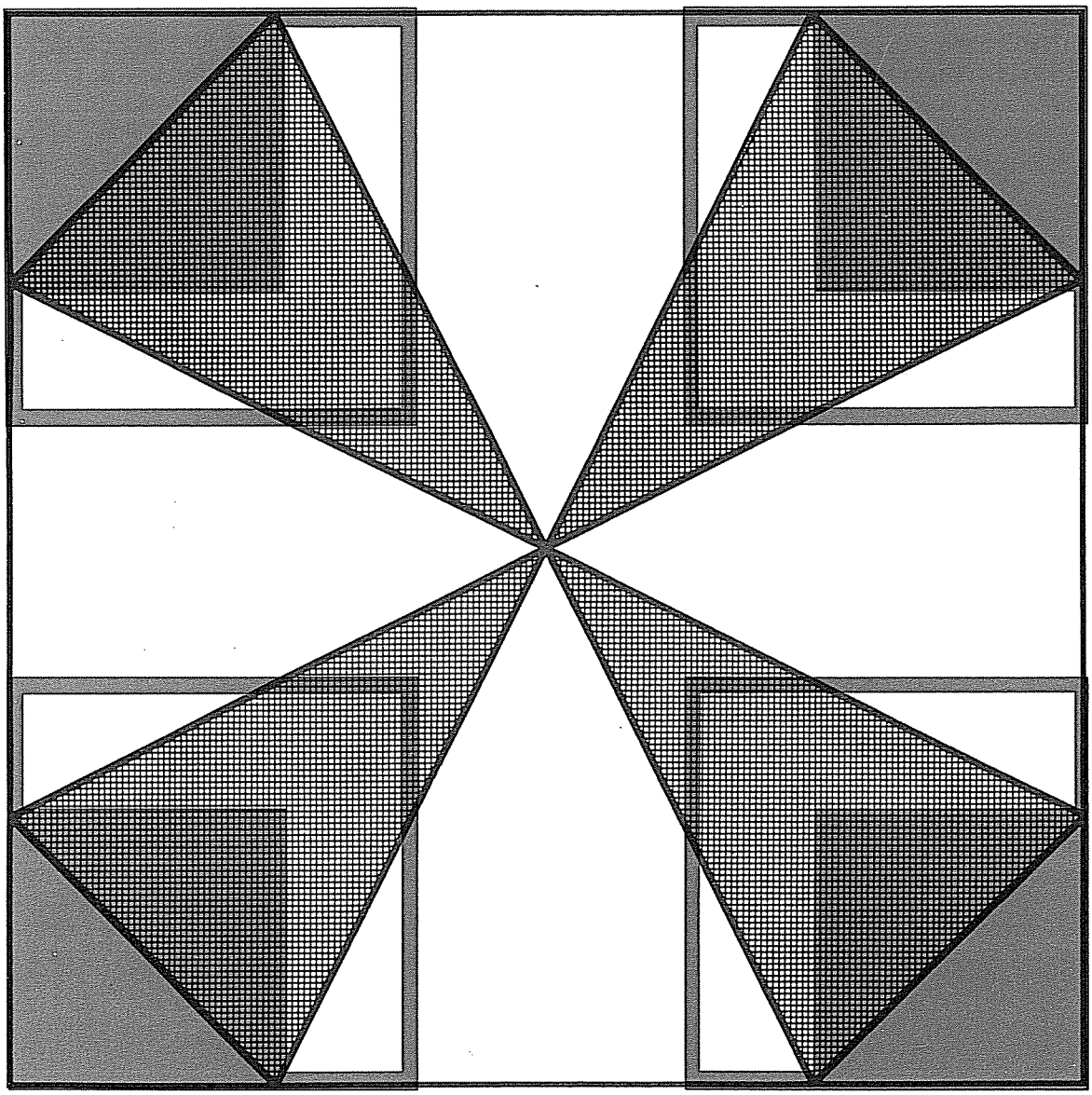
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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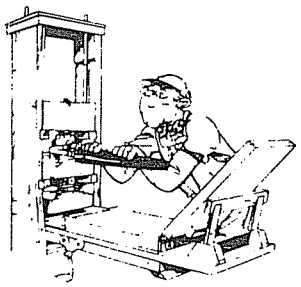
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For some time now, I have been aware that many parents, and in particular, grandparents, have a view about mathematics and the way it is taught that is in conflict with my view. I believe their view is outdated. If I am wrong in this editorial, I do hope you will take pen in hand and inform me.

For many parents and grandparents, mathematics is still abstract solutions of equations, geometric proofs, and text book problems, and/or memorization of tables and pages of calculations. But, we know that in the work-world, calculations are now routinely performed with mental arithmetic or calculator. It is becoming very rare in industry for multiplication, division or percentage calculations to be carried out using written methods. Therefore, skills development and the practice of algorithms, so much a part of the past, are making way for the more important process of conceptual understanding. Today's mathematics class spends far less time on pencil-and-paper arithmetic. Now complicated calculations are usually done on a calculator while computers are most appropriate for repetitive calculations. Increasingly, emphasis must be placed on the reasonableness of results. Computational skills become a part of the concepts of our decimmal system as used everyday in real situations. Along with the building of concepts about our number system goes the ability to read and comprehend, to write and speak the language of mathematics. The basic skills of oracy and literacy become every bit as important in the mathematics classroom as in the English classroom.

Unfortunately, two things stand out in my mind about numeracy. It is clear that mathematics teaching in the past produced a small percentage of competent mathematics students and a large group of students who did not complete school math successfully; or if they did, felt unsure about applying the formal math in the work place. The view of math for many parents, even today, is characterized by failure, loss of self-esteem and perhaps, even lost opportunity. The NCTM document of 1980 An Agenda for Action and the 1982 Cockcroft Report both made it clear change was needed and set out suggestions. Parents may not yet be aware of the math practices suggested and that are evolving. Let me try to describe some of the changes:

① Primary teachers, who teach children both language and mathematics, are now transferring the understandings gained from language to their mathematics teaching. Such things as experiential hooks, the productive value of talk in the context of co-operative group activity, the risk-taking involved in moving from ordinary language to mathematical symbolism are being explored at all grade levels.

The front page of the newspaper and magazines that I read all remind me that it is important that students be able to read, interpret and construct simple tables and graphs. There is no doubt that we have moved into a new era of communications. News reporting combines charts, maps, graphs; and some items reported consist solely of computer generated news graphics. Children are using graphs and charts, maps and tables to report their findings in both English and Mathematics classes.

② With calculators and computers in every school, more and more pupils are learning to inspect the results of their work for reasonableness. They develop simple techniques for both estimating quantities and approximating calculations. In fact, in England, teachers have been working on a CAN (calculator aware number) curriculum for the last several years. Children are encouraged to use the calculator whenever appropriate, along with other apparatus, to develop their understanding of number. Teachers report that there is a greater emphasis on discussion, mental calculation, individual methods of recording and investigative number exploration. "The calculator has become a useful tool."

③ Elementary teachers have moved forcefully into the teaching of measurement using S.I. units and into the world of three-dimensional geometry. No longer are perimeter and area of rectangles and circles the extent of the work. Visualizing transformations,

constructing and drawing models, applying ratio and proportion and determining properties of geometric figures are all included in their concepts of geometry.

④ Problem solving has become more of a focus in the 1980s. It can be separated into abstract problem solving (so familiar to all of us) and real-life problem solving. The latter can lead into investigations of a variety of social issues. Historically, these real world events have been the driving force of mathematics. Math helps us to "abstract" out the quantitative features. It has become, even more so, a key goal for all teachers.

The mathematics curriculum is changing. So too are the methods used in teaching. Because of a belief that learning is active not passive, that we must build on what students know about math and not on what we know and that students need to feel good about themselves in order to be able to talk and write about math successfully, teachers are changing their methods and ways of presenting mathematics in the classroom.

The scenario I have presented is quite a change from the views I often hear expressed by parents and grandparents!

NOTICE OF MOTION FOR THE ANNUAL GENERAL MEETING

Ottawa, Friday, May 12, 1989.

Whereas many of our past presidents have observed that the present one year term only gives them enough time to "learn the ropes" and move on

and whereas most of the other Canadian and U.S.mathematics associations have two-year terms for the office of President

and whereas a two-year term would give the President more of an opportunity to initiate projects and ideas, and see them to fruition;

thereby be it resolved that:

(i) "The term for the office of the President of OAME/AOEM be two years effective as of 1990-91 elections."

and further be it resolved that:

(ii) "Contingent upon the adoption of the motion (i) above, the number of directors be increased from three to four effective as of 1991-92 elections."

* * * * * * * *

For your information, the chart below shows how the changes would be implemented.

POSITION	89/90	90/91	91/92	92/93	93/94
Past Pres.	JS*	MLK	(A)	-	(B)
Pres.	MLK**	(A)	(B)	(B)	(C)
Pres. Elect	(A)	(B)	-	(C)	-
Directors	3	3	4	4	4

* J. Symington
** Mary Lou Kestell

PRESIDENT'S MESSAGE

When I assumed the position of President of O.A.M.E. I recognized and have continued to recognize that education in Ontario is facing change. With the number of new teachers entering Ontario's educational system, change will naturally take place; a change in attitude, a change in environment, a change in professionalism. Technological advancements as well as a renewal and implementation of sound pedagogy will also affect change. As leaders in mathematic education (I believe every member of this organization has leadership potential) we must plan for and plan to accommodate change. Change is, and will continue to be, a driving force behind the role O.A.M.E. plays in encouraging and promoting excellence in mathematics education. As members of O.A.M.E. we must ensure that change remain a positive move, both for us as an organization, and for mathematics education in Ontario.

Continual change is natural and an essential characteristic of mathematics education. Managing change will continue to be a significant part of the role of O.A.M.E.'s president. May those that follow have the continued commitment and support from their directors, councillors and colleagues. Support and work from people like Ralph Connelly and Mary Lou Kestell is certainly appreciated.

O.A.M.E. is changing. More and more educators are recognizing the fact that this organization can and does play an important role in helping teachers teach. Eleven chapters and 1820 educators are testimony to this fact. The word is spreading! O.A.M.E. began with 550 members. In 1983 there were approximately 1200 members. This change in membership reflects not only a significant increase in numbers but also a shift from mostly secondary school teachers to a membership with an ever increasing elementary teacher participation. Organizations like O.A.M.E. need new ways to renew and revitalize themselves. Let us continue to promote the value of this organization and set our sights on a membership of 1990 in 1990.

Our annual conference, to be held in Ottawa this year, is fast approaching. It is offering an excellent program. I know that many people in Ottawa are working diligently toward offering a well organized conference. For mathematics educators in Ontario this is *the* conference to attend, "Be there or be square!"

Finally, in the years I have served this organization I had the great fortune to meet many educators and form many friendships. There are two people without whose hard work and commitment this organization wouldn't be where it is today. These two people have always been with O.A.M.E. and, in a way, will always be a part of O.A.M.E. On May 13, 1973 the Ontario Association for Mathematics Education was formed and Morley and Mona MacGregor were there. The MacGregors have worked with this organization through the difficult initial years, always believing that O.A.M.E. could become a valuable organization, contributing to mathematics education in Ontario. A look at our organization today proves that their hard work paid off.

At the annual general meeting in Ottawa this year Morley and Mona MacGregor will step down as Secretary-Treasurers. I have valued their assistance and friendship throughout the years, especially this last year as President.
I'm sure those of you who know Morley and Mona would not hesitate to call them friends. On behalf of O.A.M.E. and all those whose lives you have touched I wish you all the best of health and happiness in the "retirement" years to come.

J. Symington

Morley and Mona MacGregor will be honoured at both the Friday night banquet and the reception following the address by Stephen Lewis.

The Membership and Publicity Committee has a deal for you.

I ♥ Math buttons promote the appreciation of mathematics among students and teachers. They are available in French and English at 40¢ each or $18.00 for 50.

Order from Michael Sandblom, 19 Lakeview, Box 221, Onaping P0M 2R0

Make cheque payable to O.A.M.E.

Feb. 11/89

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Thunder Bay, ON
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(807)-623-1710

We in North Western Ontario are very pleased to announce the formation of the newest chapter of OAME, namely NWOAME.

Our first AGM was held in Thunder Bay on Dec. 3/88 at Lakehead University. Fifty persons attended, representing both Thunder Bay and areas to the East and West.

An Executive was elected and a membership drive is underway.

We invite all persons in NWO who are involved in Mathematics Education at any level to join NWOAME/OAME and get involved! We also welcome suggestions and advice from other persons who have successfully launched their chapters of OAME.

We look forward to hearing from you.

Marilyn Hurrell
Chairperson, NWOAME

Jan./89

526 Oak Park Drive
London, ON
N6H 3N7

Dear Editor:

The Gazette and Abacus provide an excellent source of ideas to enhance the lessons of full or part-time math teachers.

The Teaching Tips "cards" (such as found on p.13, Dec./88 issue) will be a great help in building up handy files. I hope many of these, geared to all different levels will be included in future issues.

Keep up the good work.

Brock Rachar
former Math Coordinator
London Board of Education



Dec. 9/88

Faculty of Education
The University of Western Ontario

Dear Bob:

This letter is in response to your editorial in the last issue of the Gazette where the question is asked "Is it time to monitor how well we are meeting the objectives stated in our constitution?" O.A.M.E.'s many accomplishments have been both very useful and important - it should continue to do these things.

The question I pose is "How have the policies and activities of O.A.M.E. contributed to a significant change in the mathematics classrooms of Ontario?" My sense is that very little has changed in the last twenty-five years at the Secondary level and that it is time for some bold, new initiatives, some teacher irreverence, and some candid talk.

The mathematics teachers in this province are a very talented group and I would hope that O.A.M.E. can find ways to channel these abilities into obtaining a more effective learning environment for our students.

Yours truly,
Ron Reesor

Jan. 15/89

985 Thistledown Way
London, ON

The John McCartney Mathematics Scholarship

This award honours John G. McCartney, a teacher in the Kingston and Stoney Creek areas, who went on to become an editor of educational programs with Science Research Associates (Canada) Limited. By now Secondary Schools throughout the province will have received the application forms and a new winner will be chosen by late June. The first award was made in 1979.

This year's winner of the McCartney award is Patrick Surry of A.B. Lucas Secondary School in London.

Patrick's achievements over the last few years have been numerous and impressive. Notable in his last year of high school are his results in the regional and national science fairs, in which he won the gold medal for the Canada wide contest with a study on the design of bob-sleds.

In the Descartes Mathematics competitions he achieved a remarkable 100%, his last year's unofficial result was 99%. He was co-winner with Ravi Vakil of the Carlton University mathematics competition. He also achieved commendable results in the Canadian National Mathematics League and the Canadian Olympiad.

For these results Patrick was selected to represent Canada in the International Mathematics Contest in Australia, where he distinguished himself with a silver medal, and assisted Canada to come 10th out of the 50 countries entered. This is the best result we have achieved since 1981, the first year Canada competed.

Since returning from Australia, Patrick and Ravi have become co-authors of a new publications, Mathematical Mayhem, written especially for high school students interested in problem solving. This is a most welcome addition to the literature, since the very highly esteemed Crux Mathematicorum is not easily usable in high schools in its present form.

It is a very great pleasure to be able to report that this scholarship has been awarded to such a worthy and pleasant recipient, and especially since he has been a student of mine for the last five years.

T.J. Griffiths

TV ONTARIO'S SCIENCE EDITION SEASON PREMIER

is a half hour video program that deals with the limiting of career choices by young women due to their subject selections.

It offers positive role models and encouragement to young women to select mathematics and science courses so that their career choices are not affected.

This exceellent program is available to educational organizations for preview, for rent (at $13.00) or for purchase (at $29.00).

Phone TV Ontario at 1-800-387-8450 *and ask for Client Services. Identify the program as Science Edition, Program* 1, #BPN 325001.

YOU ARE INVITED

THE WOMEN AND MATHEMATICS BREAKFAST

OAME 89, OTTAWA

Saturday, May 13, 1989
7:30 - 8:30 a.m.

GUEST SPEAKER

Heather Jane Robertson
Director, Professional Development Services
Canadian Teachers' Federation

LOOK FOR DETAILS IN YOUR REGISTRATION PACKAGE

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Jan Cornwall	(416) 396-5614
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Viewpoint

UNDERSTANDING THE PROBLEM - A PROBLEM IN ITSELF?

by
Colin Garnham
Don Mills, Ontario

By now we all know of, and have read, the NCTM's Recommendations for School Mathematics for the 1980's on problem solving. The most popular model in use for problem solving is George Polya's four stage model. (Note the use of the word stage, and not step, as the term step implies that they are not related and the four stages of Mr. Polya are indeed related.) The four stages are:

1. Understanding the Problem
2. Developing a Plan
3. Carrying out the Plan
4. Looking Back

But, how does one Understand the Problem? Does the heading itself not suggest that if the problem is already understood then there is no need to develop and carry out the plan, but simply to apply an algorithm to the situation? Who is to Understand the Problem? Is it the problem solver, or the person reading the solution from the problem solver?

You may have read the first paragraph of the article, and the questions associated with it, and already decided that this is some *crack pot* who is getting hung up on a phrase, and is not looking at the big picture of problem solving. Understanding the Problem should be just that - UNDERSTANDING THE PROBLEM. The Understanding the Problem stage is outlined in the next paragraph, and an example is given to show how the understanding the problem stage would be used. Note that the strategies presented are common to most problem solving models based on the four stage model of George Polya.

It has been generally established that when attempting to understand a problem, you are performing some or all of the following strategies:

- write what is given
- write what is required to be proven and/or solved
- clarify the meaning of the terms and phrases that are not clear
- write the irrelevant information
- determine any and all hidden assumptions
- determine whether there is sufficient information to solve the problem
- draw a diagram or picture to simulate the problem situation

This is a lot of work to do when simply trying to understand the problem. It is essential for the educator to remember that, when you or one of your students run into a problem situation, it is probably a question that cannot be immediately answered. Whether it be a routine or non-routine type question, the algorithm or process used to solve the problem is not immediately evident to the problem solver. As a result, the problem solver has turned to the model in the hope of finding a strategy to solve the problem.

Take, for example, the following problem. It is a very traditional non-routine problem, which may be a contradiction in terms, that the majority of students will be asked to solve sometime over the course of a high school career.

> An owner hired three watchmen to guard his orchard, but a thief got in and stole some of the apples. On the way out, the thief met each watchman individually. Each watchman demanded half of the apples that the thief had plus two more in order to keep silent. The thief escaped with only one apple. How many apples did the thief originally steal?

For teachers who have taught the process of working backwards towards a solution, this is

Editor's Note: Colin Garnham is project manager, Mathematics with Nelson Canada.

not a difficult question. The student will start with one apple, add two more and double the result, add two and double the result and then once again add two and double the result to find that the thief had originally stolen 36 apples. A fairly straight forward solution if you recognize the process of solution. However, if you do not, then one of the methods most often taught to help find the solution is the four stage process. Let's put ourselves in the students shoes now and attempt to *Understand the Problem* using the points that have been listed previously as strategies to be carried out and answered.

Understanding the Problem

- Write what is given.

- There are three watchmen hired to guard an orchard.
- A thief got into the orchard and stole a quantity of apples.
- While leaving the thief met each watchman, one at a time.
- He gave each watchman one half of the apples plus two more.
- The thief escaped with one apple.

- Write what is required to be found.

- You are required to find the number of apples with which the thief had originally tried to escape.

So far so good. The student has attempted to summarize the information in some kind of point form and has also identified what is to be found. However, these are only the first two strategies in a list of strategies that is encouraged to be done in order to understand the problem. Let us continue.

- Classify the meaning of unknown words and phrases.

  Unless the student has been living an extremely sheltered life, there should be no phrases and terms that are unknown in this question and the student will be encouraged to ignore this strategy only for this particular question. It is an important part of the understanding the problem stage that all terms and phrases be clearly understood. Do not let your students skip across this section without any thought whatsoever.

- Write all irrelevant information.



Here is where the student will run into a serious problem. What is irrelevant in this question? The student has no way of answering this question until the entire problem has been solved, and by then, it is an unnecessary question to answer. The student will proceed, after the understanding the problem section, and attempt to develop a plan to solve the problem. Once the plan has been identified, then they may be able to identify and write all the irrelevant information. But this is only the case if the process of solution is successful in solving the problem. If the plan is not successful, then much of the information that may have been listed as irrelevant, may suddenly be relevant again. For example, the student who lists the number of watchman as being irrelevant will probably get the wrong answer. Of course, this student may follow the process of *Lucky Guess* or *Thy Neighbour's Answer* as the process of solution and get the correct answer, but these are rare exceptions. The only way for the student to solve the question of irrelevancy is to have the final answer, and by then, the question is redundant.

- Determine any and all hidden assumptions. Finding hidden assumptions is another question that is impossible for the student to answer until the question has been solved. How many times have you heard one of your students say "Ah, I get it now." or "The answer is so obvious that I should have gotten it sooner." Finding hidden assumptions would require that the student look up the meaning of every word in the question and account for all possibilities. Given the reality of the world around us, and especially the classroom, this is impossible. All hidden assumptions in a question will be understood by the student intuitively and will not be found by having the student look at a particular problem in any great detail.

Checking for hidden assumptions is a problem solving process on its own. It is not something that will make the understanding of the problem any easier. In fact, the teaching of the problem solving process of Making an Assumption to Draw Conclusions will be difficult for all teachers and students and will have to be done carefully. However, that is a different argument and article entirely.

- Determine whether there is enough information to solve the problem.

  How is it possible to determine whether there is enough information to solve a problem without actually solving the problem? When understanding the problem, you are to summarize information in order that a plan for solving the problem can be developed. When working in the *Understanding the Problem* stage, the plan has not been developed, and more often than not, hasn't been thought of as yet. The student must have attempted a solution to the problem before ever answering this question. Once the solution has been obtained, the answer to this question is probably "yes". Once again however, as soon as the solution has been found, the question is unnecessary. The question should only be asked, by you, if the majority of students are having a difficult time solving the problem.

Apart from the argument used above, is it our intention to give the student an excuse for not trying to solve the assigned homework problems? This is the easiest way for the student to say, and rightfully so, that their homework is not completed because the problems assigned did not contain enough information to solve the problem.

- Draw a diagram to simulate the problem situation.

  In the example of the watchmen and the thief, a diagram will probably not help you to solve the problem. However, diagrams are an excellent way of summarizing the given information. They are used continually throughout a mathematical career, especially in the field of geometry, and will be useful well into the students' future endeavors. It is essential that you encourage the use of diagrams when understanding the problem.

Of the initial *Understanding the Problem* stage, and the strategies associated with it, there are not many strategies that will help the students when attempting to solve a problem. In fact, you should be encouraging your students to carry out only the following four strategies when working in the understanding the problem stage.

- write all the given information
- write all the information needed to be proven or solved
- clarify the meaning of all unknown words and phrases
- if possible, summarize all the given information on a diagram

The *Understanding the Problem* stage may be the most important stage of the entire problem solving model. If the problem cannot be summarized successfully, a strategy cannot be found to solve the problem. However, it should only be used as an initial stage where the student is encouraged to summarize the problem and not answer various questions that cannot be answered at this stage of the problem. All the hidden assumptions that are presented in many of the various models will only tend to reinforce many of the anxieties that students feel when they are solving problems. By having the students concentrate on summarizing information in a clear and concise form, they will achieve more success than answering the types of questions, and carrying out the types of strategies that were previously outlined in the *Understanding the Problem* stage.

AN INSPIRATION TO LEARN AND TEACH MATHEMATICS

by
Dipendra N. Bhattacharya
Clarion University of Pennsylvania

This article is based on a talk "A Journey to Truth, Goodness and Beauty in Mathematics (How to Motivate the Unmotivated)", given at the 66th Annual NCTM Meeting in Chicago.

I will begin by stating what must be understood above all: The learning and teaching of mathematics is a journey, not a destination; and, motivation is the key to success.

A long time ago, my father told me that anything worthy of study must have three qualities. They are Satyam, Sivam, and Sundaram which in English means Truth, Goodness and Beauty.

Let us examine mathematics, and determine if it renders any of these qualities...Is math based on truth? I am sure, that your students will not argue with that. Mathematics has survived five thousand years of human history; surely, to do so it must be based on truth.

Does math contain any goodness? Is math good for us? Well, I am a brown man making a living in a foreign country because of it -- it has been good to me!

So, we have determined that math is based on truth, and it is good...what about beauty? Is math beautiful? At this point, most students will say, "Please give us a break! We will agree with the first two, but there is no way you can convince us that math is beautiful!" Well, with the assistance of a few examples, I will show you the beauty in mathematics...

EXAMPLE #1 One day when I was teaching high school in Canada, I was asked to substitute for the Chemistry teacher. I do not know much about Chemistry, but I thought I might be able to teach them something anyway. I asked them what the topic of the day was, and they replied "Catalytic Agents". I was fairly sure that a catalytic agent is a chemical element or compound which takes part in a chemical process, but remains unaltered after the reaction has taken place. I checked with one of their books, and then told them I was going to tell them a story to illustrate the term. A long time ago, a very wealthy king left a will stating that horses shall be divided as such:

1/2 shall go to his first son
1/3 shall go to his second son
1/9 shall go to his third son

When he died, his three sons found 17 horses in the stable. The sons, not being very bright, concluded that the only fair way to do as the will wished was to cut the seventeenth horse up with a saw! Just as they were about to begin sawing, an old (and very wise) minister happened by and was shocked by their intended actions. "What ever are you doing?," he exclaimed, shocked by their stupidity. When the three dim-witted princes explained that their only alternative was to cut up the poor horse, the minister made a very wise suggestion..."since everything I own is truly the property of my master, add my horse to the seventeen. Please allow me to help you divide them, and thus fulfil my master's last wish."

The brothers agreed to the wise man's suggestions, and the horses were divided in this manner...

17 + 1 = 18, so...
1/2 of 18 = 9, which were given to the oldest son
1/3 of 18 = 6, which he gave to the second son
1/9 of 18 = 2, which the third son took

Total = 17

So, the last son took his 2 horses, and the old minister rode off on the same horse he started out on, proving that a horse can make a very good catalytic agent!

EXAMPLE #2 Ronald Regan was born in 1911, and he is 77 years old. He took office as the President of the United States of America in 1980, and has been in office for 8 years. If a person were to add up these numbers (1911 + 77 + 1980 + 8), they would add up to 3,976.

Mikhail Gorbechev was born in 1931, and he is 57 years old. He became the Boss of the Soviet Union in 1985, and has been in charge for 3 years. If a person were to add these numbers (1931 + 57 + 1985 + 3), the answer will again be 3,976!

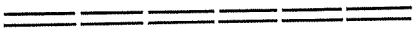
Is this some kind of eerie coincidence?

Are you mystified, or did you figure out that this strange coincidence will even work for you?

The reason, of course, is that anyone's year of birth and present age will add up to 1988! Similarly, the other two dates will also always add up to 1988! And, 1988 + 1988 = 3,976!

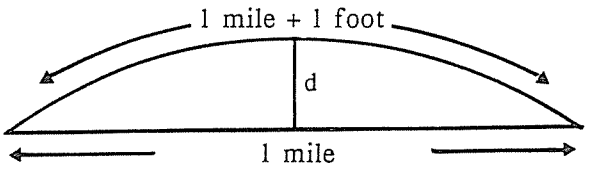
This is an example of a fixation, which is one of the biggest difficulties a person faces when trying to learn mathematics. The following examples are of fixations. You will like them...satisfaction guaranteed!

EXAMPLE #3 There are gaps between railway ties, so that they look like this:



This is done, so that they do not bend when the tracks expand due to the rise in temperature. Suppose we do not leave these gaps. Consider the following situation:

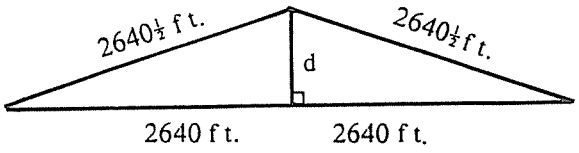
The line is one mile long, and has been expanded one foot, so that the new length is 1 mile + 1 foot. As a result of this expansion, the tracks buckle in the center, causing a bulge. Consider:



How high is bulge d = ?

Please do not calculate, just guess. Most people guessed that d is less than 4 inches. Nobody guessed more than 10 inches. Then we used the Pythagorean Theorem to find d.

1 mile = 5280 feet



$$d^2 + (2640)^2 = (2640\tfrac{1}{2})^2$$

$$d = \sqrt{(2640\tfrac{1}{2})^2 - (2640)^2} > 51 \text{ feet}$$

Being a fixation, no one guessed that d could be that big.

EXAMPLE #4 A class has 99 girls and 1 boy, how many girls must leave the class, so that the percentage of girls becomes 98%?

The answer is 50, because 99 - 50 = 49, and 49 girls out of 50 is 98%.

EXAMPLE #5 A pole is 30 feet high. A monkey climbs the pole 10 feet during the day (6 a.m. to 6 p.m.), working nonstop and comes down 8 feet during the night. How many days will it take the monkey to reach the top of the pole?

Did you notice that the answer is not 15, but 11 (or 10 & 1/2) days? Note that after 10 days, the monkey will have climbed 20 feet. So, what happens after day 10?

EXAMPLE #6 A trucker drives 20 mph for the first half of his trip from New York to Philadelphia. At what speed should he come back so that the average speed (total distance/total time), for the entire trip is 40 mph? Most people guess 60 mph. However, he will never make it! Can you figure out why?

EXAMPLE #7 Find the pattern (*) :

3 * 5 = 4
6 * 2 = 4
9 * 3 = 0

And, what is...

6 * 6 = ?
5 * 5 = ?
1 * 8 = ?

The sum of 3, 5 and 4 is 12, and so are the others.

EXAMPLE #8 "How much will one cost?"

"Twenty cents," replied the clerk in the hardware store.
"And how much will twelve cost?"
"Forty cents."
"O.K., I'll take eight hundred and twelve."
"That will be sixty cents."

What was the customer buying? House numbers! (20 cents a piece.)

Now that you have somewhat of an idea about fixations, please allow me to demonstrate it to you:

I am going to fixate you. Please try to hang tight, and not get fixated. Well, first let me tell you a little story. There is a corner store ... a Seven Eleven. One evening at 7:30 p.m., a mute person came into the store. When the clerk asked him what he wanted, the must pretended to puff on an imaginary cigarette. The clerk immediately understood, and handed him a pack of smokes. Next, a blind man came into the store wanting to buy a pair of scissors. The clerk asked him what he wanted to buy. What do you think he did to convey his wishes to the clerk? If you got fixated, you will assume that he made a cutting motion with his fingers, which he did not. He simply asked for them!

Remember, in order to be successful in mathematics, one has to be alert and awake at all times. Don't get fixated!

The session was ended with the following story...

There once was a very frustrated math teacher who had just explained the rules for adding fractions at least a dozen times. In spite of this, one student still was not catching on. The teacher became so upset by this that she yelled, "You are so stupid. You are like the blackness of coal that will not disappear even after a hundred washings!"

At this time, the school principal happened to be passing by and heard the teacher's hasty comment. He walked in to the class, and looking directly into the teacher's eyes said, "Ah, but the blackness of that same coal disappears when fire ignites it!" In other words, if teachers are good, care about their students, and know how to motivate them, just as the blackness of coal disappears when fire enters it, so does confusion.

I hope that from my talk you will be able to see the Satyam, Sivam, and Sundaram that can be found in mathematics. It is my wish that from this you may find your own way to motivate the unmotivated, to help them see the truth, goodness, and beauty in math -- to help them want to find it for themselves.

CALL FOR ARTICLES

The theme for the September, 1989 issue will be
RELATIONS, PATTERNS & FUNCTIONS.
Authors are requested to submit articles
(both short & long, practical and theoretical)
for all ages and ability levels.

COCKROACH MODEL - PART TWO
QUADRATIC FUNCTION

by
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This paper will use the framework of the cockroach model (presented in the last issue of the Gazette) to develop a unit on the quadratic function. This unit has been developed under the assumption that the students' initial exposure to the problem-solving process has been completed. Group work would be used extensively throughout the unit, with the students developing a practiced ability of explaining solutions to their peers. It may be helpful at this point to give a definition of a problem: A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining a solution (5). Therefore a question might be a problem for one student, whereas it wouldn't be for another student.

Note: The units i) ratio and proportion, and ii) quadratic equation could be integrated with this unit to give students a more holistic view of mathematics. The guideline (2) suggests 9 hours for this unit.

INITIAL (experimental) STAGE

The teacher should set the scene for a question (or questions) that will be presented to the students. Once the questions are presented the following guidelines are given for the teacher to follow:

Guidelines

- o divide the class into groups with 3-4 students in each group.
- o distribute to the groups the PROBLEM.
- o explain that you have hints to give them when they need help.
- o be sure and allow sufficient time for the groups to work on the problem.
- o you should circulate around the classroom and answer only information questions (don't give hints).
- o after the students have made an honest attempt and are asking for help, then give out the hints one at a time.
- o you should be aware that the experimental calculations may not form a uniform pattern and therefore, some teacher guidance and interpretation may be required.
- o allow approximately one period (70 min.) for the work on the problem. The first half of the next period should be spent having the students (each group) discuss their solutions with the rest of the class.

Hints:

▸ a hint may help the student to better understand the problem, but it also provides a focus for determining whether particular strategies may be appropriate.

Suggest one or more of the following strategies:

- ▸ Make a table - from the table maybe a guess could be expected.
- ▸ Make a graph - from the graph have the student estimate an answer.
- ▸ Find a relationship (pattern).
- ▸ Solve - use the relationship to find an answer.
- ▸ Look back - check the reasonableness of the answer.

The teacher should discuss the criteria for evaluating this type of experimental (problem-solving) work. An evaluation scheme was given in my last paper (1).

SECOND STAGE

The initial stage has the students conducting experiments, setting up tables of values, searching for patterns, and solving problems. This stage eliminates the hands-on experience (experiments) and has the students being given a table of values from which they progress to a solution (3). Also this stage saves valuable class time, as well as presenting results of experiments which may be difficult to perform in a classroom setting.

THIRD STAGE

At this stage problems are introduced that the students will be unable to complete because

additional skills and/or strategies are necessary. Hopefully the students will now see a need for learning some new skills. The teacher must now take the time to teach these skills and provide enough reinforcement to solidify student understanding. Routine problems can then be given as further applications for these new skills. Any algebraic and/or geometric patterns can also be established either by looking back at the previous problems or introducing new ones.

FOURTH STAGE

This stage involves summarizing the results, generalizing, and reviewing.

FIFTH STAGE - EVALUATION

1. A problem similar to the one found in the Initial Stage (done in pairs).

2. Routine questions with the emphasis being on solving problems and understanding the concepts (multiple-choice questions may be appropriate as part of the evaluation).

3. A non-routine question (at your discretion).

4. Group-work evaluation throughout the unit.
   - teacher observation
   - self-evaluation

THE QUADRATIC FUNCTION

INITIAL STAGE

Problem

The graduating class at Merry S.S. wants to donate a school crest (circular) which will be erected above the main school entrance. The principal has informed them that the structure will not support more than 30 kg. What is the size of the largest crest, if the crest is to be made of the same material and thickness as the samples (distributed by the teacher)?

Equipment

- electronic balance
- several samples of the material (circular disks) for each group (remember the thickness must be same). The samples can be made from any available material e.g. wood, plastic. It may provide for a richr class discussion if each group was given different materials.
- graph paper and rulers

Hints:

- ▸ You need to find a relationship between the radius of a disk and its mass.
- ▸ Complete a table of values - radius | mass
- ▸ Graph the results.
- ▸ Determine a relationship between the radius and the mass (either from the graph or the table).
  answer: $m = kr^2$, $k \in R$
- ▸ Use this relationship to predict an answer to the problem (note: the answer could also be predicted from the graph).
- ▸ Check that the result is reasonable.

Alternatives: experiments that will generate quadratic functions:

i) filling a conical flask with a liquid
   - volume vs height

ii) finding the distance of a freely falling object
   - time vs distance

iii) finding the smallest screen area that would allow one to show overheads at a given distance
   - distance vs area

Teacher Note: The experiment given on the previous page allowed for a hands-on experience where the students constructed a table of values and used it to help solve the problem. The next step will be to give problems where the table of results are already given.

SECOND STAGE

(1) S. P. Lat, the famous stunt clown, falls out of an aircraft which is flying at an altitude of 1 km. She falls according to the following table:

time (sec)	0	1	2	3	4	5
distance fallen (m)	0	5	20	45	80	125

i) Graph the results.

ii) Can you see any relationship or pattern from the table? (describe in words

or by way of a formula) [$d = 5t^2$]

iii) If her parachute opens in 11 seconds, then

a) how far has she fallen? [605]
b) how high is she above the ground when it opens? [395]

(2) SONG-Twelve Days of Christmas-bring in a tape recording of it. Complete the following table:

gift	represented by	total # received
partridge	1	
turtle doves	2	
french horns	3	
etc.		
drummers	12	

i) Graph the results.
ii) Which gift was received the most? [geese or swans]
iii) Can you see a relationship between the total # of gifts (T) and the number that represents each gift (N)? [$T = -N^2 + 13N$]
iv) Which gift would you like to receive?

Teacher Note: four more questions of this type should be included.

THIRD STAGE

(3) A ball is thrown vertically upward with a velocity of 22m/s and the height, h in metres, above the ground after t seconds is

$h = -5t^2 + 22t$

What is the maximum height of the ball, and when does it reach that height? [24 1/5, 2 1/5]

Teacher Note: A graphical solution is inadequate. Therefore the students will need to be introduced to the method of completing the square. More drill type questions should be included after this question to reinforce this method.

(4) On her championship springboard dive N.O. Splash's height, in metres, above the water is given by $h = -5t^2 + 8t + 4$, where t is time, in seconds, after she leaves the board.

i) Find the height of the springboard above the water. [4]
ii) Find N.O.'s maximum height. [7 1/5]
iii) Find the time to reach the maximum height. [4/5]
iv) How long is she in the air? [2]

(5) A telephone company has 30 000 telephones in a certain city where the charge is $9.00 per month. Their survey has shown that the number of telephones in use will increase at an estimated rate of 500 additional telephones for each 10 cent reduction in the rate. What monthly rate would yield the greatest revenue? What is the greatest revenue? [7.50, 282250]

Teacher Note: Several more questions of type #5 (where the equation has to be formed) should be given. For the last part of the unit the students would refer to their graphs from the previous questions and to draw more graphs to establish the patterns for:

i) $y = ax^2$
ii) $y = a(x-p)^2$
iii) $y = ax^2 + c$
iv) $y = a(x-p)^2 + q$
v) $y = ax^2 + bx + c$

FOURTH STAGE

The algebraic patterns obtained from the graphical questions at the end of the Third Stage should lead the student to be able to describe the general quadratic function and to summarize the results. This stage should also include a review of the unit.

FIFTH STAGE

1. An a Initial Stage type problem (e.g. develop a problem from the formula $L = .248T^2$, where L is the length of the pendulum (metre) and T is the period (sec). You may be more familiar with the formula in the form $T = 2\pi\sqrt{L/9.8}$) (done in pairs).

2. Routine questions with the emphasis being on solving problems and understanding the concepts (multiple-choice questions may be appropriate as part of the evaluation).

3. A non-routine type question at your discretion.

4. Group work evaluation throughout the unit.

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Teaching Tips

HOW BIG IS A TRILLION?

Although many students above grade 4 can tell you that a trillion is a 1 and 12 zeros, conceptualizing a trillion is difficult, if not impossible, even for most adults.

When asked how I would explain the size of a trillion, I remembered reading an article recounting that the American National Debt exceeded a trillion dollars. The article stated that a trillion dollar bills, attached end to end, would stretch past the sun! That helps, but not much.

Then someone asked how long it would take to count to a trillion.

Quickly! How long?

Several people thought about five or ten years. Someone suggested 100 years and the skeptics laughed.

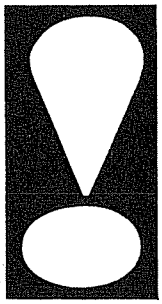
We decided to allot one second for each numeral. This is a conservative estimate since only the early numerals can be said in less than a second. Most will take far longer.

Anyway, at the rate of one number per second and provided that the counter never stops, it would take over 31 thousand years!

Now that's a long time.

Try it!!

Or better still
have your
students
work
it out



Suggested by:
Barry Onslow
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The University of Western Ontario

TEACHER ASSESSMENT PRACTICES IN HIGH SCHOOL CALCULUS: DIFFERENCES IN RESPONSE TO A COMMON MARKING TASK

by
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Responses to the Common Marking Task

A study was made of the extent and nature of the variation among the 17 teachers in the standards they applied in marking a set of examination papers. A final calculus examination, administered in June 1987 to students in a school not otherwise involved in the study, yielded a set of 20 papers that spanned a range of quality. The 17 teachers were given the 20 papers, and each was asked to prepare a marking guide and then mark the papers against it. In addition, the teachers were asked to provide written comments, should they care to make any, about the examination and performance of the students.

The Examination

The exam contained 11 questions, several of which contained three or more sub-questions. In total, the examination consisted of 37 sub-questions. The content of each question can be described briefly as follows:

1. Find the point on a quadratic function where the tangent has a specified slope.
2. Obtain the derivative with respect to x for each of 13 different functions of x - six logarithmic or exponential functions, four polynomial functions, and three trigonometric functions.
3. Find integrals of nine functions - three trigonometric functions, four logarithmic or exponential functions, and two polynomial functions.
4. Integrate using the method of parts.
5. Find the limits of three polynomials
6. Solve a problem involving a) acceleration, b) velocity, and c) the position of a particle in motion after a specified amount of time has elapsed.
7. Find the area enclosed between two trigonometric functions of the same variable over a specified range of the variable.
8. For a cubic function, a) find the coordinates of all maximum and minimum points of the function, b) find the coordinates of all points of inflection, and c) sketch the function.
9. Find the rate at which the distance between two moving objects is increasing or decreasing, given information about the direction and rate of motion of the two objects.
10. Prove that the formula (given) for the volume of a sphere can be obtained as a volume of revolution.
11. Find the radius and height of a cylinder, such that the cylinder will have a given volume and an unspecified but minimum surface area.

The exam was strongly weighted toward the testing of basic skills. According to the scheme for categorizing homework and test questions, the basic skills topics (Group I) contained most of the exam questions (26 sub-questions). The total number of sub-questions for all other content groups combined was only 11.

The Marking Guides

The marking guides prepared by the teachers indicate the maximum number of marks to be awarded for responses to each sub-question. There was considerable variation among teachers in the total number of marks allocated for perfect performance. The smallest of the maximum marks was 78, the largest 144, and the median 116. The teachers also differed in their allocations of marks to individual sub-questions. For example, wholly satis-

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Editor's Note: This is the second half of a short version of a long report under the title "Teacher Assessment Practices in a Senior High School Mathematics Course." The first half, discussing differences in use of class time, teaching content, and grading practice, appeared in the previous issue. A copy of the long report is available from the authors on request.

This research was conducted with the support of the Ontario Ministry of Education, through Transfer Grant 52-1028 to the Ontario Institute for Studies in Education.

teachers, and as few as 4 marks by one teacher. What accounts for differences such as this? For the most part, they seem to stem from differences in the number of steps or stages to an answer that are awarded marks. Another difference was in the use of bonus marks and deductions. Several marking guides indicated bonus marks for good form and for stating the answer in a complete English sentence. Several others indicated deductions for failing to include the constant of integration in answers or for failing to specify units in answers to questions involving measured quantities. These bonuses and deductions, when used, were either one mark or one-half mark.

Despite the obvious disparities found among the teachers' marking schemes, the teachers were in general agreement as to the order of importance of the examination questions and sub-questions. A coefficient of correlation was computed for each pair of teachers between the maximum marks allocated to the questions and sub-questions of the examination. All the intercorrelations were substantial, ranging from 0.76 to 0.95, with a median of 0.89. Clearly, the teachers possessed very similar views of the relative importance of the questions and sub-questions of the examination.

This does not mean that the teachers thought the exam was particularly good, at least as judged by coverage of the calculus course described in the Guideline (Ontario Ministry of Education, 1972). Several teachers objected to the strong emphasis in the exam on integration. Three teachers noted the lack of coverage of polar coordinates and complex variables. Two teachers pointed to the coverage in the exam of trigonometric functions, with one feeling it was inadequate and another thinking it was overemphasized. It was observed by two teachers that volumes of revolution, trigonometric limits and differentials were given short shrift. And three teachers objected to the preponderance of skill-type questions, and the lack of questions involving problem-solving. Note that volume of revolution, polar coordinates, and complex numbers are optional topics. (We did not suggest that the exam was a model for all teachers to emulate; it was only a means to the end of studying differences in marking behaviour. In fact, for present purposes we eliminated the section of multiple-choice questions that appeared in the exam as originally administered.)

In a draft document entitled A Handbook for the Examination Component of Evaluation in the OAC - Calculus (Ontario Ministry of Education, 1987), attention is paid to the number of marks awarded for arithmetic and algebraic simplification in answers to OAC calculus examination questions. An analysis was made of the marking guides in an attempt to assess the extent of differences among them in the proportions of marks awarded for arithmetic, algebraic simplification, and other skills and knowledge (from earlier grades) compared to the calculus skills and knowledge to be acquired in the course. (This analysis was possible for 13 of the 17 guides; four guides indicated only total numbers of marks per sub-question.) The percentages of marks for calculus as opposed to other kinds of mathematical knowledge and skill ranged from 60 to 76, with a median percentage of 66. Thus, there was some variation, but not a lot, in the extent to which knowledge and skills peripheral or prerequisite to calculus were rewarded.

Total Student Marks

The comparability of the total marks assigned each paper by the 17 teachers was assessed by computing for each pair of teachers a coefficient of correlation between the total marks assigned the 20 papers. These coefficients were uniformly high, ranging from 0.81 to 0.97, with a median of 0.92. Obviously, there is close agreement among the teachers in the relative orders into which they placed the 20 papers.

Grading achievement in calculus and other subjects involves more than rank-ordering a group of students. Determinations of fail and pass and honours are usually required. How well, then, did the teachers agree as to which papers represented failing performance, which represented passing performance, and, of the passes, which represented honours? To address this question, the total mark a teacher assigned a paper was converted into a percentage of the total mark given in the marking guide. Here we find evidence of inconsistency in standards. Three teachers assigned no paper a mark in the honours range, and one teacher assigned failing marks to seven papers. On the other hand, seven teachers assigned no paper a failing mark, and one teacher assigned percentage marks of 80 or more to 10 papers. Variation in standards is apparent, despite the fact that

the teachers ranked the papers for quality in very much the same way.

The teachers offered comments, several of which are relevant here. For example, the stiffest of the markers directed comments at student performance: the solutions were poorly developed, diagrams were missing, and the responses lacked clear, concise statements. These might be described as errors of form in the student responses. (Although the marking guide of this teacher indicated five marks for the first question, no student was awarded more than three. The apparent reason for this was the failure by all 20 students to include all the steps listed in the teacher's model answer. Thus, for example, no mark was awarded for finding the y-coordinate of the answer if the determination of this coordinate had not been made explicit, even when the student's answer did contain the correct coordinate.) Another of the hard marking teachers also noted the errors of form as a problem with student answers, but so did two teachers who were in the middle of the group as regards severity of marking. The fact that three other hard-marking teachers did not mention form of answer as a problem, suggests that this factor does not fully explain the source of severe marking standards.

In fact, no type of comment appears to distinguish the hard from the easy markers. The easiest marker described the marking exercise as boring. This teacher also described the exam questions as being all of the skill and recall type, and as not requiring higher level thinking skills. It is not apparent that adopting this point of view should cause one to be an easy marker, although another easy marker also commented on lack of problem solving questions on the exam. So too, however, did a teacher in the middle of the group for marking severity. Perhaps more in line with what might be expected, given the severity of his/her marking, was a teacher's registration of disappointment in the students' problem solving abilities. Other comments were made to the effect that the exam was too easy, was of uneven difficulty, with questions being either very easy or very difficult, was too long, was "too tricky by half", and was nicely balanced between straight-forward and challenging questions.

One factor, however, may distinguish hard from easy marking teachers. Ten of the teachers followed one textbook (published by Gage) and six others followed another textbook (published by Holt). (One teacher used a set of notes, and followed no published book.) The teachers who used the Gage text were, on average, relatively easy markers, whereas the teachers who used the Holt text were, on average, relatively hard markers.

In a final attempt to understand differences among teachers in marking standards, a study was made of the marks assigned by three teachers – the hardest and easiest markers, and a teacher at the centre – to three students – a high, middle and low scorer. It was found that these teachers differed relatively little in the percentages of marks each awarded for performance of the 26 basic skills sub-questions. Against this standard, however, the corresponding results for the other sub-questions are dramatically different. For example, one of the students was awarded about half the marks allocated by the easiest marking teacher for performance of the other-than-basic skills sub-questions, but the other two teachers assigned only one-fourth the marks they had allocated for performance of the same sub-questions. These results suggest that the main source of the difference among these teachers lies in their marking of the exam questions that test other-than-basic differentiation and integration skills.

Summary

The analysis of use of classroom time showed relatively substantial differences among the teachers in their allocation of class time to different categories of activities - administration, direct teaching, review, homework, practice, and assessment. Moreover, those with more class time available expended a smaller percentage of time on direct instruction, and allocated greater percentage to homework and practice.

Substantial differences were also found among teachers in content emphasis.

- Teachers varied widely in the number of questions assigned as homework – from under 500 to more than 1600. A large part of this variation was accounted for by differences in the number of questions on basic skills.
- Teachers varied considerably in the extent to which they emphasized different topics in their assignment of questions. For example, the number of questions on basics

ranged from 40% to 75% of the total number of assigned questions.
- The emphasis on basic skills was less in the tests than the assignments, whereas the emphasis on other content groups was greater in the tests than the assignments.

Some of the within-teacher differences between assignment and testing emphases are probably intentional, as when a teacher decides to test only at the top of a small hierarchy of skills or knowledge, ignoring the prerequisite skills and knowledge that had been included in assignments. Conversely, a teacher may teach a difficult concept and choose not to test it because most students failed to grasp it. Whether or not discrepancies between teaching and testing constitute a problem to be corrected is a matter not addressed in the present study. All we have done here is provide evidence that such discrepancies as these exist.

From the analyses of the grading practices of the 17 teachers, it was learned that examinations and term tests were the two main determinants of student grades. All students in all classes wrote a minimum of one examination, as required by provincial policy. However, the nature of this examination varied. For the four classes following an exemption policy, the majority of students took their only exam on material learned in the first half of the semester. For six other classes, the only exam was a final exam based on the entire semester's work. In the remaining seven classes, both a mid-course exam and a final exam were required. The time that students from different classes spent in an examination situation ranged from 2 to 4.5 hours. The final examination mark was weighted from 15% to 40% of the student's final grade and, when a mid-course exam was administered, the resulting mark was weighted 9% to 30% of the final grade. In some cases, the calculus content topics that were tested during the semester were not emphasized to the same extent on the final exam. This might be attributed to the fact that, while the setting of term tests was usually the teacher's responsibility, the final examination was the mathematics department's, and not necessarily the participating teacher's, responsibility.

Term testing was found to vary in the following respects: (i) number of tests (ranging from 3 to 13), (ii) total number of items comprising all tests (from 53 to 180), (iii) amount of classroom time used for test taking (from 3 to 15.2 hours), and (iv) schedule of tests (sporadic or regular). Term tests were weighted from 30% to 80% of the final grade. It is evident from this study that students taking Grade 13 Calculus in the Spring 1987 Semester from the 17 teachers in this study did not demonstrate their achievement in calculus through a common process of assessment and grading. It is reasonable to question whether or not it would be beneficial for students to have experienced similar grading processes, and to have been judged according to similar standards on similar criteria of achievement.

The empirical study of the marking process revealed the following:

- Substantial agreement among the 17 teachers as to the relative importance of the examination questions.
- Substantial agreement among the teachers as to the relative quality of the 20 student papers that were marked.
- Substantial disagreement among the teachers as to the absolute quality of the 20 student papers.
- Variation in marking standards of teachers varied, to a limited extent at least, as a function of the textbook being used.

These results pose a challenge for the Ministry of Education in a jurisdiction where there is no external mechanism - no common, province-wide examination - for aligning standards of calculus achievement. This challenge has not been lost on critics of education in Ontario, and it has not been ignored by the Ontario Ministry. A Handbook for the Examination Component of Evaluation in the OAC - Calculus (Ontario Ministry of Education, 1987) was developed for the purpose of fostering a greater degree of consistency in calculus examinations across the province. The handbook addresses several problems found in the present study - (i) the practice of granting exemptions from final examinations and the variation in value of final examinations, (ii) the emphasis on basic skills to the virtual exclusion in teaching and testing of problem solving, and (iii) the wide differences in amount of testing and other assessment activities. But the results of the present study suggest that consistency in assessment will be increased only when other steps are taken as well.

These steps include the following: Increase the consistency of what is taught; increase the

consistency with which those examination questions that test other-than-basic calculus skills are marked; increase the consistency with which displays of other-than-calculus knowledge and skills are marked; have more than one teacher independently mark every student exam paper, and set the exam mark equal to the average of the several marks; and ensure that exams are sufficiently long and numerous so that all content is covered and so that the impact on a student's grade of performance on any one question or type of question is minimized.

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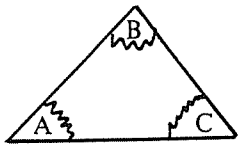
Ontario Ministry of Education. (1987). A handbook for the examination component of evaluation in the OAC - Calculus (DRAFT). Toronto.

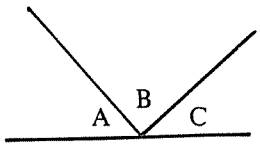
Teaching Tips

UNWRITTEN PROOFS OR PROOFS WITHOUT WORDS

I have been playing around with some unwritten proofs. I have come up with these. Perhaps you can think of others. (Proofs without words) Send them in.

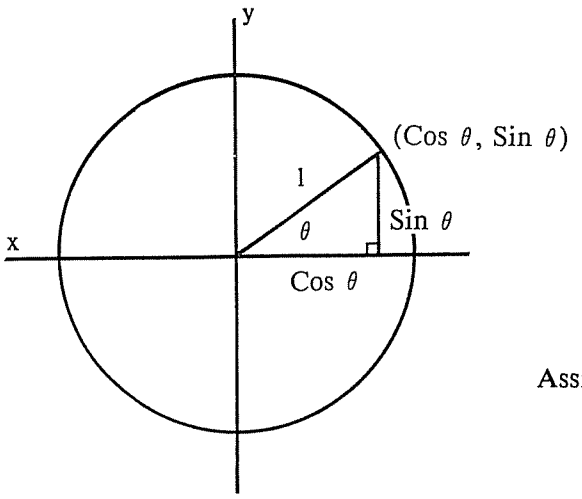
1.





$\angle A + \angle B + \angle C = 180°$

2.



$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

Suggested by:
Todd Romiens
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PROBABILITY MODELS IN ANONYMOUS SURVEYS

by
L. Keith Roy
Department of Mathematical Sciences
Lakehead University
Thunder Bay, Ontario P7B 5E1

When confronted with a question such as: "Do you cheat on your income tax?" or "Do you belong to an organization dedicated to the violent overthrow of the government?", a person's first impulse might be to answer "No." regardless of the truth. Such persons must be guaranteed complete anonymity in order to truthfully answer a self-incriminating question. Perhaps the best guarantee is a situation where no one, not even the interviewer, knows the responses of individual subjects. In such a case it is still possible to use the rules of probability and the method of randomized responses to estimate the proportion of 'guilty' individuals in the population being studies.

Here is an example of such a situation which could be demonstrated in a high school classroom.

The teacher will require

1) A ballot box.
2) A number of cards with the word 'yes' printed on each, and a different number of cards with the word 'no'. The cards should easily fit through the slot in the ballot box. The number of 'yes' cards and the number of 'no' cards should each be greater than the number of subjects (students) in the class. The subjects should not be aware of the exact number of each type of card.
3) A 1989 calendar.
4) A bag containing 5 black marbles and 20 white marbles. The colours are not important as long as there are 5 marbles of one type and 20 of another type.
5) A private voting booth. A curtain across one corner of the room will suffice.
6) A poster with a list of instructions.

The calendar, ballot box, 'yes' cards, 'no' cards, poster, and bag of marbles will be used inside the voting booth by the individual subjects.

The teacher may use any question of topical interest as long as a 'yes' answer implies guilt.

The teacher's first task is to explain the procedure to be followed. The procedure is also printed on the poster.

PROCEDURE

a) Say nothing while in the voting booth.
b) Use the calendar to determine the day of the week on which your birthday falls in 1989.
c) Without looking, reach into the bag and draw out a marble. Note the colour of the marble and return it to the bag.
d) If you drew a white marble, answer the topical question. If you drew a black marble, answer the question: "Does your birthday fall on a Sunday in 1989?".
e) Answer your question by depositing either a 'yes' card or a 'no' card into the ballot box.
f) Leave the voting booth.

It should be noted that the 'yes' cards, the 'no' cards, and the ballot box are not mathematical necessities. A subject could simply say yes or no in public (after drawing a marble in private) since no one would know which question was being answered. However the privacy of the voting booth should reassure the subjects as to the safety of a truthful response. In addition, if subjects were required to speak, vocal inflection or tone might betray their guilt or innocence. In either case it is very reassuring for the subjects to know there is a reasonable chance of answering a question concerning only a birthday. Since the number of 'yes' cards and the number of 'no' cards each exceed the number of subjects, no subject can count the unused cards during his/her turn in the voting booth to determine the current number of 'yes' responses.

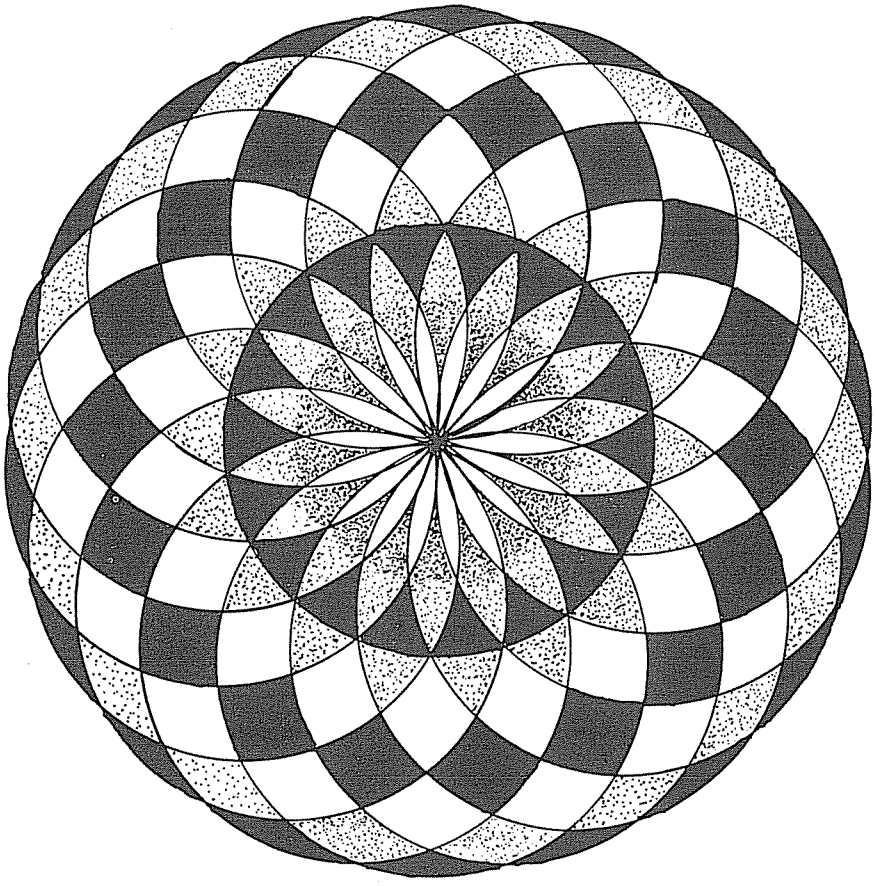
The teacher's last task, before the subjects begin to respond, is to explain how the results will be analyzed mathematically once the ballot box is opened (in public) and the number of yes answers is revealed. This again reassures the subjects that there is no trickery involved.

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NUMBER 3

Abacus

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APRIL 1989



"Before LOGO"

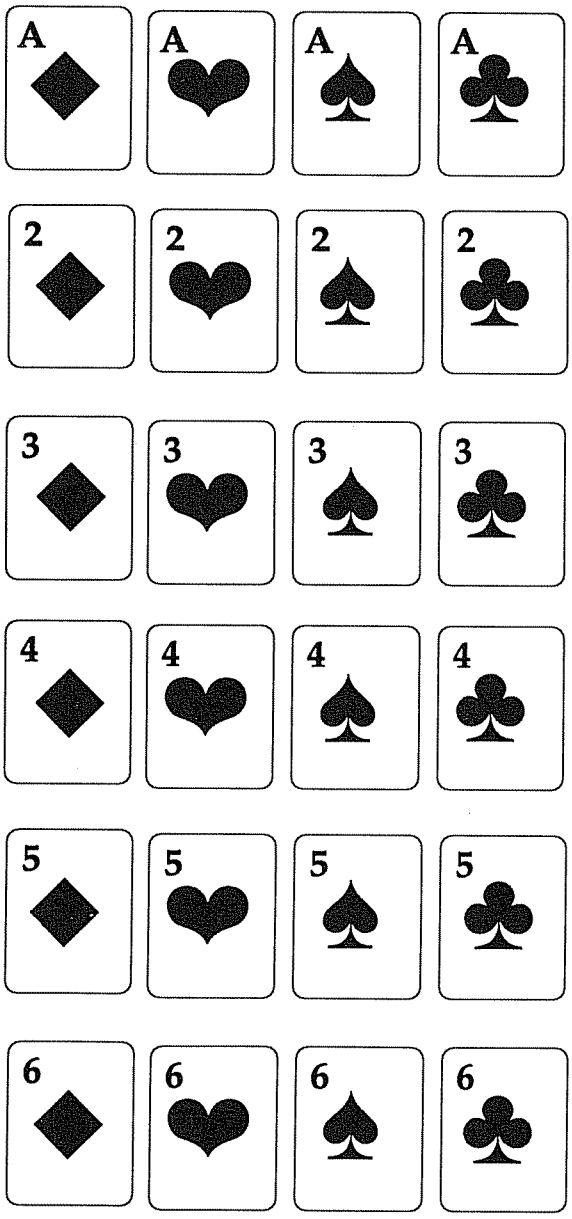
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This issue was prepared by **J. Symington** and **Don Attridge** on behalf of the **York4 Mathematics Association**

The Australian Game of 31



From a deck of cards remove the 24 cards ace through six. Arrange the cards in the following fashion.

The game is played with two players. After deciding who goes first each player takes a turn placing a card face down adding the values of the cards turned as they do. Once a card is turned down it cannot be used again during the game. The player reaching 31 exactly first wins. Should a player be forced to go over 31 he/she loses.

- Is there a strategy you can develop so that you can always win?
- Does it matter who goes first?
- Try the game and play a variety of partners.

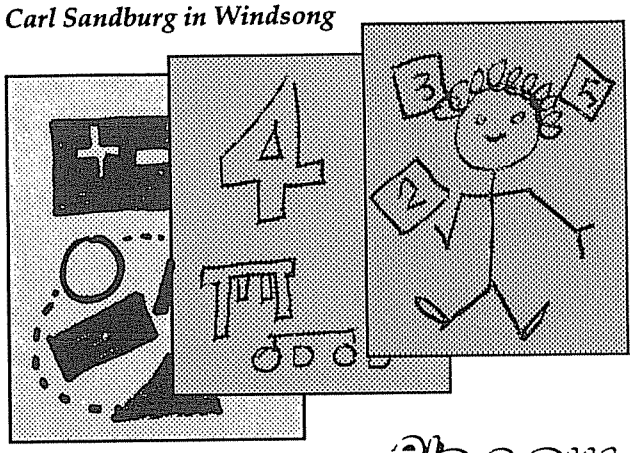
Arithmetic

- *Arithmetic is where numbers fly like pigeons in and out of your head.*
- *Arithmetic tells you how many you lose or win if you know how many you had before you lost or won.*
- *Arithmetic is seven eleven all good children go to heaven — or five six bundle of sticks.*
- *Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.*
- *Arithmetic is where the answer is right and everything is nice and you can look out the window and see the blue sky — or the answer is wrong and you have to start all over and try again and see how it comes out this time.*
- *If you take a number and double it and double it again and then double it a few more times, the number gets bigger and bigger and goes higher and higher and only arithmetic can tell you what the number is when you decide to quit doubling.*
- *Arithmetic is where you have to multiply — and you carry the multiplication table in your head and hope you won't lose it.*
- *If you have two aninal crackers, one good and one bad, and you eat one and a striped zebra with streaks all over him eats the other, how many animal crackers will you have if somebody offers you five six and seven and you say No no no and you say Nay nay nay and you say Nix nix nix?*
- *If you ask your mother for one fried egg for breakfast and she gives you two fried eggs and you eat both of them, who is better in arithmetic, you or your mother?*

Carl Sandburg in Windsong

Children will enjoy rewriting this famous poem. Each can be written on construction paper and then illustrated by the children.

A class may embark on a cooperative poem based on Arithmetic to me is ... or Measuring to me is ...



How Do You Arrange the Cards So That the Following Will Happen?

Take a 13-card deck suit from a deck of cards and arrange them in such a way that the following will occur:

A "Holding the 13 card deck face down, take the top card and place it on the bottom of the deck. Turn the next card up on the table so that it will be an Ace. Place the next card on the bottom of the deck and turn the next card up on the table so that it will be a Two. Continue this procedure - i.e. place the next card on the bottom of the deck and the following card up on the table so that it follows in sequence 3, 4, 5, ..., 10, J, Q, K."

B "Holding the 13 card deck face down, take one card from the top and place it on the bottom of the deck. Turn the next card, which is the Ace, up on the table. Take the next 2 cards (separately) and place them on the bottom of the deck. Next turn the two up on the table. Continue this procedure - i.e. place the number of cards, separately, on the bottom of the deck which will suggest the next number in sequence which will be turned up on the table. Note that 11 cards must be placed separately on the bottom of the deck before the Jack is turned up; similarly 12 before the Queen turns up."

Answers: (may be given outright, hinted at, or left to the next Abacus)

Suggestions to Answers:

Clue 1

A _ A _ 2 _ 3 _ _ _ _ _ _ _ _

B _ A _ _ 2 _ _ _ 3 _ _ _ _

Clue 2

A 7 A _ 2 _ 3 _ 4 _ 5 _ 6 _

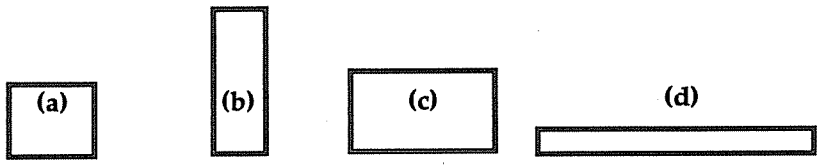
B 4 A _ _ 2 _ _ _ 3 5 _ _ _

Answer

A 7 A Q 2 8 3 J 4 9 5 K 6 10

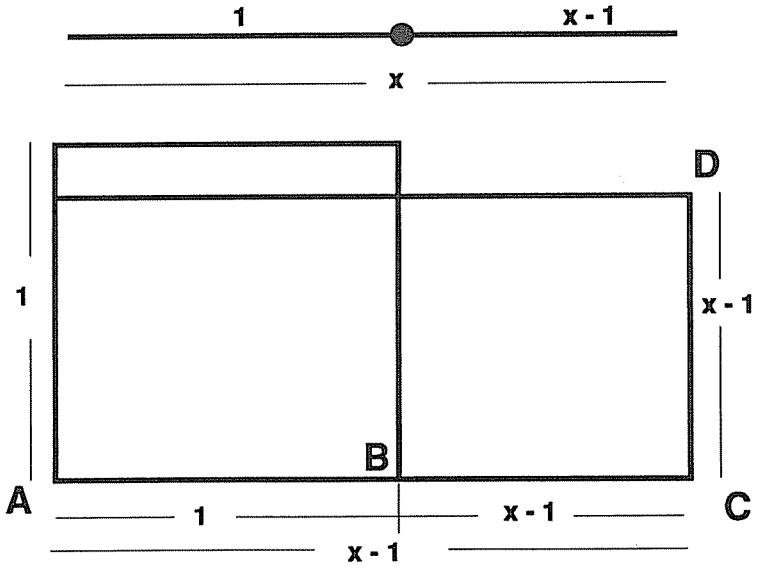
B 4 A K J 2 10 6 7 3 5 Q 9 8

The Golden Ratio



Which rectangle looks best to you? If you said (c) perhaps it is because its length and width have a particular ratio. Architects and artists often used rectangles of this shape because it was thought to be appealing to the eye. The Parthenon temple built in ancient Greece incorporates many examples of this structure. The ratio of length to width is called the "golden ratio" and is symbolized by the Greek letter ø. The value of ø is approximately 1.6 or 5:3. There are many illustrations which generate ø as indicated below:

(a) Euclid stated that "a given line can be divided into two parts so that the area of the rectangle whose length is the whole line segment and width is the shorter of the two parts, is equal to the area of the square whose side is the other longer part." Using this idea, the golden ratio ø evolves.



Let the given line be x and the part included in the square to be 1

From Euclid $(AC)(CD) = (AB)^2$

$(x)(x-1) = (1)^2$

$x^2 - x = 1$

$x^2 - x - 1 = 0$

Solving for $x = \frac{1 \pm \sqrt{5}}{2}$

A positive solution for $x = \frac{1+\sqrt{5}}{2} = 1.618033989$ or the golden ratio ø

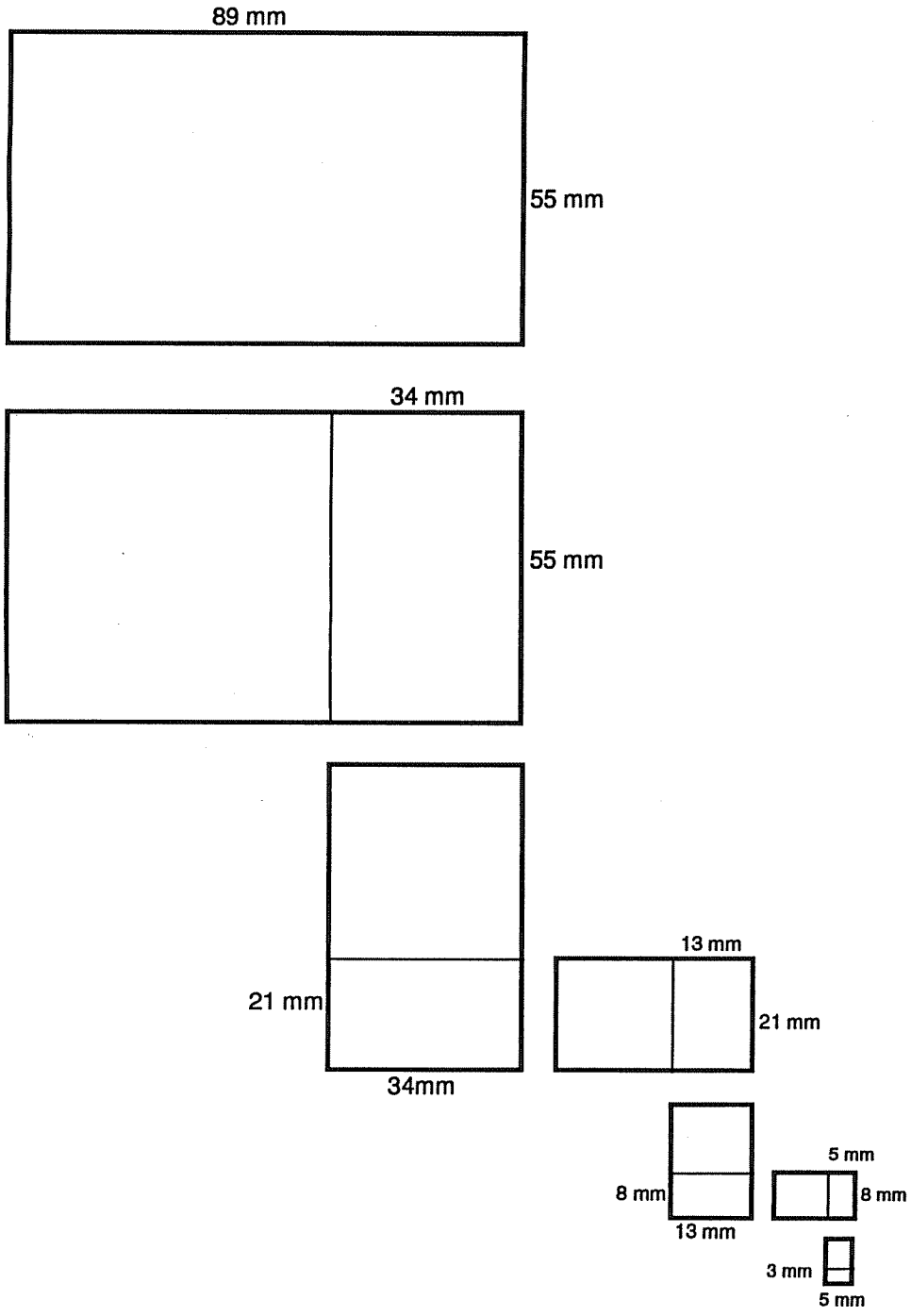
(b) The Fibonacci sequence of numbers is defined to be 1, 1, 2, 3, ... where each succeeding number is determined by adding the previous two numbers. Thus a few terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, ...

An interesting aspect to consider is the ratio of consecutive pairs of numbers.

1 : 1 = 1.000
2 : 1 = 2.000
3 : 2 = 1.500
5 : 3 = 1.667
8 : 5 = 1.600
13 : 8 = 1.625
21 : 13 = 1.615
34 : 21 = 1.619
55 : 34 = 1.618
89 : 55 = 1.618
144 : 89 = 1.618

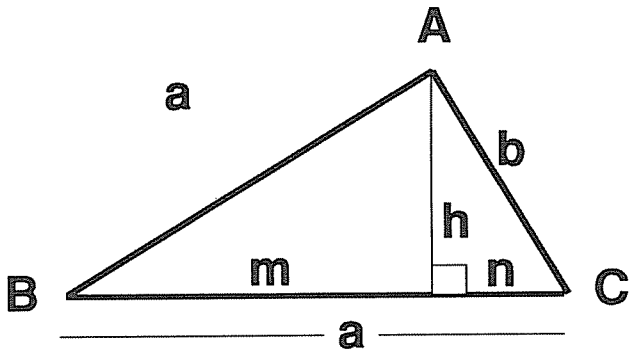
As the numbers increase in size the ratio of consecutive numbers approaches the golden ratio ø.

Consider a rectangle 89mm X 55mm where 89:55 ≈ 1.618.

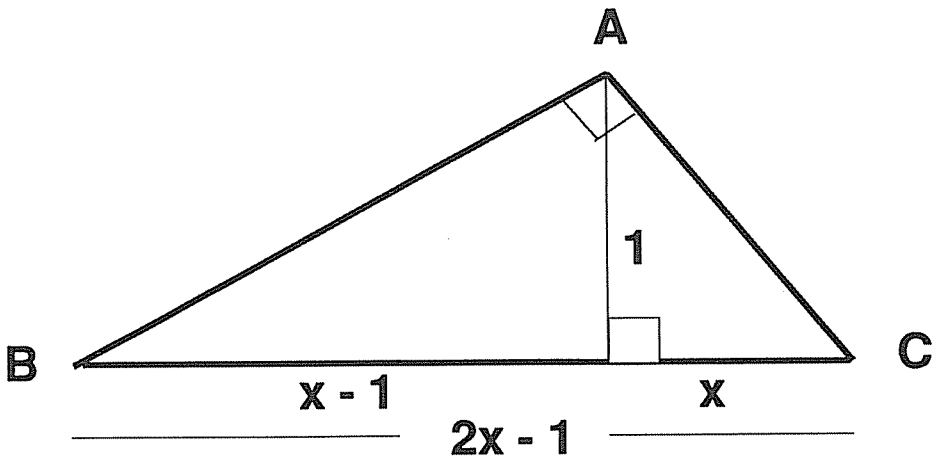


(c) **Using right-angled triangles and the Theorem of Pythagoras the following can be obtained from the given diagram:**

$m:c = c:a$ or $c^2 = ma$
$n:b = b:a$ or $b^2 = na$
$m:h = h:n$ or h^2



Suppose in the $h^2 = mn$ case that $h = 1$, $n = x$, $m = x - 1$ and $a = 2x - 1$ as below:



From above, since $h^2 = mn$,

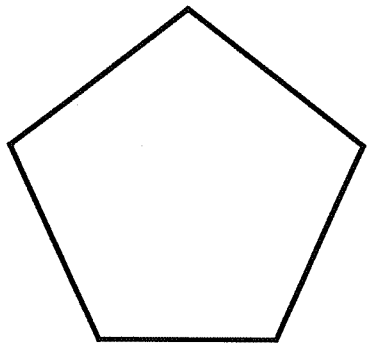
$$(1)^2 = (x - 1)(n)$$

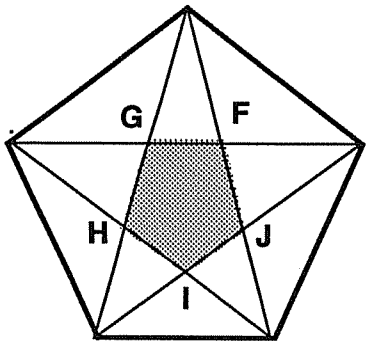
or

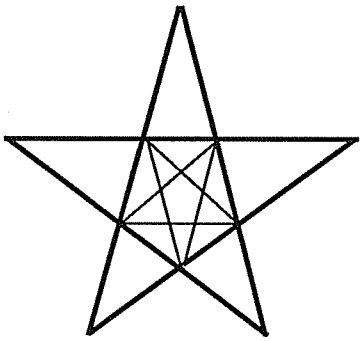
$$x^2 - x - 1 = 0$$

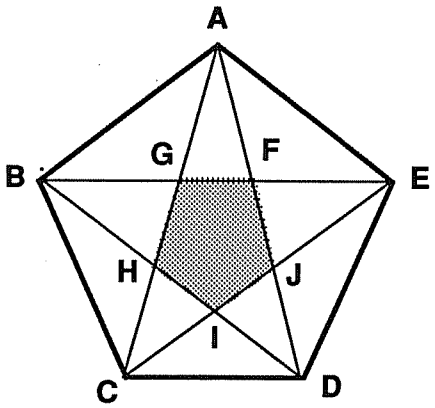
and a positive value for x is ø.

(d) Take a regular pentagon and draw in the diagonals, thereby creating a new regular pentagon FGHIJ.









Note that the following ratios approximate ø

$$\frac{AD}{AJ} = \frac{BD}{BI} = \frac{CA}{CG} = \frac{DA}{DF} = \frac{EB}{EG} = \ldots = 1.618$$

$$\frac{AD}{DC} = \frac{BE}{ED} = \frac{CA}{AE} = \frac{DB}{BA} = \frac{EC}{CB} = \ldots = 1.618$$

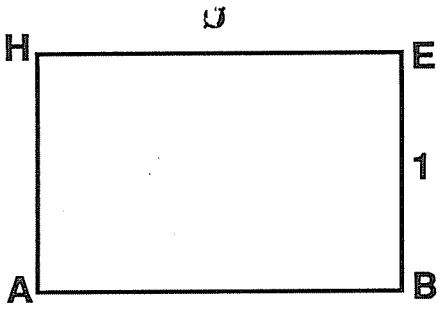
(e) Now $x(x-1) = (1)^2$ leads to $x^2 = x + 1$ or $x^2 - x - 1 = 0$.

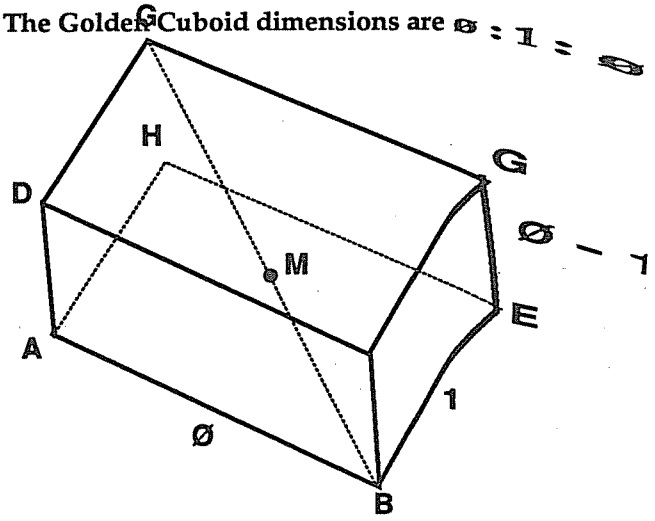
The solution is $x = \phi \approx 1.618$.

Since $x(x-1) = 1$, then $1/x = x - 1$

If $x = \phi$ then $\phi^2 = \phi + 1$
and $\phi^{-1} = 1/\phi = \phi - 1$

The Golden Rectangle dimensions are $\phi : 1$. The Golden Cuboid dimensions are $\phi : 1 : \phi$





The surface area of the Golden Cuboid is $2(\phi)(1) + 2(\phi)(\phi^{-1}) + 2(1)(\phi^{-1})$

$= 2\phi + 2 + 2\phi^{-1}$

$= 2\phi + 2 + 2(\phi^{-1})$

$= 4\phi$

The length of the diagonal BG of the Golden Cuboid is $\sqrt{(\phi)^2 + (1)^2 + (\phi - 1)^2}$

$= \sqrt{(\phi + 1) + (1) + (\phi - 1)^2}$

$= \sqrt{\phi + 2 + \phi^2 - 2\phi + 1}$

$= \sqrt{\phi + 2 + (\phi + 1) - 2\phi + 1}$

$= \sqrt{4}$

$= 2$

The radius of the sphere circumscribing the golden cuboid is

$MB = r = 1.$

The surface area of the sphere is $4\pi r^2 = 4\pi(1)^2 = 4\pi$. But the surface area is 4ϕ.

Thus the ratio of surface area of the golden cuboid to the surface area of the sphere that circumscribes it is

$4\phi : 4\pi$ or $\phi : \pi$

Magic Squares: Another Look!

If you have studied magic squares then you may have noticed the following patterns:
- any sequence of nine numbers can be arranged in the following order to make a "magic square".

8 th	1 st	6 th
3 rd	5 th	7 th
4 th	9 th	2 nd

In case you forgot - each row, column and diagonal must have the same sum.

- other arrangements which work are simple transformations of the above arrangement.
That is

6 th	7 th	2 nd
1 st	5 th	9 th
8 th	3 rd	4 th

is the same as

4 th	3 rd	8 th
9 th	5 th	1 st
2 nd	7 th	6 th

and

2 nd	9 th	4 th
7 th	5 th	3 rd
6 th	1 st	8 th

A magic square constructed with the sequence of numbers 3, 6, 9, 12, 15, 18, 21, 24, 27 looks like this:

24	3	18
9	15	21
12	27	6

Of course each row, column and diagonal adds to 45.

What sequence of numbers is used to generate this magic square? What is the magic sum?

	20	
	40	
35		

Solve each of the following magic squares by discovering the sequence of numbers and their arrangement in the square. Write the magic sum.

Easy

		15
	27	
		23

Medium

13		
17		9

Hard

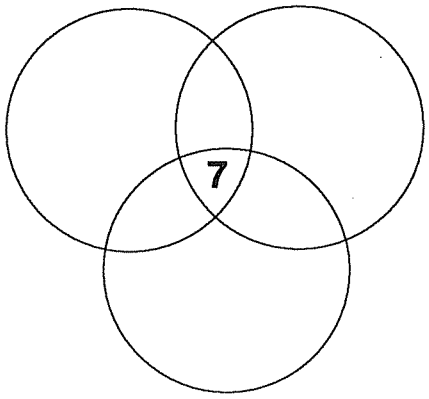
2		7
9		

Extension

Use the numbers from this sequence to fill each of the 7 sections created by this diagram so that the sum in each circle will add to the same total.

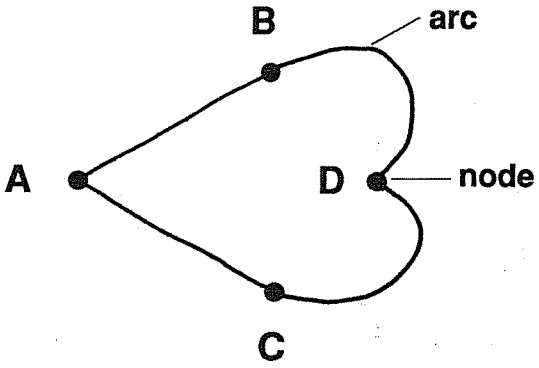
5, 6, 7, 8, 9, 10, 11

A hint is given!



Networks

This diagram is made of nodes and arcs.

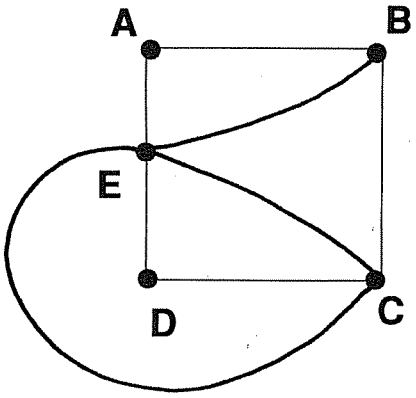


A, B, C, D are nodes

Nodes can be either even or odd

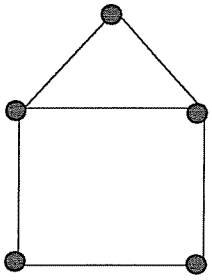
In this diagram nodes A and D are even while nodes B and C are odd. Explain why.

Which nodes are even, which nodes are odd in this diagram?

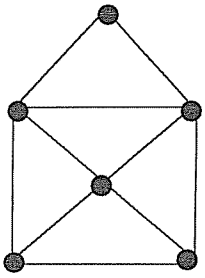


ODD/EVEN	WHY ?
A	
B	
C	
D	
E	

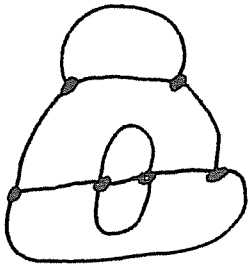
Some of the following diagrams can be traced with one continuous line without lifting the pencil off the paper or retracing any of the arcs.



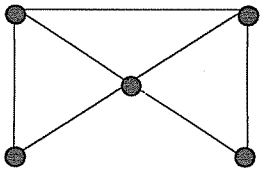
(a)



(b)



(c)



(d)

Complete the chart.

FIGURE	#ODD NODES	# EVEN NODES	Can be drawn with one continuous line ? Yes / No
(a)			
(b)			
(c)			
(d)			

Can you discover the relationship between odd, even nodes and whether the figure can be drawn with one continuous line? Write what you think the relationship is.

Create your own figures which can be drawn in one continuous line.

Problem Solving at the Puppet Theatre

Using materials commonly found around puppet theatres in primary classrooms young children can be encouraged to create and act out short (or not so short) story type problems.

Encourage children to work in groups of three or four. Allow plenty of time for them to create and plan. Provide sharing time for their production.

Cards similar to ones listed here can be used to generate story ideas.

GIVING

3 + 6 =

FINDING MORE OF SOMETHING

ODERING 5 OBJECTS

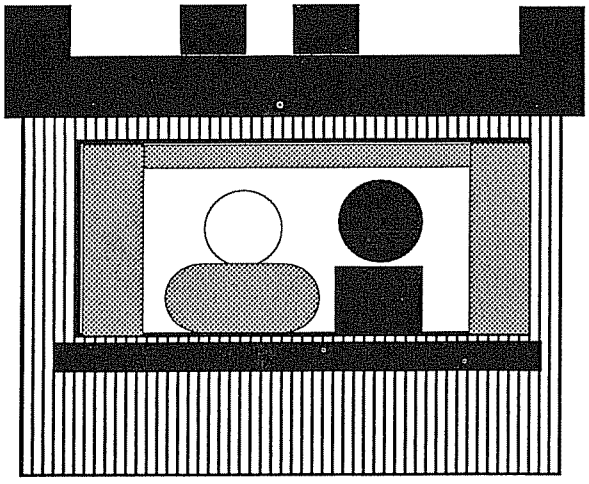
3 GROUPS of 5

SHARING 12 THINGS

Ask the children to create and act out a play which will finish with the posing of a question.

Many problem solving strategies will be used by the children in a natural way. As the teacher, sit back and enjoy the productions.

For older children, these types of activities lead to co-operative story writing.

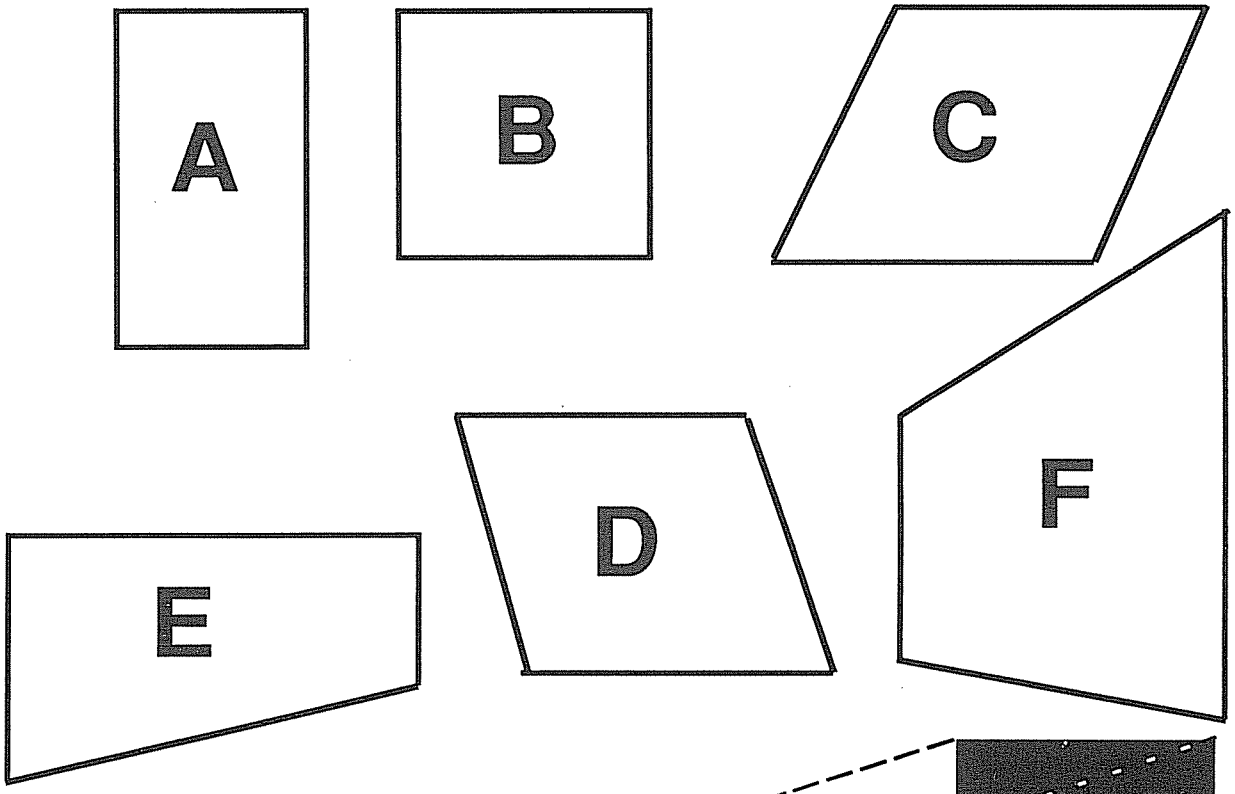


Quadshadows

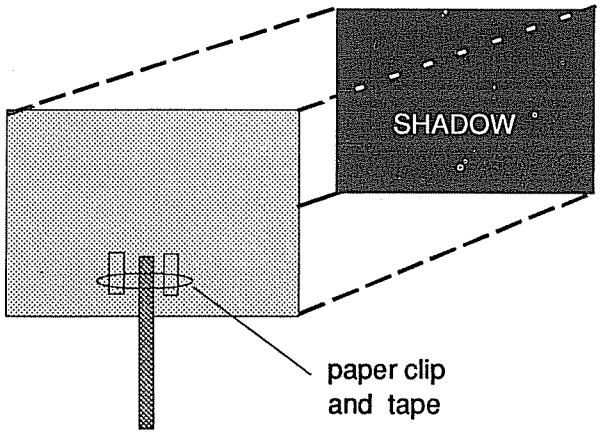
Ideas for this page came from a workshop James V. Bruni of H. H. Lehman College presented many years ago.

Enlarge and cut these 6 quadrilateral shapes out of Bristol board. Label each shape and attach a paper clip handle with tape.

Use each shape to create shadows with the following shapes:



a: square
b: rectangle
c: parallelogram
d: rhombus
e: rapezoid
f: irregular quadrilateral



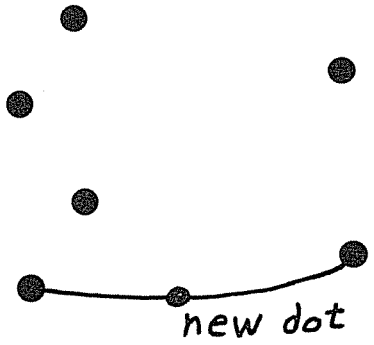
Sprouts and Super Sprouts

Sprouts

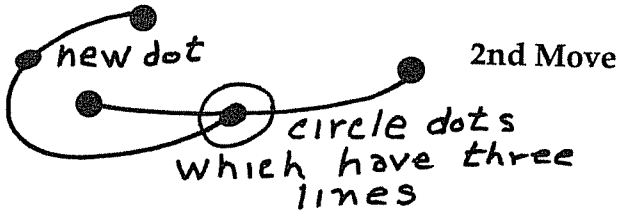
The invention of the game of sprouts has been attributed to Professor John Horton Conway and Michael Stewart Patterson who were at the time working on computer programming theory.

The game can be played with any number of dots but is best played using three dots by beginners.

- This game is for two players.
- A move consists of drawing a line joining one dot to another and then placing a new dot somewhere along the line.
- A line which is drawn can have any shape but it cannot cross itself, or cross a previ ously drawn line.
- No dot can have more than three lines ema nating from it. i.e.
- Players take turns drawing lines, placing dots
- Play continues until no further moves can be made. The last person to move, wins.



!st Move

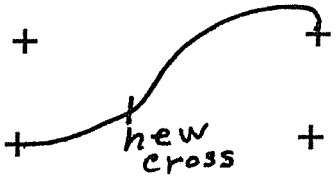


2nd Move

SuperSprouts

This game is played in a similar fashion to sprouts. Instead of using three dots, four crosses are placed on a page.

- A move consists of drawing a line joining the arms of any two crosses, and placing another cross somewhere along the line.
- Once the four arms of a cross have been used that cross becomes closed. (circle closed crosses)
- No line may cross itself or any other line
- Play continues until no further moves can be made. The last person to move wins.

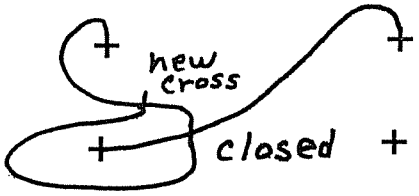


!st Move



2nd Move

Note: it has been said that the game of sprouts must last at least 2n moves where n is the number of starting dots. Prove this.

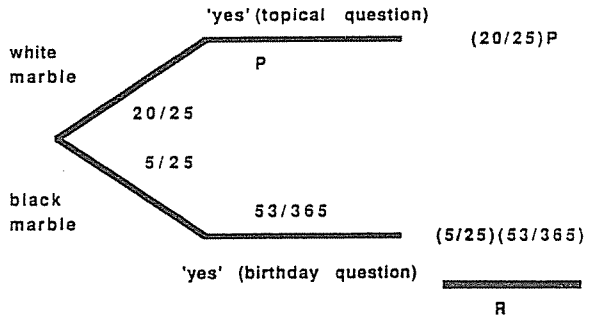


3rd Move

Mathematical Analysis

A subject has probability 5/25 of drawing a black marble and probability 20/25 of drawing a white marble.
The 1989 calendar has 365 days of which 53 are Sundays. (Each other day of the week occurs 52 times.) Consequently a person would have probability 53/365 of her/his birthday falling on a Sunday in 1989.
The probability of a 'yes' card being deposited into the ballot box is the sum of probabilities for the following two mutually exclusive sequences of events (see figure 1):

Figure 1



i) A subject draws a white marble and answers 'yes' to the topical question.
ii) A subject draws a black marble and answers 'yes' to the birthday question.

Using the product rule, the probability of (i) is the probability of a white marble being drawn times the probability of a 'yes' response to the topical question. So the probability of (i) is (20/25)P where P is the probability of a 'yes' response to the topical question. The estimation of P is the object of the whole experiment. Similarly, the probability of (ii) is (5/25)(53/365).

So the probability of a 'yes' response is
$$(20/25)P + (5/25)(53/365).$$

Once the ballot box is opened and the proportion R of 'yes' answers is revealed, the value of P can be determined from

$$(20/25)P + (5/25)(53/365) = R$$

$$P = (5/4)R - (53/1460)$$

At this point the teacher allows the subjects to enter the voting booth one at a time.

Suppose that when the ballot box is opened, there are 12 'yes' responses and 18 'no' responses. The estimate of the proportion of 'guilty' individuals is

$$P = (5/4)(12/30) - (53/1460) = 677/1460$$

There are problems connected with the above procedure of which he teacher should be aware.

There is the possibility that a subject might be tempted to remove marbles from the bag so that the probabilities of drawing the two colours of marbles will be altered midway through the experiment. There is also the possibility that a subject might deposit more than one card, or no card at all, into the ballot box. The total number of responses should equal the number of subjects. At the end of the experiment the teacher should count the marbles in the bag and the cards in the ballot box in order to determine whether either of the above situations had invalidated the experiment.

The above analysis could also give rise to a negative value for P. The equation

$$(20/25)P = R - (5/25)(53/365)$$

requires that the amount subtracted from R be $\leq R$. If the teacher suspects that R might be a small fraction, then the number of black marbles should be reduced and the number of white marbles correspondingly increased. For example, if the teacher anticipates that R will be near but not less than 1/50, an appropriate number of black marbles is 3. Then $(3/25)(53/365) = 159/9125$ which is less than 1/50. The equation involving P would be

$$(22/25)P = R - (3/25)(53/365)$$

There is a danger in using only a small number of subjects in such a survey. Firstly, the value of R could turn out to be zero, that is, no 'yes' answers are obtained. In such a case the method outlined above will not be able to generate a reasonable estimate for P. Secondly, the *actual* proportion of subjects who chose a black marble could be very different from the proportion of black marbles in the bag, a situation which would produce a considerable error in the estimate for P. Using a larger number of subjects would tend to decrease the chances of such problems occurring.

There is an alternative method of estimating P where the subjects never have to respond to the topical question.

Replace step (d) in the procedure with:

dd) Consider the topical question and establish in your mind whether you are 'guilty' or 'innocent'.
If you are 'guilty', answer the question: "Did you draw a white marble?".
If you are 'innocent', answer the question: "Did you draw a black marble?".

In this version of the experiment the probability of a 'yes' card being deposited into the ballot box (see figure 2) is

$$P(20/25) + (1-P)(5/25)$$

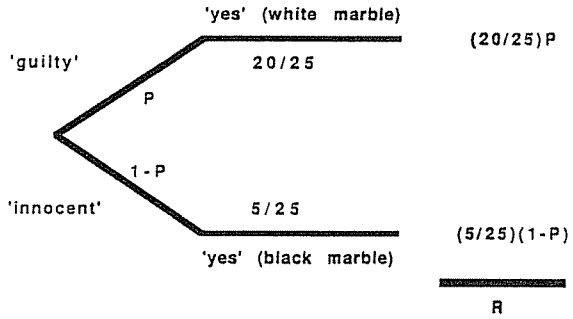
so that

$$R = P(20/25) + (1-P)(5/25)$$

$$P = (5R-1)/3$$

Again, the relative numbers of black and white marbles might have to be adjusted depending on the anticipated value of R. As well, for the reasons already discussed above, a large number of subjects should be used.

Figure 2



If the question of topical interest were one where the subjects could readily admit their 'guilt' at the end of the experiment, the teacher could compare the estimate of P to the actual proportion of 'guilty' individuals.

The reader may wish to consult an article by Paul Mullenix in the UMAP Journal (Vol. II, No. 4, 1981, pages 29-35), entitled *Randomized Response Technique: Getting in Touch With Touchy Questions*, which gives a more detailed and very readable summary of the various techniques associated with the method of randomized responses. The techniques used above are much akin to what Mullinex calls the Simmons or unrelated question model. The Mullinex article also lists a number of original papers on the subject of randomized responses.

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ON CHERCHE DES MANUSCRITS

Nous avons besoin de vos idées!
On encourage les lecteurs à soumettre des manuscrits qui examinent ies techniques dans la classe ou des questions de l'enseignement de mathématiques (écrits en français) pour la critique.

Les instructions pour les auteurs peuvent être trouvées au début de la revue.

ON CHERCHE DES CRITIQUES

Afin de faciliter la politique de la gazette à propos de la publication d'articles en anglais et en français, il nous faut des critiques pour les articles français. Veuillez envoyer votre nom à:

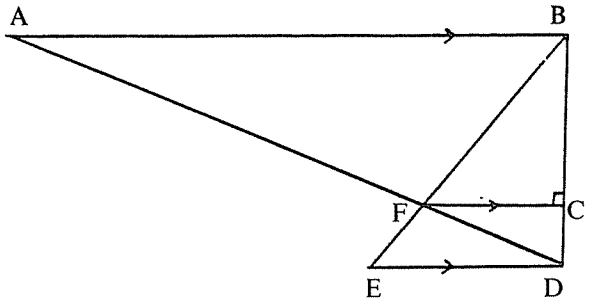
The Editor
The Ontario Mathematics Gazette
c/o University of Western Ontario
Faculty of Education
London, Ontario N6G 1G7



PROBLEMS

by
Ronald Lancaster
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Oakville, Ontario L6J 3L6

Question 1



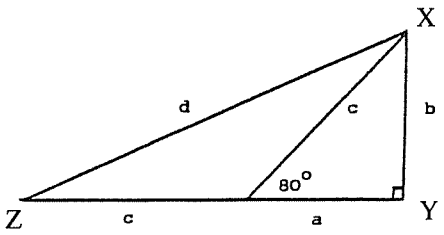
a) Referring to the diagram given above, if ED = 4, DB = 5, and CF = 3, find the length of AB.

b) If ED = x, DB = y, and CF = z, find the length of AB in terms of x, y, and z.

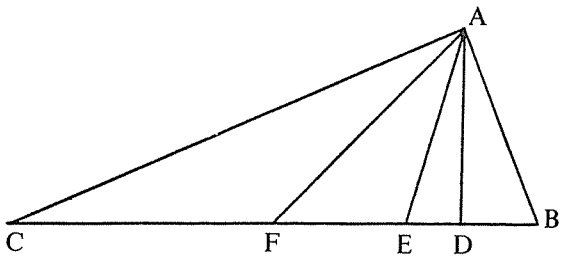
Question 2

Prove that sin80° = 2sin40°cos40°

HINT: Start with the diagram given below and use an indirect proof.



Question 3



In ΔABC, the altitude AD, the angle bisector AE, and the median AF divide <BAC into four equal parts.

Prove that ∠ BAC = 90°.

SOLUTIONS AND COMMENTARY

Question 1

a) Using the fact that ΔBFC ~ ΔBED, you should be able to show that BC = 3.75. Therefore the length of CD is equal to 5 - 3.75 or 1.25
Using the fact that ΔFCD ~ ΔABD it can be shown that AB = 12.

b) Using the procedure outlined in part a), it can be shown that AB = $\frac{xz}{x-z}$

I recently gave this question to my Grade 12 mathematics class on their similar triangles test. All of them got the correct answer to part a) and most of them came up with the answer to part b) too. They were all surprised though that the answer in part b) did not involve y. I was also surprised that the length of AB did not depend on the length of DB. Several students wrote computer programs that displayed the diagram. They varied the length of DB and we saw first-hand that the length of AB remained the same. This really is a neat problem!

Question 2

This problem is also from one of my Grade 12 tests from this semester. We had not yet developed the double angle formulas. I had used a similar diagram in class to find the exact value of sin15° and so my students immediately realized that ∠ XYZ = 40°.

Note that $a^2 + b^2 = c^2$
$(a+c)^2 + b^2 = d^2$

Subtracting the first equation from the second gives

$(a+c)^2 - a^2 = d^2 - c^2$

Expanding and simplifying it follows that

$$2ac + 2c^2 = d^2$$

Now suppose that $\sin 80° \neq 2\sin 40°\cos 40°$

Therefore $$\frac{b}{c} \neq 2\left(\frac{b}{d}\right)\left(\frac{a+c}{d}\right)$$

$$\therefore d^2 \neq 2c(a+c)$$

$$\therefore d^2 \neq 2ac + 2c^2$$

This contradicts the above result derived by using the Pythagorean Theorem. Therefore $\sin 80° = 2\sin 40°\cos 40°$ as required.

I have shown this proof to several other teachers and they were surprised to see an indirect proof used in a question like this. I use the indirect proof for proving many trigonometric identities and find that it works very well. Quite a few of my students end up using it too.

Question 3

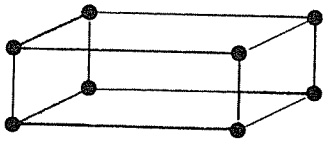
This is one of the best geometry problems that I have ever seen. I originally gave this question to my Grade 12 students as a bonus question on a test. I let them take it home and they had two days to work on it. I received no complete solutions. I was not able to solve the problem either. I sent out an S.O.S. to many other people and I ended up receiving solutions from Rob Smythe (Computer Sciences Head at Oakville Trafalgar High School) and Ian McGee and Ross Honsberger from the University of Waterloo. I requested a solution that did not use trigonometry. I would encourage you to work on this problem. I will run a solution in the next issue of the Gazette.

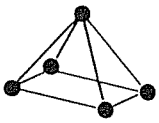
Teaching Tips

3-D GEOMETRY

There are various ways to have your class make models of different prisms and pyramids. Most teacher's editions include patterns for the nets of common 3D shapes.

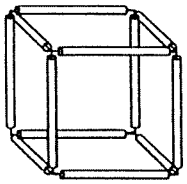
As an alternative, try using whole dried peas for vertices and round toothpicks for the edges. The peas need to be soaked overnight. Once the shapes have been built the peas dry out and the shape becomes quite sturdy, although still delicate. A group of 4 students can easily construct 4 different pyramids and 4 different prisms in one class period.





Other possibilities:

a) toothpicks and plasticine lumps as vertices
b) toothpicks and miniature marshmallows as vertices
c) plastic straws and pipe cleaner used at the vertices



I have done the peas and toothpicks with grades 5 to 8.

Suggested by:
Barb Kaiser
Princess Elizabeth P.S.
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THINKING THROUGH PROBLEM SOLVING III
Assessment of Students' Work

by
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A difficult aspect of teaching process problems is assessment. If some of our objectives in introducing students to problem solving are:

- to develop and improve students' ability to select appropriate problem solving strategies
- to encourage students to attempt new problems and persevere in a determined fashion to find solutions
- to initiate cooperative learning
- to improve students' attitudes towards problem solving
- to enable students to interpret/evaluate their own thinking

then how do we decide if these objectives have been obtained?

In a previous issue, Norrie (1988) discussed some of Randall Charles' assessment techniques which can be used to observe students while they are in the process of solving problems, (see fig. 1) as well to analyze the finished problems (see fig. 2).

Figure 1
Observing Problem-Solving

1. Selects appropriate solution strategies
2. Accurately implements solution strategies
3. Tries a different solution strategy when stuck (without help from the teacher)
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, selects and implements a solution strategy, checks solution)
5. Shows a willingness to try problems
6. Demonstrates self confidence

When recording observations teachers might prefer to use a checklist, anecdotal comments or a mixture of both. Do not attempt to observe more than six children during any one lesson.

Figure 2
A Point System for Scoring Written Work

U Understanding the problem
- 0 completely misinterprets the problem
- 1 misinterprets part of the problem
- 2 complete understanding of the problem

S Choosing and implementing a solution strategy
- 0 no attempt or a totally inappropriate strategy
- 1 partly correct strategy based on part of the problem interpreted correctly
- 2 a strategy that could lead to a correct solution if used without error

A Getting the answer
- 0 no answer or wrong answer based on an inappropriate solution strategy
- 1 copying error, computational error, partial answer for a problem with multiple answers, answer labeled incorrectly
- 2 correct solution

Questioning should usually accompany observations. The questions need to be open ended and ask about the process rather than the product. At first such questions may well be viewed with suspicion since many students believe that teachers only ask *why* when an incorrect answer has been given. With the right climate, however, students do become comfortable with the process, and we, as teachers, become better able to assess students' true understanding.

Another method of assessment is self-evaluation. This process is time consuming for the student, but allows for self-reflection of the thinking processes while providing valuable information for the teacher. Such an assessment asks students to describe *how* the solutions were obtained and *why* particular strategies were chosen.

Norrie indicated the difficulty inherent in the assessment of process problems and acknow-

ledged that, in his school board, more inservice than anticipated was required before teachers felt comfortable with the procedure.

This article offers an alternative means of school based inservice through a problem solving assessment contest in which three problems are presented together with various student responses. Using the outline provided in figure 2 entrants are to assess each student's solution e.g. 1 2 0 and **submit a copy of the form at the end of this article.** Entrants are encouraged to collaborate with colleagues and submit a group answer. The benefits of such an endeavour will come from discussing differences of opinion with staff members and providing justifications. Individual solutions will, of course, be accepted.

A panel of 'experts' will be asked to provide solutions and these, together with a rationale, will be printed in the September issue. The response most closely aligned to the 'expert' viewpoint will be published in the September issue. The winner(s) will receive a book on problem solving.

There is often more than one correct answer and the experts choice is offered, with tongue in cheek, for more discussion and deliberation. Assessment of problem solving must be taken seriously and can only become reliable through consultation and reflection. It can be discouraging to find problem solving assessed by multiple choice numeric answers (which has been evident on some Canadian provincial examinations) or by assigning marks according to a set format taught by the teacher, allowing no room for creativity. As teachers we want creative solutions. The difficulty arises because we often have diverse opinions regarding the value of alternative solutions. To this end we invite our readers to test their assessment techniques against those of the experts'.

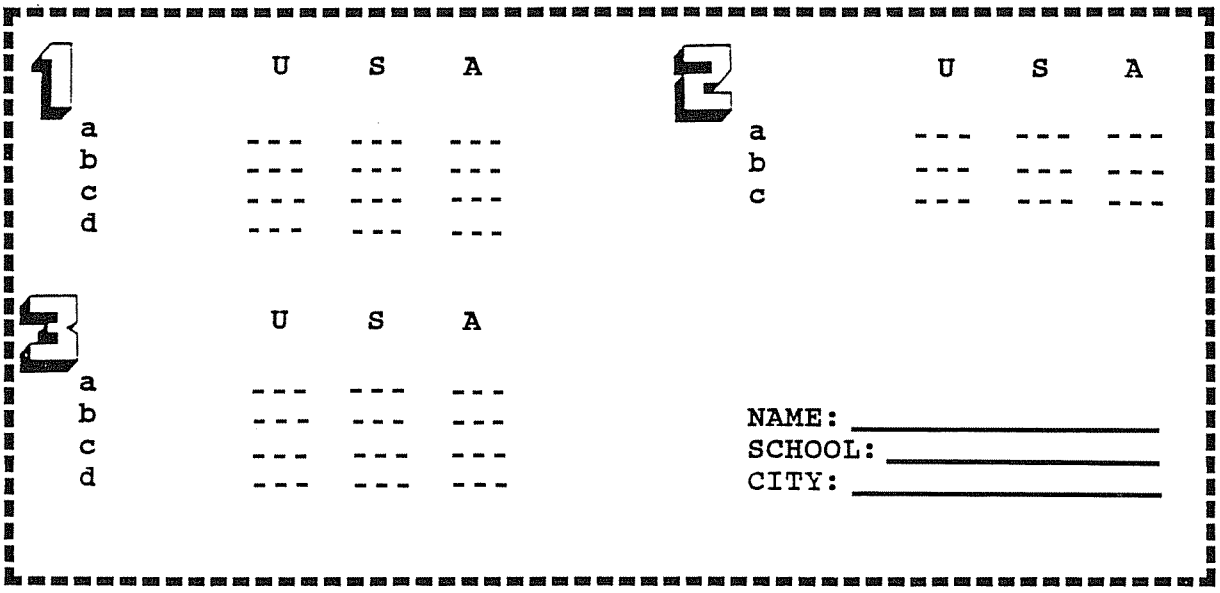
We hope you enjoy and discuss these problems with your colleagues and respond to the challenge.

References

Norrie, A. Problem-Solving: Development, Implementation, Evaluation and Beyond! Ontario Mathematics Gazette, 27 (1), 1988.

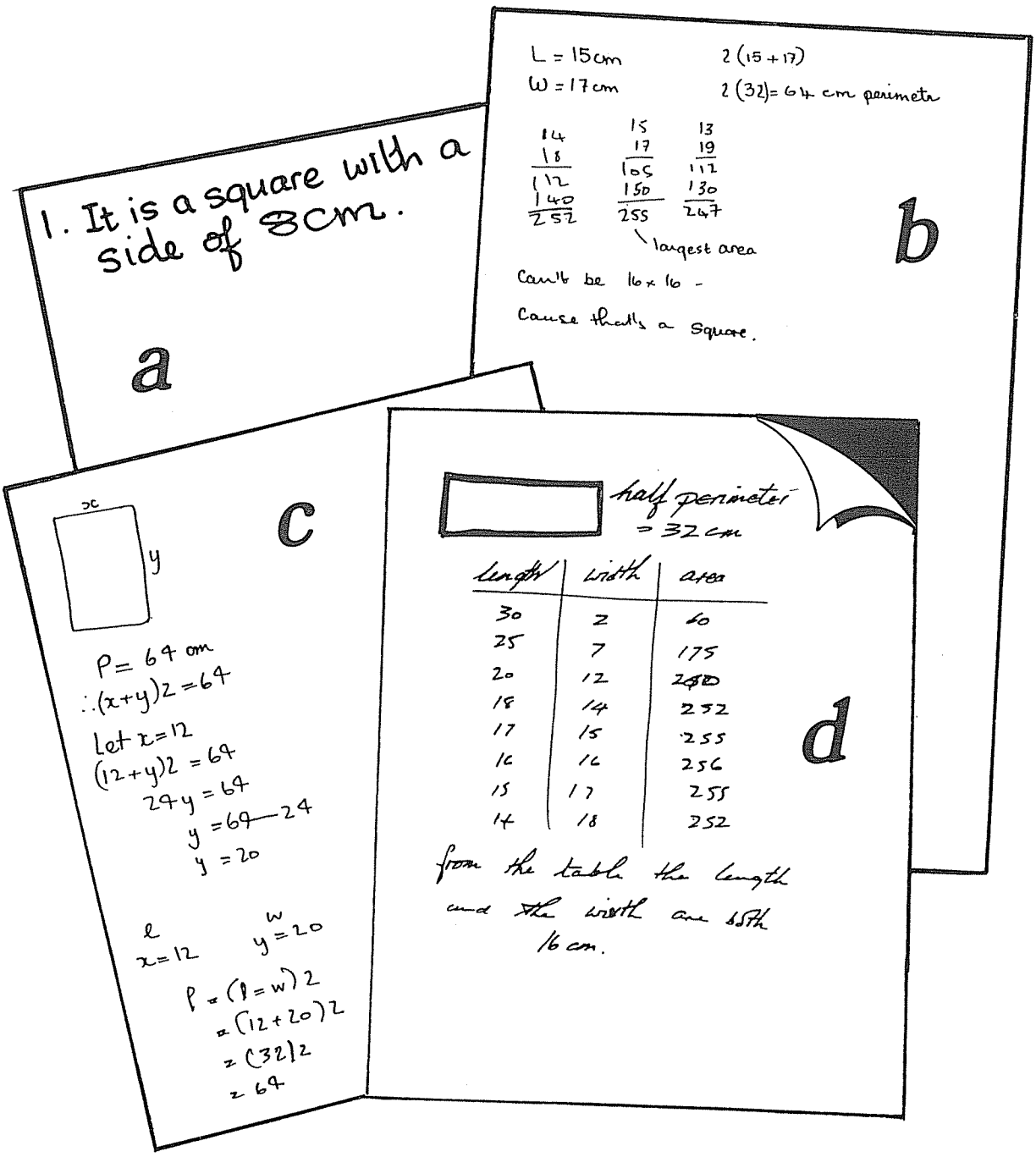
Charles, R. I. Evaluation and Problem Solving. Arithmetic Teacher. January, 1983.

In the case of multiple 'winners', the correct entry with the earliest post mark will be declared the winner. The judges decision is final.



1

The perimeter of a rectangle is 64 cm. Find the length and width that gives the largest area.



2

Bass is responsible for keeping the fish tanks in the wharf shop filled with water. One of their 55l tanks has a leak, and along with evaporation, loses 3l of water each day. Every three days, Bass adds 7l of water to the tank, and on the 30th day he fills it. How much water will be added on the 30th day to fill the tank?

let amount of water to be added be x
7L is added each day
3L is taken away each day

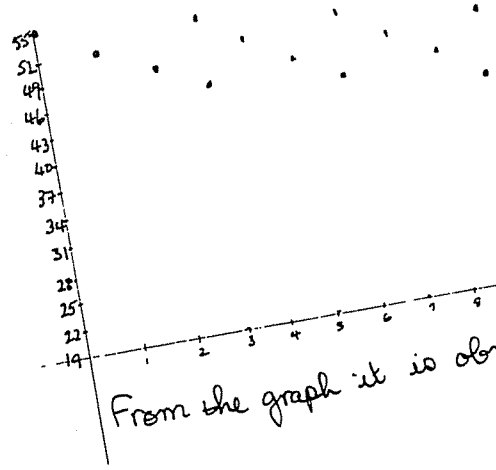
After 30day
$x + 7 - (3) = 30$
$x + 7 + 3 = 30$
$x = 20$
$\therefore$ 20L must be added.

a

Loss = 9 − 7 ℓ/day
= 2ℓ/day
$\therefore$ TOTAL LOSS = 2 × 30 ℓ
= 60ℓ

THIS IS A TRICK QUESTION
SINCE THE TANK HOLDS 55ℓ
$\therefore$ BASS MUST ADD 55ℓ

b

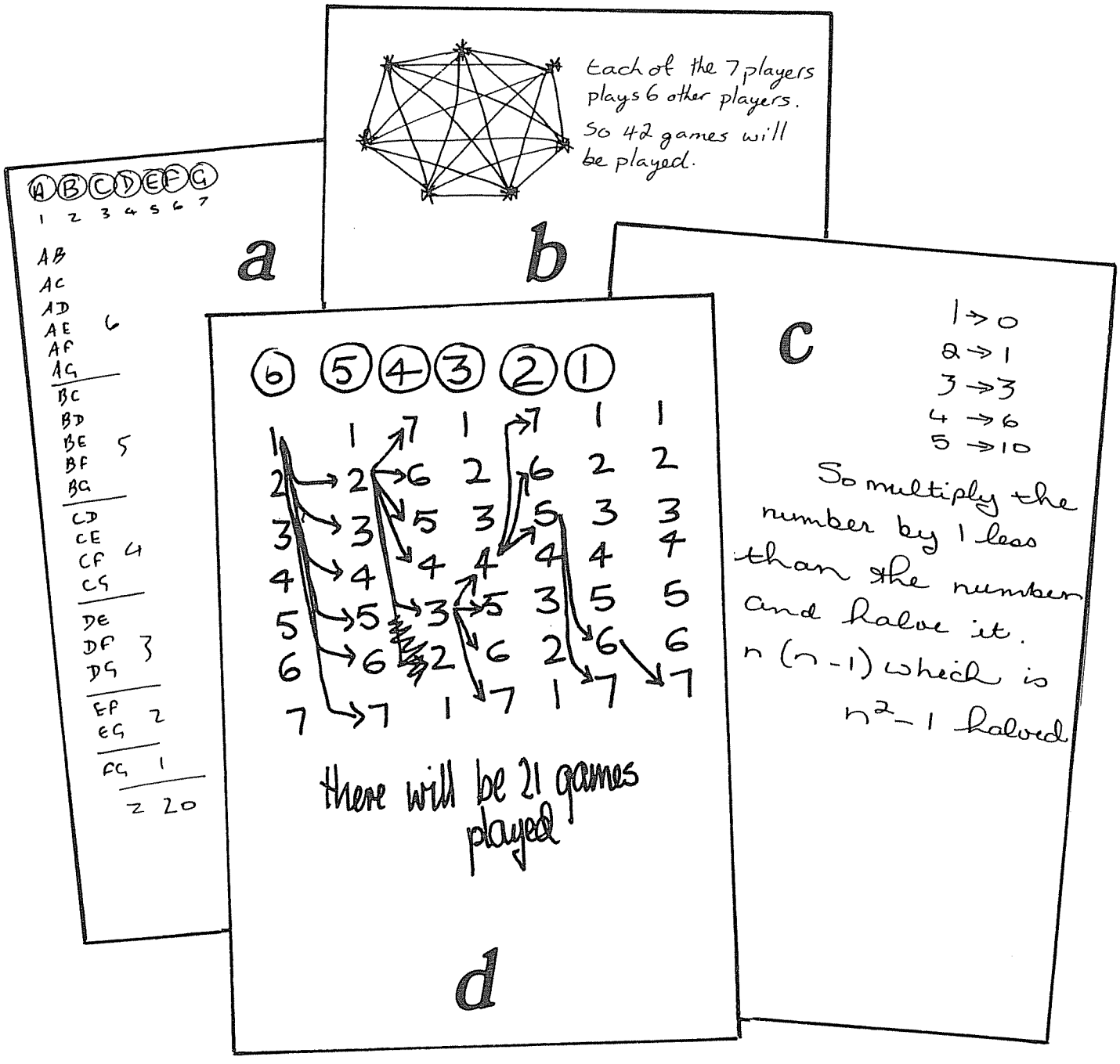


From the graph it is obvious the answer is 20ℓ

c

3

A round-robin, singles badminton tournament is to be organized. In this type of tournament every person plays against every other person. How many games will there be if 7 players enter the tournament?





NEW "NEW MATH" AND OLD "NEW MATH": TIMES CHANGE

by
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Mathematics education is under pressure to be transformed in ways as profound as those seen in the "new mathematics" reforms of the late 1950s to early 1970s. The pressure is coming from three clearly identifiable quarters: one is the subject content itself, second is the impact of technologies on mathematics and its applications, the third is the international economic restructuring of the past decade and a half.

I will make the argument that what is defined as "mathematics" and what is defined as "doing mathematics" is tied quite directly to the economic, political and social environment. These definitions in turn redefine what teachers do in their classrooms.

The reform of mathematics curriculum in the 1980s has several distinct characteristics. First, it is international, with concurrent drives for reform in, for example, Australia, North America and Britain. Second it is occurring following a period of concern with the mathematical skills of school leavers as related to the workplace, and a general focus on the transition from school to work. Third, the most vocal advocates for reform are outside the ranks of professional and academic mathematicians. Fourth, the reform is set in a context of economic efficiency. Fifth, the nature of mathematics is said to be individualistic. Sixth, the focus of mathematics is to be "problem solving" of a particular type: the manipulation of data to reach reasonable solutions based on enough, too much or insufficient information.

Allan Pitman joined the mathematics education group at The University of Western Ontario in January 1989. He was previously a Senior Lecturer in the School of Education at Deakin University, Australia. He has worked extensively on issues relating to reform of school curriculum and of education systems and is consultant to the University of Wisconsin-Madison in the evaluation of the Ford Foundation's Urban Mathematics Collaboratives project.

Comparison invites itself with the last major reform of school mathematics curriculum to sweep the world: the "new mathematics" of the 1960s. It is instructive to list some of its characteristics. First, it was international in its effect. Second, the "new mathematics" reform occurred at a time of intense military and scientific competition between the west (particularly the United States) and the Soviet Union. The symbolism of the struggle was the launch by the Soviets of Sputnik I, and the intense reaction which followed. In the United States, mathematics and science reform was supported through a Defense (Education) Act. Third, the mathematical nature of "new mathematics" was defined by professional mathematicians. As Moon (1986) quite rightly points out, the group known as the Bourbakists were instrumental both in defining the nature of mathematics and in the rapid dissemination of that definition. Its interpretation into schools was mediated by educators, who tied the formalism of the Bourbakist view of mathematics to theories of mathematics learning, particularly those of Piaget and Bruner. Fourth, the discourse of reform was embedded in the language of military power and the struggle between ideologies. Fifth, the nature of the mathematics was individualistic in that individual expertise was to be created to address the research problems to be confronted in maintaining military-scientific parity or superiority over the Communist bloc. Sixth, the form of mathematical knowledge was formal and based on strict assumptions about formal logic and its mathematical structure.

The present reform, then, is occurring in a context of political pressure based on the demands of the industrial and commercial sectors at a time of international economic restructuring. The "new mathematics" reforms was located in context of political pressure based on the relation between scientific expertise and military capabilities at a time of rapidly changing military and strategic balances. Politically, power has been redefined: there has been a shift from a concept of power based upon military

supremacy to one of power based upon economic success.

The "New Mathematics"

The content of school mathematics curriculum had remained remarkably static for most of the century prior to the late 1950s. Mathematical work dealing with the structures of mathematical systems dating from early in the twentieth century had produced no impact on what was taught in schools. With respect to content, then, the "new mathematics" was to represent a revolution in the way in which mathematical content was understood. Coming as it did in most countries (with the prominent exception of the United States) with the recasting of school mathematics as a unity instead of separate subjects of arithmetic, algebra, geometry, trigonometry and calculus, its basis in set theory provided a theoretical framework to this unity.

The "new mathematics" defined mathematics as a set of formal logical systems, with set theory as the underlying structure which provided a unity to the discipline. Set theory is isomorphic to (that is, it has the same form as) formal logic. It was at that time that formal logic in Australian schools was coopted by mathematics from English, where it was taught as Clear Thinking.

Intellectually, the underpinning of the "new mathematics" is to be found in France. The Bourbakists, as Moon points out, provided a cohering theoretical position and ideology for the New Mathematics movement. This group of mathematicians, largely French and with close ties to the United States, published anonymously under the name of N. Bourbaki from the middle 1930s. This network provided the mathematical substance of the reform, and the mechanism of its transmission, at least at the high level of the initiators of the reform efforts in the various countries.

The influence of the Bourbakists' work on the new mathematics in the school curriculum can be seen in reflecting upon the subdivisions of Part 1 of their The Fundamental Structures of Analysis, quoted by Moon:

1. Set theory
2. Algebra
3. General topology
4. Functions of a real variable
5. Topological vector space
6. Integration

(Moon 1986, p. 6)

The formalistic nature of the mathematics was readily wedded to then current theories of mathematical concept development in children in the production of school mathematics curricula. The theories of Jean Piaget provided points of connection between the Bourbakist position and assumptions about forms of logic constructed by children. The interpretation of Piaget's stages of development through preoperational, concrete and formal stages placed children's transitions from one to the next at or near the transitions from preschool to elementary school and thence to secondary schools. In consequence, interpretations of the work of Jean Piaget and Jerome Bruner were of the greatest importance in a further reinforcement of the division between elementary and secondary mathematics curricula.

Thus the elementary school curriculum was heavily influenced by a variety of notions about concrete thinking and the introduction of a range of concrete aids to mathematical learning. The extent to which materials such as Dienes' blocks or the Cuisinaire materials provided concrete manifestations of common world mathematics is rather problematic. The notion of a concrete operation, as applied to the Cuisinaire rods, was used as the theoretical basis for a quite abstract mathematical system. Confusion entered when it was not clear whether Cuisinaire was the model of mathematics through which children were to develop mathematical concepts, or a model for them to do so, or whether Cuisinaire in some sense was mathematics.

The secondary curriculum on the other hand assumed that children were at the formal stage of mathematical thinking, and was on the whole very abstract in its content. From Year 7 onward, mathematics courses rested on the formal consideration of set algebra, with a proliferation of symbols and often incredibly complex language. The thrust of such curricula was unashamedly in the direction of the "under-standing" of the abstract, formal structures of mathematical systems. The isomorphism of set algebra to the algebra of formal logic was a rallying point: it was at this time that logic as Clear Thinking was lost to the Victorian English curriculum and coopted by mathematics as a form of problem solving.

Who defined the "new mathematics," and what was the context of its introduction? The demands for curricular reform came from the tertiary academic community, or at least a section of it. Governmental pressure was mounting in the context of an arms race which pitted two ideologically opposed camps against each other. Communism was being confronted in turn in Korea, Malaya, Africa, Cuba, Vietnam, Indonesia and elsewhere. It was a time when the so-called Domino Theory of communist expansion through South East Asia was seen by many as a real threat to Australia. It might be argued that at the same time, the Soviets held a world view of themselves being increasingly encircled by an atomic-armed cordon of land and sea bases to the West, South, East and across the North Pole. In international relations, power was military power. In memory, people think of the Russian launch of Sputnik I as the stimulus for the wave of educational reform that swept the Western world at that time. It is more appropriate to think of Sputnik as a symbol of the time and recognise that the reform movement was already under way. What did flow from the advent of Sputnik was the release of huge amounts of funds for education: in the United States, where education could not be directly funded federally, the Congress passed the Defense (Educational) Bill.

What then was the purpose of this support of educational reform? There were several thrusts. One was to emphasize the nature and values of Western democratic society. A second was to prepare more and better scientists and engineers. A third was the increase in the number of teachers being trained to respond to the post war "baby boom" and increased retention rates. The shortage was particularly acute in mathematics and in the sciences.

The Mathematics-as-Problem-Solving Movement

A new radical challenge is now being mounted to what is understood as mathematics. It takes the form of a particular notion of problem solving being presented as the basis of mathematics. No longer is a formal logic the centrepiece, but in its place is the manipulation of data in order to reach decisions. While the strict formalism of much traditional mathematics (and in this I include the "new mathematics") has not quite been replaced by Feyerabend's "anything goes" there is certainly an undercurrent implicitly sympathetic to that idea.

The subject of mathematics has made rapid advances, particularly in the areas of discrete mathematics and in probability and statistics. In this, the computer has been a significant contributor in enabling forms of analysis otherwise impossible to perform, as in the case of the exhaustive proof of the four colour map problem. The advent of powerful computers has made sophisticated analyses of large data bases readily available, even at the school level. Calculators have changed the relation between time needed in computation and in problem solving.

The response to these pressures to reform mathematics curriculum has lacked the theoretical coherence of the "new mathematics" phenomenon: there does not appear to be a well expressed view of the nature of mathematics emerging. Rather, there is a view of the subject which is obscured by the general discussion of relevance, motivation and retention of students. The present redefinition of mathematics is, I believe, as fundamental as that of the "new mathematics," but lacks as yet a justifying theoretic framework. The new view is that mathematics is an empirical activity, and consists of reaching firm of tentative solutions to problems based on data and its manipulation. Such a position is closely tied to the economic transformations of recent years and the associated redefinition of power as economic in nature: just as the formalist, abstract mathematics of the 1960s was appropriate to the perceived demand for research scientists and mathematicians, so the empiricist position is a response to the changing nature of the workplace.

An interesting insight into the fundamental difference between the responses of curricular constructors of the 1960s and 1980s is gained by seeking what is "new." In the 1960s, it was a changed view of what constituted mathematics which was the new content. The retained content was recast as expressing in a very formal sense the underlying principles of a unified mathematics which was based on the new basic content: sets. In the 1980s, the new content is not of the same type. Rather, it is different content which is to replace the old: statistics and probability, for example are characteristic of the new, and are to replace redundant content rather than underpin it. What is fundamental in this reform is not in

the listing and formal structuring of content, but is in the processes which are claimed to constitute "doing mathematics." The proponents of change want mathematics characterised by cooperative learning tied with competitiveness, a concentration on handling data, a willingness to explore solutions, capacity to reach solutions based on too much, sufficient and event insufficient information. These are skills that are commonly demanded for the workforce to cope with rapid changes in the workplace.

The economic crisis which has been associated with the international restructuring of production and trade is asserting a direct influence on the present reforms. This has at least two motive forces. One is the changes in production due to the introduction of new technologies to production, and the consequent redefinition of required work skills. The other is the loss of old industries and the creation of new ones as international trade restructures national economies.

The movement for the implied redefinition of mathematics as an empirical, "laboratory" science (in contrast to the "theoretical and abstraction oriented reform of the "new mathematics") is world wide, and there are commonalities in rationale and direction which cross ideologies of governments: in Thatcher's Britain, Reagan's USA and Hawke's Australia the thrusts are similar. Both the left and the right are basically arguing for the same reforms in content, for the same reasons. The future is now, in the sense that preparation for the future is the purpose of schools. Discussion is based on the notion of economic war. Everywhere the human capital argument seems to hold sway.

Equity and quality we are told are not in competition. In the United States, for example, the American documents A Nation at Risk (National Commission on Excellence in Education, 1983) and Everybody Counts (NRC, 1989) captured well the push that we can have both more and better mathematics in schools. The NCTM Standards document (1987) provides a detailed statement of the position. This concern provides a glimpse of a shift in the context of reform: whereas the "new mathematics" was embedded in the ideological struggle of a "good" democratic West and an "evil" communist East, the "problem-solving" movement is enmeshed in the economic struggle between "inefficient" and "efficient" countries. Thus the requirement that schools demonstrated the inherent superiority of the system in overcoming inequalities is being supplanted by the demand that they produce graduates who are able to raise national economic efficiency.

An emergent curricular outcome of this position is the demand for a new type of streaming. It is one in which the content is "the same" for all students, but the depths of study will vary according to abilities and developmental readiness. The clearest example of this to date is the University of Chicago Mathematics Project, which defines a secondary curriculum in which all students are exposed to the same content - but the depth of treatment is the differentiating feature. In its own subtle way, a push for a core curriculum is consistent with this trend. And trend it is: we forget that this current reform has been under way for nearly a decade now.

What then appear to be the purposes of the current support for educational reform? As with the old "new math" there are several. They are however not the same. One is to emphasise the need for efficiency and competitiveness in industry and commerce. A second is to produce workers better able to adapt to current and future changes in the workforce. A third is to provide more teachers of mathematics and science to support increased participation rates of senior secondary students in mathematics and science courses.

Conclusion

Both the old and new "new mathematics" are responses to a complex of pressures which stem from the changes in the subject itself, from the influence of new technologies, from political and economic transformations and from shifts in the relative influence of psychology and other interest groups on the development of curriculum.

The present reform differs from the old in that it does not have the intellectually rigorous underpinning of the 1960s reform. The content changes respond to the material changes in schools and the rest of social activity wrought by the availability of calculators and computing facilities. It reflects the changed definition of political power, from one of military superiority to an economic one, which is layed out by increased attention to

collaborative activities between schooling and industry. The structures of the emergent curricula pay attention to, rather than build from, specific theories of child development.

The attempt at reform is profound in its attempt to change the meaning of school mathematics. Whereas the old "new math" attempted to provide a rigorous theoretical underpinning to all mathematical activity, the new "new math" aims to transform the subject in schools from a study and application of formal logical systems into an empirical model more attuned to the idea of a laboratory science.

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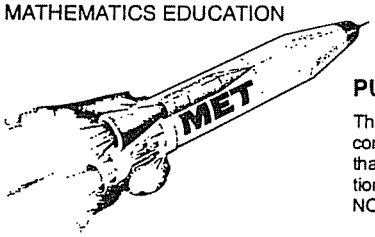
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MATHEMATICS EDUCATION TRUST

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PURPOSE

The Mathematics Education Trust will receive contributions and use these funds for projects that further excellence in mathematics education and support the purposes for which the NCTM was organized.

KINDS OF PROJECTS

Innovative projects showing creativity and adventurousness are encouraged; they may be supported from special funds. Possible kinds of projects include special interest conferences, nontext learning materials, research projects, the publication of monographs, consultants for teachers, lecturers for students, scholars in residence, exchange teachers, and teacher centers. Projects funded to date include the preparation of career education materials, lecturers for students, videotapes, scholarships, and classroom units.

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INTERVIEW

Taped Talks with Bob Smith

... Dr. Michael Mitchelmore

This interview took place last March when Dr. Michael Mitchelmore visited the Faculty of Education, The University of Western Ontario. Dr. Mitchelmore is presently working in a computer centre in Munich, Germany. He has researched and written extensively about children's understanding of geometric concepts, particularly three dimensional representations.

R.S. Mike, tell us a little bit about your teaching background.

M.M. I started as a math teacher in Ghana. I went there on a three-year contract and stayed for nine years because I got involved with curriculum development.

R.S. I taught in Nigeria in 1965 and it seems to me the text-book I used was by Durell, written in the twenties. Surely you had something better than that in Ghana?

M.M. We had a book by Snell and Morgan, written in the fifties. (Snell was actually teaching at the school where I went to teach.) But this was still based on the British mathematics curriculum, and there was a need for something more suited to the African situation. Teachers from several schools joined together to develop a more appropriate text.

R.S. The academic content still had to meet the examination requirements laid down and monitored by British institutions?

M.M. That's right. There was a West African Examinations Council that ran exams very similar to those in England. We had to prepare a syllabus that was acceptable to them.

R.S. Where did you go when you left West Africa?

M.M. After a couple of years at Ohio State University in Columbus, I went to Jamaica, first to do research for my doctorate and then to work there. I worked in the Ministry of Education for a while and then moved to the University of The West Indies. One of my responsibilities was to assist the teachers' college tutors in the development of their mathematics curricula.

R.S. This role also gave you an opportunity to do some interesting action research. Take a moment to describe this work.

M.M. Sure. One year I gathered together twenty student teachers from two colleges and we looked at the general topic of teaching geometry. I was especially interested in the problems children have in visualizing and drawing figures and how this problem might affect their learning of geometry.

R.S. You encouraged your student teachers to go out in their two or three week practicum and teach some geometry.

M.M. That's right. Every teacher had to do a small research study during their practicum. They usually didn't know what to do, so, they were very happy for guidelines, a framework, and a few bits of advise as well as some money.

R.S. They taught a variety of topics in grade 3 through grade 10. Results were gathered and some general observations made. What were those observations?

M.M. The major observation was that although children did have difficulties with the spatial concepts involved in learning geometry, they could learn geometry provided the teaching was appropriate.

R.S. You felt good about the project.

M.M. I did. So did the student teachers. They had enjoyed the experience and felt good that the children had done so well.

R.S. Did this feeling of doing well come because it was geometry or because they were involved in research?

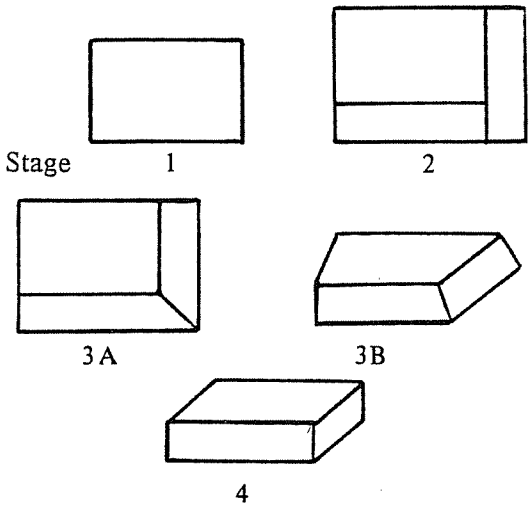
M.M. Funnily enough, both geometry and compulsory research study are usually regarded by student teachers as a burdensome waste of time! I think the fact that they were working together made a big difference. They all reported that children liked geometry. Children like playing with ideas, even those who have difficulty with arithmetic. They seemed to be able to grasp the more general ideas, providing the teaching was based on concrete objects, visible patterns and interesting activities.

R.S. Mike, let's change our direction, and go back to your doctoral work at Columbus, Ohio. Please take a minute to briefly outline it.

M.M. My research was concerned with 3-dimensional representations - how do children draw mathematical solids, cubes, cuboids, cylinders and the like. This problem had been of interest to me from my time in Ghana. There I found that intelligent and high achieving math students had great difficulty with geometrical problems in three dimensions. They often couldn't get started, because they couldn't represent the problem situation on paper. I wanted to know why this was so difficult. As a start, I tried to document the stages through which children go in drawing simple solids. After this research and a couple of validation studies, I think I've managed to get a fairly secure idea of the pattern of development. Figure 1 illustrates some typical drawings of various stages in drawing a cuboid.

Figure 1

Drawings of a cuboid:



R.S. What were your experimental groups and where did you do your validation studies?

M.M. I developed the test in Jamaica, came back to Ohio State and tested some children in Columbus; then I had the chance to go to England and test some children there; and later still I tested a small group in Germany.

R.S. From these four samples, did you come up with any generalizations?

M.M. The same sequence of development seemed to occur in all the four samples. The rate of development was constant (about a quarter of a stage per year) in all four samples. But the stage reached by the average child was very different in the different cultures. The difference between the average Jamaican child and the average English child was about 6 years. The average difference between an American and Jamaican child was about 2 years.

R.S. Could you suggest some reasons for this difference?

M.M. There are lots of suggestions as to what the causes may be. The obvious thing, it seemed to me, was in the demand of the different cultures. In the schools that I visited and in the lectures that I experienced in Ohio, the emphasis on visual representation was far less than in the English education system.

R.S. If I could put it in popular terms then, not only do the English want literate and numerate citizens, but what we might call "graphical" citizens. What results did you find in your sample of German children?

M.M. I only managed to test one group of 11 year olds, but they were even better than English ones. The general environment in Germany seems to be very technically oriented and I think children feel that from an early age.

R.S. At Stage 3, children are beginning to develop a visual representation of a cuboid that is more in line with what we would expect at an adult level, your Stage 4. What were some common difficulties?

M.M. In Stage 3A and 3B, the children have begun to use a diagonal to represent the depth dimension. But they do not use parallels to represent lines which on the solid itself are parallel.

R.S. Why is that?

M.M. I've conducted several experiments to try to answer that question! In one, I took a group of children that were drawing at Stage 3A or 3B, and observed very closely what they did when they drew a cube. I thought perhaps they didn't know what parallels were, but found very quickly that this wasn't the case; they could recognize parallels and they could draw parallels. I thought that they perhaps didn't notice that the edges on a cube were parallel; this was also false. I thought that in their haste to do the drawing they didn't notice that there were parallels on the cube; this was also not the case. I found that what they were trying to do was to draw a quadrilateral in which the opposite sides were equal in length. Now that should give you a parallelogram of course. But they didn't know that they could do that very easily by drawing parallelograms. They tried all sorts of different ways to get the opposite sides equal in length (usually by some sort of iteration process) but very few children drew parallels.

R.S. They didn't look at the cube and see that the edges were parallel?

M.M. No. They were operating on the picture itself to try and produce something which satisfied an important property that they knew about - the equal opposite sides - and they totally ignored another known property - the parallel opposite sides - which would have been more useful. Some of the children did draw one or more parallels, and they were usually able to draw the edges more accurately than the other children. I guessed that some time later in their lives they give a better drawing method.

R.S. They would do this themselves, when they reach that point in their development?

M.M. That was what I conjectured. In other testing that I've done, I've occasionally noticed children making similar discoveries tell you their drawing doesn't quite look right, but cannot say why. Every now and then a child will try something different and produce a better representation of the solid and immediately notice that it is better. They have discovered something about their drawing and not about the solid that they are trying to represent. It's almost an accidental thing.

R.S. As a teaching strategy, you suggest that the pupil would be able to understand and learn better, not by going back to the real world and looking at the solid in front of them but rather to direct their attention to the drawing and its shortcomings?

M.M. That's right. It seem rather heretical but there it is!

R.S. It is obvious Mike, that we have uncovered a fascinating topic to you. Thank you for sharing some thoughts with us. Good luck in your continuing investigations.

M.M. It was a pleasure talking to you, Bob.

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PATTERN AND PLACE: AN INTRODUCTION TO THE MATHEMATICS OF GEOGRAPHY
K. E. Selkirk, Cambridge University Press, New York, 1982.

This book is a result of several projects in the United Kingdom, in particular, the School Mathematics Project, and the School Council Project: The Mathematics Curriculum - A Critical Review. Although the book is 6 years old I would guess that few mathematics teachers know of its existence. This is unfortunate because it provides a wide spectrum of applications (albeit all to do with geography) of many of the topics that are part of presently existing math courses as well as new courses such as the OAC Finite Mathematics.

Because of the fact that it deals with many different mathematical topics and how they relate to various aspects of the study of geography, it will require a little bit of study on the part of mathematics teachers who are unfamiliar with the geographical concepts that are presented. However, the explanations are usually sufficiently clear and accompanied by enough illustrative diagrams to make the effort required very worthwhile.

For example one discussion centers on the explanation of why there are problems with political unrest in Corsica (a French island possession) but not in Sicily (an Italian island territory). By using transformations to translate, reflect, rotate and enlarge the outline maps of the two countries it is easy to illustrate that Corsica is much further (relative to the size of the country) from Paris (the capital) than Sicily is from Rome. The diagrams that accompany this discussion make a very nice illustration of the use of transformations and help show an application of a topic that can be difficult for many students to grasp if presented in an abstract sense.

Other mathematical topics which are utilized in applications include (but are not limited to): coordinate geometry, graphing, area, vectors, points and planes, geometric constructions, conic sections and networks. Even a topic like network theory is treated in a very practical way be considering the problem of a sanding truck sanding each road in a subdivision exactly once.

It would be an exaggeration to claim that this book should be on every math teacher's bookshelf; however, if you are looking for a source of applications that are comprehensible to the students and a little different to the ones that most of us are familiar with then this book is worth a serious look.

Reviewed by Eric Wood
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"BREAKING THE CODE"
by Hugh Whitemore performed by the NAC English Theatre at the National Arts Centre, November 24 - December 10, 1988.

Editors Note: This review marks a first for the Gazette! We have never published a review of a play before and yet this one seemed so compelling that we decided to include it in the book review section. Special thanks to David Handley for bringing it to the attention of our readers.

It is a rare occasion when a film or play is produced with mathematics or a mathematician as its focus. I can remember an old movie on Omar Khayyam, which emphasized romance and poetry rather than the Persian's mathematical works. However the play I saw on its last day in the nation's capital combined the tragic story of Alan Turing's personal life with the splendid achievements of his magnificent brain. More than anyone else, Turing was responsible for cracking the German High Command's Enigma Code (for

full details you are recommended the book "Alan Turing, the Enigma" by Andrew Hodges).

The play was first presented on Broadway in 1986; if the actors there were as professional as the ones in the Ottawa performance, then New York theatre-goers were treated to a tremendous show. Tom Wood played the title role of a seedy little man in a rumpled suit, who was only able to relate to other people when he was describing his mathematical investigations. His explanation of the relationship between the Fibonacci sequence and the spirals on fir cones came as a revelation to most members of the audience. Many of Turing's contemporaries - Hilbert, Russell, Gödel - were "name-dropped" and a couple of quotes that I remembered were - "This room stinks of math" and "the teaching of math is NOT the only way to judge the quality of a school." Tom Wood was able to enliven his basically tragic story with many humorous moments.

The tragedy, as those of you who heard Peter Hilton's address at the Scarborough Conference (or heard the extended version in his lecture at other OAME (NCTM) gatherings) was, of course, Turing's revelation to the police that he was having a homosexual affair. This disclosure came while Turing was giving information to the police on a rather minor burglary of his home. His trial and conviction made headlines in the "News of the World" (albeit only in the Northern edition, to Turing's chagrin). Even here some humour was found - Turing/Wood noting that he was worried about the headline "Accused had powerful brain." It was the past tense that worried him.

Among the supporting actors I must cite Barbara Worthy as Pat Green - the girl he could have married (except that he did not wish to live a lie) and Donald Davis as Dilwyn Knox, another of his co-workers on the Enigma Code at Bletchley.

Director Bob Baker received the Dora Mavor Moore award for outstanding direction of "B Movie - the play" in Toronto, Edmonton, Edinburgh and, later this year, New York. He deserves similar honours for "Breaking the Code."

If you should ever have the chance to see this piece of mathematical theatre, I enjoin you to do so -- both for its theatrics and its mathematics.

David M. Handley

NOTE: Alan Turing committed suicide, at the age of 41, in 1954. If he were still alive today, one wonders how much mathematics he would have created!

CALL FOR ARTICLES

The theme for the December, 1989 issue will be
STATISTICS & PROBABILITY.
Authors are requested to submit articles
(both short & long, practical and theoretical)
for all ages and ability levels.

Obituary

PETER ULLRICH - 1946-1989

The education community is greatly saddened by the sudden passing of Peter Ullrich, January 12, 1989.

Peter began his career at Vincent Massey School teaching mathematics and guidance. He was a founding member of the Ottawa Association of Teachers of Intermediate Mathematics as well as council member and president of C.O.M.A. He was an elementary mathematics' consultant with the Ottawa Board of Education (1978-1984). He made an outstanding contribution to O.A.M.E. serving as a Provincial Counsellor and Director from 1976-1982. He contributed to the ABACUS and was an organizer of the 1977 Annual Conference held in Ottawa. In 1977 Peter received the Queen's Silver Jubilee Medal in recognition of his contribution to Mathematics Education.

Always seeking a new challenge, in 1980 he started touring elementary schools with three PET computers. Computer education quickly became his main career interest and in 1984 he was appointed Elementary Computer Education Officer, a post he held for four years. Through his efforts the groundwork was laid for computer education in the 1990s.

In September 1988 Peter was assigned to Alta Vista School where he taught mathematics and science.

Peter's talent for organization was an asset to many community organizations with which he was involved. He served as coach, manager, and president of both soccer and hockey associations. Most recently, he was director of the Ottawa International Hockey Tournament.

Peter was a devoted husband and father. He will be remembered as caring, kind and generous; his sense of humour, stories and easy manner were his trademarks. Peter packed a great deal of living into forty-two years, more than most of us in a lifetime.

Peter's dedication and commitment to children, staff and community will be remembered. We will miss him.

The Peter Ullrich Memorial Fund has been established and will be used for a student in the COMA Region for educational purposes. Anyone wishing to make a donation may send a cheque, payable to "The Peter Ullrich Memorial Fund", to M. Ann Jones, Ottawa Board of Education.



COUNCIL COMMENT

by
Bob Robinson
114 Dromore Cr.
Hamilton, ON L5N 3L1

These comments are based on the Council Meeting held at the Ramada Inn - Airport West, on February 11, 1989.

IN MEMORIAM:

It is with great sadness that we were informed of the sudden passing of Peter Ullrich on January 12, 1989. Our friends in the north-east region will sadly miss Fabien Lemieux, an education officer with the Ministry of Education. Fabien passed away on February 9, 1989.

LEADERSHIP SEMINAR, 1989

The Leadership Seminar will be held from Wednesday evening, October 25 through Friday, October 27, 1989 at the Ramada Inn - Airport West. The theme will be: "Moving Into the Nineties - Implementing the Standards."

Annual Meeting 89 - Ottawa
Plans are on target and going extremely well. The organizers hope we come and join them in their celebrations in Ottawa.

Annual Meeting 90 - Hamilton
All chairpersons have been selected. A planning meeting with NCTM is scheduled for March 31 and April 1, 1989 in Hamilton. The 1990 Hamilton meeting will commence Wednesday evening, May 9 and continue through noon on Saturday, May 12, 1990 at the Hamilton Sheraton. If any readers of this "Council Comment" are planning to attend the NCTM annual in Orlando, please contact George Knill or Bob Robinson. We need some help at the Hamilton Regional Meeting table to promote the conference.

OAME Delegate to NCTM
Pat Rogers will be the delegate and Jan Cornwall and Bob Robinson will be the alternates to the delegate assembly in Orlando.

NCTM
Jan Scully reported how to get materials from NCTM and encouraged membership before the end of March in order to obtain a free copy of the new "standards" document. Jan encourages OAME chapters to find a member interested in arranging a NCTM display and the sale of NCTM materials at their mini conferences.

Conference Guidelines
Arnie Carefoote is putting together a set of conference guidelines. Any input that you can provide Arnie would be appreciated.

CHAPTER REPORTS

COMA
The annual January mini-conference was held on January 23, 1989 at Confederation High School. The workshop/conference attracted 150 participants. Planning has begun on another mini-conference, in the Fall of 1989. The tentative dates are October 4 and 5.

Golden Section
The new chapter representative is Leslie Khan. He informed council that they will be hosting an afternoon mini-conference at Brock University at the end of April. The chapter will host a meeting to honour Warren Sebben, retiring consultant for the Lincoln County Board of Education, at 4:00 p.m. on April 25 at Brock University. The guest speaker is Anthony Ralston. Contact Eric Muller for more information.

GVMA
A "Special Spring Conference" will be held Saturday, April 8 (9:30 a.m.-4:00 p.m.) at the William G. Davis Computer Centre, University of Waterloo. Six innovative computer labs will be open for brief demonstrations and ample opportunity for hands-on experience. Information can be obtained from Dean Murray (416) 878-2839 or Tom Rajnovich (519) 576-5100.

NOMA
NOMA welcomes our new chapter NWOAME and will assist them any way they can. Mickey Sandblom has circulated information and an invitation to join the "gang" in Ottawa in May. Transportation pools are being encouraged. In order to promote OAME and NOMA throughout northeastern Ontario, and to recognize student excellence, NOMA is sponsoring cash awards to top individuals in the Canadian Mathematics Competition for the first time this spring.

NWOAME
The first annual meeting was held on December 3, 1988 at Lakehead University in Thunder Bay. The following Executive 1989-90 was elected:
Chair: Marilyn Hurrell (OAME Rep.), Vice Chair: Bill Otto, Secretary Treasurer: Greg Geddings, Councillors: Wayne Fletcher (Thunder Bay Secondary), Doug Cole (Confederation College), Gerry Vervoort (Lakehead University), Dale Cooper, Bill McKay (NWO East-Secondary), Molly Stoktini, Rachel Lalaone (NWO East-Elementary), Still Required: a councillor for NWO West-Secondary. A mini-conference and general meeting is planned for the third week in September.

PAME
PAME is working on amalgamation with representatives from the Dufferin Peel R.C.S.S.B., the Halton and Etobicoke Boards of Education. Because of the change in membership, we are looking for a new name for their organization and are running a contest to this end. Mathfest 88 was held on October 27 and many new memberships were generated. They are presenting "March with Math" on March 2, 1989 at Phillip Pocock Secondary School from 3:30 to 6:00. The Peel Math Contest sponsored by PAME was written on November 16, 1988. Over 2500 students participated. Contribution of good contest questions would be appreciated. PAME has run into difficulties with the scoring of the contest results. Any suggestions on what works well can be directed to Mary Lou Kestell.

PINE RIDGE
A mathematics mini-conference is scheduled for Wednesday, April 5, 1989 from 3:45 to 8:00 p.m. at General Vanier Secondary School in Oshawa. Information

can be obtained from Ron Firth at 666-3821 or Toronto line 681-1282.

QSLMA
A very successful Fall conference was held in Kingston on October 27. Plans are being made with COMA for a joint conference in the Fall. The spring conference will be held on April 6 at the Royal Military College in Kingston.

SAME
A professional Activity Day with the theme "Problem Solving: Thinking Skills In Mathematics" was held at the Sheraton Parkway Hotel on Friday, February 10, 1989. Over 400 Scarborough teachers attended. SAME's next big event will be the "Math Circus" at Maplewood High School on April 13, 1989.

SWOAME
The November Conference was a huge success. Thanks are extended to the publishers, registrants and speakers. SWOAME, although a new chapter, is looking at streamlining its operation and re-examining its boundaries for membership. They are going to present another professional activity in April.

Y4MA
Y4MA has been arranging speakers and facilities for their second mini-conference in early October 1989. A general meeting is being organized for June. At this time, a speaker will address the NCTM Standards Document. Following the address there will be an election of officers.

COMMITTEE REPORTS

Primary/Junior Committee

Provincial Reviews

The provincial review in mathematics for grade 6 is on target. There has been some input by OAME on the Consultative Committee and the Validation Committee. Clarification will be sought regarding future meetings of provincial review committees and further input of OAME in this process.

Formative Years Renewal

OAME desires to be pro-active regarding this renewal and will make suggestions to the Ministry. Discussion will take place at our June Meeting. If you have any suggestions or concerns in this regard, send them directly to Susan Stuart, Chairman, Primary/Junior Committee, 5902 Onwatin Lake Road, Hanmer, ON, P0M 1Y0.

Problem Solving Award

Presentations to award winners will be made in Ottawa.

Publications

The new Abacus editors are Colin and Anna Garnham. Best wishes to each of these individuals. The Leadership Seminar Committee will investigate having a session for prospective new writers. How do they take the first step? There is still a need for articles to be sent to the Editors of the Abacus & Gazette especially at the Elementary levels.

Membership

Thousands of "I Love Math" buttons have been sold. This has been a most worthwhile promotional enterprise for Council. Mickey Sandblom spearheaded this drive and buttons may be obtained from him. The committee is also investigating design and availability of an OAME lapel pin.

Futures

The production of a "Survival Binder" is under discussion at OSSTF and we are awaiting approval. Other alternatives will also be investigated. The history of OATMP-OMC-OATM-OAME will be written by Jack LeSage. Each chapter has been asked to produce a history. Anyone out there who can assist Jack in this huge project please send information to or contact: Jack LeSage, 9 Mount Baton Road, Barrie, ON, L4M 1T4.

Women In Mathematics

The final copy of the goals of the committee will be presented in Ottawa. A breakfast meeting has been scheduled for 7:00 a.m. Saturday at the Annual Meeting in Ottawa. Heather-Jane Robertson, author of the Idea Book published by CTF, will be the speaker. The committee is encouraging people to send articles to the Gazette or Abacus on issues pertaining to Women and Girls in Mathematics. Olive Fullerton is compiling a booklet which will include names, contact and descriptions of speakers and topics which address women and girls in mathematics issues. If you have any input, contact Olive at the Faculty of Education, York University. A conference sponsored by the coalition for Equality in Mathematics, Science and Technology will be held at Glendon College (York University) on May 24-27, 1989. This will be a working conference.

Intermediate/Senior Committee

Provincial Reviews

An announcement from the Ministry should be forthcoming by the end of February with regard to future reviews in mathematics. The International Assessment of Educational Progress (IAEP), a mathematics and science assessment for 13 year olds was released on January 31, 1989. Ontario (English) students performed at about mean and Ontario (French) performed slightly below the mean in overall proficiency. Copies of the report will be sent to each council member and chapter representative when they become available.

Availability of Mathematics Teachers in the Future

A questionnaire is being prepared and will be sent to Directors of Education. It is hoped that the responses will be received by June 1, 1989.

Select Committee on Education

The committee will investigate the non-streaming issue for grade 9 students in mathematics.

A FOND FAREWELL

This was the last Council meeting for Morley and Mona MacGregor. Words cannot express how much they will be missed. Oldtimers (like myself) and young alike will miss their presence at future meetings of Council. Those of you who have had the privilege of working with Morley and Mona will have a chance to wish them all the best in the future at the Ottawa Conference.

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MEMBERSHIP DUES

Subscriptions to the Gazette alone are no longer available; however, members of O.A.M.E./A.O.E.M. receive the Gazette and Abacus free. Membership dues for 1 year are $25.00 in Canada, $27.00 in United States and $30.00 outside Canada and United States. Membership dues for 5 years are $112.00 in Canada, $122.00 in United States, $140.00 outside Canada and United States. Orders for memberships should be sent to:

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