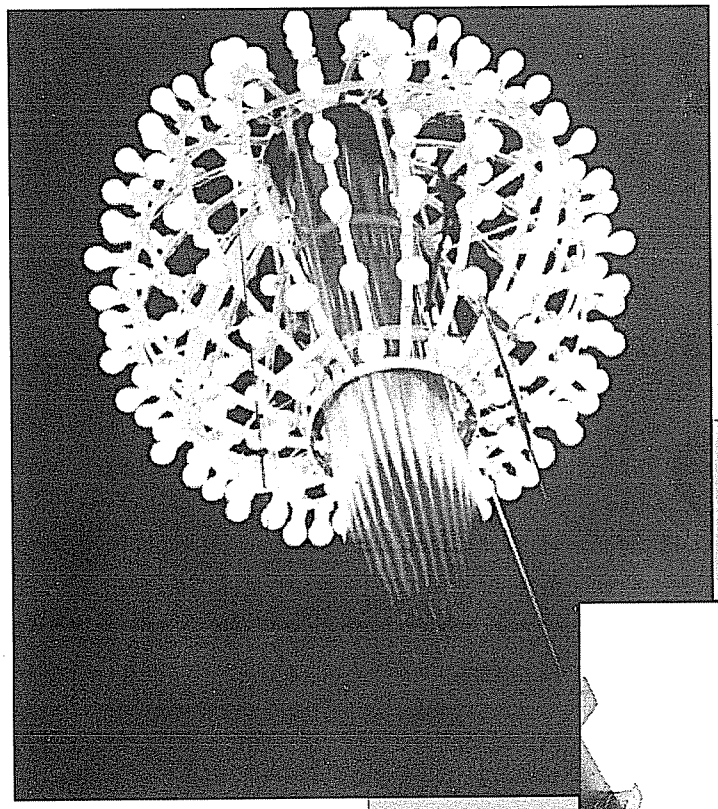
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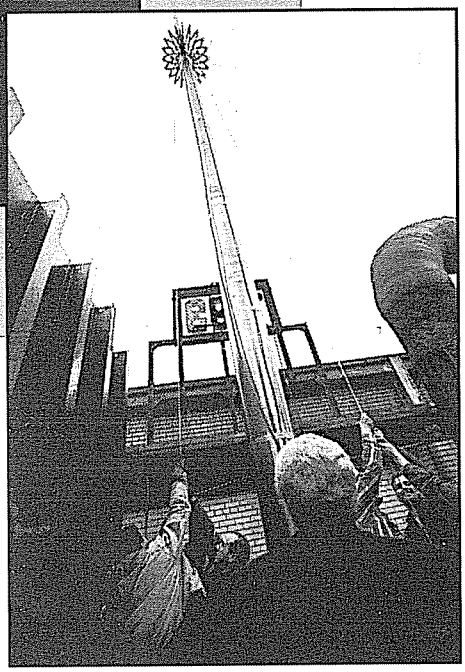
Ontario Mathematics Gazette

New Year's Eve in Times Square

See Math and the World around us, page 27

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ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
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Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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▲ Editorial

I'm in teacher heaven.

In my Math Modelling course, we were working on ordinary annuities: drawing a time sequence diagram, setting up the corresponding geometric series and then number crunching using Sum = $A(r^n - 1)/(r - 1)$). How much will we have after twenty years? How much did we invest? How much interest will we have earned? The usual stuff.

I distributed a two part assignment. The first part had several ordinary annuity questions with the final answers supplied. The second part, which I emphatically stated was homework for the next class, consisted of present value of an annuity questions, again with answers supplied. Trust me, I made this clear. Today's homework consisted of part 1 only.

The next class, we developed the method for the present value of an annuity. (For example, "How much would that car have cost, if we paid cash?") The use of the geometric series sum formula here is a little more difficult: it's not quite as easy to identify A, r, and n. But we persevered through a couple of examples, and most students became comfortable with the process.

Not Shannon. She raised her hand and said, "Jack, I didn't use your method at all. I did these questions after last class using the old method."

"Well, gee, Shannon," I replied, "I admire your initiative working ahead like that, but that method just won't work here."

"Yes, but Jack, I got all your answers!"

"Uh, right, class is about over. Lunchtime, everybody. Shannon, leave me your solutions and I'll try to figure out how you got the right answers using the wrong method."

Well, Shannon was absolutely right. And very clever. I'll tell you what she did in the next issue. For now, though, I'm just basking in teacher heaven. Shannon did more than she was asked to do—great motivation. And she modified existing tools to solve a new problem—great insight. Terrific, Shannon.

Just in on email: "Dear Colleague: I just left a lecture by John Coates at the Newton Institute at Cambridge University. He announced that the proof of Andrew Wiles has a problem and that Fermat's Theorem remains unproved. Cordial regards, Pablo Ferrari"

Good, there's still hope that I can be the first to crack it. Do you think my chances are more than marginal? Maybe I'll give it to Shannon!

On behalf of all those who help to prepare the Ontario Mathematics Gazette, I wish you a safe and happy holiday season.

Jack

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▲ LETTERS TO THE EDITOR

Dear Editor

On the inside cover page of the OAME Gazette, I have noted that photocopies of OAME journal material for use other than those indicated must be reported to OAME.

As the Junior Division Consultant for our Board of Education, I produce a newsletter each term. I wish to report that in this term's newsletter, I have included three pages from the September ABACUS—pages 2, 3 and 6. These ideas for the integration of design and technology into mathematics programs will no doubt be of assistance to our teachers but will also draw their attention to the ABACUS publication.

I trust that this reproduction of OAME material will meet with the approval of the Association.

Yours truly,
Ann Larsen
Junior Consultant

Editor's Reply

This feedback to OAME via the Gazette is important. It provides evidence that the material is useful to our readership. For me personally, it's a pleasure to tell a contributor his/her material has found an audience!

Editor's Note:

The following letter from Gail Rappolt was addressed to the EXECUTIVE AND COUNCIL OF THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATORS. The Executive asked that it be reprinted here.

August 25, 1993

Dear Member:

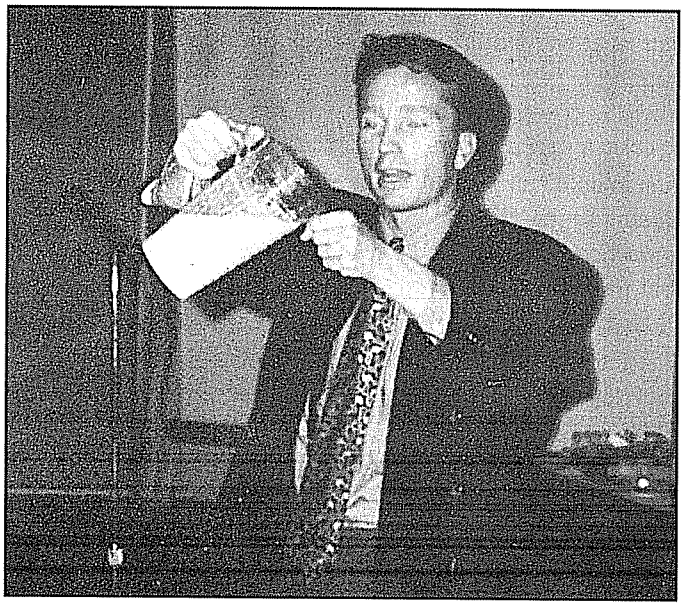
On behalf of the Ministry of Education and Training I would like to extend my sincere appreciation to your presidents, executive and council for your involvement and support of the development of provincial standards (benchmarks) over the last two years. From the outset you were positive, helpful, and constructive in your criticism. You have used meeting time to listen to explanations, supported the project financially by funding expenses for writing teams, as well as providing much needed moral support for the project.

Some of you worked on revisions through the spring of 1992 and responded to the validation draft in the fall through your boards and associations. Many of you used the validation process as a teaching tool in your own schools - and at the same time provided valuable feedback for the revision process. Some of you have contributed to the project as team leaders, as part of the validation/revision and as part of the summer rewrite in 1993. In addition, you have provided teachers for writing teams and given up evenings and weekends to meet our deadlines. A special thank you for allowing the Ministry to use sections from the *Focus on Renewal* document. Congratulations too, on the wonderful conference in Windsor last spring!!

The team this summer, under Judy Crompton's competent leadership - and it truly has been "leadership" - has taken another giant step for the province. I hope you feel, as I do, that the new format and the inclusion of sections from the *Focus on Renewal* document strengthen the standards, while maintaining their philosophy and principles. It is my hope that the standards as produced will be another step toward a deeper understanding of mathematics and increased mathematical competence for Ontario students.

I have mentioned many times the support and understanding of both presidents, Anne and Jeri - but it deserves to be mentioned again!! They have been patient with deadlines, tolerant of innumerable changes and constructive in their criticisms. Best wishes to Ron Lancaster, your new president, and to all of you in your endeavours with your organization this year.

Sincerely,
Gail Rappolt
Field Co-ordinator
Provincial Benchmarks Project
Learning Assessments Branch



President's Message

RON LANCASTER



Several weeks ago I spoke with several angry parents whose daughters I taught last year. They were in my calculus class and they are now both taking a first year calculus course at the same university in Ontario. The parents were angry because the professor had indicated to their daughters on the first day of classes that calculators could not be used at any time in the course. Both of these parents use mathematics in their professional work and they were shocked to find that this professor was not making use of technology. Both parents felt that this professor was probably teaching the exact same calculus course that they took 25 years ago. This may or may not be the case but I found it frustrating to see that my students were not able make use of many of the things that I had taught them. We had used a graphics calculator throughout the year to carry out explorations and to verify results that we had obtained using paper and pencil techniques. I attempted to show them some honest and real-to-life applications of calculus. I doubt very much that these students will be exposed to any real world applications in their university course this year. This is sad because the vast majority of students who take a first year calculus course do not go on to specialize in mathematics. They will specialize in other areas where they will use mathematics as a tool. Peter Taylor from Queen's University has written quite a few articles about this issue and he has questioned how useful the typical first year calculus course is to these students who go on to specialize in other subjects. His comments are quite in line with the movement in the United States towards a more lean and lively calculus.

I had a similarly frustrating experience with the same university in the late 1980s. Throughout the 1980s I had students coming back to me complaining that I had not taught them any techniques of integration. They indicated that many of their friends had been taught these techniques and they were finding that their university professors were assuming that they had seen it. Each year I would point out to these students that the topic of integration was not in the guidelines issued by the Ministry of Education. I would further point out to them that as far back as 1972 the topic of integration has never ever appeared in the guidelines. The complaints kept pouring in and I finally decided to visit the university and speak to them about this issue. I went with the mathematics coordinator from my board and I gave the mathematics department a presentation on what I was doing in my Calculus course. I discussed how I was spending time on all sorts of topics that were related to the differential calculus. It was an attempt on my part to enrich horizontally, rather than vertically. Many of the professors who came had no idea what was supposed to be taught in the high school calculus course and they found the presentation to be useful. At the end I asked them if they thought that I should abandon what I was doing in order to cover the topic of integration. There was a unanimous feeling that the students would be the losers if I did this. The communication that occurred between us that afternoon was very healthy and the situation improved a great deal. I rarely have students coming back from that university to complain about this issue. The complaints now come from either students who are taking physics at that university or from students who take calculus courses elsewhere.

To be fair to university professors, there are certainly a number of valid complaints that they have about what is being taught in high school mathematics classes. As it stands right now there is no real forum for airing these complaints. I clearly do not have the time to visit every university to discuss the integration issue or the use of technology and how could university professors possibly get around to every school in the province. All of this is about to change thanks to Dr. Steve Halperin, a professor at the University of Toronto. Several years ago Dr. Halperin came up with the idea of having a forum for improving the lines of communication between universities, other levels of education and the business community. Dr. Halperin has spent a tremendous amount of time working on this idea and he has enlisted the help and support of some hard-working and influential people. His dream is now a reality and the organization is called the Mathematics Education Forum. The Forum involved eight constituent groups and OAME is well represented on both the steering committee and in the Forum membership. Arnie Carefoot, an OAME Councillor, was on the initial steering committee and Marg Warren, another OAME Councillor, has now taken over his position.

The Forum is going to be more than a link between secondary schools and universities and the intention is to involve all levels of education. Look for a complete report on this Forum in the next issue of the Gazette.

▲ Have Your Say: Another Perspective on Evaluation and Achievement

TODD ROMIENS
MATHEMATICS CONSULTANT
WINDSOR BOARD OF EDUCATION,
WINDSOR, ONTARIO

I would like to react to the article, "My Perspective on Evaluation and Achievement" by Ed Barbeau that appeared in the 'Have your Say' section of the September, 1993 Ontario Mathematics Gazette.

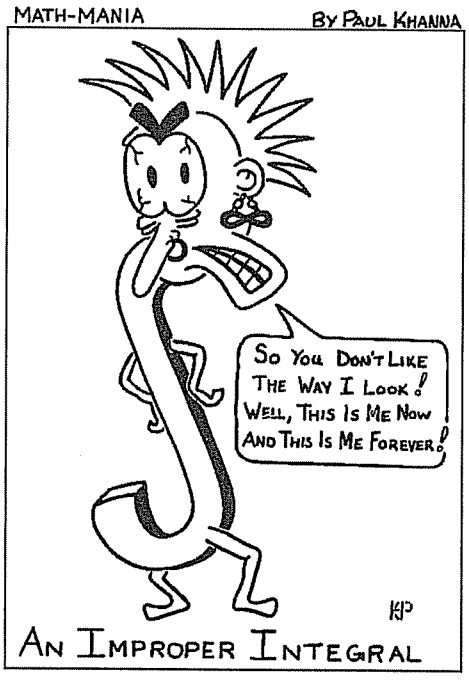
The author seems to confuse two models of learning. He seems to advocate a mastery learning model when he says "no student would be permitted into high school mathematics without having cleared this examination." This model supports a lock step, mechanical approach to learning. It implies that mathematics is linear, that is, you cannot learn mathematics except in logical, preset order, and there can be no progression until hurdles placed in front of students are overcome. This model is driven by achieving goals. It implies a regime that tries to force learning and is not consistent with what we know about how learning best takes place. No one learns under coercion.

In place of this model, I believe we need to engage students in the ownership of their learning. I support an interpretive model that is based on a belief that learning is a process of growth over time. All students enter a classroom with knowledge about math (or any subject) based on previous experiences. Teachers take students from where they are and move them forward, working together on goals that must reflect the real life requirements of today's society.

The author's example of the experiment with the high school correspondence course seems to support the interpretive model, yet he advocates a mastery learning model. Is there a contradiction here?

I would also take issue with his analysis of how secondary teachers teach and assess mathematics. He says "much of the school grade is based on superficial tests, which may involve filling in the blanks or multiple choice answers." My experience is that this certainly is not the case. Most (probably all) teachers of mathematics are deeply concerned about their students and how they are assessed. With the current emphases in education, it is even more crucial that the area of assessment be addressed. I have given workshops and attended workshops on alternate methods of assessment recently that have been oversubscribed, which indicates the great interest of teachers to improve assessment techniques in their own individual classrooms. Most teachers are concerned with the performance of students in their classes, and are working very hard, under difficult conditions to move students forward in their abilities to do relevant mathematics.

The overall impression left with me after reading my colleague's article was that he is trying to put a "quick fix" solution on a complex problem so as to make life easier for university personnel. I think it is time that the universities paid attention to what is happening across the province in education and mathematics education in particular. It's time they took a hard look at the people they are getting and made their courses to fit the learners. The perception of many K-OAC teachers is that universities seem to be exempt from changes in education and in this case, assessment. What is needed is more contact with the university community so that there can be a greater understanding of the problems faced by secondary teachers, especially in the area of assessment and evaluation.



Editor's Note: Paul Khanna is a second year Engineering student at the University of Guelph.

▲ A Fairy Tale: Car Hoods and Door Slams

GARY FLEWELLING
MATHEMATICS CONSULTANT

Gary wrote this cautionary tale out of concern for the way the public misuses the results of large-scale tests.

Once upon a time automobile associations from around the world gathered and decided that it would be a good thing to assess, in a comprehensive way, the quality of automobiles. To this end, and amid great fanfare, they launched The First International Motor Car Study (FIMS). Car companies from around the world were invited to participate in FIMS. Companies like Rolls Royce, Honda, and Mercedes accepted immediately. Soon after most other car companies, including the "Big Three" from North America, joined the study. They joined out of fear that non-participation in the study would label them as companies with something to hide. Only the Lada car company accepted the invitation on the basis of having nothing to lose.

Autometricians from the participating countries met and after much heated discussion decided on the assessment criteria, test items, and assessment techniques to be used in FIMS. They decided to judge the quality of a car by the number of coats of paint on its hood and the noise level generated by slamming the driver's door shut. Though it was acknowledged that there were many other important car quality criteria, testing for them was dismissed for such reasons as cultural differences, language differences, and cost. Some experts fought especially long and hard to keep carburetors on the test. You see, carburetor test items were plentiful, easy to administer, and had a long history of use. A slim majority of the experts voted down their inclusion on the test when it was pointed out that the cars being tested no longer had carburetors.

To simplify and standardize the testing procedure it was agreed to test cars when they reached the 50 metre mark on their respective assembly lines, or as the experts called them, developmental continua (DC's). Testing at the 50 metre mark on a DC upset the several companies that didn't use assembly lines in the manufacture of their cars. For political reasons, these companies didn't complain too loudly and remained in the study. Some experts argued unsuccessfully for a longitudinal study that would have seen the same cars re-tested at the 100 metre mark. No one, by the way, thought it very useful to test the cars at the end of the assembly process and no one even considered testing the cars over time, on real roads, by their owners.

By and by the tests were administered and the results collected and evaluated. The results of FIMS surprised and shocked many. Rolls Royce, for example, with a bare metal hood at the 50 metre mark was found to be five coats of paint behind the Lada in the hood test. Spokespeople for Rolls Royce tried to explain that they had discovered that paint was better applied at the 150 metre mark, but these statements, were perceived as self-serving rationalizations and Rolls Royce, was roundly condemned in the papers for shoddy assembly practices. Rolls Royce nevertheless, because of its limited production levels and elite clientele, decided to ride out the storm and took no remedial action. Honda on the other hand with only primer on its hoods at the 50 metre mark and smarting from being four coats of paint behind Lada, made expensive changes to its developmental continuum. No one at Honda thought to remind themselves, that, pre-FIMS, they outsold Lada 100 to 1.

Mercedes, strangely, was deemed the winner of the slammed door noise level test. Though no door is hung on a Mercedes at the 50 metre mark on its DC, they were able to successfully argue that a non-existent door made the least noise when slammed. This revolutionized car construction over night, with all car companies, except Rolls Royce, producing doorless cars in the next model year. This unfortunately, was to cause the maiming and death of tens of thousands of unbuckled family pets. The problem was to be made only slightly more tolerable when the more socially responsible car companies offered to stuff any pets so lost in a procedure that came to be known as "porsching the pooch".

Five years, later, when The Second International Motor Car Study (SIMS) was conducted, all participating companies demonstrated dramatic improvements. Honda, with 5543 micro-thin layers of paint on its hood at the 50 metre mark and revolutionary puppy-proof lace door curtains received critical acclaim by the media and took top honours. No one took much notice of the fact that since FIMS Honda had laid off 85,000 workers and was producing only 110 post-SIMS units per month.

Finally, accountability had come to the automotive industry. The automobile associations quivered in anticipation of future studies. And few car companies lived very happily ever after.

▲ A Look at One Standardized Test: The CTBS

GARY FLEWELLING



Gary's on (writing/travelling) leave for the 93-94 school year

Out of curiosity I decided to look at one of the better known standardized mathematics tests, The Canadian Test of Basic Skills, and find out how well it measured achievement of the learning outcomes for the Transition Years. I did this out of concern that some of those pushing for increased use of standardized tests in Ontario just might be tempted, for reasons of expediency, to use something like the CTBS as one of the tests.

You see listed on this and the next page the learning outcomes for Grades 7, 8 and 9. These are the same learning outcomes that appeared in the June, 1993 focus issue of the Gazette. Note that about one-fifth of the outcomes have a circle beside them. I placed a circle beside a learning outcome if the CTBS contained one or more test items related to the outcome. Notice the scarcity of circles in all but the Number Sense and Numeration strand. Also notice that the circles are, on average, one-third shaded. I shaded each circle according to my (generous) interpretation of how well each outcome was tested.

Based on the position of the circles, the number of circles, and the amount of shading in each circle, I conclude that the CTBS tests the easiest one-fifteenth of the Transition Years mathematics curriculum and that it has nothing to say about the tougher fourteen-fifteenths of it.

A school or board using a test like the CTBS to judge the health of their grade 7-9 mathematics program would be equivalent to a doctor using a blood pressure test, solely, to judge the overall health of a patient. The point I am trying to make here is that if we are not very careful of the tests we use, programs and students could suffer.

Patterning and Algebra: Learning Outcomes

7	8	9		
			i	create , describe, analyze, and extend a variety of patterns derived from number and geometric concepts, using a variety of materials.
7	8	9		ii use concrete and physical representations to establish an understanding of variables.
7	8	9		iii represent simple patterns with tables, graphs, and rules.
7	8	9		iv model real-life situations using numerical expressions.
7	8	9		v create and evaluate numerical expressions with and without brackets using the order of operations.
	8	9		vi create, describe, analyze and extend a wide variety of patterns using a variety of materials.
	8	9		vii predict and verify a pattern beyond the point where physical modelling is practical.
	8	9		viii represent simple patterns with tables, graphs, and rules (use words and algebra).
	8	9		ix describe in words and with variables, rules derived from simple patterns.
	8	9		x interpret algebraic expressions that model real-life situations using word phrases.
	8	9		xi translate word phrases modelled from real life situations into algebraic expressions.
	8	9		xii evaluate algebraic expressions by substitution with and without calculators (whole numbers then integers).
	8	9	◐	xiii solve linear equations informally.
		9	◔	xiv do algebraic manipulation involving addition/subtraction of like terms, addition/subtraction of polynomials, distribution of a constant monomial, with and without the use of physical methods.
		9		xv to model real-life situations using linear equations.
		9		xvi use a balanced situation to model and write equations.
		9	●	xvii solve equations using formal methods using the algebraic skills listed above.
		9		xviii model real-life situations using inequations.
		9		xix solve inequalities using informal and formal methods.
		9		xx substitute into real-life formulas and then solve the resulting equation.
		9		xxi graph relations using data modelled by real-life information.

Spatial Sense and Geometry: Learning Outcomes

7	8	9		
7	8	9		i construct triangles, quadrilaterals and regular polygons using a variety of tools and strategies.
7	8	9		ii classify, identify, name and describe triangles, quadrilaterals, pentagons, hexagons, octagons and decagons.
7	8	9	◔	iii identify properties of simple 2-D shapes, such as, congruent parts, angle sum, and line and rotational symmetry.
7	8	9	◔	iv identify and apply the properties of parallel and perpendicular lines and congruent figures.
7	8	9	◔	v classify, identify, name, describe, and construct simple prisms, pyramids and regular polyhedra.
7	8	9		vi create, analyze, describe and extend linear geometric patterns, identifying the pattern generator and pattern symmetry.
7	8	9		vii identify figures that will tile the plane and create tessellations of the plane identifying associated symmetries and patterns.
7	8	9	◔	viii graph ordered pairs in the first quadrant.
	8	9		ix carry out basic constructions using a variety of tools and techniques.
	8	9		x identify basic properties of translations, rotations, reflections and rotations and use these properties to both construct the transformation of simple shapes on dot or plain paper and identify the transformation linking a figure and its image.
	8	9		xi perform basic constructions using a variety of tools and strategies (e.g., paper-folding, transparent mirrors, set square, protractor, compass, computer).
	8	9		xii create and analyze designs using geometric patterns in a variety of real-life contexts and cultures.
	8	9		xiii create similar figures and scale drawings, identifying the properties of similar figures and using these properties to interpret scale drawings (e.g., maps and scale models).
	8	9		xiv sketch 3-D figures on dot paper, construct simple 3-D figures using cubes from their top, side and front views, and identifying views given models.
	8	9		xvi tessellate in 3-D, identifying figures which will fill space (relating to packaging problems).
	8	9		xv graph in all four quadrants.
		9		xvii plot the translation, reflection (in the axes and y=+/-x) and rotation (multiples of 90 degrees) of points in the coordinate plane and identify these transformations from their mapping rules.
		9		xviii identify and apply the angle properties of supplementary and complementary angles, and intersecting and parallel lines.
		9		xix identify the properties of angle and perpendicular bisectors.
		9		xx draw similar figures in the coordinate plane and identify dilatations from their mapping rules.

Measurement: Learning Outcomes

7	8	9	◔	i use a personal set of references or anchors for a variety of measures.
7	8	9		ii round measures to appropriate accuracy in all measurement activities.
7	8	9	◔	iii estimate and measure the perimeters of regular and irregular 2D figures whose boundaries are line segments or portions of curves.
7	8	9		iv construct regular and irregular figures with given perimeters using a variety of materials.
7	8	9	◔	v estimate and measure the areas of regular and irregular 2D figures whose boundaries are line segments and/or portions of curves.
7	8	9		vi construct regular and irregular 2D figures with given area using a variety of materials.
7	8	9		vii identify the inter-relation between figures with fixed perimeters (area) and their corresponding areas (perimeters).
7	8	9		viii solve real-life problems involving the use of perimeter and area, using appropriate degree of accuracy.
7	8	9		ix compare and order a variety of containers by estimating and by measuring their capacities in mL and L.
7	8	9	◔	x compare and order angles by estimation and measurement of their angles in degrees.
7	8	9		xi estimate and measure masses of objects in grams and kilograms.
	8	9		xii state and apply the formula for the perimeter of a rectangle (including squares)
	8	9		xiii state and apply the formula for the area of a rectangle (including squares)
	8	9		xiv state and apply formula for the circumference of a circle and estimate the value of pi.
	8	9		xv state and apply the formula for the area of a circle.
	8	9		xvi determine the area of regular and irregular figures using a variety of techniques.
	8	9		xvii state and apply the Pythagorean Theorem.
	8	9		xviii state and apply the formula for the area of a parallelogram, triangle & trapezoid.
	8	9		xix identify the change in perimeter and area of shapes if their dimensions are changed by given factor.
		9		xx make scale drawings and models.
		9	◔	xxi state and apply the formula for the volume of rectangular prisms and cylinders.
		9	◔	xxii by estimation and measurement determine the surfaces areas of rectangular and triangular prisms and cylinders.
		9		xxiii measure volumes by displacement methods or capacity measures.
		9		xxiv identify the relationship between the volume of a prism (cylinder) and its corresponding pyramids (cones)
		9		xxv solve problems involving surface area, volume and capacity.
		9		xxvi identify the change in volume of shapes when one or more dimensions are changed by a given factor.
		9		xxvii using models, solve practical problems involving irregular 3D figures (e.g., surface area of a vehicle, volume of wood in a tree, capacity of kidney-shaped pool)

Data Management and Probability: Learning Outcomes

7 8 9	i	collect graphs from various sources and then classify them according to their type.
7 8 9	ii	identify the characteristics of each type of graph.
7 8 9	iii	identify the type of information that is associated with each type of graph (continuous, related or unrelated)
7 8 9	iv	recognize the suitability of different types of graphs for displaying specific data.
7 8 9	v	read and interpret graphs.
7 8 9	vi	construct graphs from given data or data found in other sources.
7 8 9	vii	pose and answer questions about the graphs constructed and those found.
7 8 9	viii	develop intuitive concepts of "chance" (probability) using spinners and other similar activities.
7 8 9	ix	determine and express the "chance (probability)" of events as fractions using the terms "fair/unfair", "likely/unlikely".
8 9	x	determine mean, median, and mode from collected data.
8 9	xi	use mean, median or mode to analyze tabular data.
8 9	xii	identify the various methods used to collect data from a variety of sources and then state the advantages and disadvantages of each method.
8 9	xiii	create a survey based on learner-decided (teacher assisted) problem.
8 9	xiv	analyze questions as to bias/non-bias.
8 9	xv	conduct a survey using good sampling techniques.
8 9	xvi	organize and interpret the information from a survey.
8 9	xvii	display information from a survey using an appropriate graph.
8 9	xviii	interpolate and extrapolate information from graphs.
8 9	xix	identify patterns and trends and make predictions from graphs.
8 9	xx	estimate populations by taking samples of various sizes.
8 9	xxi	explain why actual results do not always match predicted results.
8 9	xxii	predict outcomes by creating a probability experiment or simulation to model a problem.
9	xxiii	create a formula or an equation from data displayed in a chart/table.
9	xxiv	design and conduct an experiment to test an hypothesis.
9	xxv	collect, organize and display the data gathered from an experiment.
9	xxvi	when possible, create an equation to represent data of an experiment. (Note: many problems will not generate an equation.)
9	xxvii	given an equation, display data in the coordinate plane.
9	xxviii	pose and answer questions about the data.
9	xxix	determine probabilities of events from their relative frequencies as obtained from: i) an individual's results, and ii) class results.
9	xxx	explain why actual results do not always match predicted results.
9	xxxi	identify the relationship between sample size and the accuracy of expected results.
9	xxxii	conduct simulations to predict probability of occurrence of complex events.

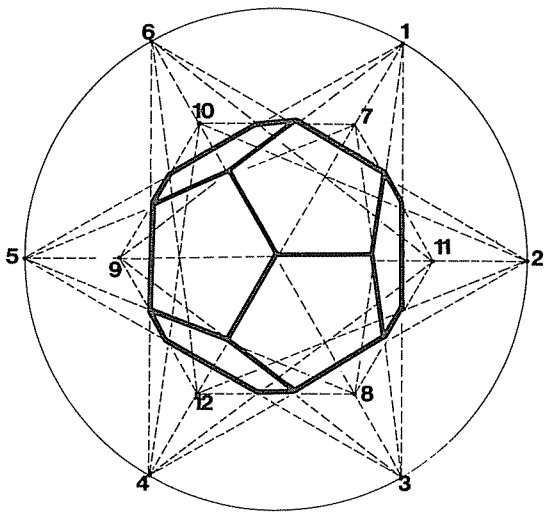
***Number Sense and Numeration:* Learning Outcomes**

7 8 9	i	identify and use place values in context from millions to thousandths.
7 8 9	ii	identify the effect of an operation on a pair of numbers. (e.g. 5 ÷ 0.2 > 5)
7 8 9	iii	apply in applications the order of operations.
7 8 9	iv	perform whole number and decimal computations using paper and pencil. Maximum limits: – addition, three 3-digit; -subtraction, two 3-digit; – multiplication, 2-digit by 2-digit; -division (short), 3-digit by 1-digit
7 8 9	v	recognize and apply the appropriate type of computation for a given context, that is, mental calculation, estimation, paper-pencil computation, or calculator.
7 8 9	vi	approximate sums, differences, products, and quotients of whole numbers and decimals using a variety of strategies.
7 8 9	vii	use a calculator for awkward computations.
7 8 9	viii	judge the reasonableness of answers calculated by paper-and-pencil or calculator.
7 8 9	ix	understand the meaning of integers and identify contexts in which integers occur.
7 8 9	x	add and subtract integers with models.
7 8 9	xi	identify equivalent and approximately equivalent fractions.
7 8 9	xii	add and subtract fractions with and without models.
7 8 9	xiii	multiply and divide fractions with models.
7 8 9	xiv	express hundredths as fractions, decimals, and percents.
7 8 9	xv	estimate and solve problems involving percents and whole percents of numbers (e.g., sales tax, discounts, tips).
7 8 9	xvi	use their number concepts and skills in a wide variety of applications.
8 9	xvii	identify and use place values in problem contexts.
8 9	xviii	use calculators for awkward computations, compacting numbers where necessary.
8 9	xix	solve problems involving addition and subtraction of integers without the use of models.
8 9	xx	solve problems involving the multiplication and addition of fractions.
8 9	xxi	solve problems by using equivalent rates and comparison of rates.
8 9	xxii	use tables and the unit method to solve ratio, rate and percent problems (e.g., if 8% of a number is 50, find 7% of the number).
8 9	xxiii	relate squares and cubes of numbers to the areas of squares and the volumes of cubes.
8 9	xxiv	relate and estimate the square roots and cube roots as the lengths of squares and cubes.
9	xxv	record and interpret numbers in scientific notation.
9	xxvi	approximate the value of expressions using scientific notation or compaction of numbers.
9	xxvii	solve problems involving the addition, subtraction, multiplication and division of integers
9	xxviii	solve problems involving the addition, subtraction, multiplication, or addition with rationals.
9	xxix	solve ratio and percent problems algebraically.
9	xxx	understand powers involving integral exponents.
9	xxxi	use the power laws (in numerical cases only).

Editor's Note: Our apologies for the small font. For easier reading, consult the June, 1993 Gazette.

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Award (Check One)

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I/We nominate (Name of Nominee) _____

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Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to Don & Carol Attridge, Executive Directors OAME/AOEM, 1 Southdale Drive, Markham, Ontario L3P 1J6 Fax (416) 472-3272

▲ Zipf's Law or When is a Law a Law?

AL NICHOLSON
FENELON FALLS S.S., FENELON FALLS, ONTARIO

Al Nicholson was born, raised and educated in Saskatchewan. He has taught mathematics and music in Fenelon Falls for 25 years.

In each American presidential election, the people of New Hampshire have an early say on who will be the next president. For the past forty-five years, every elected president has won the New Hampshire primary, one of the first tests of a candidate's strength in the lengthy presidential campaign. Political scientists note that this

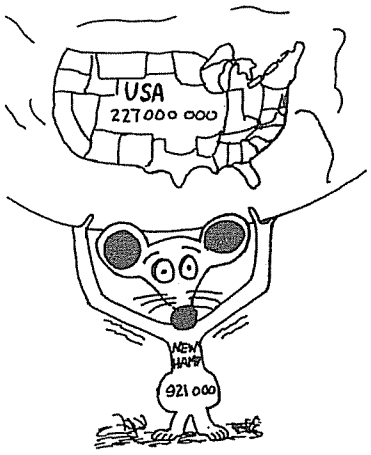


Figure 1

state which ranks 42nd in population with only 921,000 people, exerts an undue influence on the nation's destiny (fig 1). A student of mathematics will wonder how much information one might deduce about the USA from the rank and population of one of its smaller states. Quite a bit it turns out but, unfortunately, not the name of the next president!

In this article, we will use Zipf's law to show how the rank and size of one province of a country can give us an estimate of the size of the whole country. It can also estimate the sizes of the other provinces and the number of provinces. There are two main reasons that this problem is useful in the classroom.

First, it provides an accessible classroom mathematical modelling example which can be used as a vehicle for data handling and error analysis.

Second, since in some situations the law gives very good results and other times it does not, students can begin to understand that "laws" have limitations and must be used with care and knowledge.

We will introduce the law and then apply it to Canada and its provinces before returning to our New Hampshire question. This material could be used at any level in Junior or Senior High School. We will then look at a refined version of the law which uses exponents, logarithms and analytic geometry and would provide a good review for these topics in senior classes.

The Law

G.K. Zipf [1] was an eclectic thinker who noted that if we rank the words of any language, the most commonly used word is used (roughly) twice as often as the second most used word; three times as often as the third word. Zipf's law states that the use of a word is inversely proportional to the usage-rank of the word. It has been noted that Zipf's inverse rank law can be applied to populations of geographic units [2,3]. The law states:

If we have a complete set of geographic regions (continents, countries, provinces, cities), the population of a region is inversely proportional to the rank of its population.

We will discuss several applications of the law and its extensions. The law is easily understood, readily applied and brings to light the broader question of the connection between a mathematical model and the reality it is trying to portray.

Canada and its Provinces

Our first example is a look at the provinces of Canada. In order to use the law we must consider all the provinces. This is the reason for the use of the word "complete" in the statement of Zipf's law. If we knew that in Canada, Saskatchewan ranked 6th with a population of 992,000 and New Brunswick ranked 8th, we could estimate the population of New Brunswick thusly:

$$\frac{x}{992\,000}=\frac{6}{8} \quad \text{and find } x = 744\,000$$

The actual population is 707 000. Thus Zipf's law provides an answer with an error of about 5%. Using Saskatchewan's population, we can estimate the population of all ten provinces. This is done in table 1. For the rest of this paper, all population figures will represent thousands. [1]

[1] All population data used in this paper are taken from *The 1990 Information Please Almanac* (Houghton Mifflin Co., Boston, 1989).

Table 1

Province	Calculated Population	Actual Population	Error
Ontario	5 952	8 816	32%
Quebec	2 976	6 515	54%
British Columbia	1 984	2 825	30%
Alberta	1 488	2 352	37%
Manitoba	1 190	1 046	14%
Saskatchewan	992	992	0%
Nova Scotia	850	859	1%
New Brunswick	744	707	5%
Newfoundland	661	576	15%
Prince Edward Island	595	124	380%

Calculated and actual population values of the provinces of Canada. (Saskatchewan as basis)

We see that Zipf's law produces some successes, some dubious results and an incredible error.

A more convenient way to use the law is to use the maximum population. Then we have

$$\frac{P_{max}}{P_r} = \frac{r}{1}$$

where P_{max} is the population of the largest region. P_r is the population of the region with rank r. Rearranging, we have

$$P_r = \frac{P_{max}}{r} \qquad (1)$$

In our example of the provinces of Canada, we would use the largest province, Ontario. The results are shown in table 2.

Table 2

Province	Calculated Population	Actual Population	Error
Ontario	8 816	8 816	0%
Quebec	4 408	6 515	32%
British Columbia	2 939	2 825	4%
Alberta	2 204	2 352	6%
Manitoba	1 763	1 046	69%
Saskatchewan	1 469	992	48%
Nova Scotia	1 259	859	47%
New Brunswick	1 102	707	56%
Newfoundland	980	576	70%
Prince Edward Island	882	124	611%

Calculated and actual population values of the provinces of Canada. (Ontario as basis)

Some of the errors are smaller but several are much larger. We will discuss the large errors at the end of the paper but we note here that in the social sciences, precision is not easily attained.

Total Population of a Group of Regions

If we let P_{tot} be the total population of the geographic regions we are considering then we have,

$P_{tot} = P_1 + P_2 + P_3 + \ldots + P_n$ where n is the number of regions in our group. Using equation (1) we obtain

$$P_{tot} = P_{max}\left(1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \ldots + \tfrac{1}{n}\right)$$
$$\approx P_{max}(\ln n + 0.5)$$

The last step is a rough approximation that can be obtained by integrating

$$\int_1^n \frac{1}{x}\,dx$$

If we use the population of Ontario at 8 816 and equation (2) with $n = 10$ we calculate Canada's population thusly:

$$P_{tot} = 8\,816(\ln 10 + 0.5) = 24\,708.$$

The population of Canada (excluding Yukon and the NWT) is 24 812. The calculated answer is within 1% of the actual. This accuracy using Zipf's law is the exception rather than the rule.

New Hampshire as the Bellwether of the USA

We return to study New Hampshire: population 921 and rank 42 of 50 states. We ask and answer the following questions:

1. What are the populations of the two largest states?

2. What is the population of the USA?

Answers: 1. From (1) we have

$$P_{max} = P_r \cdot r$$

Hence the population of the largest state is

$$P_{max} = (921)(42) = 38\,682.$$

Using (1) we find that the population of the second largest state if max 19 341.

2. We use (2) and find
$P_{tot} = (38\,682)(\ln 50 + 0.5)$
$= 170\,666.$
Hence the population of the USA is 170 666.

The actual figures (errors) are: California 23 668 (63%), New York 17 558 (10%) and USA 226 546 (25%). We leave it to the reader to discuss whether, armed with the

population and rank of New Hampshire, Zipf's law gives us quite a bit of information or it only feeds us useless misinformation.

A More Refined Version of Zipf's Law

Equation (1) can be written

$$P_r = P_{max} \cdot r^{-1}$$

A general form which we hope would give better results is

$$P_r = A \cdot r^b \qquad (3)$$

where A and *b* are constants for a particular group of regions. The study of this equation would provide a good opportunity in class to review logarithms and the analytic geometry of the line.

We take the logarithm of each side of equation (3) and obtain

$$ln\ P_r = b \cdot ln\ r + ln\ A \qquad (4)$$

Now if a scatter plot of $ln\ P_r$ and $ln\ r$ is made for the populations (for example) of the provinces of Canada and a line of best fit is found we obtain figure 2. To find the line, it is suggested that your students use an "eyeball" approach. An analytic approach such as least squares could be used if the students have sufficient background. Finding the slope and *y*-intercept of this line will produce estimates of the values of $ln\ A$ and *b*. The estimate of A is found using $A = e^{lnA}$. For the line in figure 2, we find that A is 14 186 and *b* is -1.48. The calculated results and errors are shown in Table 3.

The errors are considerably less than those found in the calculations displayed in tables 1 and 2.

Discussion

Zipf's law gives us a principle by which we may get a feeling how populations are distributed in geographical regions. It is not useful if we wish to obtain a precise population count. Nevertheless, there are several important lessons the high school student will learn studying this law.

First, no scientific law is perfect. Special conditions are always presupposed. For example, the value of 9.8, the acceleration due to gravity, is dependent on how far one is from the center of mass of the earth. Calculated results in theory will nearly always differ from measured results of experiments. Too often students will think that these type of differences are only due to sloppy experimental technique; they do not realize that theory is restricted. Zipf's law would provide a good starting point for discussing these issues.

Second, some inaccurate laws are useful and some very accurate laws are useless. It is true that every newspaper has an even number of pages - but of what use! Theory and laws connected with the social sciences will never be very accurate. Economists can not even agree on whether interest rates should go up or go down. Yet, the search for mathematical models gives economists much insight into how the economy works. For a simple social theory of a complex problem, Zipf's law does quite well.

Third, one must use care when applying a law and when extending a law. Equation (2) can be rearranged to obtain

Table 3

Province	Actual Population	Rank r	ln r	ln Pop	Calculated Population	Error
Ontario	8 816	1	0	9.08	14 186	61%
Quebec	6 515	2	0.69	8.78	5 085	22%
British Columbia	2 825	3	1.10	7.95	2 790	1%
Alberta	2 352	4	1.39	7.76	1 823	22%
Manitoba	1 046	5	1.61	6.95	1 310	25%
Saskatchewan	992	6	1.79	6.90	1 000	1%
Nova Scotia	820	7	1.95	6.76	796	7%
New Brunswick	690	8	2.08	6.56	653	8%
Newfoundland	592	9	2.20	6.36	549	5%
Prince Edward Island	124	10	2.30	4.84	469	278%

(PEI is not included in the scatter plot.)

Calculated and actual population values of the provinces of Canada. (Extended Zipf's Law)

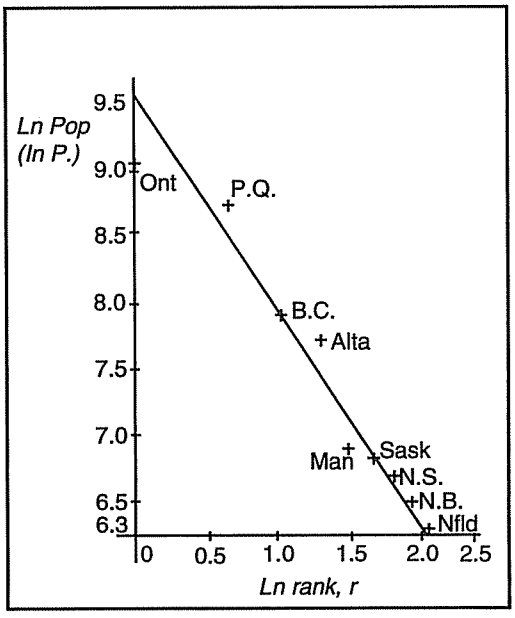


Figure 2

$$n = exp\left(\frac{P_{tot}}{P_{max}} - 0.5\right) \qquad (5)$$

This can be used to find the number of regions if we know the population of the largest region and the total population. Using our example of Canada and its provinces, we use $P_{tot} = 24\,812$ and

$$P_{max} = 8\,816 \text{ and find } n = exp\left(\frac{24\,812}{8\,816} - 0.5\right) = 10.1$$

This is very close to the 10 provinces we know so well. However, if we use (5) to calculate the number of states in USA and use $P_{tot} = 226\,545$ and

$$P_{max} = 23\,668 \text{ we find } n = exp\left(\frac{226\,545}{8\,816} - 0.5\right) = 8\,706$$

which is far off the correct number, 50 states.

Whereas the basic Zipf's law, equation (1), gives us some idea of how populations are scaled and the sum formula, equation (2) gives a ballpark figure for the total population, this inverted equation (4) is probably not useful at all.

Using Zipf's Law in Class

Zipf's Law can be used for continents in the world, countries in a continent, provinces in a country, cities in a country, counties in a province, or towns in a county. Three results might appear: (a) Zipf's law (1) applies quite well; (b) the refined form (3) must be used; or (c) the largest one or two regions are disproportionately large and the law's pattern is destroyed. In this third case, the refined form will often work well if the extra-large regions are not included in the calculation.

How well the law works and which form to use are worthwhile topics for discussion by our mathematics classes. The law works best for a country that has developed within a set of natural boundaries. Countries that are federations or have annexed neighbouring countries do not fit the pattern as well. Metropolitan cities that have annexed many smaller communities distort the picture and cause the third situation mentioned above. The results for PEI are poor because its natural boundaries limit growth and this matters when it is included in the calculations for Canada.

The NCTM STANDARDS [4] encourage the making of connections between mathematics and other disciplines. If we study the slope b in equation (4) for different countries, we see that it is correlated to the degree of urbanization of the country. An investigation of this concept would be a good project for geography and mathematics.

Another project we have used is to find the constants A and b in equation (3) for several nesting geographic units. The units we used were:

1. countries in North America
2. provinces in Canada
3. counties in Ontario
4. townships in Frontenac County.

The values of A and b will vary over these four levels, but it is striking that the structures are so similar. This similarity at different scales means that populations are fractal like. [2]

At any level in the intermediate-senior grades, the basic Zipf's law, equation (1), provides an opportunity to use a formula, tabulate data and calculate and analyze errors. For instance, after doing the Canada-province calculation of table 2, the student should be invited to consider the questions:

1. Does this law work?
2. Is this law useful?
3. Do Newton's laws work?
4. How can one distinguish between experimental errors and errors of theory?

Conclusion

Zipf's law and its extensions provide an opportunity to use mathematical skills while analyzing an easily understood real life problem. Students appreciate an accessible application. This law illustrates the use and the limits of the use of mathematics in scientific research.

If we can calculate the population of the USA from that of New Hampshire, maybe it is fair that the New Hampshire primary has so much influence on the destiny of the whole country.

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2. **Tanner, Judith M., Ed.,** *Statistics: A Guide to the Unknown,* Holden-Day, San Francisco 1972.
3. **Lloyd, Peter E.,** *Peter Dicken Location in space: a theoretical approach to economic geography,* Harper & Row, New York, 1972.
4. *Standards NCTM,* Reston 1989.

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[2] Readers familiar with the iterated function system fractal images of Michael Barnsley might like to try a system where discrete maps are chosen with probabilities of 1/2, 1/3, 1/4, 1/5, etc. What emerges as the computer works is an evolving country. A few small towns that grow in size. These in turn spawn smaller satellite towns and eventually we see large metropolitan sprawls with sparsely populated regions - a fractal country!

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what: An instant(?) problem-based curriculum for intermediate algebra.

how: You send me the problems, I make the resource book, we have a free-for-all at the session, and you take the resource book away.

I am convinced we need to be moving at all levels towards a problems-based concept of curriculum. This means letting go of the deadly skills-based organization of our current curricula. This means not taking anything into your classroom unless you think it's fun and neat and good and worth the hour-unless you can get out of bed in the morning and say, hey we're doing that neat problem today, and bounce into the shower. [Would an English teacher take a poem into class that he or she didn't think was neat and good? Nor should you.]

Okay so we lose that nice safe system of building skill x on skill y right where it should be in the order of things. But that nice safe system isn't really working. Anyway this has got to be a lot more fun...

I don't mean math contest type problems here! I don't mean brainteasers that only the executive of the chess club will know what to do with.

I mean problems that will explode into small group discussions right away, where half-baked ideas and conjectures can be put up on the board for class scrutiny, where team-work and communication is what counts, where exploration and hands-on stuff can bring insights, where the struggle is more to see what to say and how to say it, and where in the working of it all out, some of the basic algebraic skills are needed. In fact, if you get out your intermediate and senior curriculum guidelines and read the AIMS you'll see exactly what I want!

Let me give you three examples. The first is a bit off the wall, but I chose it because in some of our problems, I really want to get away from the standard stuff and do more where the emphasis in an organization, analysis, and presentation. It's a card trick found in Ball and Coxeter Mathematical Recreations and Essays - and don't let the title of this classic (first edition 1892) fool you - it's got enough in it to fuel an undergraduate curriculum.

EXAMPLE 1: I give a deck of cards to a student and ask her to shuffle and deal 26 cards in a face-up pile. As she deals I watch the cards go down with great concentration. I ask her to turn pile over - it is now face down and will be called the discard pile. She now deals the remaining deck, from the top, face up, into a number of piles, as follows. Suppose the first card dealt is a 7. Then she says "7" and covers it with 3 cards (face up), calling out 8, 9, and 10, as she deals. Then she starts a new pile. If the next card is a 4, she says "4", and covers it with 6 cards, calling out, 5, 6, 7, 8, 9 and 10. If she lays down a 10, she just announces "10" and leaves it in a pile by itself. An ace counts 1, and all face cards count 10. At the end, she will usually run out of cards before she can finish a pile. In that case, she just picks this pile up and places it (face down) on the discard pile.

Now she chooses any 3 piles, and you instruct her to turn them over (so they are face down) and then turn over the top cards. All non-chosen piles are to be gathered up and put on top of the discard pile (face down). Now the values of the three turned-over cards are added and you quietly announce the name of the card that is in the position in the discard pile. For example, if the three cards were 4, 7, and Q, you would give the identity of the 21st card in the discard pile. The student then counts out 21 cards and is suitably awed (and so is the class—try it!) by your powers of "memory".

That's not all! You offer to vary the trick. Instead of counting to 10, why not count to c=13? And instead of taking 3 piles, why not choose only p=2? In fact you can ask them to provide reasonable values for c and p (though there's only a small feasible range) and provided you know these at the beginning, you can still do the trick.

The problem of course is, how does it work? Somewhere around there's presumably a "formula" involving c and p. What is it?

There's lots of good stuff here: finding the formula is just the fun stuff in the middle. But at the very beginning, there's the inspiration to decide to look for such a formula, and at the very end, there's the task of presenting the package and writing everything down in a good way. These are skills we must start to emphasize more.

I was shown the second example last summer at the CMESG meetings at York, and I believe it was attributed to our Editor Jack Weiner. Anyway, it's a lovely open-ended

little gem which uses just the right skills for us:

EXAMPLE 2: $2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}$

The third example is important in applications of functional analysis in the social and life sciences, but I find that university students in first year have a lot of trouble with this kind of problem. I wish they had started thinking this way long ago.

EXAMPLE 3: Suppose that y is a function of x, and the graph of the function is given by the curve shown below. Consider the problem of finding the value of x which makes

$$\frac{y-2}{x}$$

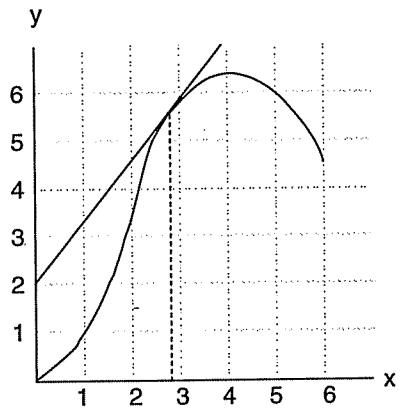
a maximum. The construction on the graph (the tangent line) shows that the maximum is attained at $x \approx 2.6$.

(a) Explain carefully why this construction solves the maximization problem.

(b) What expressions of the form:

$$\frac{ay+bx+c}{ry+sx+t}$$

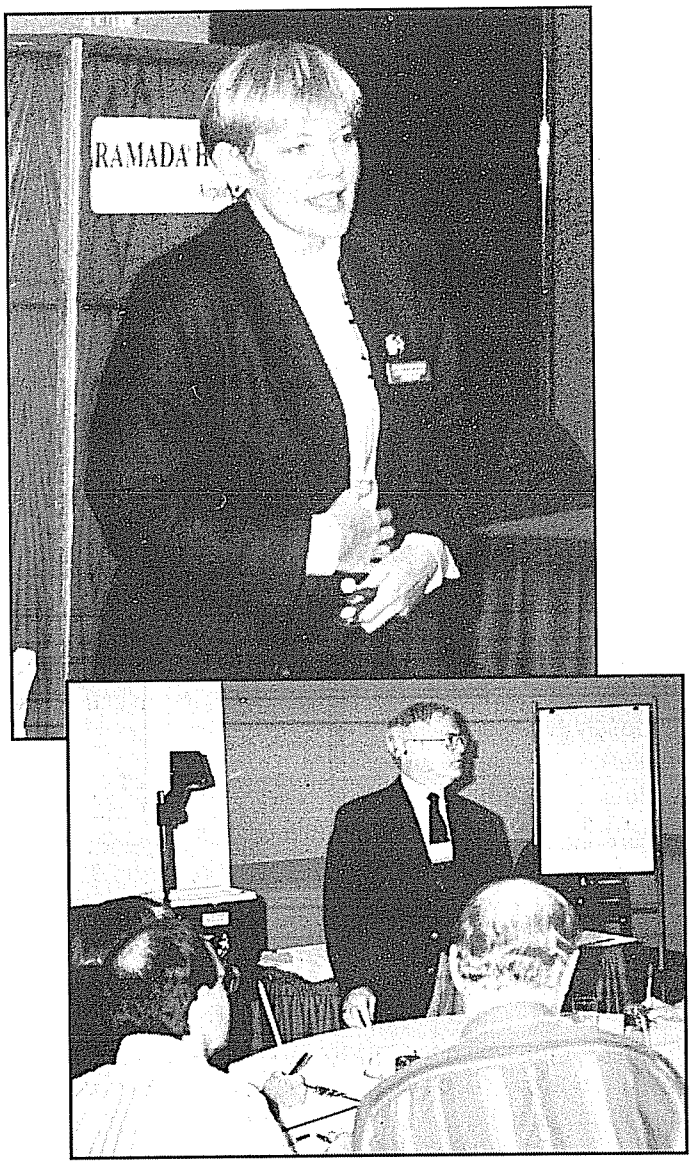
for given constants a,b,c,r,s and t, can be maximized (or minimized?) by this geometric approach?



Let me emphasize: I'm not talking about Friday afternoons here. If I were teaching grade 9-10 and had a set of problems like these, I would spend the whole time on them. And let the skills fall out as they will. [Or more precisely, as the motivated student and the wise teacher see the need and opportunity. And of course, I would have to offer the right kind of support to those students who can see that their skills are weak - but from grade 9 up I want the onus for this initiative to be on the students!] This is roughly how curricula are designed in the humanities, and science could gain much by studying their model. In a poetry course, for example, the "problems" are the poems, and they are carefully chosen for their artistic merit; over the years, the important skills emerge. [You may be wary of an argument that involves an analogy between poetry and mathematics, but the more experience I have with each of these, the more I think we each have to learn from one another.]

So here's what you have to do. Send me a contribution, at most one page, for the resource book. It could be a single problem, with or without commentary, or a collection of two or three problems. Include your name and address and phone number at the top left of the same page. I will screen them (I reserve the right not to use everything I get) and assemble them into a booklet which I will put into the hands of everyone who comes to the session, whether they are a contributor or not. At the session we will talk for an hour or so about the problems in an informal way.

Deadline for submissions is March 15, 1994. Act now, and let's make a start at putting some new life into our curriculum!



▲ THE REVISED WITPO GAME

MIKE ZICHOWSKI
NEWTONBROOK S.S., WILLOWDALE, ONTARIO



Mike Zichowski has been teaching in North York for sixteen years. He has written over two hundred articles and has produced educational programs for cable and City T.V. He is currently working with his O.A.C. Algebra Geometry class to create a dating service using vector algebra.

WHAT IS THE PROBABILITY OF

The first snowfall of the season had brought the usual misery of traffic congestion, and I had arrived late for my classes. Since there was no time to start a new lesson I decided to play WITPO with my senior Finite Math classes; fortunately we had finished the probability sections of the course. I quickly grabbed a red die and the question cards for this game from my desk drawer and entered the portable. They sensed something different was about to happen as I was not carrying the textbook. And when I began calling their names in groups of five to occupy a portion of the classroom, they realized that today's lesson would be a brand new experience. While I was formulating the groups I persuaded a student to draw the game board on the blackboard. Each group selected a captain who proceeded to the front of the room for a die toss to determine the order of play (largest value went first). I had purposely selected the groups so that our math geniuses would not be on the same team. My team selection had obviously not affected the excitement that pervaded the classroom as cheering erupted during the inaugural die toss. An added incentive was the announcement that the winning team would receive an extra two marks on their next chapter test.

And so the game began in earnest. With each die toss a question card was selected, its question read twice to the group, the time limit announced at usually 60 or 90 seconds, and if the answer received from the group was correct an appropriate number of steps were marched off along the game board. If an incorrect answer was received, the value of the roll would be halved for the next respondent (any group could answer). This was the most challenging part of the game. If a wrong answer was encountered, the M.C. (i.e., the teacher) must select the quickest hand in the air for the next answer and award the proper steps along the game board if that answer is correct. We interrupted the game several times to calculate an answer that the students perceived as difficult. I had the student with the correct answer demonstrate his/her solution for the entire class before we proceeded with the next roll of the die. It was interesting to see that even our "hot-shot" math students, who frequently responded much before the allotted time limit for the question, got the answer wrong. These blunders caused a genuine team approach to the question posed, and as the competition became heated, I sensed much team discussion and problem-solving before a final solution was given. It was co-operative learning at its best as I relished these intense problem-solving moments. It seemed that all that commuter traffic anger was cured by this enjoyable game. Before the class was dismissed a game evaluation sheet was distributed so that their suggestions and ratings could be analyzed. (See enclosed student reviews).

The introduction of games into the OAC Finite Math course continues to be a hot, contentious topic among educators. The results submitted by my students indicate the need for such diversions from the course material and that the exercise is a valid activity. The REVISED WITPO GAME tends to reinforce and develop many of the concepts of the probability units of study in the finite math course. It is a great way to review basic probability skills such as independent events, the binomial theorem, permutations and combinations.

THE PLAY AND RULES:

The game requires a die, a playing board (provided with this article), magnetic board markers, a timing device and a deck of questions on cards (provided). Answers appear on the question cards to allow the teacher or a student to verify the student's answer. To avoid repeating questions during the game, your question deck should include at least 40 questions. WITPO is designed to be played by groups of 3, 4 or 5 members. A typical class will have four or five teams. Calculators and notebooks are allowed. The game requires about one hour to complete with the teacher assuming the role of M.C.

RULES:

1. Each team rolls the die to determine the order of play. The largest number goes first.
2. The first team rolls the die. The teacher reads the

question twice to the entire class and announces the time limit for the question. If the first team gives the correct answer then its marker on the game board is moved the appropriate number of spaces as indicated by the die.

3. If the number on the die matches the number on the playing board where the marker lands, additional spaces equal to the number on the die are marked off.
4. If the question is answered incorrectly, then the question has half the value of the initial roll and any team may answer (quickest hand up). If the die shows a one, three or five then round up to the nearest whole number before halving.
5. The play continues in this manner until one team reaches the final circle.
6. "Lose a turn" : if this occurs then leave the marker in this space. Do not read the question card; the team loses its next turn with the die.
7. Used cards should be returned to the bottom of the deck.
8. Calculators are permitted, and no distractions are allowed during the question period. A time limit is imposed on each question issued.
9. If a player rolls a "one" on the die and the team gives an incorrect answer, then no subsequent halving of the question value is allowed for the other teams.

CONCLUSION:

The finite math course provides a perfect setting for utilizing games in the curriculum. The REVISED WITPO game may be used as a supplemental review activity when the probability chapter concepts are completed. Question cards may be devised to include other topics, such as: matrices, statistics, series and sequences, rather than just probability. Games which emphasize counting principles, and allow the students to develop a few basic skills and apply some combinatorial notation make studying mathematics enjoyable. WITPO provides students with a background of concrete experiences upon which abstract university mathematics can be based.

STUDENT REVIEW:

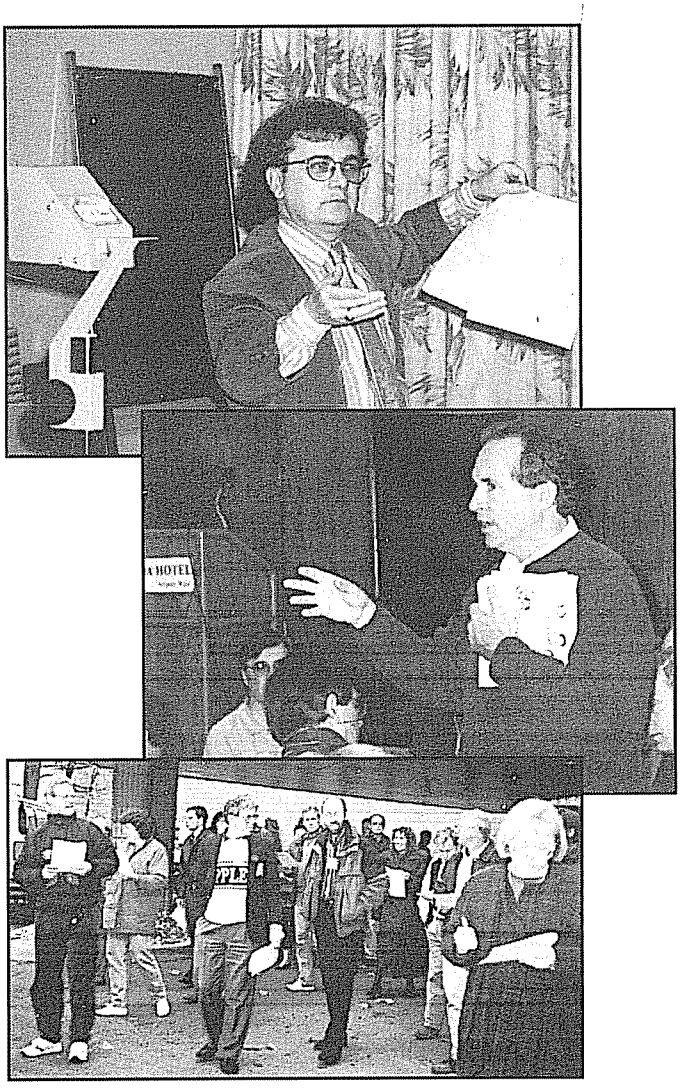
Rating: 8.8/10

THEIR SUGGESTIONS FOR IMPROVEMENT:

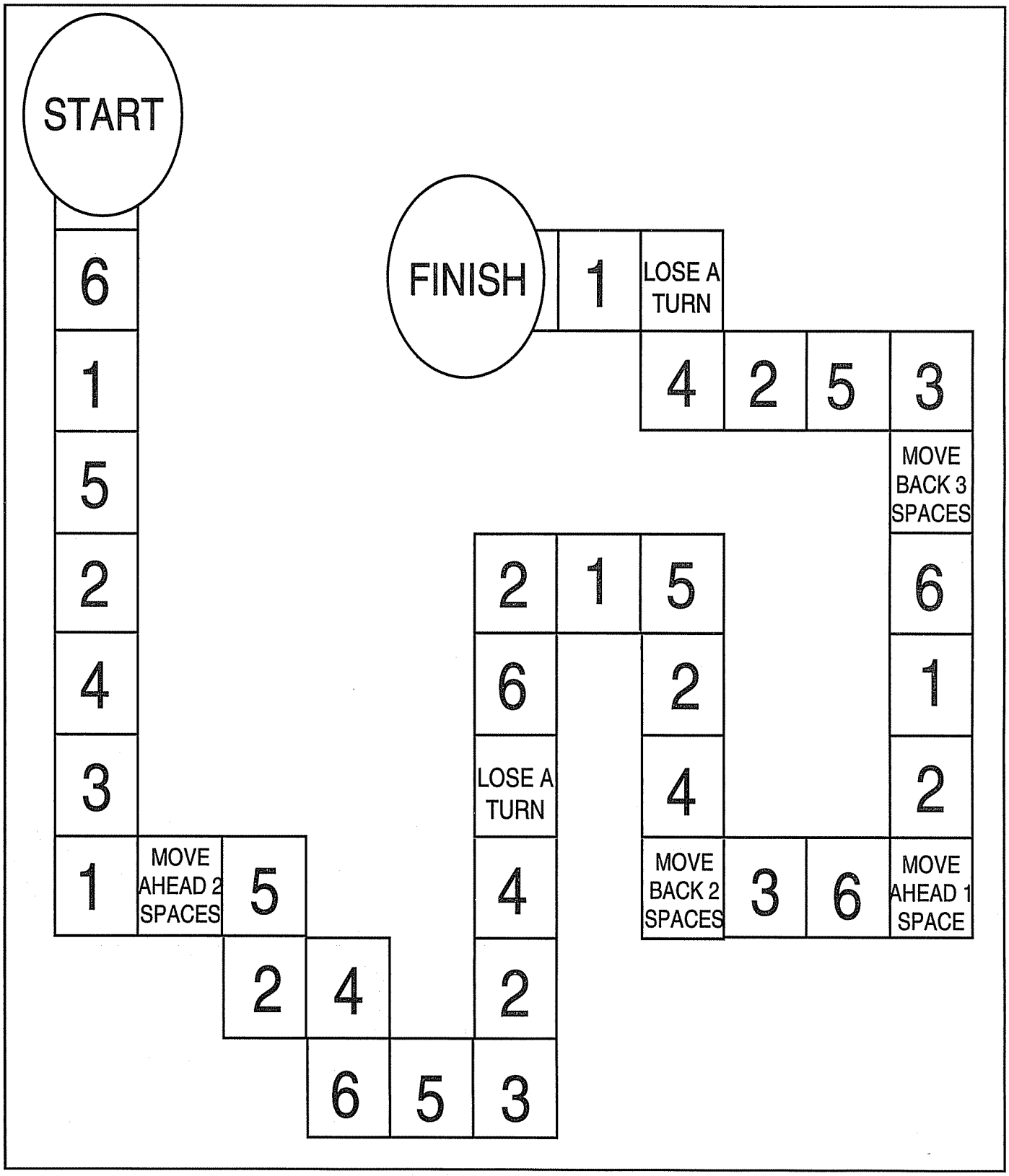
- sketch a large playing board on the blackboard
- speed up the pace of the game by not taking up wrong answers, or by not allowing other teams to answer after an initial wrong response
- use coloured cards on table of each group matching the colour of their marker on the board
- reduce the length of the game board
- teacher should not pick the teams
- more penalty boxes are needed in the game board
- do not take up wrong answers, provide written solutions for distribution at the end of class; game should not be used to go over wrong answers
- every student agreed that this game strategy should be part of the finite math curriculum

THEIR COMMENTS:

It's fun; it's very helpful; a good method of review; please extend game to calculus class; it's a fun way to do the most challenging part of the course; it is good practice, better than text review questions; great way to test each other in a fun way; should be played before the exam or test; it's a good incentive definitely; the board spacing could be improved.



▲ THE REVISED WITPO GAME



Abacus

From the desk of the Abacus editor

Pat Blake, a consultant with the Halton Board of Education, has kindly provided an entire package for the December Abacus. This package stems from her popular workshop, "Problem Solving : Making the Connection", and includes a valuable bibliography.

May all of you have a happy holiday season.

Amy



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PROBLEM SOLVING: MAKING THE CONNECTION!

The major goal of mathematics programs is to provide for the development of problem solving skills. We would like children to approach a task by thinking about it (rather than asking how to do it), to search for alternatives, to test out assumptions, to communicate ideas, and to reflect on the process. Through games, puzzles, literature, everyday experiences etc., children have many opportunities to make connections about the applications of mathematics. Experiences need to be multi-modal; let children act out the problem, build it, bake it, measure it, etc. Children's attitudes towards problem solving are also critical in their attaining success. Problem solving should be as enjoyable as reading a good book! The following activities and the list of resource books provide some sources for both teachers and parents in making this important connection. Enjoy!

Level: Primary or Junior (see individual activities)

Outcomes: Students will

- develop a variety of strategies to solve problems
- choose and implement a viable strategy
- use mathematical concepts related to number, geometry, spatial sense and measurement to reach a solution
- communicate their thinking in a variety of ways (pictures, role playing, models, charts, etc.)
- demonstrate a positive attitude towards problem solving

Assessment and Evaluation

Collect information about the student's attitude, process, and product through observation, conferencing, learning logs, audio tapes, rubrics, and oral presentation.

Can the student restate the problem in his or her own words?

Does the student select an appropriate strategy?

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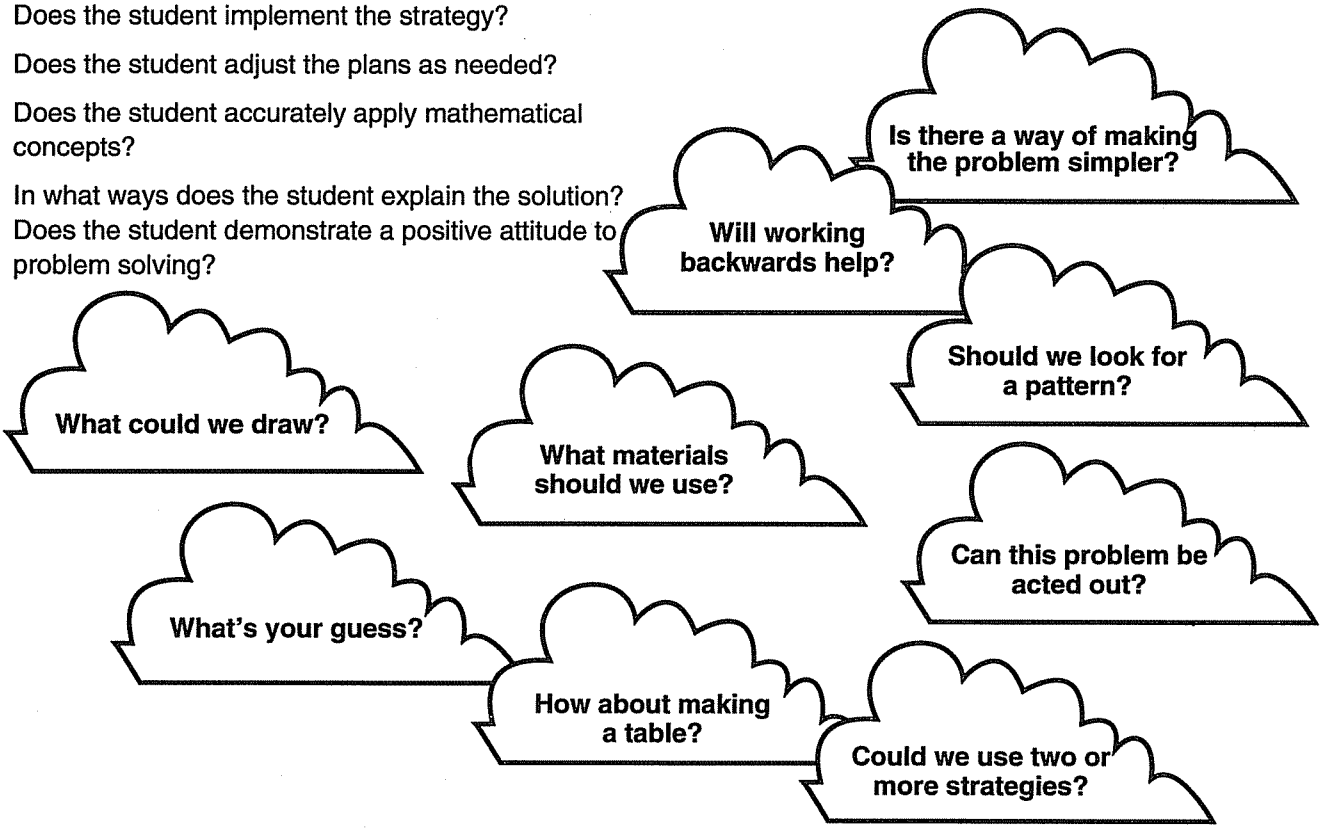
Edited by Amy Parker, School - Folkstone P.S., 104 Folkstone Cres., Brampton, Ont. L6T 3M5
Business (905) 792-2266, Fax (905) 792-8725, Home (519) 822-3034

Does the student implement the strategy?

Does the student adjust the plans as needed?

Does the student accurately apply mathematical concepts?

In what ways does the student explain the solution?
Does the student demonstrate a positive attitude to problem solving?



Problem solving begins with lots of opportunities for students to talk together, to ask probing questions and to "play" around with the problem. Thinking about thinking is a critical process. Teaching students to reflect on decisions - to ask questions - requires that teachers model the questions. Try building a visual chart of the process and the "thinking" questions.

THINK ABOUT THE PROBLEM

What is this problem all about?

How will I retell the problem in my own words?

Is this problem like any problem I've solved?

PLAN HOW TO SOLVE

What problem solving strategies do I know how to use?

Which strategy might work best?

Can concrete objects, the computer, or the calculator help me?

(See the clouds above.)

TRY OUT YOUR PLAN(S)

Is my strategy working?

What will I do next?

LOOK BACK

Does my solution provide a reasonable answer for the problem?

How do I know I'm done?

How else can I check?

USE MATERIALS

String Along

Level: Primary or Junior

Materials:

About 4 metres of string tied in a circle (use rope for younger children).

Activity:

Form a circle of 5 to 6 students with 1 circle of string.

Instruct students to hold onto the string with 2 hands. They must use both hands at all times and slide their hands back and forth when necessary. (Younger students may need to keep the string on the floor.)

As a group, complete the task posed.

"Make the string into:

- a shape with 4 sides...a different 4-sided shape
- a shape with 4 equal sides
- a shape with no sides the same length
- a shape with as many sides as you can make

OR

- a square
- a trapezoid
- a rectangle twice as long as it is wide
- a cube

Discuss each shape as it is completed and brainstorm additional words to describe it. "Is there more than one solution? Why?"

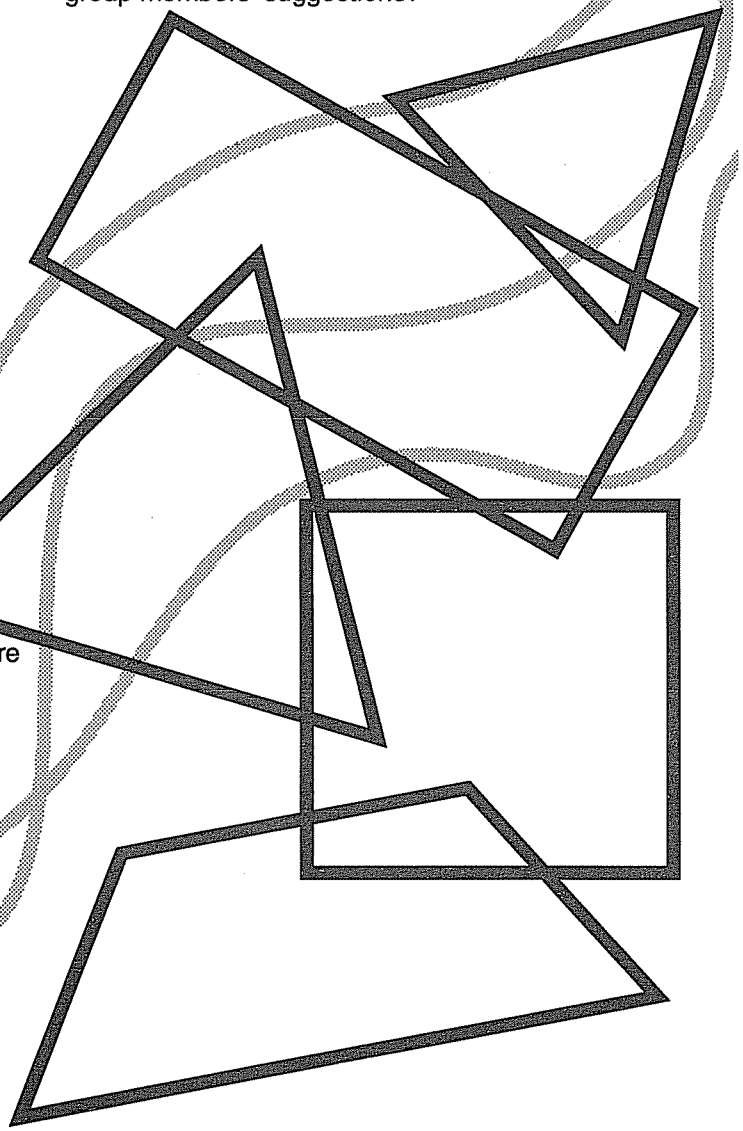
NOTE: Focus on the characteristics of the shape, not just the name.

Extension:

Have each group work cooperatively to create its own challenge. The group should first draw a shape, eg.,◇. Then, using the cooperative strategy round table, each person writes a word or phrase to build a "rich" description of the group's shape. When complete, the group invites another group to listen to its description and try to create the shape.

Evaluation:

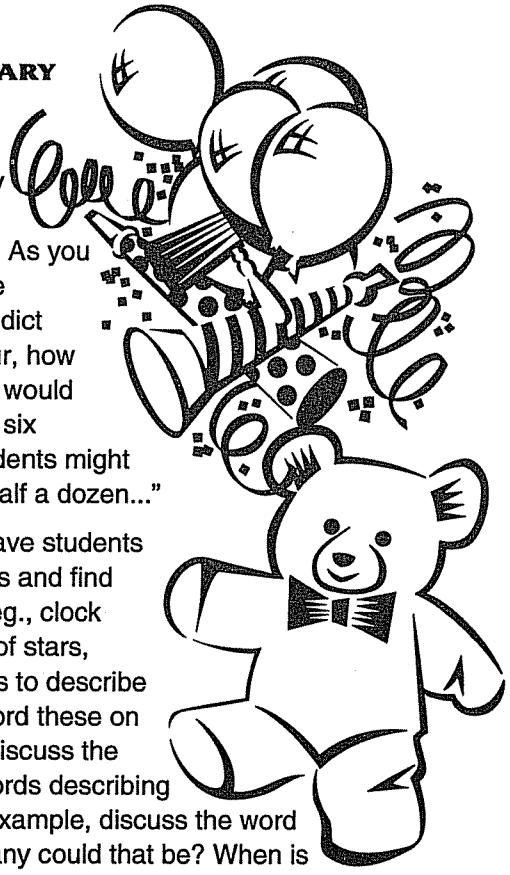
Do the students relate the characteristics to the appropriate shape? Do the students use appropriate language to describe the shape? Do the students refine their language as other groups respond to their descriptions? Do the students respond positively to their group members' suggestions?



USE LITERATURE

P. Bear's New Year's Party

by Paul Owen Lewis
Silvia Mattacchiane & Co.,
Pickering, Ont.
ISBN #09692640-8-9



Level: Primary

Activity:

1. Read the story to the children for enjoyment. As you read, allow the children to predict after each hour, how many animals would arrive, eg., "at six o'clock..." Students might reply "6", or "half a dozen..."
2. Reread and have students look at pictures and find connections, eg., clock face, number of stars, animals, words to describe number. Record these on chart paper. Discuss the meaning of words describing number. For example, discuss the word "lots". How many could that be? When is "a lot" a lot? When is "a lot" not enough?
3. Brainstorm and record all the different ways to find the answer to the question on the last page: "How many guests came to the party?" (Provide calculators too.)
4. Have small groups take the guest list or book and find the solution.
5. Share strategies and solutions with the whole group.
6. Discuss other ways to solve this problem.

Extension:

1. Have students pose their own questions to solve, eg., "How many guests were there at 4 o'clock?", or "How many more goats than zebras were there?"
2. Have students create their own party book following this pattern.
3. Act out the story with finger puppets.

What it Feels like to be a Building

by Forrest Wilson
Preservation Press,
Washington D.C., 1988
ISBN #0-89133-147-6

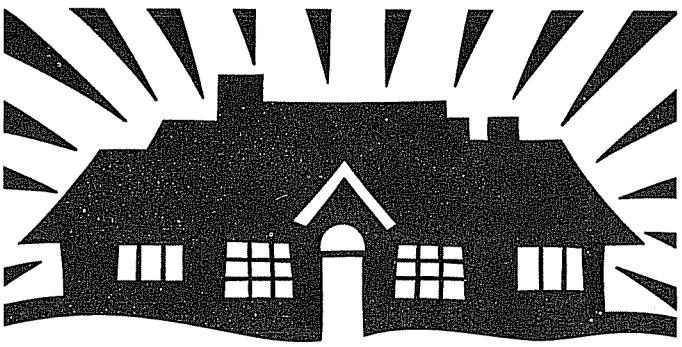
Level: Junior

Activities:

1. Read the title and respond in writing or with movement.
2. Through pantomime, tableau, or with physical objects, eg., cubes, 3-D shapes, blocks, popsicle sticks, geoblocks, lego, create your own visual representation of building words, eg., column, brace, arch, gables.
3. Find examples of the "building" words in your environment, magazines, and/or resource books.
4. In a group, build using junk (boxes, paper tubes, plastic containers). Do not use any tape or glue. How can you use the knowledge you gained in the book to help you?

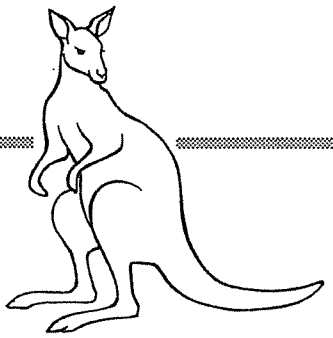
See if your structure can withstand the hurricane test. (Wave a piece of heavy cardboard back and forth 6 times over and beside your building.)

Write a learning log to tell about your experiences.

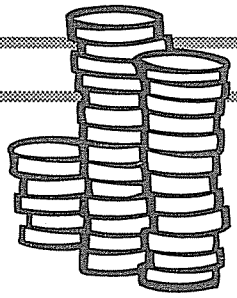


USE PROPS, MANIPULATIVES, ROLE PLAYING...

Level: Primary



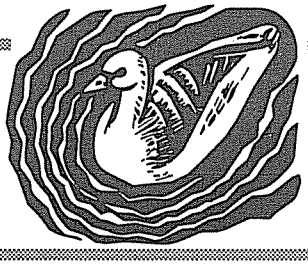
A rabbit, a kangaroo, an iguana, and a cow decided to have a race. The iguana was the slowest. The rabbit was faster than the cow but slower than the kangaroo. Who won the race? Who came in second, third, and fourth?

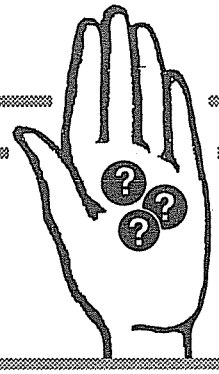


If you have a column of coins 10 centimetres high, how rich are you? (What do you need to consider?)

(What if you had a handful of coins?)

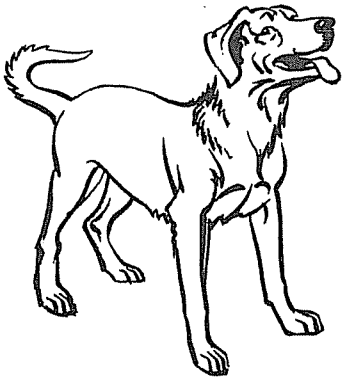
There are 4 animals in a pond. Some are frogs and some are ducks. There is a total of 10 legs. How many animals are ducks and how many are frogs?





You have some pennies, nickels and dimes in your pocket. If you put three coins in your hands, how much money could there be? How many different combinations could you have?

Level: Junior



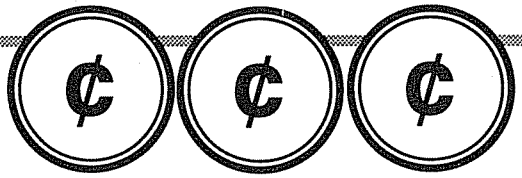
Suppose you had 64 metres of fence with which you were going to build a pen for your large dog, Bones. What are some different pens you can make if you use all the fencing? What is the pen with the least play space? What is the biggest pen you can make - the one that allows Bones the most play space? Which would be best for running?

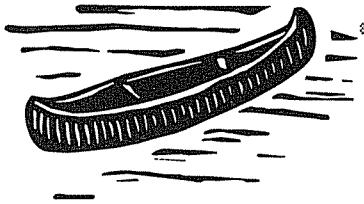


Cindy has a pocketful of change. She would like to buy a gift which costs 75¢. How could she pay for the gift so she could eliminate the most change from her pocket?

(What are all the things you need to consider?)

"$500.00 worth of pennies laid side by side would make a line 1 kilometre long." Do you agree or disagree?





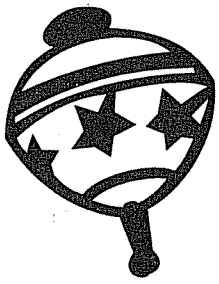
A man and two boys want to cross a river and only one small canoe is available. The canoe can hold either the two boys or the man. What must they do in order for all three of them to get across the river?

USE DESIGNING

Design a Spinning Top

suitable for gift giving in the holiday season!





Level: Primary or Junior

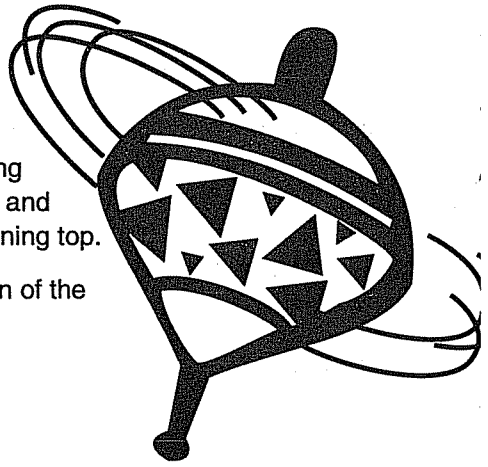
Materials:

a collection of spinning tops

bristol board, scissors, markers, straws, skewers, screws (different lengths), paper cups, plasticine, pencils, yogurt lids, compass, egg cartons plus any other pieces of beautiful junk.

Activity:

1. Provide students with a collection of spinning tops to play with.
2. Brainstorm about what makes a top spin. Develop appropriate language related to shape, balance, length, etc.
3. In pairs, have students use a collection of "beautiful junk" to create a spinning top. Share the creations.
4. Brainstorm what changes might be made to affect the spinning. (What if I change the length of the shaft, the shape of the spinner, the material, the height of the spinner, the way it is launched, etc.?) Investigate and discuss the findings.
5. Have students look back at what has been learned and design a top that will spin for a long time. Record the number of seconds various tops spin. Think about why!



Extension:

1. Investigate the effect of combining different patterns and colours on a spinning top.
2. Find out the origin of the "dreidel".

Resources for Problem Solving

Primary-P, Junior-J

Burns, Marilyn and Tank, Bonnie, *A Collection of Math Lessons* Grades 1 through 3, (also, Grades 3 through 6), Math Solutions Publication, 1988

Burns, Marilyn, *About Teaching Mathematics,* A K-8 Resource, Math Solutions Publications, 1992 **(P.J.)**

Burns, Marilyn, *Math for Smarty Pants*, Little Brown and Co., 1982 **(J.)**

Burns, Marilyn, *The Good Time Math Event Book.* Creative Publications, 1977 **(J.)**

Burns, Marilyn, *The I Hate Mathematics Book*, Little Brown and Co., 1975 **(J.)**

Clyne, Margaret and Griffiths, Rachel, *Books You Can Count On, Linking Mathematics and Literature,* Heinmann, 1991 (Irwin) **(P.J.)**

Cook, Marci, *Domino Math*, Creative Publications, 1988 **(P.J.)**

D'Angela, J. and Lessard, P, *The Geoboard Collection* (Grades 4-6)(Grades 7-9), Exclusive, 1990.

Graham, Noel and Blinko, Janine, *Mathematical Beginnings, Problem Solving for Younger Children*, Claire Publications, 1988 **(P.)**

Hyams, S., Challenges for Children, *Problem Solving For Young Children,* Claire Publications, 1989 (P)

Jones, Grant, *Problem Solving: What You Do When You Don't Know What to Do*, Grades 4 to 9, Exclusive, 1991

Roper, Ann, *Cooperative Problem Solving with Pattern Blocks* (Gr. 4-6), Creative Publications, 1989 **(J.)**

Roper, Ann, *Primary Cooperative Problem Solving with Pattern Blocks*, Creative Publications, 1990 **(P.)**

(See other cooperative problem solving titles which use a wide variety of manipulatives from Creative Publications.)

Seymour, Dale, *Problem Parade* (Grades 4-6), Dale Seymour Publications, 1984

Whitin, D. and Wilde, S., *Read Any Good Math Lately?*, Heinmann, 1992 **(P.J.)**

Arithmetic Teacher magazine, National Council of Teachers of Mathematics **(P.J.)**

Resources for Parents to use with Children

GAMES

Abalone (Exclusive)

Board Games - Monopoly, Junior Monopoly

Card Games - Uno, The Klutz Book of Cards

Computer Software - Tetris

Chess

Dominoes

Memory

Puzzles

Tangram Puzzles (Tangoes)

Backgammon

Harvest Time

Checkers

Connect Four

Jenga

Number Jumbler (Discovery Toys)

Ready or Not (Exclusive)

The Twenty-Four Game (Exclusive)

Triominoes

BOOKS

Blum, Raymond, *Mathemagic*, Sterling Publishing Co. Inc., 1991

Burns, Marilyn, *I Hate Mathematics*, Little Brown & Co., 1982

Burns, Marilyn, *Math for Smarty Pants*, Little Brown & Co., 1982

Haines, Max, *Mystery Challenge*, Wasn't That a Party Inc., 1992, Vol. 1-4

Metzner, S. and Sharp, R., *The Sneaky Square and 113 Other Activities for Kids.* Tab Books, 1990

Pinkney, John, *Big Book of Puzzles, Riddles and Muddles,* 5 Mile Press Ltd., 1992

Stenmark, Jean, *Family Math,* Addison Wesley, 1986

Townsend, Charles Barry, *World's Most Baffling Puzzles,* Sterling Publishing Co., 1992

Wells, David, *The Penguin Book of Curious and Interesting Puzzles*, Penguin Books, 1992

The Dell Book of Logic Problems #1, Dell, 1986

The Dell Book of Logic Problems #2, Dell, 1986

CHILDREN'S LITERATURE

Counting Books

Anno, M., *Anno's Counting House,* Philomel Books, 1982

Anno, M., *Anno's Flea Market,* Harper & Row, 1984

Crossley, K., *Under the Sun and Over the Moon*, Doubleday, 1987

Crowther, Robert, *The Most Amazing Hide and Seek Counting Book*, Viking Penguin, 1981

Gretz, Suzanna, *Teddy Bears One to Ten*, Ernest Benn, 1969

Hutchins, Pat, *1 Hunter*, Greenwillow Books, 1982

Johnston, Tony, *Whale Song*, G.P. Putnam's Son, 1987

Lottridge, C., *One Watermelon Seed*, Oxford University Press, 1979

Macdonald, Suse & Oakes, Bill, *Numblers,* FitzHenry & Whiteside Ltd., 1989

Ockenga, Starr, *World of Wonders: A Trip Through Numbers*, Houghton Mifflin, 1988

Sendak, Maurice, *One Was Johnny*, Harper & Row, 1962

Twinn, M., *There Were 10 in a Bed*, Child's Play, 1979

Number Books

Anno, M., *Anno's Mysterious Multiplying Jar*, The Bodley Head, 1983

Birch, David, *The King's Chessboard,* Dial Books, 1988

Bogart, Jo Ellen, *10 for Dinner,* Scholastic, 1989

Burningham, John, *The Shopping Basket*, Collins Publishing Group, 1980

Carle, Eric, *The Hungry Caterpillar*, Scholastic, 1981

Carle, Eric, *The Rooster Who Set Out to See the World,* Franklin Watts Inc., 1987

Hutchins, Pat, *The Doorbell Rang*, Mulberry Books, 1986

Schwartz, David M., *How Much is a Million*, Scholastic Inc., 1989

Geometry Books

Anno, Mitsumasa, *In Shadowland*, Orchard Books, 1988

Anno, Mitsumasa, *Upside Downers*, Putnam & Grosse Group, 1971

Hoban, Tana, *Circles, Triangles, and Squares*, MacMillan, 1974

Hoban, Tana, *Look Again*, MacMillan, 1971

Hoban, Tana, *Shapes and Things*, MacMillan, 1970

Hutchins, Pat, *Changes, Changes*, MacMillan, 1973

Jonas, Ann, *Round Trip*, Greenwillow, 1983

Tompert, Anne, *Grandfather Tang's Story, a Tale Told with Tangrams*, Crown Publishing, 1990

Measurement Books

Carle, Eric, *The Very Hungry Caterpillar*, Putnam Publishing Group, 1969

Caldone, Paul, *The Little Red Hen*, Seabury Press, 1973

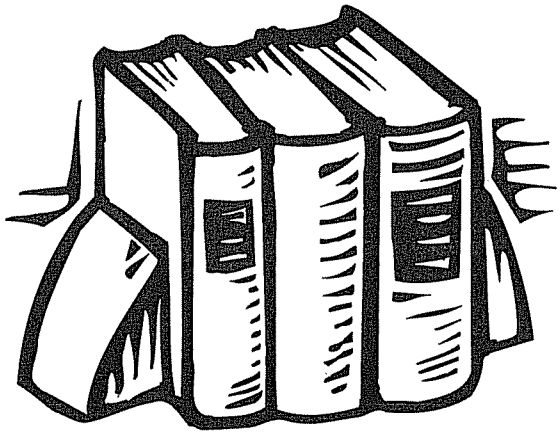
Krauss, Ruth, *The Carrot Seed*, Harper & Row, 1945

Lionni, Leo, *Mouse Days*, Pantheon Books, 1980

Lotteridge, Celia, *Name of the Tree*, Greenwood Books, 1989

Marshall, James, *Yummers!*, Houghton Mifflin, 1973

Sendak, Maurice, *Chicken Soup and Rice,* Harper and Row, 1962



QUESTIONS AND ANSWERS FOR THE REVISED WITPO GAME

What is the prob. of the sum of two dice being a multiple of four? Ans. 1/4 Time: 60s

What is the prob. of the sum of two dice equalling 7 or 11? Ans. 2/9 Time: 60s

What is the prob. of the sum of two dice equalling 2,3 or 12? Ans. 1/9 Time: 60s

What is the prob. of guessing correctly on four true-false questions in a row? Ans. 1/16 Time: 60s

What is the prob. of tossing four coins and getting exactly two heads? Ans. 3/8 Time: 60s

What is the prob. of tossing four coins and getting exactly three heads? Ans. 1/4 Time 60s

Three cards are drawn from a deck without replacement. The prob. of getting 3 aces is: Ans. 1/5525 Time: 60s

Three cards are drawn from a deck without replacement. The prob. of getting 3 diamonds is: Ans. 11/850 Time: 60s

Three coins are tossed, the prob. of obtaining 2 heads and a tail is Ans. 3/8 Time: 30s

Three coins are tossed, the prob. of obtaining at least two heads Ans. 1/2 Time: 30s

A bag contains 6 red, 5 blue, and 4 green chips; the prob. of drawing one red chip is: Ans. 2/5 Time 30s

The prob. of drawing a red even numbered card from a full deck is Ans. 5/26 Time: 30s

Two dice are thrown. What is the prob. of rolling a total under five or a double? Ans. 5/18 Time: 60s

In a family of 4 children the prob. of more than 2 boys is: Ans. 5/16 Time: 60s

Editor's Suggestion: Photocopy pages 19-21, cut out the individual rectangles and enlist some volunteers to help you paste them on to forty-five cardboard squares.

Two dice are tossed, what is prob. of tossing a double or a sum of seven? Ans. 1/3 Time: 60s

Mr. Smart, Mr. Wise, Mr. Brain work independently on a problem. If the probabilities that they will solve it are 1/2, 1/3 and 3/5: What is the prob. that the problem will be solved? Ans. 4/5 Time: 60s

What is the prob. of throwing two fives in a row with a pair of dice Ans. 1/81 Time: 60s

If P(A) = 1/3, P(B) = 1/4 and P(C) = 4/5, then P(A and not B and not C) is: Ans. 1/5 Time: 60s

Two dice are rolled. What is the prob. that the total is greater than five? Ans. 13/18 Time: 60s

Three coins are tossed, what is the prob. of obtaining at least two tails? Ans. 1/2 Time: 60s

One chip is drawn from a bag of 3 white, 4 red and 5 black. What is the prob. that it is not red? Ans. 2/3 Time: 60s

What is the prob. of tossing 4 coins and getting exactly 3 heads? Ans. 1/4 Time: 60s

A bag contains 5 blue, 4 green, 6 red chips. What is the prob. of not drawing a green chip? Ans. 11/15 Time: 60s

Two dice are rolled, one blue and one red. What is the prob. that the number on the red die is less than the number in the blue die? Ans. 15/36 Time: 60s

A die is rolled 3 times: what is the prob. of getting at least two fours? Ans. 2/27 Time: 60s

Two dice are rolled and show a sum of ten. What is the prob. that the second die shows a six? Ans. 3/6 Time: 60s

Two dice are rolled. What is the prob. that the difference in the dice is two? Ans. 2/9 Time: 60s

The prob. that a student will pass history is 0.95. The prob. that a student will not pass French is 0.40. What is the prob. that a student will pass both courses? Ans. 0.57 Time: 60s

Two dice are rolled. What is the prob that at least one 4 or a total of 7 appears? Ans. 5/12 Time: 90s

Three hockey players calculated their respective probs. of scoring a goal in a game as 0.90, 0.72, 0.68. What is the prob. that at least one of them will score? Ans. 0.99 Time: 60s

Person A tells the truth 3/4 of the time. Person B tells the truth 5/6 of the time. If they are asked about the same fact, what is the prob. that they contradict each other? Ans. 1/3 Time: 60s

From a pack of 52 cards, you draw a card. What is the prob. of drawing a King, Queen, Jack or a Spade? Ans. 11/26 Time 60s

If the probs. of a man and woman both 50 years old living till age 51 are 0.80 and 0.85 respectively, what is the prob. that neither will live until their 51st birthday? Ans. 0.03 Time: 60s

A sack contains 5 yellow, 2 blue, 3 green marbles; three marbles are drawn without replacement. What is the prob. of drawing 3 yellow marbles? Ans. 1/12 Time: 60s

From a pack of 52 cards, three cards are dealt. What is the prob. that all cards are hearts? Ans. 11/850 Time: 60s

A three digit number is formed by randomly selecting 3 of the digits from 1 to 9 without repetition. What is the prob. that number is composed of odd digits. Ans. 5/42 Time: 60s

Three dice are thrown. What is the prob. of throwing a sum of three? Ans. 1/216 Time: 60s

Two dice are thrown. The prob. of getting not a total of five, nine or eleven is:Ans. 13/18 Time: 60s

Four dice are rolled. What is prob. of throwing exactly one four or a total of ten? Ans. 11/36 Time: 90s

Two dice are rolled. What is the prob. that the total is greater than 8? ans. 5/18 Time: 60s

Two dice are rolled. What is the probability of getting a sum less than 4? Ans. 1/12 Time: 60s

Reggie's batting average is 0.4. What is the prob. of Reggie getting at least one hit in two times at bat? Ans. 0.64 Time: 60s

Three dice are thrown. What is the prob. of not throwing a sum of 3? Ans. 215/216 Time: 60s

What is the prob. of correctly answering exactly two of three true-false questions (assuming you are guessing)? Ans. 3/8 Time: 60s

The prob. of winning against Vanier is 3/5 against Jackson is 1/3 and against Westview is 1/10. What is the prob. of losing all 3 games? Ans. 6/25 Time: 60s

▲ Assessment and Evaluation of Students in MAT 1W1

DEAN A. MURRAY
MILTON DISTRICT HIGH SCHOOL
MILTON ONTARIO



Dean A. Murray has been teaching for 19 years, 16 as head of mathematics at Milton District High School. He is programme chair for GVMA and a member of the Pascal, Cayley, and Fermat Committees for the Canadian Mathematics Competition. In this article, Dean shares his department's methods of assessing students in the destreamed grade 9 mathematics course.

As schools destream their grade 9 mathematics courses, a different approach to assessment and evaluation is required. Each department should attempt to include a variety of assessment strategies to supplement the paper-and-pencil testing. As suggested in the June 1993 OAME Gazette, additional strategies may include performance assessment, open-ended questions, investigations, journals, observations, conferencing/interviewing, portfolios, and student self-assessment.

At Milton District High School we decided our Assessment and Evaluation methods needed to meet the following criteria:

- satisfy the Ontario Provincial Standards of Performance for mathematics
- meet administrative criteria
- be a reliable method to select students for general or advanced level grade 10
- provide information that would make reports meaningful and understandable by students and parents
- be objective
- be efficient

What follows is our evaluation system for the new grade 9 courses and how we responded to the challenge. The report cards were designed to follow our assessment and evaluation system. They have been printed in a format that is easy to read, and which illustrates our measurements and criteria to make sense to students, parents and administrators.

Note that our grade 9 is non-semestered. The dates in the four reports can be modified for a semestered grade 9.

All grade 9 courses at Milton D.H.S. are offered with one level of difficulty. Mathematics courses in Grades 10, 11, and 12 will be taught at two levels of instruction: General and Advanced.

Mathematics teachers will make a preliminary recommendation for each student at the end of the first term. In April teachers will review their initial recommendation. Parents will be notified regarding the level of instruction that their child should consider taking next year in mathematics. At the end of second term, a final recommendation will be made and printed on the report card.

Terms One and Two:

Marks will consist of two components: the Formal and the Continuous.

THE FORMAL

The first component of evaluation is the traditional paper-and-pencil testing and examinations. It will serve as the most important criterion in determining whether a student will be better suited for MAT2G1 or MAT2A1 in the following year. At the end of either term a mark less than sixty percent for tests and examinations would normally indicate the student should take MAT2G1. A mark between sixty and seventy percent would indicate the student may proceed to MAT2A1 or may choose to take MAT2G1. A student who earns a mark over seventy percent would be recommended for MAT2A1.

Students and parents should refer to the school course calendar for a description of all mathematics courses. Advanced level courses in mathematics are designed to provide fundamentals in mathematics for the student considering university studies, and as such, are at a more abstract level than general level courses. General level courses provide a mathematics background stressing practical applications taken from everyday experience. General level courses meet requirements for admission to Colleges of Applied Arts and Technology. However, students are encouraged to take advanced level courses where possible to provide more opportunities for post-secondary learning.

The first component will be determined using the following:

Term One		Term Two	
Tests	- 35%	Tests	- 35%
Exam (January)	- 15%	Exam (June)	- 15%

THE CONTINUOUS

The second component of assessment is based more on a broad range of assessment practices. The headings will be student self-management, portfolios, and student self-assessment.

1. Student Self-Management: a mark out of 30

A mark out of 10 for each of the sub-headings (dependability, self discipline, and participation) will be assigned by the teacher using the following:

5 • student always meets expectations

4 • student usually meets expectations

3 • student often meets expectations

2 • student needs improvement in this area

1 • student rarely meets expectations.

The Student self-management criteria to be observed are:

Dependability: a mark out of 10

• arrives on time

• maintains good attendance

• brings necessary equipment and supplies

• completes homework

• maintains an organized notebook, as described in the mathematics policy and guidelines

Self Discipline: a mark out of 10

• works well without supervision and stays on task

• controls behaviour

• cooperates with classmates and teacher

• responds appropriately

• follows classroom routines

• is courteous and willingly helps teachers and students

• follows directions and instructions

• seeks extra help for problems immediately upon having the difficulty, either in the mathematics clinic or from the teacher

Participation: a mark out of 10

• completes assignments

• takes an active role in the classroom

• assumes responsibility for own learning

• asks and answers relevant questions

• contributes to class lessons and decision making

2. Portfolios: a mark out of 10

Portfolios will be evaluated. A mark out of 5 will be given by the teacher, based on completeness, organization, neatness and the quality of the work. Throughout the year, the teacher will look in students portfolios for signs of growth in attitudes, mathematical understanding, and achievement.

Student self-assessment promotes ownership of learning, and independence of thought. It is an opportunity for students to reflect upon their own work. Students will assign a mark out of 5 for their personal portfolio. The mark is determined by assessing their individual effort and performance, using the portfolio guidelines.

3. Student self-assessment: a mark out of 10

The mark will be calculated as follows:

Step 1: • Student self-assessment

The student will assign a mark out of 5 using the criteria below:

5 marks • students who attempt to do their best. They always attempt their homework and study for tests and examinations. They attempt enrichment problems regularly, and usually try work beyond what is expected. They try to keep the learning environment in the class productive and positive.

4 marks • students who continue to work effectively daily. They always attempt their homework and study for tests and examinations. They try to keep the learning environment in the class productive and positive.

3 marks • students who try to be good mathematics students. They attempt their homework and study for tests most of the time. They attempt to keep the learning environment in the class productive and positive.

2 marks • students who put forth some effort but are not attempting to be good mathematics students. They frequently do not complete their homework and often do not study for tests.

1 mark • students who seldom attempt their homework and are not trying to be good mathematics students. They often do not study for tests. They attempt to keep the learning environment in the class unproductive.

0 marks • students who are not trying to be good mathematics students and do not attempt their homework nor study for tests often. They regularly attempt to keep the learning environment in the class unproductive.

Step 2: • Peer assessment of the student

The student will receive a mark equal to the average mark rounded to the next highest number of the marks given by their three peers. Two of the student's peers will be selected by the student and one by the teacher. Each peer will assign a mark to the student with reasons using the same criteria as outlined in step one.

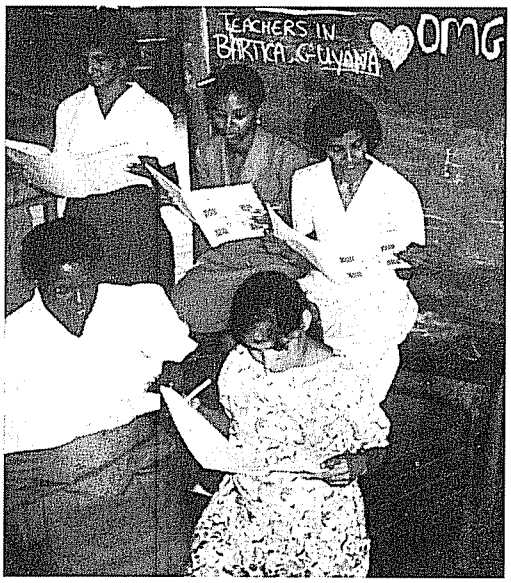
Step 3: • Grade out of 10 calculated by simply adding the results of Step 1 and Step 2: that is,

the sum of Student self-assessment + Peer assessment of the student.

School Reports-for grade 9

Milton District High School issues four reports during the school year for grade 9 students. In November, and again in April, an anecdotal progress report is issued. At the beginning of February, and again in June, a report card is issued with marks given for each course.

Editor's note: the November and January reports are included with this article.



Deidre McConnell, a concurrent Education and Physics student at Queen's University, supplied teachers and students at a school in Bartica, Guyana with some suitable reading material.

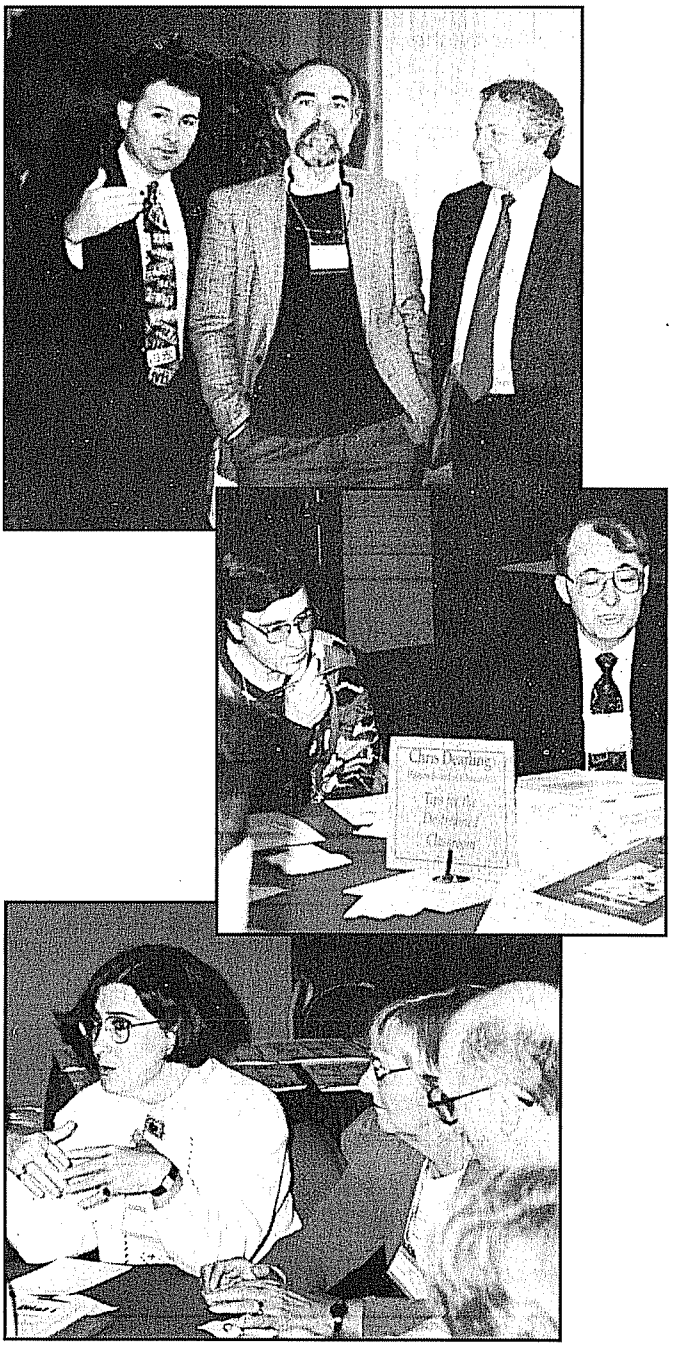
Calculation of report card marks for MAT1W1

Final Report Card Mark for Semester One: The sum of the Formal and Continuous evaluations.

Final Report Card Mark for Semester Two: The sum of the Formal and Continuous evaluations.

Final Report Card Mark in June: [Final Mark for Term One plus Final Mark for Term Two] divided by 2.

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MILTON DISTRICT HIGH SCHOOL

GRADE 9 REPORT CARD NOVEMBER, 1993

MATHEMATICS

STUDENT: _____ HOMEFORM: ___ SUBJECT TEACHER: _____

SELF MANAGEMENT																	
DEPENDABILITY			SELF-DISCIPLINE			PARTICIPATION			PORTFOLIO			SELF-ASSESSMENT			TEST RESULTS		
E	S	N	E	S	N	E	S	N	E	S	N	E	S	N	E	S	N

E: EXCELLENT S: SATISFACTORY N: NEEDS IMPROVEMENT

Dependability:
- arrives on time
- maintains good attendance
- brings necessary equipment and supplies
- completes homework
- maintains an organized notebook, as described in the mathematics policy and guidelines

Participation:
- completes assignments
- takes an active role in the classroom
- assumes responsibility for own learning
- asks and answers relevant questions
- contributes to class lessons and decision making

Self Discipline:
- works well without supervision
- stays on task
- controls behaviour
- cooperates with classmates and teacher
- responds appropriately
- follows classroom routines
- is courteous and willingly helps others
- follows directions and instructions
- will seek extra help for problems immediately upon having the difficulty, either in the mathematics clinic or from the teacher

Portfolio:
- shows signs of growth in attitudes, mathematical understanding, and achievement
- is complete
- is organized and neat

Self-Assessment:
- completes homework
- studies for tests
- works with others productively
- attempts enrichment problems

COMMENTS:

Your assistance in monitoring your son's/daughter's progress is very helpful. Should you wish to discuss this report, please telephone 878-2830 to make an appointment for Parents' Night . Parents' Night is Wednesday, November 17.

Teacher's signature: _____

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MILTON DISTRICT HIGH SCHOOL

GRADE 9 REPORT CARD **JANUARY, 1994**

MATHEMATICS

SELF MANAGEMENT						
DEPENDABILITY	SELF-DISCIPLINE	PARTICIPATION	PORTFOLIO	SELF-ASSESSMENT	TESTS AND EXAMINATION	FINAL MARK SEMESTER ONE
OUT OF 10	OUT OF 10	OUT OF 10	OUT OF 10	OUT OF 10	OUT OF 50	OUT OF 100

The first component of evaluation is the traditional paper-and-pencil testing and examinations. It will serve as the most important criterion in determining whether a student will be better suited for MAT2G1 or MAT2A1 in the following year. At the end of either term a mark less than sixty percent for tests and examinations would normally indicate the student should take MAT2G1. A mark between sixty and seventy percent would indicate the student may proceed to MAT2A1 or may choose to take MAT2G1. A student who earns a mark over seventy percent would be recommended for MAT2A1.

Students and parents should refer to the school course calendar for a description of all mathematics courses. Advanced level courses in mathematics are designed to provide fundamentals in mathematics for the student considering university studies, and as such, are at a more abstract level than general level courses. General level courses provide a mathematics background stressing practical applications taken from everyday experience. General level courses meet requirements for admission to Colleges of Applied Arts and Technology. However, students are encouraged to take advanced level courses where possible to provide more opportunities for post-secondary learning.

The student named above is being recommended for _____ for September.
The recommendation will be reviewed following the Progress Report in April.

COMMENTS:

Teacher's signature: _____

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▲ Mathematics and the World Around Us

RON LANCASTER, ST. MILDRED'S-LIGHTBOURN SCHOOL, OAKVILLE, ONTARIO



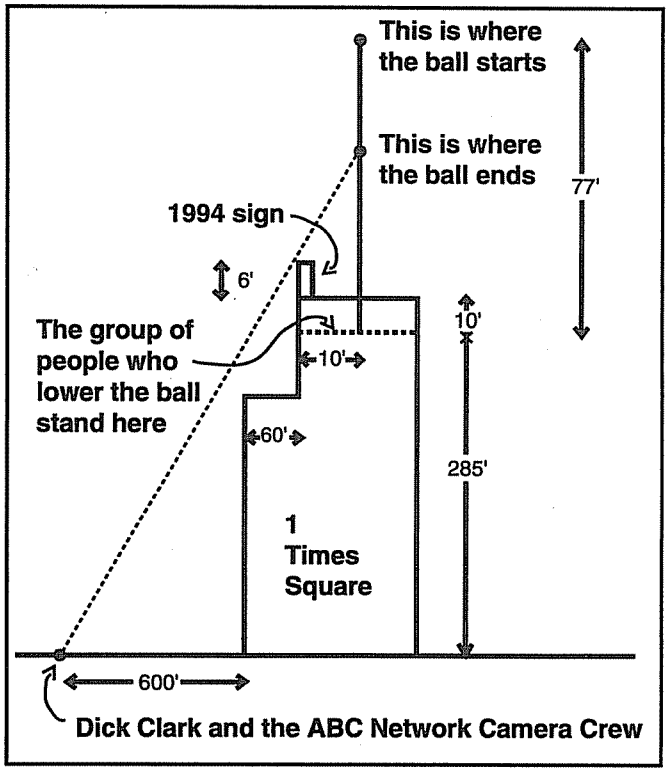
Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of the Gazette as an aid to helping their students see the connection between mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

Last year Shelly and I celebrated our second New Year's Eve together on the roof of 1 Times Square in Manhattan and the questions that follow are all related to the lowering of the ball in Times Square. With the new year just around the corner, I thought that it might be interesting to create an entire column related to the celebration of New Year's Eve and I hope that you are able to use some or all of these questions with your students.

Perhaps I should first explain how we ended up on the roof. Shelly grew up on the Upper West Side in Manhattan and her best friend through high school was Tama Starr. Tama and her brother own Artkraft Strauss, a company that was started by their grandfather. Artkraft Strauss makes all of the huge electronic signs in Times Square and this company has been in charge of lowering the ball in Times Square ever since 1907. Each year Tama invites a small group of people to watch the ball come down and needless to say I was ecstatic when she asked us if we wanted to be included in the group. It was an incredible experience to look down upon a crowd of hundreds of thousands of people and then to stand right beside the ball as it was dropped.

I was expecting to see a rather sophisticated system for lowering the ball and I was surprised to find that it was attached to a rope and lowered by a group of people who held one end of the rope. The release is done so smoothly that there is no need for any specialized equipment. The ball is lowered at one minute before midnight and millions of people all over the world watch the descent on television. If you watch the lowering of the ball this year, you will see that the ball continues to drop until it appears to be just above a big 1994 sign. At that point the ball is turned off and the 1994 sign is turned on. If all goes well this should happen at exactly midnight. If you look closely at the ball you will see that it is actually a spherical shell with 180 30-watt light bulbs attached to it.

The diagram and the photographs given below should help you to understand the geometry of the set-up.



The crew that drops the ball needs to know in advance how far the ball should be lowered. There is not much of an opportunity to rehearse the lowering of the ball with the ABC Network and they have found it essential to know this information beforehand.

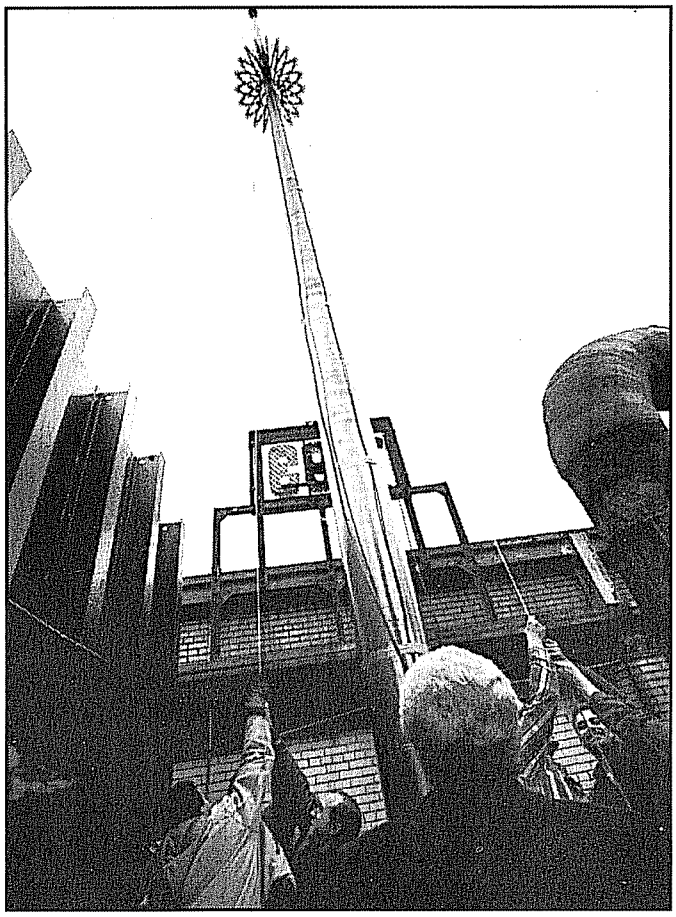


Photo © 1993 by Scott Humbert (212) 673-4795

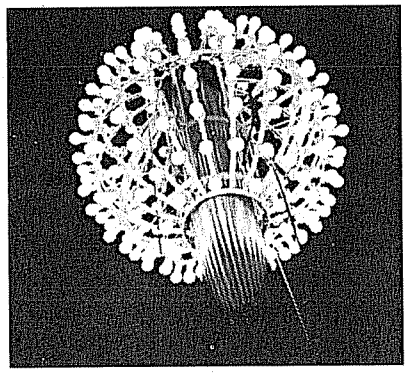
Questions

1 a) How far should the ball be dropped?

b) If the ball is dropped at a constant rate over the one minute interval, find the rate at which the ball should be lowered.

c) The ball is 6 feet in diameter. What is the volume of the ball?

d) Look closely at the photograph of the ball given below and try to figure out how the 180 light bulbs are arranged around the ball. You might find it helpful to make a scale model.



e) How much does it cost to light up the 180 bulbs for the one minute descent? What information do you need to know to do this question?

2. January 1, 1993 was on a Friday. Find the day of the week on which January 1 lands for the next 100 years. Is there a pattern? Will there be any change in this pattern after the year 2100? Is there any day of the week on which January 1 does not fall?

3. Have you ever wondered how people estimate the size of a crowd like the one in Times Square? There have been a number of reported cases where officials have simply made up the number without giving the matter any thought, but as noted in the excerpt from the article given below, ("Crowd counts a duel of truth and propaganda", Kitchener-Waterloo Record, Saturday, October 16, 1991) there is a formula that is often used to estimate the size of a crowd.

"Probably the most accurate formula for crowd counting - and the one used by the Park Service in Washington - is to measure the boundaries of the gathering, then divide by the number of square feet occupied by a standing adult."

The figure that is generally used for the space occupied by an adult in standing-room conditions is 2 square feet. The rule is not very well explained in the above excerpt and it might have been better had they just said "calculate the surface area and divide by 2." The article appeared on two pages and that was in fact the title that they used for the continuation of the article on the second page.

a) Many of the hundreds of thousands of people who watched the ball drop were standing along 7th Avenue. They stood in partitions on opposite sides of 7th Avenue and enough space was left in part of the street for emergency vehicles to get through. When Shelly and I looked down we were amazed to see that the crowd extended all of the way up 7th Avenue from 42nd Street to Central Park at 59th Street. In Manhattan there are 20 blocks in one mile. Use the rule given in the above article to estimate the number of people who watched the lowering of the ball on 7th Avenue.

b) Carry out an investigation to verify the accuracy of the claim made in the above newspaper article that the space occupied by an adult in standing-room conditions is 2 square feet.

Comments and Answers

There may be some teachers who are concerned about the use of non-metric units in the above information. As a general rule I do not use non-metric units, but I feel justified in using them in this setting. This is the exact information given to me by Mama Starr and it is not unusual for measurements from the United States to be in non-metric

units. The United States has not yet adopted the metric system and until it does it seems reasonable that we should be able to do some basic work with its units. The problems are too good to be thrown away because of the non-metric units, so if you are really concerned about this issue then you can easily convert all of the measurements to metric units.

1. a) There are a variety of ways that this question can be answered. For students in Grades 7 to 10 you might have them make a scale diagram and then have them measure the desired distance with a ruler. The actual distance can then be found by using the scale of their diagram.

   Students in other Grade levels (and perhaps even some from Grades 7 to 10) can solve this problem by using similar triangles. One way of starting this is to let x represent the distance from the point where the ball started to the point where the ball ended.

   Using similar triangles it can be shown that

$$\frac{77\text{-}x\text{-}10\text{-}6}{10} = \frac{285+77-x}{600+60+10}$$

   By solving this equation you will find that $x \approx 56.4$. Therefore the ball ended up about 20.6 feet above where the crew was standing. We were standing on top of some equipment above and off to the side of the crew and this answer seems pretty reasonable to me. I recall that when the ball stopped moving it was about 12 feet above us.

   b) In one minute the ball travelled 56.4 feet. Therefore the ball was dropped at the rate of about 56.4 feet per minute. You might want to ask your students if this is more or less than 1 mile per hour.

   c) Students will need to use the following formula for the volume V of a sphere whose radius is r.

$$V=\frac{4}{3}\pi r^3$$

   Since $r = 3$ it follows that $V \approx 113$ cubic feet.

   d) Students will be able to come up with a variety of possible arrangements for the 180 light bulbs. To be honest, I am not sure what arrangement is used. We will be in New York City in the near future and I will see if I can find out how the light bulbs are arranged.

   e) The students will need to know the cost of electricity in Manhattan. You could ask your students to try to find out what this cost is or you could just have them use the rate for Ontario. They will find that the rate is given as a price per kilowatt hour. In one minute the 180 30-watt light bulbs used up 5 400 watts of electricity. This is equivalent to using 0.09 kilowatts in an hour. This number should be multiplied by the rate in order to find the cost of lighting the bulbs for the one minute time interval. The total cost was probably somewhere in the area of 1 cent.

2. This is a wonderful way to introduce your students to the concept of something that is periodic. For the time being I will leave it to you and your students to see if you can find the pattern. The pattern does change after the year 2100 because that year is not a leap year. A century year is a leap year only if it is divisible by 400.

3. a) In order to do this problem your students will need to know the width of 7th Avenue. I do not have this information at this point in time, but I recall that 7th Avenue is a one-way street with 5 lanes and wide sidewalks on both sides. Have your students find the width of a typical street in their area either by calling city hall or by actually measuring the width. They will then be able to estimate the width of 7th Avenue from this result. Remind them to include the sidewalks in the calculations. I will obtain the actual measurements and report on what I find in the next column. You can then see how well your students were able to estimate the width of 7th Avenue.

   b) Your students might want to carry out this investigation at a pep rally or you might bring them outside one day and have them form a crowd. I have found in the past that the figure of 2 square feet is pretty accurate.



▲ About Problem Solving

JOHN GRANT MCLOUGHLIN, DEPARTMENT OF MATHEMATICS, MEMORIAL UNIVERSITY CORNERBROOK, NEWFOUNDLAND

Welcome to the problem-solving column. I am pleased to have been invited by Jack Weiner to reintroduce the column following its brief absence. Many of you will recall that Ron Lancaster wrote this column for several years prior to shifting his attention to "Mathematics and the World Around Us". You may have attempted to verify the solution to a pyramid problem that was included in the April 1993 Gazette. Ron wishes to acknowledge the verifications submitted by Terri Laliberte, a teacher at Saugeen District S.S. in Port Elgin, and Ranjit Zachariah, an OAC student (at the time) at University of Toronto Schools.

This regular feature is intended to serve as a medium for the exchange of interesting problems and solutions. Also, it is hoped that readers will take an active role in shaping what is written. You are encouraged to share problem-solving experiences from your own classrooms. Each issue of the Gazette will include some problems that you may wish to play with. An effort will be made to include problems of interest to students and teachers at all levels. Written responses from the readers will influence future directions that the column may take. Please take time to actively participate as problem solvers and share your feedback. Suggestions, solutions, proposals, and anecdotes are welcomed.

The following problem appeared as Question 23 in the Grade 8 Gauss contest written earlier this year.

Using the digits 1, 2, 3, 4 and rearranging them, it is possible to form twenty-four different four-digit numbers, including the number 1234. The sum of these twenty-four numbers is

(A) 24000 (B) 5555 (C) 24568
(D) 29616 (E) 66660

During a recent visit to University of Toronto Schools, Jean Collins outlined to me some of the methods that students had used to solve this problem. She mentioned that one student wrote the digits in ascending order and descending order, then averaged these numbers and multiplied by 24. While appreciating the creative approach taken by the student, it struck me that it would not generalize to all sets of four digits. Consider the digits 1, 2, 3, and 6, for example. You will find that the averaging method employed by the creative student will not produce the correct sum.

Our brief discussion had already modified my perception of this problem. It no longer represented a mere question on a contest paper. Instead it suggested directions for further exploration. When I mentioned this problem to Peter Taylor (a mathematics professor at Queen's University) at a mathematics education conference, he pointed out that the averaging method would work for the digits in a number such as 2956. That's neat, isn't it! What mathematical principles are at the crux of this problem?

Here is the challenge:

Examine why the averaging method works for some sets of numbers but not others. Then write down a four digit number and quickly determine (without averaging) whether the averaging method would work. You may wish to extend the exploration to five or six digit numbers.

In conclusion, I'd like to share two intriguing problems dealing with arithmetic sequences. The first problem was introduced to me by Gerry Rising, a Professor of Mathematics Education at SUNY at Buffalo. The second problem was featured in the team questions' section of Hodie Dies Numerorum, an annual mathematics contest hosted by University of Toronto Schools. Jean Collins challenged me to attempt this problem. Apparently the problem was not intended to be so difficult. The content required is basic but the fact is that no definitive answer had been confirmed. Give them a try.

A. The entries in each row and column are in arithmetic sequence, find the value of *.

			*	
	74			
				186
		103		
0				

B. Find the minimum number of elements that must be removed from the set {1,2,3..., 100} such that no triple of the remaining numbers form am arithmetic sequence.

NOMINATIONS FOR OAME/AOEM COUNCIL 1994-95

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1994, to:

Jeri Lunney
Past-President, OAME
Box 564, 157 Elgin Street
Almonte, ON K0A 1AO

Include the nominee's qualifications with the Nomination Form. The nominee must be a current member of OAME. Their names and qualifications will be published in March, 1994. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in Kingston in May, 1994.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of: _____ **President-Elect**

_____ **Director**

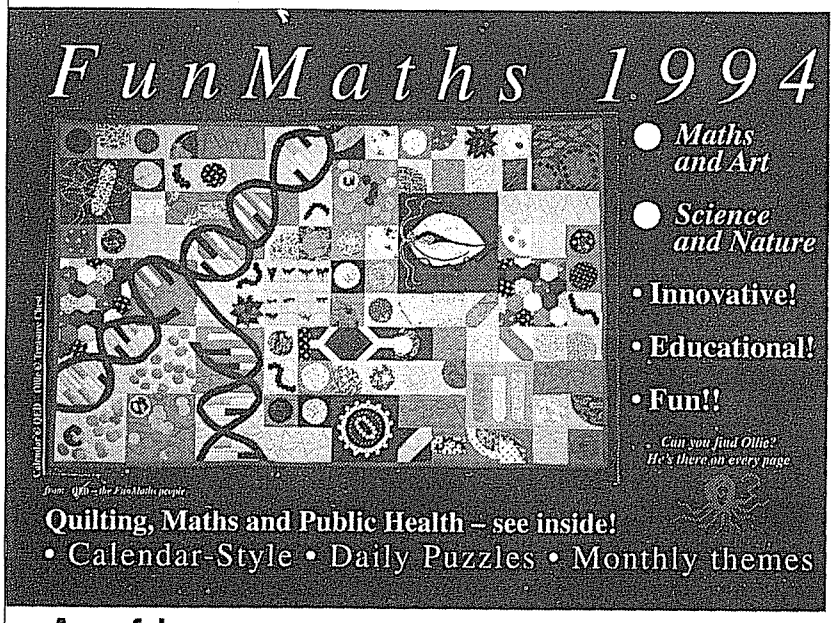
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Nominator _____

Address _____

_____

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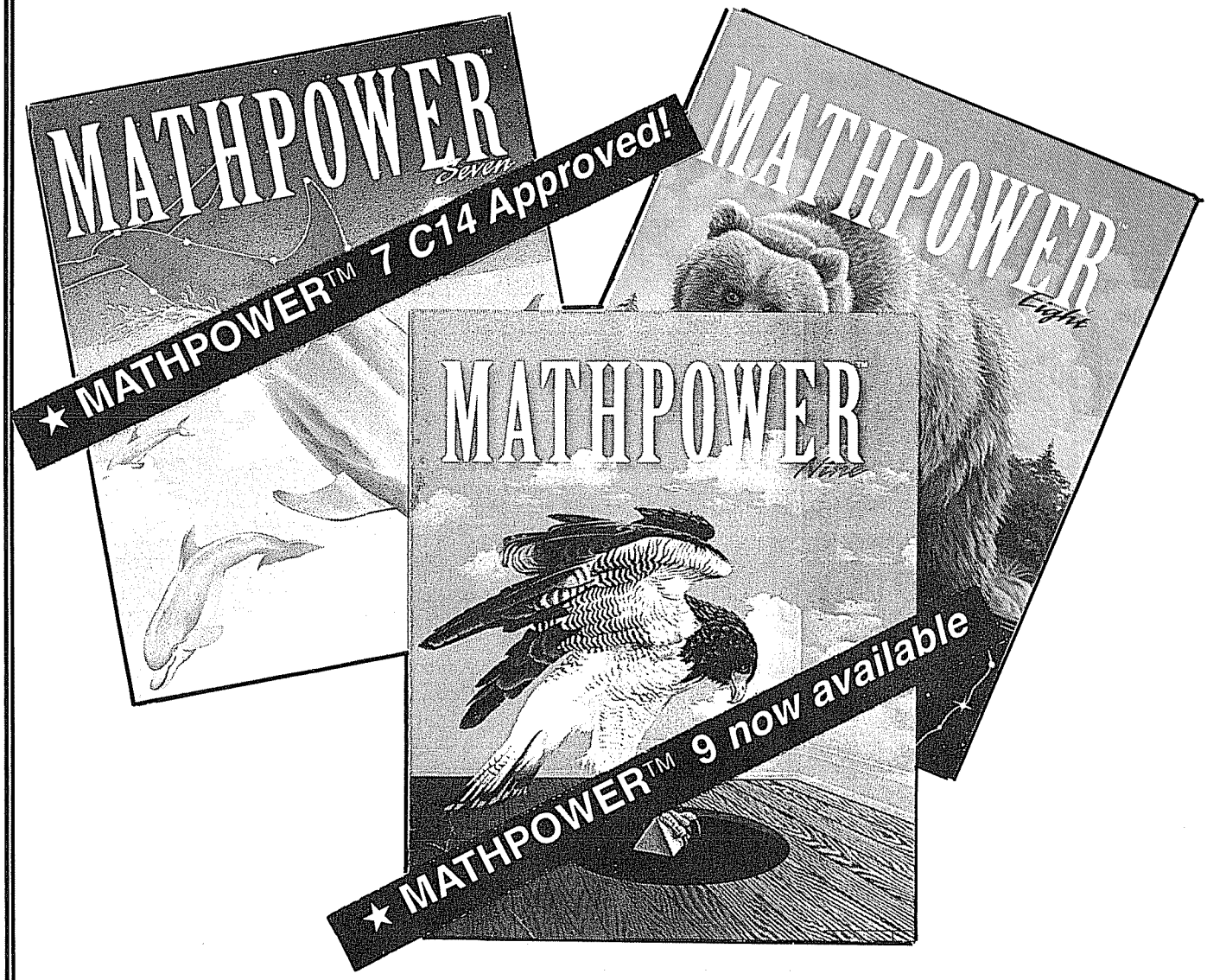
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▲ The Magic of Math: New TVO Series Brings Mathematics To Life

ROSEANNE MEANDRO, TVONTARIO

Whom do you call if you've got a problem to solve? Mathica, problem solver extraordinaire! And starting this fall, you'll be able to watch Mathica work her magic on your students in a new TVO primary math series called Mathica's Mathshop.

Mathica is a curious and eager elf who finds a job in the Mathmagician's mathshop in the Land of Tales. Mathica's task is to help solve all the problems of the local residents: What shape is the witch's hat? In how many ways can 10 maidens share four umbrellas? How many tickets must the king sell to his ball in order to pay his rent?

Mathica's Mathshop is a lively and charming series of ten 15-minute programs geared to grades 1 and 2. Based on a classic tale, each program relates mathematical problems involving number, measurement, and geometry to everyday situations.

"We wanted to create a series that helps children recognize a problem, motivate them to think and reason on their own, and explore different ways in which a problem may be approached," says producer Jennifer Harvey. Teachers will find this video particularly useful in developing analytical skills in young children and encouraging them to apply these skills to other areas.

Another interesting feature that will appeal to teachers is its use of the whole-language approach to math. Each program is presented in the style of a fairy tale, encouraging children to listen, read, and write. The series also allows children to learn about values like friendship, sharing, and cooperation as they follow Mathica's problem-solving adventures.

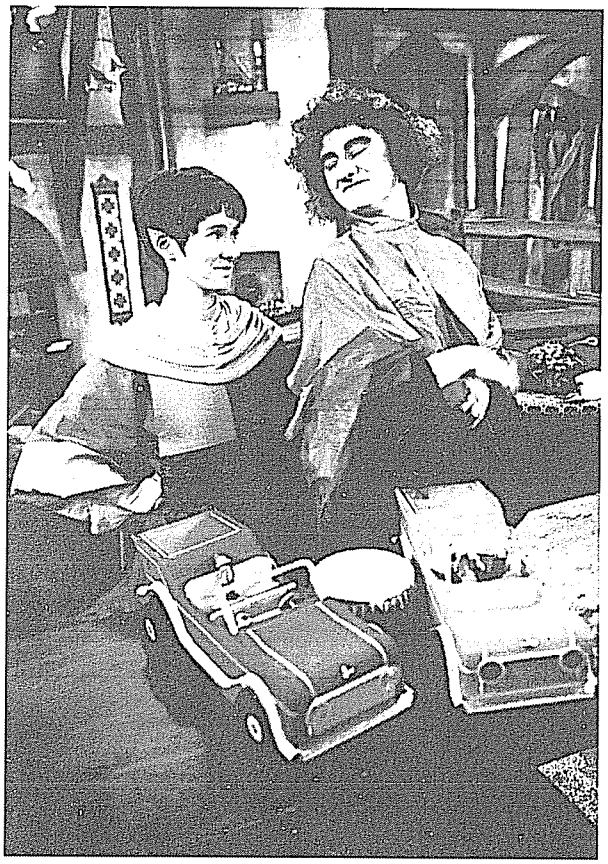
Mathica's Mathshop is a must-have video for bringing math to life in your classroom. Mathica's enthusiasm, combined with live-action characters, humour, enchanting stories, and colourful graphics, will help foster a confident attitude to math in your students and make it an involving and exciting experience.

A teacher's guide chock-full of ideas for pre- and post-viewing activities for students and utilization tips for teachers complements this new series.

Watch for Mathica's Mathshop on TVO Mondays at 9:45 a.m., starting 27 September. It's copyright-cleared, so you can tape it for later use in the classroom.

If you miss the series, video copies and teacher's guides are available through TVOntario. For ordering information, call TVO's Client Services at (416) 484-2600, ext. 2665, toll-free

(800) 668-9974, or fax (416) 484-4425.



▲ COUNCIL COMMENTS

BOB ROBINSON
GOLDEN SECTION CHAPTER REPRESENTATIVE



These notes and comments are a result of the OAME Council Meeting on October 2, 1993 at the Ramada Inn, Airport West in Mississauga. Any correspondence should be sent directly to the editor.

INTRODUCTION

The President, Ron Lancaster welcomed the new members to council. Mention was made of Jack Weiner's article in the current issue of the Mathematics Teacher. Gary Flewelling was also commended for his contribution to the excellent Transition Years focus issue of the Gazette. A search committee has been established to seek sites for the 1997 and 1998 annual meetings with discussion of whether we should offer to host a regional meeting of the NCTM in either of those two years.

AWARDS

The awards committee is seeking nominations for the following: Life Membership Award, Morley MacGregor Award, Mona MacGregor Award, and the Kenneth D. Fryer Award. Qualifications for these awards are found in the September issue and on page 8 in this issue.

NOMINATIONS

Nominations are being accepted for positions of President-elect, Director, and Councillor. See the nomination form on page 30 in this issue.

PUBLICATIONS

The publications committee is most pleased with the high professional quality of the Gazette. Jack, Amy, and the crew at the University of Guelph were highly commended. Council is also pleased with the professional quality of the ads, coordinated by Bob Robinson.

EQUITY

Once again, a breakfast is being planned for the Annual Meeting in Kingston. The guest speaker will be Ann Jones, a former President of the OAME. A folder for equity related materials is being assembled. Discussions pertaining to equity issues have taken place with persons involved with mathematics contests. A Gender and Science and Technology Conference was held recently in Waterloo. A 2-volume resource entitled Gender in Science and Technology for Sustainable and Equitable Development is available, free of charge, from the International Development Center, 250 Albert St., Box 8500, Ottawa, ON K1G 3H9.

PUBLICITY/MEMBERSHIP

The committee is proceeding with a joint OAME/NCTM membership drive initiative. They are also planning to send out welcoming letters to new members on council. Ways that the committee can assist chapters with membership and publicity services at their mini-conferences are also being investigated.

EARLY YEARS/FORMATIVE YEARS

A submission to the April Abacus is being prepared. It will deal with the Data Management and Probability strand using the theme "What's Fair at the Fair?". Activities, assessment suggestions and an annotated resource list will be included.

TRANSITION YEARS

A parent pamphlet is being planned which will follow a question and answer format. An integrated lesson resource document for Patterning and Algebra is in its early discussion phase.

SPECIALIZATION YEARS

The committee will be submitting reviews of appropriate software for upcoming issues of the Gazette. A workshop to assist Grade 10 teachers is being planned for Kingston. It will address how the Specialization Years section of the Focus on Renewal Document can be completed. OMCA will be contacted regarding this endeavour.

NCTM

A new catalogue of materials is available from Barry or Jan Scully. Members of NCTM can also receive a 20% discount on special products such as mugs, stickers, pencils, etc. Ralph Connelly (council member) and Lorna Wiggan (former council member) are running for NCTM Director. The annual meeting of the NCTM will be in Indianapolis this spring.

OTF

The last meeting of the Ontario Teachers' Forum centered around issues pertaining to the common

curriculum. For financial reasons, the OTF has suspended its activities including the Ontario Teachers' Forum.

MATHEMATICS EDUCATION FORUM

Members of the forum are from eight constituent groups. There is also a steering committee consisting of one member from each of these groups. Marg Warren, Arnie Carefoote, and Radford de Peiza (official OMCA representative) are councillors who are members of the forum. The forum is setting up three working groups to deal with 1. OAC Calculus 2. Statistics, Data Management, and Probability 3. Geometry issues (K-OAC). All groups will be addressing the issue of technology.

OAME'94

Exciting plans are underway for our 21st Annual Meeting in Kingston. Registration forms and other information will be available in the new year. We are told that the prices will be "moderate" with accommodation packages available including breakfast and lunch for as little as $55. A gala reception is being organized for Friday evening and an outstanding program is being assembled. Promotional material is available from David Handley, RR 4, Madoc, ON K0K 2K0 (613) 473-2478

OAME'95

Planning has already started for our "annual" in Brantford.

CHAPTERS

CHAMP

They completed the writing of their CHAMP Mathematics Contests in November. The contests are designed for students in grades 1-8 and provide excellent preparation for those students who will participate in the Canadian Mathematics Competition in February. Since the contests are marked by the "home" school, the students benefit from receiving almost immediate feedback.

COMA

A January mini-conference is being planned for January 14, 1994 at Sir Guy Carleton from 4:00 to 6:00 p.m.

GOLDEN SECTION

Eric Muller, Brock University has produced a wonderful resource for elementary school entitled Niagara Falls Math Trail. Copies can be obtained from the Niagara Parks Commission (905) 356-2241, ext. 266 or 272 or the office of conference services, Brock University (905) 688-5550, ext. 3369.

GRAND VALLEY

GVMA expects to have a new set of OAC Algebra and Geometry exams available for the May Annual in Kingston. They also hope to have a collection of Grade 9 destreamed exams in publication at that time.

ISOMA

Contest preparation sessions are being held at the following locations:

- Hillfield-Strathallan College (Hamilton) Dec. 3, 1993
- Branksome Hall (Toronto) Feb. 10, 1994
- Country Day School (King) Feb. 17, 1994
- Upper Canada College (Toronto) March 3, 1994

NWOAME

The Lakehead University/NWOAME Math Awards Day is going to be held in the spring. The afternoon, including a luncheon preceded by some "Table Top Math", awards presentations and math workshops, recognizes student/school success within the region in various math contests.

PINE RIDGE

A team of Grade 7, 8 and 9 teachers has written a curriculum document which reorganizes the mathematics program to reflect the common curriculum and benchmark strands. This guideline will also contain sample lesson plans, activities, and other resources to help teachers provide a cohesive approach to the mathematics program for the transition years. A mini-conference is planned for April 7, 1994. The theme will be Real World Mathematics.

SAME

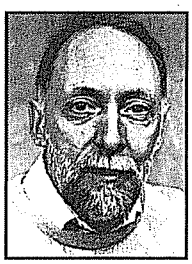
At the spring SAME meeting Richard Kinney was given an award for his lifetime contribution to SAME and mathematics education in Scarborough. At the Scarborough Math Awards night, several teachers were honored for their commitment to mathematics education. Among those recognized was Arnie Carefoote a longtime OAME councillor, and Roy Wood, a Scarborough teacher has developed a software package called SoundMan which integrates math and physics. Roy can be contacted on ScriBE, the Scarborough Board of Education bulletin board 396-6761.

SWOAME

The chapter is beginning to work on materials for the specialization years. They will also be supporting a math awareness week in February.

▲ AIRLINE RESERVATIONS AND THE BINOMIAL DISTRIBUTION

L. KEITH ROY
DEPARTMENT OF MATHEMATICAL SCIENCES
LAKEHEAD UNIVERSITY, THUNDER BAY, ONTARIO



Educated at Queen's, Keith teaches Statistics at Lakehead University. A frequent contributor to the Gazette, he has given many invited talks and workshops for groups of students and teachers.

A small regional airline uses a fifteen-passenger aircraft for short flights. Knowing that a number of reservations for a given flight will end up being cancelled at the last minute, the airline purposely allows as many as twenty reservations to be made for the fifteen seats available. The airline bases this strategy on an analysis of past experience which resulted in the following information:

(a) On the order of twenty percent of persons with reservations either cancel their reservations at the last minute or simply do not show up for their flight. Further, these late cancellations or "no-shows" occur independently of each other.

(b) The events that 16, 17, 18, 19 or 20 reservations will be made for a particular flight have probabilities 0.06, 0.03, 0.03, 0.02, and 0.01 respectively, while the probability of at most 15 reservations is 0.85.

Using the above information, the probability that too many persons with reservations will show up for any particular flight can be calculated with the aid of techniques from the OAC course in Finite Mathematics.

The information (a) indicates a binomial situation and, if "success" is defined to be the event of a person with a reservation actually showing up for her/his flight, the probability of x individuals with reservations actually showing up is

$$f(x) = \binom{n}{x} (0.8)^x (0.2)^{n-x}$$

for x= 0,1,2,...,n where n is the total number of persons who have made reservations. Then, for instance, the probability of seventeen persons with reservations showing up for a flight where a total of nineteen reservations had been made would be f(17) with n = 19. Using either the above formula or tables of the Binomial Distribution results in a probability of 0.154. Furthermore, the probability that more than fifteen persons with reservations will show up for a flight where a total of nineteen reservations has been made is

$P(16 \le x \le 19) = f(16)+f(17)+f(18)+f(19)$
$= 0.218+0.154+0.068+0.014 = 0.454$ using n = 19

The conditional probabilities that too many persons with reservations will show up for a particular flight are tabulated below for the various given values of n.

n=16 $P(x=16) = 0.028$

n=17 $P(16 \le x \le 17) = 0.096+0.023 = 0.119$

n=18 $P(16 \le x \le 18) = 0.172+0.081+0.018 = 0.271$

n=19 $P(16 \le x \le 19) = 0.218+0.154+0.068+0.014 = 0.454$

n=20 $P(16 \le x \le 20) = 0.218+0.205+0.137+0.058+0.012 = 0.630$

Consequently, the probability of occurrence of both the event "nineteen reservations are made for a particular flight" and the event "too many persons with reservations show up for that particular flight" would be (0.02)(0.454) = 0.009.

The probability that too many persons with reservations will show up for a particular flight is the sum of such calculations for the various values of n, that is

(0.06)(0.028)+(0.03)(0.119)+(0.03)(0.271)+(0.02)(0.454)+(0.01)(0.630) = 0.029

So only about three percent of the airline's flights will be overbooked!

O.A.M.E. 1993-1994 Council

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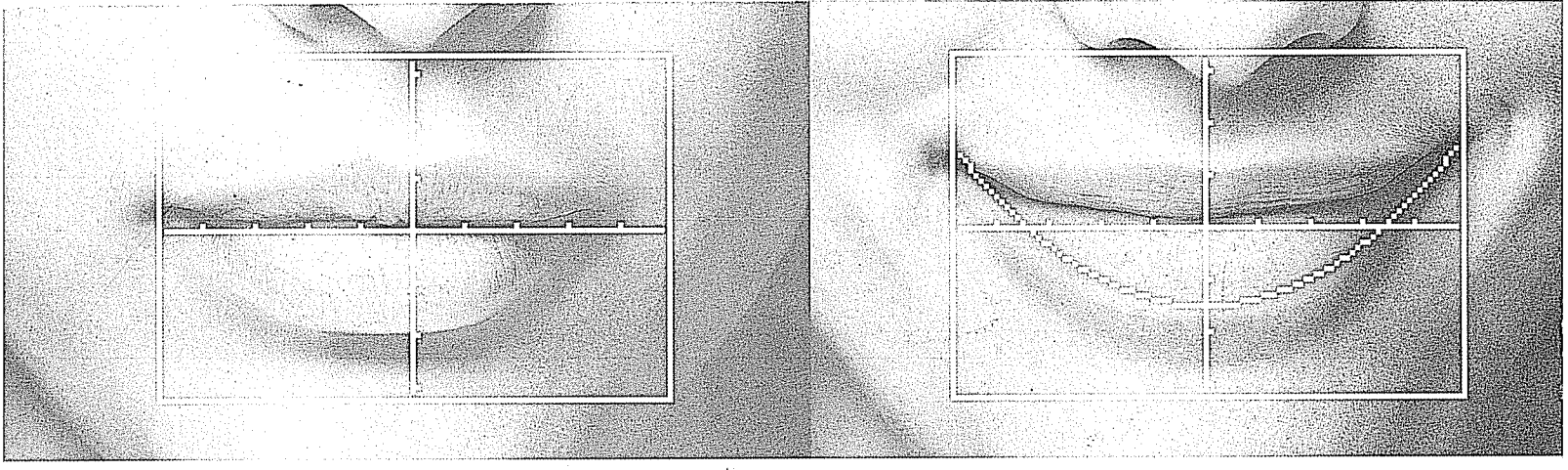
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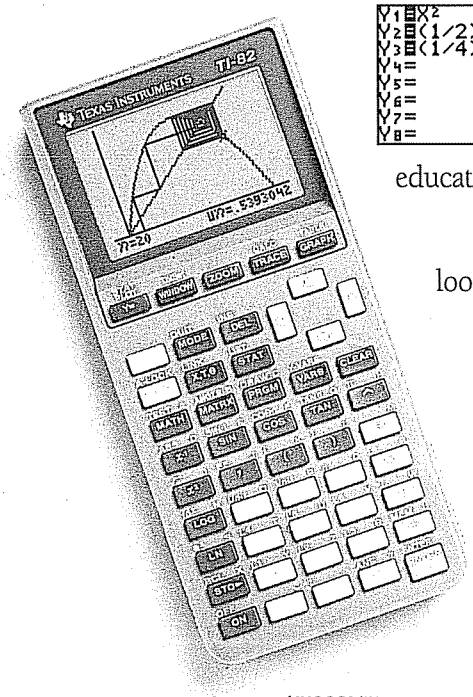
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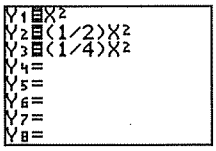


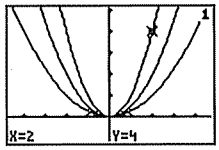
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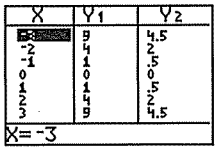
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