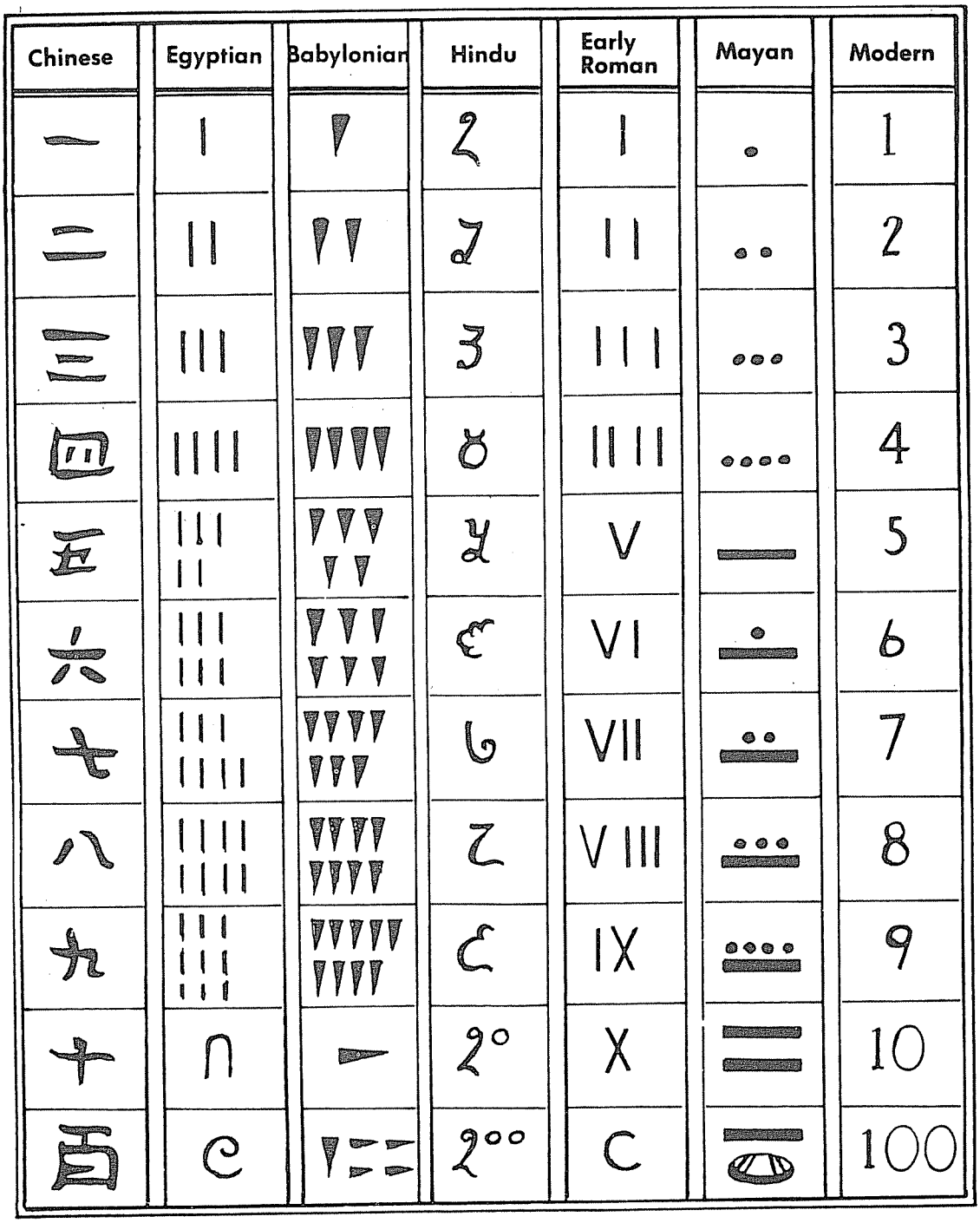
ontario mathematics gazette

VOLUME 26 NUMBER 3 MARCH 1988



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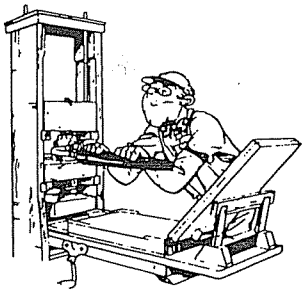
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To Evaluate or Not to Evaluate

(Ed. Note: Elaine Harvey, Chair of OAME's Women and Math Committee, was asked by Clifford Gravelle to prepare this editorial. With two articles in this issue coming from the efforts of the Women and Math Committee, it seemed appropriate for this editorial to appear in this issue as well.)

One of the good things about the women's movement is that it is identifying many of society's assumptions and questioning them. One of those assumptions is that students' achievement must be evaluated. Let's set aside for a moment the question which would most restrict our exploration of this topic: the question, Could a school exist without evaluation? Let us consider instead why we would want to dispense with evaluation.

Evaluation by the teacher implies that the teacher has power over the student. Feminists do not agree that one person should have power over another person. Many women and a considerable number of men have been restricted from reaching their creative potential because of personal and/or business relationships in which others have had power over them. That implies an hiearchical structure rather than a co-operative arrangement. Individuals will gain self confidence and achieve fulfillment through empowerment of themselves, but not through exerting power over others. So the basic concept of teacher evaluation of the student is anethama to feminism.

The women's movement cannot claim sole responsibility for this theory. It is implicit in many other philosophies such as humanism and student-centred teaching; but women have identified the basic patriarchal/hierarchical structure of our society, and by doing so have provided new insights into this topic.

Evaluation by the teacher implies that all people are not equal. The teacher by the act of evaluation assumes a superiority which implies that the students are inferior. While the teacher likely does in fact have more experience and more of certain kinds of knowledge (hopefully particularly in mathematics), teachers are well aware and have often been afraid to voice the concern that some of their students will have more intellectual ability than they do. Some will have less. The superiority that a teacher assumes through the act of evaluation is a false superiority as many students and a few teachers realize. Most adults can recall at least one instance in their schooling of receiving a mark that they considered unfair. Such a mark has often resulted in such a feeling of frustration and powerlessness on the part of the student that it has had a profound effect on the rest of her life. Certainly we are aware of the implications of a failure in mathematics on the later career opportunities available to students.

Dr. Roberta Hamilton, women's studies professor at Queen's University, has stated, "Dominant power relations are accompanied by ideas that legitimate those relations." A dominant power relation between teacher and students is legitimzied in part by the evaluation process. And the evaluation process is readily accepted by a society such as ours that is in love with numbers. We will evaluate not only cars and houses but also sexuality and mathematical knowledge. We attach numbers to anything. We continually quantify qualities. We will guess at numbers (in probability, for example) and then treat them as if they are exact. We mathematics teachers frequently go to great extremes because we love numbers so much that we calculate final marks by intricate computerized programs. Marks are one way of maintaining the dominant power relation between teacher and students.

If we want to continue to exert power over our students at the expense of the creative process and the well being and development of our young people, then we must continue to evaluate as we have in the past. If, on the other hand, we are motivated to empower our students (and at the same time empower ourselves), we will dispense with evaluating them altogether. "Whoa, there!" you may say. And I will respond that if the system does not allow us to dispense with evaluation at present, then at least we can occasionally replace teacher evaluation of the student which is external with student self evaluation which is internal and empowering.

PRESIDENT'S MESSAGE

by

Ralph Connelly

One of the advantages of finding oneself as both President of the organization and the acting editor of its publication is that you can relinquish the Editorial page (as I've done this month to Elaine Harvey of the Women and Math Committee), and still have an opportunity to get your two cents worth (OK so maybe it **is** a tad overpriced at that!) through the President's page. Did I say **one** of the advantages? I should have said **the** advantage as I sure would be hard-pressed to come up with any other advantge of simultaneously carrying the two jobs!!

As I write this we have just finished the last OAME Council meeting for this year-Bob Robinson will supply more details of that in his column in this issue. For some of the councillors, this marked the end of their term and they have chosen not to run again. I wish to take this opportunity to thank them for all their time and efforts on behalf of OAME. Thanks also to the councillors who will be returning next year, and to those who have indicated a willingness to let their names stand for election to Council. OAME's real strength has been in the devotion and commitment of its volunteers, and we have much to be proud of in our accomplishments.

In attempting to list those accomplishments, there is the clear danger of leaving something or someone out, so I'll apologize in advance for any omissions and attempt to identify some of the highlights of the past year. With the now completed conversion of our "subscriptions" to memberships, our membership count stands at an all time high of 1714! More good news along this line was the forming of two new chapters of OAME- The South-Western Ontario Association for Mathematics Education (SWOAME), and the North-Western Ontario Association for Mathematics Education (NWOAME). Of course, York Region Mathematics Association (YRMA), which actually became "official" last school year should also be recognized as they began their chapter activities this year. The growth in the number of chapters, and their vitality, bodes well for OAME. Obviously congratulations need to go in many directions for this growth- to the Membership/Publicity Committee; to the members who worked so hard to get new chapters off the ground, to the successful Scarborough conference, M & M, etc.

The Publications Committee has been very busy seeking a new **Gazette** editor. Cliff Gravelle's sudden death made this task all the more urgent. For the most part, the membership of OAME did not know Clifford all that well, but many are aware of the superb job Clifford was doing with the **Gazette.** Although he was going through a very difficult time, he still put forth a monumental effort for the **Gazette,** and for that we are deeply indebted to him. Indeed, much of this issue of the **Gazette** is material that Clifford had obtained.

Bob Smith, from the Univ. of Western Ontario, has been appointed the interim editor for a term of one year.

Both the Int/Sr and Pr/Jr Committees have had a very busy year- appointing people to represent us on various committees, overseeing the "Problem-Solving Project", etc. The Futures Committee seemed to wind up with many more projects than I'm sure they anticipated, and have come through admirably. The "Survival Binder" is ready for the next step; the "100 Year History" of mathematics education associations in the Province has suffered some setbacks but is still being actively pursued; and the Committee found itself with an extra task of being asked to come to grips with bringing some order to the "special projects" funding that has been steadily expanding. The Women and Mathematics Committee continued their work in obtaining speakers for the Annual, writers for the Gazette, and more women to run for Council.

I've found my year as President to be a most challenging one. I confess I often felt "overextended" with my new responsibilities as a Department Chairman at Brock, President of OAME, and "Reinheriting" the **Gazette.** I extend my sincere thanks to Morley and Mona, the Executive, Council, and to all who helped see me through. I should also say that I have been especially impressed by, and grateful for, the exceptional work of Jay Symington, the incoming President. OAME couldn't be in better hands! I confess that it's very comforting to know that there's someone as strong as Jay coming along to "pick up all the fumbles" I've made along the way this year. Jay, good luck, and you certainly have all my support (or, if you prefer, I'll just stay out of your way!!) My thanks to the membership for giving me the opportunity to serve OAME as your President. It's been a great honor (or is that honour?)

GEOMETRY AT THE PRIMARY LEVEL

by
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Kingston, Ontario K7M 2Z8

"Shared Discovery", a support document to The Formative Years from the Ontario Ministry of Education, states that teachers must "provide pupils with a secure, stimulating environment" which helps create an atmosphere where the child's curiosity and need to manipulate things manually are met. This is not always easy to achieve, but as teachers realize we must invariably start where a child is and help direct him/her on from there.

Whether or not they agree with all of Piaget's steps of progression in learning, probably most teachers of young children recognize the importance of play. This is not just the "tag at recess" kind of play, but supervised, guided activity in which the child handles concrete material and interacts with other children. Because of my belief in this process of learning for young children, I suggest that the introduction to geometry to Kindergarten pupils be through the handling of concrete objects rather than beginning with 2-dimensional shapes, such as a circle or square, which many four and five-year olds recognize.

At the Kindergarten level, pupils need numerous opportunities to sort, using articles they bring in such as buttons, bottle caps, toys, leaves, nails or things in the classroom, e.g. children, articles of outer clothing, crayons, blocks, beads. They begin by sorting according to their rule with an opportunity to explain that rule. Later they sort according to a given rule, e.g. buttons may be sorted according to colour, number of holes, shape, with or without shank, transparent or opaque, type of material, what it might be used on, etc.. After considerable practice in sorting objects from the child's environment, geometric solids (cone, triangular-based pyramid, rectangular-based pyramid, cube, sphere, cylinder, rectangular prism, triangular prism) are introduced as another set of objects to be sorted, first to their own rule and then to a given one.

Rules for sorting solids may be as simple as "Which roll?" or as difficult as "Which have curved edges?" By repeated opportunities to handle the solids in guided play, children become familiar with their features and will notice similar shapes in their environment. Kindergarten children can compile charts and/or booklets of shapes that match the shape they have handled. The ability of children in a class will help to determine whether Kindergarten children call a triangular prism by its correct name or a reasonable substitute such as a "tent" or "roof top".

The use of directed play, i.e. handling the solids,is a reasonable place to begin a geometry programme at any age in the primary division. To develop the skills and concepts as outlined in The Formative Years (pages 6 and 7) I have successfully used the following with children ages six to nine:

1. Use a "cheat sheet". This page will have drawings of the solids with their correct names which the children may use as reference for spelling and/or clarification.
   (see Figure 1)
2. Hang large bristol board "empty solids", labelled, in the classroom.
3. Make a Riddle Box. Laminate numbered riddles on various shapes of bristol board. Leave numbered answer strips in box.
   (see Figure 2)
4. Make solids from plasticene or play dough; make skeletons with pipe cleaners and straws, plasticene and straws, small marshmallows and toothpicks or crackers and peanut butter. (They love this one!)
5. Glue coloured yarn along the edge of individual sets of solids made from "nets" to aid in the understanding of "edge".
   (see Figure 3)
6. Draw faces, happy, sad, puzzled etc. on the faces of a solid to help children remember what a face is.
7. Graph various attributes. e.g. number of faces, edges, vertices, of solids.
8. Provide opportunities for comparing solids.
   a) Use hoops, string or yarn to group solids with common features. Children explain the reason for the grouping.
   (see Figure 4a)
   b) Compare two solids in chart form.
   (see Figure 4b)
9. Create creatures, toys, space objects etc. using shapes collected.
10. Visit a shopping mall or park and collect names of objects which are similar to a cone, cylinder, sphere, etc..
11. Use gummed paper to show the faces of a solid. e.g. six pre-cut squares show the faces of a cube
12. Provide challenges for exceptional students.
    (see Figure 5a and 5b)

The transition to 2-dimensional geometry can be easily made by having children trace the faces of solids and identify the features of a circle, square, triangle and rectangle. Hexagon and octagon may be added, as well as various quadrilaterals and kinds of triangles, if any of the children are ready for more precise differences. To help establish names and properties of 2-dimensional shapes the following are helpful:

1. Geo-boards, commercially made or teacher made with assignment cards.
2. Tangram puzzles - commercial product or bristol board.
   (see Figure 6)
3. Colour picture according to code - see **Journeys in Math 2** (Ginn) - Working Together book, page 38.
4. Shape Bingo - Make as many different cards as you wish and play as Bingo.
   (see Figure 7)
5. Perception Skills Activities.
   (see Figure 8)
6. Art activities, e.g. "tiling" with a limited number of shapes.

Aspects of 2-dimensional geometry that require special attention in the upper primary level are symmetry perimeter, area, reflections (flips), translations (slides), rotations (turns) and grids.

Mirrors are especially helpful to establish the concept of symmetry. If a trip to a shopping mall is part of your unit, you may find a mirrored wall useful. Small plastic mirrors are invaluable in determining whether or not a figure is symmetrical. Letters of the alphabet, especially capitals, and objects in nature such as leaves, shells and tree shapes present interesting patterns to determine where or if a line of symmetry belongs. Geo-board activities and completion of symmetrical seasonal patterns, e.g. jack-o-lantern or Christmas tree will help sharpen skills.

The gymnasium is an excellent place to begin the concept of flips, slides and turns as children develop their own body awareness. e.g. "Lie down, leave your feet on the spot marked by tape or where two lines on the floor cross and **turn** your body one-fourth of a circle to your left or **slide** your body the width of itself." When this concept is transferred to classroom activities, it is wise to use unusual shapes, like scissors for tracing in flips, slides and turns. This results in a more noticably different change from the original position than if a shape such as a square from a tangram puzzle is used.

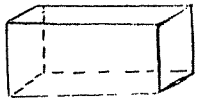
The popularity of the game Battleships has introduced many children to grids. A simplified version of laminated bristol board is adequate. This concept may also be introduced in the gymnasium. e.g. "All seven year olds walk four steps North on a yellow line." **Prime Time Maps,** by Jerry Aten, (Good Apple) and **Point-Counterpoint** by Ken Rand (Creative Publications) have excellent grid activities. "Find Your Name" helps develop the skills of reading and recording co-ordinates. (see Figure 9)

Many opportunities for "hands-on" measuring are necessary to establish the difference between the concepts of area and perimeter. When moving to activity assignments or work sheets, the use of squared centimetre paper is most beneficial. Open-ended tasks such as "Cut out as many different shapes, each with a perimeter of 18 cm/area of 30 sq. cm as you can." are most challenging and rewarding.

Even the computer is a valuable asset to a Geometry programme. An Apple machine with the Turtle provides a variety of levels of challenge. The children can progress from following teacher-written instructions to working out how to make a triangle, circle, etc. to developing their own programme.

There are many commercial products available to complement a geometry programme at the primary level. A few are necessary, such as a set of wooden and/or hard plastic solids; many can be substituted with teacher/volunteer-made products. The most important essential is teacher enthusiasm to excite young children to become increasingly aware of shapes in our environment and their relationship to us and each other. Geometry at the primary level is fun!

Figure 1



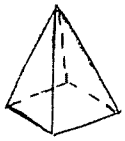


Figure 2

6.
Which solid has four triangular faces?

3.
Which solid has square faces?

Figure 3

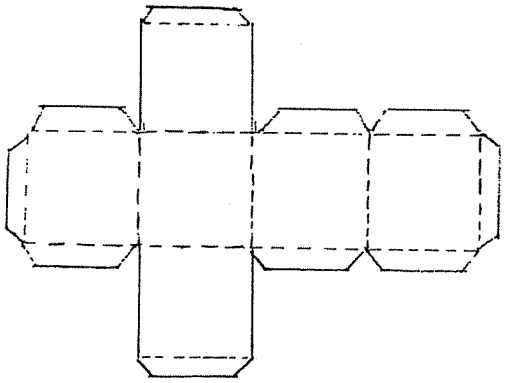


Figure 4a

Why have these solids been grouped together?

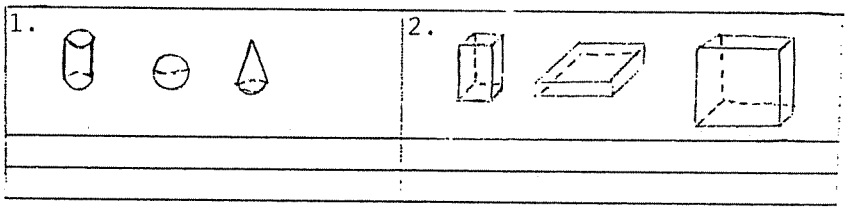


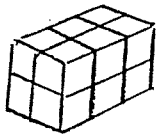
Figure 4b

	rectangular prism	cube
Number of faces		
Number of edges		
Number of vertices		

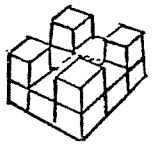
Figure 5a

How many blocks were used to make each of the following?

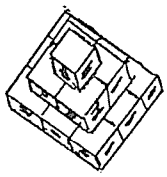
A. ____



B. ____



C.



D.

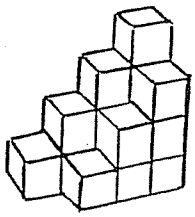


Figure 5b

If you folded this pattern together, it would form a cube. Imagine the sign in the first row of the chart is on the face of the cube closest to you Complete the chart. Make a model of the cube to check your answers if you like.

Front	Back	Top	Bottom	Left	Right
=					
≠					
+					

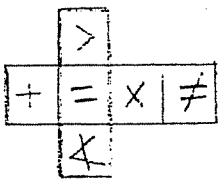


Figure 6

Cut out the shapes. Arrange them to make a square. Arrange them to make a plus sign.

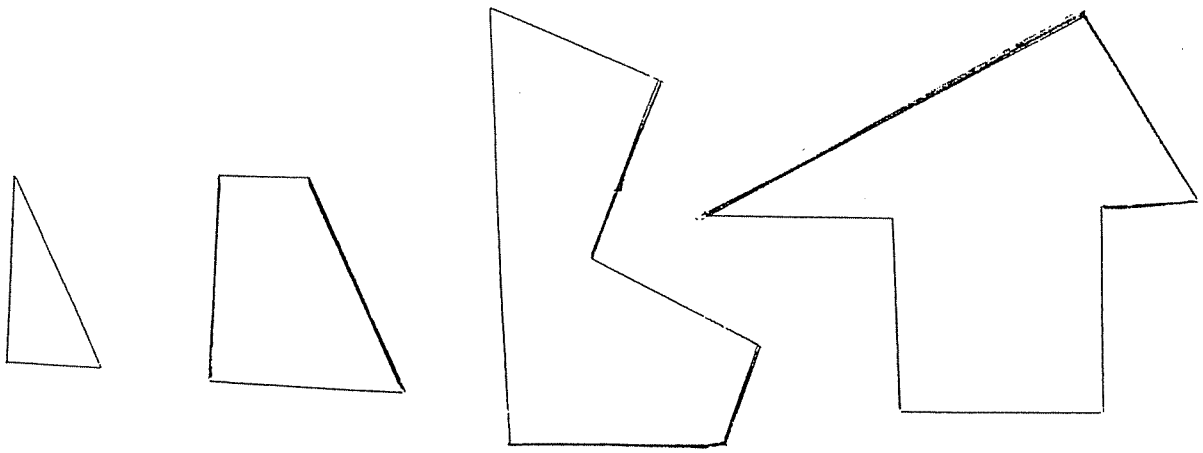


Figure 7

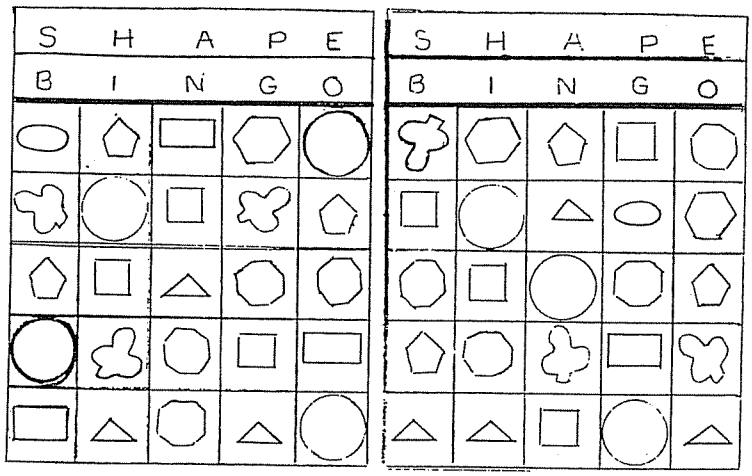
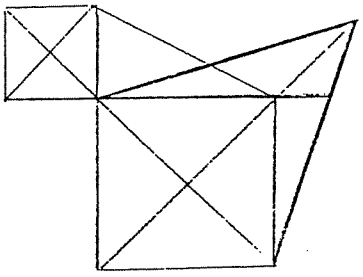


Figure 8

How many triangles can you find?



How many squares can you find?

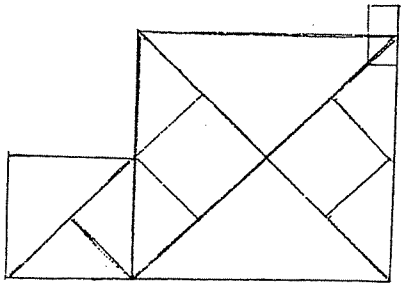
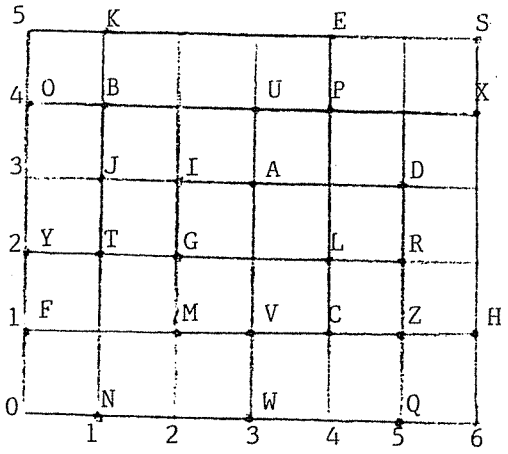


Figure 9

FIND YOUR NAME



My name is

	Across	Up
M		
A		
R		
Y		

MATH CAN BE FUN: EVEN IF YOU HATE IT!

by
V. Sleightholm

Dear Teacher,
Thanks for sending me all those cards and stories from the class. The doctor says my leg is healing well and I can soon come back to school. My mum helped me with those math papers you sent home. I think I got all of the adding questions right-there sure were alot,
weren't there? We had some trouble, though, with the "crocodile mouths". My mum got kind of crabby about those and she said that in real life you don't need to know how to do math like that. So I guess no one cares if $5 > 3$ or if $23 < 37$. See you next week.

Love,
Melissa

Melissa's letter should make us painfully aware that too much of the math that we do in our classrooms is **not** the kind of math that will serve any purpose **outside** of the classroom. In the elementary grades, when children are not yet ready to specialize, the focus of the math curriculum ought to be on *real life math* rather than on the single-correct-answer types of questions which pervade our classrooms. Through real life math activities, children have the opportunity for versatile thinking and creativity in applying their math skills. With classroom math, children are evaluated for accuracy, neatness, speed and ability to abstract - skills which are much more easily evaluated than versatility and creativity, but far less useful to the student in life outside the classroom. Knowing the formula for finding the curcumference of a circle, for example, is of little use in buying a round tablecloth. Neither is it necessary to be able to calculate area in order to buy the correct amount of wallpaper. Consider the number of times in a day when adults are required to use real life math skills in juggling bank accounts, doing comparative shopping and filing income tax returns. I used to wonder how my accountant could work with exactly the same numbers as I did and arrive at a much more favourable "bottom line" on my tax return form? His explanation made it clear that there *is* room for creativity, even in something which appears to be a straightforward calculation task.

One really simple way to ensure that children have the chance to explore math creatively is to assign the "brain teasers" and puzzle corners - usually found at the end of the chapter in math textbooks - **first.** These activites will provide just as much practice in arithmetic calculations as doing a whole page of questions - and, they are a lot more fun. There are many commercially prepared mathematics workbooks with wonderfully challenging activities and yet these often sit unused because teachers worry that they don't provide enough activities to "drill and reinforce skills".

Perhaps the hidden message in this is that teachers perceive boring and repetitious worksheets to be better reinforcers of skills than are challenging and interesting activities. Consider the following two methods of reinforcing the skill of addition:

Method 1: - a worksheet that has 50 addition questions
Method 2: - a worksheet that contains many numbers and asks the question, "Which of these numbers is divisible by 3?"

The first worksheet needs no explanation as we are all too familiar with this method. The second method may sound as if it is designed to reinforce skill in division but this is only an added benefit of the activity. A quick explanation will show the children the "magic" of the divisible-by-three rule...
ie. "Add the digits in the number. Is the sum divisible by 3? If it is, then the original number is divisible by 3."
Once they've been told the rule, children will be motivated to test it out, and not only will they practice their addition (and division) skills but they will find it an interesting change from the usual type of worksheet.

As well as providing interesting ways for students to practice computation skills we should be giving them opportunities to develop their creativity in working with numbers. One easy way to do this, for students of any age, is to give them the **answer** and let **them** suggest the questions...
ie. "The answer is 246. What are the questions?"
Or, Using 4 fours and any signs of operation how many ways can you arrive at the answer 1987?"

Real life math problems don't come neatly packaged with clear instructions, but instead they can be obscure and require many more skills than simple computation. (Consider income tax returns!) To solve real life math pro-

blems, one must be able to think creatively as well as logically. In any classroom there are many opportunities for **teachers** to practice their real life math skills. Why not share these opportunities with students? For example, the ordering of consumable supplies requires considerable skill in estimation and averaging. Students could participate in this process and experience the results of ordering too many, or not enough, pencils and notebooks for a term. Attendance registers require skill in addition and calculating percentages. Using an overhead, students could be involved in this monthly activity. At least once in each school year teachers are required to calculate students' ages in years and months. Why not use this as a math activity and have each student calculate his or her own age?

As well as these externally imposed opportunities for real life math, teachers can create word problems involving many "steps" and requiring both logic and creativity.
ie. "We have been given permission to spend X dollars on an area rug for our classroom." (Please don't do this type of exercise unless a rug **will** actually be purchased.)

This problem solving activity involves:

- comparison shopping - colour, price, durability (provide catalogues, brochures and newspaper ads)
- calculation of area
- dollar value in terms of quality, practicality, durability, etc.

Or, if this type of possibility does not present itself, then consider (with your principal's permission, of course) alotting some of your textbooks or consumable budget to classroom luncheons. It is quite possible that everything one ever needs to know about fractions can be learned by baking enough chocolate cakes to serve thirty, not to mention measurement, estimating, averaging and calculating cost!

If you have a computer and printer in your classroom or school, it will be necessary to order an appropriate amount of printer paper. Students could be given the responsibilities of:

- assessing how much paper is used each day over a period of a week
- calculating the monthly requirement
- projecting how much paper will be needed for a 6 month period

The use of graphs and averaging would be a meaningful part of this task.

These types of real life math activities do not preclude the need for skill in computation. Instead, they will demonstrate to students the need for automaticity in number facts and basic, "classroom math" skills. In implementing a real life math program, teachers can ensure that students have mastered basic skills through pre- and post-testing. In this way, skill drill would be considered remedial rather than being the main focus of the curriculum.

Society is demanding that graduates of our education system be able to **think** - logically and creatively. In a recent series of six articles published in the Toronto Star, parents, business and industry, as well as the Ministry of Education, made it clear that rote learning, memorization and convergent thinking are of minimal use in adult life compared to the ability to apply creativty and logical problem skills to everyday life experiences. Logic and creativity, in the form of real life math activities, should therefore be the mainstay of the mathematics curriculum.

FIBONACCI SEQUENCE: MODULAR OR CYCLIC PATTERNS

by
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This activity makes use of the calculator to find numbers congruent to the Fibonacci Numbers. To find a specific Fibonacci number, mod n, the Fibonacci number must be divided by n; its remainder is then the required number. One cannot simply divide by n on the calculator as decimals appear if n does not divide the number with remainder 0. The student must use the fact that dividend = divisor x quotient + remainder. If the remainder is not 0, then the whole number part of the quotient is to be multiplied by the divisor and this product then subtracted from the divident; this leaves the required remainder.

For example find 233 mod 6. Using the calculator,

$$6 \overline{)\ 233} \quad 38.83333333$$

So to find the whole number remainder,

$$6(38) = 228$$

Then $233 - 228 = 5$

Thus $233 \equiv 5 \pmod 6$.

Have your students generate Table 1 and observe the patterns.

Table 1

Fibonacci Number	Mod 2	Mod 3	Mod 4	Mod 5	Mod 6	Mod 7	Mod 8	Mod 9
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
2	0	2	2	2	2	2	2	2
3	1	0	3	3	3	3	3	3
5	1	2	1	0	5	5	5	5
8	0	2	0	3	2	1	0	8
13	1	1	1	3	1	6	5	4
21	1	0	1	1	3	0	5	3
34	0	1	2	4	4	6	2	7
55	1	1	3	0	1	6	7	1
89	1	2	1	4	5	5	1	8
144	0	0	0	4	0	4	0	0
233	1	2	1	3	5	2	1	8
377	1	2	1	2	5	6	1	8
610	0	1	2	0	4	1	2	7
987	1	0	3	2	3	0	3	6
1 597	1	1	1	2	1	1	5	4
2 584	0	1	0	4	4	1	0	1
4 181	1	2	1	1	5	2	5	5

6 765	1	0	1	0	3	3	5	6
10 946	0	2	2	1	2	5	2	2
17 711	1	2	3	1	5	1	7	8
28 657	1	1	1	2	1	6	1	1
46 368	0	0	0	3	0	0	0	0
75 025	⋮	⋮	⋮	0	1	6	⋮	1
121 393				3	1	6		1
196 418				3	2	5		2
317 811				1	3	4		3
514 229				4	5	2		5
832 040				0	2	6		8
1 346 269				4	1	1		4
2 178 309				4	3	0		3
3 524 578				3	4	⋮		7
5 702 887				2	1			1
9 227 465				0	5			8
14 930 352				2	0			0
24 157 817				2	5			8
39 088 169				4	5			8
63 245 986				1	4			7
102 334 155				0	3			6
165 580 141				⋮	1			4
267 914 296					4			1
433 494 437					5			5
701 408 733					3			6
1 134 903 170					2			2
1 836 311 903					5			8
2 971 215 073					1			1
4 807 526 976					0			0
					⋮			⋮

Note the following lengths of the cycles:

Mod 2 - 3
Mod 3 - 8
Mod 4 - 6
Mod 5 - 20
Mod 6 - 24
Mod 7 - 16
Mod 8 - 12
Mod 9 - 24

The following patterns are of interest:

Mods		Cycle Lengths
2 divides 4	and	3 divides 6
2 divides 6	and	3 divides 24
2 divides 8	and	3 divides 12
3 divides 6	and	8 divides 24
3 divides 9	and	8 divides 24
4 divides 8	and	6 divides 12

From these patterns we conjecture that if a divides b, then the cycle (mod a) divides the cycle (mod b).

Now consider the presence of 0's in the cycles:

Mod 2	every 3rd number is 0
Mod 3	every 4th number is 0
Mod 4	every 6th number is 0
Mod 5	every 5th number is 0
Mod 6	every 12th number is 0
Mod 7	every 8th number is 0
Mod 8	every 6th number is 0
Mod 9	every 12th number is 0

We conjecture that if a divides b, then the "lengths" between zeros also divide:

2 divides 4	and	3 divides 6
2 divides 6	and	3 divides 12
2 divides 8	and	3 divides 6
3 divides 6	and	4 divides 12
3 divides 9	and	4 divides 12
4 divides 8	and	6 divides 6

CHALLENGE:

Find other patterns in the Fibonacci sequence.

A MATHEMATICIANS'S LONDON

by
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Everything you've ever wanted to see in London you can see: the Tower of London, Westminster Abbey, St. Paul's Cathedral, Greenwich; but it's much more fun to see the tourist sights from the perspective of the history of mathematics. As you visit the great landmarks, parks and historical sites of London, you can look for traces of the mathematicians who have lived and worked here.

Sir Christipher Wren

Stalking mathematicians in London can become addictive, particularly if the mathematician is Sir Christopher Wren (1632-1732). Wren is best remembered as England's greatest architect. It was he who designed St. Paul's Cathedral, site of the wedding of Prince Charles and Princess Diana. Wren was also a famous scientist and mathematician in his time. Newton, a contemporary, characterized him as one of the "greatest geometers" of their day. Sir Robert Hooke, another contemporary, compared Wren to Archimedes.

In the history of mathematics Wren is given credit for the rectification of the cycloid, finding the length of an arc of the curve, an insignificant achievement when compared to his architectural accomplishments in London alone.

The Great Fire of London of 1666 in which most of the City of London was destroyed, including St. Paul's Cathedral and 89 parish churches, gave Wren his opportunity to make his remarkable architectural imprint on the city. Wherever you go in London, you are bound to find traces of half a century of Wren's work as royal surveyor for Charles II in the rebuilding of the city.

The Monument, a 202-front column of stone, was designed by Wren in collaboration with Sir Robert Hooke. It stands 202 feet from the spot in Pudding Lane where the Great Fire started.

Wren was responsible for the building of 51 parish churches in London. Although some were destroyed in the World War II blitzkrieg, many survived or were rebuilt. St. Bride on Fleet Street with its wedding cake steeple of four octagons stacked one above the other, has Wren's tallest spire. St. Mary-le-Bow on Cheapside is famous for its Bow-Bells. Anyone born within the sound of the bells is deemed a true Cockney.

St. Paul's Cathedral, Wren's greatest masterpiece, took thirty-five years to build. Wren was an elderly man when his son fit the top stone in place in 1710. The Cathedral, built of golden-colored Portlandstone and covering 78000 square feet, is enormous. Walk down the 180-foot nave, as Princess Diana did on her wedding day, and experience the building's awesome beauty. Climb up to the Whispering Gallery and whisper to the wall, "How wonderful!" and the words are heard directly across by your companion. Descend into the crypt and now you will understand the epitaph on Wren's tomb, "Si monumemtum requiris, circumspice." (If you seek a monument, look about you).

But we must not overlook Sir Christopher Wren's secular buildings. Chelsea Royal Hospital, was commissioned by Charles II as a home for aged or disabled soldiers. Old pensioners in their summer red coats or their winter blue coats continue to be residents.

In Greenwich is the more splendid Royal Naval College, built later as a home for seamen, but converted to a naval college in the nineteenth century. The Old Royal Observatory, also in Greenwich, was found by Charles II in 1675 "to perfect navigation and astronomy". A fairly simple structure, it was, in Wren's words, intended "for the observator's habitation and a little for pompe." In the Octagon Room, occupied by the first Royal Astronomer, Sir John Flamsteed, are seventeenth and eighteenth century telescopes reminiscent of those used by Flamsteed (1646-1719) and the second Royal Astronomer, Sir Edmond Halley (1656-1742). The Greenwich time ball graces the top of a turret of the Observatory. The great orange ball drops precisely at 1 o'clock each day, just as it did in the nineteenth century to aid navigators in setting their chronometers.

Nearby, in the courtyard, is the Prime Meridian of the world. Straddle the brass strip, with one foot in the Eastern, and the other in the Western Hemisphere.

It is easy to see why, in Wren's portrait at the National Portrait Gallery, the artist Kneller depicted him with a pair of compasses in hand, seated near a table on which rests a plan of the west end of St. Paul's and a copy of Euclid's **Elements.**

It is no wonder that the New York Times architertural critic Paul Goldberger sees in Wren's work "the elegance of mathematics and the elegance of architecture join (ed) in exquisite harmony".

Mary Fairfax Somerville

Born in Scotland, Mary Fairfax Somerville (1780-1872) lived in London for twenty years. In 1824 Mrs. Somerville and her husband moved to Chelsea College when Dr. Somerville, an army surgeon, was assigned to Chelsea Royal Hospital. Walk about Chelsea, with its narrow cobblestone streets and houses bearing the traces of famous men and women. Oscar Wilde lived at 15 Tite Street, Thomas Carlyle at 24 Cheyne Row, and George Eliot at 4 Cheyne Walk. Find the Chelsea Physic Garden on Royal Hospital Road. An old botanical garden founded in 1673 by the Worshipful Company of Apothecaries, it was the scene of some of Dr. Somerville's great struggle to study mathematics and science. Recall that she could barely read at the age of ten but by the end of her life of ninety-three years she had earned the admiration of the scientific world for her ability to write clear scientific and mathematical exposition.

Encouraged and assisted by the leading intellectuals of Scotland to study higher mathematics, then by her husband who was a member of the Royal Society, Mrs. Somerville is best known for an early work, her translation of Laplace's **Mecanique Celeste** into English. In 1827 she was persuaded to translate this monumental 2000 - page compendium of all the discoveries about the mechanics of our solar system made by Newton, D'Alembert, Euler, Lagrange and Laplace himself. She promised to try, but stipulated that if the translation were not adequate, it would be destroyed. What finally emerged was much more than a translation. Detailed explanations, diagrams, and a preface in which all the mathematics necessary to understand Laplace's work were added. The work was published in 1831 and won great acclaim from the scientific community. The Royal Society voted to place a portrait bust of Mrs. Somerville in its Great Hall, although it did not invite her to become a member. When I first saw the bust in 1980 at the Royal Society on Carlton House Terrace, it was, in the words of the librarian, Mr. N.H. Robinson, "guarding the ladies' cloakroom."[1]

In 1981 the bust had been moved to a window in the dining room. An undergraduate student in a study group that I took to London wrote in her journal:

> We visited the Royal Society. We saw a lock of Newton's hair and a ruler made from his apple tree. One of the most important things we saw was that Mary Somerville had been transferred up from the Ladies' Washroom to head cook in the dining room. Her father would be glad to know that she's still in the kitchen.

Mrs. Somerville never was elected to membership in the Royal Society, but in 1835 she and Caroline Herschel became the first two women members of the Royal Astronomical Society. She continued writing and publishing into old age. At the time of her death she was revising a paper on quaternions. Somverville College, a women's college in Oxford, was named after her in 1879.

Charles Babbage

If you join the hordes of tourists gazing at wax figures of Emperors and murderers at Madame Tussaud's, or if you mingle with the more enlightened crowd at the Planetarum next door, after you leave, walk down Baker Street which is close by. You won't find Sherlock Holmes' legendary rooms at 221B, but just off Baker Street, at 1 Dorset Street, you'll find the house that Charles Babbage (1791-1871), the father of the computer, lived in for many years. It was here that Babbage waged war on the organ grinders and other "street nuisances" whose noise he claimed disturbed his work on the Analytic Engine, considered to be the forerunner of the earliest computer. Babbage relentlessly, and unsuccessfully, brought these street musicians to court year after year.

Babbage's place in the history of mathematics dates back to his Cambridge days. He entered in 1810, finally settling down at Peterhouse College, and soon found that he was a better mathematician than his professors. He did not need to spend much time studying for the Tripos and Fellowship examinations, so Babbage spend his time in the Great Cambridge libraries studying the work of Euler, Lacroix and Lagrange in the publication of the academies of Paris, Berlin and St. Petersburg. To his dismay, he discovered that British mathematics had stagnated since the death of Newton. An irrational loyalty to Newton's dot notation in calculus had prevented British mathematicians from reaping the benefits yielded by Leibniz's superior notation -- the modern notation using d's, the d's of differential. In 1812, Babbage founded the Analytical Society with the aim of revitalizing British mathematics by promoting "the principles of pure D'ism in opposition to the Dot-age of the University.

With two other gifted undergraduates, John Herschel and George Peacock, Babbage's first project was a translation of Lacroix's **Differential and Integral Calculus** into English. Still, reform came slowly, so the trio stepped up its campaign. This time they prepared a set of calculus problems with complete solutions in the modern notation, assuming that the tutors at Cambridge would take the easy way out and see them -- which they did. And so English mathematicians forged ahead and caught up with or surpassed the mathematicians of western Europe.

The Tower of London

The Tower of London is a grisly, yet pleasant place to visit. Through Traitor's Gate passed many unfortunated men and women condemned to die in the Tower: Sir Thomas More, the great scholar, author of **Utopia,** who refused to support Henry VIII's divorce from Catherine of Aragon or his Protestant reformation; Ann Boleyn, Henry VIII's second wife, who bore him no sons and was found guilty of multiple adultery.

The Beefeaters in their extravagent scarlet and gold uniforms police the grounds. These Yoemen Warders, distinguished veterans of Britain's armed forces, provide entertaining, tours at 30 minute intervals starting from the Middle Tower, close to the main entrance.

Go to the Jewel House where you can admire the crown jewels and coronation robes and regalia which have been in the Tower since the coronation of Charles II in 1661. Here the Yoemen Warders are more serious as they whisk you past Britain's great royal treasures.

But don't forget to visit the Bloody Tower. It was here that Sir Walter Raleigh spent the years from 1603 to 1616; it was on Tower Hill that he was executed in 1618 by James I. Raleigh plays an important role in our mathematical look at the Tower.

It was Raleigh who in 1580 hired the mathematician Thomas Harriott, newly graduated from Oxford University, to be his private mathematical tutor. In 1585 Raleigh sent Harriott on the first Virginia expedition as historian and surveyor.

When Harriott returned from Virginia the following year, he brought back two strange plants called tobacco and potatoe, and a manuscript, "A Briefe and True Report of the New Found Land of Virginia". It was the earliest eye-witness account of the first Virginia colony, containing descriptions of its flowers, plants, trees and wild life and of the native American culture. Harriott's work, together with the water color paintings of the expedition's artist John White, constitute the most valuable documents that we have on the history of Virginia.

After Harriott returned to England, Raleigh introduced him to Henry Percy, the ninth Earl of Northumberland. The two became friends, and the "Wizard Earl" even gave Harriott a life pension of 120 pounds per year.

Believe it or not, mathematicians have been sent to the Tower of London! In 1605 both Harriott and the Earl of Northumberland were implicated in the Gunpowder Plot. Harriott was released shortly afterwards, but the Earl remained in the Martin Tower for seventeen years.

So it was that Harriott visited both Raleigh and the Earl of Northumberland in the Tower. To Raleigh, Harriott brought materials for his "History of the World"; to Northumberland, Harriott brought news of the latest mathematical problems.

In 1607 the Earl assigned living quarters to Harriott at Syon House, where he lived and worked until his death in 1621.

Go to Syon Park along the Thames, just across the river from Kew. Syon Park is known for its outstanding gardens, sixty-three spectacular acres originally landscaped by Capability Brown in the eighteenth century. Look for the portrait of the Wizard Earl in Syon House, whose Great Hall was recently featured in the "Stately Homes of Britain" exhibit at the National Gallery of Art in Washington, D.C.

As a mathematician, Harriott is considered to be the first of the great English algebraists. His only publication, on the theory of equations, **Artis Analylicae Praxis** did not appear until ten years after his death. In it he demonstrated the solution of linear, quadratic, cubic and quartic equations; he showed how to write equations, given the roots; he explained the relationship between roots and coefficients in equations; he presented numerical methods of solving equations. His work has been compared to that of the great late sixteenth century French algebraist Francois Viete, but Harriott's notation is thought to be better. Compare Harriott's a • a to Viete's A quadratum, and his a • a • a to Viete's A cubum. (We have to wait for Descartes to see x^2 or x^3). Harriott also gets credit for inventing the symbols for "greater" and "less', $>$ and $<$.

Thomas Harriott was not the only mathematician to visit the Tower of London. Sir Isaac Newton was there for a good part of his life.

Newton in London

In 1696 Newton was appointed Warden of the Mint and moved to London. In 1699 he became Master of the Mint, a position that he held until he died in 1727.

Where was the Mint located at that time? It was in the Tower of London, where it remained until 1811.

On the trail of Newton in London, you can find the sites of houses in which he lived. When he first came to London to work at the Mint, he lived at 87 Jermyn Street, just back of Fortnum and Mason.

In 1710 he moved to St. Martin's Street near Leicester Square. The Westminster Reference Library replaces the house he occupied for about fifteen years. It was in this neighborhood that he used to meet the Protestant Heugenot mathematician, Abraham De Moivre, at Slaughter's coffee house. De Moivre recalled that Newton had described to him how he had studied mathematics during his university years:

> Took Descarte's Geometry in hand, tho' he
> had been told it would be very difficult,
> read some pages in it, then stop't,
> began again, went a little farther than
> the first time, stop't again, went back
> again to the beginning, read on till
> by degrees he made himself master of
> the whole . . . . . [12]

In 1725 Newton moved to the country, to Pitt Street in Kensington, not far from Harrod's, where Bullingham Gardens succeed the house in which he died.

Newton was elected to the Royal Society in 1672 and served as its President from 1703 until his death. It was the Royal Society which published Newton's **Principia** in 1787, although Edmond Halley, Clerk of the Society at the time, paid the costs out of his pocket. The Royal Society is the place to go for look at Newtonian memorabilia. Here you'll see the printer's manuscript for the first edition of the **Principia** and the first edition itself, Newton's reflecting telescope and the original mansucript describing it, and a slate on which Newton scratched a sundial as a boy.[13]

The trail ends at Westminster Abbey, where Newton lay in state in the Jerusalem Chamber after he died. The inscription on the monument that marks his tomb reads: "Let Mortals rejoice That there has existed such and so great an Ornament to the Human Race."[14]

Lord Bertrand Russell

So it is in a mathematician's London ... One minute you've admiring the Henry VII Chapel in Westminster Abbey, or the Rose Window, and before you know it you're strolling through Richmond Park. The view that you stop to admire is the view that Henry VIII saw as he waited for a flare to be sent up from the City, the signal that Ann Boleyn had been executed. It's the same view that Lord Bertrand Russell saw when, as a boy, he lived in Pembroke Lodge with his grandparents and Richmond Park was his backyard. Russell's grandfather, Lord John Russell, had twice been prime minister to Queen Victoria and Pembroke Lodge was the grateful monarch's gift to him.

As you dine on the terrace of Pembroke Lodge, now a cafe in Richmond Park, you can recall that it was here that little Bertie studied algebra, wanting to know what the x's and y's really were, and thinking that his tutor knew but wouldn't tell him.

It was here in Richmond Park that his brother Frank gave him his first lesson in Euclid's geometry. Bertie wouldn't accept the axioms on trust; he wanted them proved. Frank insisted that unless the axioms were accepted, they couldn't continue.[15]

The doubts raised in the eleven year old boy's mind about the foundations of mathematics persisted and dominated his life from that day until he and Alfred North Whitehead completed the monumental **Principia mathematica** published in 1910-1913.

National Portrait Gallery

Lord Russell's portrait hangs at the National Portrait Gallery, just one of a magnificent collection of portraits of historical personalities. Here you'll find images of many of the famous persons who have dominated our mathematical tour of London, including Sir Christopher Wren, Sir Walter Raleigh and Sir Isaac Newton.

What's especially useful about a visit to the National Portrait Gallery is that the collection is arranged chronologically by historical periods, offering a unique opportunity to study British history. For those of us who need some help in distinguishing between Tudors, Elizabethans, Cavaliers and Roundheads, the Restoration and the Glorious Revolution, Georgian Britain, the Regency and the Victorian age it's the most colorful basic British history course available anywhere.

Conclusion

You have had a preview of coming attractions in London from the perspective of the history of mathematics.

Now, as a more informed traveller, you should love your trip to London even more than would be expected.

Footnotes

1. To make arrangements to visit the Royal Society, write in advance to:
   Mr. N.H. Robinson, Librarian
   The Royal Society
   6 Carlton House Terrace
   London SW1Y 5AG
   England

2. To find out when the lecture-demonstration is scheduled, or to arrange for a special presenatation for a group, write in advance to:
   Mr. John Stevenson
   Education Service
   Science Museum
   South Kensington
   London SW7 2DD
   England

3. The Rosetta Stone is in room 25, to the right of the exit from the bookstall.

4. The Rhind Papyrus is located in Room 62, just past the mummies in Rooms 60 and 61.

5. In I Kings 7:23 and II Chronicles 4:2
   Then he made the molten sea,
   it was round, ten cubits from brim
   to brim, and five cubits high,
   and a line of thirty cubits
   measured its circumference.

6. The room of Writing is numbered 56, and is next to Room 64!

7. The table of square roots is numbered 92680.

8. The multiplication table is numbered 92703.

9. The tablet containing the collection of geometry problems is numbered 15285.

10. The quadratic equations are on the tablet numbered 13901.

11. To make arrangments for a group lecture on ancient Egyptian mathematics in the Egyptian Gallery, write in advance to:
    Mr. George Hart
    British Museum Education Services
    1 Bedford Square
    London WC1B 3DG
    England

12. Christianson, Gale E. **In the presence of the creator, Isaac Newton and his times.** New York: The Free Press, 1984, p. 62.

13. See footnote 1.

14. If you enter from the West door, walk through the nave to the West side of the choir screen to see Rysbrack's work commemorating Newton. It's very ornate; there are cherubs galore, the Queen of the Sciences weeping profusely and a reclining Newton. Close by you'll find the graves of Lord Kelvin, Charles Darwin, Sir William Herschel and Sir John Herschel.

15. These anecdotes come from volume 1 of **The Autobiography of Bertrand Russell,** 1872-1914. Boston: Little, Brown, & Co., 1967.

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CONSECUTIVE NUMBERS AND y = mx + b

by
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An interesting relationship exists between straight lines when the slope (m) and the y-intercept (b) of each line are consecutive numbers.

There are two cases in which m and b are consecutive numbers:

1. $m > b$ $(m = b + 1)$
2. $m < b$ $(m = b - 1)$

Case 1 (m b)

Line 1: $y = m_1x + b_1$

$m_1 = b_1 + 1$

Therefore: $y = (b_1 + 1)x + b_1$ (1a)

Line 2: $y = m_2x + b_2$

$m_2 = b_2 + 1$

$y = (b_2 + 1)x + b_2$ (2a)

Equating (1a) with (2a) gives:

$$(b_1 + 1)x + b_1 = (b_2 + 1)x + b_2$$
$$b_1x + x + b_1 = b_2x + x + b_2$$
$$b_1x + x - b_2x - x = b_2 - b_1$$
$$b_1x - b_2x = b_2 - b_1$$
$$(b_1 - b_2)x = b_2 - b_1$$
$$x = (b_2 - b_1)/(b_1 - b_2)$$
$$x = -(b_1 - b_2)/(b_1 - b_2)$$

Therefore $x = -1$

Substituting $x = -1$ into (1a) gives:

$$y = (b_1 + 1)(-1) + b_1$$
$$y = -b_1 - 1 + b_1$$
$$y = -1$$

Therfore all lines $y = mx + b$ with $m = b + 1$ intersect at the point (-1, -1).

Case 2 (m b)

Using a proof similar to the one used in Case 1, it can be shown that all lines which contain the relationship $m = b - 1$ intersect at (-1, 1).

CALGARY JUNIOR HIGH SCHOOL MATHEMATICS PROJECT

by
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formerly Supervisor of Mathematics, Calgary Board of Education
and
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Professor, Univeristy of Calgary

Abstract

The project tested students in Grade 7 and 8 to determine their ability (level of cognitive thinking) in the areas of fractions and ratios. Also the cognitive demands of the student texts, tests, and curriculum guides were determined. It was found that a significant gap exists between the levels of cognitive thinking of students and the demands of the curriculum and resources -- the students being very much lower than demands.

Next, teaching strategies for these topics were devised in consultation with classroom teachers -- strategies that reflected a concrete process -- oriented approach (CPO) stressing making conclusions, generalizations, and giving explanations as well as the computational skills required. The classroom teachers taught the program using these novel strategies while other teachers taught control classes.

Final tests were administered to compare achievement.

Overall, the project findings have demonstrated that a concrete process-oriented approach can result in significantly improved achievement in, and attitude towards fractions and ratios while enhancing the development of general mathematical strategies and maintaining computational facility.

Summary Report

In phase one of the Calgary Junior High School Mathematics Project (CJHMP), assessments were made of student cognitive levels in grade seven and eight fraction and ratio contexts and of the corresponding cognitive demands made by curriculum guide objectives, authorized textbooks, teacher presentation and teacher made tests. Of the 435 grade seven students tested in the context of fractions, 55.8% were at the concrete level, 38.2% were transitional, and only 6.0 percent were formal. For grade eight (312 students), the figures were 44.5% concrete, 43.3% transitional, and 12.2% formal. In the treatment of fractions by regular textbooks, teachers and tests in the two grades, the cognitive demands were found to conform to the concrete level less than 4% of the time and to the formal level more than 68% of the time. Similar patterns were found for ratios. These findings indicated a considerable gap between student cognitive level and curriculum demand in the contexts of grade seven and eight fractions and ratios.

This figure shows the percentage of tasks at the three cognitive levels as presented by the regular teacher, teacher-made tests, textbooks, project teachers (CPO) and curriculum guide compared to the percentage of students with cognitive development at each of the three levels.

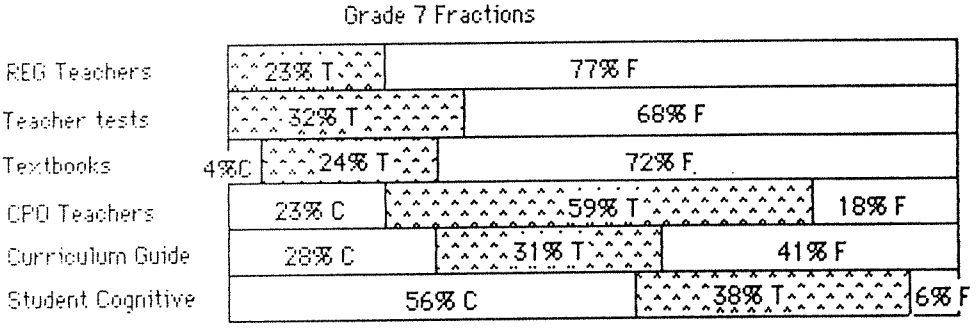


Figure 1

Phase two of the CJHMP analyzed the effects of a concrete process-oriented approach to the teaching of fractions and ratios in seventh and eighth grades. The effects measured included achievement in fractions and ratios as well as attitude and ability to apply general mathematical strategies. Six teachers taught 246 grade seven students in ten experimental classes while 234 grade seven students in nine classes were taught by four teachers using regular classroom teaching methods. At the grade eight level, 120 students in four experimental classes were taught by two teachers while 213 students in eight regular classes were taught by four teachers.

The students had been matched on standarized school-administered tests. All 313 students were pretested on achievement in fractions and ratios, attitude toward fractions and ratios, and general mathematical strategies. Both groups of grade seven students were taught fractions and ratios for approximately twelve weeks. For the grade eight groups, the length of treatment was approximately eight weeks. At the completion of each unit all of the treatment groups were retested using the same achievement and attitude instruments as those administered initially.

Two-factor repeated-measures analysis of variance were used to compare treatment groups on attitude towards fractions and ratios (anxiety and enjoyment) as well as the ability to apply general mathematical strategies.

The treatment group results, in general, were higher than those of the regular group results, though the differences were not always statistically significant at the grade 8 level:

a. the students experiencing the concrete process-oriented treatment scored higher on the achievement tests and on the problem solving strategies tests than did the regular students.
b. the students in the experimental group showed a more positive change in attitude towards mathematics than did the regular students.
c. the greatest change in both achievement and attitude occurred among the students in the transitional cognitive levels.

Overall, but particularly at the grade seven level, the project findings have demonstrated that a concrete process-oriented approach can result in significantly improved achievement in, and attitude towards, fractions and ratios while enhancing the development of general mathematical strategies and maintaining computaional facility.

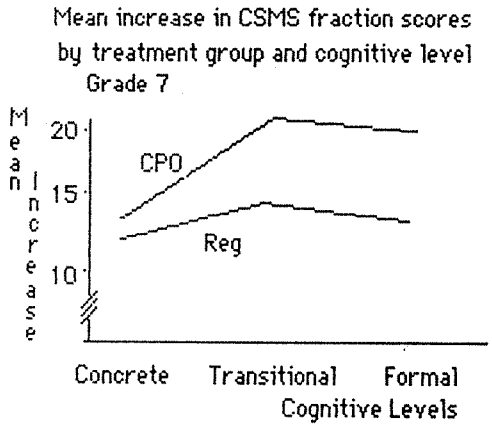


Figure 2

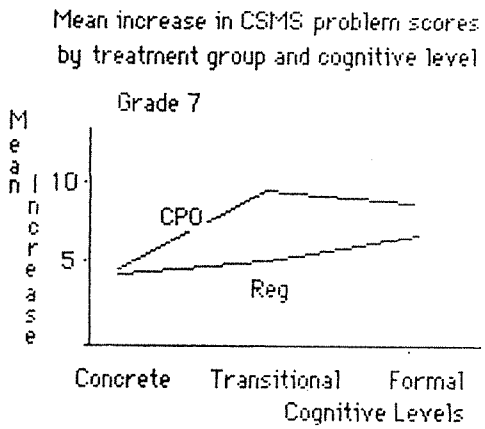


Figure 3

A Brief Synopsis of the ACLIC Project Relevant to the Fractions Issue

The **Assessing Cognitive Levels in Classrooms** (ACLIC) project was similar to the CJHMP project in that it assessed and contrasted the levels of the demands placed on the students and the corresponding student response levels with respect to the elementary mathematics curriculum.
The final report was completed in 1985.

One of the striking points in the CJHMP report was the gap between the student response level and the curriculum demand level with regard to concepts of and operations with fractions. When one considers the efficacy of teaching operations with fractions to grade 6 students, a look at the student response level to Operation and to Numeration tasks in general is worthwhile. In the ACLIC project, the demand levels of the curriculum, the teaching resources and the classroom presentations were contrasted with the student responses levels. Both students were designed to assess the understanding, the cognitive aspect, of the topic and not the student achievement in the skill area.

It is to be noted that in the ACLIC, the demands of only those skills/concepts identified in the Provincial Curriculum guide were assessed. The Alberta Elementary Curriculum does not include operations with fractions, therefore no inference can be drawn about operations with fractions directly. However, we do know that the demand levels in fractions are higher than in whole numbers. We can conclude that in those cases in which demand levels in whole numbers exceed the corresponding student response levels, the gaps would be even larger were the assessment made in fractions.

The following tables show the mismatch between the curriculum demands and the student response to be significant in both Numeration and Operations. Whereas, in Numeration, only about 12% of the students responded at the early formal level, from 50 to 91% of the curriculum demands were at the level. In Operations the match was fairly good in all curriculum demand categories except the Classroom where the demand was 22% early formal and 78% in sharp contrast to the response levels of 3% early concrete, 28% late concrete, 57% early formal and only 9% formal[1]. The reader may correctly conclude that since these gaps exist in work with whole numbers, then the gaps would be much larger and more significant in operations with fractions.

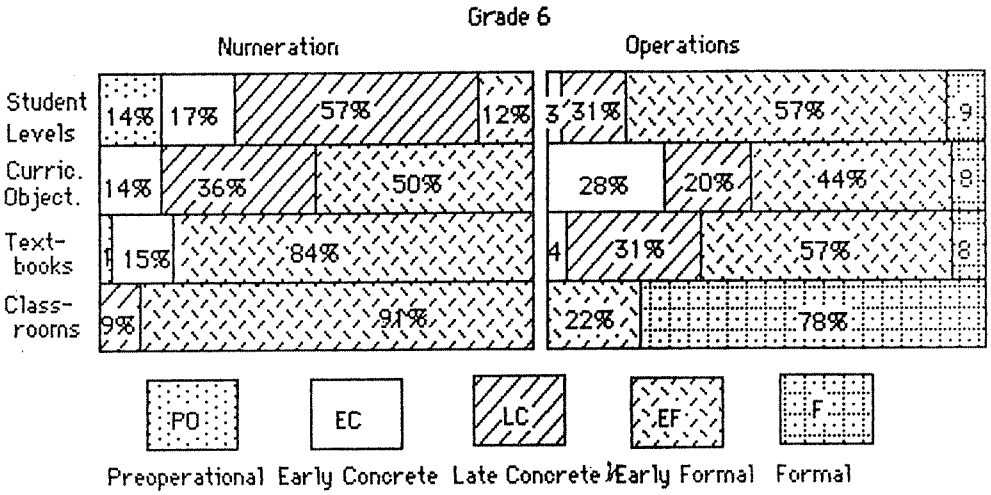


Figure 4

It is now some seven years since the completion of the **Calgary Junior High Mathematics Report** and two years since the completion of the **Assessing Cognitive Levels in the Classroom Report.** A reasonable question posed to the two writers is "So what do you recommend for the placement of fractions in the curriculum?" We shall try to answer that objectively.

First, let us make it clear that there is nothing contained in the two reports that suggest topics be delayed so as the gap between the curriculum demands and the student response level be decreased. We do not know the optimum width of the gap between the response and the demand levels. We do believe, however, that the broader the gap the more the students will have difficulty.

The Junior High study did show that the gap could be bridged through use of an appropriate type of instruction -- we called it 'concrete process-oriented'. That is, by using instruction designed to encourage the students to give explanations, draw conclusions and generalizations, as well as to develop the computational skills, resulted in higher achievement and increased ability to use general mathematical strategies at the seventh grade level.

The Elementary study did not address the question of bridging the gap.

We can relate what has happened in Alberta. Traditionally operations with fractions started in Grade 2 and continued through Grade 6, by which time the four operations were covered. In 1977 all **operations** with fractions were dropped from the grade 2 through the grade 6 curriculum. The concept of fractions, equivalent fractions and a general understanding of fractions built through the use of models remains prominent in the curriculum. In general, there was little opposition to the dropping of operations with fractions from the curriculum. The increased emphasis on fractions seems to fill any need previously satisfied by operations with fractions. The two writers in their positions of supervisor of mathematics and members of various Provincial Mathematics Curriculum committees, experienced little negative response from parents, elementary teachers, junior high teachers or senior high teachers.

In 1978, the Junior High Mathematics Curriculum included addition and subtraction of fractions at the Grade 7 level and multiplication and division of fractions at the grade 8 level.

The two studies in 1980 and 1984 seemed to vindicate the curriculum planners.

Grade 7-9 teachers endorse the action of the lower grades omitting the teaching of operations with fractions. Many maintain that the students in grade 7 and 8 have a better attitude toward fractions. The students don't have a

defeatist attitude. While there is no hard data available yet, teachers maintain the performance in fractions is better than when the operations were taught for six years. It seems that the students in grades 7 and 8, in two years, can master the material quite readily and meet the normal grade 9 demand. The CJHMP report showed that greater success can be had if a concrete process oriented approach is used than if a traditional teaching approach is used.

In the years that followed there was some negative response to the splitting of the operations between the seventh and eighth years. As a result the tentative Junior High Mathematics Curriculum dated 1987 has included all four operations with fractions in grade 7.

With regards to action taken by British Columbia, the draft **Mathematics Curriculum Guide 1-8** produced by the Ministry of Education and dated 1986, has virtually left the operations with fractions to Grade 7 and 8. The illustrating of division with fractions is listed for grade 7 with the major work outlined for grade 8. The Guide lists for grades 2 through 6 a series of objectives designed to provide background and understanding of fractions in preparation for the operations.

It was with some interest, that while attending the NCTM Annaul Meeting in Anaheim in 1987 that we learned a number of school systems in the United States have taken this step of delaying operations with fractions to grades 7 and 8, emphasing the concept of fraction in the lower grades.

In conclusion, the two studies do support the placement of the operations with fractions in grades 7 and 8. The CJHMP study indicates the demand levels of fractions are higher than the response level of most students in the respective grades and hence would be significantly higher than the response level of most grade 4, 5 and 6 students.

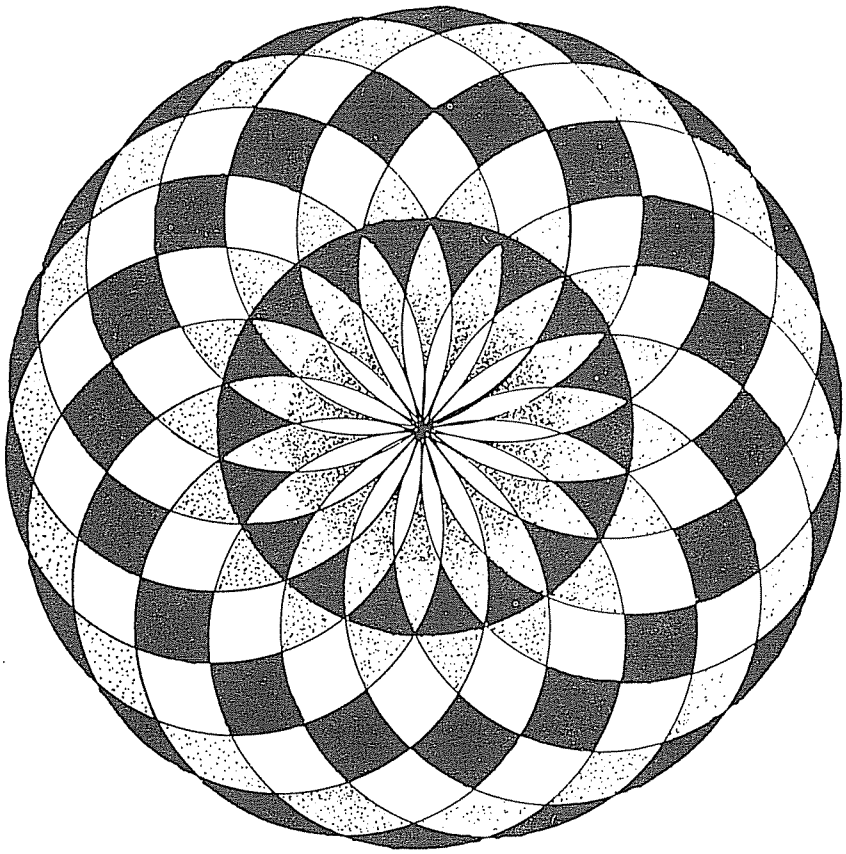
[1]Note the levels in the ACLIC study were labelled differently than in the CJHMP study. For rough comparisons one may equate the Concrete, Transitional and Formal of the CJHMP to Early Concrete, Late Concrete and Early Formal, and to Formal, respectively, in the ACLIC study.

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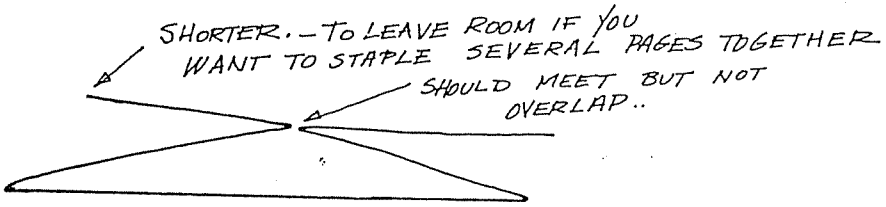


"Before LOGO"

EDITOR: RON RIPLEY
FACULTY OF EDUCATION
QUEEN'S UNIVERSITY

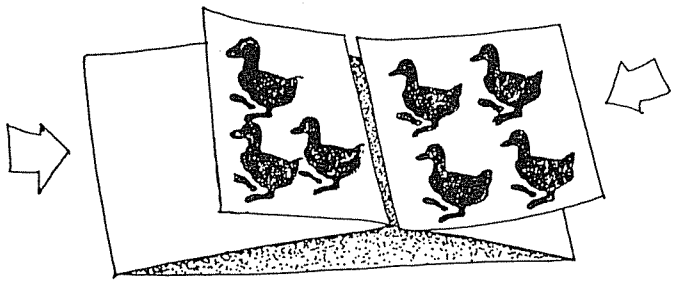
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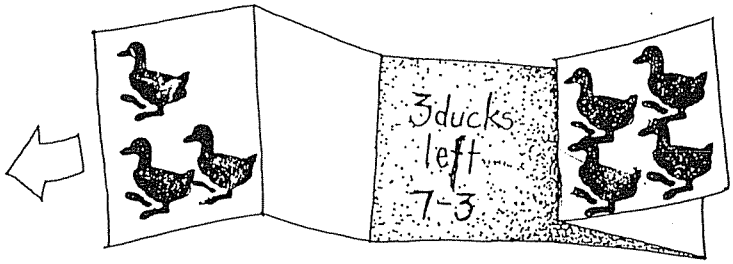
Problem Posing

Steve Lewis and Gary Flewelling
Wellington Board of Education



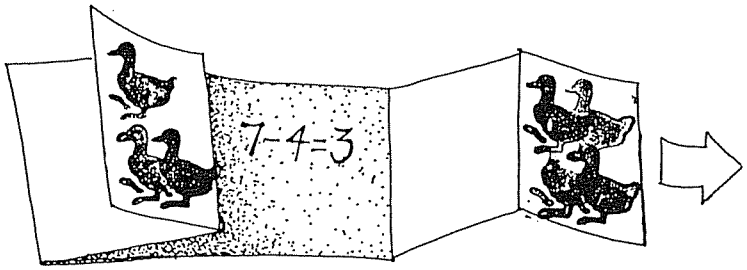
Students should be encouraged to ask and solve questions of their own devising. Here are a few ways in which it can be done.

- two sets of objects can be drawn by hand or by using a rubber stamp

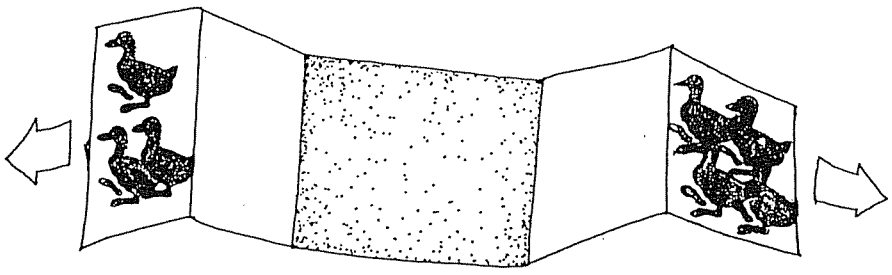


- in the centre section, write a story putting objects with context

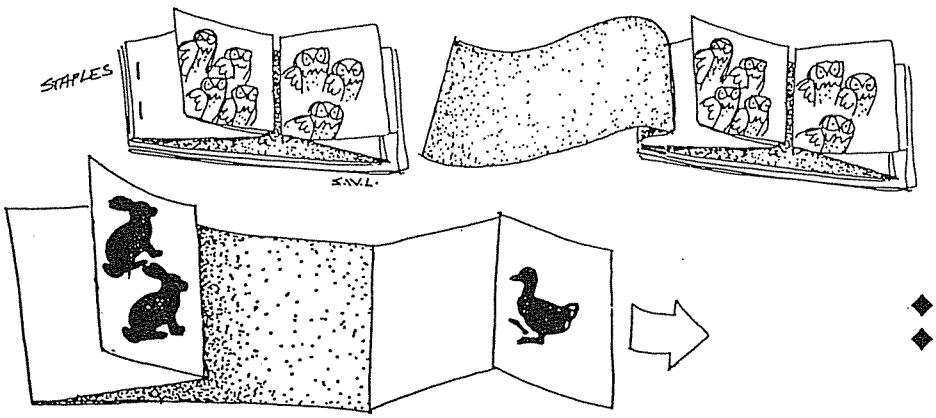
and/or



- students write their own number statements, with or without answers, for each set of pictures



- addition and subtraction action can be demonstrated

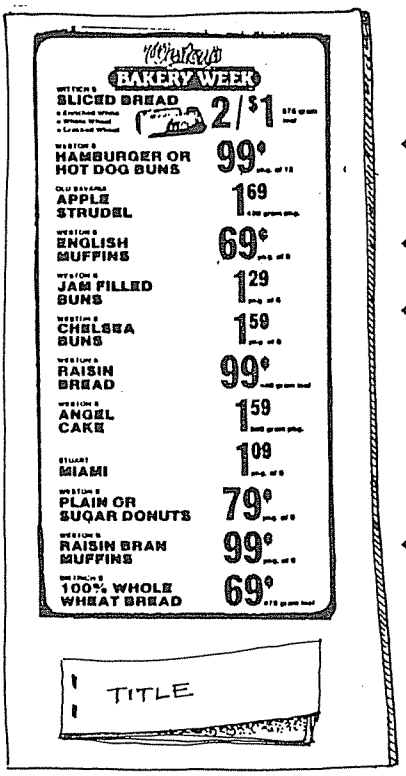


- make a book of a number of such pages
- add a construction paper cover and a title
- use different stamps
- student writes related problem

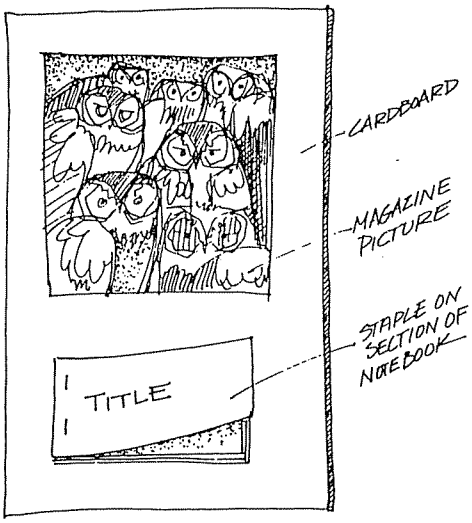


Challenge students to find fractions (or shapes or numbers or ...) as they occur in a picture of their choice (catalogues are excellent!).

- glue pictures on a sheet of paper or in special "fraction" (or shape or number or ...) book
- students write statements or stories about their picture
- language, math in "real world" built on a foundation of increasing visual awareness (for children at many levels of development)



- glue picture to a card
- each page is a new problem
- pictures can be from magazines, newspapers, menus, flyers, catalogues and so on
- answers could be written out and stored separately or written on the back of the card.



Making Room for Decimals

Doug McDougall
Waterloo County Board of Education

Objective:

The student will relate decimals to the concept of area.

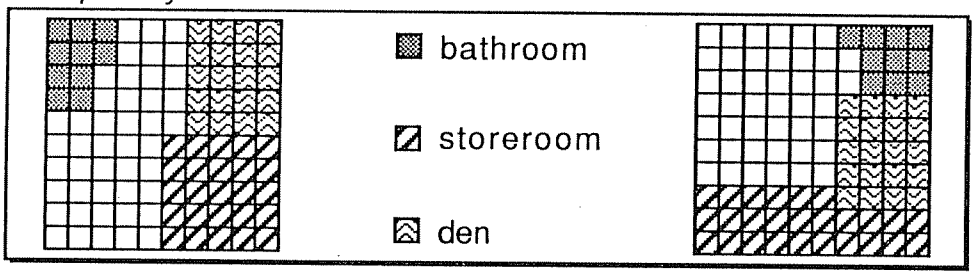
Materials:

10 x 10 grids
Coloured pencils or crayons
Pencils

Activities:

1. a) Using a 10 x 10 grid to represent a basement, show one way of dividing up the space so that:
   - 0.10 is occupied by a bathroom
   - 0.20 is occupied by a den
   - 0.25 is occupied by a storeroom
   - the remaining space is occupied by a recreation room.

Sample Layouts



b) What decimal of the basement is your recreation room?

2. a) Using a 10 x 10 grid to represent a classroom in your school, design a classroom which contains:
   - a reading area
   - desks
   - bookcases
   - storage areas
   - activity areas

b) Make a chart listing the areas of the classroom and the amount of space they occupy.

3. Each square on the grid represents one square unit of area. Have the students find the area occupied by each of the rooms in the basement created in Activity 1. The students should notice that the sum of the areas in each case equals 100 square units.

Hockey Playoff Action

Gary Flewelling, Doug McDougall, Stu Telfer
Grand Valley Mathematics Association

A scale drawing of a hockey rink (see next page) can present you with a mathematical opportunity that you can exploit as both a measurement, estimation and problem solving activity. Have the students perform the needed measurements and use the given scale to convert their answers.

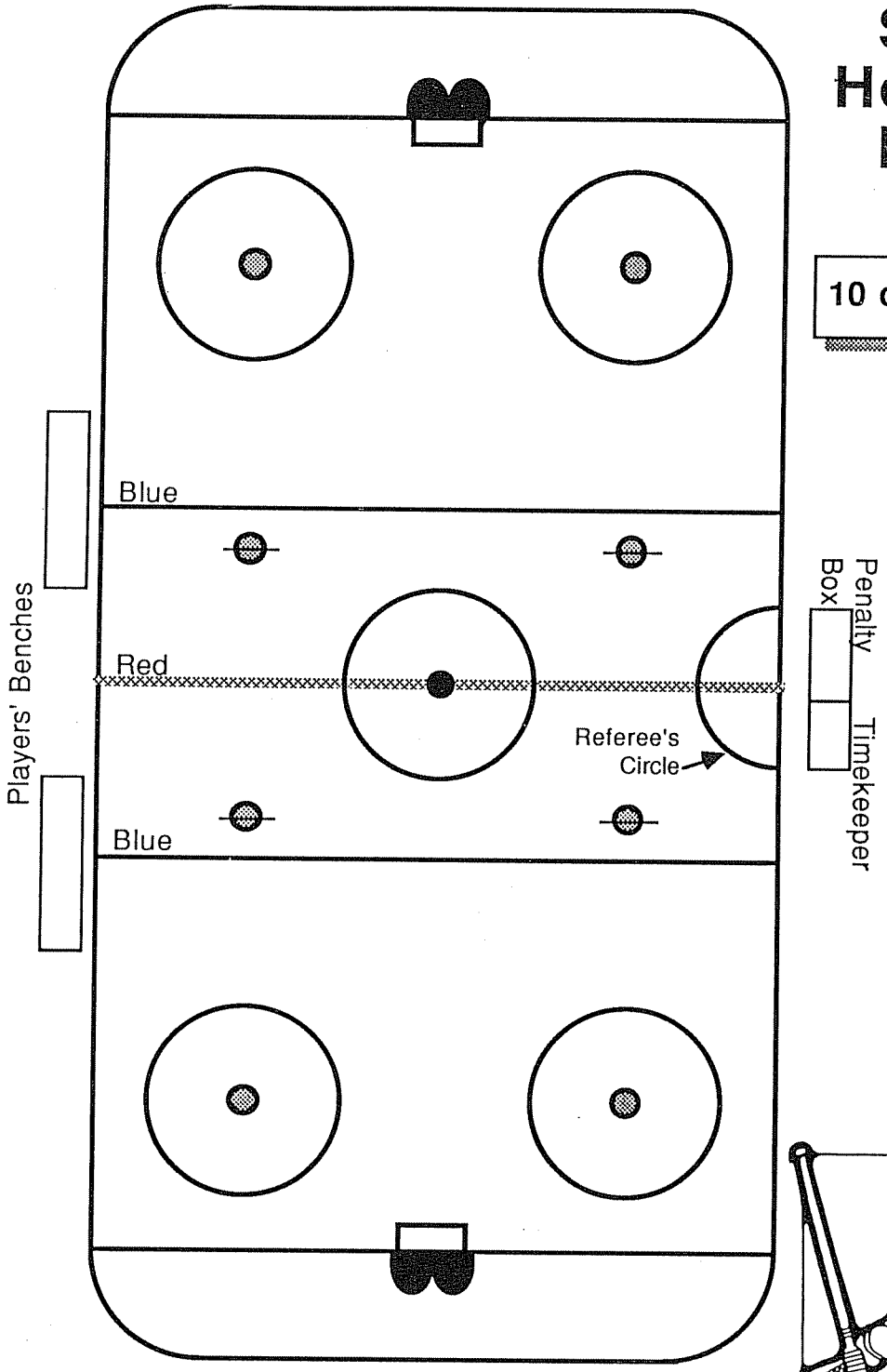
Below is a list of some possible questions that can be asked of students based on the diagram. There are many more that we are sure that you can add to the list. Also, you can extend the activity by having your students create the questions!

Note: some questions may require additional information that the student must find and other questions may have more than one answer

Sample Questions

- ❄ What are the dimensions of the hockey rink?
- ❄ What is the distance between the blue lines?
- ❄ What is the distance between the two nets?
- ❄ How far does a goaltender skate from his net to the player's bench?
- ❄ What is the radius of the centre face-off circle?
- ❄ Advertising space is rented along the boards at a cost of $100/m. How much money could be collected if all available space was sold?
- ❄ Estimate the area of the ice surface.
- ❄ What is the longest pass that can be made without being offside?
- ❄ If the ice surface is flooded to a depth of 3 cm, estimate the amount of water needed to flood the rink.
- ❄ A zamboni clears a track 2 m wide. How many trips around the ice surface are needed to clear the ice at intermission?
- ❄ How long is a shot from the right face-off circle to the net?
- ❄ A slapshot travels at 150 km/h. How long does a goaltender have to react if the shot is taken from: the blue line? centre ice? the back of the face-off circle?
- ❄ How many pucks wide is the goal crease?

International Size Hockey Rink



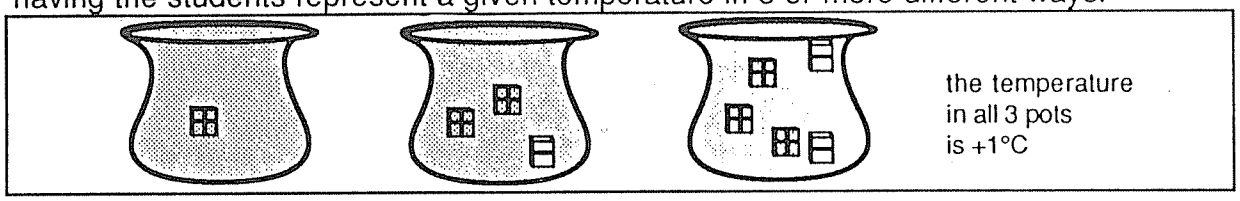


A Consistent Model for Integers

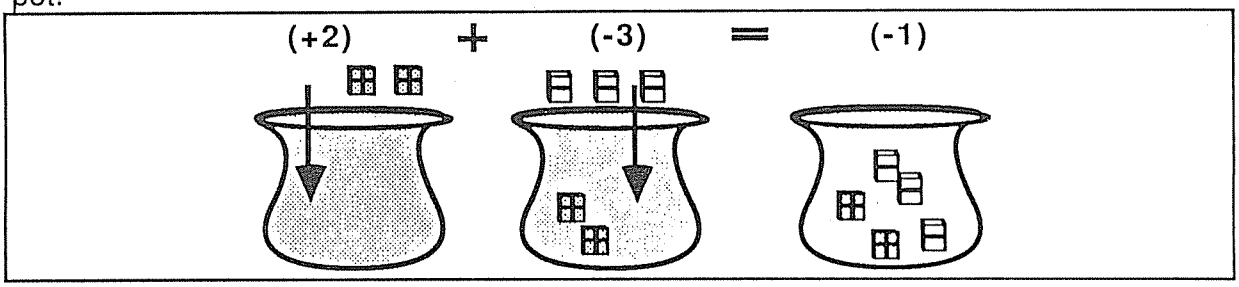
Stu Telfer
Waterloo County Board of Education

A practical model that can be used to illustrate the four basic operations with integers is, what I call, the "cubie model". The model is not new and it has appeared in many different forms, but it does offer a simple, concrete introduction to integers. The worksheets introduce addition and subtraction of integers (core topics in grade 7 and 8) but the idea can be easily extended to include multiplication and division. Apart from having the students draw the individual cubes, you may wish to have them use coloured disks (green for negative, red for positive) to represent the cubes.

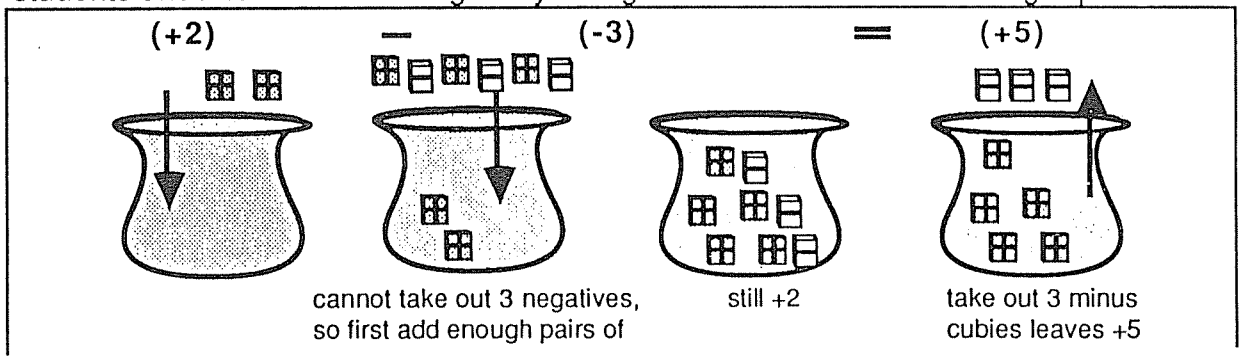
Worksheet #1 : The basic notion of the cubie model is that a pot has an initial internal temperature of 0°C. By adding a "plus cubie" to the pot, the temperature rises 1°C and by adding a "negative cubie" to the pot the temperature drops 1°C. The first worksheet requires that students are able to state the temperature of the pot after several cubies have been introduced. It is important that students realize that any given temperature can be represented in many different ways. Extend the activity by having the students represent a given temperature in 3 or more different ways.



Worksheet #2 : Addition is now a simple matter of the student adding cubies to the pot!

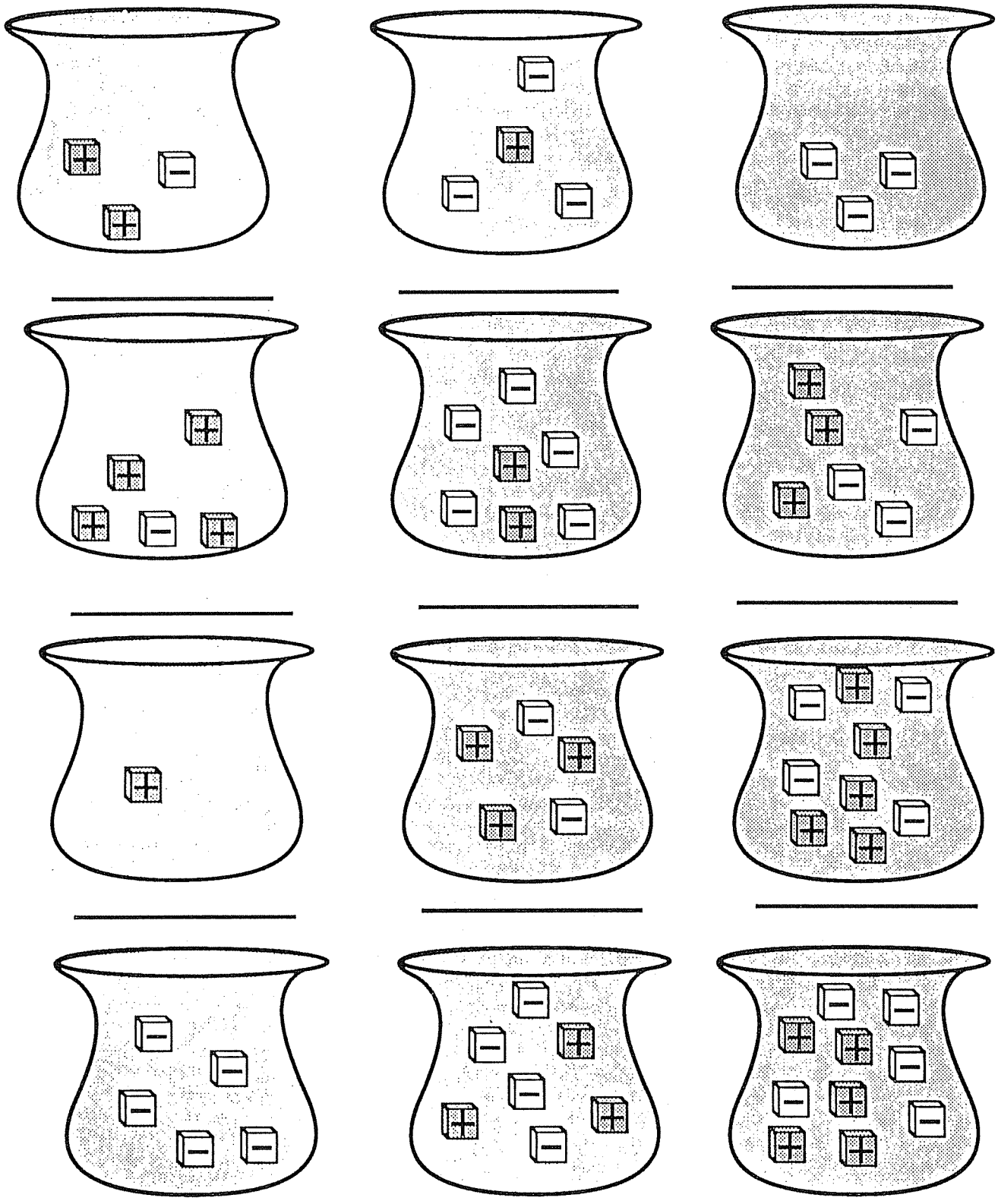


Worksheet #3 : Subtraction builds on the student understanding of 'taking away'. Some questions may require that students alter the number of cubies in the pot to allow for subtraction (see worksheet #1). With a little practice, this model can help students understand how 'taking away a negative' is the same as 'adding a positive'.



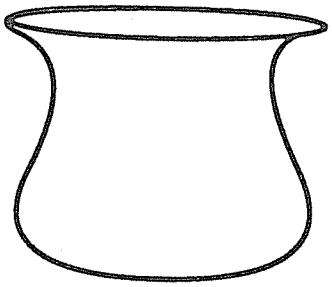
Worksheet #1

State the temperature in each of the following pots:

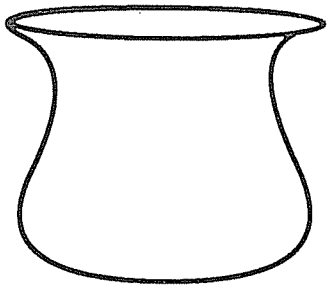


Worksheet #2

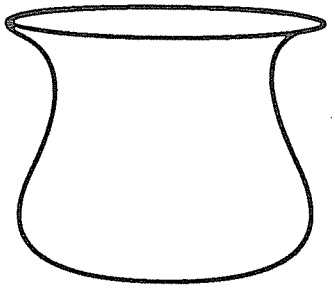
Represent the following additions and state the answer:



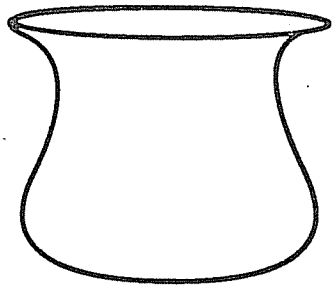
(+3) + (+2) = ____



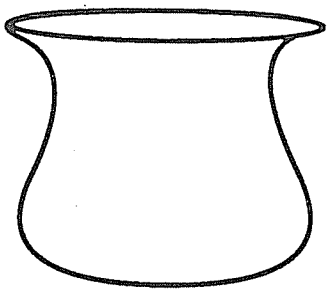
(+5) + (-3) = _____



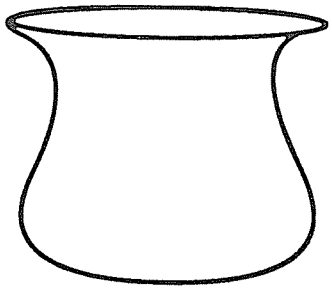
(-4) + (+4) = ____



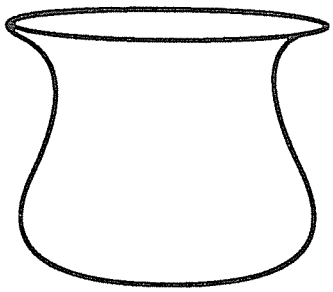
(+3) + (-5) = _____



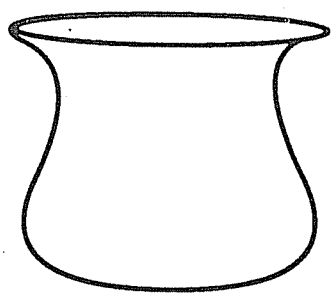
(-2) + (-4) = ____



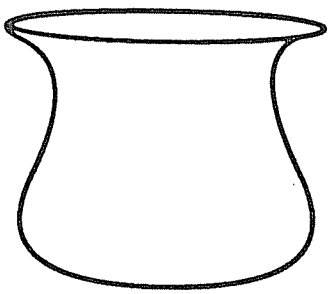
(-1) + (-3) = _____



(+2) + (-5) = ____



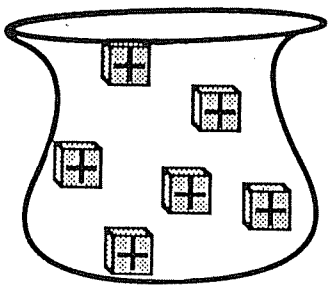
(-3) + (+7) = _____



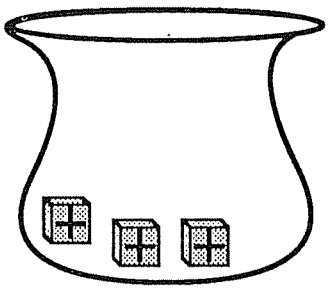
(+2) + (-5) = ____

Worksheet #3

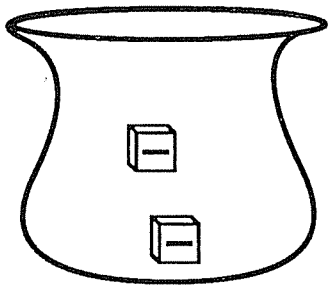
Represent the following subtractions:



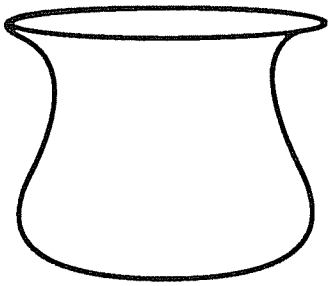
(+6) - (+4) =____



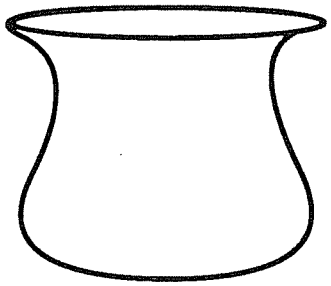
(+3) - (+4) = ____



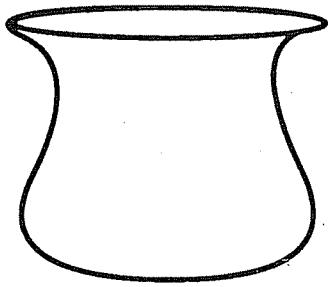
(-2) - (+3) = ____



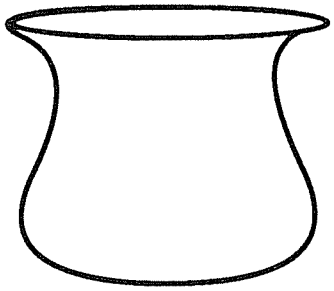
(-3) - (-5) = ____



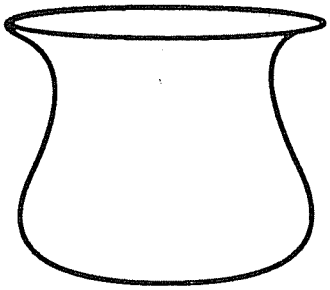
(-2) - (+4) = ____



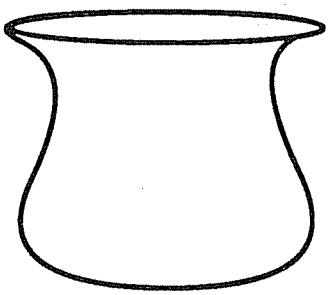
(+1) - (-2) = ____



(+5) - (+6) = _____



(- 3) - (-5) = ____



0 - (-4) = _____

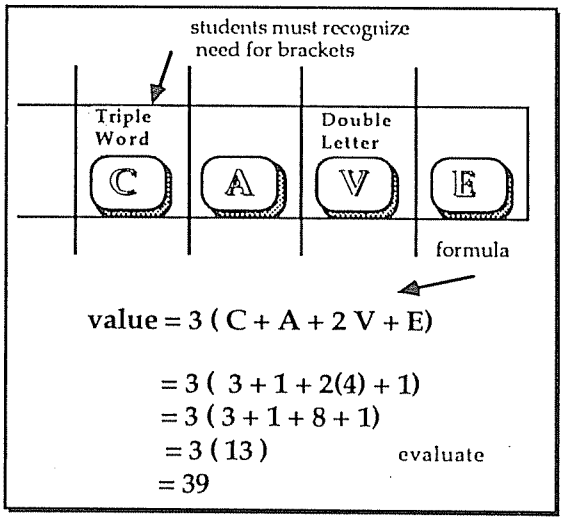
SCRABBLE© - An Introduction to Formulas

Stu Telfer
Waterloo County Board of Education

The use of a Scrabble© board can be a novel way to introduce formulas and substitution.

Often, we ask students questions such as: Evaluate $4a - 2c^2$ when a = 4, c=-3
Such a formula has no meaning or context. Using a Scrabble© board allows the student to not only give meaning to a formula, but also allows him/her to **create the formula**!

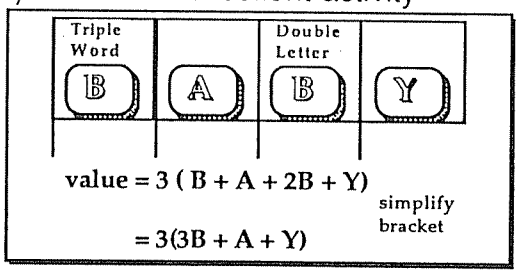
What the activity can do:



- allows the student to generate formulas (from simple to complex)
- reinforces procedures for substitution
- improves the students understanding of the use of brackets (since he/she must use brackets effectively to generate the formulas)
- provides a vehicle for estimation, mental arithmetic and problem solving
- **Worksheet #1** introduces how to create a formula from a word on a SCRABBLE© board. The letter values (same as in real game) are shown in the upper right corner of the worksheet but may be changed easily.
- **Worksheet #2** is an assignment based on a 'revised' SCRABBLE© board. **Note**: the values of the letters are NOT shown. Tell students to refer to worksheet #1 or give other values (try integers, fractions, decimals, etc.. to challenge them).
- **Worksheet #3** is a blank 'revised' SCRABBLE© board

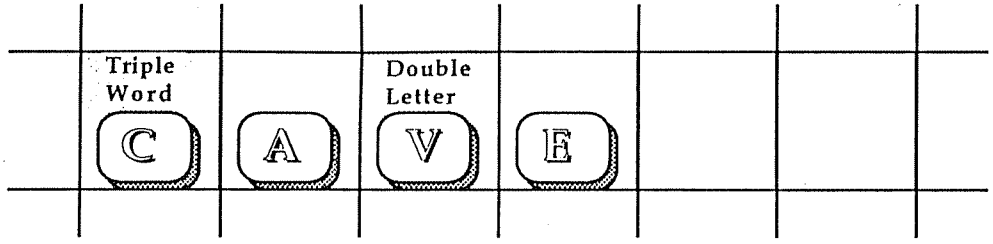
Extensions

- Problem Solving: Use the blank board and ask students to find the best location to place a particular word to maximize their score (i.e. find the best location to place the word MATH, based on specified letter values). This is an excellent activity utilizing reasoning and mental arithmetic.
- Let your students create their own scrabble boards with operations of their choice!
- Use words that will introduce the concept of collecting like terms.



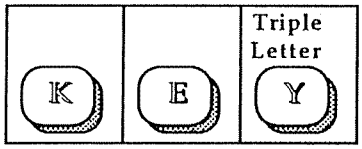
Worksheet #1

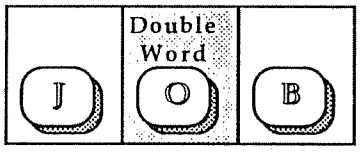
Scrabble© Number Phrases

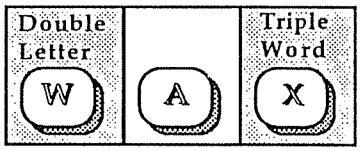


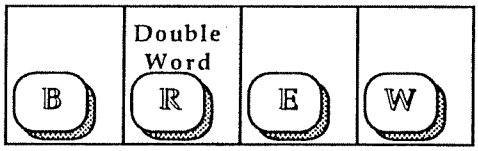
A	1	N	1
B	3	O	1
C	3	P	3
D	2	Q	10
E	1	R	1
F	4	S	1
G	2	T	1
H	4	U	1
I	1	V	4
J	8	W	4
K	5	X	8
L	1	Y	4
M	3	Z	10

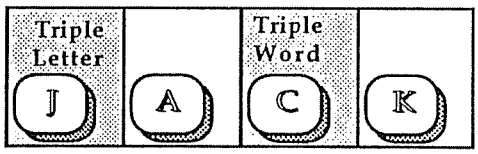
value = 3 (C + A + 2 V + E)
= 3 (3 + 1 + 2(4) + 1)
= 3 (3 + 1 + 8 + 1)
= 3 (13)
= 39

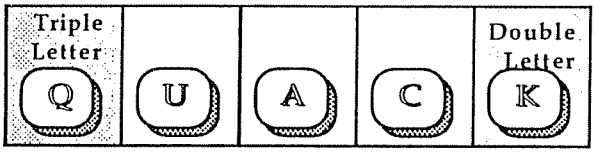


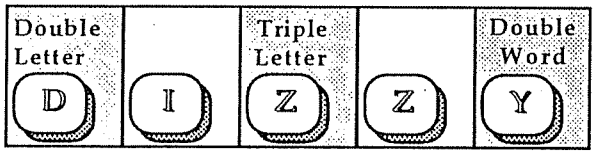








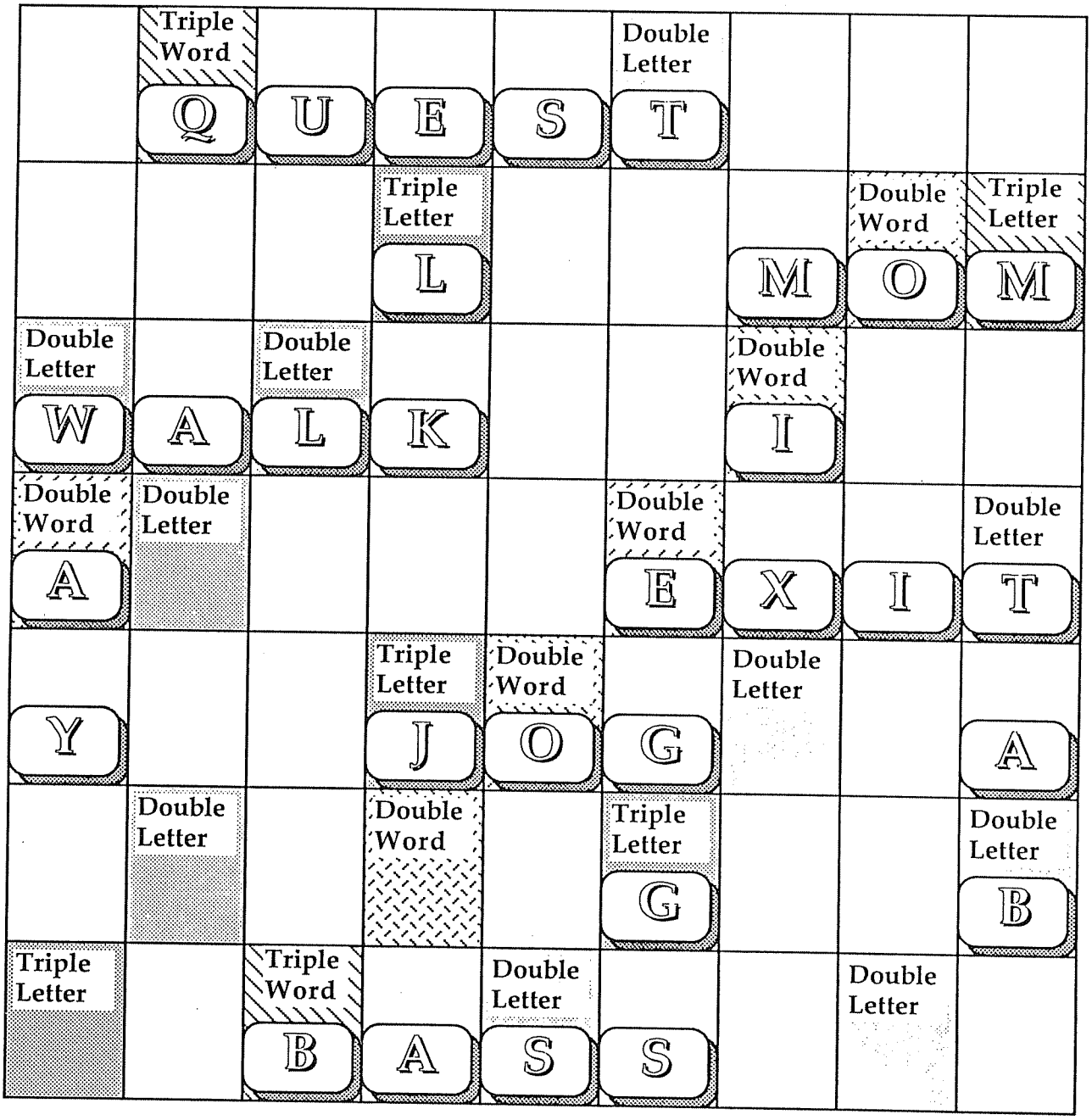




Worksheet #2

For each of the 11 words on the board,

- write an expression
- evaluate by substitution using values that the teacher assigns



Worksheet #3

MATH SCRABBLE © Board

	Triple Word				Double Letter			Triple Word
			Triple Letter				Double Word	
Double Letter		Double Letter				Double Word		
Double Word	Double Letter				Double Word			Double Letter
		Triple Letter		Double Word		Double Letter		
	Double Letter		Double Word		Triple Letter			Double Letter
Triple Letter		Triple Word		Double Letter			Double Letter	

COUNT - UP

Topic: Addition and Place Value

Grade Level: 2 - 3

Time: 10 minutes

Number of players: 2

Materials needed: Gameboard (on next page)
1 Die, 4 Markers

Object: To be the first player to reach 100

Rules:

1. Each player has a two markers
2. The first player rolls the die and places a marker on that number on the gameboard. Players take turns rolling the die.When there's a score over 10, two markers are used:one for the ones digit and one for the tens digit.
3. The game ends when one player reaches 100.

Variations:

1. Use two dice.
2. Start at 100 and subtract the numbers until one player reaches zero.
3. Use one board and let the players co-operate to reach the goal.

Some Criteria for Selecting Games:

1. Does the game teach what you want?
2. Is the game based on chance so that each child has a chance to win?
3. Is it enjoyable for the children?
4. Are the rules few and simple?

100	9
90	8
80	7
70	6
60	5
50	4
40	3
30	2
20	1
10	0
Start	

Player A

Count up ★

10	0
20	1
30	2
40	3
50	4
60	5
70	6
80	7
90	8
100	9
Start	

Player B

MATH'S MULTIP ⊥E CHOICES

by
Judith Finlayson

Ed. Note: This article has been reprinted from ***Homemaker's magazine*** *with the permission of the publisher and the author.* ***Homemaker's magazine*** *is published by Comar Communications, Ltd., 2300 Younge St., Toronto, Ontario.*

The world has been transformed in the past 20 years. Geared to microchips, floppy disks and video display terminals, today's society demands a higher degree of mathemtical skill than ever before. But, sadly and even dangerously misinformed about the realities of the working world, many teenaged girls across the country are repeating their mothers' mistake, a mistake that has propelled the majority of Canadian working women into low-paying, dead-end jobs. In high school, they are dropping out of science and math.

Today, a background in math is required for most high-paying technical jobs in fields such as computer technology and microelectronics, as well as for many apparently unrelated professions such as law, interior design and urban planning. Some companies require Grade 12 math for all entry-level positions, even for janitorial jobs.

The need for mathematical competence has been heightened not only by the extraordinary technological change of the past two decades, but by social change as well. Women have entered the workforce in unprecedented numbers. They can also expect to stay there - from 25 to 45 years, even if they choose to marry and have children as well as a job outside the home. And they should anticipate changing careers at least twice during that time.

As Donna Steward, educational co-ordinator for Women Skills, a Vancouver organization devoted to education and research on women's work, warns: "Whole fields of work are shrinking or disappearing entirely. We export enormous amounts of work to countries where labor is cheap, or we give it to machines. One result is that greater levels of competence are required, even for low-level jobs."

It's not surprising that a research report published last year by the Economic Council of Canada stressed the value of flexibility in today's workplace. Not only are better-educated and highly skilled women more likely to benefit from technical change, but the report also concluded that their adjustment may be dependent upon how successfully they enter nontraditional occupations. It is important to note, however, that both higher education and nontraditional work are increasingly linked with competence in math.

Consider, for instance, that a minimum of high-school math is often required for entrance to university courses such as nursing, teaching and law. It is also mandatory for many social-science courses such as psychology and sociology, as well as for admission to a substantial number of community college courses. The problem is, female students tend to drop maths and sciences as they progress through high school. This fast has led educators to conclude that math is an "invisible filter" denying females entry into the growth-related industries of the future.

Math and science avoidance in females is generally acknowledged as a serious issue, but unfortunately there are no national statistics that document the full extent of the problem. Research by the Toronto Board of Education, however, shows that even at the introductory level, the ratio of students in computer science courses is two-thirds male to one-third female. By Grade 13, approximately two-thirds of girls in Toronto schools have dropped out of maths and sciences.

"I'm still seeing the Cinderella myth at work" says Arlene Day, a resource teacher for equality in education with the Manitoba Teachers' Society. "Even though they see their mothers working outside the home because the family needs the money, girls are refusing to believe that the same thing will happen to them. They're still aiming for clerical jobs. Most are not even acquiring the computer skills that are necessary to be successful at office work." And her view is echoed in *What Will Tomorrow Bring?,* a 1985 Canadian Advisory Council on the Status of Women report that concluded "Adolescent girls still see their lives in very traditional and romanticized terms."

Statistics confirm that the majority of women (almost 60 percent) hold clerical and service-sector jobs, which are generally low-paid, offer little potential for advancement and may be in danger of becoming obsolete. Although women have made serious inroads into some male-dominated professions such as business, medicine and law, they are still segregated outside the more scientific fields such as engineering and computer science. According to Statistics Canada, at the univesity level, the majority of women remain concentrated in the traditional fields of study such as education, nursing and the humanities. The pattern also holds true for community colleges, where most women continue to study secretarial science, community and social services, nursing, education and the arts.

"To some extent, women have succumbed to the myths about women's work," comments Donna Steward. "They want to be helpers and to work with people. They may be avoiding nontraditional jobs and careers that require a sound basis in math because they haven't seen the human context to these jobs. Social service agencies need

to balance their books. And no one builds bridges alone. You're part of a team."

Women who avoid math may be ignoring more than the human context of working with numbers. Mathematical training has been linked with high salaries and job security in fields that have been targeted for future growth. Engineering technology, a profession that is 92 percent male, is one example of this trend. Two years after graduating from community college, an engineering student can expect to earn $20,000 a year. Perhaps more importantly, engineering technologists who reach the senior level will likely make more than $30,000 and, if they rise in management, they can earn up to $50,000 annually.

Compare these salaries to those in a female-dominated field. Ninety-nine percent of secretaries are female. Not only is their average salary just $14,100 two years after graduation from community college, but according to a 1983 Labour Canada report, even those who reach senior levels earn on average under $20,000.

This kind of wage discrepancy alarms educators who see girls avoiding math. "Nowadays, a math and science background is necessary for most of the higher-paying jobs," says Linda McClelland, a science teacher at Crescent Heights High School in Calgary. "And girls are losing out on these credientials at the same time that more and more women are entering the workforce. In addition, there is a rising number of women supporting families on their own who really need to earn a decent wage."

Tasoula Berggren, an instructor of calculus and linear algebra at Simon Fraser University in Burnaby, British Columbia, points to at east 82 careers for which math education is a prerequisite. Last November, she organized what she hopes will become an annual conference, Women Do Math, for girls in Grade 9 and 10 and their parents. Four students from each 85 Vancouver schools were invited. "I thought we would get 100 people," Berggren recalls, "and 300 registered, with many more schools asking to bring students."

Berggren designed the conference not only to introduce girls to women professinals but to provide an introduction to basic mathematical concets. "Once they see the applications of calculus - how a formula can give them the volume of a lake - they find it exciting. They say, 'This is great, I'm enjoying math!'"

She stressed the necessity of constant parental encouragement, something that Myra Novogrodsky, co-ordinator of women's and labor studies at the Toronto Board of Education , observes does not come naturally. She is conducting a new program designed to make parents of Grade 7 and 8 girls aware of the importance of math and science education to their daughters' futures.

"I usually begin the workshops with a true-or-false quiz designed to test awareness," she says. "I've discovered that a lot of people haven't thought much about the implications of social change. They still think that most girls will live in a nuclear family and be secondary wage earners, if they work outside the home at all."

One result of this misconception is that many parents have lower career expectations for their daughters than for their sons. Their attitude is reinforced by negative role modelling, which can include apparently innocuous statments such as "Women don't have a head for figures" or "Her mother can't balance a chequebook." These stereotypes can seriously undermine the confidence of girls who may have an interest in technical subjects or non-traditional work.

"By Grade 10 I knew I was mechanically oriented, but people said that physics was too hard for me and I believed them," recalls Heather Bears, who is currently studying electronics technology at Red River Communtiy College in Winnipeg. As a result, after graduating from high school, she spent two unsatisfactory years in the workforce doing odd clerical or child-care jobs. Career counselling finally revealed her scientific aptitude and motivated her to return to high school as an adult student. Not only did she make up her physics course, but she earned straight A's.

Today, as a second-year electronics student, she still feels that negative effects of gender roles. "When I entered the course there were only two other girls and approximately 100 guys. There was a real sense that we were bucking the system, and it was scary. At its most basic, I'm only five-foot-two and most of the male students are in the six-foot range."

Although Bears admits that is is difficult being a pioneer - "some teachers pick on us and others favor us" - the satisfaction of doing what she finds fulfilling is worth the price. "If I had one word of advice for girls in high school it would be, 'Don't be afraid to enter a man's world.' I believed people who said I couldn't do it because I was a girl, and that's what held me back."

Mary Elizabeth Morris, a math teacher at Castle Frank High School in Toronto, believes that the "my mother/myself syndrome" can also influence a girl's career expectations.

"A woman who does low-level work could undermine her daughter's success because she might not convey the sense that work can be a rewarding experience," she says. "If her mother is a poor role model in terms of job satisfaction, a girl may cling to the cinderella myth because she doesn't see work outside the home as desirable."

Studies such as *What Will Tormorrow Bring?* show that professional mothers tend to be positive role models for their daughters. But mothers who don't work outside the home can also encourage their daughters to develop an interest in traditionally male domains by organizing scientifically oriented excursions, such as a visit to a science museum, or by doing traditionally masculine tasks.

"We live on a farm, so my mother is a real handyman," says Robin Chant, a Grade 12 student at MacGregor Collegiate Institue in MacGregor, Manitoba, who excels at maths and sciences. "I think one of the reasons I do well in math is because, like her I enjoy figuring things out."

Chant was the only girl in her physics class last year and there is only one other girl in this year's math class, compared to nine boys. "Most of my girlfriends have dropped math because they think it's too hard," she says. "They all want traditional jobs as secretaries and day-care workers. They plan to get married and have kids. I'm different because I really want to have a career."

Myra Novogrodsky believes that if mothers are to help their daughters overcome their negative outlook toward math, they must become aware of and overcome their own negative feelings. If parents "suffered" through math class themselves, they may convey their anxiety and inadvertently undermine their children's performance. Equally important are the role models tha girls receive outside the home.

"It's hard for girls to accept the message that they can have high career aspirations and study maths and sciences if they don't see any other women doing it," says Linda McCleeland. "We need more female math and science teachers as role models, as well as more women in nontraditional careers,"

All the women math and science teachers interviewed for this article strongly agreed. Moreover, those who kept statistics on the ratio of males to females in their classes reported that the fact that a women was teaching the subjects had a positive effect on girls.

"In the past there was usually only one female student in senior-level physics," recalls Shelagh Pryke, who teaches all the physics classes at Kwalikum Secondary School in Qualicum Beach, B.C. "Now as many as 42 percent of my students are girls, and I know the fact that I'm a woman who is married with a family has played a role in this change. The girls see that it's socially acceptable to be a woman who is interested in science."

Lydia Picucha, a math and science teacher at Mount Elizabeth Junior and Senior Secondary School in Kitimat, B.C., shares this point of view. "I've been teaching here for seven years and I know my female students relate to the idea of a woman who enjoys her work and takes her career seriously. As a result, most of my female students -about 70 percent - have continued with science into Grade 11, when girls normally start dropping out."

The lack of female teachers as role models is complicated by the way maths and sciences are taught in schools. Mathematics, for example, may alienate girls because it is typically taught in a masculine style. John Clark, coordinator of mathematics at the Toronto Board of Education, says, "Math is usually presented as a search for the right answer rather than as a process of enquiry. Some sociologists believe that females have a more collaborative style. They want to work by consensus and talk with other people."

Whether or not there is any inherent difference between the male and female aptitude for mathematics remains a hotly debated issue. However, there is no doubt that the way girls are socialized undermines whatever natural ability they might have. For example, the kinds of throwing, jumping and mechanically oriented play that boys engage in actually prepares them for an understanding of maths and sciences.

Consider the game of baseball. Most boys catch balls better than most girls simply because by constantly playing ball sports they have learned how to estimate where the ball will land and, therefore, how to position their hands. What is less obvious is that this skill requires an understanding of the relationship between distance, force and velocity that serves them well once they begin to study physics.

At the Institute of Child Study, a school that operates in conjunction with the University of Toronto's Faculty of Education, teacher Robin Ethier confirms that there is a division of play along gender lines by the time the children arrive at kindergarten. "The boys choose blocks and sand to build large spaces, whereas the girls prefer small paper projects," she says. "The few girls who prefer large motor-skill projects really stand out. They are identified as tomboys."

Even so, Anne Cassidy, the Grade 5 teacher at the school, says she is not aware of a gender difference in her

students' appraoch to maths and sciences. To some extent, she believes the school's emphasis on intuitive and personalized learning has helped minimize the difference. Classes are small and teachers strongly encourage children to learn through their own activities. For example, to teach the law of averages she might ask her class to count up the pennies all seven grades collected for UNICEF over Hallowe'en. When she asks her students to work out approximately how much each grade collected, they soon realize that to get an average they must divide the total number of pennies by the number of classes. In the end, they discover the mathematical formula all on their own.

This kind of hands-on learning validates the children's own observations about the world. It also reinforces their sense of themselves as autonomous problem solvers, a skill linked with success in math. Parents can play an important role in helping their children develop this problem-solving ability by transforming daily activities into informal lessons in maths and sciences. Children should be encouraged to play mathematically oriented games such as backgammon and chess. Cooking is an excellent activity for teaching fractions as well as the principles of chemistry. Similarly, carpentry teaches measurement and spatial concepts, and comparing sizes and prices at the supermarket can even turn shopping into a learning experience.

"People make the mistake of trying to introduce new math concepts with paper and pencil," according to Dr. Ada Schermann, principal of the Institute of Child Study. "Start with a game or a fun activity such as cooking, gardening, or playing a mathematically based card game like 21. Then children don't think they're being taught, and the learning comes naturally."

Girls' poor problem-solving abilities have been linked to the fact that they are not usually encouraged to assert themselves as individuals. So perhaps it's not surprising that they begin to retreat from maths and sciences during their teenage years. During this period their willingness to consider a nontraditional career also wanes.

It must be the responsibility of parents and teachers to erase the myth that an interest in math makes a girl "different" or "unfeminine." From preschool to high school, maths and sciences should be as natural and nonthreatening subjects of study as English and history. Without a solid grounding in these subjects, the doors of opportunity will slam shut for yet another generation of young women - and, unfortunately, unemployment figures are numbers *everyone* understands.

Be sure to buy your
I ♥ Math buttons
at OAME '88 in
N.F. in May.
See you there.

THERE IS A DIFFERENCE

by

M. Elaine Harvey

Women need to identify their own teaching style and validate it. The field of education originated with men's assumptions, men's language, men's values, men's identification of subjects, men's curricula, men's policies and men's teaching methods. Men's perspective continues to control the field from Minister of Education Christopher Ward to the majority of directors of education, chairpeople of the Boards of Education and principals of schools to writers of textbooks and producers and directors of audio-visual material. And they insist that any debate about women be conducted in their language, with their rules and according to their values not because they want to subordinate women but because they do not recognize that the language, rules and values are theirs and not women's. Women need to identify and construct their own educational tradition and when they can enjoy respect within it equal to that which men now receive, then we will be able to cull the best from both and develop a balanced system that serves both males and females equally and more completely. In order to do this, women do need to identify their own teaching style and validate it.

To offer you a different perspective on the role of the teacher in education, here is a short poetic statement in which I have transformed Robin Morgan's story in **Anatomy of Freedom** into a tale regarding a mathematics teacher and a principal. This is an attempt to express these ideas in a non-linear way. It is hoped that you will have a gut reaction to this which does not stand up to analysis, but which may result in your seeing the role of teacher from a new perspective. Whatever you read into this is yours and yours alone. Do not expect that some other reader will interpret it in the same way. Feminists respect other people and their ideas, and recognize that there are indeed different interpretations--different realities.

Year after year, the woman/teacher/employee of the man/principal/boss was assailed by questions--in the early days from without, but as the students stopped coming, the questions began from within.

It was not, after all, as though she had a real right to be surprised. He had made it clear from the beginning that he was fit only to be a principal, and that he must sit withdrawn in his broadloomed office. If the world came and asked anything of him, as students, or if the world came and gave to him, as donors of praise, that was the business of the world, and not his concern. Even though he had never said all this in words, the empty spaces deep behind his eyes had clearly told her so. The handmaiden/teacher of the holyman/principal could not say that he had ever deceived her. Yet year after year she was assailed by questions.

She remembered that, in the early days, the questions students asked had seemed, at first, terribly difficult for her to answer. They asked, "What is the subject of mathematics all about?" They asked, "Why do we have to study it?" They asked, "How can we use this in our lives?" They asked, "Will you give us a spare?" They asked, "Will you let us out early?" A few even asked, "What is it like to be a woman/teacher/employee in this patriarchal system?" She remembered with a sad smile how her replies had changed over the years. In the beginning she gave them brief answers to their questions which indicated that she thought they would all become engineers eventually, but she did so with an expression on her face which deliberately implied that she knew far more than she could ever tell. Then, later, she began to invent answers; sometimes the answers would vary from student to student--which the students interpreted as further weighty difficult mathematical concepts. Thus she added to the legend of math anxiety, when actually she was merely experimenting with various responses to see which was most effective in hiding her own ignorance. Only much later, when students had begun to ask, "Why is the principal so severe in his counsel to us?" or "Why is there no mercy tempering his judgments?" or "Is he angry with us? Does our behaviour disturb his mission?"--only then did the handmaiden/teacher of the holyman/principal begin to reply simply, "I don't know," to all their questions. At first this response felt to her like a betrayal of her defense of the holy man; then it felt to her like a shameful admission of her own ignorance; at last she grew indifferent to its meaning anything but the truth.

This handmaiden/teacher has struggled to find her own answers in education. In the same way women teachers can search for and identify their own teaching style and validate it. I am going to try to help you do this because I am finally beginning to put my ideas regarding teaching into words after years of feeling the conflict between what I perceived was expected of me by both administration and students, and what I felt within my heart was right. My struggle to maintain my own integrity has not been easy. It has left me open to question by administrative officials who value authoritarian teachers who control the classroom more than those caring teachers who encourage mutual respect and individual responsibility in their students. It has left me open to question also by students who have come to expect an authoritarian school situation in which they must be coerced into learning and in which they play the expected student pranks--forging notes from parents, making up excuses for missing class, asking for spares, wanting delays on tests, cheating at exams and so on.

It has taken me a long time to understand why my struggle to maintain my integrity in the teaching profession has been so difficult. But I know now that it has to do with the fact that education is patriarchal and that I was taught

to teach by male professors and as a result of research that was dominated by masculists who do not value those human qualities that feminists are aware of as much as they value power and control.

The greatest sense of frustration that people have in their lives is a sense of powerlessness. We control our young people to a greater extent and for a longer period of time than many other societies and all other animals do. Watch an adult in charge of a child in a public place restrict that child in ways that are completely unnecessary either for the child's safety or for other people's comfort. Can you understand why a fully grown human being, albeit a student, must ask an adult for permission before he/she can urinate? Students have many unpleasant behavioural methods of reacting against their lack of power at school. We must give up the traditional power and authority of the teacher to empower both the teacher and the students.

Teaching disempowers young people and makes them dependent on the teachers. The basic assumption of teaching is that young people have no direct access to mathematics. Teachers along have that knowledge and are authorized to teach. The teaching profession is built upon and pre-supposes patriarchy. It is impossible to liberate the educational system from patriarchy and retain a teaching definition of the teacher. A great deal of pressure is placed on teachers to maintain an authoritarian stand by administrators, parents and even students because that is their perception of the school system, and they cannot imagine that the system would survive by any other approach.

Mediating, on the other hand, transforms leadership from power over others to empowerment of others and also of ourselves. Leadership is called forth from within the classroom rather than imposed by the teacher in a way that deprives the class of its own self-articulation and individuals of their creative, imaginative and intuitive spirit. The abdication of the teacher's power as domination has nothing to do with servility. Mediating means exercising power in a new way, as a means of liberating one another. Service to others does not deplete the person who mediates, but rather causes her to become more liberated. Mediating overcomes competitive one-up, one-down relationships and generates relations of mutual empowerment.

Consider the following potentially power-limiting occasions for students:

(a) going to the washroom
(b) arriving late for class
(c) skipping class
(d) not completing homework

You might reconsider your methods of dealing with these situations. I have tried to give students more responsibility and more freedom than I had done in the past. This is empowering for the students. At first they take advantage of their new-found freedom, but soon they begin to accept the responsibility that goes along with the freedom. 83% of one of my Grade 10 classes admitted taking advantage of their freedom at the beginning of the course, but only 37% were still doing so three months later. I was pleased that so many had learned to accept their responsibilities in such a short time for it has been a long tradition that students try to get away with whatever they can. Consider the number of adults who years later still boast about their childhood pranks at school, and note how many that never "grow out of it" become our representatives in Parliament. In the case of the students the irresponsible behaviour is their reaction against what they consider unreasonable authority. I have found it helpful to discuss with students their need for taking responsibility on themselves in order to be granted the freedom. Sometimes I have had private discussions with individual students that have been helpful. It is really important to treat the student as an adult, and say nothing to her that you would not say to another adult. It is especially important that discussions involve two-way communication, and that the teacher hears and respects what the student has to say. This also empowers the student.

Here is an actual discussion that took place in a Grade 10 class regarding homework:

Student: Are you going to check homework?
Teacher: Do you think I should?
Student: Yes.
Teacher: Why?
Student: So that we will do it.
Teacher: Why don't we have a vote to decide whether I should check the homework?
Student: That's a good idea.

As a result of the vote and after some further discussion, it was decided that homework would be checked occasionally. If you try to determine who has the leadership role in this conversation, I think you will see that it shifts between student and teacher. In fact, all of the students and the teacher had a part in reaching the final decision here, but it was the teacher who offered the opportunity for the co-operative decision which empowers all who participate. This is an example of mediating as opposed to authoritarian teaching.

You might, however, carry the mediating one step further by asking yourself, "Why do I assign homework anyway?" Don't be satisfied with the standard answers-the course must be completed, students all work at different paces and they must have covered the same amount of work to do tomorrow's lesson, when they are busy they keep out of mischief, etc. You might even have a class discussion about this question. Giving the students a voice in the decisions that affect their lives decreased their frustrations and empowers all who contribute to the decision-making.

As you learn to really hear what your students are saying, you have more reason to expect them to hear what you have to say. We have not really heard and taken seriously the awkward question, "What good is mathematics anyway?" There are probably many other students' questions that we have not really heard after the first couple of years of our teaching careers. One of the cardinal rules for those who want to be listened to is not to say too much. Teachers like to talk a great deal. Make what you say more effective by saying less.

But the teacher must identify her role and communicate this to the students.

> As teachers we have two choices: to lend our weight to the forces that indoctrinate women to passivity, self-deprecation, and a sense of powerlessness ... or to consider what we have to work against, as well as with, in ourselves, in our students, in the content of the curriculum, in the structure of the institution, in society at large. And this means, first of all taking ourselves seriously: recognising that central responsibility of a woman to herself, without which we remain always the **Other,** the defined, the object, the victim; believing that there is a unique quality of validation, affirmation, challenge, support that one woman can offer another. Believing in the value and significance of women's experience, traditions, perceptions. Thinking of ourselves seriously, not as one of the boys, not as neuters, or androgynes, but as women.
>
> - Adrienne Rich, "Taking Women Students Seriously", **On Lies, Secrets and Silence,** Virago, 1980.

AUTOMATH

by
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Are your students 'into' cars? Mine are. It is a subject they never get tired of. I can get an animated discussion going any time I merely mention the name of a car, the current interest rates on car loans or the high cost of insuring young drivers. Everyone has an opinion or an incident to relate about an automobile, whether it be their own or a friend's. Cars (and trucks and motorbikes) are symbols of status and power or just enjoyment among students. It is now time for us, as math teachers to tap this source of built-in motivation among our pupils.

Years ago I transferred to a vocational school in the face of the certainty of closure of the collegiate I was then in. It was goodbye to the 'safe' and very predicatable world of mathematics teaching as I had come to know it: integers, algebra, some problem-solving of sorts, and preparing students for further mathematical studies. A far more challenging career was just around the corner. In my new school I was immediately confronted with students who were not particularly adept at mathematics or even, in some cases, arithmetic. What they wanted, indeed, what they demanded, was to be shown right away what good percents, decimals, fractions & square roots could possibly be in their daily lives. May I quickly add that their attitude was not an aversion to mathematics (as some suppose) but a deeply ingrained frustration caused by i) lack of success and ii) not knowing how to put what mathematical skills they had to practical use.

My most memorable class that first year (and where the whole car thing got started) was a group of ten students assigned to me as a 'special' class for remedial purposes. They came from all grades: 9, 10, 11 & 12. Their math abilities were as varied as their personalities. We had 35 minutes together each day during one of the lunch periods. How was I to address the curriculum needs of four courses at once? Answer: I didn't. At the time I was considering the purchase of a new car myself. The showroom dealers were giving away the glossy folders by the handful. As soon as I saw them I knew I had found something to take back to the classroom!

The rest, as they say, is history. What followes in this article is an outline of some of the topics in car owning which have been tried and found to be very useful in the classroom over the past six years. As well they have been well-received by teachers in presentations made at conferences such as the O.A.M.E. Annual. The material is suitable just about anywhere - basic, general and, yes, I'd even make time for it in advanced level classes - in any grade 9-12. (A full resource kit is available, but it is no longer feasible to mail these free. A charge of $5.00 will reimburse the author and the Board of Education for paper, postage and time spent assembling each kit.)

MAKING A DISPLAY

As an introduction to the topic of car buying you can easily create an attractive bulletin board exhibit. Your own 'home made' posters can be assembled from a variety of sources:

1) Newspaper advertisements, both black & white and colour.
2) Cut up a few of the folders and attach with list price shown, options listed, etc.
3) Any of the paper magazines such as 'Motor Trend', 'Car and Driver'.
4) 'Consumer Reports' annual auto edition (April) contains a wealth of charts and other statistical data.

Attach your pictures etc. to bristol board and display prominently in the classroom.

FOLDERS

1. Obtain sufficient copies of several makes of cars and trucks. These are readily available from a local dealer. If you are doing the topic in mid-winter you can pick some up at the annual Toronto Auto Show.

2. Prepare the folders for student use. Attach a card to each folder of the form:

Price	$ _____
Down Payment	$ _____
Rate	_____ %
Term	_____ Yr.

Options: a) Write the actual prices and a down payent on the cards yourself. See your dealer.

b) Let the students do this. At the time of the Auto Show a comprehensive price list appears in the newspaper. Make copies of this list. Have your class research the prices and decide their own down payments. This takes longer but is less teacher directed.

READING THE FOLDERS

For discussion -

1. Select a car or truck that appeals to you. Why do you like it? Does the price seem high?
2. What items are included as standard equipment? Which are optional?
3. List three (3) options you'd like to have. How much do you think each costs?
4. What is the fuel consumption in m.p.g. and in litres/100 km on

   i) highway? ii) in town?

   ii) combined rating?
5. What percent of the stated price is the suggested down payment?
6. What are the significant dimensions of the vehicle, ie., wheelbase, headroom space, legroom, etc.?

BASIC NET PRICE

In this activity, students are given the folders. Using price and down payment they can calculate net price as follows:

Net Price = Price on folder (P) plus 7% tax
minus down payment (D)
or P + .07 x P - D
or 1.07 x P - D

FINANCING

Enclosed you will find a number of tables and work sheets. The two ideas you want to get across are:

a) **Monthly Payment** - Most people don't have $10,000 or more cash to hand over all at once. Therefore, they pay a portion of the price down and discharge the rest over a period of time, usually 2 to 4 years, by monthly instalments. If you have a computer mortgage program & printer, your students can input car price, interest rate and loan time to get their own personal repayment schedule. Alternatively you can use the payment tables & work sheets. This is a great calculator exercise good for 2 or 3 class periods. The principle behind the work sheets is:

Net Price - 1000 x monthly payment factor
= the monthly payment

AUTOMOBILE FINANCING

	Make & Model	List Price	Tax (7%)	Total Price	Down Payment	Basic Net Price
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						

b) **Interest** - Stress to the class this fact: if you drive a vehicle off the lot without paying for it in full, you're going to pay more in the long run. The question is, how much more? The formula is:

Interest = monthly payment x no. of months
minus net price

Investigate the effect on interest by varying the rate and loan time. eg. $12,500 station wagon at 9% for 2 years. or 7% for 3 years.

A consumer decision: is it better to have small (perhaps more affordable) payments even though interest cost will be higher?

MONTHLY PAYMENT PER 1000

Example: A new Honda cost $8495
Rate is 10% over 3 years

Monthly payment = 8495/1000 * 32.27
= 274.13365
= 274.13

Interest = 274.13 * 12 * 3 minus 8495
= 1373.68

	1 yr	2 yr	3 yr	4 yr	5 yr
6%	86.07	44.32	30.42	23.49	19.33
7%	86.53	44.77	30.88	23.95	19.80
8%	86.99	45.23	31.34	24.41	20.28
9%	87.45	45.68	31.80	24.89	20.76
10%	87.92	46.14	32.27	25.36	21.25
11%	88.38	46.61	32.74	25.85	21.74
12%	88.85	47.07	33.21	26.33	22.34

CAR PAYMENT
by Math 2B1

Name of Vehicle_____

Base Price $_____

Tax (7%) $_____

Licence $_____

Total $_____

Down Payment $_____

Net Price $_____

Interest Rate _____ %

Time to Repay _____ yr

Monthly Payment $_____

Annual Interest = _____ + _____

(up to 5) _____ + _____ + _____

Total Interest $_____

APPLYING SKILLS TEST: CARS

Show Work
Name: _____

1. The price of a luxury car with accessories is listed as follows: basic car $3560, air conditioning, $412; power brakes, $52; tinted glass, $43; am-fm radio, $117; steel belted radial tires, $650. What is the total cost?

2. Jack learned that the end-of-the-year model car is now priced at $3330. It was originially listed at $3999. How much had the price been reduced?

3. Anne bought a car selling for $5430. After making a $2000 down payment she paid off the loan in 36 payments of $100 each. How much did the car really cost?

4. Canada Trust agreed to finance Karen's new car. The showroom price of the car was $4560. She had to pay 1/3 down, 13% interest, and make 36 monthly payments. How much was each monthly payment?

5. John wants to buy a 5 year old used car selling for $2800. For used car financing, he has to pay 15% down, 15% interest and make 30 monthly payments. How much does John have to pay down? How much is each monthly payment?

6. Linda needs to finance $2110 after making a down payment on a car. Loan company A offered a loan for 36 months at 13%. Loan company B offered a loan for 24 months at 18%. Should Linda choose the company with the lower rate of interest? Prove your answer is correct by showing your calculations.

LICENCE PLATES

1. Plate to owner chart from 1983.
   eg. Find your fee if your birthday is in May. When do you have to renew your licence?

   ans: $68, May 1984
   Note: annual licence fee is now $54. rather than $48. See if your students can draw up a **new** table for the current year. Also, you are now required to pay your fee annually by your birthday.

2. Licence Plate combinations - a simple exercise. As a variation, let I and O be allowed for letters. Also, ask why in Ontario do we now see plates of the form 123 ABC as well as ABC 123?

3. Obtain copies of "Ontario Licence Plate and Permit Guide" to see who uses what form of plate.

ONTARIO



PLATE TO OWNER *is coming*

On Dec. 1, 1982, a "Plate-to-Owner" system will be introduced in Ontario. Under this new system, licence plates will be registered to the owner, not the vehicle.

Benefits of the new system include:

1. The end of long lineups for validation stickers after the conversion year.
2. Annual renewals based on vehicle owner's birthdate.
3. An end to wrongful identification of vehicle owners.
4. Convenient replacement of lost or stolen permits at any local licence issuing office.
5. Availability of more accurate and up-to-date records of vehicle transactions.

Ministry of Transportation and Communications

Ontario

Hon. James Snow, Minister
Harold Gilbert, Deputy Minister

Here are the passenger car fees for 1983

If your Birthday is in.	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
*Your fee is.	$24	28	32	36	40	44	48	52	56	60	64	68
This will cover you for.	6 MTHS	7 MTHS	8 MTHS	9 MTHS	10 MTHS	11 MTHS	12 MTHS	13 MTHS	14 MTHS	15 MTHS	16 MTHS	17 MTHS
Your licence plate is valid until.	Jun 83	Jul 83	Aug 83	Sep 83	Oct 83	Nov 83	Dec 83	Jan 84	Feb 84	Mar 84	Apr 84	May 84

*Northern Ontario residents pay half this amount.

EXERCISE: LICENCE PLATE COMBINATIONS

How many different license plates are there of the form:

1. KTX 363 eg. 26 x 26 x 26 x 10 x 10 x 10 = 17 576 000
2. CB 6789 _____
3. GEOG 131 _____
4. 12 KM 38 _____
5. 43892 _____
6. AB _____
7. Any 3 odd numbers with any 3 letters from second half of the alphabet _____
8. A personalized plate having 2 digits and 4 letters _____

HOW TO USE "ONTARIO LICENCE PLATE & PERMIT GUIDE"

1. Select a category, e.g. - Historic Vehicle
2. Note fixed items: HV
3. Note range: A to Z 001 to 999
4. Calculate combinations: 1 • 1 • 26 • 999 = 25974
5. Double your answer if plates are reversal: 25974 x 2 = *51,948*

SAMPLE: ONTARIO LICENCE PLATE & PERMIT GUIDE

Lieutenant Governor of Ontario
1 - 1

Public
AAA-001 - ZZZ-999 001-AAA - 999-ZZZ

Federal Government Cabinet
CAN-001 - CAN-999 001-CAN - 999-CAN

Ontario District Court Judges
CCJ-001 - CCJ-999 001-DCO - 999-DCO

Federal Court Judges
FCJ-001 - FCJ-999 001-FCJ - 999-FCJ

Federal Government Vehicles
FDA-001 - FDZ-999 001-FDA - 999-FDZ

Historic Vehicles
HVA-001 - HVZ-999 001-HVA - 999-HVZ

Medical Doctors
MDA-001 - MDZ-999 001-MAA - 999-MZZ

House of Commons Members
MHC-001 - MHC-999 001-MHC - 999-MHC

STUDENTS QUESTIONING STUDENTS - A TECHNIQUE TO INVITE STUDENTS' INVOLVEMENT

by
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Overview

1. What is SQS all about?
2. An algebra problem and possible questions a student might ask his/her peers about it
3. Types of questions asked
4. Advantages of SQS to students and to teachers
5. Creating an environment conducive to learning
6. Implementation of SQS
7. And then your questions all in 30 minutes

Let us look at the equation $x + 2 = 5$. Will you plese jot down or think about questions a student might ask about the solution of the equation, if he/she were presenting it to the class? What questions do you think a student might ask? The essence of SQS is: students question their peers about the problems they are presenting, instead of merely explaining the problems one after the other.

The question is: how can SQS be used to teach talented students the required content in a manner that develops critical thinking skills, allows for imagination, and improves student self-concept, while bringing about healthy social relationships? When you leave this session, you will be able to answer that important question.

A fundamental principle in education is that students learn best when they actively participate in the learning process, that is, when they are part of the action in the classroom. Students are part of the action indeed when they question and answer their peers. When students question their friends, they become more attentive listeners to the lesson. Students are encouraged to listen as though their best friends were speaking.

Students realize that they must have good comprehension in order to ask intelligent questions about a topic. Answering a question often requires knowledge of a specific point, as opposed to asking a question which usually requires understanding of various aspects of the material being discussed.

In 1981, it was reported in a **New York Times** article that psychologists Jean Piaget and Jerome Bruner stated that discovery teaching should help children to discover what is in their own minds rather than, to quote Dr. Druner, "what is out there." To achieve this, students must learn to think. And with SQS, they formulate concepts in their minds, see relationships of information, and verbalize their thinking.

Philosophy of SQS

Mortimer Adler, in his book, **The Paideia Proposal,** (Macmillan Publishing Co., New York, 1982) states: "All genuine learning is active, not passive. It involved the use of the mind, not just the memory. It is a process of discovery, in which the student is the main agent, not the teacher." This philosophy underlies the philosophy of SQS. The teacher is the catalyst in large and small group discussions.

Questioning

In questioning, students must try to relate different aspects of a topic in order to ask intelligent questions. Each question a student asks has a level of difficulty. Gifted students ask more original questions, some not even thought of by the teacher during the lesson. For example, the trigonometric identity, $\sin 2x = 2\sin x \cos x$, was presented to the class by a student. He asked his peers the question, "Why can't we divide both sides of the equation by 2?" Although I have taught trigonometry more than 15 years, I have never posed that question to my students! Additionally, the questioner might stump the class at first; the class is, therefore, further challenged. Sometimes the questioner gives a hint. Sometimes the questioner rewords the question, thus clarifying his/her own knowledge and that of the class. The questioner, in the style of the teacher, is encouraged to sincerely commend a peer for his/her correct response. For example, "That's good logic, Jane!" Such commendation helps students to accept and respect each other's ability, and enhances their self-concept; it also helps to bring about healthy social relationships.

With SQS, students retain their learning for permanence and not for the moment. The same question may be asked during several presentations of homework or classwork problems. The question, with the answer given,

serves as a review and foundation for related topics.

Now I would like to ask you a question. Which number is neither positive nor negative? Of course, the number is "zero." Did anyone not hear the question? There are numerous questions having the answer "zero." For example, a question might be: What number may not be used as a divisor? You can think of others. You see, if a student does not hear the question and hears the answer only, he/she learns nothing. But if the student answers the question in a complete sentence and uses the key words of the question, he/she brings about continuity of thought and the student not hearing the question learns something. I should like to quote from a letter written to me in 1983 by a former student, Larry, who worked on a special award-winning project:

> ... Answering questions in full sentences may have seemed tedious at the time, but it really has helped me in thinking more clearly, and expressing my thoughts better (and I don't have to tell you how important that is for college interviews).....It's funny, but your SQS method was really just a variation on the scientific method (finding a problem, asking questions, finding solutions, and drawing conclusions), something which helped me not only on my special project, but also in my four years at Science....

Types of Questions Asked

What types of questions do students ask in their various classes? The students use adverbs such as: why, what, where, when, how.

Illustrations:

1. **Hygiene**
   Why is it necessary to eat a balanced diet each day?
2. **English**
   What are the main sources of conflict in Shakespeare's "Hamlet?"
3. **Social Studies**
   Where in the U.S. Constitution can we find the Elastic Clause?
4. **Biology**
   Explain **when** photosynthesis takes place.
5. **Health Education**
   How does one prepare to become a good baseball player?

During the school year, I model questions using such verbs as: discuss, demonstrate, show, summarize, agree or disagree, suppose it were. The students gradually demonstrate high-level thinking skills in originating thought-provoking questions by using this "vocabulary." The questioner can exhibit inventiveness and a high degree of originality in the formation of questions. The questioner and the class are stimulated to think critically and analyze subject material logically. Their minds are highly challenged. "Recent studies have found that the predominant use of higher-level questions during instruction has a positive affect on student achievement. Questions help students understand content....Students who use questioning learn more subject matter than students who do not." (Walsh, D.: "Socrates in the Classroom: Strategies for Enhancing Critical Thinking," **American Educator,** Summer 1985)

SQS enhances the learning process since the questioner must be able to determine if the student response is correct. The questioner must know well the aspect of the topic he/she is questioning. While an incorrect response is being given, other students listen attentively, and the questioner calls on another student to rectify the error. It might be necessary for still other students to pitch in to bring about a correction. Lively discussions of quality can develop. Thus there is student-student-student interaction which is the foundation of SQS. A question-and-answer discussion is initiated by and conducted by students, and the students are brought together as a unit.

Formulation of Questions

The students are encouraged to originate questions about their homework problems as they do them and to write them down. Nevertheless, the questioning is casual and sometimes the grammatical errors are quite creative. Sometimes the questioner makes a mistake, which can openly be pointed out in front of the class. I strive to make my classroom a stress-free, cooperative learning environment where questions and answers can be risked and where there is no need for embarrassment.

Benefits of SQS to Students

1. Not only do the students gain confidence in participating in the "action of learning," but also confidence in talking with their peers.
2. A student reported: "SQS helped me especially because I am shy and now I am not afraid to ask questions in all my classes."
3. SQS helps students to improve their belief in themselves academically and personally.
4. A social attitude is created among the students. The students introduce the next speaker by name. This procedure helps students become acquainted with each other.

5. Larry, from whose letter I have already quoted, state: "The one-to-one relationship in asking questions about a problem helps from excellent out-of-class friends."
6. Another student: "Since the class is a friendly environment, the student feels safe, alert and willing."
7. Still another student: "There is not only personal accomplishment in having a successful student-student exchange, but there is also group effort to want to learn and understand together."

SQS GIVES EVERYONE A PIECE OF THE ACTION!

8. The students learn how to correct each other's mistakes diplomatically.
9. The students are dramatic and some are quite humorous in their presenatations.
10. An on-task, social, educational atmosphere is created.
11. The students enjoy seeing each other take the role of the teacher. And I must admit that I enjoy seeing 30 young "student teachers" perform in each class during a week's time.

Benefits of SQS to the Teacher

SQS provides positive feedback to the teacher of the students' understanding of the subject. Importantly, the teacher learns from the students. As has been mentioned, the students bring up points which never occur to the teacher. It is really amazing to see the connections that the students make between various aspects of the course. So everyone is constantly learning, the teacher along with the students. To quote Anna in **Anna and the King of Siam:** "If you are a teacher, by your pupils you will be taught." I imbue my students with the idea that each one is a teacher and that I am a student, too.

Creating an Environment Conducive to Learning

It is important to create a comfortable classroom climate to bring about an easy and successful implemenation of SQS. At the beginning of the school year, I decorate my room attractively and regularly have a bouquet of flowers on my desk. I discuss mathephobia (fear of doing mathematics) with my students and have them write a math autobiography, which explains their like or dislike of the subject and any fears they may have when confronted with mathematics.

Dr. Sidney Simon of Boston University, Boston, Ma., USA, introduced me to the term "IALAC." (I am lovable and capable.) I ask the students if they feel lovable and capable and may call on two or three students a day to answer the question. Students are imbued with the idea that they are all lovable and capable and that they must believe in themselves. I ask the students what it means "to realize one's potential." It is surprising to learn that some students have no idea what the phrase means. Immediatley, I start to imbue the students with Norman Vincent Peale's statement, "You can if you think you can." We discuss a commitment to excellence. Throughout the year, I demonstrate my positive expectation of my students. Students whose work is excellent in mathematics and/or SQS leadership receive certificates of merit. We discuss responsibility, integrity and initiative. Also respect for each other. I explain that I show the same respect and courtesy to the lady who cleans the brass in our school as to our principal.

Early in the school year, we discuss active listening. For example, students are required to face the speaker and be ready to explain in their own words what they have just heard. Students try to detect errors in responses and try to correct them. It is rewarding to observe how students quickly catch each others' errors because they have actively listened.

In addition, I introduce the concept of "creative tension" (credit to Jane Martin, Director, Staff Development, St. Louis, MO., USA), by stressing the importance of each student being highly motivated to learn. We discuss ways in which students can be involved. Let me repeat that students who are involved learn more and can retain material longer than students who are not, and can apply the material to new situations. Students are encouraged to be enthusiastic, on their toes, and ready to share their ideas.

The above approach, which takes 4 or 5 class periods of 40 minutes each, sets the stage for SQS.

SQS is introduced during the first homework review in September. I would say to the class, "We are all avid readers. We can read what is written on the blackboard. I would like you, Ken, (who put the problem on the blackboard), to ask questions of your classmates about your problem, instead of reading and explaining the problem yourself." Further instruction in the formulation of questions and in the implementation of SQS would be given the following days.

Implementation of SQS

Perhaps it is easiest to initiate SQS at the beginning of the school year, although I feel that it can be initiated at any time. Two students, to date, have written that SQS should be introduced at earlier ages. Let me add that, although this conference focuses on the gifted and talented, I feel that SQS can absolutely be used to great advantage with all students at all levels.

Suppose I were teaching the commutative axiom of multiplication.

Given: $2 \times 4 = 4 \times 2$

I would first ask the students the question, "What do we call the numbers multiplied together in a multiplcation example?" Hopefully a student would answer in a complete sentence, "The numbers multiplied together in a multiplication example are called factors." I would explain the purpose of answering a questin in a complete sentence and using the key words of the question. That student would call on another student to repeat the answer to see if he/she is actively listening. Students would be encouraged to ask a second question about the factors in the question. The question could be: "What changes when we apply the commutiative axiom of multiplication?" The student offering that question would call on another student for the answer. I would again stress complete sentences. The student responding would answer, "When we apply the commutative axiom of multiplication, the order of the factors changes."

"Suppose it were" questions are popular with the students. I would suggest that we ask a "suppose it were" question about our equation. An alert student would ask, "What axiom are we applying if the equation were $2 \times 4 = 2 \times 4$?" He/she would call on a student for the answer. (This question with the answer given would reinforce material taught the preceding day.) A student would answer: "If the equation were $2 \times 4 = 2 \times 4$, we would applying the reflexive axiom of equality." Students are encouraged to call on nonvolunteers, so that all the students are involved in the "action of learning."

As the year progresses, instruction in asking questions is not always necessary. Students listen attentively to my questions and to those of their peers and seem to proceed naturally to originate their own questions. Throughout the year, I model the various types of questions students can ask. I feel that children are naturally curious and will ask questions. But unfortunatley, teachers and parents sometimes stifle that natural curiosity to question. And it is questioning that I believe is the heart of teaching!

Concluding Remarks

I can only see positive aspects of the use of SQS. My students report to me without exception that my class has more student involvement than any mathematics clas they have ever experienced. A statement from the first edition of William W. Purkey's book, **Inviting School Success: A Self-Concept Approach to Teaching and Learning,** (Wadsworth Publishing Co., Belmont, California, USA, 1978) sums up the intrinsic value of the use of SQS. The statement follows:

> When students are invited to assert themselves in socially acceptable ways in the classroom, their feelings about themselves and their abilities are likely to improve along with their acadmeic performance.

Former students Kimmi and Eina (1983) had the following to say about SQS in their evaluations of their courses with me:

> Kimmi: I feel that SQS is a method that should be used in every classroom across the nation....(First Kimmi wrote "in every classroom in the world," but she crossed the phrase out, evidently becoming more conservative, on second thought!)
>
> Einar: SQS should be used in every subject to promote greater academic results.....

I have not as yet had the opportunity to test scientifically whether SQS

1. stimulates student thinking
2. enhances the rate of learning
3. affects the quality of discussion

and, in addition provides personal and social benefits for students. I do hope to test SQS during the new school year.

Lastly, I should like to quote from Albert Einstein: "The important thing is not to stop questioning."

HISTORY AND PHILOSOPHY OF MATHEMATICS

The editor of this section welcomes comments, queries and notes from the readers of the Gazette on matters concerned with the history, philosophy and heuristics of mathematics. Write to Prof. E.J. Barbeau, University College, University of Toronto, Toronto, Ontario M5S 1A1.

One can distinguish three main reasons for including a topic in the school curriculum:

(1) It provides useful information and skills for the prosecution of daily life. For this reason, we teach reading, writing, grammar, civics, and health
(2) It provides the basis for broadening one's outlook and resources for a richer use of leisure. Thus, the curriculum includes literature, music, art, sports and foreign languages.
(3) It provides a sense of context and a deeper understanding of the nature of our society, so that as citizens we can intelligently assess social and political developments and form sensitive and informed opinions about current issues. Literature, science, history, geography and econmics are areas in this category.

Some students, of course, will require a basic background to prepare them for later studies or even material which might open out new possibilities for their future careers. Thus we include courses which might open out new possibilities for their future careers. Thus we include courses which might turn out not to be relevant to a majority of students, but which are absolutely fundamental for future scientists, politicans and professionals. More advanced courses in history, English, language and science play this role.

Mathematics teachers are privileged to teach a discipline which fulfils **all** of these roles. The ordinary citizen who buys and sells, earns a salary and pays taxes, invests, measures, sews and builds, interprets maps and diagrams, requires basic arithmetical and geometric tools, including ratio and proportion, which should be provided by the time Grade 6 is completed. As for music, so in mathematics there is much which can appeal to the amateur -mathematical games, logical puzzles, geometrical deducation, numerical and pictorial patterns. Such recreational mathematics should be a larger part of the curriculum, not only for the pleasure it might bring but also because it is often through recreational mathematics rather than the more technical standard curriculum that a student can glimpse the true character of what a mathematician really does. It is at this point that one can discuss creativity, use of notation, techniques for systematic analysis and exposition of ideas. Thirdly, mathematics has always been at the forefront of intellectual and philosophical development. Euclidean geometry, analytic geometry and calculus, the logical paradoxes of the early twentieth century all led to drastic alterations in our worldview and provided the impetus for the redefinition and astounding growth of knowledge in areas apparently far removed from mathematics. This process still continues and there is hardly an area in the humanities, the sciences, business and politics in which mathematical tools and perceptions have not played some role. Finally, the greatest part of the secondary curriculum provides a foundation for the minority destined to become scientists, mathematicians, statisticians, engineers and economists.

It is the third reason I would like to emphasize in this column. The intellectual importance of mathematics is due to its power as a discipline in which key ideas can be identified, honed and applied with great generality. Variable and function are seminal ideas through which a seemingly disparate collection of difficult problems can yield to routine methods of solution. The curriculum offers two occasions on whcih this point can be powerfully thrust forward, when algebra is introduced in middle school and when calculus is studied in the final year. Let us illustrate this for calculus.

One type of problem which engaged the attention of both Greek and medieval mathematicians was the determination of areas and volumes. This was usually a tough business and one had to come up with the most ingenious methods which usually suffered from the flaw that they could be applied only to a limited number of very special cases. Archimedes' **Method** based on the principle of the lever is but one example. (For others, consult H. Eves, **An introduction to the history of mathematics** (Holt, Rinehard and Winston), C. Boyer, **The history of the calculus and its conceptual development** (Dover, paper) or M.E. Baron, **The origins of the infinitesimal calculus** (Pergamon). One of my favourites is Cavalieri's method for determining the volume of a solid. Bonaventura Cavalieri (1598-1647), a student of Galileo, was a professor of mathematics at the University of Bologna. He died when Newton, and Leibniz, the creators of the calculus, were still children. Let us use his method for finding the volume of a sphere.

Cavalieri's method is based on the following principle, which your students should find intuitively appealing. Suppose that we have two solid figures bounded by a pair of parallel planes (ie they are resting on one plane and a parallel plane rests on the top of them). Suppose further that the cross sectional areas cut off from the two of them by any plane parallel to the given planes is always the same. Then the two figures must have the same volume. This is illustrated by Cavalieri's own diagram. (Figure 1)

Two preliminary results are needed:

(1) The volume of a cylinder is equal to its base area times its height (A cylinder is any solid figure who base and top surfaces are parallel congruent plane figures and whose lateral boundary is made up of straight lines joining corresponding points of the circumference of the base and top figures. An ordinary box and a tin can are two examples.)
(2) The volume of a cone is equal to one third of its base area times its height. (A cone is any solid figure with a plane figure as base, a single point not on the plane of the base as apex and a lateral boundary made up of lines joining the apex to points on the circumference of the base. A dunce cap and an Egyptian pyramid are two examples.)

The strength of Cavalieri's method lies in the fact that we need only establish these two results for special cases. In the case of (1), the result is evidently true for a cylinder with a rectangualr base. To establish it for an arbitrary cylinder, we take as a comparision solid a rectangular cylinder of the same base area and height. Imagine them resting between a common pair of parallel planes. (Figure 2) Since the cross sectional areas are the same at every level, the two volumes are identical, and the required result follows.

The case of the cone is a little more difficult, since we need to find a particular cone for which we can verify the result. This can be done in the following way. Let ABCD be the bottom and EFGH the top face of a cube, with A adjacent to E, B to F, etc. (Figure 3) The cube can be subdivided into three congruent cones (lopsided pyramids), each with its apex at a certain vertex of the cube and its base a face of the cube; one of these cones has base EFGH and apex A. You may wish to make three models of the cones out of bristol board to illustrate this to your students. The faces of the cone consist of the base, a 1 x 1 square, two 1 x 1 x $\sqrt{2}$ right triangles and two 1 x $\sqrt{2}$ x $\sqrt{3}$ right triangles.

We now know that the formula for the volume of a cone holds when the cone has a square base and height equal to a side of the square. If the base is kept fixed, the volume is proportional to the height, so we can deduce that the formula holds when the cone has a square base and arbitrary height. Now take an arbitrary cone. Compare it to a pyramid of equal height and equal base area. Since the areas of cross sections vary as the square of the vertical distance from the apex, the cross sectional areas of the two figures are equal at every level, so the volumes of the cone and pyramid are equal. (Figure 4)

Now we are in a position to introduce the sphere, or, more conveniently a hemisphere of radius r. Let the hemisphere rest on its circular base. It is straight forward to check that the cross sectional area t height x is $\pi(r^2 - x^2)$. So now we have to cook up a comparison solid whose volume can be identified and whose cross sectional areas at various levels are the same. Our choice is a right circular cylinder with base radius r and height r with an upsidedown cone (with same base radius and same height) removed. (Figure 5) By what we know about the volumes of cones and cylinders, the volume of the comparison solid is $\pi r^3 - \frac{1}{3}\pi r^3$. Since the volume of a sphere of radius r is twice the volume of the hemisphere, its volume is $4/3\pi r^3$.

This is a very elegant way of finding the volume of a sphere, which should be a source of delight to high school students at all levels. But what does the success of this method depend on? One has to find a suitable comparison solid whose volume is already known and whose cross sectional areas agree with those of the given solid when the two are suitably oriented. This requires considerable ingenuity, and each particular problem merits its own special treatment. Of course, many volume problems will simply be intractable.

Now look at it as a problem in calculus. It is a mundane exercise in finding the volume of a solid of revolution which could well appear in the A section of the exercises on applications of integration. Calculus is a tool of great power and generality, whose sway subjugates as technical exercises a vast array of problems. It is the awe of this we should never lose as teachers and which we should not fail to convey to our students.

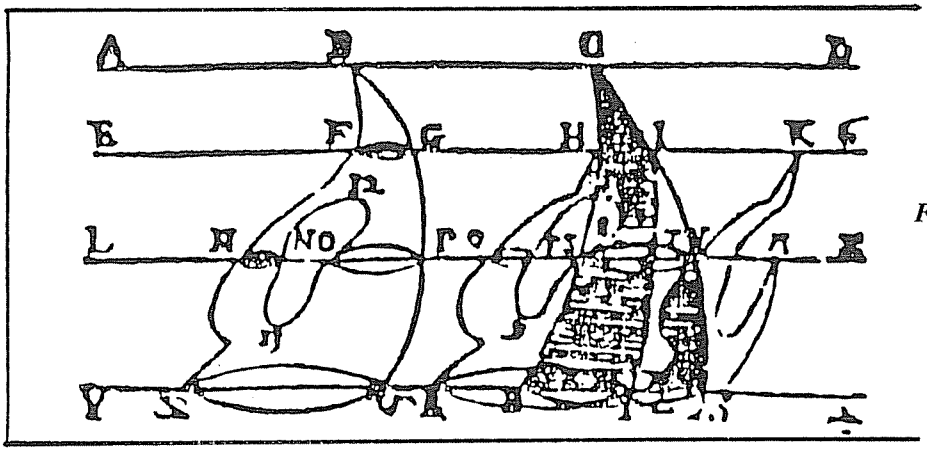


Figure 1,

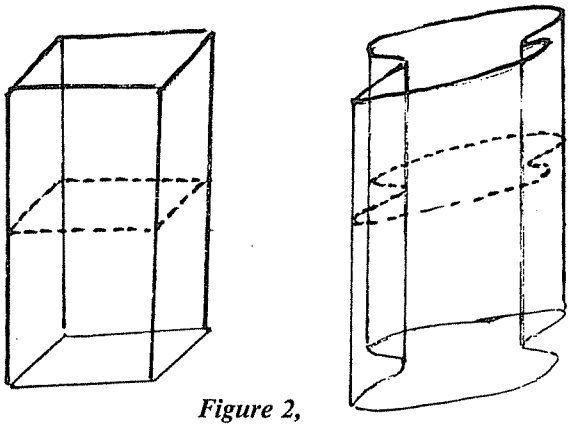


Figure 2,

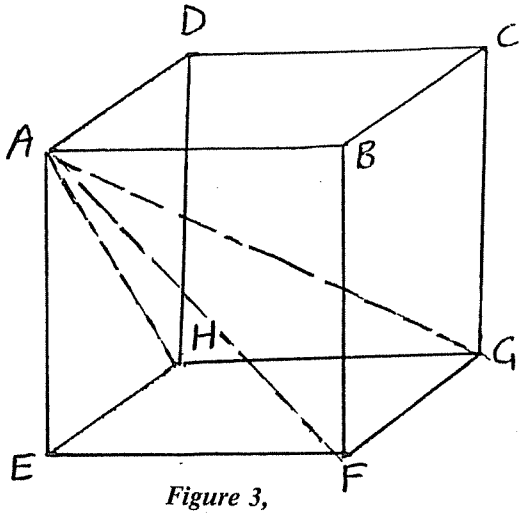


Figure 3,

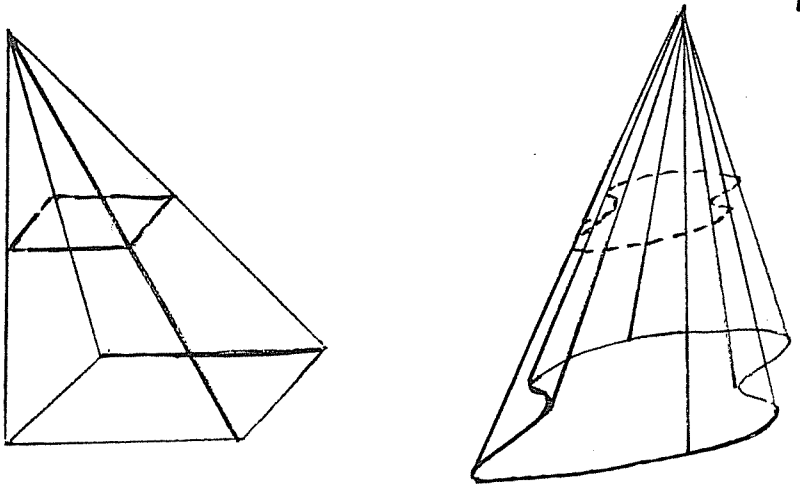


Figure 4,

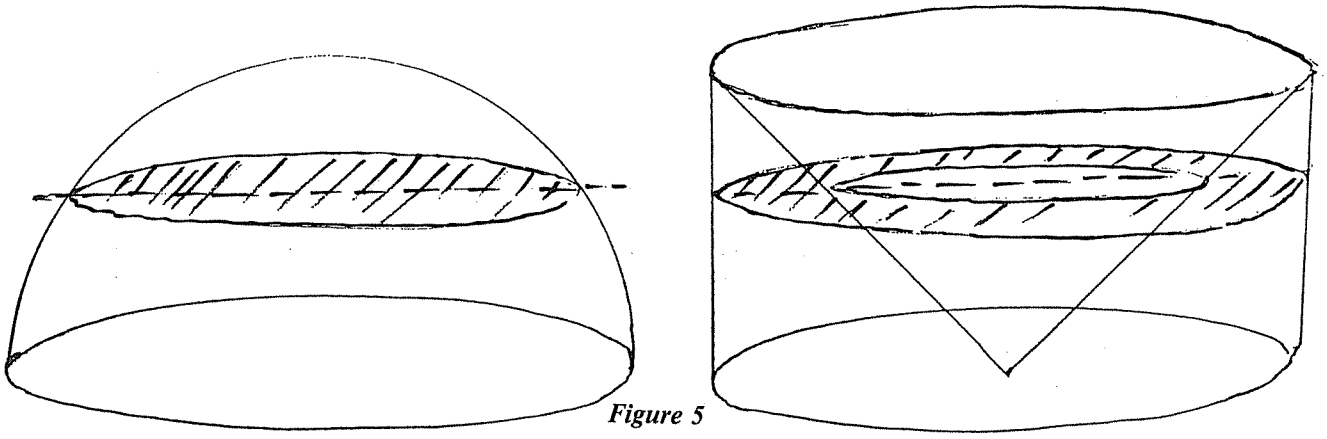


Figure 5

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME in conjunction with the Grand Valley Mathematics Association established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

In 1984-85, the inaugural year, the award was presented to two schools:
La Salle Secondary School (Sudbury)
O'Neill Collegiate and Vocational Institute (Oshawa)
There will be one such award presented in each subsequent year. In 1985-86, it was awarded to Overlea Secondary School (East York) and in 1986-87 Glebe Collegiate (Ottawa) was the recipient.

At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in Niagara Falls.

Please complete a copy of the form below and forward it to:

Mr. M.B. MacGregor
Sec./Treas. OAME
247 Bright Street
Sarnia, Ontario
N7T 4E9

Deadline for nomination submission: April 30, 1988

Kenneth D. Fryer Award
Nomination Form
1987-1988

I/We nominate School: _____

Address: _____

Principal: _____ Math Dept. Head: _____

Nominated by: _____

Please attach a separate sheet to the nomination form outlining the reasons for your nomination.

O A M E

Secretary - Treasurer

The Ontario Association for Mathematics Education requires a Secretary-Treasurer to work with the President and Executive "to promote excellence in Mathematics education in the Province of Ontario".

RESPONSIBILITIES INCLUDE:

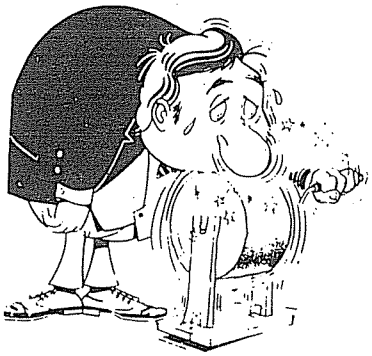
- Preparation for and attendance at four Executive meetings, three Council meetings, Leadership Conference and Annual Meeting.
- Attend at least one meeting of each Chapter annually
- Monitoring publication timelines
- Maintenance of updated membership list
- Mailing of Organization's material (minutes, publications, notices, ballots, etc.)
- Supervision and organization of Annual Elections
- Communications directed by Executive and Council
- Manage finances
- Preparation of regular financial statements

The successful candidate should possess the following skills:

- Good public relations
- Keyboarding
- Organizational/Clerical
- Accounting

This position is renewable annually, pending ratification at the Annual meeting. The position commences in May, 1989. Current honorarium is $2,400.00. CANDIDATES SHOULD APPLY IN WRITING STATING QUALIFICATIONS INCLUDING REFERENCES PRIOR TO APRIL 30, 1988 TO:

DAVE DAVIDSON, CHAIRMAN, SEARCH COMMITTEE,
P.O. BOX 385,
COLLINGWOOD, ONTARIO L9Y 3Z7



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: Your comments are welcome and should be addressed to the Editor. These comments are boxed on the Council Meeting held at the Ramada Inn - Airport West on Saturday, 1988 01 14.

OMCA POSITION PAPER

Alex Norrie, the liaison from the Ontario Mathematics Co-Ordinator's Association to the OAME Council, presented their latest position paper entitled "Mathematics Education For Elementary School Teachers: A Revolution For Ontario". The inadequacy of the teacher education system pertaining to mathematics was outlined. The following three points were discussed:

- the need to revitalize preservice training
- strong recommendations for inservice
- availability of post graduate work in mathematics education.

For more information and a copy of the recommendations contact Alex Norrie at the Peel Board of Education (see Primary/Junior Committee Report).

ANNUAL MEETINGS

1988 Annual (Niagara Falls)

The program is complete and has been distributed. Registration is at the Ramada Inn and at the time of writing, pre-registration was over 200. You are encouraged to register early to avoid disappointment.

1989 Annual (Ottawa)

It will be held at Ottawa University with the dinner-dance at the Westin hotel. A good French program will be offered and the keynote speaker will be Stephen Lewis.

1990 Annual (Hamilton)

The "annual" will be an NCTM regional meeting which will be held at the Sheraton-Hamilton and the Hamilton Convention centre. Overall suggestions may be sent to the General "Chair", Bob Robinson and program suggestions can be sent to either of the program "Chairs", Alex Norrie or Ralph Connelly.

LEADERSHIP SEMINAR

1987 Leadership Seminar

Dino Dottori reported that the leadership seminar was highly successful. The theme of "The Future" was most appropriate as over 70 people who attended will attest. Inquiries have already been received concerning the 1988 seminar.

1988 Leadership Seminar

The '88 seminar will be held at the Ramada Inn - Airport West on 1988 10 19-21. For more information contact the Sarnia office.

CHAPTERS

COMA/AMOC (Carleton - Ottawa Mathematics Association/Association Mathematique d'Ottawa-Carleton)

Permission has been received to name a memorial trophy to honor the late Don Fox. The Don Fox Memorial Trophy will be presented annually to the winning team from Ottawa-Carleton in the Descartes Contest given to senior mathematics students.

A mini conference was held on 1988 01 18 with a fine dinner provided. It was well attended.

The very popular publisher's display will take place on 1988 04 12.

Due to the planning for the '88 Annual there will be no leadership conference held in Ottawa next fall.

GOLDEN SECTION (Niagara Peninsula including Hamilton)
Their efforts are being directed toward the '88 annual in Niagara Falls.

G.V.M.A. (Grand Valley Mathematics Association)
A spring conference is planned for 1988 04 14 at Eastwood Collegiate beginning at 4:00 p.m. The keynote speaker will be Doug Henderson who will discuss Grade 12 Geometry. Don Fraser will be the featured dinner speaker.

The Executive of GVMA is discussing the impending shortage of mathematics teachers as we approach the 1990's.

NOMA (Northern Ontario Mathematics Association)
The NOMA Conference for 1988 will be held at LaSalle Secondary School in Sudbury on Friday, 1988 09 30 and Saturday, 1988 10 01. They hope to attract even more teachers this year, particularly from outside the Sudbury region.

NOMA intends to give financial support to the Mathematics Olympiad that is sponsored by the Sudbury Board of Education for their grades 7 and 8 students.

NOMA certificates will be distributed to all schools in the Northeast. These certificates, available in English and French) are awarded to students from grades 1 to 13 for outstanding achievement in mathematics.

As an innovative gesture NOMA distributed "I MATH" buttons at the fall conference. They were extremely popular and more will be ordered.

Many "Northerners" are planning to make the trip south to Niagara Falls in May to OAME '88.

PAME (Peel Association for Mathematics Education
They ran a successful conference last fall when they had Claire Zeller as the major speaker.

PAME conducted a mathematics contest in which 700 students took part. Businesses were canvassed to provide money for the trophies. They hope to build up more interest in this contest and to this end will run a coaches clinic.

A "Marvellous Math Day" is being planned during Education Week and on 1988 04 21 a special Publishers Display will be held.

PINE RIDGE MATH ASSOCIATION (Oshawa/Peterborough Area)
A very successful P.A. Day occurred on 1987 11 03. Father Stan Buzuszka of Boston College gave a keynote address in the morning and a very amusing Mathematics talk in the afternoon. About 200 teachers attended.

They are preparing for their annual mini conference at Vanier High School in Oshawa on 1988 04 13. This dinner meeting will include at least 9 workshops.

PRMA is again co-sponsoring a Mathematics Award night in June '88 to honour top students in Mathematics.

Linda Bain from O'Neill C.V.I. and her committee have prepared and distributed a booklet for preparation of Mathematics contests to participating schools.

QSLMA (Quinte-St. Lawrence Mathematics Association)
A General Meeting will be held in Kingston on Friday, 1988 03 25 and Saturday, 1988 03 26. Registration will begin at 12:30 p.m. A wine and cheese/publisher display will be from 4:00-6:00 p.m. with a dinner at 6:30 p.m. Computer workshops will operate from 9:00 a.m.-11:30 a.m. on Saturday. Accommodations at very reasonable rates are available. Contact Jim Whitley (Chapter representative) for more information.

SAME (Scarborough Association For Mathematics Education)
The Seventh Annual Mathematics Mini-Conference took place at Maplewood High School on Thursday, 1988 01 12 from 3:45 p.m. to 8:00 p.m. The theme was Geometry. Two each of Primary, Junior, Intermediate, and Senior workshops were offered before dinner with seven different workshops following dinner. They had 150 teachers attend this worthwhile inservice.

On Wednesday, 1988 02 10, the Scarborough Secondary Teachers held a Professional Development Day conference at the Sheraton Parkway Hotel entitled "Future Directions In Mathematics". It was co-ordinated by the

Program Department of the Scarborough Board and SAME. TEN sessions were presented with the plenary address by Don Fraser.

A "Market Place" to share ideas with other Elementary teachers is planned for 1988 03 29 at the Manoogian Centre.

"Math Camps" will be run by Paul Zolis once again this year. Contact Paul Zolis at the Scarborough Board for further information.

SWOAME (South-western Ontario Association for Mathematics Education)
This new chapter is "on its way". Their membership drive has started with about 50 members at the time of writing.

YRMA (York Region Mathematics Association)
This relatively new chapter held its first evening session last fall. Over 95 teachers attended with Brendan Kelly the major speaker. They are looking toward a late September or early October date for their next Mathematical activity.

NWOAME (North-western Ontario Association for Mathematics Education)
This is our newest chapter and they are conducting a membership drive at this time.

COMMITTEE REPORTS

Publications Committee

Due to the sudden passing of Cliff Gravelle our **Gazette** editor, Bob Smith, University of Western Ontario (Faculty of Education) has been appointed the interim editor for a term of one year. Thanks go to Bob for stepping in and providing this editorial leadership. We also thank Ralph Connely for assisting and providing a rather smooth transition during this difficult time. Cliff will be missed by us all. It is so unfortunate that he spent such a short time with us but during that time Cliff added many new and innovative features to our major publications.

Intermediate and Senior Committee

Mickey Sandblom is our representative on the OAIP Intermediate Committee.

The committee will research the effect that an open Grade 9 and Grade 10 mathematics course may have on our students.

The OAIP Senior document is in draft form and appears to reflect a more traditional form of evaluation.

The committee will seek more information and discuss the issue of the non mathematics specialist teaching mathematics in the Intermediate and Senior divisions.

There is some concern that the failure rate may increase for the General Level student because of the requirements of OSIS.

Primary and Junior Committee

Proposals have been received for the "Problem Solving Project" which is being co-ordinated by Chester Carlow. Information will be forthcoming as the project progresses.

The committee endorsed and gave full support to the OMCA positive paper on "Mathematics Education For Elementary School Teachers". The council has asked the President to notify the Minister and his Deputy of our position as well as informing the teachers' federations. Concern was expressed about Queco qualifications. This will be pursued.

The committee expressed their concern that there is too much of a delay in support documents coming out in French. A letter will be sent to Ministry expressing our feelings.

Women and Mathematics

Watch for the special program stand at the Annual '88 in Niagara Falls.

Futures

A "Special Projects Committee" has been formed which consists of the Secretary-treasurer(s), Past President, President and the "chair" of the Futures Committee.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
1987-88 EXECUTIVE - COUNCILLORS

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Pres. Elect - J. Symington, 50 Copeland Rd., Brampton L6Y 2S7 (H) 416-454-4221 (B) 416-884-4802

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