Curriculum Development.*** *Unpublished paper prepared for Mathematics Centre, Curriculum Branch, Victorian Ministry of Education.*

*National Council of Teachers of Mathematics (1980).* ***Mathematics for the Eighties: An Agenda for Action.*** *Illinois: N.C.T.M.*

*National Council of Teachers of Mathematics (1985).* ***The Impact of Computing Technology on School Mathematics.*** *Report of an N.C.T.M. Conference.* ***The Arithmetic Teacher.*** *April, pp. 14-18.*

*Romberg, T.A. (1984).* ***School Mathematics: Options for the 1990's.*** *Chairman's Report of a conference held at Madison, Wisconsin, 5-8 December 1983. Washington: Department of Education.*

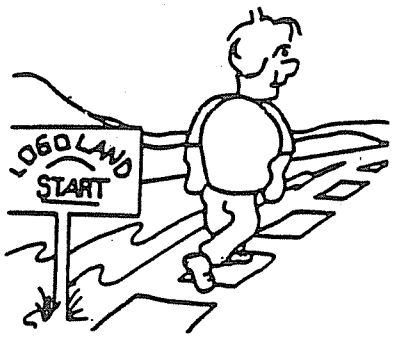

# SAME: OF MICE AND MATH 1987




**Dr. Ed Barbeau**

# LOGO: AS A TOOL TO ENHANCE THE MATHEMATICS CURRICULUM

*by*
*Rudy V. Neufeld*
*LOGO/Mathematics Consultant*
*London Board of Education*




This article is an introduction to the use of LOGO AS A TOOL TO ENHANCE THE EXISTING MATHEMATICS CURRICULUM.

## A. 'LOGO': WHAT IS IT?

In the classroom, the computer is nothing more or nothing less than a tool to enhance learning. LOGO is simply a computer language which makes this tool easier to use.

The name LOGO is derived from the Greek word for THOUGHT.
LOGO is the language of learning.
It can be used by people of all ages and at all levels of skill. It is referred to as the language with "no threshold and no ceiling". It is very easy to get into and use meaningfully by primary aged children however it is also being used as a powerful learning tool at the post-secondary level.

The computer language LOGO was developed at the Massachusetts Institute of Technology (MIT) by Seymour Papert. Papert was influenced by the learning theories of Jean Piaget.

The creature on the screen is called a 'TURTLE'. This tradition can be traced back to the work of Grey Walter, a neurophysiologist who experimented in Britain in the 1960's with robots which he called 'tortoises'. This work inspired the first turtle designed at MIT.

## B. THE LOGO LEARNING ENVIRONMENT: PRINCIPLES OF LEARNING

The turtle-graphics component of LOGO is a powerful and motivational learning tool for geometry and other mathematical concepts for the following reasons:

1. The child is in control of and teaches a robot called a turtle.
   One of the best ways of learning a concept is to teach it.

2. A concept is introduced through manipulatives.
   A number takes on meaning when it results in movement on the screen or on the floor. The abstract becomes concrete.

3. The child is encouraged to take risks.
   In the LOGO environment there are NO mistakes.
   'Bugs' occur when something does not work as the user expected.
   A bug is AN OPPORTUNITY TO LEARN.
   A positive LOGO learning environment is created.
   Every plan does not work out smoothly but it may give rise to other ideas.

4. Emphasis is on ...
   THE THINKING PROCESS TOWARDS AN ANSWER
   not on ...
   THE 'RIGHT' OR 'WRONG' ANSWER

5. Just a few, simple commands are required before LOGO can be used as an effective, user-friendly tool to assist in learning.

6. A safe, defined learning environment (usually called a microworld) can be established. Here students can do some real learning through open-ended guided exploration.

   The role of the teacher changes.
   As teachers, we often tell too much. We take away the joy of learning.
   Teachers are facilitators and challengers, rather than lecturers.

7. Creativity in problem solving is encouraged.
   The learner becomes more actively involved in the learning process.

8. Procedural thinking skills are developed.
   Students are encouraged to break a complex problem into smaller components or chunks. This is a key to problem solving across the curriculum.

RAWITSCH'S RULES:

1. LOGO IS NOT EASY
   ...it requires thinking
   ...thinking requires a lot of effort
2. LOGO IS NOT IMPORTANT
   ...the language itself is not important
   ...what can be done **with it** is important
3. LOGO IS NOT SAFE
   ...it is unpredictable
   ...the best-laid teacher plans may go awry
   ...a creative teacher must remain open to all kinds of opportunities when they occur

**C. THE PROCESS**

In activities where LOGO is used, the student is encouraged to:

1. ...First THINK THROUGH the commands. Predict what should happen.

2. ...Then WALK THROUGH the commands on the floor or with pencil on paper.

3. ...TRANSLATE these steps and turns into simple LOGO commands.

4. ...COMMAND the turtle on the screen.

5. ...TALK to a partner about WHAT HAPPENED and WHY it happened.

**D. GENTLE STEPS IN USING LOGO IN THE MATHEMATICS CLASS**

STEP #1: FLOOR & PAPER ACTIVITIES
This is an important step for learners at all levels.
It prepares the learner for Step #2.
When the learner has difficult with directions and shapes on the screen, it is natural to go back to walking through the design on the floor or on paper. The abstract becomes concrete.

STEP #2: THE JUMP TO THE SCREEN
This step introduces a code that is simple enough for non-readers to use but is powerful enough to discover and investigate mathematical ideas. This step is a smooth transition between the turtle on the floor and the turtle on the computer screen.

STEP #3: STEPS INTO LOGOLAND
This step investigates some LOGO commands. It also makes sure that the user is familiar with turns and calculations used in LOGOLAND. Angle patterns in various shapes are investigated. Step #3 creates a solid base for following concepts in Step #4.

STEP #4: THE TURTLE IN SCHOOL
In this step, the student teaches the screen turtle some lessons. The student is 'programming' the screen turtle. The student must thoroughly understand a lesson and present it in a logical way. The student will be open to turtle reactions and then adapt the lesson. The student will learn math concepts and problem solving methods by teaching and then testing the turtle! THE STUDENT LEARNS BY TEACHING!!

# WOMEN AND MATHEMATICS DEPARTMENT
# EDITORIAL
# AN INTERNATIONAL ADDICTION

*by*

*M. Elaine Harvey, Women's Editor*

Dr. Rosalie Bertell, director of research for the International Institute of Concern for Public Health, claims that our society has an international addiction--that we solve problems by threat or by violence. Problem solving has a strong position in the mathematics curriculum. But are problems solved in the mathematics classroom by threats or violence?

Let us first consider teacher behaviour:

> Teachers check to see that homework is done and punish the offenders by recording the offense, taking off marks, phoning the parents, giving a detention or reporting the situation to guidance or the vice principal.
>
> Teachers deal with a persistently talkative student by threatening to change his seat, put him in the hall, kick him out of class, phone his parents or arrange that he drop the course.
>
> Teachers deal with a student who is frequently late for class by demanding a written explanation, refusing to admit her to class, beginning the period with a test, phoning the parents or reporting to guidance or the vice principal.
>
> Teachers deal with a student who does poorly on a test by demanding that she work harder,stay after school for extra "help" (i.e. more work), get a tutor, check her homework more regularly and/or watch her attendance and application in class.
>
> Teachers do not notice the student who as a result of this authoritarian, threatening system has become quiet, introverted and obedient, and given up his creative spirit through lack of self confidence.

All of these problems are usually preceeded by making the students aware of possible punishment by means of written rules and regulations, oral communication on "expectations" or carrying out a punishment as an example to other students. Whether you name these "rules and regulations", "expectations" or "disciplinary guidelines", they are threats or retaliation for students who do not succumb to the authority of the teacher.

Although physical violence by adults against young people for punitive purposes is no longer readily condoned, it is still legal to strap a student under controlled circumstances, and society continues to debate the advisability of that ultimate physical violence--capital punishment. While physical punishment is seldom meted out in to-day's schools, the psychological violence and stress created by teachers (and perhaps also the administrators and/or the parents) who threaten or carry out punishments for unfinished homework, unacceptable behaviour, lateness or failure on tests may be even more hurtful than the strap, particularly when added to the self-inflicted frustration and disappointment that students experience in not living up to expectations.

Now let us consider student behaviour:

> Students feel powerless under an authoritarian school system in which teachers use threats and/or violence (psychological and occasionally physical). In general, those that must submit to authoritarian control find their own method of obtaining power over others. Students cleverly do this by challenging the system.
>
> Students copy homework or make up wild unusual excuses that any humanitarian would accept.
>
> Students act up in class in order to challenge the teacher's authority, and then promise to reform in such persuasive tones that one cannot but accept the promise.
>
> Students are habitually tardy in order to assert themselves.
>
> Students say they don't care about passing when they fail a test in order to strike out against those who use marks as an authoritarian threat.
>
> They play typical student pranks, skip classes, forge notes from home, lie about having to leave early, all in order to try to retain some control over their lives in the school setting where they feel powerless.

These student challenges are threats to the traditional authoritarian educational system that may result in psychological and occasional physical violence against those in authority.

Neither teacher nor student is free of the addiction that Dr. Bertell identified--solving problems by threat or by

violence. Teachers and students through discussion together could use the problem solving techniques taught in mathematics classes to find solutions to the behavioral problems of both that do not involve threats or violence. They might consider the following steps from the mathematical problem solving model:

1. Recognize the problem.
2. Gather all the data.
3. Consider several approaches to solutions.
4. Choose one of those methods.
5. Attempt to solve the problem.
6. Evaluate the result.
7. Try another solution if the previous one didn't succeed.
8. Evaluate the new result.
9. If necessary, repeat #7 and #8 until a satisfactory solution is found.

This problem-solving model overcomes the pattern that many people have unknowingly adopted--solving their problems by using the same unsatisfactory method over and over again without ever learning from previous failures. By using the mathematical problem solving model to identify and solve behavioral problems, teachers and students will learn togethr more effective ways of coping with the stresses of life within the school system as the system becomes less authoritarian and more cooperative, and we will have played a small part in reducing the number of threats and acts of violence in the world.

# SAME: OF MICE AND MATH
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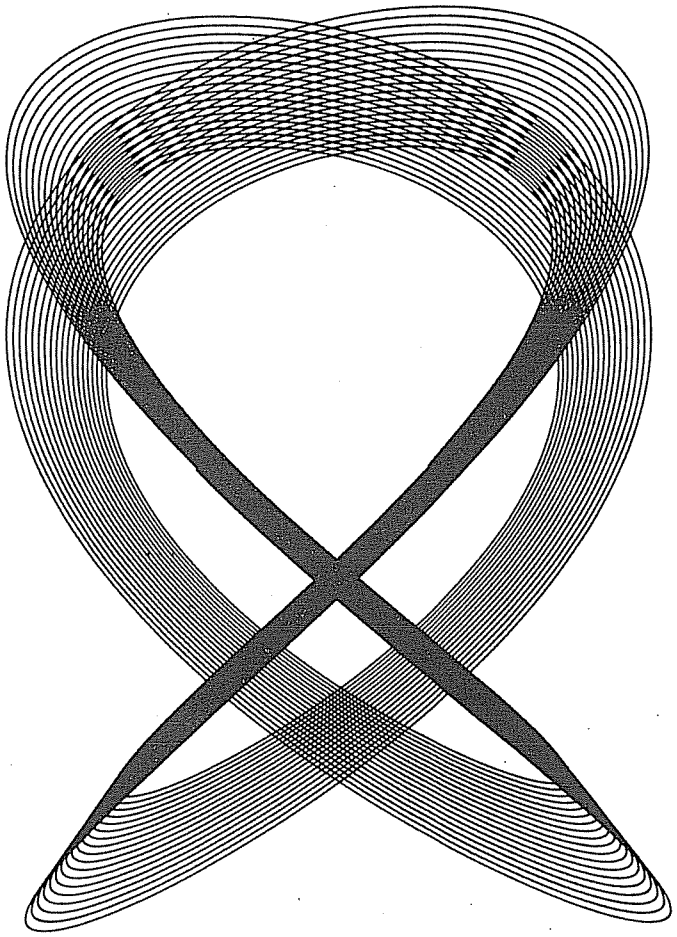
**Dr. Olive Fullerton**

Volume II
Number 1

# Abacus

*A publication of the O.A.M.E.*

September 1987



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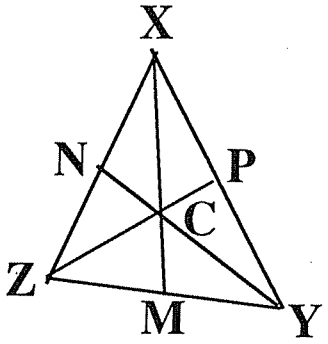
*This issue is sponsored by QSLMA. Other chapters wishing to sponsor an issue please contact the editor.*

# A Generalization

Again this past weekend, somebody asked me why I called my yacht 'CEVA'. "There are two reasons," I usually respond, "One, it is an acronym and two, it's the name of a little known 17th century mathematician and since this vessel is owned by a little known 20th century mathematics teacher, it seemed appropriate".

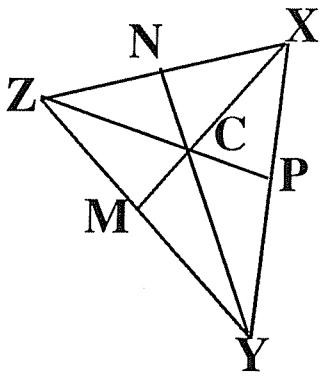
I used to re-introduce my grade 10 mathematics classes to geometry with accurately drawn diagrams of the various centres and circles associated with the triangle. Now, mathematicians are always generalizing particular results, so why not do the same for the median-centroid result.

By definition, a median is a line segment joining a vertex of triangle to the midpoint of the opposite side. The 3 medians are concurrent and this point is the centroid. Necessarily, too, the medians divide each side of the triangle in the ratio 1:1.




I get my students to draw a large triangle, XYZ and pick any point, C, within the triangle. They then join X to C and produce it to M, Y to C, etc. Unless by chance C is the centroid, M,N and P will not be the mid points of the sides. For convenience I will call XM, YN, ZP. Cevians.
I ask my students to find, by measuring, the 3 ratios

$$\frac{XP}{PY}, \frac{YM}{MZ}, \frac{ZN}{NX}.$$




I ask, "Are they equal? etc. Finally, I lead them to the product. Its approximately 1! Eventually we produce "If any 3 cevians of a triangle are concurrent, then the product of the ratios, taken in order, in which they divide the opposite sides, is 1".

i.e., If XM, YN, ZP are concurrent,

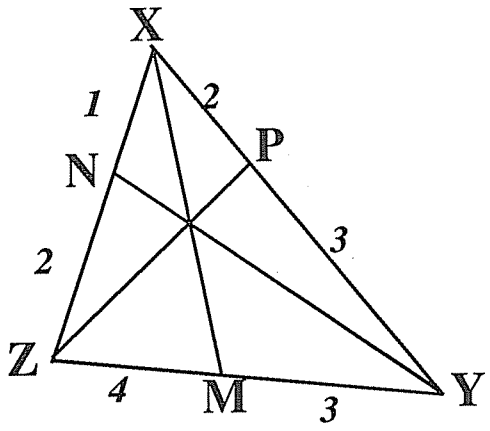
Then $\frac{XP}{PY} \cdot \frac{YM}{MZ} \cdot \frac{ZN}{NX} = 1$

I then set the class onto verifying the converse statement i.e. suppose $\frac{XP}{PY} \cdot \frac{YM}{MZ} \cdot \frac{ZN}{NX} = 1$

Does it follow that XM, YN, ZP are concurrent? I set the students to think of 3 easy ratios whose product is 1.

Suppose one picks $\frac{2}{3}, \frac{3}{4}, \frac{2}{1}$.

It's a simple matter, with a calculator to determine the points M, N and P. A carefully drawn figure suggests that indeed XM, YN and ZP are concurrent.




It's a good exercise in grades 9 and 10. Medians are but special cevians and the centroid is that point of concurrence in the degenerate case.

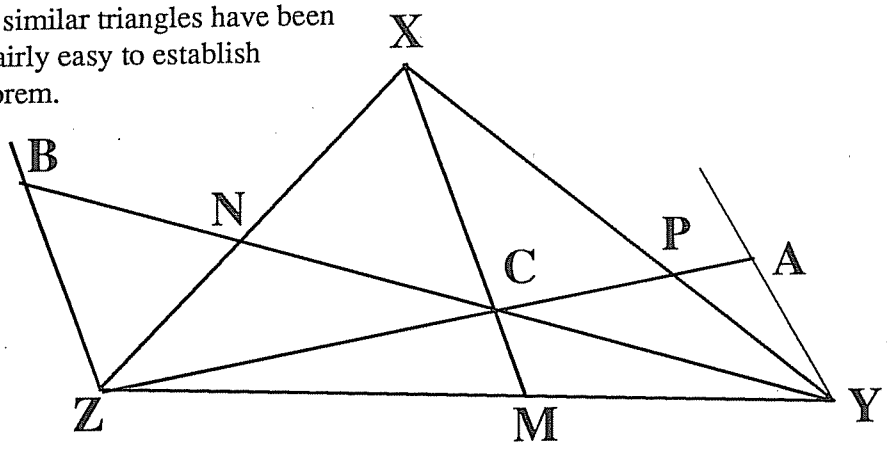
Def. A cevian of a triangle is a line segment formed by joining any point of a side of a triangle to the opposite vertex.

## Theorem:

Three cevians XM, YN, ZP of $\Delta XYZ$, are concurrent if and only if

$$\frac{XP}{PY} \cdot \frac{YM}{MZ} \cdot \frac{ZN}{NX} = 1$$

N.B. When similar triangles have been taken, it is fairly easy to establish Ceva's Theorem.




Draw YA // MCX // ZB where A and B are on the productions of ZP and YN respectively.

$\Delta XPC \sim \Delta YPA$, $\Delta ZYA \sim \Delta ZMC$, $\Delta YBZ \sim \Delta YCM$ and $\Delta BNZ \sim \Delta CNY$

So,

$$\left.\begin{array}{l} \dfrac{XP}{PY} = \dfrac{XC}{AY} \\ \left\{\begin{array}{l} \dfrac{YM}{YZ} = \dfrac{CM}{BZ} \\ \dfrac{ZY}{ZM} = \dfrac{AY}{CM} \end{array}\right. \\ \dfrac{ZN}{NX} = \dfrac{ZB}{XC} \end{array}\right\} \quad \text{Product} = 1$$

So,

$$\frac{XP}{PY} \cdot \frac{YM}{MZ} \cdot \frac{ZN}{NX} = 1$$

The converse is easily proved too - use the indirect method.

***David Wing***
*Frontenac Secondary School*

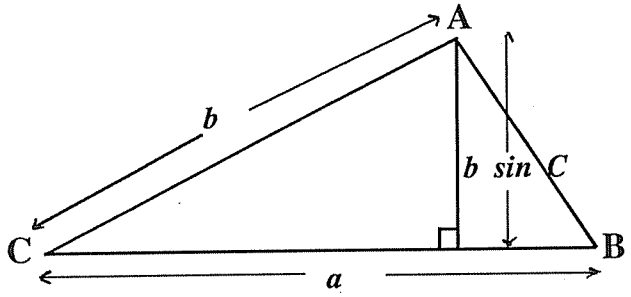
# GEM:

# THE SINE ANGLE SUM FORMULA

One of the drawbacks of introducing the trigonometric ratios as functions is that it takes some time to reach the point where students can use trigonometry for problem solving. There is still a lot to be said for introducing trigonometry, after a discussion of similar triangles, as the angle dependent ratios of sides of right-angled triangles. My favourite mnemonic for remembering the names of the ratios is "Some Old Hen Caught Another Hen Taking Oats Away". By reciting this, you can write down the equations: $s = \frac{o}{h}$ ; $c = \frac{a}{h}$ ; $T = \frac{o}{a}$.

I have a large poster in my classroom depicting a bossy looking hen, wings tucked onto her hips, crossly scolding another hen that is scampering away with a satchel slung over her shoulder from which oat grains are spilling. The mnemonic is written on the poster and it attracts attention. Students are always curious about it, and by the time we come to the trigonometry unit, they have it memorized.

From the very first lesson, students are able to use trigonometry in practical applications. By the second lesson, the students are able to deduce that the height of $\Delta ABC$ is b sin C and, therefore, that its area is



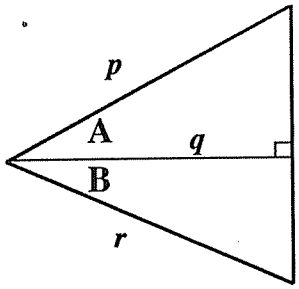

1/2 base x height = 1/2 ab sin C

It is worth pointing out here that this formula uses two sides and the included angle and is the area formula that must necessarily exist because any two triangles having the same measures for two sides and the included angle are congruent.

Now for the gem. If your students have been prepared as above, you could now prove the "sine angle sum" formula in a way that they would find readily acceptable.

To begin with, in order to consider sin (A+B) you need a model of the angle A+B. Obviously, you make A and B adjacent angles. Now draw a line perpendicular to the common ray and extend it both ways to intersect the other two rays forming A and B. Let thelengths of the intercepts of these three rays be p, q and r as shown. Then the area of the whole triangle is 1/2 pr sin (A+B).

But this is simply the sum of the areas of the two right angled triangles of which it is composed.



Thus:
1/2 pr sin (A+B) = 1/2 pq sin A + 1/2 qr sin B multiply by 2 and divide by pr

$$\sin (A+B) = \frac{q}{r} \sin A + \frac{q}{p} \sin B$$

$$= \sin A \cos B + \cos A \sin B$$

***Peter Battey-Pratt***
*Ernestown Secondary School*

# A Problem That Won't Quit!

George, who was 1/3 his father's age 4 years ago and whose present age now makes a sum of 64 with his dad's, is mixing a 40% salt solution with a 25% salt solution to get 600 kg of 28% solution. He plans to sell this mixture by driving from Montreal to Cornwall at 80 km/h and then from Cornwall to Belleville at 110 km/h. The total distance is 395 km, and the time is 4 hours. He will sell the mixture for $1.50/kg, and the money will be composed of a total of 54 $10 and $20 bills. George then will invest part of his money at 12% and part at 16% in order to earn $112 interest for one year.

1. How old is George?
2. How much 25% salt solution will he need?
3. How far is it from Cornwall to Belleville?
4. How many $20 bills will he have?
5. What amount will he invest at 16%?
6. Why would a teacher ever dream up a crazy problem like this?

***Bruce Black***
*Frontenac Secondary School*

# Flashing Lights

I have designed an advertising sign with three flashing lights on it. The lights flash at different but regular intervals. The orange light flashes every 2 seconds, the green light flashes every 3 seconds, and the blue light flashes every 4 seconds.

How often do the orange and blue lights flash simultaneously (at the same time)?

How often do all three lights flash simultaneously?

What would be the shortest simultaneous flash-time if their separate times of flashing were 3, 4, and 5 seconds?

If there were four lights with different flash-times of 2, 3, 4, and 6 seconds, what would be the shortest simultaneous flash time?

Can you design a three-light board with different flashing time for each light and a shortest simultaneous flash-time of 8 seconds?

Can you design a four-light board with a shortest simultaneous flash-time of 30 seconds?

How many different three-light boards have a shortest simultaneous flash-time of 24 seconds if:

(a) all three lights have different flash-times?

(b) two of the lights may have the same flash-time?

Is it possible to design a three-light board with different flash-times for each light, and having a shortest simultaneous flash-time of 4 seconds? 5 seconds?
6 seconds? ... ?

Make up some problems of your own flashing light signs and have a class-mate solve them.

***Hugh Allen***
*Faculty of Education*
*Queen's University*

# ANALYTIC GEOMETRY
## Quiz 2

Answer all of the following questions for equation 1 or 2 or 3

| | **Equation 1** $x - 2y + 6 = 0$ | **Equation 2** $2x - y - 5 = 0$ | **Equation 3** $y = 2x - 1$ |
|---|---|---|---|
| (a) Find the x-intercept | | | |
| (b) Find the y-intercept | | | |
| (c) State the coordinates of the point where the line crosses the Y-axis | | | |
| (d) State the coordinates of the point where the line crosses the x-axis | | | |
| (e) Find the slope of the line. | | | |
| (f) State the slope of a line parallel to the given line. | | | |
| (g) State the slope of a line perpendicular to the given line | | | |
| (h) Give an equation of a line that has the same Y-intercept as the given line | | | |

Note: The quiz will be marked out of 10. There will be one mark for each of the eight parts. If you use equation 1 for your answers, you will get an additional 2 marks. If you use equation 2 for your answers, you will get an additional 1 mark. If you use equation 3 for your answers, there will be no additional marks.

(I use this type of quiz so that students who understand the concepts but cannot apply them to the difficult questions are able to tackle the questions and pass the test; and yet the best student is also challenged. I encourage students to answer all three strands if they have time and I give them their best mark.)

***M. Elaine Harvey***
*Frontenac Secondary School*

# FUNCTION GRAPHICS

by Jack Sontrop
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*One future direction indicated by Ontario Ministry of Education Memorandum 91 is the use of graphics editors in a wide range of activities. I first wrote graphics programs on my home computer to dazzle myself, my family, and visitors to our rec room. This article aims to promote such activity and to foster awareness of mathematical concepts involved in certain types of applications.*

New visitors to the computer labs at Loyalist College are usually spellbound by a plotter performing the magical ritual of precision drawing. Compared to a single graph of a particular function, a composite of multiple graphs of related functions can create a striking visual impact. Composites like those reproduced in this article are difficult to produce without the aid of a computer. They are also difficult to produce with commercial computer software such as MacPaint, AutoCad and AccuCad. Yet programs to produce such composites are relatively simple and can be easily modified to create interesting illusions of curved or jagged surfaces, depth and tension. The creation of unique, function-based, graphic objects can be a challenging and satisfying experience.

To illustrate, start with the set of cubic polynomials represented by $\mathbf{y = K\,x\left(x^2 - a^2\right)}$ with intercepts at $(-\mathbf{a},0)$ $(0,0)$ and $(\mathbf{a},0)$. Figure 1 below shows sixteen such polynomials plotted over the domain $-\mathbf{a} \leq \mathbf{x} \leq \mathbf{a}$. Each graph consisted of 128 segments. $\mathbf{K}_1$ through $\mathbf{K}_{16}$ formed an *arithmetic* sequence, terminating with $\mathbf{K}_{16} = 0$.

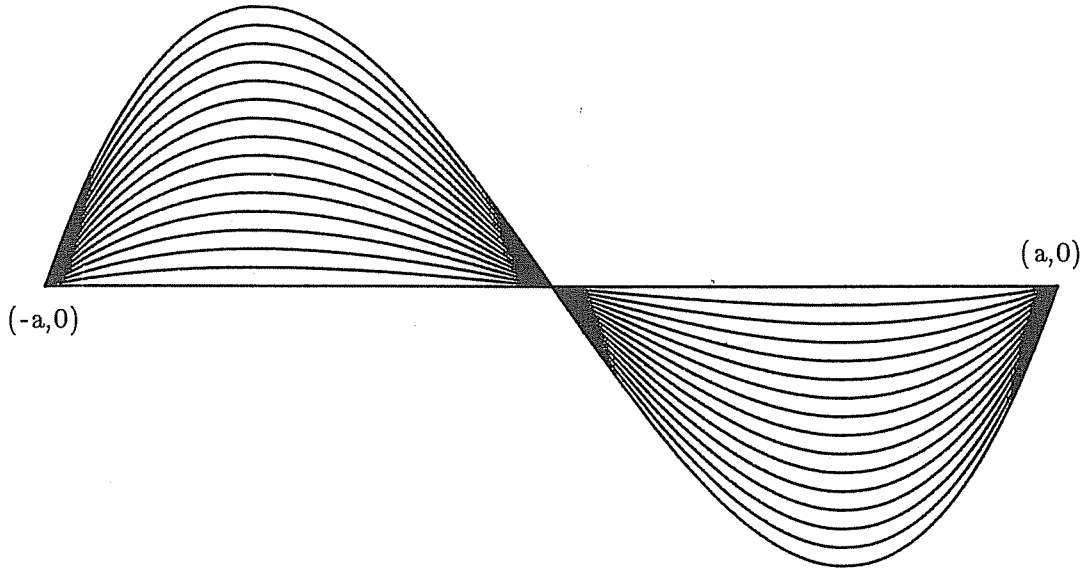



Figure 1: Family of Cubic Polynomials: $\mathbf{y = K\,x\,(x + a)\,(x - a)}$
The **K** coefficients formed an arithmetic sequence. $\mathbf{K \geq 0}$

On the next page is an algorithm for producing plots similar to Figure 1. You will have to translate it into your favourite programming language. If it's been a while since you graphed a function, it may help to get out the calculator and graph paper and do a few hand plots until you are thoroughly accustomed to the steps involved in producing graphs of functions. You may also need to review the topics of programming loops, sequences (arithmetic and geometric) and translation of coordinates.

ALGORITHM FOR FIGURE 1:

1. Initialize the output device. If it's a video display monitor, specify the screen resolution, colour, etc. If it's a plotter, specify graphic limits, grid size, scale, pen colour, pen size, etc.

2. Get (input, read, fetch, assign or scan) **a**, the absolute value of the x-intercepts.

3. Get $x_i$ and $x_f$, the limits of the domain over which each plot will be made.

4. Get **n**, the number of segments used to plot each polynomial.

5. Compute $x_{incr}$, the common difference in the arithmetic progression of abscissae as **x** proceeds from $x_i$ through $x_f$.

6. Get **p**, the number of similar polynomials to be plotted.

7. Get $K_1$ and $K_p$, the values of the numerical coefficient **K** to be used in the first and **p**'th plot respectively.

8. Compute $K_{incr}$, the common difference in the arithmetic progression of numerical coefficients as **K** proceeds from $K_1$ through $K_p$.

9. Compute $X_{off}$ and $Y_{off}$ which are offsets required to translate coordinates (Step 11) onto the grid established in Step 1. Note that if you establish a grid such that address (0,0) is near the centre of your monitor display or plotter, you can set both of these parameters to zero.

10. Initialize the numerical coefficient **K** to $K_1$.

11. Repeat the following:
    a. Initialize abscissa **x** to $x_i$.
    b. Compute its ordinate mate **y**, according to the functional relationship.
    c. MOVE the cursor or pen to $( x + X_{off} , y + Y_{off} )$.
    d. For segment number 1 to **n** perform the following:
        Increment abscissa **x** by $x_{incr}$;
        Compute its ordinate mate **y**, according to the functional relationship.
        DRAW the cursor or pen to $( x + X_{off} , y + Y_{off} )$.
    e. Increment **K** by $K_{incr}$.

    until the **p**'th plot is finished.

Once you have implimented the basic algorithm, refine it to do some simple error checks. For example, what's to be done if the calculated coordinates are off the grid? (If your graphic output is to a monitor, this may not be a problem.)

Next you could modify Step 8 to create a graphic like Figure 2. Notice that the spacing between successive graphs (at $x = \pm a/2$) is no longer constant as it was in Figure 1. This may be important if you wish your composite to have a 3-D effect like Figure 6. Experiment with *geometric* rather than arithmetic increments to **K**. My increments were determined as follows: let the initial smallest difference between successive **K**'s be **d**; then let the sequence of subsequent *differences* be **2d, 3d, 4d** ..... **4d, 3d, 2d, d**. It is left to you to determine the relationship between **d, p,** $K_1$ and $K_p$ for odd and even values of **p**.

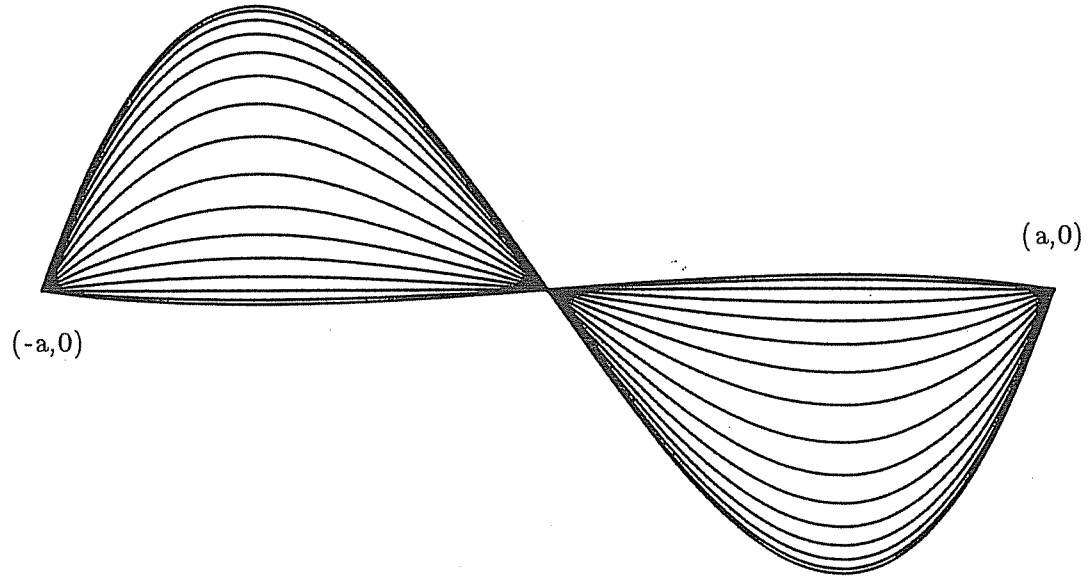



Figure 2: The **K** coefficients did *not* form an arithmetic sequence.

By keeping **n** small and extending the values of $x_i$ and $x_f$ beyond the intercepts, you can create a more jagged effect like Figure 3. Interesting patterns can be produced with a colour monitor. Experiment with different colour combinations. Don't hesitate to use *low* resolution to give a "painted with a wide brush effect".

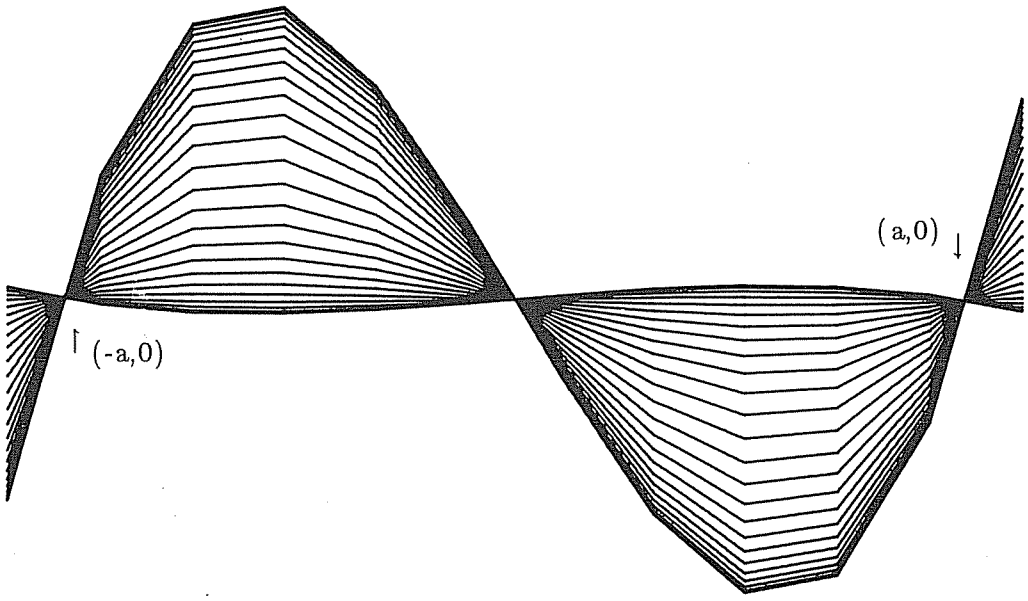



Figure 3: $\mathbf{n} = 11$; $\mathbf{x_i} < -\mathbf{a}$; $\mathbf{x_f} > \mathbf{a}$

The symmetry of the composite can be most easily upset by choosing unequal absolute $x_i$ and $x_f$ values. This is not illustrated here. A second way is to put in a selective y-scaling factor at Step 11. For example (Figure 4) at all abscissae greater than or equal to zero, scale down and invert the ordinates. In BASIC the code is simple:

```
IF X >= 0  THEN Y = -0.35 * Y
```

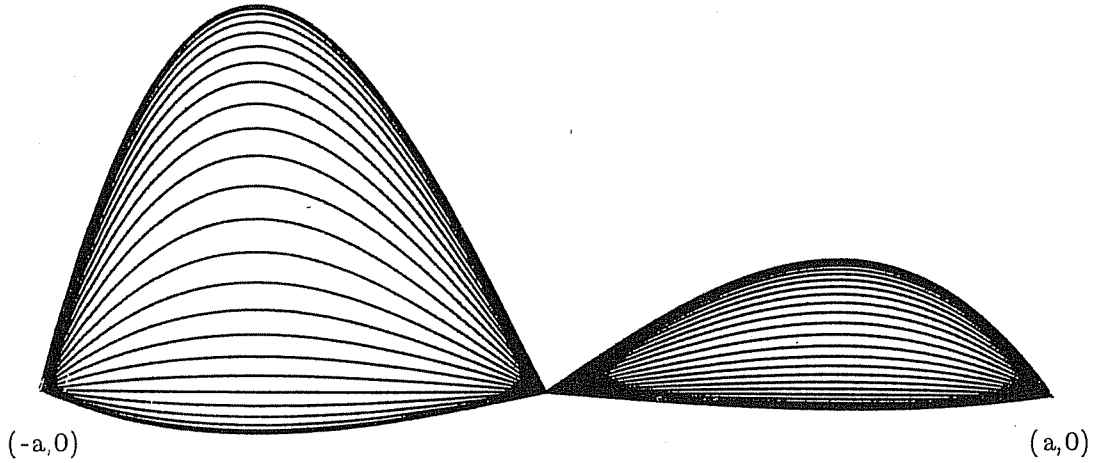



Figure 4: At $x \geq 0$ the ordinates were scaled down and inverted.

For those Artsies who dislike symmetry, there are lots of interesting non-symmetrical functions with which to experiment. Figure 5 was produced using the polynomial function $y = K x^3 (x + a) (x - b)$ with intercepts at at $(-a,0)$ $(0,0)$ and $(b,0)$. Make sure you modify Step 2 of the algorithm to get **a** and **b** from the user. Then if you want symmetry again (Figure 6) set $K_1$ equal to $-K_p$.

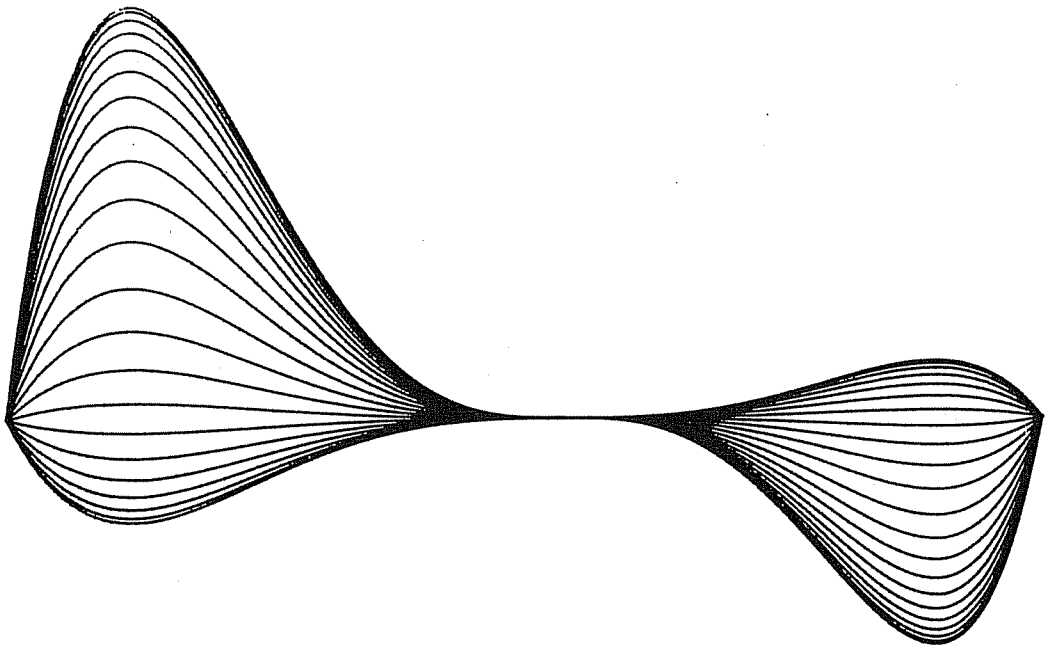


Figure 5: Polynomials: $y = K x^3 (x + a) (x - b)$ $( x_i = a;$ $x_f = b;$ $n = 128 )$

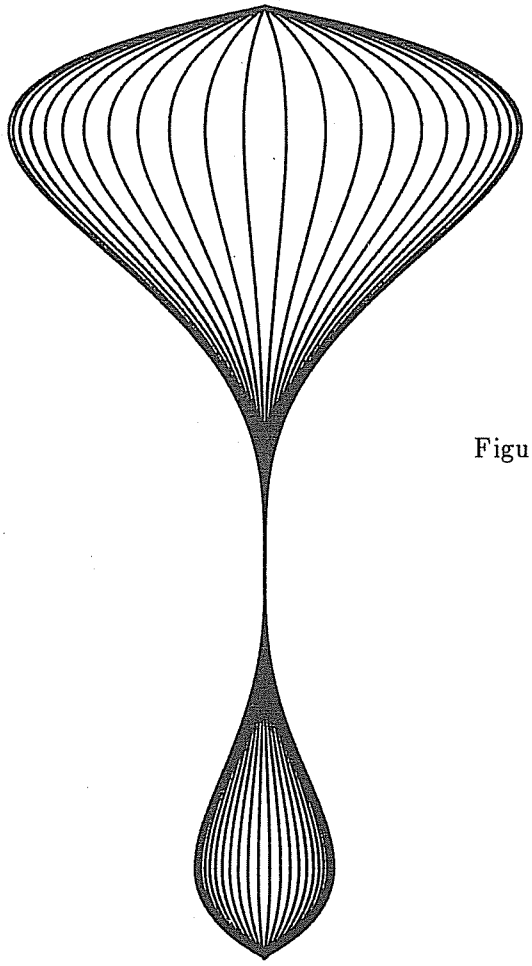


Figure 6: Polynomials: $y = K x^3 (x + a) (x - b)$
$-K_1 = K_p$. Graphs rotated 90 degrees.

Once you master polynomials, you may want to go on to more challenging functions. Another challenge is to make your programs run efficiently. There may be some heavy number crunching involved in programs that use trigonometry to produce composites like Figures 7 through 10. Modularize your programs. Use functions and procedures (subroutines). That way you won't be reinventing the program every time you have a new idea and you can cannibalize old programs to make new ones.

Finally, a warning - you could get so caried away with graphics, that you'll end up drawing something that everyone will want a copy of. See Figure 11.

*All figures in this article were produced on an Hewlett Packard Model 7221A plotter using programs that I recently converted from BASIC to C. These programs now reside on a DEC VAX 11/750 running under UNIX 4.3 bsd.*

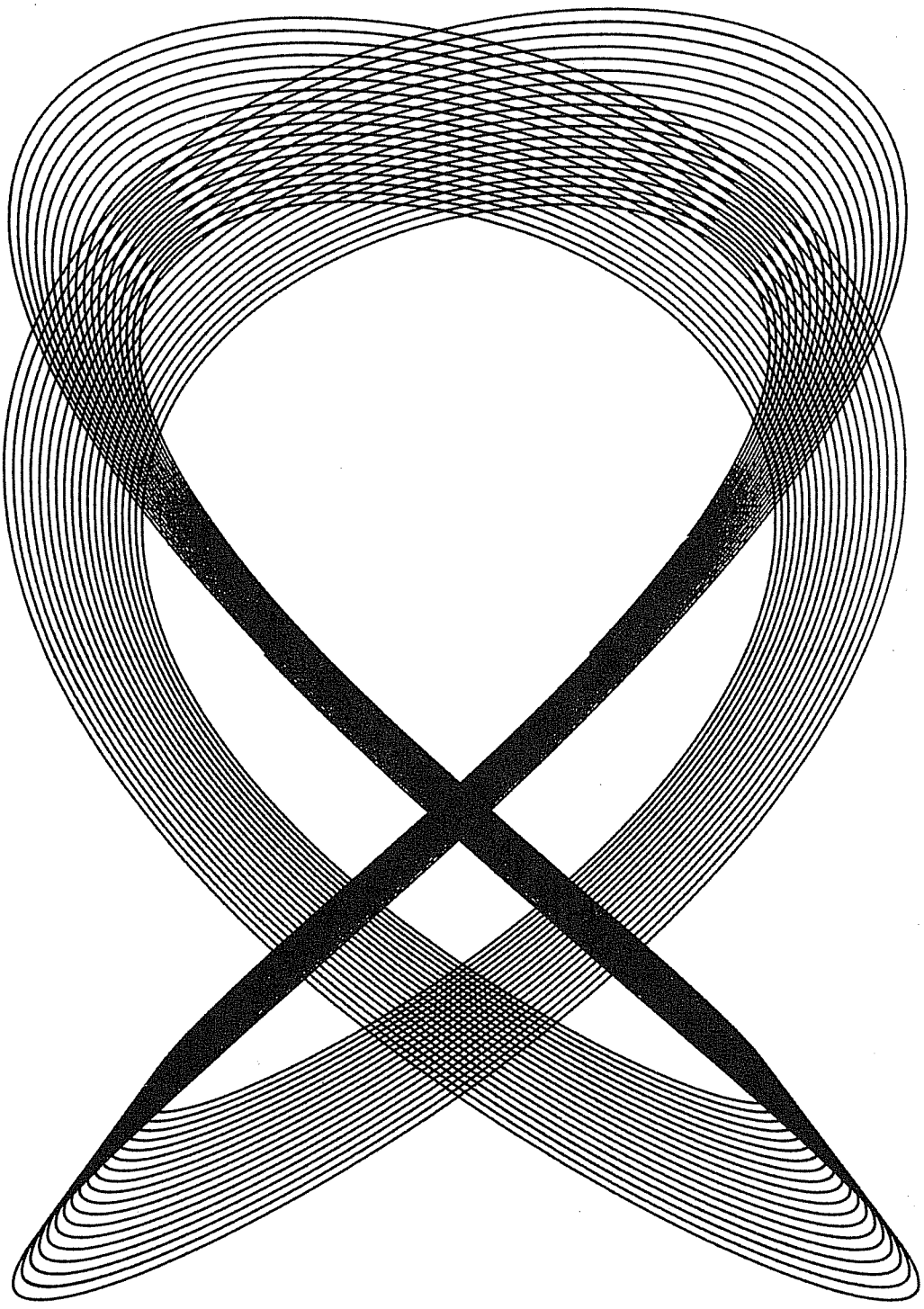


Figure 7: Lissajou Figure - Vibration in a Plane $\quad x = A_x \cos\left(2\pi f_x t + \Theta_x\right)$

$y = A_y \cos\left(2\pi f_x t + \Theta_y\right)$

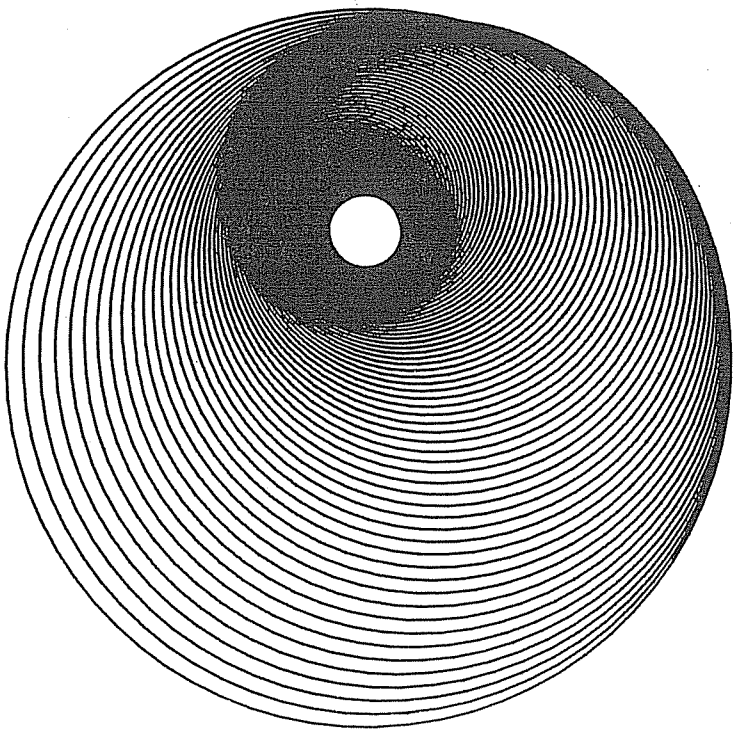


Figure 8: Circles of decreasing radii along a spiral path.

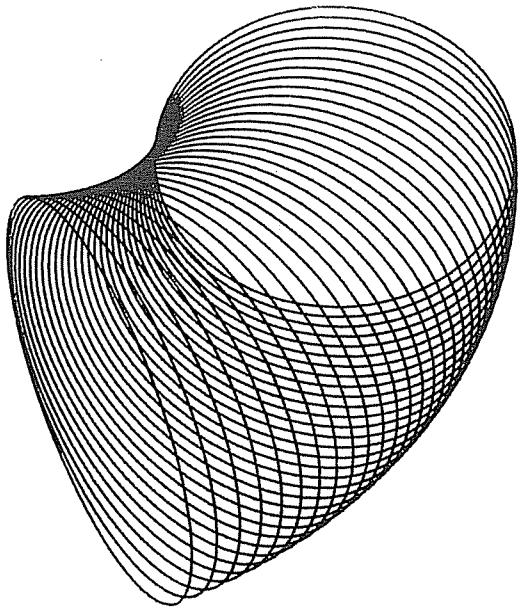


Figure 9: Ellipses of varying size, eccentricity and orientation along an elliptical arc path.

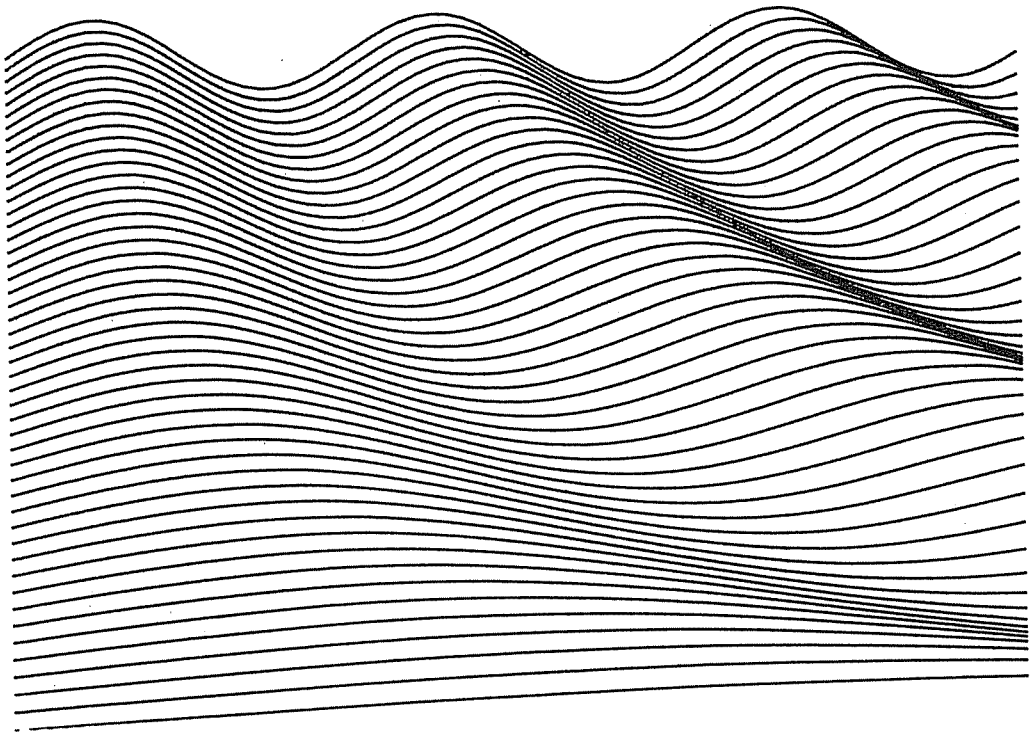


Figure 10: $\mathbf{y} = \mathbf{K} + \mathbf{A} \sin\ 2\pi \mathbf{f}\, \mathbf{x}\ ;\quad \mathbf{f}_1 \leq \mathbf{f} \leq 12\, \mathbf{f}_1$. Amplitude **A** was the same (constant) for each function. Vertical space between the graphs as determined by **K**, is a geometric progression. From bottom to top each *change* in **K** was 99 percent of the previous change.

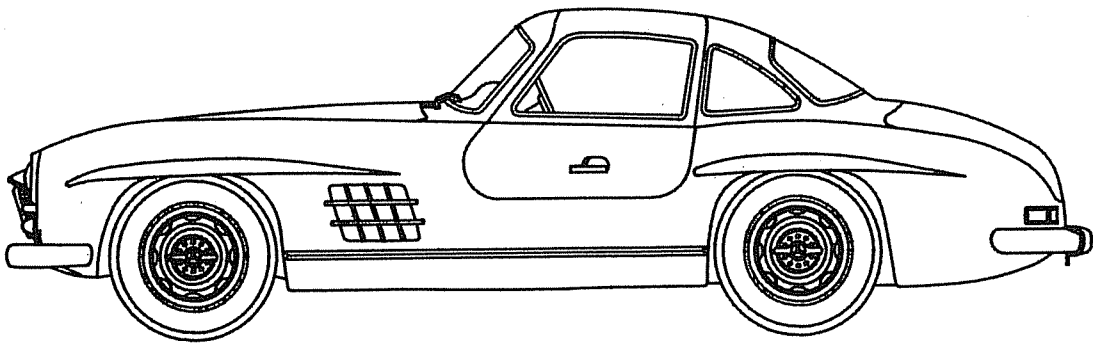


Figure 11: *1955 Mercedes-Benz 300-SL.*
The car was defined as a composite of 582 lines and arcs.

# GIRLS AND MATHEMATICS CONTESTS: PART II

*by*
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In Part I, we noted several patterns that have emerged from our analysis by sex of recent Descartes and Euclid contests. In this article, we speculate briefly on the reasons for these observations; specifically, why ....

- do girls participate in math contests less frequently than boys?
- don't girls win such contests very often?
- do girls leave questions untouched more often?
- do girls choose the 'distractor' answer more often?

**1. Participation:**

Actually, the rate at which girls participate in mathematics contest seems to be increasing, if the University of Waterloo contests are indicative. The fraction of female competitors in 1987 ranged from around 28% on the Descartes contest (grade 13) to around 48% on the Pascal (grade 9). It is also worth noting that in the Australian Mathematical Contest, which consists of three separate papers, one each for grades 7-8, 9-10, and 11-12, the participation rates of males and females have been roughly equal since its inception in 1976.

There are several possible contributing factors to non-participation in contests. Some individuals experience higher levels of anxiety than others, and 'math anxiety' seems to affect students' self-confidence particularly badly; there is also evidence that females, especially, tend to underestimate their performance levels [1], [2], [5]. Even though anxiety tends *not* to account for a large portion of *actual* performance discrepancies [3], it is hardly a state of mind conducive to voluntary competition in contests. A second possibility is indicated by studies which show that *past success* and *teacher* are more significantly related than gender to self-confidence in geometry problem solving [4]; further support for the importance of the teacher's role is provided by the fact that when they do occur, differences in mathematical performance are found between the boys and girls in some high schools, but not others [8], [10]. A 1984 survey of Ontario teachers [11] asked whether it is important to "Offer special encouragement to girls to do well in mathematics" and to "Try to convince students that math is as appropriate for girls as for boys"; 47.6% of the grade 8 teachers queried and 50.5% of the grade 13 teachers rated the first statement "of little or no importance" (the lowest of four possible ratings), while 31% and 43.8% respectively, did so for the second one. Differences in participation rates may well be partly due to the attitudes of some teachers.

Trying to answer why fewer girls enter contests may be somewhat akin to looking for reasons why there aren't more female participants in *'les 24 heures du mans'* (a famous European car rally). Like high school mathematics, driving a car and navigating with road maps are things many of us do reasonably competently, yet women do not tend, much, to enter competitive events involving driving. Perhaps, as one (male) University of Waterloo math graduate answered when asked why more high school girls don't enter math contests, "They've got better things to do!"

**2. Winning:**

While there is a wealth of literature on gender - related differences in mathematical abilities/achievement, there is little agreement on how such abilities should be defined, nor on how differences should be measured; in fact, many older studies have been re-analyzed, and doubt cast on the original conclusions [6]. Further, different measures used on the same data may imply different conclusions. Benbow and Stanley, on their on-going **Study of Mathematically Precocious Youth,** use a measure called 'effect size' in comparing scores; it involves normalized differences in mean scores, and indicates significant male-female differences in areas such as 'mechanical reasoning' and space relations' [7]. When a similar measure was applied to the 1985 Descartes contest scores [8], it also indicated significant differences favouring males on 10 of the 12 questions. However, when an 'area of overlap' measure of the discrepancies was used, the results did not disprove the null hypothesis (no gender difference) at the 5% level. It should be noted that sample means and their differences are *very* sensitive to a few observations that are markedly higher than the rest; thus distribution *means* may suggest larger differences than measures which compare the *entire* distributions.

One obvious reason for fewer girls winning is their lower participation rates; beyond that, the waters get muddy pretty quickly. Forgetting gender for the moment, what distinguishes winners from the merely competent? A profile emerges of a student who progresses rapidly through the standard mathematics curriculum (often precociously), participates extensively in extra-curricular mathematical activities, and has great intrinsic zest for such competitive events. There is certainly documented evidence that course-taking influences mathematical performance in many areas (see [9], for example); other inhibitory factors for non-winners may include greater caution, reluctance to guess, inexperience (see below), or just generally being slower to 'get into gear'. Finding con-

clusive evidence about the very complex set of ingredients in the recipe for a winner is no easy task.

**3. Attempts:**

Some interesting patterns in students' attempts, and their relationship to performance, emerged in our study of the 1985 Descartes contest, and were re-affirmed by the 1986 Descartes and Euclid results. While significant differences in attempts favouring both males and females were found, they tended to favour males on questions involving spatial visualization, and females on more abstract problems involving matrices and series; this may reflect courses taken. Further, it was not clear whether the differences arose from interest levels or from the order in which the problems were organized.

Some very intriguing anomalies were found, in particular, in the patterns of attempts and productivity on two related 1985 problems, #4 and #10. To do #4 requires knowing the equations of both the ellipse and the hyperbola (i.e. conic section equations); thus one would expect a student who is successful on #4 to stand a better chance on #10, which involves a parabola. Yet female attempts on #10 lag *farthest* behind male attempts for those females who were *most* successful on #4. Further, while those females who attempted #10 scored consistently higher as their scores on #4 increased, this was *not* true for males attempting #10; in fact, many males who did *not* attempt #4 were productive on #10. Clearly, sorting out the effects of gender, knowledge, self-confidence and ability is a difficult assignment indeed.

The mean numbers of attempted questions on the 1985 Descartes did not differ much overall (70.1% for females versus 70.9% for males); however, the female attempts were somewhat less productive, leading to lower scores. Explanations for this diminished productivity in a contest situation may include a disinclination on the part of girls to persist when confronted with negative feedback, in this case of the internal sort (i.e. frustration) [12]; a preference for more abstract problems in which they are more interested, but for which they are not necessarily better prepared (female attempts were higher than male attempts, overall, on the more difficult and abstact half of this contest, and fewer girls than boys take Grade 13 algebra [11]); or as was alluded to earlier, their high school background (matching male-female pairs within each school and grade level was found to reduce but not eliminate the gender discrepancies on the 1985 Descartes).

**4. Distractors:**

In multiple-choice questions, girls choose the 'distractor' answers more frequently (a distractor is an incorrect response reachable by a simple but fallacious argument); they also leave more blanks. In a sense, these seem almost contradictory tendencies, since one might attribute the choice of distractors to haste or lack of care, whereas leaving more blanks might be due to an over-cautious approach.

What characterizes a student who is misled by a 'plausible incorrect alternative'? One possibility is simply lack of training; a student who has not competed much in previous contests will perhaps not have acquired the healthy skepticism necessary to reject an apparently easy solution, or to try a problem another way as a check. Another contirbuting factor may be a more 'holistic' thought process, into which enter elements other than the 'pure logic' of the mathematical solution. A remarkable example of this can be found in [13]; in this study, the researchers started with a simple arithmetic word problem that had two distractors, one of which seemed to attract more females than males and led to significant differences in their scores. They repeated the experiment with several slight variations, and discovered that if the stronger distractor was eliminated, the male and female choices among the remaining four answers were virtually identical; further, when the problem was re-worded so that the (previously wrong) distrator became the *correct* answer, there was also no significant gender difference. Certainly food for thought!

**Conclusions:**

Whether they are affected by disinterest in competing, lack of confidence, differential course-taking, or something else, no doubt both participation rates and performance of young women in mathematics problems will continue to improve if we try to create a more nurturing environment for them. To that end, in Part III, we will try to put forward some practical ways to overcome some of the above-mentioned problems.

*References:*

1. *Furst, D., Tenenbaum, G., Weingarten, G., "Test Anxiety, Sex, and Exam Type", Psychological Reports, 1985, V. 56, pp 663-668.*

2. *Buckley, P., Ribordy, S., "Mathematics Anxiety and the Effects of Evaluative Instructions on Math Performance", paper presented at Midwestern Psychological Association, Minneapolis, MN, May 1982. (ERIC Report ED 222 334, SE 039 200.)*

3. Harress Dew, K.M., Galassi, J.P., Galassi, M.D., "**Math Anxiety: Relation with Situational Test Anxiety, Performance, Physiological Arousal, and Math Avoidance Behaviour**", *J. of Counseling Psychology, 1984, V. 31, No. 4, pp 580-583.*

4. Peterson, K. et al, "**Geometry Students' Role-Specific Self-Concept: Success, Teacher, and Sex Differences**", *ERIC Report ED 229 219, SE 041 299, 1983.*

5. Chipman, S.F., Brush, L.R., Wilson, D.M., Eds. "**Women and Mathematics: Balancing the Equation**", *Hillsdale, NJ: Earlbaum, 1985.*

6. Caplan, P.J., MacPherson, G.M., Tobin, P., "**Do Sex-Related Differences in Spatial Abilities Exist? A Multilevel Critique with New Data**", *American Psychologist, 1985, V. 40, No. 7, pp 786-798.*

7. Holden, C., "**Female Math Anxiety on the Wane**", *Science , V. 236, May 1987.*

8. Bischoping, K., **University of Waterloo Mathematics Faculty Internal Report,** *1985.*

9. Johnson, S., "**Effects of practice and training in spatial skills on sex-related differences on performance on embedded figures**", *Unpublished thesis, Fairfox, Virginia, George Mason University, 1976.*

10. Sherman, J., "**Sex-Related Cognitive Differences**", *Thomas Publishers, 1978.*

11. Raphael, D., Wahlstrom, M., McLean, L., "**Are Boys Better at Math than Girls?**", *Orbit, V. 15, No. 2, April 1984, pp 6-9.*

12. Xonge, G.D., "**Sex Differences in Cognitive Functioning as a Result of Experimentally Induced Frustration**", *Journal of Experimental Education, 1964, V. 32, No. 3, pp 275-80.*

13. Maier, N.R.F., Burke, R.J., "**Response Availability as a Factor in the Problem-Solving Performance of Males and Females**", *J. of Personality and Social Psychology, 1967, V. 5, No. 3, pp 304-310.*

# SAME: OF MICE AND MATH 1987



**Tom Griffiths**

# WHY JOANNIE DOESN'T WANT TO ADD

*by*
*William Higginson*
*Faculty of Education*
*Queen's University*
*Kingston, Ontario K7L 3N6*

**Select Bibliography**

Belenky, Mary Field; Blythe McVicker Clinchy, Nancy Rule Goldberger and Jill Mattuck Tarule **Women's Ways of Knowing: The Development of Self, Voice and Mind.** New York: Basic, 1986, 0-465-09212-8, 256 pp.

Brush, Lorelei R. **Encouraging Girls in Mathematics: The Problem and the Solution.** Cambridge: Abt, 1980, 0-89011-542-7, 163 pp.

Carmichael, Hilda W.; Dale J. Burnett, William C. Higginson, Barbara G. Moore and Phyllis J. Pollard **Computers, Children and Classrooms: A Multisite Evaluation of the Creative Use of Computers by Elementary School Children.** Toronto: Ministry of Education, 1985, 446 pp.

Chipman, Susan F.; Lorelei R. Brush and Donna M. Wilson **Women and Mathematics: Balancing the Equation.** Hillsdale, N.J.: Earlbaum, 1985, 0-89859-369-7, 381 pp.

Eisner, Eliot (Ed.) **Learning and Teaching the Ways of Knowing.** Eighty-fourth Yearbook of the National Society for the Study of Education, Part II Chicago: NSSE/University of Chicago Press, 1985, ISSN 0077-5762, 304 pp.

Gardner, Howard **Frames of Mind: The Theory of Multiple Intelligences.** New York: Basic, 1983, 0-465-02508-0, 440 pp.

Gilligan, Carol **In a Different Voice: Psychological Theory and Women's Development.** Cambridge: Harvard, 1982, 0-674-44544-9, 184 pp.

Hewlett, Sylvia Ann **A Lesser Life: The Myth of Women's Liberation in American.** New York: Warner, 1986, 0-446-385115-5, 475 pp.

Keller, Evelyn Fox **A Feeling for the Organism: The Life and Work of Barbara McClintock.** New York: Freeman, 1983, 0-7167-1504-X, 235 pp.

Lenz, Elinor and Barbara Myerhoff **The Feminization of America: How Women's Values are Changing our Public and Private Lives.** Los Angeles, Tarcher, 1985, 0-87477-369-5, 276 pp.

Lessing, Doris **Prisons We Choose to Live Inside.** CBC Massey Lectures Toronto: CBC Enterprises, 1986, 0-88794-244-X, 76 pp.

Moore, Barbara "Equity in Education: Gender Issues in the Use of Computers: A Review and a Bibliography." **Review and Evaluation Bulletins 6** (1), 1986, Education and Technology Series. Toronto: Ministry of Education, 68 pp.

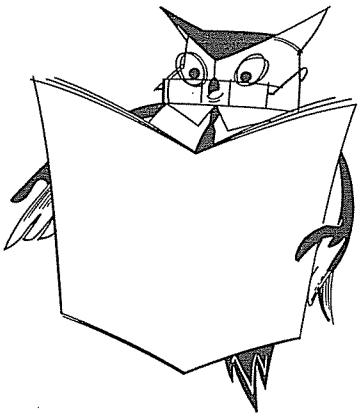
Skolnick, Joan; Carol Langbort and Lucille Day **How to Encorage Girls in Math and Science: Strategies for Parents and Educators.** Englewood Cliffs, N.J.: Prentice-Hall, 1982, 0-13-405662-0, 192 pp.

Tittle, Carol Kehr "Gender Research and Education". **American Psychologist 41** (10) October 1986, 1161 - 1168

Tobias, Sheila **Overcoming Math Anxiety.** New York: Norton, 1978, 0-393-06439-5, 278 pp.

Toronto Board of Education, Mathematics Department **Mathematics: The Invisible Filter: A Report on Math Avoidance, Math Anxiety and Career Choices.** Lorna Wiggan, Senior Author. Revised edition, July 1985 (January 1983), 52 pp. Bibliographies. **A particularly thorough, well-written and thought-prevoking report. Available as part of the "Invisible Filter" kit from the Mathematics Department of the Toronto Board of Education [155 College Street, M5T 1P6] for $18. The kit includes five related pamphlets. Two posters, "Mathphobia Can Cost You a Career" and "Dropping Math? Say Good-by to 82 Jobs" are available for $2 each from the same address.**

*Author's Note: A presentation at the 1987 Conference of the Ontario Association for Mathematics Education Scarborough, 1987 05 08.*



# HISTORY AND PHILOSOPHY OF MATHEMATICS

*by*
*E. J. Barbeau, Editor*
*University College*
*University of Toronto*
*Toronto, Ontario M5S 1A1*

***Editor's Note:*** *The writer of this section welcomes comments, queries and notes from the readers of the Gazette on matters concerned with the history, philosophy and heuristics of mathematics.*

A key topic in the calculus curriculum is logarithms. Nowadays, the logarithm is generally defined as the indefinite integral of 1/x, although there are treatments which begin with the exponential function and handle the logarithm as the inverse of this. However, the prevalence of handheld calculators has led to the demise of the idea of the logarithm as a computational tool. This is unfortunate in that the students no longer have the direct practical experience of working with logarithms as their parents and grandparents and in that two fundamental ideas about logarithms get downplayed:

(1) $\log_b x$ is the power to which the base b must be raised in order to obtain x;

(2) logarithms are a means of converting a problem in multiplication to one in addition, and a problem in exponentiation to one in simple multiplication, through the characteristic properties

$$\log (xy) = \log x + \log y$$

$$\log (x^c) = c \log x.$$

There is a cultural loss here which poses a dilemma for the teacher. While it is not practical nor desirable to return to the extensive use of log tables for calculations, we should somehow recapture the broader context in which students should perceive and understand logarithms. One way is to look back into history. An excellent reference is **A history of numerical analysis from the 16th century to the 19th century,** by Herman H. Goldstine (Studies in the history of mathematics and physical sciences 2, Springer-Verlag, 175 Fifth Avenue, New York, NY 10010). This book is accessible to a thoughtful student and could well be considered for the school library.

In some way, the sixteenth century is like our own. It was a time of great ferment and development, a period of scientific advancement, commercial activity, exploration and philosophical reorientation. Demands of science, astronomy and business for convenient and effective mathematical tools were answered by the invention of notation and algorithms still in use today, the universal adoption of Arabic numeration and a number of publications designed to bring mathematical techniques to a wider public. One important bit of "software" was developed almost simultaneously by Joost Burgi (1552-1632/33), John Napier (1550-1617) and Henry Briggs (1561-1631).

The procedure used by Briggs to create a table of logarithms is particularly transparent and depends on being able to take repeated square roots. I do not know Briggs' algorithm for finding sqaure roots, but in the classroom one might show the students the paper-and-pencil computation which was regularly taught over thirty years ago. Suppose it is desired to find the square root of 10. Write down the number whose root is to be found, pairing off the digits on either side of the decimal point:

$$1\,0\ .\ 0\,0\ 0\,0\ 0\,0\ 0\,0\ \ldots$$

The first digit of the root is the highest number whose square does not exceed the first block (of one or two digits) of the number to be SQiRTed. Let us interrupt the algorithm at a later stage and see how one gets the next digit:

```
        3.  1  6  2
      ┌──────────────
      10. 00 00 00 00
       9
      ──
61     1  00
          61
          ──
626       39 00
          37 56
          ─────
6322       1 44 00
           1 26 44
           ───────
6324?        17 56 00
```

Observe that 3756 = 626 x 6; 12644 = 6322 x 2.

Bring down on the left **twice** the result obtained across the top so far; thus 6324 = 2 x 3162. The next digit of the root is the highest digit (?) for which 6324? multiplied by ? does not exceed 175600 (the 175600 is obtained in a way similar to that in ordinary long division). Since 63243 x 3 = 189729 and 63242 x 2 = 126484, the digit chosen should be 2. Write this in the top at the right, to get 3.1622 and continue on.

To make sure you have the hang of it, here is a worked example:

```
        2  8  7
      ┌──────────
      8 23 69
      4
      ──
48    4 23
      3 84
      ─────
567     39 69
        39 69
```

I have found that high school students are quite interested in this, and it is both a mathematical and expository challenge for your better students to explain why it works. Perhaps some reader would like to comment on the origin of this algorithm and on the algorithm that Briggs himself might have used.

Let us return to Briggs who made a table, writing in the left column the results of taking repeated square roots of 10 and in the right column the exponent (or logarithm) of 10 yielding the root:

| | |
|---|---|
| 10.000000 | 1 |
| 3.162277660 1638793319 9889354.. | 0.5 |
| 1.778279410 0389228011 9730413.. | 0.25 |
| . . . . . | |
| 1.000000000 0000005112 7659728.. | 0.00000000000000222044.. |
| 1.000000000 0000002556 3829864.. | 0.00000000000000111022.. |
| 1.000000000 0000001278 1914932.. | 0.00000000000000055511.. |
| . . . . . | |

If we look closely at this table, we find that the left hand column is a number of the form 1 + x, where x is very small. Taking the square root yields $(1 + x)^{1/2}$ which is very close to $1 + \frac{x}{2}$. Each entry in the right column is half its predecessor. As a consequence, for x small, log (1 + x) is proportional to x. From the table, we can make the computation that, for small x, log (1 + x) ≐ 0.43429448..x (the constant of proportionality is actually $\log_{10} e$ - do you see why?).

Having thus got the constant of proportionality, we are now in a position to construct a table of logarithms for the natural numbers. All we need to do is to find the logarithms of the primes; the logarithms of the other numbers can be found by adding together the logarithms of the primes involved in its prime factor decomposition.

Let p be a prime. Let us SQiRT p a large number, n of times to find that $p^{2^{-n}} = 1 + u$, where u is very small. Then $\log p = 2^n \log (1 + u) = 2^n(0.43429448..)u$ gives us the logarithm of p.

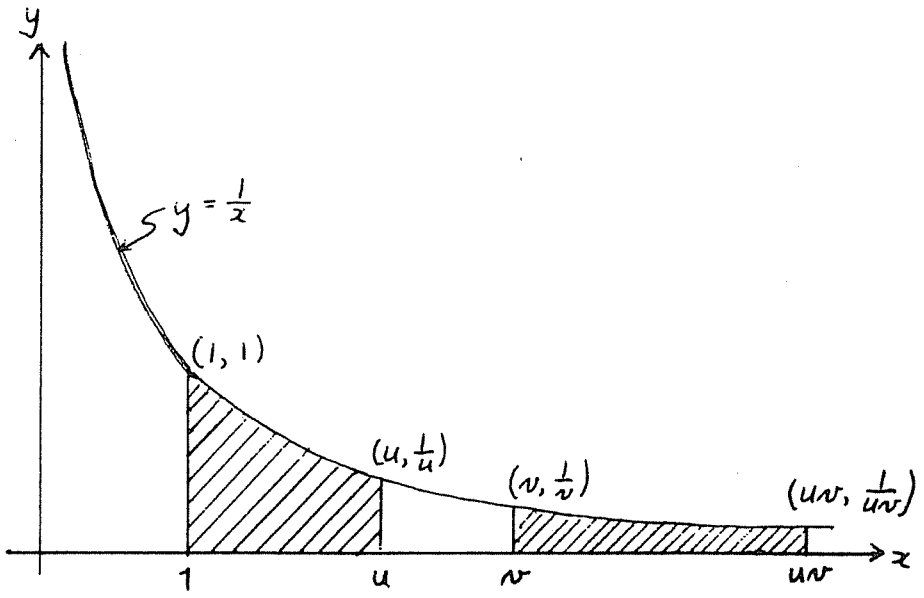
This historical excursion exposes the student to quite a lot of mathematics. He gains a hands-on experience of the behaviour of log (1 + x) for x close to 0 and an understanding of the interaction of logarithms with exponents and products. A computer science student could be given the task of writing a programme which will construct a logarithm table by the technique of Briggs. Finally, there is the task of analyzing the constant of proportionality in modern terms.

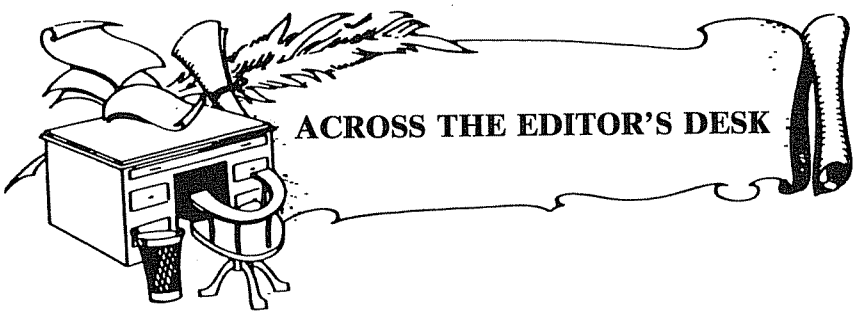
At approximately the same time as the development of the logarithm, Gregory of Saint Vincent made an observation about the rectangular hyperbola which was to shed a different sort of light on the role of the logarithm function. In the diagram, the graph is that of the curve $y = \frac{1}{x}$. Let A(x) denote the area under the graph between the abscissae 1 and x (we will look only at the situation where $x > 1$). Compare the regions under the graph between the abscissae 1 and u, and between the abscissae v and uv. The right hand region is obtained by expanding the left horizontally and contracting the left vertically by a factor v; thus, both regions will have the same area. In other words $A(u) = A(uv) - A(v)$

or $A(uv) = A(u) + A(v)$

so that the function A has the characteristic property of the logarithm. It did not take the eighteenth century analysts long to argue that A had to be a logarithm for some base, and so was born an alternative formulation of the logarithm, which we now make use of.

This is discussed in a marvellous little book called **The calculus: a genetic approach** by Otto Toeplitz (University of Chicago, Phoenix paperback, 1963), another good acquistion for the school library.






May 29, 1987

Clifford Gravelle, Editor
College of Education, Brock University
St. Catharines, Ontario L2S 3A1

Dear Mr. Gravelle,

It seems that we will forever, in Northern Ontario, be trying to convince the rest of the world that we are still a part of Ontario even though we don't live in Toronto.

The article "Math Whizzes" which appeared on page 51 of the March issue of The Gazette is a reprint of an article which appeared in the October issue of Education Ontario which was a reprint of an article which appeared in The Toronto Star during the summer of 1986 . . . . . I have been chasing down these reprints all year to correct the same error.

The article is a report on the Ontario math students who represented Canada in Poland at the International Mathematics Olympiad during the summer of 1986. If you ask "where's the beef", it is in the fact that the article reports that "four of the six Canadian team members are Ontario winners", and goes on to name the four Toronto students. I hasten to point out that there were five team members from Ontario. For the **second year in a row, Giuseppe Russo, a student at Lasalle Secondary School in Sudbury, was a team member and won a bronze medal this past summer.** Actually, Lasalle has had students on the team for the past **three** years. Russo and former team member Frank D'Ippolito are now undergraduates at the University of Waterloo.

Hopefully I can now call off my scouts who are out looking for any more reprints of the "offending" article.

Yours from the "far North"

John Savage
Lasalle Secondary School
Sudbury, **ONTARIO** P3A 2G1

April 27, 1987

To the Editor of the Ontario Mathematics Gazette

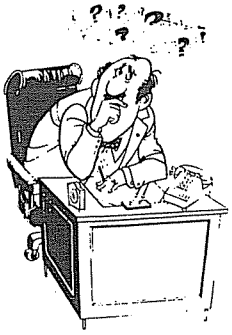
In the March, 1987 issue, you reprinted an article from **Education Ontario** on Math Whizzes (page 51). Unfortunately, the second and third last paragraphs of the article may cause some misconceptions, which should be corrected.

The third last paragraph suggests that Ravi Vakil and Steven Siu were guaranteed a place on the International Mathematical Olympiad team in Cuba on the basis of their performance in Warsaw. This is not true. While their Warsaw performance will certainly have a bearing, the final selection of team members depends on a number of criteria including solving problems by correspondence and performing well on competitions during 1986-87. No student is guaranteed a place on the team until the final selection is made in mid-May.

The second last paragraph suggests that the USAMO is the critical qualifier for membership on the IMO team. (Incidentally, the 367,000 figure applies to the number writing the AHSME competition). The Canadian Mathematics Olympiad is most important, since it is the one competition likely to be taken by all the possibilities. However, we do pay a lot of attention to AHSME, AIME and the Descartes competitions.

There are a number of places in which the article skirts the pale of accuracy, and it is unfortunate that Ms. Widerman did not check her text with someone involved with the IMO before seeing it into print.

University College University of Toronto
Toronto, Ontario M5S 1A1



# PROBLEMS

*by*
*Ronald Lancaster, Problem's Editor*
*Oakville-Trafalgar High School*
*Oakville, Ontario L6J 3L6*

1. Write the polynomial $x^8 + x^4 + 1$ as a product of four different factors.

2. Investigate the remainder that is obtained when a prime number is divided by 30.

3. a) Is it possible to find three real numbers x, y, and z for which

$$x + y + z = 1$$
$$x^2 + y^2 + z^2 = \tfrac{1}{2}$$

b) Repeat part a) for each of the following cases.

i) $x + y + z = 1$
$x^2 + y^2 + z^2 = \tfrac{1}{3}$

ii) $x + y + z = 1$
$x^2 + y^2 + z^2 = \tfrac{1}{2}$

iii) $x + y + z = 1$
$x^2 + y^2 + z^2 = 5$

**Answers on Page 44**

# OPEN LETTER

To: Friends of Jim

From: Mr. Paul Zolis, Co-ordinator of Mathematics
Scarborough Board of Education
Scarborough, Ontario M1P 4N6

Dear Friend:

On January 22, 1987, Jim Fencott passed away.

Jim was Co-ordinator of Mathematics in Scarborough from 1972 until his retirement in July of 1985. Not long after he "retired", Jim joined CTF's "Project Overseas" team in The Gambia, West Africa. There, Jim devoted his considerable energies to the task of upgrading the math skills of under qualified teachers and high school students.

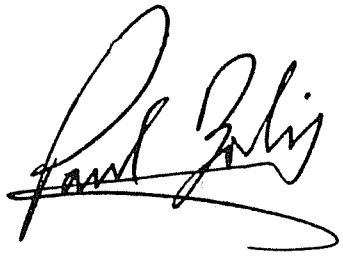
Beginning this year, the Program Department of the Scarborough Board of Education will be initiating two new awards to honour Jim: **The Jim Fencott Math Award (Scarborough) and The Jim Fencott Education Trust (Gambia).** The former will be an annual award open to Scarborough students who excel in Mathematics at the grade 8 level. The latter fund has been created in recognition of Jim's deep commitment to the advancement and support of Mathematics education in The Gambia. Each year a grant will be made in Jim's name through C.O.D.E., (Canadian Organization for Development Through Education) to one outstanding student in Gambia, for the purpose of providing books and/or supplies that will enhance the teaching of Mathematics in that country.

We are requesting donations for those of you who knew Jim, and with whom he worked to improve the quality of Mathemaics education, in order to establish these two special awards.

If you would like to contribute to this worthwhile endeavour to encourage deserving students and teachers of Mathematics in Scarborough and in The Gambia, and to honour the memory of an outstanding educator, please forward your gift to the address shown below. All cheques should be made payable to the **JIM FENCOTT MATH FUND.** You may use the enclosed coupon to indicate how you wish your donation to be allocated. **Please note that your entire donation is tax deductible.**

**Your participation in this effort is greatly appreciated!**

Yours sincerely,



**Please mail to:**
Mrs. Sue Ames,
Program Department
Scarborough Board of Education
140 Borough Drive
Scarborough, Ontario M1P 4N6



Name: _____

Address: _____

_____

Cheque Amount:$ _____

Please indicate below how you would like your donation to be used:

☐ Jim Fencott Math Award (Scarborough)

☐ Jim Fencott Education Trust (Gambia)

☐ Both

Please indicate below if you are willing to participate in future fundraising events:

☐ YES ☐ NO

Please indicate if you would like a receipt mailed to you:

☐ YES ☐ NO



## WHAT'S GOING ON

## DONALD ATTRIDGE

*by*
*Dave Davidson, Mathematics Department*
*Collingwood Collegiate Institute*
*Collingwood, Ontario L9Y 2J2*

Don attended elementary and secondary schools in Meaford, Ontario, where his father was a dentist. He graduated from the University of Western Ontario in 1956 with a BA degree in mathematics. Later he obtained an MA in mathematics from York University.

Don has had thirty years teaching experience. He taught for 8 years at Orangeville District High School before accepting a position with the York Central High School Board, later to be incorporated into the York County Board of Education. In York County he has taught at Bayview Secondary School, Thornhill Secondary School, Thornlea Secondary School, Markham District High School, and Unionville High School. He was the head of the mathematics department at Thornhill, Thornlea, and Unionville. Don has done two stints overseas. He taught for one year at Neuchatel Junior College in Switzerland, and two years for the Department of National Defense in Lars, Germany.

Don co-authored secondary level mathematics textbooks published by Ginn and Company. He has played an active part in the writing and marking of the contests sponsored by the University of Waterloo. He has been an enthusiastic supporter of the Waterloo Co-op program, and has taken advantage of this program by having co-op students teach in his deaprtment.

Don has been an active member of OATM/OAME all through his teaching career. He was a member of the council of OATM, a member of the executive, represented OATM in the Ontario Education Association and the Ontario Mathematics Commision. He was the last president of OATM, presiding at the inaugural meeting of OAME which combined the forces of the Ontario Mathematics Commission and the Ontario Association of Teachers of Mathematics.

In 1974 Don became a member of the Society of Descartes Medallists, in recognition by the Rene Descartes Foundation, University of Waterloo for "signal service to mathematics in the secondary schools of Ontario".

When the York Region Board of Eduction opened the new Unionville High School for the Arts in 1985 it was fitting that Don was picked to be its first head of the mathematics department. Music has always played a big role in Don's life. One of his extra curricular activites has been singing with the Dukes of Harmony for the past few years. You may have heard them at one of their appearances at Thompson Hall.

Don is presently the Mathematics Consultant (K-13) for the York Region Board of Eduation. He is indeed a worthy recipient of a Life Membership in OAME.

## LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall: (i) have ten or more accumulated years of membership in OAME/AOEM, and (ii) have demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

If there is someone you wish to nominate for life membership in OAME/AOEM, you should submit his/her name and any information supporting the nomination to:

Dave Davidson, Life Membership Committee
7 Ferguson Road
Collingwood, Ontario L9Y 2Z9

Nomination should be submitted to Dave by November 15, 1987.

# the grand valley mathematics association

## G. V. M. A. PUBLICATIONS

| | |
|---|---|
| Handbook of Trigonometry (1975) | $2.00 |
| Solutions to the Handbook of Trigonometry (1978) | $2.50 |
| A Handbook of Geometry (1979) | $3.00 |
| Survey Test in Mathematics (1979) | $1.00 |
| "Mathematics For Life" (1983)<br>(a resource package for grade 12 general level consumer and business mathematics) | $7.00 |
| "Mathematics For Life" Problems Supplement (1983) | $1.50 |
| Recreational Math Problems for High School Students,<br>Books I and II (Revised 1984) | $3.00 |
| 50 Grade 13 Calculus Problems (1984) | $1.50 |
| Grade 10 Sample Exams (1984) | $1.50 |
| *Mathematics For Everyday Life (1985) | $5.00 |
| Living With Math (Mathematics for Work and Home) (1985) | $7.50 |
| Grade 11 General Business Math Resource Package (1986) | $9.00 |

*In the first week of July, 1984, G.V.M.A. brought together three teams of teachers to create readily implementable resources for the new grade 11 basic and general level courses. The basic level materials, "Living With Math", and "Math for Everyday Life", were first published in 1985 and were very well received. The general level material was expanded and polished by a fourth team in July, 1986. While designed to meet the requirements of the new ministry guidelines, these three booklest contain a wealth of resource materials which can be used effectively in basic and general mathematics courses in any grade level from 9 to 12.

| | |
|---|---|
| Grade 9 Sample Exam Bank (1986)<br>(exam questions based on the core material in the new general and advanced level ministry guidelines) | $3.00 |

To receive these publications, send your order and payment (cheques or money orders made payable to the "Grand Valley Mathematics Association") to:

Rachel Hotson
Faculty of Mathematics, University of Waterloo
Waterloo, Ontario, N2L 3G1

## McCARTNEY SCHOLARSHIP

Mona and Morley MacGregor and S.R.A. are pleased to announce that Rocky Lee, Woburn Collegiate, Scarborough, Ontario, was selected as winner of the John G. McCartney Scholarship for the year 1987-1988.

Details of the presentation will appear in the December Gazette.

# THE RENE DESCARTES FOUNDATION

*Editor's Note: This excerpt is reprinted with the author's permission, Robert Whitton, News Bureau, June 6, 1987.*

At its twentieth annual dinner on June 6, the Rene Descartes Foundation for the Advancement of Mathematics of the Faculty of Mathematics at the University of Waterloo, honoured five secondary school mathematics teachers for their contributions to mathematics education.

Two persons, Peter Crippin (Woburn Collegiate, Scarborough) and Donald Roberts (Beamsville District Secondary School) were admitted to the Society of Rene Descartes Medallists, while the remaining three, Michael McNally (Lower Canada College, Montreal), Dean Murray (Milton District High School, Milton) and James Schurter (Listowel District Secondary School, Listowel) were presented with Teacher Recognition Awards.

Rene Descartes Medals, named after the father of analytic geometry, are awarded to persons who have made significant contributions to the mathematics education in Ontario. Speaking of the recipients, Ronald G. Dunkley, Chairman of the Foundation, said: "Peter Crippin has been a dynamic force in the development of the mathematics program at Woburn to its present position as probably the premier secondary school in Canada. Their school teams win national prizes every year, their students are prominent not only in Canada, but as an integral part of the Canadian effort in international competitions, and their graduates routinely proceed to the most demanding university programs. He is an absolutely unique person in the development of young minds and a valued creator of challenging problems for students at all levels. Donald Roberts has devoted his talents to areas of professional development in the Niagara peninsula, where he has organized student workshops, numerous area teacher development programs, and has co-chaired two provincial mathematics meetings. In additon he has been instrumental in continuing the long tradition of excellence in the Beamsville mathematics program."

Teacher Recognition Awards are made by the Rene Descartes Foundation to persons who have established a record of excellence in the classroom and who have, by their example, encourged their students to pursue the study of mathematics. Speaking of the three recipients, Professor Dunkley said:
"Michael McNally has been a leader in the Montreal area and has developed the Lower Canada College program to a pre-eminent position among Canadian schools. Graduates of the program have proceeded to studies at most major Canadian universities and have nothing but praise for the inspiration they have received from Mr. McNally. He is also a leader in teacher organizations in Quebec and has served as president of the Quebec Association of Mathematics Teachers. Dean Murray has established a record of excellence in the classroom as a teacher of mathematics and computer studies. His work in developing curriculum has made an impact throughout Halton County. As program chairman for the Grand Valley Mathematics Association he has been instrumental in ensuring that twice-yearly conferences provided the highest level of stimulation to teachers in Mid-Western Ontario. James Schurter has been teacher, assistant head, and department head at Listowel in a twenty-year career. Under his guidance the mathematics program has improved steadily and now ranks as one of the best in that part of the province. In the community he is active in minor sport and in the scouting movement. In the classroom he is an enthusiastic, devoted teacher who has the rare ability of enticing students to achieve beyond normal expectation."

The Faculty of Mathematics enjoys a close relationship with the secondary schools of the province, and indeed beyond. From our wide contacts with the schools, we have developed an appreciation for the valuable work being carried on by the dedicated teachers of mathematics. Many of these teachers have devoted their lives to the cause of school mathematics and the development of excellent students. It would be unfortunate if their work was not recognized and rewarded.

For this reason, the Board of Directors of the Rene Descartes Foundation has established two awards to honour mathematics who, over a period of years, have been examples of excellence to their students and peers. The Teacher Recognition Award honours teachers who have been an inspiration to their students and who have, year after year, sent well-prepared students on to further study. The Rene Descartes Medal honours persons who have made signal contribution to mathematics education in the schools of Ontario. Recipients of the Medal are admitted to lifetime membership in the Society of Descartes Medallists.

# LEADERSHIP SEMINAR

**MEMO TO:** Teachers of Matheamtics

**FROM:** Mr. Dino Dottori, Past President
The Ontario Association for Mathematics Education

**RE: CONFERENCE INFORMATION**
- A uniqe mathematic seminar

The 16th Annual Mathematics Leadership Conference

We invite teachers to come and celebrate with us a mathematics experience. The complexion of the teaching environment, the resources we select and the self-confidence we encourage, are the elements that help turn a child's investigation into a good investment for the future.

1) This conference is a 2 day activity that is held every year at the Ramada Inn Airport West on Dixie Road in Mississauga during the month of October

2) This cost effective math conference ($217 in '87) offers an extensive range of math opportunities to it's participants.

3) This conference will be of interest to:
   * Teachers who have special interest in mathematics
   * Principals and vice-principals who would like to influence the math programs in their schools
   * Teachers who are serving on board committees or family of school committees in mathematics
   * Chairman/Heads of Mathematics who would like to provide leadership to their feeder schools

4) **This year the committee is endeavouring to have a secondary topic in each time slot**

It is our sincere hope that this 2 day conference of mathematical sharing will help make the study of mathematics a good investment.

**LEADERSHIP**

| | | | | | |
|---|---|---|---|---|---|
| **O.** | 1 | 9 | 8 | 7 | **A.** |
| **A.** | 7 | 8 | 9 | 1 | **O.** |
| **M.** | 9 | 1 | 7 | 8 | **E.** |
| **E.** | 8 | 7 | 1 | 9 | **M.** |

**SEMINAR '87**

**PRESENTS:** *MATH FUTURES - A GOOD INVESTMENT*
**DATES:** OCTOBER 21, 22, 23, 1987
**PLACE:** AIRPORT RAMADA INN
DIXIE ROAD AT 401
MISSISSAUGA, ONTARIO

## CONFERENCE REGISTRATION

**$217 in '87**

The registration fee of $217 includes: (A Program will be mailed to you)
*Shared Accommodation 2 nights
*5 Meals over 2 days
*2 Complimentary Socials and
10 sessions on Mathematics Education
**REGISTER NOW** FEE $217.00

Mail cheque payable to OAME along with your name and address to: Morley MacGregor, 247 Bright Street
Sarnia, Ontario N7T 4E9

Name _____ School Board _____

Mailing address _____

_____

For further information call **Dino Dottori, 705-474-7210**

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# E = OMG ?

## We're looking for the next Editor, Ontario Mathematics Gazette

NEEDED: Editor for OMG for a two year term, beginning with the September, 1988 issue

APPLY TO: M. MacGregor
247 Bright Street
Sarnia, Ontario N7T 4E9

BEFORE: February 1, 1988

INTERESTED: For information call
Paul Lessard, (416)525-2930, Ex. 233

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## POSTER CONTEST

The Ontario Association for Mathematics Education (OAME) invites students of Ontario Elementary and Secondary schools to submit posters on the topic "Unsolved Mathematics Problems".

Up to four prizes of $100, $75, $50, $25, will be awarded by the judges on the basis of artistic merit and representation of the topic.

The judges decisions will be final.

All posters become the property of the OAME, which may publish and distribute winning entries, giving credit to the students.

Some of the entries will be exhibited at the OAME 1988 annual conference in Niagara Falls.

The final date for receiving entries will be February 29th 1988, and the results will be published in the March issue of the Ontario Mathematics Gazette. Winners will be notified by mail prior to the publication of results.

Entries should be sent to: Mr. M. MacGregor, OAME
247 Bright Street
Sarnia, Ontario N7T 4E9

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# THE CAYLEY CONTEST WINNER FOR 1987

*Editor's Note: This excerpt is reprinted with the author's permission, Marjory Adkin, **The Standard**, April 14, 1987.*



**Amy Wong's armload of text books might be heavy but her dedication to learning has made her Canada's top Grade 10 mathematician.**
— Staff photo by Mike Conley

Amy Wong knows how it feels to be the very best in Canada. The 15-year-old Grade 10 student is now the best mathematician for her grade in the nation.

A student at A.N. Myer Secondary School in Niagara Falls, Amy received word last week that she was first in an annual University of Waterloo math contest. She defeated 18,712 competitors from across Canada in the hour-long multiple choice exam with a score of 144 out of 150.

Until she received the highest math mark in her class last year, Amy had never heard of the Waterloo competition but still managed to finish first in Niagara zone and 210th in Canada in the Grade 9 contest.

"I heard I was in the top 80 three weeks ago (for the Grade 10 contest), and I thought I was about 50th," she said of this year's contest, which was writen in February. "I just stood there and stared" when told of the first place win, Amy said.

For her efforts, which are the result of writing sample contests from 1964 to 1985 since last fall, Amy will spend a week at Waterloo in mathematical pursuits and will receive a gold medal for her win.

As well, she has been invited to take part in a special contest which involves writing a two-hour exam with full solution answers this Thursday. She will have to write several of them during the next two years to qualify for an international competition, said A.N. Myer assistant math head Sabina Knight.

Amy's interests don't end with math. She is quickly developing an interest in computer programming and excels in sciences. As well, art is a special hobby for the new Canadian who moved from Hong Kong with her parents and younger brother in 1985.

She is not the only winner at A.M. Myer this year. Both Grades 9 and 10 math teams entering the contest came first in Niagara zone, which includes six school board jurisdictions.

# SAME: OF MICE AND MATH 1987



**Lalie Harcourt**

## BOOK REVIEW

*by*

*Book Review Editor, R.S. Smith, Faculty of Education*
*University of Western Ontario, London, Ontario N6G 1G7*

Spencer, Donald D. (1982). **Computers in Number Theory.** Rockville, Maryland: The Computer Science Press Inc.

Many mathematics teachers have been pressed into service over the past few years as teachers of the new Computer Science courses. Finding interesting problems, of a mathematical nature, that allow the students to develop both programming and problem solving skills, is not always easy. This inexpensive paperback is a great resource for this purpose.

Although it is certainly not suitable for textbook, it does, in fact, present a brief overview of the BASIC language and some of its most important features. However, it is when the book starts to discuss problems from elementary number theory and their solution by computer that is becomes most interesting. Prime numbers, amicable numbers, perfect numbers, abundant numbers, deficient numbers are all explained in some detail, along with the BASIC programs needed to find them. There are also some less well known "magic curios" such as Armstrong numbers. An example of this category would be the number 153 since $1^3 + 5^3 + 3^3 = 153$. The idea can be extended to the n'th powers of n-digit numbers and finding out whether they actually exist provides an excellent exercise in programming.

Other chapters include Fibonacci numbers, magic square, modular arithmetic and unsolved problems of number theory. Each topic is presented as if the reader is completely unfamiliar with it, and as such is well explained. The explanation of the theory is generally illustrated by a short program and each chapter includes a set of review exercises for student solution.

In spite of the fact that the book is designed to be used in a computer science context, many of the ideas that it presents could be used quite profitably by a mathematics teacher as high interest curiosities, or alternatively as the basis of enrichment for above average students.

## MATH GREMLINS

*by*

*Clifford Gravelle*

***Editor's Note:*** *Please correct these gremlins*

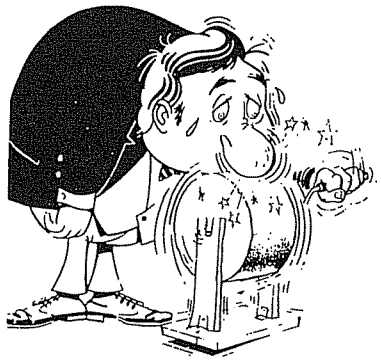
1) La regle de 72: Une application des logarithmes, Volume 25, Number 2, March 1987, Page 27.

| i | $\log_e (1 + .01i)$ |
|---|---|
| 1 | 0.00995 |
| 2 | 0.01980 |
| 3 | 0.02956 |
| 4 | 0.03922 |
| 5 | 0.04879 |

and

2) "Macpainting" the inverse trig functions, Volume 25, Number 2, March 1987, Page 31.

** Many texts use $[-\pi, -\pi/2) \cup [0, \pi/2)$ as the range of $y = \text{arcsec}(x)$. By doing so, the derivative formula simplifies to $y' = 1/(x(x^2 - 1)^{1/2})$. With the range used in this article, $[0, \pi/2) \cup (\pi/2, \pi]$, we have $y' = 1/(|x| (x^2-1)^{1/2})$. The trade-off: simpler range or (slightly) more complicated derivative formula.



# COUNCIL COMMENT

*by*
*Bob Robinson*

*Editor's Note: Council Comment appears in an issue of the* **Gazette** *following OAME/AOEM Council meetings. Your comments are welcome and should be addressed to the editor.*

## MEMBERSHIP FEES

Effective 1987 12 01, the following fees will be in effect:

| | Canada | U.S. | Outside U.S. |
|---|---|---|---|
| 1 year membership | $21.00 | $ 23.00 | $ 27.00 |
| 5 year membership | 95.00 | 105.00 | 125.00 |

All of the above are in Canadian funds.

## POSTER CAMPAIGN

A poster campaign is underway under the leadership of Tom Griffiths. Watch for the advertisements in the **Gazette.**

## 1987 LEADERSHIP SEMINAR

A memo has been sent to the Directors of Education and/or Superintendents in each local Board of Education from Dino Dottori, the seminar chairman and present past president of OAME. The theme this year is "Math Futures - A Good Investment". The seminar fee includes two nights accommodation at the Ramada Inn - Airport West, five meals over two days, two complimentary socials with mathematics educator colleagues, and more than ten sessions on leadership in mathematics education. The major publishers and education materials companies will be present and specific time will be allowed in the program to personally discuss the current mathematics products and also what "future" resource matherials we would like to have at your disposal. This is one of the best values of the school year. Contact your Director of Education, Superintendent of Curriculum, Dino Dottori (Ontario Ministry of Education, North Bay office), or Morley MacGregor in the Sarnia office of OAME for more information. The dates of the leadership seminar are 1987 10 22-23.

## ANNUAL MEETINGS

### 1987 ANNUAL (SCARBOROUGH)

A unanimous vote of thanks was extended to the conference organizers of the "Scarborough Annual". A financial report will be presented at a later Council meeting.

### 1988 ANNUAL (NIAGARA FALLS/ST. CATHARINES)

Plans are being finalized for our next annual to be held 1988 05 5-7. This will be an excellent opportunity to see the Niagara Peninsula and hear the fine speakers the "Golden Section" is lining up for you. For more information, contact Don Roberts, Warren Sebben, or Ed Haines.

### 1989 ANNUAL (OTTAWA)

The committees are formed and the University of Ottawa has been booked as well as local hotels. The publcicity committee is running a contest for a conference logo. Contact Jerry Lunney for information.

### 1990 ANNUAL (HAMILTON)

This annual will be held in conjunction with an NCTM regional meeting. The conference "chair" and program "chair" will be announced in September after the council has offered suggestions to the OAME Executive.

## NCTM REPRESENTATIVE

Don Attridge and Shirley McIntyre have been appointed representatives to the NCTM.

## CHAPTERS

**COMA/AMOC** (Carleton Ottawa Mathematics Association/Association Mathématique d'Ottawa Carleton)

COMA hosted a very successful 3rd Annual Publisher's Display at the Embassy West Hotel in Ottawa on April 7, 1987. There were 15 publishers participating and over 130 people attended. This event has now become an annual affair and will be repeated next year.

The Second Annual Fall Mathematics Conference, on a theme of Leadership and Current Issues in Mathematics, will be COMA's first activity of the 1987-88 school year. The conference has undergone a 50% increase in length and number of participants. It will be held at the Glen House in Gananoque on October 2 and 3, 1987, and will accommodate 75 participants on a first come basis.

**COMA Executive 1987-88**

| | |
|---|---|
| Past President | Jerry Lunney |
| President | Dave Hunter |
| Vice President | Fran Outerbridge |
| Secretary | Ron Bender |
| Chapter Rep. to OAME | Peter Lenczewski |
| Treasurer | John Lebrun |
| Communication Officers: | |
| OBE Public | Dave Hunter |
| OBE Separate (Eng.) | John Lebrun |
| OBE Separate (Fr.) | Richard Landry |
| CBE Public | Ron Bender |
| CBE Separate (Eng.) | Bill Murphy |
| CBE Separate (Fr.) | TBA |
| Private Schools | Fran Outerbridge |
| Algonquin College | Sandra Menna |
| Carleton University | Ken Williams |
| Univeristy of Ottawa | Mayor Alvo |
| Ministry of Education | Gerry Hurtubise |

**GOLDEN SECTION** (Niagara Peninsula including Hamilton)

Their mini conference on April 23 went quite well with approximately 100 people attending. They are involved with the planning of the '88 Annual.

**GRAND VALLEY MATHEMATICS ASSOCIATION**

The April conference went well and plans are underway for a Fall conference.

Summer Writing Project: Final plans are being made to create a resource document to support **Mathematics for Business and Consumer,** grade 12.

The writing team will meet for 4 days (June 28 - July 2) at the University of Guelph.

Writing Team: Lorraine Banks - Oshawa
Mike Foster - Mississauga
Joe Glabb - Guelph
Myrna Ingals - Unionville
Doug Rutledge - Kingston

The final document should be available for purchase at the fall conference.

**NOMA** (Northern Ontario Mathematics Association)

NOMA will be holding its fall conference at Lasalle Secondary School in Sudbury on September 25-26th. Recently NOMA distributed NOMA certificates to every elementary school in the North Eastern and North Central regions. This certificate is intended to be awarded to a student for mathematics excellence at the end of year's activities or ceremonies held in June. This project involved sending one certificate per class to in excess of 300 schools.

**PAME** (Peel Association For Mathematics Education)

The PAME spring barbecue was successful for those who attended, but unfortunately, attendance was low. There were too many other events taking place on the same night.

**PAME Executive**

| | |
|---|---|
| Co-chair | Janet Mitchell |
| | Marg Warren |
| Treasurer | TBA |
| Secretary | Carol Danbrook |
| Past Chair | Bob Jones |

The fall conference is currently being planned for October 27, 1987. At the present time plans are being made for a conference to assist teachers and/or students with preparations for writing math contests. Watch for details.

**PINE RIDGE MATH ASSOCIATION** (Oshawa/Peterborough Area)

A very successful mini conference was presented on April 14, 1987 at General Vanier Secondary School in Oshawa. There were 10 different workshops. It was attended by 200 teachers from Durham, Northumberland and Victoria boards of education. The evening ended with a buffet prepared by the students of the school.

Plans are underway for their annual P.A. day on November 13, 1987. Father Stan Buzuska from Boston has agreed to speak on that day in Oshawa.

**PRMA Executive 1986-87**

President - Maureen Ricciuto, Dunbarton High School, Pickering
Secretary - Cathy Hunt, Henry St. High School, Whitby
Treasurer - Les Dow, Consultant, Durham Board of Education
OAME Representative - Kumar Pasricha, Alternative Education, Pickering

**QSLMA** (Quinte-St. Lawrence Mathematics Association)

They conducted a very successful conference last April. Plans are going ahead for a mini conference on October 27 in Gananoque and a spring conference on 1988 03 25-26.

**SAME** (Scarborough Association For Mathematics Education)

Prior to the hosting of the '87 Annual, SAME held its sixth annual P.D. afternoon-dinner-evening on October 21, 1986 and recorded the best turnout ever.

On November 13, 1986, SAME sponsored an after school professional activity session to highlight the use of "LOGO" in the math programs.

Even with lots of energy being devoted to the May OAME Conference, SAME conducted a highly successful Professional Development Day for February 13, 1987 for our elementary and secondary teachers at the Holiday Inn, Don Valley.

**SAME Executive**

| | |
|---|---|
| President | Shirley Fairfield |
| Vice President | Rad de Peiza |
| | Bill Haehnel |
| Secretary | Darlene Stephenson |
| Treasurer | Fred Huggett |

Plans are presently underway for the 1987-88 Program. Two mini conferences (October 20, 1987 and April '88).

A Professional Activity Day (K-13) on February 10, 1988 at the Holiday Inn, Don Valley. This full day progrtam will be a joint project between SAME and the Scarborough Board Science Department.

SAME lost a valued member and friend this year with the sudden death of Jim Fencott, their former Mathematics Co-ordinator. The winner of the Jim Fencott Memorial Award is Ashutosh Gupta from Albert Campbell C.I.

A scholarship fund entitled "The Gambia" has been established. For more information, contact Sue Ames or Paul Zolis in the Program Department of the Scarborough Board.

**COMMITTEE REPORTS**

**Publications Committee**

David Handley, Executive Liaison
Paul Lessard, Chairperson
Jack Weiner
Stuart Telfer
Sam Pravica

Ron Ripley was introduced as the new editor of the **Abacus.** Welcome aboard Ron! Ron is on the Faculty of Education, Queen's University and invites input from the "Chapters of OAME" and the membership in general.

Cliff Gravelle continues as the editor of the **Gazette** and we are looking forward to more excellent issues of our publication. The publications committee welcomes your comments regarding the format of the **Gazette** and the **Abacus.**

**Primary and Junior Committee**

J. Symington, Executive Liaison
Susan Stuart, Chairperson
Roland Morrow
Olive Fullerton
Shirley Fairfield
Eileen Collins
Ann Kenny
Bob Robinson

See the OAME response to the Early Primary Education Project (EPEP) which appeared in the last issue of the **Gazette.** The committee will be corresponding with the Ministry regarding having primary specialists in mathematics education.

The place of fractions in the Primary and Junior divisions will be pursued further and a policy statement on the use of calculators may be produced.

**Intermediate and Senior Committee**

Mary Lou Kestell, Executive Liaison
Doug Henderson, Chairperson
Kumar Pasricha
Enzo Carli
Ed. Haines
Ann Jones
George McNabb
Mary Lynn Jeffries
Chester Carlow

This committee will look at the problem of non-specialist teaching mathematics at the secondary level. Your comments should be directed to this committee.

We are awaiting the results of a study regarding the use of calculators from OISE. The data is currently being collected.

The committee will be examining videotapes of mathematics as a result of the letter received from Ed Barbeau.

**Women and Mathematics Committee**

Ralph Connelly, Executive Liaison
Elaine Harvey, Chairperson
Lorna Wiggan
Jim Whitley
Jeff Shifrin ?

Suggestions for speakers are being sent to the '88 Annual program committee in Niagara. The committee is looking into ways to encourage women to pursue opportunities to become councillors and board members of OAME. Lorna Wiggan will represent us on a provincial committee concerned with equal opportunity.

**Membership, Publicity and Promotion Committee**

Dino Dottori, Executive Liaison
Betsey Friar, Chairperson
Don Roberts
Robert Jones
Steve Martin
Mickey Sandblom

The membership brochure is being updated and will be available for distribution during the '87-'88 school year. The committee is looking for suggestions for exciting promotion materials. Ideas can be sent to Betsey Friar. Suggestions for ways to increase membership should be sent as well. One avenue they will pursue is how to get student members who are at Faculties of Education.

**Futures Committee**

Arnold Carefoote, Executive Liaison
Paul Pogue, Chairperson
George Knill
Jerry Lunney
Peter Lenczewski
Tom Griffiths

A history of mathematics education associations in Ontario is being compiled by Joan Routledge. If you have any information which would assist OAME in this endeavour, contact Paul Pogue or Joan Routledge directly.

The committee is exploring the production of a survival binder which will be of use to teachers in the classroom (particularly in secondary school).

The committee will provide to the executive and council a set of guidelines regarding future projects.

**ICME**

ICME will be held in Quebec City in 1992.

**DESCARTES MEDAL**

Don Roberts was congratulated on being awarded the prestigious Descartes Medal.

**AWARDS FOR IMPROVING THE TEACHING OF MATHEMATICS PROBLEMS AT THE ELEMENTARY SCHOOL LEVEL**

This proposal was made by Chester Carlow. Watch for further announcements.

**FUTURE OAME MEETING DATES**

| Executive | Council |
|---|---|
| 1987 09 03 | 1987 10 17 |
| 1987 11 20 | 1988 02 13 |
| 1988 04 TBA | |

## ANSWERS TO THIS MONTH'S PROBLEMS

1. $x^8 + x^4 + 1$
$= x^8 + 2x^4 + 1 - x^4$
$= (x^4 + 1 - x^2)(x^4 + 1 + x^2)$
$= (x^4 + 2x^2 + 1 - 3x^2)(x^4 + 2x^2 + 1 - x^2)$
$= (x^2 + 1 - \sqrt{3}x)(x^2 + 1 + \sqrt{3}x)(x^2 + 1 - x)(x^2 + 1 + x)$
$= (x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)(x^2 - x + 1)(x^2 + x + 1)$

2. The remainder is **always** prime! This can be proven as follows.

Let q be the quotient and r the remainder obtained when the prime number p is divided by 30, so that $p = 30q + r$. Suppose that r is a composite number. It can easily be seen that every composite number less than 30 shares a common factor $s > 1$ with 30. It follows that $p = 30q + r$ is divisible by s. But this cannot happen since p is prime. Therefore r is not a composite number and so r is prime as claimed.

3. If $x + y + z = 1$, then by squaring both sides of this equation, it follows that

$$x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = 1$$

Using the well-known result $2ab \leq a^2 + b^2$, it follows that

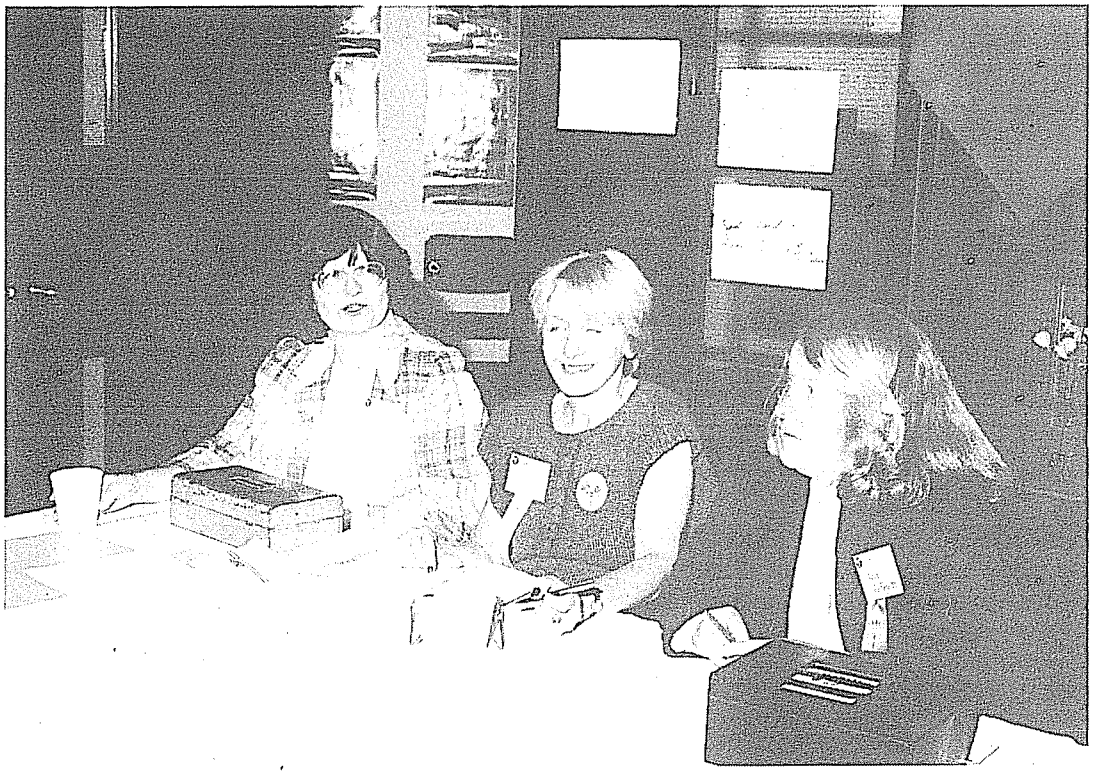
$$1 \leq x^2 + y^2 + z^2 + x^2 + y^2 + x^2 + z^2 + y^2 + z^2$$
$$= 3(x^2 + y^2 + z^2)$$

Thus $x^2 + y^2 + z^2 \geq \frac{1}{3}$

a) It is not possible to find three real number x, y, and z for which $x + y + z = 1$ and $x^2 + y^2 + z^2 = \frac{1}{2}$ since $\frac{1}{2} > \frac{1}{3}$.

b) All cases are possible.

# SAME: OF MICE AND MATH
## 1987





# ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
# 1987-88 EXECUTIVE - COUNCILLORS

**Past Pres.** - Dino Dottori, 640 Lancelot Ave., North Bay P1A 3A8 (H) 705-474-9146 (B) 705-474-7210

**Pres.** - Ralph Connelly, 11 Berkwood Pl., Fonthill L0S 1E0 (H) 416-892-5942 (B) 416-688-5550 Ex. 3351

**Pres. Elect** - J. Symington, 50 Copeland Rd., Brampton L6Y 2S7 (H) 416-454-4221 (B) 416-884-4802

**Director** - Arnold Carefoote, 50 Carslake Cres., Scarborough M1J 2A7 (H) 416-431-9671

**Director** - David Handley, R.R. 4, Madoc K0K 2K0 (H) 613-473-2478 (B) 613-962-9581

**Director** - Mary Lou Kestell, 223 Ellis Ave., Toronto M6S 2X4 (H) 416-767-2688

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Collins, Eileen, 299 Limeridge Road West, Unit 3, Hamilton L9C 2V4 (H) 416-387-4629

Davidson, Dave, 7 Ferguson Rd., Collingwood L9Y 2Z9 (H) 705-445-5394 (B) 705-445-3161

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Henderson, Douglas, 146 Stonehaven Dr., Waterloo N2L 6C5 (H) 519-579-0813

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Weiner, Jack, Dept. of Mathematics & Stat., Univ. of Guelph, Guelph N1G 2W1 (H) 519-823-1852 (B) 519-824-4120 Ex. 2157 (messages Ex. 2155)

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Jones, Ann, 14 Rockcliffe Way, Ottawa K1M 1B1 (H) 613-745-7423

Lessard, Paul, 60 Leinster Ave., S., Hamilton L8M 3A1 (H) 416-549-6491 (B) 416-525-2930 Ex. 233

McNabb, George, 1796 Windle Dr., Sudbury P3E 2Y8 (H) 705-522-0768 (B) 705-566-2280

Roberts, Don, 83 Dorchester Dr., Grimsby L3M 1B1 (H) 416-945-8321 (B) 416-563-8267

Sandblom, Michael, 19 Lakeview, Box 221, Onaping P0M 2R0 (H) 705-966-2112

Wiggan, Lorna, 250 Scarlett Rd., #1802 Toronto M6N 4X5 (H) 416-767-1424

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GOLDON, Haines, Edward, 22 Robbie Burns Rd., Beamsville L0R 1B0 (H) 416-563-7270

GVMA, Telfer, Stuart, 286 Faraday Court, Waterloo N2L 6A5 (H) 519-884-6901

NOMA, Kenny, Ann, 158 Village Cres., Sudbury P3A 5A1 (H) 705-566-9124

PAME, Jones, Robert, 25 Torrance Woods, Brampton L6Y 2T1 (H) 416-459-0696

PINE RIDGE, Pasricha Kumar, 1176 Northgate Cres., Oshawa L1G 7C5 (H) 416-576-7960 (B) 416-839-1125

OSLMA, Whitley, J.A., R.R. 3, Gananoque K7G 2V5 (H) 613-382-7986 (B) 613-547-5820

SAME, Fairfield, Shirley, 12 Oscar Ct., Willowdale M2K 2A4 (H) 416-222-3123

OAME Office - M&M MacGregor, 247 Bright Street, Sarnia N7T 4E9 (H) 519-337-8382

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Ripley, Ronald, Editor - ABACUS, 217 Glengarry Rd., Kingston K7M 3J6 (H) 613-544-2872