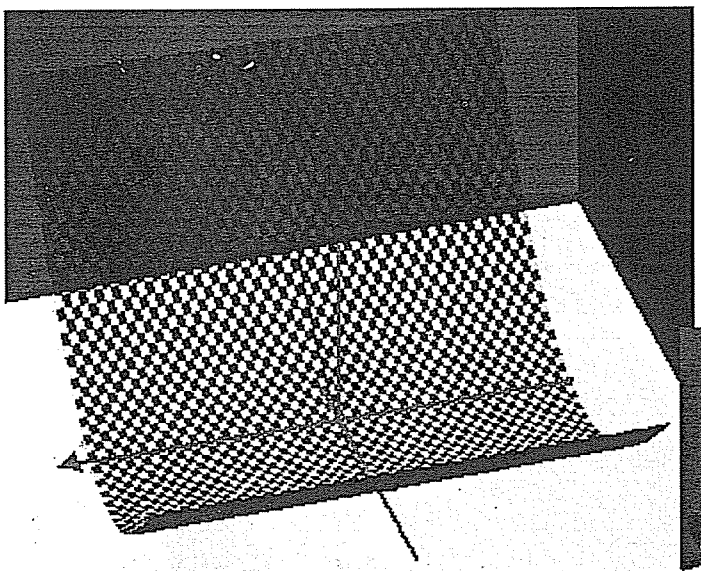
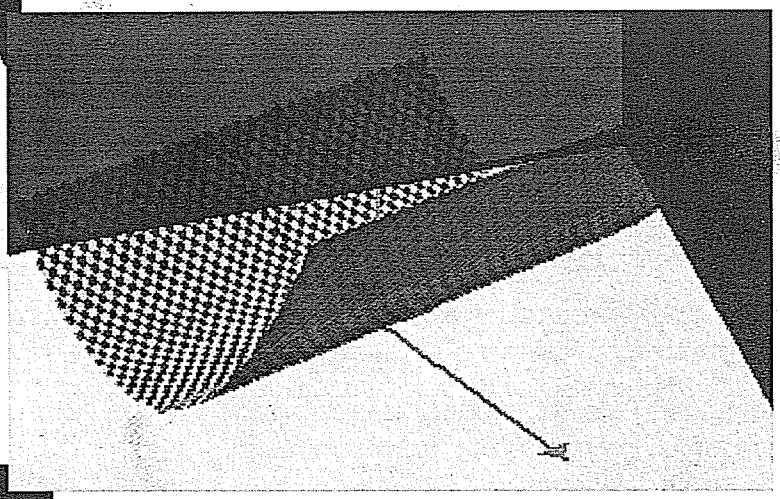
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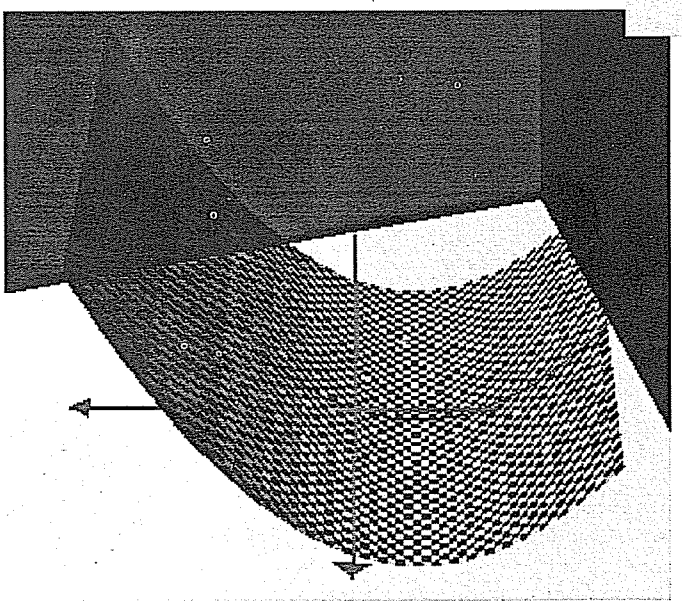
Ontario Mathematics Gazette

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$$z = \frac{x(x+1)}{2}$$



IN THIS ISSUE

Developing a Specialization Years Mathematics Curriculum

Thoughts on University Entrance Exams

The Attraction of the International Baccalaureate in Mathematics

How Big is a Million?

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

"How do you write tests, homework & classwork?"

"by hand?"
(The old way)

Final Exam

1. State the Domain of $y = \frac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse o

3. Use a half-angle identity to find

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $\frac{-1}{2}\sqrt{2-\sqrt{2}}$

4. If $f(x) = 2x-5$ and $g(x) = \frac{2}{x+3}$

a) $\frac{-(5x+11)}{x+3}$ b) $\frac{1}{x-4}$

5. If $\sin\theta = -\frac{1}{2}$ and θ is in qu

a) $\frac{-\sqrt{3}}{3}$ b) $\frac{\sqrt{3}}{3}$

"struggling with your word processor?"
(The hard way)

Final Exam

1. State the Domain of y=x-10.. / sqrt(x^2-16)

2. Which relation is the inverse of y=(x-2)^3 ?

a) y=cbrt(x-2) b) y=cbrt(x) -2 c) y=cbrt(x+2) d) cbrt(x) +2

3. Use a half-angle ider
cos 67.5°:

a) 1/2sqrt(2-sqrt(2)) b)

4. If f(x)=2x-5 and g(x)

a) -(5x+11) / x+3 b)

5. If sin θ=-1/2 and θ i

a) -sqrt(3) / 3 b)

6. Given f(x)= x^2 / x+3 and

a) 4x^2-2x / x+3 b)

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FINAL EXAM

1. State the Domain of $y = \dfrac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of $y = (x-2)^3$?

a) $y = \sqrt[3]{x-2}$ b) $y = \sqrt[3]{x}-2$ c) $y = \sqrt[3]{x+2}$ d) $\sqrt[3]{x}+2$

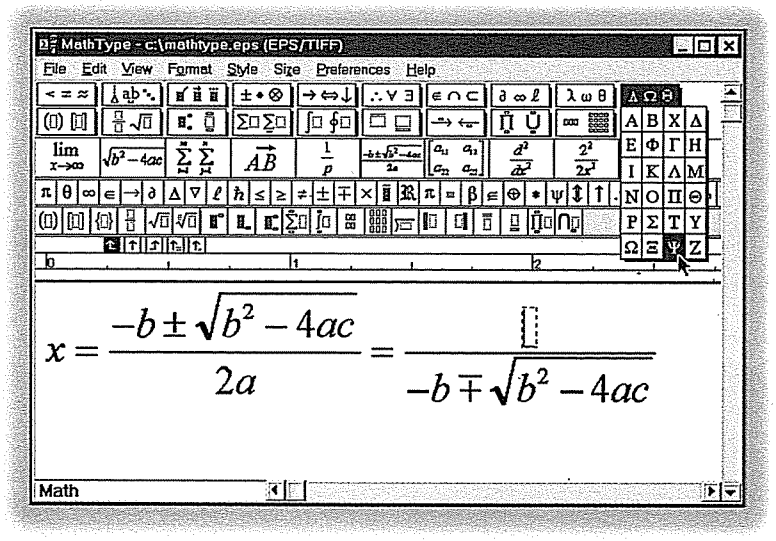
3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $\frac{-1}{2}\sqrt{2-\sqrt{2}}$ c) $\frac{-1}{2}\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

4. If $f(x) = 2x-5$ and $g(x) = \dfrac{2}{x+3}$, find $[g \circ f]\,(x)$:

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Articles for the Gazette should be submitted to John Egsgard at the address on the left.

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Your article should be typewritten or word-processed, double-spaced on one side of 21.6 cm x 27.9 cm paper with 3 cm margins. Diagrams and figures should be drawn by computer, if possible, or neatly drawn in black ink in camera-ready form. Please submit four copies to the editor and keep one as protection against loss. Submissions are blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript. Computer files of text and graphics, where possible, will be requested later if your article is accepted.

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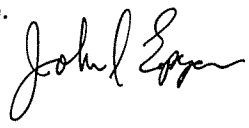
▲ EDITORIAL

JOHN EGSGARD



With all of the negative publicity about education nowadays it is refreshing to read the articles of this December 1996 issue of *The Ontario Mathematics Gazette*. Three of the articles give positive suggestions for the curriculum itself. In the article *Developing a Specialization Years Mathematics Curriculum*, Geoff Roulet challenges us to make ourselves heard in the curriculum debate. In *Thoughts on University Entrance Exams,* Peter Taylor suggests a very unique way of examining potential candidates for university. In *The Attraction of the International Baccalaureate in Mathematics* article, Jean Paul Ginestier gives one answer to the challenge contained in Roulet's article and indicates an alternative for Taylor's suggestions. Since no issue of the *Gazette* would be complete without teaching ideas, this issue finishes with my notes on teaching, a problem solving article on Triangular Numbers, suggestions on teaching mathematics in our destreamed grade nine situation, and an article entitled *"How Big is a Million?"*.

Remember there is always a place for your comments and suggestions in a *Letter to the Editor*. But better still please share your teaching ideas with your peers through a brief or a long article.



H.M.S. Coxeter, Canada's most famous mathematician, to be 90 in 1997

On February 9, 1997, Harold Scott MacDonald Coxeter, professor emeritus of the University of Toronto, and world famous geometer, will be 90 years of age. To mark this great occasion the Mathematics Department of the U of T has announced the foundation of a special scholarship fund in his honour. Coxeter scholarships will be awarded to top students at the university. Anyone wishing more information about the scholarships or who would like to make a contribution to the Coxeter Foundation should contact Professor Steve Halperin, chair, Department of Mathematics, University of Toronto, Toronto, ON M5S 3Gs, email halperin@math.utoronto.ca.

▲ PRESIDENT'S MESSAGE



BARRY ONSLOW

One of the difficulties in writing a President's Message that will not be read for another two months is that many events which are about to happen will be history in December when you receive this *Gazette.*

The OAME 1996 Leadership Conference, Connecting Assessment to Learning, will have been held November 7-9. I would like to thank Ron Sauer for co-chairing this conference with me, and Bonnie and Dave Alexander for their hard work helping to prepare and organize the conference. This year over one hundred delegates are attending and we had to close registrations in mid-October. We hate to turn people away from conferences, but it is important to keep the Leadership Conference small so that discussion rather than transmission is the priority. One reason for so many people registering early for the conference was the quality of the speakers. A very sincere thank you to Judy Crompton, Shirley Fairfield, Don Fraser, Sue Gordon, Joan Green, Nancy Iannizzi, Nancy Moore, Peter Saarimaki, Ron Sauer, and Jan and Barry Scully for taking the time to prepare quality sessions for the conference. All our speakers are extremely busy people who are dedicated to improving the teaching and learning of mathematics.

The OAME/OAJA Junior Assessment Document, produced primarily by Nancy Moore and Nancy Iannizzi, is now available. Nancy and Nancy, with the help of several of their colleagues, have produced a fantastic collection of resources which will greatly facilitate the assessment and instruction of authentic problems for teachers of the late Formative Years. Details of how to order the assessment package will be available in the very near future. A most sincere thank you, Nancy^2.

Gareth Haddock, with the assistance of Jack Weiner and Guelph University, is busy preparing materials for the OAME Web Site. More news will be forthcoming in future editions of the *Gazette.* However, if you would like information on the OAME Web Site or have materials that you think should be included, please contact Gareth whose address appears on the inside back cover of this issue. A very sincere thank you Gareth, our new ISOMA Chapter Representative, for taking on this responsibility.

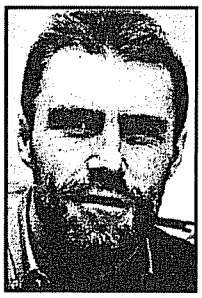
At the Board of Director's Meeting of October 5, 1996, it was moved by Kaye Appleby and seconded by Rob McLeish, "that OAME approach the College of Teachers to request that participation in the OAME Annual Conference be recognized for creditation for maintenance of certification". The motion was carried unanimously. I will be contacting Margaret Wilson, Registrar of the Ontario College of Teachers, on your behalf to request that participation as a speaker and/or delegate at our Annual, Leadership, and Chapter Conferences be recognized as valuable professional development and worthy of recognition for maintenance of certification.

Our Special Projects and Specialization Years Committees are taking a proactive stance in developing a vision for mathematics teaching and learning at the secondary level. We will be working closely with the Ontario Mathematics Coordinators' Association to develop this vision. Margaret Warren, chair of the Special Projects Committee, offered a glimpse of what might be forthcoming at the Board of Director's Meeting, October 5th. In a well crafted presentation she suggested a vision of the future secondary school student *exploring* mathematics. Process was certainly as important as product, and perhaps more so. Marg's evidence was compelling. She cited data presented by John Von Newmann – that at the beginning of this century, it was possible for any reasonably well informed mathematician to understand all the mathematics known at that time. By 1950, even the most brilliant mathematicians had access to only ten percent of the knowledge in the mathematics field, and by the year 2000 that amount will be reduced to a fraction of one percent. Actions such as exploring, modelling, formulating, transforming, manipulating, inferring, drawing conclusions, and communicating will be crucial components in the mathematics classrooms of the next century. Those who want to read more on this topic will be interested in Geoff Roulet's article, *Developing a Specialization Year's Mathematics Curriculum,* also in this issue of the *Gazette.*

We are looking for energetic, enthusiastic people to serve on the OAME Board of Directors. Consider running for one of the positions advertised in this issue and help shape the future of mathematics in Ontario. It's time consuming, but a lot more enjoyable and fulfilling than sitting on the side lines. You can make a difference – if you are willing to play. Get involved!

▲ Developing A Specialization Years Mathematics Curriculum

GEOFFREY ROULET
FACULTY OF EDUCATION
QUEEN'S UNIVERSITY, KINGSTON



Geoffrey Roulet's 13 years of secondary school mathematics teaching in Timmins were followed by terms as the Board's Mathematics Consultant and as an Education Officer with the Ministry of Education in Sudbury. For the past 6 years he has been at Queen's University, working with teacher candidates and conducting research on activities through which students can build mathematical understanding.

In the creation, introduction, and support of mathematics programs, neither teachers, educational administrators, parents, nor the general public should allow themselves to be manipulated into false choices among

- the old and the new mathematics
- skills and concepts
- the concrete and the abstract
- intuition and formalism
- structure and problem-solving
- induction and deduction

While the above identification of false dichotomies was made over two decades ago by the National [USA] Advisory Committee on Mathematical Education (Hill et al., 1975, p. 136) in reference to the rather heated debates accompanying the decline in popularity of the "new math" program (1955-75), I believe they provide an important warning for us today as the province moves to restructure the Specialization Years. There is a danger that those of us who are involved in, and concerned about, mathematics teaching and learning, become distracted and divided by debates on details and thus fail to develop any concrete curriculum proposals. In the absence of any clear message from the mathematics education community we might find the Ministry of Education and Training developing a Specialization Years program that satisfies few if any of us.

In 1989 the National Council of Teachers of Mathematics [NCTM] published the *Curriculum and Evaluation Standards for School Mathematics*. This policy document has stimulated considerable debate about mathematics education and had a significant influence on the parallel Ontario document, *Focus on Renewal of Mathematics Education,* issued jointly by the Ontario Association for Mathematics Education [OAME] and the Ontario Mathematics Coordinators Association [OMCA] (1993). Unfortunately when debate focused upon the Specialization Years (grades 10-12) consensus could not be reached and the Ontario document addresses only the Early, Formative and Transition Years of school.

Over the past few years a number of groups (OAME, OMCA, Mathematics Education Forum, committees from the universities and CAATs) have addressed issues concerning the Specialization Years mathematics curriculum, but in each case the debate has failed to produce a complete and acceptable policy statement. It seems to me that too often the discussions are conducted within small isolated circles and position statements become too highly developed before they reach the more general population. We need to expand the conversation to include more players. In particular we need to hear from the secondary school mathematics teachers of this province, since it is they who in the end will determine the success of any new curriculum policy. It is with such a purpose in mind that I am writing this position paper. I hope that it might stimulate further discussions in mathematics department work areas, at OAME chapter meetings, and in future pages of the *Ontario Mathematics Gazette.*

Constructing a New Curriculum

I believe that the OAME/OMCA's original approach to developing a curriculum, that is, first addressing general principles as in the *Focus on Renewal* (1993) document and in the 1994 draft and consultation paper for a Specialization Years extension, was sound. The ideas expressed in the draft documents may not have received universal acceptance but they were a starting point. Recent work, such as the efforts by a committee representing some of the universities to "flesh out our

entrance expectations for university students, as could be appropriately measured on the basis of an exam" (Bland, 1996, June), suggest that we may be abandoning efforts to build a consensus on guiding principles and are jumping into a debate concerning specific content items. This change of focus is dangerous, for we are likely to get mired in conflicts about small details while others outside the mathematics education community set the curriculum for Ontario. The constrained view of mathematics held by many curriculum generalists can be seen in the introductory statements, concerning our subject, found in *The Common Curriculum* (Ontario Ministry of Education and Training, 1995).

> *Mathematics is **used** in many fields - in the arts and social sciences, as well as in science and technology. For example, the ability **to use** statistics is important in social planning; a knowledge of geometrical principles is **useful** in the visual arts, music, and graphic design; and measurement is an important **skill** in the design and building industries. We **use** mathematics in many kinds of work and in our daily life. (p. 70)* (Bold type added)

Here we have mathematics presented, not as a way of thinking, but as simply a collection of procedures useful for solving problems in other domains. Do we want such an image of our subject driving mathematics curriculum development? We need to "get our act together" to ensure that curriculum policy reflects the thinking of those who have some passion for mathematics as a discipline and knowledge of its content and history.

Content details are important and there are significant issues here to be addressed, but this should be done in light of agreed upon organizing principles. If we do our curriculum building from the top down in stages, each of which flow from the previous level, I believe that much of the apparent present conflict will disappear. If we have agreement at stage n and we then build to stage n+1, a consensus can hopefully be maintained. Fortunately this analogy to mathematical induction does not need to be carried through $\aleph_0$ levels, but curriculum planning does involve a sequence of stages.

We do not want to repeat history. We often proudly claim that many of the "new" directions suggested in the NCTM's *Standards* appeared five years earlier in Ontario's 1985 *Curriculum Guideline: Mathematics: Intermediate and Senior Divisions* (Ontario Ministry of Education). But, in this document, the course content lists were in general not linked back to the introductory "Process Components" section and the results of provincial reviews suggest that this separation of principles and content significantly reduced the potential impact of the guideline.

Potential for Agreement on Principles

On the surface there appear to be some wide divisions in the Ontario mathematics education community. I do not see these differences as unresolvable for there are some significant common themes running through our various documents and policy proposals.

The "Process Components" section (pp. 17-21) of the 1985 guideline (Ontario Ministry of Education) calls for an experiential approach to teaching and learning. Students should develop their mathematical understandings through explorations, possibly with manipulative materials and computer software. Language and communication play an important role in learning and the guideline tells us that students should be involved in oral and written presentations of explanations and meanings. This emphasis on mathematical reasoning is further developed in a discussion of the place of proof and a focus on extended problem solving activities. Teachers are asked to link mathematics to the world beyond the classroom by developing skills and concepts through applications and mathematical modelling.

The first four of the NCTM's (1989) standards: problem solving, communication, reasoning, and connections echo the 1985 Ontario guideline. These themes are also developed by the OAME/OMCA (1993) *Focus on Renewal* and the draft documents for the Specialization Years produced by OAME/OMCA (1994a, 1994b). Reports from the meetings of the Discussion Group, Secondary School Curriculum: Appropriate Expectations for University Bound Students (Bland, 1996, June), and this committee's report to the Ministry of Education and Training (Bland, 1996, September) set out as basic principles the themes of problem solving, communication, reasoning (form and test hypotheses, proof, abstract thinking) and connections.

> *Fundamental outcomes for the student are the ability to think critically - to be able to form and to test hypotheses; the ability to argue rigorously based on firm logical principles; knowledge of a variety of specific problem solving techniques, as well as more general approaches to solving problems; and the power to think abstractly - to recognize underlying patterns behind*

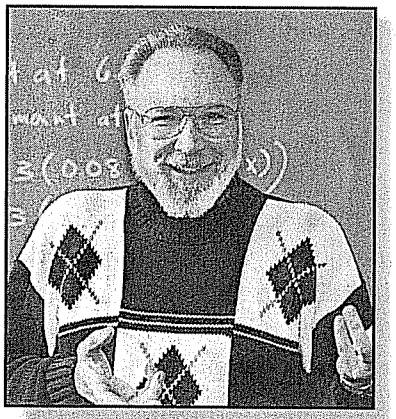
problems and to use this effectively to find more powerful attacks on the problem. (Bland, 1996, September, p. 1)

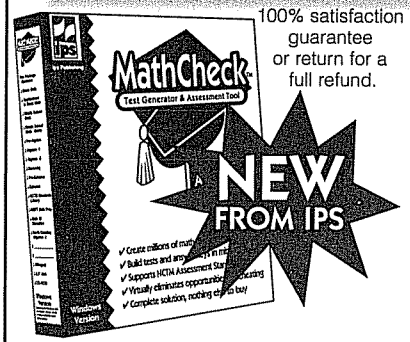
In addition to the common goals identified above, the documents have also consistently suggested that technological developments in our society mean that we need to alter both the content and instructional methods of our programs. Using calculators and computers in teaching and learning is a recurring theme.

Proposals made by individual members of Ontario university Mathematics (and Statistics) departments also address the above common themes. Peter Taylor's (Queen's University) articles, "A Proposal for OAC Calculus" and "A Problems-Based Curriculum for Intermediate Algebra" in the December 1992 and 1993 issues of the *Ontario Mathematics Gazette* certainly call for student exploration, conjecturing, reasoning and communication and these themes run through *Calculus: The Analysis of Functions* (1992b), his Ministry approved text for OAC-Calculus. Section 3. "Broad goals for the high school curriculum", of Ed Barbeau's (University of Toronto) proposal, distributed at OAME'96, contains paragraphs that promote connections, reasoning, communication and problem solving. Barbeau (1995) presents this image of mathematics in his book, *After Math*.

> *There is a strong experimental side to mathematics. It is the science of structure, and the first step is to discover what structure is there by simple observation. This requires sensitivity to detail and imagination in guessing possible patterns. Only then can we formulate conjectures and prove theorems. (p. 31)*

The Third Queen's-Gage National Mathematics Institute (August, 1996), which involved university mathematicians, education faculty, teachers from primary to senior grades, and school board mathematics consultants, developed similar visions of mathematics and its teaching and learning. Its document, *Tomorrow's Mathematics* (in press), presents mathematics "as a set





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If, in fact, this apparent agreement on general principles, by those involved in mathematics education – kindergarten to university, does exist, I suggest that OAME publish a list of broad goals and encourage the use of these for guidance in subsequent Specialization Years curriculum work.

Continuing the Curriculum Development Process

Once there is an agreed upon set of guiding principles our next step is to identify outcomes, for all secondary school graduates not just those headed toward university and college programs that include mathematics study. If the focus is only on those progressing to tertiary mathematics study, there is the risk that courses for the remaining high school population will become just the "high level" program with the hard topics eliminated. As individual outcome statements are developed we need to imagine teaching and learning activities compatible with our guiding principles. Failure to visualize such instructional strategies should signal a need to carefully examine the proposed outcome.

I see outcomes as being stated at two levels of specificity. Broad general outcome statements would apply to all graduates while more specific outcomes, differentiated according to post secondary career or educational aspirations, would be grouped under these. This second stage of outcome statements would serve to define the course expectations for the different series (current Ministry terminology for levels) of the program.

An example:

In January, 1996, the OAME Council endorsed a draft list of Mathematics Graduation Outcomes. Item P1 under Patterns and Relations states,

> *See, recognize, describe patterns and make generalizations and predictions about them*

This outcome might be further developed by more specific sub-outcomes such as the following:

> ***P1a*** *Recognize recurring features in numeric calculations or physical processes, summarize their observations in words, and test their conclusions with further examples*
>
> ***P1b*** *Recognize a pattern in data generated by a numeric or physical process, describe their observations in words, develop an algebraic expression for their observations, test and possibly refine their expression, and prove that their expression models the data and generating process*

The second of these (P1b), which actually includes and extends the first (P1a), would be expected of those students continuing with post secondary mathematics study.

Such outcome statements can serve as a framework for identifying and justifying content. For instance the effects on area and volume when scale is changed might be a topic that supports P1a and work with geometric sequences and exponential functions would fit with P1b. Outcome statements can help check the validity of topics, suggest instructional emphasis, and identify links between units. If a proposed content item does not support the development of any listed outcome, we need to ask, "Do we really need this topic or have we missed an important outcome?" The above outcomes and our common guiding principles suggest that, for the content items identified, students should build their skills and knowledge through experimentation, developing hypothesis, and presenting arguments for their conjectures. When a number of separate content items are seen to support a particular outcome we have identified a possible means of connecting curriculum topics.

If the prescriptive sections of any new Ministry guideline were restricted to the identification of guiding principles and outcomes, groups of teachers could collaboratively develop interesting units addressing specific content items and employing instructional methods that realize the common themes of problem solving, communication, reasoning, connections, and the use of technology. Thus, while providing province-wide direction, such guidelines would not restrict the creativity of dedicated professionals nor force the Ministry into false choices as noted earlier in the opening quote. A central data base and distribution system could ensure that all boards and schools had access to teacher developed materials and from these be able to build course modules working towards the attainment of graduation outcomes.

OAME Progress to Date and Future Steps

The draft OAME/OMCA Specialization Years documents of 1994 were presented to the teachers of Ontario and feedback was received. With revisions to reflect teachers' comments the OAME Specialization Years Committee should be able to put together a list of guiding principles that have general acceptance across the province. The committee and OMCA have also begun the task of defining broad and more specific outcomes. A first run through this task needs to be completed and a draft list of outcomes must be released to the OAME membership for comment. I believe that these tasks could be accomplished without getting side-tracked in arguments on small points, such as which methods for factoring quadratic polynomials should be taught.

There is obviously some urgency here if the members of OAME are to have a voice. When Ministry efforts turn to the development of individual subject curricula for the Specialization Years, the OAME, as the representative of professionals concerned with the teaching and learning of mathematics, must be able to present proposals that have been fully debated and endorsed by the membership.

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Your newspaper is lacking some of its pages.

If page 19 is missing and the newspaper should contain exactly 32 pages,

then what other pages are missing?

Your automobile has 5 new tires. You decide to rotate the tires every 15 000 km.

How many kilometeres will be on each tire after your car has travelled 60 000 km?

Thoughts on University Entrance Exams

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He has taught OAC Calculus and, for 20 years, has been running enrichment sessions for high school students and teachers at Queen's. He has published a reform calculus text which is used at the high school as well as the university level, and is known as a good source of innovative problems.

I spent some time this summer in Toronto meeting with a small group of high school, college and university math teachers talking about high school curricula and university and college entrance exams – whether they're desirable or needed, what material they should test, how and when they should be administered, etc. These are perplexing questions, and we agreed on some things and disagreed on others and it's not clear what the fruits of our deliberations will actually be – perhaps the decisions will be made at another time and in another way with a different rationale, perhaps political, perhaps economic etc. etc.

When the meetings were over, one thing that struck me was how easily I had seemed to put aside my own vision of what a math curriculum should be and bend to the committee pressure of deciding whether this topic of that should be included. But that's a matter for another article.

Well, no matter – for me it was an interesting exercise and served to sharpen some of my own thinking, particularly about entrance exams. This is an important and timely issue – I wouldn't be surprised if we saw such exams, at least in Ontario, in the very near future. I should like to propose a fundamental principle, offer some support for it, and then extract a few corollaries. I sent an early draft of these remarks to Geoff Roulet and he made some very fine comments which I have summarized at the end.

First let me be clear what I am talking about. What we have in mind here is a set of exams, maybe three or four in various subjects, which would be set by the universities and designed to measure the preparedness and potential success of incoming students. These exams could be used as a component in the admissions process, as one of a number of factors in the award of entrance scholarships, and perhaps as a diagnostic tool for the student. If such a set of exams was introduced, one of them would almost certainly be in mathematics and another in English (or French). There are good reasons for that – mathematics and language are both fundamental to almost every discipline that is studied in university. Now here is the principle.

FUNDAMENTAL PRINCIPLE. A university entrance exam should not be based on the curriculum of any particular high school course.

There are a number of points that might be made in defense of this principle. First of all, if the exam is based on the current curriculum, it can hardly fail to intrude on the professional role of the teacher, and most high school teachers I have talked to view this as quite undesirable. And for good reasons – a course is a unit with its own sense of integrity. At the beginning of the course, the teacher has certain objectives, and as the weeks go by she works towards these guided by her own experience with the material and the progress of her students. And then at the end she examines them in a way that reflects their joint experience together. If an outside agency comes along and examines her students on the material she is teaching, this is almost certain to subvert her program to some extent, especially if (and this will inevitably be the case) her work as a teacher is partly judged by how well her students do.

In addition, an exam that is administered part way through the year before the the teacher has put things together, is nothing less than insulting to her.

[Note that it is quite another matter if the entire curriculum is directed toward an external examination, as was the case with the Ontario "Departmentals" of the

50's and 60's. Such exams are designed to test (and even drive) a prescribed curriculum, and are administered at the end of the course. Such a system has its pros and cons, but my impression is that there is little desire on the part of Ontario teachers to return to it.]

ANYWAY – the university is not really interested in whether or not students can cope with a particular course, be it grade 12 Ontario, Nova Scotia, or the British GCE. The university is even happy to have students who have spent the last few years tooling around the continent on the back of a motorcycle studying Zen, as long as they can solve mathematics problems at the right level, organize material, learn what is missing, and present results in a coherent readable manner. That's what the exam should test, and it should do that as much as possible in a curriculum independent way.

It follows from this that the technical skills that are either tested directly on the exam or are needed to solve the problems should be basic to most curricula and reasonably elementary. In this context, it's worth emphasizing that not all that many technical mathematical skills are really needed to navigate in the world. Even technically sophisticated professions, such as engineering, use very little of the mathematics that they learn in their formal education, and what they do use is relearned to suit the context. I have observed that when university mathematicians set tests and curriculum – and I include myself here – they often forget this, and behave as if they are preparing future math majors. Such students of course form only a tiny fraction of the high school population.

It is even the case that standard math exams do not reflect well the work that a mathematician does. The great majority of my professional work does not require the solving of tricky problems or manipulation of algebraic expressions. Rather it involves an incisive reading of some preliminary materials, a careful organization of data and discoveries, perhaps a search for some simplifications, a drawing of some appropriate conclusions, and then setting everything down in a simple coherent manner. In fact, that's what any professional needs to be able to do, and that is most of what is required to succeed in university. (Though it goes without saying that it is essential to be able manipulate standard algebraic expressions in a reliable manner.)

Here is my sample exam. It is designed for students entering a program in the mathematical or physical sciences, or engineering. In fact, it would also be not unreasonable for students entering programs such as biology and economics.

UNIVERSITY ENTRANCE EXAMINATION MATHEMATICS

The exam is three hours long. There are three questions, each of which requires a certain amount of preliminary thought, organization, and analysis. Marks will be based on the clarity, coherence and simplicity of your solution, as well as on its correctness. Appropriate use of diagrams, graphs, and tables will often contribute to the readability of your presentation. What you do, do well – it is much better to answer one or two questions well than to do a mediocre job on all three.

1. Suppose I take a set of positive integers and denote their average value by a. Now form a second set of integers by adding 1 to every integer in the first set. Now for each of the two sets, calculate the average of the squares of the numbers, and then let the difference of these two averages be d. The problem is to find and understand any simple relationships between a and d.

   For example, if the first set consists of the numbers 4, 6 and 11, the average is a = 7. Then the second set will have the numbers 5, 7 and 12. The squares of the first set are 16, 36 and 121, and their average is $\frac{173}{3}$, and the squares of the second set are 25, 49 and 144 and they have average $\frac{218}{3}$. The difference between these two averages is:

   $$d = \frac{218}{3} - \frac{173}{3} = \frac{45}{3} = 15$$

   The question in this case would be to find how 15 and 7 are related.

   (a) Repeat these calculations for other suitably chosen sets of numbers which have the same average a = 7. Now perform the calculations for other simple sets of numbers with other values of a. In each case observe the value of d, and make an algebraic conjecture about the general relationship between d and a.

   (b) Provide an algebraic argument to support your conjecture.

   (c) How does your result generalize if instead of adding 1 to the numbers in the first set, you add 2? What happens if you add 3? Formulate and prove a general result.

2. A rectangular box is exactly large enough to hold forty soft-drink cans arranged in a 5x8 array. It is observed that by rearranging the cans, it is possible to fit forty-one cans in the box. Show how this can be done and include the calculations needed to show that it works.

3. There are three cylindrical tanks, A, B and C. A starts with 150 litres of water and B and C start empty. You are asked to draw graphs of the amount of water in each tank over a 20 minute period. Here's the situation. At t = 0, a pump is turned on which pumps water from A into B at a constant rate of 10 litres/min. This continues for 15 minutes, at which time A is empty. At t = 5, when B has 50 litres, a stopcock is opened in the base of tank B and water starts to flow out into C. The level of water in tank B immediately starts to drop until, at t = 10, it reaches an apparently constant level of 20 litres, and this is maintained for the next 5 minutes. Of course, at t = 15, the flow of water into B is terminated, and the level in B is again observed to drop until the tank becomes empty at t = 20. At this point, all 150 litres are in C.

   On a common set of axes, draw graphs of the amount of water in each tank against time for $0 \leq t \leq 20$ and label your three graphs A, B and C. Point out and explain any special geometric features of, or relationships among, your graphs (height, slope and concavity).

My Comments on the Exam

There are 3 questions because I want the student to have something like an hour to spend on each. Some will spend the whole hour just getting to a solution. Some will get there in 10 minutes and will have time to prepare a careful write-up. For example, in #2 there are a number of interesting decisions to make in how to organize and present the calculations. Some might choose to do only two problems and get an hour and a half on each. In the instructions I claim that quality counts more than quantity, and we should adopt a "marking scheme" that reflects this. For example we might ask the student to select the two "best" questions submitted, and these would be worth 40 marks each, the remaining question worth 20 marks. So that a student who submitted only two questions, both done at a 75% level would get the same mark (60%) as a student who did all three questions each at a 60% level.

Now I'll make some comments about content and process. The first tests what the student knows, and the second tests what the student can do with this knowledge.

Content. The three questions cover the three areas I want to examine: algebra, geometry and analysis. The algebra question asks the student to identify arithmetic patterns in numbers, and then formulate some precise results and provide an algebraic argument. The geometry questions does the same thing with spatial relationships. Here I am interested in questions that have to do with optimal shapes and configurations, proportional relationships etc. The analysis question tests the student's facility with function graphs and interpretations of rate of change. Perhaps I should say something here about the analysis curriculum. I don't imagine the test requiring knowledge of calculus, but the relationship between slope of a graph and rate of change are quite fundamental and should be covered early in the curriculum. For linear functions, this should be done in grade 10. Then in grade 11 we can introduce the notion of slope of a graph as the slope of the tangent to the graph, and interpret that as the instantaneous rate of change at that point. Then we move to an understanding of concavity as telling us whether the rate of change is increasing or decreasing. Of course we don't need anything formal here; all this can be done on an intuitive level. Examples of temperature change, flow rates, distance-time graphs can be given.

There are some important areas I have regretfully omitted having to do with simple combinatorics, probability and statistical analysis of data. There is perhaps some uncertainty at the moment just where in the curriculum these should appear, and I might leave this matter for further discussion.

Process. The first question tests first of all whether the student can read and understand a construction or a procedure, and construct similar examples (part a), and then identify the simple pattern and provide a formal argument (part b), and then generalize the pattern found (part c). A university bound student should be able to do (a) and make some progress with (b) and (c). I would expect considerable variation in the form of the answer to (b). The second question is a test of the student's ability to organize an argument and identify the crucial calculations that need to be done to support it. The third question is quite a challenging question of its kind, and requires the student to read clearly and capture the simple pieces of a complex situation.

If I have erred with the level of this exam, it will be that I have made it slightly too hard. On the other hand, it should be compared, not with high school, but with university exams and it is designed to be comparable in difficulty and sophistication to the latter.

How is the student to prepare for the exam? I would produce a list of some 20 problems in each of the three areas and put this into the students' hands at the beginning of grade 10. Then they will have it by their side over their next three years of study. The nature of this exam could "inform" the math curriculum at all senior levels, perhaps not so much in terms of content, as in process and style. It might encourage teachers at all levels to set and grade (indeed to learn how to grade – something we all find difficult!) problems of this type.

In fact, I envision this math exam as part of a small number of number of entrance exams – perhaps three or four with some choice as to which ones the student might take. And these should all be fairly general – not only should they not be tied to specific courses, they should not even be bound to stay within the framework of any particular discipline. For example, I can imagine an exam that would test general quantitative literacy, problem solving and expository skills – an amalgam of math, science and language. Such a test would be hard for the student to prepare for in the conventional sense, but it might provide a measure of what the student had accomplished over a period of a few years. It is not unreasonable for an exam like this to test, not only what the student has studied formally in school, but what has been read, talked about, thought about, played with in life itself. That, after all, can be a good component of success at university.

Comments from Geoff Roulet

"I have no problems with your examination suggestions from a philosophical point of view, but I do see some problems in having them accepted. Of course this is no reason not to propose such a process and is possibly a positive comment on its value.

"On a fundamental level, it seems to me that you are changing the rules of the game. You are asking students to learn and understand, rather than gather credits, so that a student might easily study hard and still not do well. I think that students, parents, and teachers like to think that there is a correlation between effort and outcome. The present exams, which measure how much material one has retained, make that correlation possible, if not certain.

"I would expect not to see your suggestions receive overwhelming approval, but that is all the better reason to put them forth."

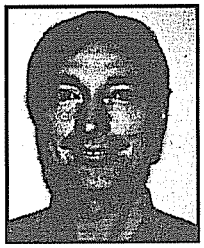
"Your examination would certainly measure understanding and deep facility with mathematical ideas. The results would be much more meaningful than the 95+%'s that students are receiving in OAC courses today. I'm not sure that the marks would always measure potential for university success. It is my impression that there are still a significant number of university mathematics courses that essentially involve the learning of a limited set of fixed algorithms. For such courses the present OAC grades might be valid indicators of potential to succeed. (PDT: I agree. And it's worth pointing out that present OAC grades are already quite good predictors of success in university – partly because both are highly correlated with ability. My interest in the above exam comes from the message it might send to curriculum planners and teachers, as Geoff points out below.)

"I began by rejecting your desire to separate the examination from curriculum as just a dream. It seemed to me that an entrance exam, after a few years, would become the curriculum. After all, if I as a teacher will be judged based upon my students' success on the examination, I am likely to adjust my course to meet the exam expectations. But after studying your sample exam I see that it would be very difficult to select course content that would ensure success. Thus I think that such an exam could remain separate from the curriculum in a content sense.

"This is a significant point, so let me emphasize it. It is essentially a truism that testing drives the curriculum. Usually this means content, i.e. the teacher delivers a course that emphasizes a set of discrete algorithms that apply to a limited set of questions that are expected on the exam (How to solve the leaky-trough related-rate problem). With your exam we may still get the test driving the curriculum but in this case it would be the approach or the process that was driven rather than content. The teacher would want to cover the standard content in a more open-ended problem-solving manner. I see this as an excellent outcome of your proposed exam."

The Attraction of the International Baccalaureate in Mathematics

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Jean-Paul Ginestier has taught mathematics in a number of different places - including Cameroon, England, and Switzerland, during an exchange in 1984-85. He lived in Toronto from 1975 until 1995, where he taught both in the public system (Harbord Collegiate) and in the independent Toronto French School. Most of the schools he has taught in had a very international mix of students, and three of them were bilingual. He is also a co-author of the Gage "Algebra and Geometry" text (which includes past International Baccalaureate questions in its problem sets).

What is the International Baccalaureate?

The International Baccalaureate Organization (IBO) has its headquarters in Switzerland, while the Curriculum and Assessment Centre is in Wales. The International Baccalaureate (IB) is a university-entrance diploma which is offered by 655 schools, both public and independent, in 85 countries around the world. IB Diploma holders have been accepted in universities in over 60 countries. In North America, good IB results have provided students with advanced placement status in many of the best universities.

Founded in 1967, the IB, which is available in English, French or Spanish, was to fulfill international schools' wishes to establish a **common curriculum** that would nevertheless respect each student's culture. International educators were motivated by the ideal that the students' new shared experience would foster tolerance and inter-cultural understanding among them. (Previously, students were separated into small classes according to their nationality in the last two years of high school: for instance, the English would prepare A levels, the French the Baccalauréat, the Germans the Habitur, the Americans the SATs, and others would have to choose one of these options).

Although the IB continues to fill that first mandate quite successfully, it has also grown into something else: more than a third of its activity is now in North America, where it is viewed both as a "European" influence on education (including math education) but more importantly a **solid external standard**. There is no equivalent on this continent other than, perhaps to some extent, the American Advanced Placement tests; these, however, have a much narrower syllabus, and do not represent the culmination of a two-year comprehensive programme.

The IB in Canada

There are a whopping 62 IB schools in Canada. Observe that this represents close to 10% of the world's IB schools, although the Canadian population represents only 0.5% of the world. Of the Canadian schools, 9 are in Ontario[1].

Why such popularity? Clearly, in my view, it is because Canadian education is even more regional than its US counterpart. Thus, while Americans do, at least sometimes, discuss education at the federal level, and do have the SAT and AP instruments, do have texts that are used throughout the country, do have a national forum in which to discuss mathematics education (the NCTM), Canada has none of these. Not only does it lack any educational voice at the national level, but Canadian publishers even produce different textbooks for different provinces. There isn't even a Canadian Association of Mathematics Teachers!

The IB seems, then, to be the only viable instrument for Canadians who feel the need to measure their achievements at all outside of their provinces or boards of education. The IB Diploma is definitely a challenge: it is not an automatic payoff for any student who enrolls in

[1] ***Ontario IB Schools:***
in Brampton: Notre Dame Secondary School
in Ottawa: Ashbury College, Elmwood School
in Toronto: Collège Français à Jarvis; Toronto French School; Upper Canada College; Victoria Park Secondary School; Weston Collegiate
in Windsor: Académie Sainte Cécile

the course, as High School diplomas have become throughout North America through continuing mark inflation. It should be mentioned here that Canadian schools have performed quite favourably in the IB. The IB is now recognized as a valid university entrance qualification in Ontario, attaining advanced placement status in some instances[2].

The IB Diploma

IB Diploma candidates are required to select subjects from 6 different groups, 3 of which are taken at Higher Level, and 3 at Subsidiary Level, as follows:

Group 1: first language
(almost any language is examinable)

Group 2: second modern language

Group 3: individuals and society

Group 4: sciences

Group 5: mathematics

Group 6: arts and electives

The IB hence provides a broad education while allowing for a degree of specialization through the Higher Levels. In addition, candidates must follow a course in Theory of Knowledge, complete a Research Paper in a subject of their choice, and engage in programmes of Creativity, Action, and Service.

The academic subjects are generally examined, in two separate papers each (taken on different days), at the end of the two-year course. However, up to two of the Subsidiary level examinations can be taken at the end of the first year. Also, most subjects have some form of internal assessment which is externally moderated.

Mathematics in the IB

Mathematics is thus a compulsory subject, and is available in 3 main programmes: Higher Level Mathematics, Subsidiary Mathematical Methods and Subsidiary Mathematical Studies. The first is designed for students interested and competent in mathematics, who may wish to continue in this or related fields at university; the second is designed for students who expect to need some mathematics at university for further studies in subjects such as chemistry, economics, business administration, medicine; the third is designed for students of varied backgrounds and abilities (at present, only Math Studies has an internal assessment component). There is also a fourth programme, Advanced Mathematics, for students who wish to go beyond Higher Level.

Comparing either the Higher Level or the Mathematical Methods programmes to the Ontario curricula in mathematics, the following can be stated: the IB syllabi are much broader - they contain much of the OAC Calculus, Algebra and Finite Mathematics course contents, and more, and present these in an blended way (hence the "European influence"). A good example of this blend is the use of calculus in the study of continuous probability distributions. Students are not compelled to analyze merely simple rectangles and triangles, since they can calculate probabilities, as well as expectations and variances of any function for which they can handle the relevant integral.

Here are examples of IB examination questions.

Example 1.(Higher Level, May 1996):

The line L passes through the point P(2,3,1) and has direction

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}.$$

A second line M passes through the point Q(4,2,0) and has direction

$$\vec{v} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}.$$

(a) Write down, in parametric form and using t as the parameter, the equation of line M.

(b) Find the vector $\vec{w} = \overrightarrow{RP}$, where R is any point on the line M.

(c) Write down the vector $\vec{u} \times \vec{w}$ and hence express $|\vec{u} \times \vec{w}|$ in terms of t.

(d) Deduce that $|\vec{u} \times \vec{w}|$ is minimized when $t = -\frac{3}{5}$ and find its minimum value.

Example 2: (Math Methods, May 95)

The matrix M is given by $M = \begin{pmatrix} 17 & 18 \\ -16 & -17 \end{pmatrix}$.

(a) Find M^2; M^3; M^{-1}

(b) Solve for X the equation $M^{99}X = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$.

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[2]Should your school be interested in obtaining more information, contact:
International Baccalaureate North America
200 Madison Avenue, #2007
New York, NY 10016, USA
Tel: (212) 696 4464 Fax: (212) 889 9242
e-mail: IBNA@ibo.org

(c) Let $P = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$, $Q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Calculate P^2, PQ, QP, Q^2.

(d) By expanding $(kP+Q)^2$, find in terms of k the values of a, b, c and d such that
$(kP+Q)^2 = aP^2 + bPQ + cQP + dQ^2$,
and use the results of part (c) to simplify the result.

Note that in the latter part of the example 1, which appears to be a "vector" question, the student is expected to use calculus, or some other technique, to find a minimum.

In both the above questions, observe the progression which helps the candidate along; for instance, in example 1, in part (d), the answer is given, so that students will know whether or not they have made a mistake in their solution. Although the questions might involve some challenging mathematics, they are intended to be "user-friendly"; the main reason for this is that the IB is taken world-wide by many candidates who are not writing in their first language.

In example 2, the student is expected to infer properties of the matrix M from part (a) and hence use this information to answer part (b). The latter two parts explore a particular aspect of linear combinations of matrices, although this topic is not specifically mentioned in the syllabus.

It should be emphasized also that, unless a method is specified in an IB question, candidates may use any method of their choosing in their solutions, even if it is not in the syllabus.

Alternatives to the IB?

The IBO is committed to maintaining rigour, depth and internationalism of the Diploma, in part by drawing on international dimensions of current best practice and innovation. The syllabuses are enhanced and renewed regularly, and a balanced variety of teachers play a significant role in this activity. In designing syllabuses, an attempt is made to avoid bias towards particular local systems of education, cultures, or gender. The IBO endorses the view that internationalism, in the context of education, has intrinsic value, encompassing the diversity of human practices, values and identities.

Many Canadians are attached to the notion that education should be locally controlled. It is interesting to observe the situation in Switzerland. Each canton is even more ferociously independent than Canadian provinces, but the culmination of secondary education is validated by Federal "Maturity" examinations. Universities, too, have a federal status.

Although a local rather than a national outlook on education has its attractions (certainly in subjects such as History), it is arguable that some subject areas should be as universal as possible, and mathematics is definitely one of these. After all, mathematics is, already, an international language.

Should Canada decide to unite its forces and create national curricula, the IB would lose a lot of its attraction there, although it would most surely be upheld in those schools interested in maintaining an international outlook. Mathematics might well be the best subject in which to establish a first Canadian national curriculum. A real challenge for whomever wants to pick it up!

The WINNER OF
SATURDAY NIGHT
& FABULOUS
INTERNATIONAL
SUNDAY BRUNCH FOR 2
at the Four Points Hotel,
Mississauga
site of OAME's Leadership '96

Drawn from all who enroled as
OAME members between June 15,
1996 to September 30, 1996

was
Linda Goodale
Lockerby Composite School,
Sudbury
Congratulations Linda!

▲ TEACHING IDEAS FROM THE EDITOR

Algebra: an Easy Method of Factoring Trinomials

During my last 14 years (out of 47) of teaching I used the following method to factor trinomials $ax^2+bx+c \quad (a \neq 1)$, especially for my weaker students.

$ax^2+bx+c=\frac{(ax+d)(ax+e)}{a}$ where de = ac and d + e = b. Note that the a in the denominator compensates for the extra a in the numerator..

Example 1: Factor $3x^2-2x-8$.

Solution: $3x^2-2x-8=\frac{(3x+d)(3x+e)}{3}$

where de = (3)(-8) = –24 and d + e = -2. .

But two factors of -24 with a sum of -2 are 4 and -6

Thus, $3x^2-2x-8=\frac{(3x+4)(3x-6)}{3}=(3x+4)(x-2)$

as the 3 divides into the second factor.

Example 2: Factor $12x^2 + 28x + 15$.

Solution: $12x^2+28x+15=\frac{(12x+d)(12x+e)}{12}$

where de = (12)(15) = 180 and d + e = 28.

But two factors of 180 with a sum of 28 are 10 and 18.

Thus,

$$12x^2+28x+15=\frac{(12x+10)(12x+18)}{12}=(6x+5)(2x+3)$$

as 2 divides into the first factor and 6 into the second factor.

One advantage of this solution is that your students can find d and e by applying the method they use to factor $x^2 + bx + c$.

When I taught this method I did not use variables d and e so that the solutions would omit the first 3 lines. In Example 1 the students would ask themselves what two numbers multiply to give 24 and have a difference of 2 (difference because of the minus sign with 8). The 6 was given the minus sign because it was the larger of 4 and 6 and the -2 was negative. In example 2 they would ask themselves what two numbers multiply to give 180 and have a sum of 28 (sum because of the plus sign with 15).

Ontario Mathematics Coordinators Association
Ontario Association for Mathematics Education

LINKING ASSESSMENT AND INSTRUCTION IN MATHEMATICS: CONNECTING TO THE ONTARIO PROVINCIAL STANDARDS TRANSITION YEARS

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- **Sample Student Solutions illustrating Levels of Performance**
- **Notes to Teachers**

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Abacus

What better time in the school year than now to take a look at some real geometry and to think about interesting games for those cold winter days. From my own elementary school days I recall the joy of cutting out paper snowflakes and stars to decorate the school as well as the special board games the teacher would allow us to play at recess time. Conversations with elementary grade teachers suggest this journal publish some ideas that could be taken home too. I am indeed pleased to present activities that will meet the need.

At the May '96 OAME conference in London I invited **Jane Felling** from **Box Cars & One-Eyed Jacks, Edmonton, Alberta** to write an article for us. She is co-author of a three volume series of books by the same name. The box cars are dice and these books are all about math games for kids using dice and cards. The games described in each volume cover numeration, arithmetic operations (fraction dice are available), data gathering and problem solving. The authors have combined their experiences in elementary and special education teaching to provide fun and motivation in elementary math programs from kindergarten to seniors. A game can be chosen to introduce a concept, practice a skill or master a topic.

Jane is a promoter of math backpacks. For the workshop "find a seat and get ready to play" was her opening line. Most of the participants in the session left with math backpack ideas.

Why Math Games Backpacks?

1. Many primary students are used to taking home readers or other activities. Why not "add" some math for a change.
2. Children are normally eager to play games. Math games are as willingly played by our most reluctant math students as by our math whizzes.
3. They are a welcome change for parents who often are frustrated by the struggle to get homework completed. Parents and children become partners in a game and the setting is relaxed. Cards and dice replace paper and pencil.
4. Games can be individualized for student's needs.

Getting Started

Use the Meet the Teacher meeting to discuss the purpose of at-home practice with math games. **Odd or Even** is an excellent choice for a starter or first game in the mathpack. Communication is important. Jane recommends that an introductory letter go home to parents in the first backpack to suggest the parents try 10 to 15 minutes a day and to provide the rules for returning the packs to school as well as guidelines about the games component of the child's math program. To strengthen the home link, give parents an opportunity to comment on each math-game experience. **Addition War** is an example of a game that is suitable for players of different skill levels. **Combo Five** is reprinted with permission from **Box Cars & One-Eyed Jacks Vol III, 1996.**

Grades 4 and up will find **Make a Tetrahedral Gift Box** by **Catherine Ross, London, Ontario** gives experience with manipulatives in the spatial sense and geometry strand. What better way to observe geometrical properties than by using compass, scissors and folding tricks to create a useful three-dimensional figure. Recall that a tetrahedron is a geometrical shape with four triangle faces.

A Publication of the OAME/AOEM • Volume 35 Number 2 • September 1996
Edited by Frances Schatz, 56 Oxford Street, Kitchener ON N2H 4R7
Tel: (519) 578-5948 Fax (519) 578-5144

All elementary school children enjoy working with tangrams and the junior grades can have fun making their very own tangrams with the help of **Tangrams Through Paper Folding** by **Robert Jones, LWCDinc., St. Louis, Missouri.** This is an example from his book **Paper Folding** that he wrote to provide hands-on activities to help junior students discover math concepts.

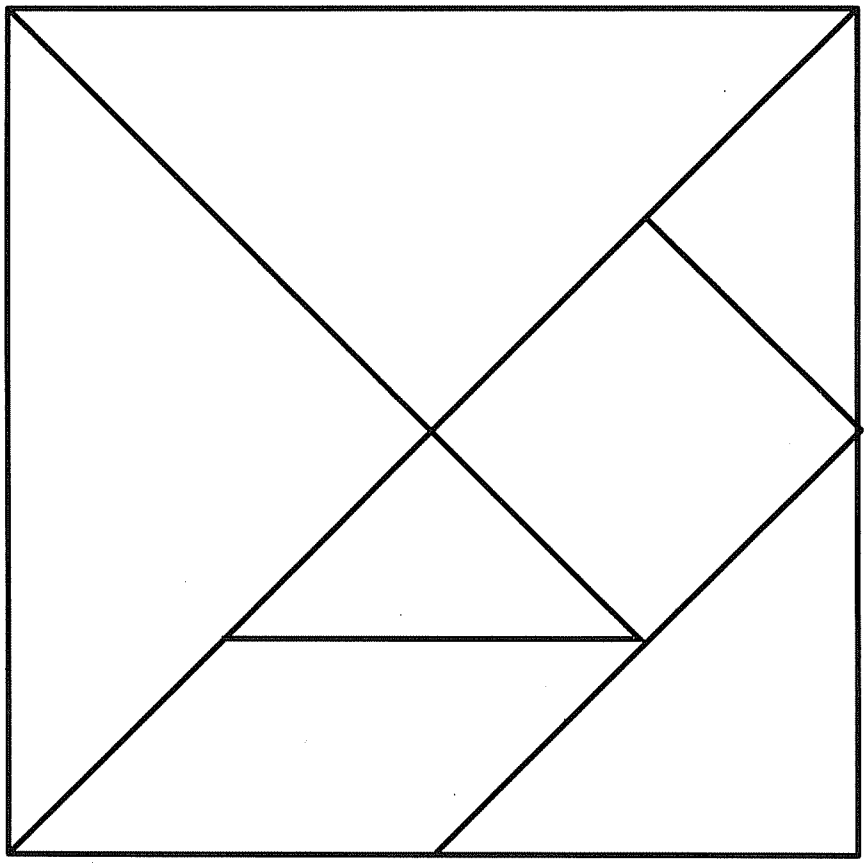
Thank you Catherine Ross, Jane Felling and Robert Jones for providing *Abacus* readers with motivational and fun mathematics.

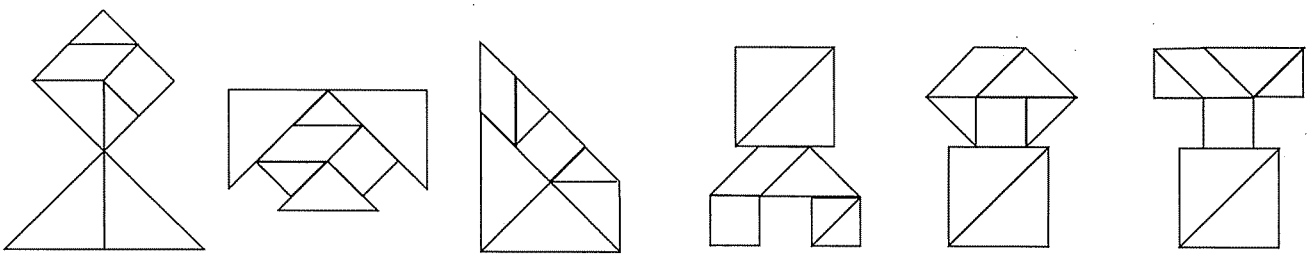
Readers, don't forget to have your students send in entries for the September issue contests. I can see *Abacus* is going to have an interesting title page beginning with the March, l997 issue. Submissions, comments, special requests and questions are always welcome.

Frances Schatz

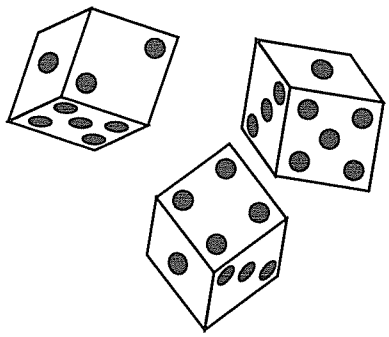
SOLUTIONS

Tangram Puzzles





Odd or Even

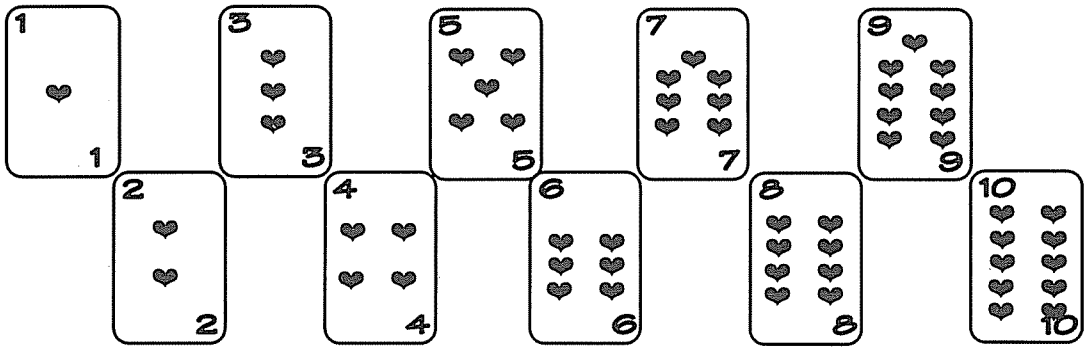


Skills: Addition to ten, odd and even

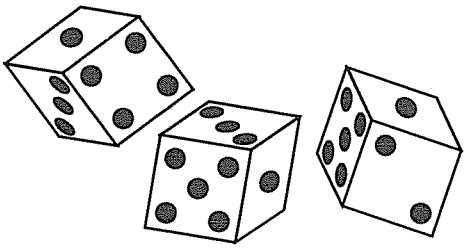
Players: Two or more

Equipment: Each player cards (Ace=1) - 10
Two dice with faces 1-6

Start: Each player arranges the ten cards as follows:



Before players begin, they predict which set of numbers they will eliminate first: either the odd or the even set. Players then take turns rolling one or two dice and begin eliminating sums of their rolls.



Example:

- Roll 2 and 4: remove 6
- Roll 5 and 3: remove 8

Players continue to take turns until one player has removed all cards. This player earns 10 points for doing so. If the player also made a correct prediction, then the player earns 5 more points.

- Play continues to 50 or l00 points.

Variation: Encourage players to add or subtract the dice before removing cards. Have players remove up to two cards per roll.

Addition War

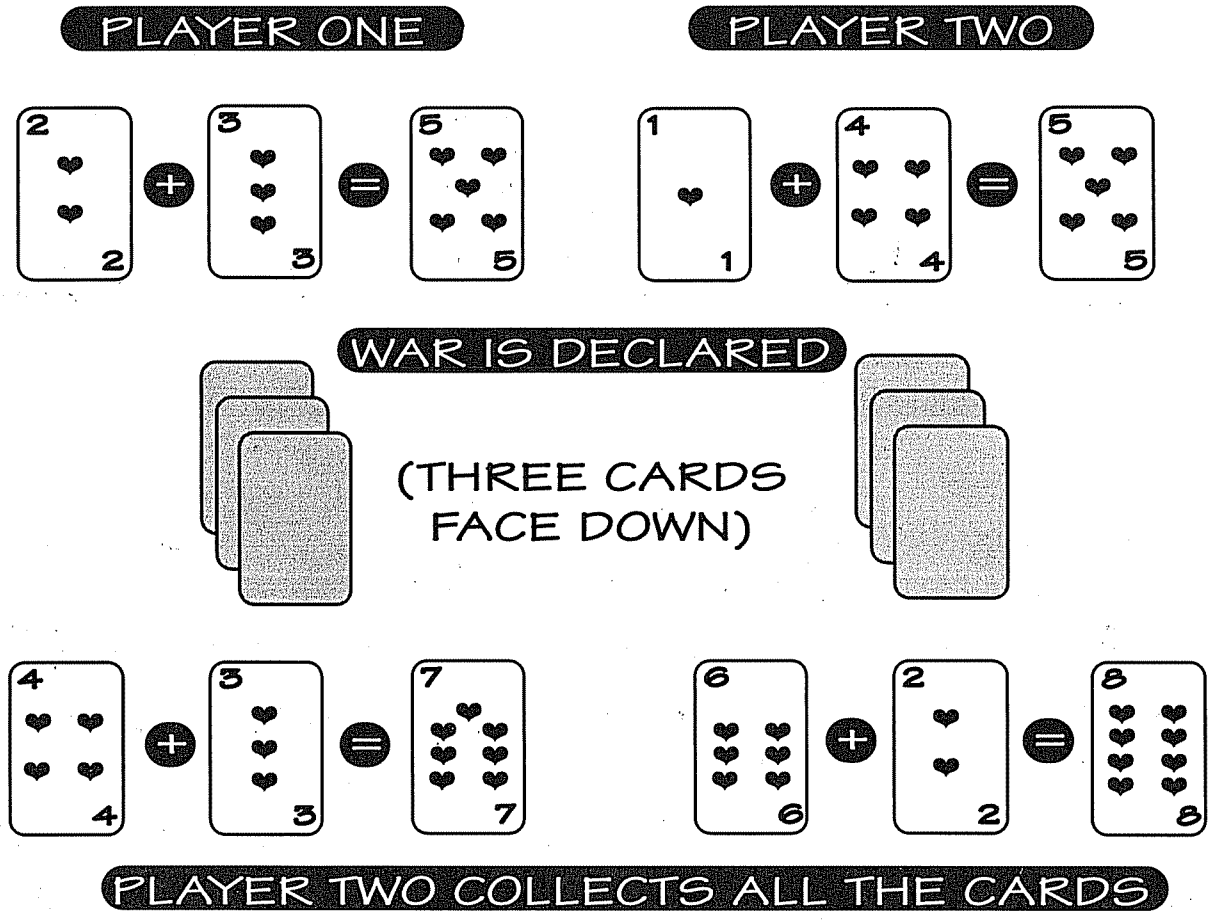
Skills: Addition

Players: Two

Equipment: Grades 1-2 (ace=1), 5 Grades 2-3 (ace=1), 9

Start: Players divide cards evenly between themselves. Each player turns over two cards and adds them together. The higher sum gets all the cards. In the event of a tie (each player has the same sum), WAR is declared. Each player deals out three more cards face down and then turns over two more cards. These two cards are added together. The higher sum wins all the cards. Play continues until one player has collected all the cards dealt.

Example:



Variation: Vary the number of cards to modify the level of difficulty; for example, three cards/player gives a sum such as 23 + 6.

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The Ontario Association of Mathematics Education requires a NCTM Representative

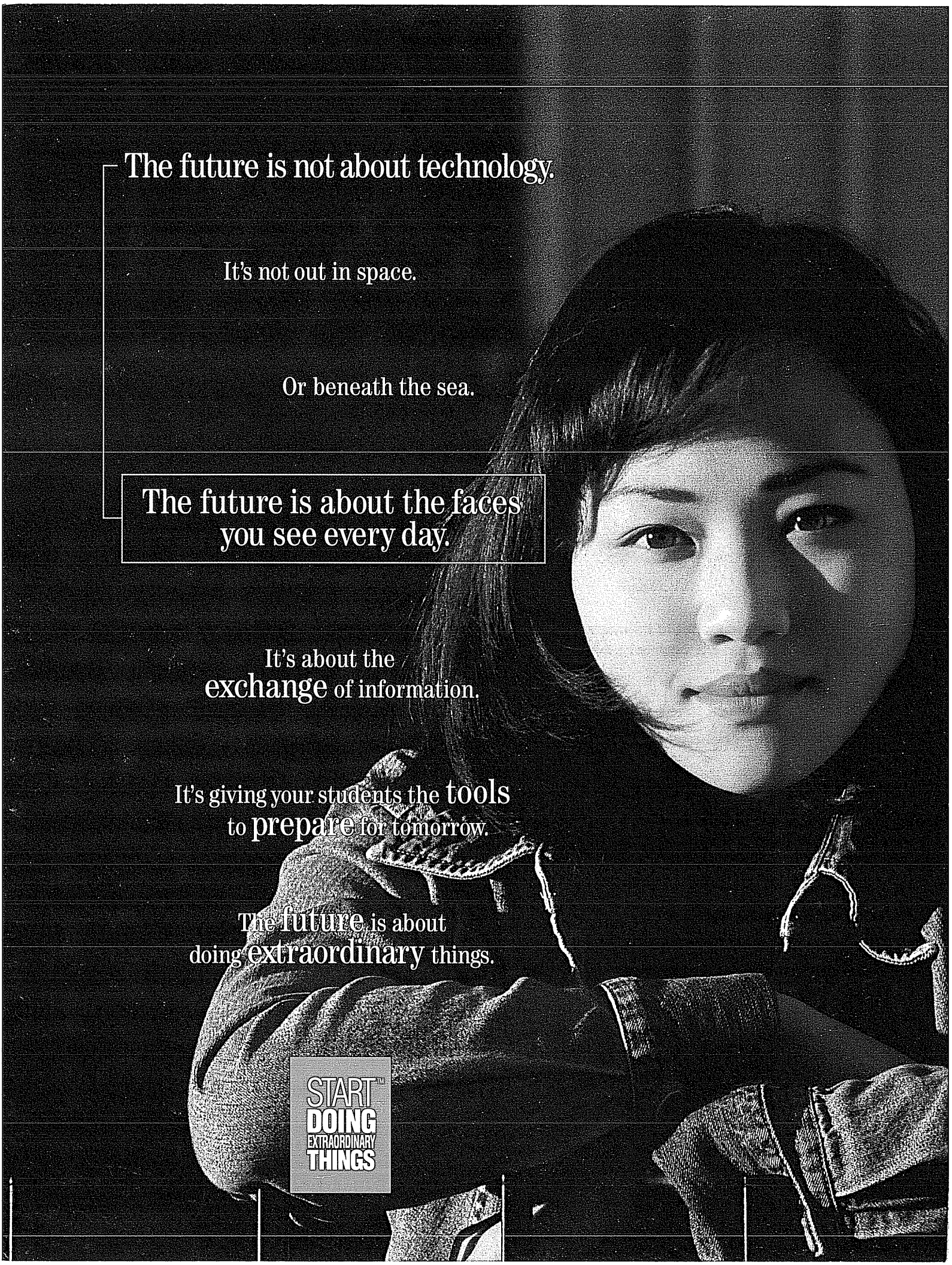
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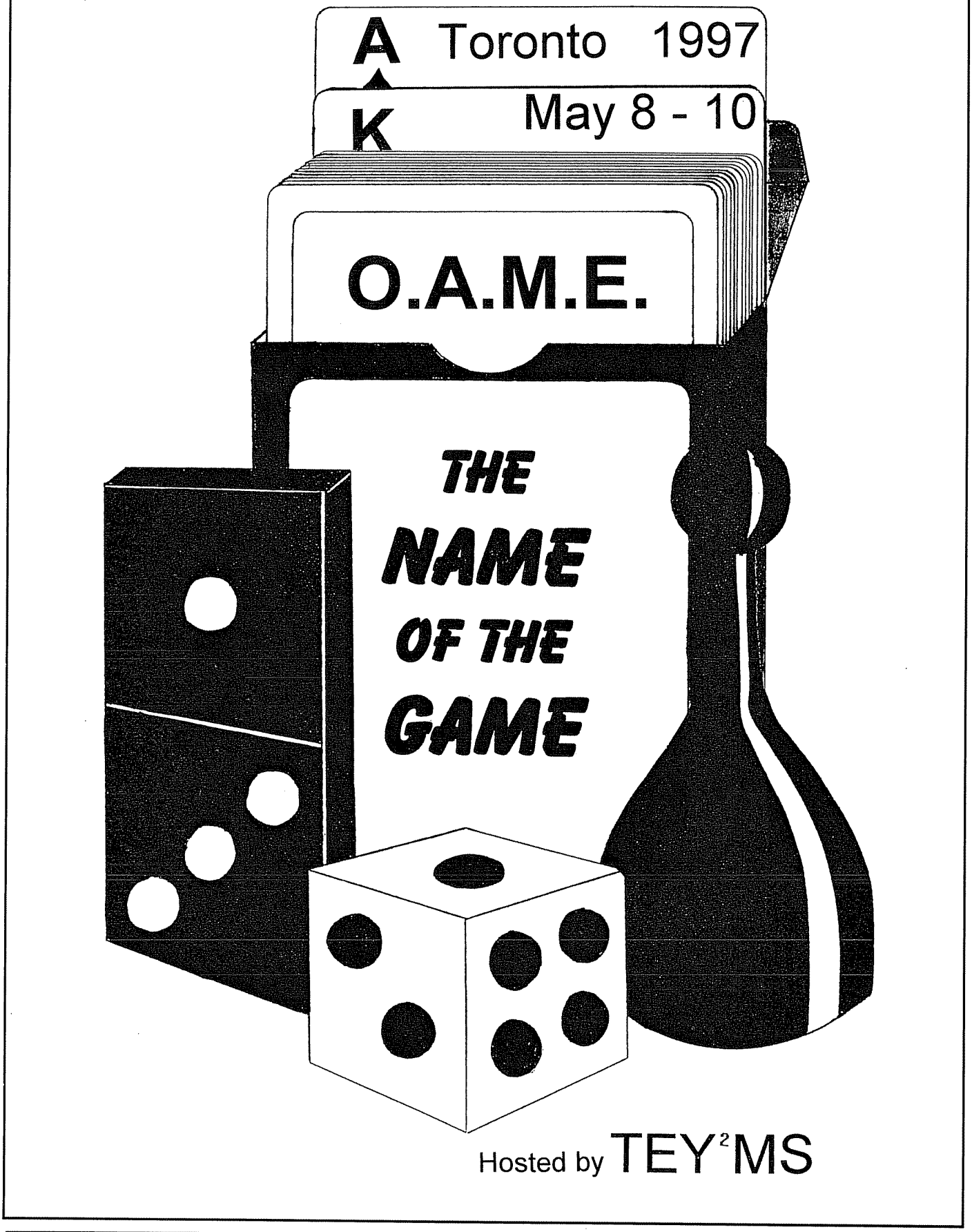
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TEXAS INSTRUMENTS


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O.A.M.E.
THE
NAME
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GAME
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Combo Five

Level: Grade 3 and up

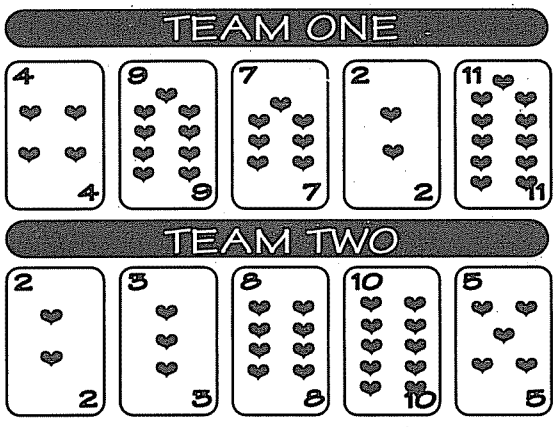
Skills: Mixed operations (add, subtract, multiply, divide), problem solving

Players: Teams of 2 vs. 2

Equipment: One 20-sided die, cards Ace(1)-10, Jack(11), Queen(12), King(0)

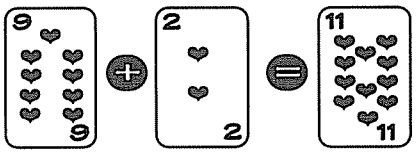
Start: Both teams take 5 cards and place them face up. The goal of the game is to equal the rolled target number each round. To begin, one team rolls the target number for the round. This number will be used by both teams. Teams now begin finding combinations that equal the target number rolled - all operations may be used. A single card cannot be taken off. Teams may take off two, three, four or five card combinations. Teams may also take off a two card and three card combination, or two-two card combinations leaving one card behind for the next round. Each card may only be used once in any combination (ie., in the following example 4 can only be used once and not again in a second combination).

Example: Cards drawn are as follows:



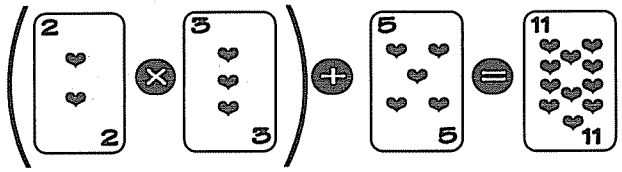
Target rolled = 11

Team One made the following combinations and removed the cards as follows:



leaving behind the 11 card as it was not used in any combination.

Team Two made the following combinations and removed the cards as follows:



leaving behind the **8** and **10** cards. Cards used in any combination are placed in the team's point pile and will be counted at the end of the game to determine the winner. After each round both teams replenish their five cards. Team One in the above example will draw four new cards and Team Two will add three new ones as **8** and **10** were left behind.

A new target is rolled and players work with their new target and card combinations.

Variations: To decrease the level of difficulty have players look for adding and subtracting combinations only.
Example: Target rolled = 6.
Team One cards are 6, 8, 2, 7, 5;
Team Two cards are 3, 9, 4, 8, 10.
Team One makes the combination and removes the cards for: (7+5)-6 and 8-2.
Team One was successful in using all 5 cards.

Score: After a set period of time teams count their cards in their point piles. Winner has the most points.

Make a Tetrahedral Gift Box

Materials:

Paper - any size and colour.
Pencil
Compass
Scissors
Sticky clear tape

Steps:

1. Draw the largest circle your compass will make. Mark the centre of the circle with a dot. Cut out the circle.
2. To get the first side of a triangle, fold any point on the outside (circumference) of the circle so that the point touches the centre dot.
Fig I

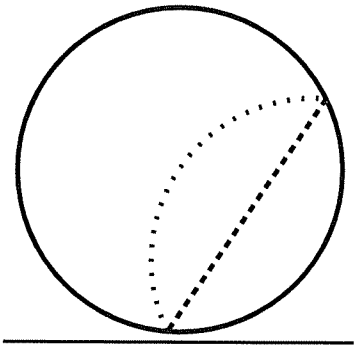


Fig 1

3. Start the second fold at one end of the first fold line so that circumference touches the centre dot.
4. Make the third side of the triangle by folding the remaining circumference to touch the dot.
Fig 2

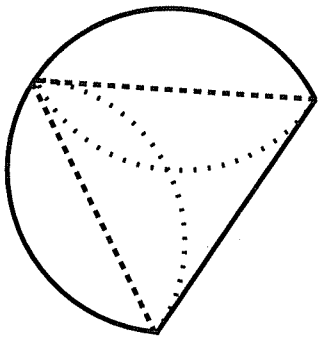


Fig 2

Note: If your triangle doesn't have three sharp, well-formed angles, unfold and start again.

5. Fold one of the sides in half to find its midpoint. Fold the opposite vertex down to touch this midpoint. Sharpen the fold line by running your thumbnail over it. You now have a small triangle between two others.
Fig 3
6. Fold the outside triangles on top of the inside triangle. Crease the fold lines.
7. To form a tetrahedron, join the vertices of the original large triangle as shown in **Fig 4.** Put your present inside, tape the sides shut, and add a celebration bow if you like.

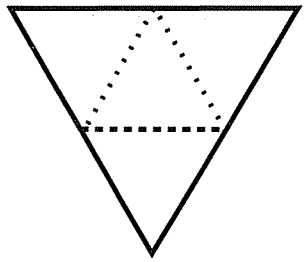


Fig 3

Variations: If you want a gift box that is larger, trace a large round object onto paper. Cut out the circle and fold in half twice to find the circle's centre at the intersection of the two fold lines. If you want a gift box for a flat present, do steps I-6 then:

7. Unfold to get the original large triangle. Fold the vertices in to touch the dot at the centre of the circle. Crease the fold lines and now you have a hexagon. Fold the sides up and interlock to make this flattened shape. Put your present inside and tape your box shut.

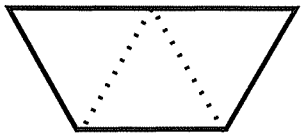
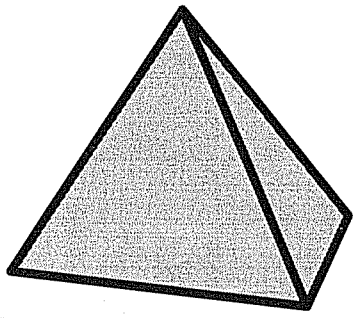


Fig 4

For other ideas:
Ross, Catherine S. *Triangles: Shapes in Math, Science and Nature.* Toronto: Kids Can Press, 1994.



Tetrahedral box
4 Triangular Faces

Tangrams Through Paper Folding

To Make a Square

Take a sheet of paper and fold it **Fig A**. Cut off the portion to the right of the triangle.

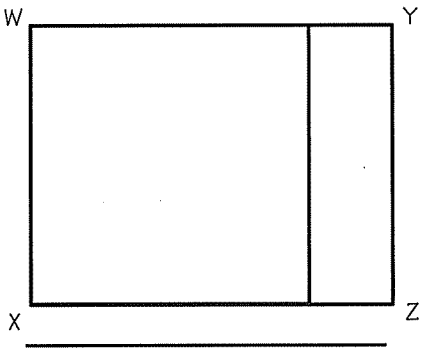
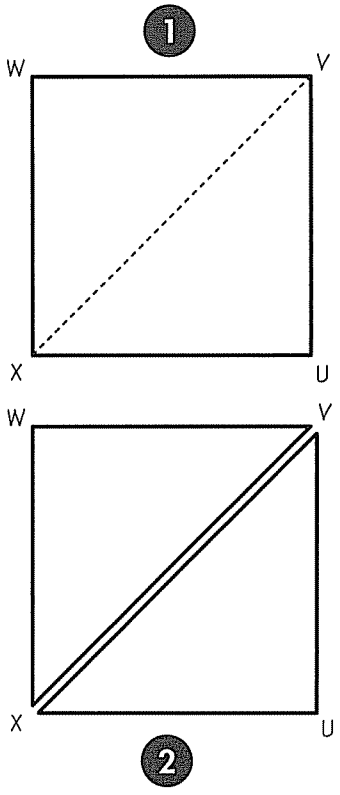
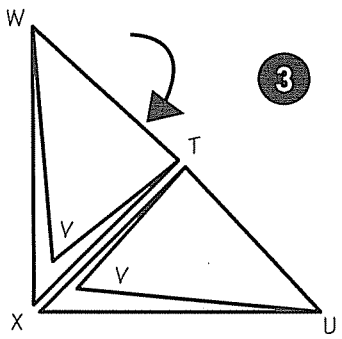


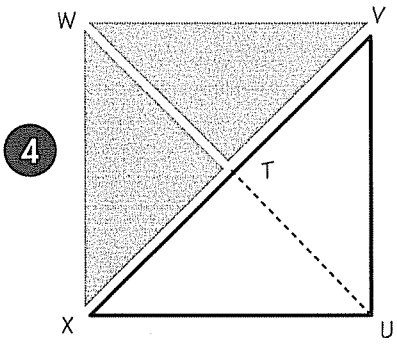
Fig A

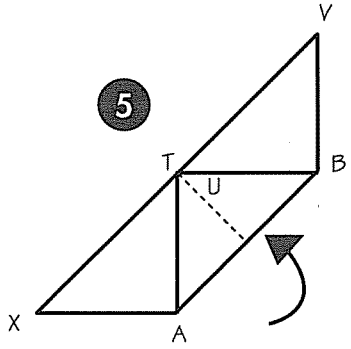
Steps:

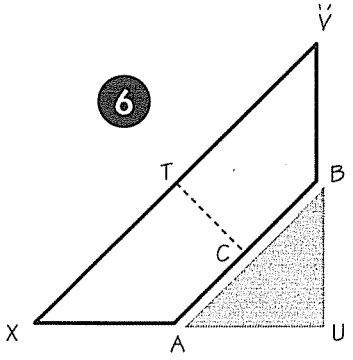
1. Take your square WXUV. Fold a crease through points X and V.
2. Cut the square along the crease to produce two triangles.
3. Fold both triangles as shown and crease. Open both triangles back up.
4. Cut one triangle on the crease that was formed in the last step. Set these two pieces aside.
5. Take the other triangle and fold it so that corner U lines up with the crease on edge VX.
6. Unfold the small triangle that was folded in step 5 and cut along the crease.Set this shape aside for now.
7. Fold tip V of the remaining shape so that the fold passes through point B and the tip V falls on the crease at point T.

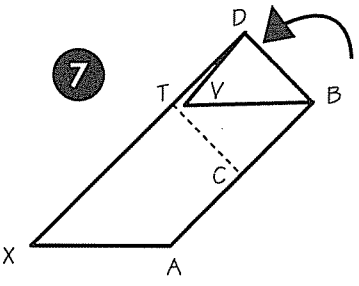


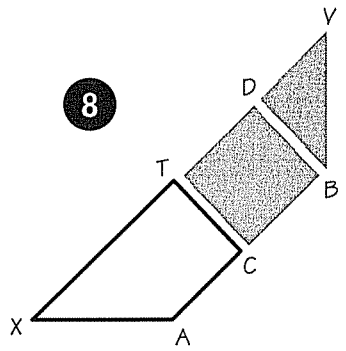


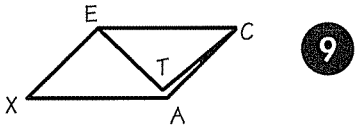


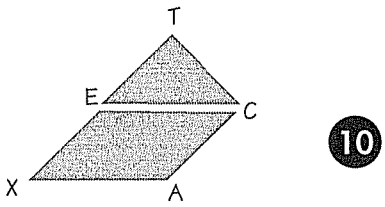












8. Unfold and cut the shape along the two creases BD and TC. This gives two more pieces for your tangram.

9. Fold the remaining shape so that corner T is on corner A and the crease goes through corner C and crease.

10. Unfold the shape and cut along the crease. Your 7-piece tangram puzzle is now complete.

Solutions shows the location of each of the seven tangram puzzle pieces that were produced using steps I-10.

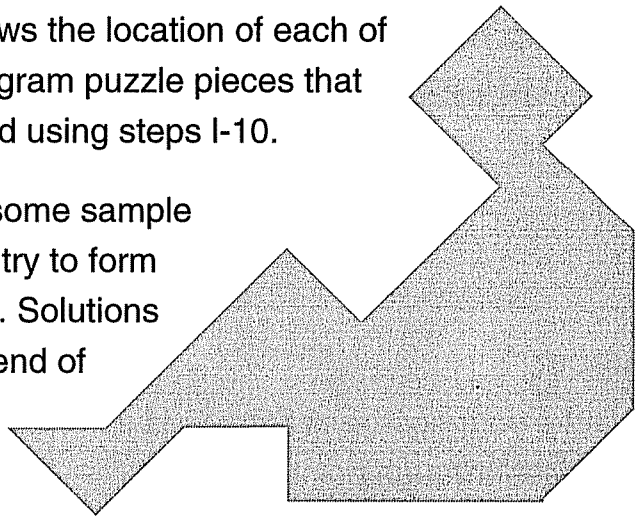
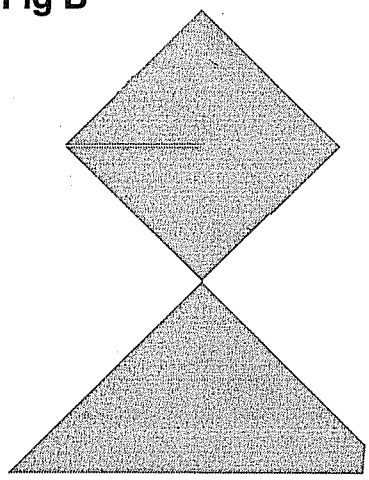
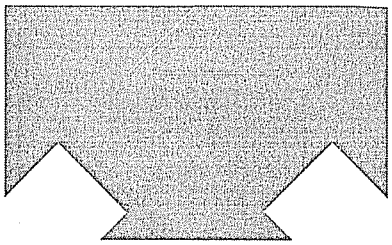
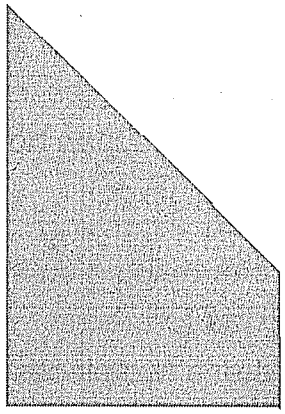


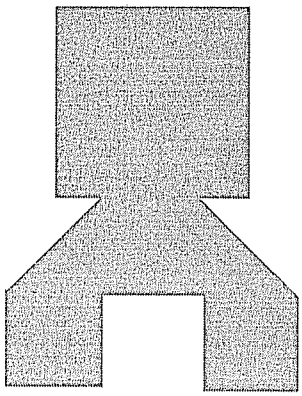
Fig B provides some sample shapes you can try to form with your pieces. Solutions are provided at end of editorial notes. "Person Reclining" is a challenger.

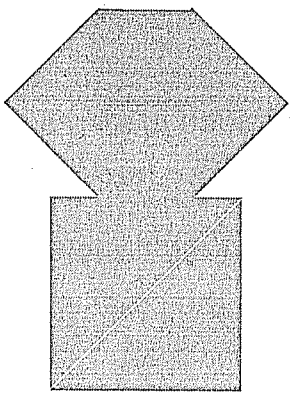
Fig B

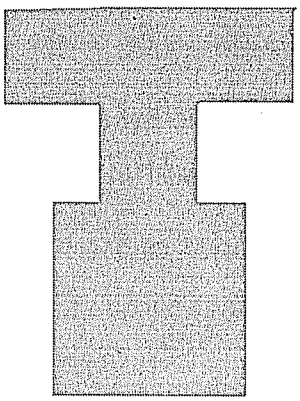






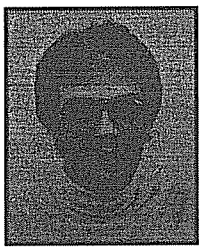






▲ Connecting Mathematical Ideas Using Triangular Numbers

JOHN G. MCLOUGLIN
AND PETER BROUWER



John Grant McLoughlin teaches in the mathematics and education departments at Okanagan University College. His mathematical interests include problem posing, recreational mathematics, and the history of mathematics. John is currently completing his Ph.D. in Mathematics Education. His research focuses on undergraduates perceptions of mathematics within the context of a problem solving course.



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Engaging in mathematics may be seen as the process of searching for patterns and expressing them using mathematical concepts and language. Finding patterns involves making connections between the world we live in and mathematics, as well as making connections within mathematics itself. That today's students should make these connections has become an imperative. The NCTM Standards (1989) recommends that the mathematics curriculum "should include the investigation of the connections and interplay among various mathematical topics and their applications so that all students can —

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;
- use and value the connections among mathematical topics;
- use and value the connections between mathematics and other disciplines." (p. 146)

This article will consider a variety of connections within mathematics that may be developed by exploring a common sequence, that of triangular numbers.

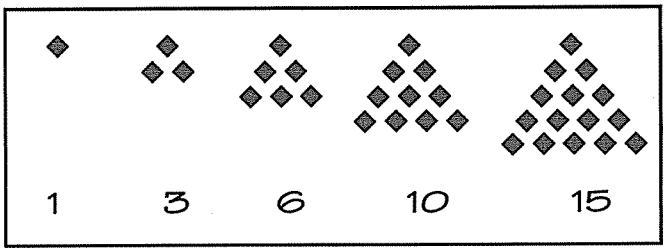


Figure 1 - Illustration of Triangular Numbers.

Triangular numbers are an example of figurate numbers, or numbers which can be represented by objects in the pattern of a specific geometric shape. Triangular numbers represent the number of objects that can be arranged in an equilateral triangular pattern (see Figure 1). They are formed by adding successive natural numbers beginning with 1. The triangular number sequence begins with 1, 1+2=3, and 1+2+3=6, respectively. The sum of the first n natural numbers gives the nth triangular number. Carl Friedrich Gauss cleverly developed a formula for such a sum (see Figure 2).

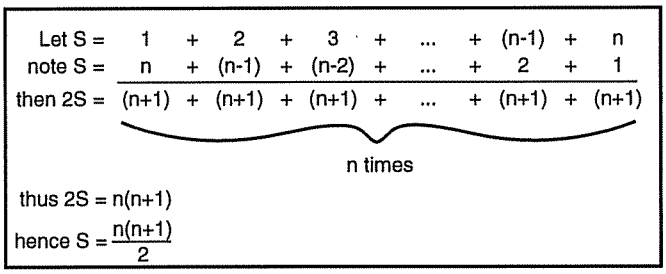


Figure 2 - Gauss' solution.

Let us suppose that such a formula was unfamiliar to us. Perhaps we did not observe that the triangular numbers represent sums of the form 1+2+3+...+n. How might we determine an expression for the nth triangular number? What other mathematical techniques might be used?

One possibility might come from experimenting with the sequence and making a conjecture that $\frac{n(n+1)}{2}$ is the correct expression. Mathematical

induction could be employed to verify this conjecture. The application of arithmetic sequences (Gauss' method) or mathematical induction is familiar to many students of mathematics. Let us consider a lesser known approach - one that may be applied in a number of situations. This approach, called the method of finite differences (see Seymour and Shedd, 1973), involves the use of a difference table. Difference tables are constructed by subtracting successive terms of a sequence. The resulting differences form a new sequence. The differences between the terms in this new sequence are then calculated. This process is repeatedly applied until a sequence of identical terms results. Figure 3 illustrates the method of finite differences applied to the case of triangular numbers.

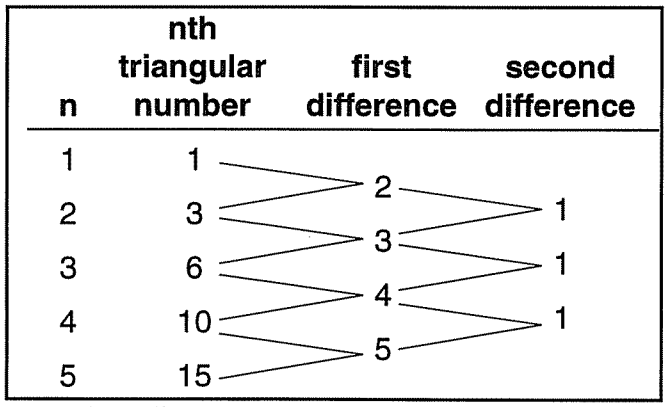


Figure 3 - Difference Table for the Triangular Number Sequence.

Observe that the second differences are constant. This tells us that the numbers in the sequence can be described by a second degree polynomial (Guillotte, 1986).

Readers may wish to satisfy their curiosity by checking this for a selection of second degree polynomials. To examine the general case, consider the standard quadratic of the form $f(n) = an^2 + bn + c$. Select equally spaced values of n and calculate f(n) for these values. One will find that the second differences are constant. In fact, this common second difference will be a multiple of a.

Let us reconsider the first three terms of the triangular number sequence, that is, f(1) = 1, f(2) = 3, and f(3) = 6. The general quadratic equation $f(n) = an^2 + bn + c$ must describe the numbers in this sequence for some set of values a, b and c. Substituting these values into the general equation produces the following system of equations:

$f(n) = an^2 + bn + c$

$f(1) = a + b + c = 1$

$f(2) = 4a + 2b + c = 3$

$f(3) = 9a + 3b + c = 6$

Figure 4 - System of Equations for Finding the Coefficients of the Second Degree Polynomial.

Solving, we find that $a = \frac{1}{2}, b = \frac{1}{2}$, and c = 0.

Therefore, the equation is $f(n) = \frac{n^2}{2} + \frac{n}{2}$, or $f(n) = \frac{n(n+1)}{2}$, as it is commonly expressed. Note that while we chose the first three terms in the triangular number sequence, any three consecutive (or equally spaced) terms would suffice for this analysis. For a more elegant way to solve for a, b and c given the nature of the finite difference relationship, see Guillotte (1986).

The equation holds for the values listed, namely, 1, 3, 6, 10 and 15. How do we know that it holds for all terms in the sequence? Induction could be used for the general case. Instead, could we provide a convincing argument that $f(n) = \frac{n(n+1)}{2}$ is true for all positive integer values of n that builds upon the pictorial presentation in Figure 1?

Successive triangular numbers are formed by placing a row of (n+1) objects below the array of objects representing the nth triangular number (see Figure 1). Observe from Figure 3 that the first differences are 2, 3, 4, and so on. It follows that each second difference equals (n+1) - n = 1. That is, the second difference is constant. Therefore, there exists a quadratic equation that provides an exact fit to the data. Since $f(n) = \frac{n(n+1)}{2}$ is the second degree equation which holds for n = 1, 2, 3, 4 and 5, it must be that general equation. Hence, all triangular numbers can be

generated by substituting positive integer values into this equation.

The use of a difference table lends itself to various problems involving sequences. For example, what is the next term of the sequence: 1, 2, 4, 8, 15, __ ? The 15 is not a misprint! Our suspicion is that the track down which many teachers lead students will foil most students in nonroutine situations such as this. Let us take the initiative to diversify students' skills, thus enhancing their abilities as problem solvers. Readers are encouraged to investigate this sequence, if they have not done so already. Although we are not saying that there is a single correct answer to this problem, the finite differences approach will yield a next term of 26, assuming the pattern of differences continues indefinitely.

One cautionary note is necessary when dealing with sequences. No finite number of terms uniquely determines the next term or a formula for the general term. For an illustration of this problem, consider what the next term in the following sequence is: 1, 2, 4, __ ? Certainly a strong case can be made for either 7 or 8. Perhaps the number that one has in mind as the next term is not the same as that proposed by others. The responsibility lies with the problem poser to keep an open mind and invite solvers to present solutions that may challenge what was expected.

Mathematics educators are encouraged to incorporate alternative mathematical presentations in their teaching. Such initiatives are likely to enhance the interest of students and their appreciation of mathematics by allowing them to see the connections that exist within mathematics. It's interconnectedness and inherent beauty can lead teachers and students alike to explore mathematical topics in greater depth.

The topic of triangular numbers is not commonly featured in the learning experiences of many school mathematics students. In an effort to enrich one's appreciation for this subject, a few problems dealing with triangular numbers are posed.

1. Consider three consecutive triangular numbers. Find the relationship between the square of the second number and the product of the first and third numbers.
2. If t is a triangular number, show that 8t + 1 is a perfect square. Is the converse true? (Morgan and Lerman, 1992)
3. Prove that every even perfect number is also triangular. A perfect number is a number that equals the sum of its proper divisors (e.g., 6 = 1 + 2 + 3, 28 = 1 + 2 + 4 + 7 + 14).
4. The Pythagorean triple (3,4,5) features the second triangular number and the second perfect square. Find another Pythagorean triple that includes a triangular number and a perfect square of the same rank.
5. Find a formula for the sum of the first n triangular numbers.
6. Show that the sum of any two consecutive triangular numbers is a perfect square. Try to demonstrate this fact both algebraically and geometrically.
7. Show that any number that is all 1's in base 9 can be written as a triangular number in base 10. (Barton, 1989)
8. Prove the following generalization: Let T(k) be the kth triangular number. Then if n is triangular, so is $n(2k+1)^2 + T(k)$ for every positive integer k. (Jordon, 1990)
9. Select any natural number. Express it as the sum of as few triangular numbers as possible. For example, 2 = 1 + 1, 19 = 1 + 3 + 15, 146 = 10 + 136. Comment on your findings. (Bezuszka, 1977)
10. Explore triplets of triangular numbers whose sum is also a triangular number. Can you express the relationship in a general way?
11. Prove that the sum of the reciprocals of the triangular numbers is 2.

An invitation to solve such problems is intended to help develop increased insight into the nature of triangular numbers. Additionally, while our discussion here has focused on triangular numbers, it is our hope that readers will be moved to investigate other figurate numbers such as square, pentagonal, hexagonal, cubic or square pyramidal numbers. Although similar methods can be used, some of these investigations will move the discussion beyond formulas that involve second degree polynomials.

Ideally, some students will pursue the subject further. A variety of problem solving approaches should be encouraged. Computers, calculators, additional readings, and our own minds are powerful means of exploration and discovery. The connections formed here may motivate a lifetime of mathematical discovery and appreciation.

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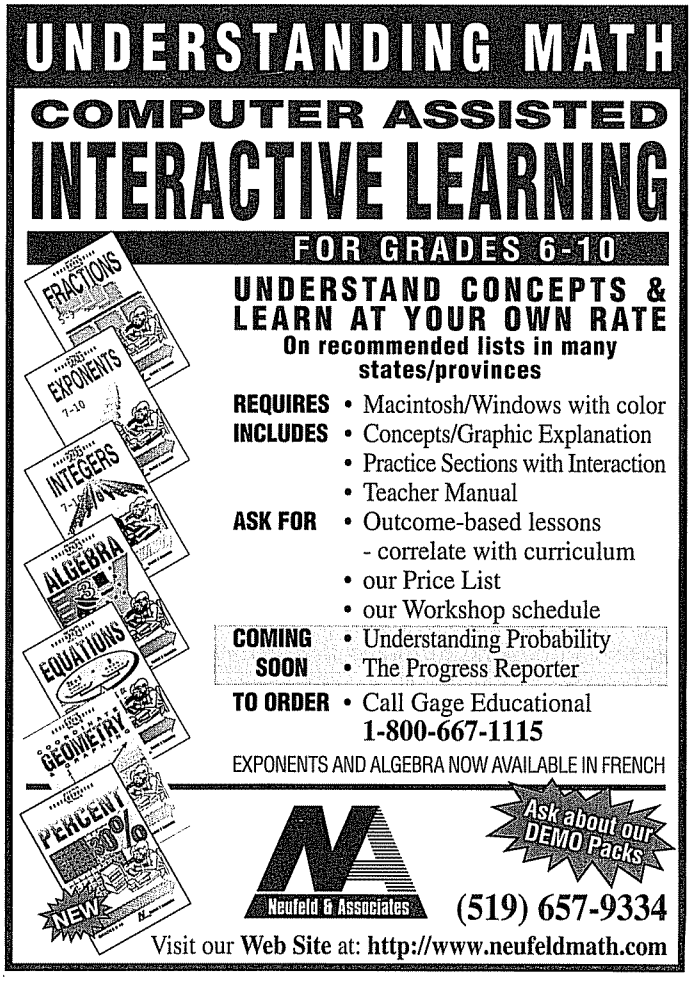
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▲ Destreaming in Grade 9: Teachers' Strategies and Sources of Ideas

DAVID H. HAIMES



David Haimes was formerly a mathematics department head in a secondary school in Australia, and has also taught mathematics in a secondary school in Ontario. Since completing his Ph.D. at the Ontario Institute for Studies in Education in 1993, he has done contract work for both the OISE Trent Valley Centre and the Trent-Queen's Concurrent Teacher Education Program. His present designation is: Research Associate, OISE Trent Valley Centre.

A group of 26 grade 9 mathematics teachers from a variety of boards and schools in the central region of Ontario was surveyed in the spring of 1995. The survey focused upon the teachers' sources of ideas and their teaching strategies for implementing destreaming. Follow-up interviews with 19 of the respondents identified three teachers with whom case studies were then conducted. Data from the case studies were gathered through interviews, classroom observations, and the collection of artifacts.

At the time of writing, little in the way of literature had been published which focused upon the implementation of destreaming, especially in mathematics. Wheelock (1992) suggested that successful heterogeneous grouping in mathematics, as in other subjects, required new approaches to curriculum, instruction, and assessment. In addition to these measures was the importance of providing for special needs students (Oakes and Lipton, 1992). An enriched curriculum for all students and the adoption of cooperative learning techniques were among measures for effective destreaming recommended by Slavin (1991, 1993), Oakes (1992), and Drake and Mucci (1993).

Collection of data for the study was guided by the following questions:

1. What are teachers' main sources of ideas in implementing destreaming?
2. How are teachers catering to the needs of the wide variety of students within their classes and, in particular, the identified and exceptional students?
3. What teaching strategies are being employed by teachers? Are they utilizing cooperative learning? Is problem solving a focus of their lessons?
4. What techniques are teachers employing to assess student progress?

Sources of Ideas

Responses from the survey indicated that teachers' three most important sources of ideas for implementing destreaming were: programs developed in the mathematics departments of their own schools, meetings conducted with teachers of grade 7 and 8 classes, and conversations with colleagues.

A range of program development within departments was apparent, including many initiatives to address the needs of the students of varying abilities present within heterogeneous classes. Among them was the development of computer software for a mastery learning package which would allow students to progress at their own rate.

Administrators in one board set up a series of professional development workshops involving teachers of grades 7, 8, and 9 from their schools. These workshops focused both on teaching strategies and on re-designing the curriculum in order to accommodate the common curriculum and to eliminate overlap between these grades. At another school, meetings with teachers of grade 8 classes from feeder schools focused on what they taught in geometry and in other hands-on sections of their program.

Many respondents obtained ideas for teaching destreamed grade 9 classes from teachers within their own schools. This was through discussions with colleagues who had taught these classes previously, sharing of materials gathered by those who had attended workshops, grade 9 teachers bouncing ideas off one another and, in some cases, interaction with teachers of other subjects.

Several teachers reported the use of other resources in developing their programs. In the main, these took the form of grade 9 texts apart from the one used in their own school, and other printed materials, including those produced by the National Council of Teachers of Mathematics.

Provision for Identified and Exceptional Students

Most teachers made special provision for both high and low achieving students. With low achieving students, some were able to enlist the aid of special education or resource teachers, and of peer tutors. This occurred either in the regular classroom, or through withdrawal of individual students for part or all of the lesson. In one school, a small group of learning disabled students was withdrawn from the destreamed grade 9 classes for their core subjects and taught as a separate group. At another, a teacher indicated that she sometimes had one or two identified students work on their own modified program. Teachers also took steps to modify worksheets, assignments, and assessment tasks for low achievers. In addition, manipulatives such as algebra tiles were often used specifically with these students.

High achieving students were usually accommodated by offering them more challenging work. In a few cases, this took the form of an accelerated program but, generally, it took the form of enrichment. This included the use of contest problems, assigning non-routine problem-solving tasks, devising open-ended assignments and projects, and the inclusion on assessment tasks of challenge questions which attracted bonus marks. One teacher reported students being withdrawn one or two lessons a week for a special enrichment program, while another had set up a withdrawal program for a half period a week, but not necessarily during a mathematics lesson. In the case of the latter, students spent this time solving contest problems and were given encouragement to participate in the Canadian Mathematics Competition. Students in another teacher's class were given the option of studying a self-directed unit on income taxes once they had satisfactorily completed the requirements of an equations unit. There were also instances of teachers commencing their lessons with a challenging puzzle or problem solving activity, a measure designed especially to generate interest for the more able students.

Teaching Strategies

In general, teachers were employing an eclectic mix of teaching strategies, as they endeavoured to provide a balance between group activity and individual work. Although the use of structured cooperative learning

techniques was not substantial, there was a strong tendency among respondents to allow students to work in pairs or small groups for at least part of each lesson. Activities which occurred in pairs or small groups included homework and quiz review, working with manipulatives such as algebra tiles, working on assignments and other desk work, and working on portfolios.

Of the teachers who did report the use of cooperative learning strategies, few did so regularly or in a structured manner. Generally, they found it more appropriate for specific topics such as word problems, and data gathering and analysis. One teacher reported the use of a "Jigsaw" strategy for independent investigations, while there were others who assigned specific roles to students when they worked in groups. A teacher who did employ cooperative learning techniques indicated that he found them more difficult to set up in mathematics classes compared with those in subjects such as history, his first teachable.

There was little evidence to suggest that teachers were making problem solving a focus of their lessons. Where problem solving did feature, it took the form of extension work for more able students or those who completed assigned work early. Applications questions at the end of a unit and the assigning of the problem of the day or week were other activities which involved problem solving. One teacher did report using problems on a regular basis to generate the mathematics for his lessons, while another often utilized investigations designed to assist the development of mathematical ideas.

Some specific teaching strategies emerged as a consequence of the three case studies. One teacher had developed a portfolio for a word problems unit, where the focus was the use of algebraic expressions and equations to solve practical problems. The same teacher made use of a computer package and the school's computer laboratory to assist student learning in a statistics unit. A second teacher, together with her school's librarian, had developed a history of mathematics project which involved library research. This teacher had also developed a cooperative statistics assignment and a transformations unit where she had students work in homogeneous groupings. The third teacher with whom a case study was conducted structured his weekly schedule of lessons to include a regular session for projects. Projects which he had employed included the construction of a mathematical board game, one in which students simulated the investment of money in the stock market, one on tesselations, and another involving the construction of scale models. This teacher also held a monte carlo day and involved his students in conducting surveys when they were studying a statistics unit.

Student Assessment

Examination of data revealed that the majority of respondents were utilizing assessment strategies other than unit tests and examinations. These included assignments, projects, portfolios, student notebooks, and conferencing with students. Of the teachers who utilized group work, some awarded group marks for assignments, others assessed individual attributes such as communication and behaviour, and some incorporated peer evaluation. Most respondents reported the inclusion of a component from the affective domain in their assessment, such as attitude, effort, and preparedness for class. This component usually accounted for 20% of the student's assessment and was usually assessed with the aid of checklists. Some teachers had developed tests with multiple sections, one of which was often a problem solving or challenge section.

At the school of one of the case study teachers, exams, tests, and major quizzes had three sections, A, B, and C. While Section B was compulsory, students had a choice of answering section A or C. Section A consisted of routine questions, while Section C contained more challenging problems. Generally, students headed toward the Advanced level course in grade 10 completed Section C, while those likely to take General or Basic level completed Section A. Where necessary, teachers guided students on their selection of the appropriate section. On progress, mid-term, and final reports, the teacher indicated whether the student was headed toward Advanced, General, or Basic level in grade 10.

At a second case study school, the final examination was worth 20% of the student's mark. However, if a student's exam mark exceeded her/his term mark, the exam counted for 30% instead of 20% for that student. This measure was adopted to encourage students to approach the semester exam in a more serious manner.

Teachers' General Comments

Teachers involved in the study were invited to offer some general comments about teaching destreamed grade 9 classes. The prime concern raised was the difficulty experienced in meeting the needs of the wide variety of students, especially those who were headed toward Advanced level in grade 10. It was felt that destreaming tended to benefit the lower rather than the higher ability students, and some teachers expressed frustration in progressing through the curriculum while trying to accommodate the weaker students. Teachers who had the assistance of peer tutors and special education teachers considered this a bonus which allowed for individual and small group enrichment or remediation. Others commented that, if destreaming was to be successful, class sizes needed to be kept to a maximum of 25. An individual teacher bemoaned the lack of direction given by the Ministry of Education and his local board office, claiming that only vague guidelines were provided and that, to implement such a radical change, far more guidance was necessary.

Conclusion

From teachers' responses, it appeared that they were relying mostly upon their own resources and those of their colleagues when implementing destreaming. New approaches to curriculum were not evident, as teachers were expected to follow the Ministry guidelines, and these were essentially a hybrid of the previous grade 9 Advanced level course. However, new approaches to both instruction and assessment were in evidence, as were efforts to cater to the needs of the wide variety of students in destreamed classes. Teachers, especially those involved in the three case studies, were making a concerted effort toward providing an enriched curriculum for all of their students. While an eclectic mix of teaching strategies was apparent, there was a reluctance to adopt formal cooperative learning techniques. This may be due more to an inadequate knowledge of these techniques than an outright rejection of their appropriateness.

Two interesting issues which emerge from this research might be worthy of further investigation. The first concerns why teachers felt that destreaming tended to be of more benefit to the lower rather than the higher ability students. If this perception was demonstrated to be correct, the question then arises as to how this situation can be rectified. The second issue is that concerning why formal cooperative learning techniques recommended by Robert Slavin and others are not being implemented by teachers. Perhaps the two issues are related.

In order to facilitate the teaching of destreamed classes, my recommendation is that the Ontario Ministry of Education and Training develop a grade 9 curriculum which is supportive of the teaching of mathematics to students in heterogeneous classes. A curriculum which is interactive and emphasizes problem solving would help teachers shift their focus when teaching these classes. Meanwhile, teachers of mathematics, along with those of other subjects, should be encouraged to explore the variety of cooperative learning techniques promoted by Slavin and others as a means to accommodate the wide variety of student abilities represented in their grade 9 destreamed classes.

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Note

Readers who would like more information regarding any of the ideas and strategies presented in this article are invited to contact the author by email: demmes@oncomdis.on.ca.

Is the following statement written between the quotation marks true?

Is it false?

"This statement is false."

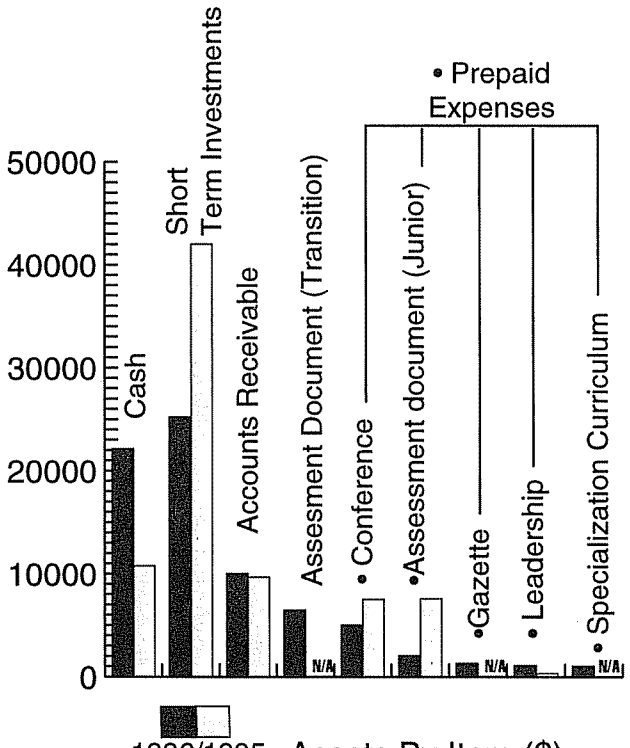
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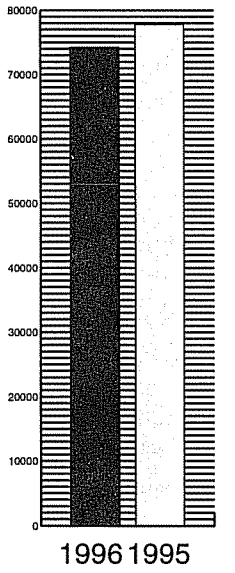
NOTICE TO READER

I have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Ontarienne Pour L'Enseignement Des Mathematiques) as at August 31, 1996 and the Statement of Operations and Accumulated Surplus for the year then ended from information provided by the Directors. I have not audited, reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

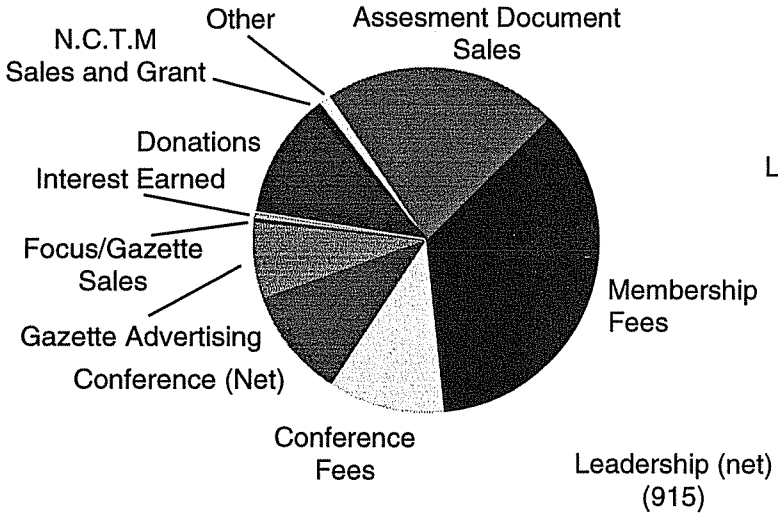
Markham, Ontario, September 16, 1996 ROBERT M. DAVIES Chartered Accountant

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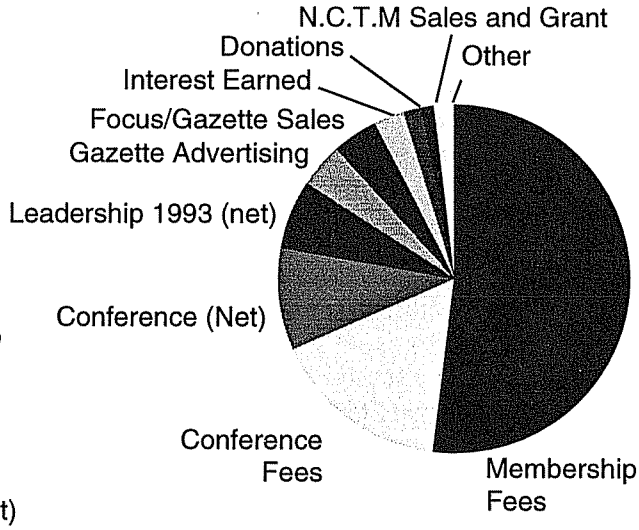




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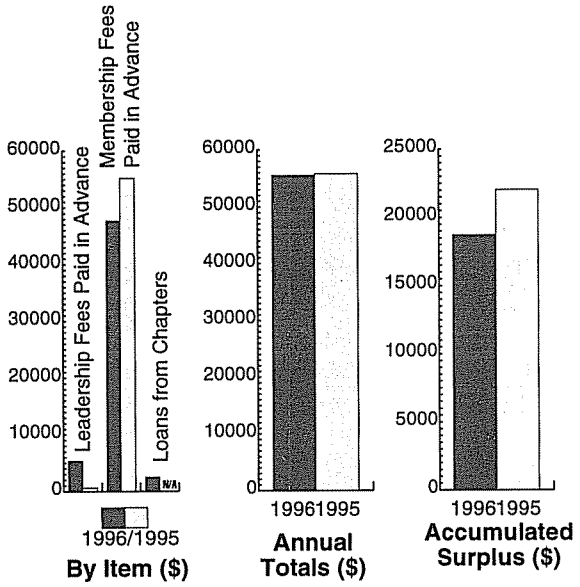


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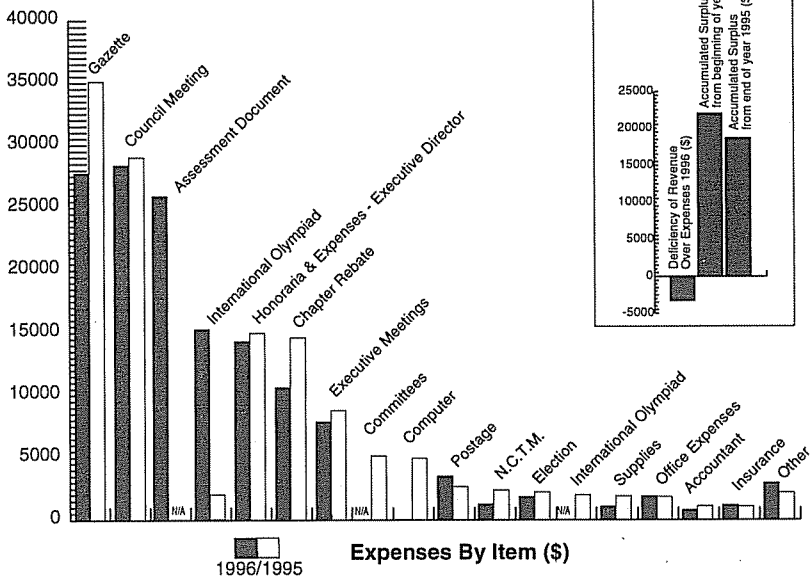


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Liabilities & Surplus 1996/1995



Expenses 1996/1995



Special Projects Call For Submissions for OAME Resource Publication

"Teaching Mathematics Through Technology"

The OAME Special Projects Committee intends to compile a Classroom Resource document which will showcase how mathematics is currently being taught through technology in classrooms across Ontario.

Do you have an activity/unit which you are currently using and that you think would be of use or interest to other mathematics teachers across the Province?

Please submit it to the OAME Special Projects Committee. If your submission is published, you will receive a complimentary copy of the resource package. All contributors will be appropriately acknowledged.

- **must submit hard copy and disk**
- **must be original or contributor must have gained copyright permission from the author**
- **deadline for return of submissions January 13, 1997 to:**

Marg Warren Program Services
H. J. A. Brown Education Centre
Peel Board of Education
5650 Hurontario Street Mississauga
Ontario L5R1C6

- ***Volume 1 ready for sale at May annual conference***
- ***everyone who is published will receive a free copy***
- ***submissions may be of the form of single activities, entire units, articles***
- ***we are hoping to receive submissions for each division***

If you need additional information, please call Marg Warren at 905 - 890 - 1099 ext 2546 or e-mail at warrenm@cbogate.peel.edu.on.ca

How Big is a Million?

MONICA CAMPBELL
ST. PAUL'S CATHOLIC SCHOOL, LONDON



Monica Campbell has been an elementary school teacher for six years with the London and Middlesex County Roman Catholic School Board. She has taught in both Junior and Transition Years classrooms. Monica finds mathematics a challenging and rewarding subject to teach. "Certainly many students need motivating in a math class," Monica believes, "but I feel a math lesson can be as exciting or stimulating as any other subject if well thought out and prepared. " The reward for her is the enthusiasm of her students and their steady progress throughout the year.

How big is a million?
How long is a kilometre?

These are questions that can be of interest to your students especially if they use an exciting method to answer them. In this article I want to share with you my method of bringing these questions off the paper into real life.

What is a million? An abstract concept such as this is hard for students to imagine, and for most adults too. I asked my class what they thought a million was? Here are some responses from my grade seven class at Holy Cross Elementary School in London.

"A million is the stars in the sky."
"A million is the leaves on a tree."
"A million is the people at a concert."
"A million is the zits on my face."

These answers indicated to me that my students did not have a clear idea of what a million was.

So here is how I proceeded to help them understand this concept.

I asked them how long it would take someone to draw a million dots on paper. The majority of students estimated that twenty minutes to half an hour would be plenty of time. So we decided to draw a million dots to check their estimates. Each day we added to our dots grouping them in hundreds to keep track of our progress. We started in October working during our spare time which varied daily. We finished six months later: thirty-seven metres of paper and eight boxes of markers later. But it was worth it. Not only did the students see and feel what a million was but they learned a number of other mathematics concepts along the way. They used skill in measurement to determine the amount of paper that they would need. Grouping and counting helped the students to keep track of the dots as they progressed. Area was involved as they tried to decide where and how they would display their project with an area of 37 x 1 square metres. The whole project was an excellent example of problem solving by the class.

But not only math was involved in the project. My students integrated the project into their Language Arts program by writing and directing a play about the fun they had during their construction of "One Million Dots". One unexpected reward for my students was a write up in the local paper as they were featured in the news. They felt proud and happy to be able to show off their work. Will they remember what a million is? Yes! They will remember it for a lifetime.

I used a second project with my grade eight class at St. Paul's Catholic School in London to answer the question "What is a Kilometre?". My students thought that everyone knew what a kilometere was. Hadn't they all walked one? But they decided that they would really know what a kilometer was if they constructed one, every centimetre of it.. To make the project more interesting the class decided to build their kilometre out of pennies. They collected over 53 000 pennies sticking each one on to masking tape. This was a very meticulous job. Was this a waste of time? Not at all for not only did my students experience first hand what a kilometre was but they used their model throughout the year to aid in understanding other mathematical concepts. For example at each stage in the process they used fractions to calculate how much further they needed to go. Money took on a whole new meaning as they related money to the distance they had completed. Pennies were sent to them from other cities leading to a lesson on distance and the use of maps.

My students became very creative in collecting pennies especially after they decided to donate their pennies to a local charity. One of our class mottoes was "Set your Goals High, Then Exceed Them". That is exactly what they did as a total of $600 was raised through this project.

These are two projects that my classes have enjoyed. You can probably think of many more such projects. One more project that I have been thinking of would answer the question "How Much Does My Teacher Weigh?" Perhaps I can get my students to match my weight with nickels. That should be a challenge for all of us!

▲ Directors Dialogue



PETER SAARIMAKI
COORDINATOR OF MATHEMATICS
TORONTO BOARD OF EDUCATION

Introduction

These notes were taken at the OAME Board of Directors meeting held Saturday, October 5, 1996. Comments about OAME and its activities should be directed to the President. If you have comments about my comments, ... they can be sent to the editor.

General

A few new Directors were welcomed to the second meeting of this term, namely: Gord Cook (QSLMA president, filling in for Doug Evans), Maureen Dunn (TEY²MS rep) and Chris Usih (SAME president).

Your representatives spent a large portion of the weekend time working in committees. With only three meetings a year, much has to be accomplished in a very short time frame.

A very short period at the beginning of the meeting was taken up by administrivia ... the Executive Directors really handle that very efficiently and quietly so that valuable time is not wasted. Have you noticed elsewhere who won the free weekend in the hotel as a lucky new member? The prize is to be offered again, so get your friends to enter ... maybe you can be their guest next year. Local chapters will again be looking for new members and encouraging previous members to rejoin.

*** CHALLENGE! Rearrange the letters of NEW DOOR to form one word. *** First correct response wins a prize!!

A guest visitor, Jack Weiner, announced the opening of OAME's web site. Gareth Haddock from ISOMA will be the web-site editor. Visit, participate and revisit often, as materials and information will be updated regularly. If you have any materials to be included in the site, contact Gareth (see back page). By now there should be juicy bits about the 1997 conference, even before the official program booklet is sent out. We are very thankful to Guelph's Math and Stats department for hosting it at

http://www.mathstat.uoguelph.ca/oame

Ministry Directions

The OAME executive continues to meet with Ministry officials to provide input, comments and suggestions to the on-going Ministry initiatives, including the provincial tests (all Grade 3 students and a sampling of Grade 6 students) and the Secondary Reform.

Nominations for OAME/AOEM Board of Directors 1997-98

Each year, elections are held for President-Elect, 3 Vice Presidents (1 year term) and 6 to 9 Directors (3 year term) of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 15, 1997, to:

Susan Stuart
Past-President, OAME/AOEM
Nipissing University, Faculty of Education
100 College Drive, North Bay, P1B 8L7
FAX (705) 474-1947

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March, 1997, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Toronto in May, 1997.

Nomination Form

I nominate: _____

For the position(s) of: ☐ President-Elect ☐ Director ☐ Vice-President

Nominee's Address: _____

_____

Telephone: (h) ( ) _____ (w) ( ) _____

Nominator's Address: _____

_____

Telephone: (h) ( ) _____ (w) ( ) _____

The OTF has NOT budgeted monies to run the OTF Curriculum Forum, a meeting of association reps, for several years, but this year decided to run one meeting. It was set up in early November to prepare a response to the Ministries proposals for Secondary reform. Reps from ALL provincial organizations, including OAME, met and sent a message to the Ministry regarding the need for curricular integrity in all the reforms. Contact me, as OAME's OTF rep, for more specifics .

OAME '96 conference

Kaye Appleby reported on the very successful conference last May in London. The many volunteers are what make such events work, and although they were not paid for their efforts, they all received rewards in the form of increased professional development and contacts with their colleagues.

OAME '97 conference

The Name of the Game is well underway with keynote and featured speakers lined up, exciting workshops and extracurricular events planned, and opportunities for teachers and their students to participate even before the conference next May. A teacher resource on *"Designing Math Games"* is being used by many classes to prepare student work for exhibit at the conference.

Although the conference officially starts on the Thursday (May 8), there will be a public celebration of Mathematics through multicultural and student games on the Wednesday night.

OAME Leadership conference

Plans are underway for 1997, following a very successful 1996 event. In early November, over 100 delegates heard from Joan Green, CEO of the EQAO (the TEST makers) and Don Fraser and then spent almost a day and a half in their grade groupings exploring critical issues with acknowledged experts from the mathematical community.

1997 may look at the whole issue of Secondary Reform and one question organizers have is whether elementary issues can be deferred for a year or whether the K - OAC spectrum should continue to be addressed. Send your opinions to Judy Crompton, president-elect.

Futures committee

This committee looks to the functions that OAME supports and sponsors each year. As a result of their work, we know the following future plans:

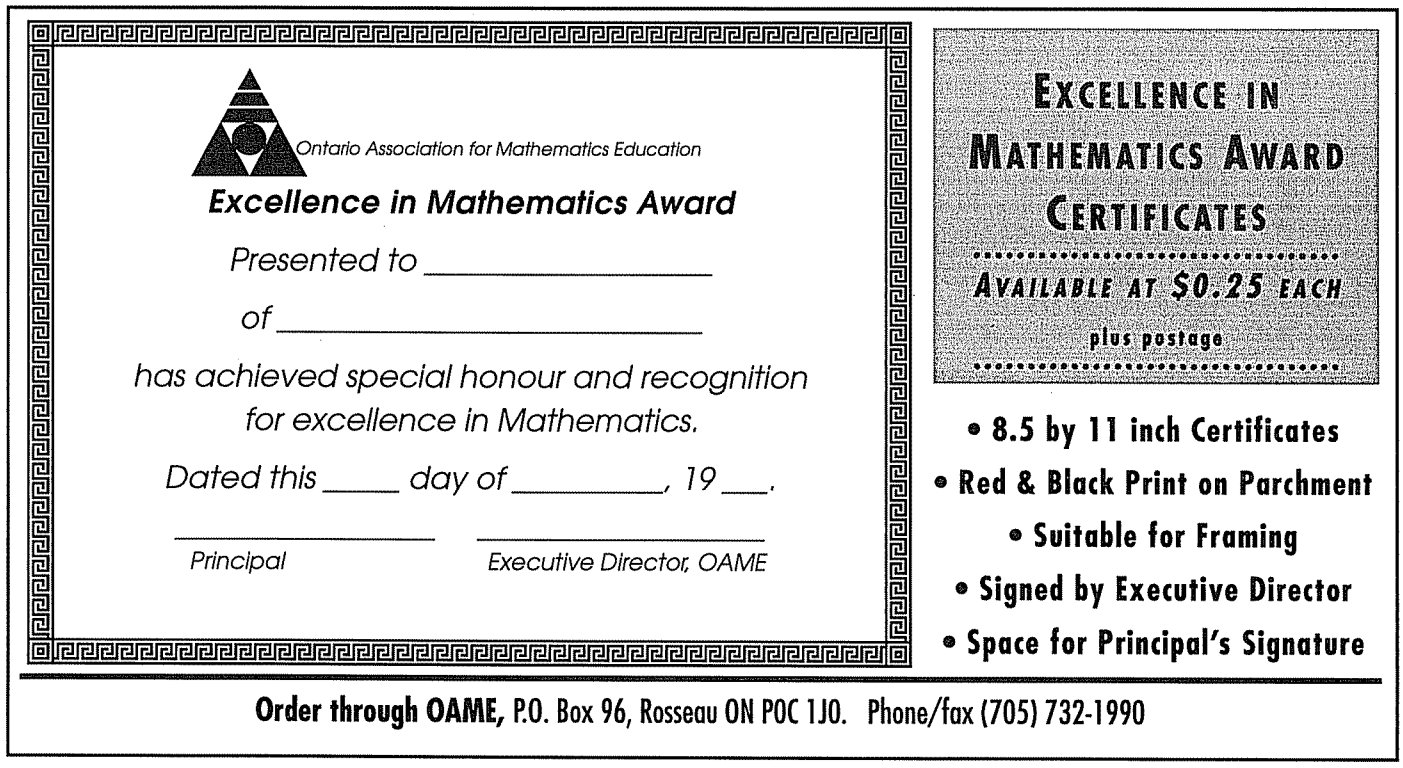
- **Ontario Mathematics Olympics: 1997:** will be hosted by MAC^2 at Kempenfeldt Centre at Barrie (June 6, 7) Chapter reps should be receiving a preliminary package in early November. Directors were reminded to publicize local and regional play-downs as much as possible and to recognize the continuing contributions of Rockwell International.
- **OMO: 1998:** likely hosted by ISOMA
- Annual Conference: 1998: hosted by NOMA, North Bay
- several key speakers have already been booked - check with OAME's web site for details
- **Annual Conference: 1999:** still looking for host chapter
- **Annual Conference: 2000:** hosted by COMA in Ottawa

Resources

John Kersley again provided directors with a packet of interesting and useful materials, including a large TI-92 poster and overhead, articles and cartoons.

Special Projects

Marg Warren made an impassioned presentation on the works of the Special Projects committee, the Specialization Years committee and the Ontario Mathematics Coordinators



Association regarding their collective plans for Secondary Education Reform. After a special two day work group in October, a framework will be made available for boards to develop into the full mathematics program for secondary students.

All of this is proactive to the Ministry's plans to eventually produce Curriculum Guidelines and Course Profiles. The options for curriculum development includes "partnerships with other organizations with expertise in specific areas" (**Curriculum for Ontario Secondary Schools**, A Discussion Document, MOET, Sept. 20, 1996). Check the OAME web site for the framework and opportunities for your input.

COMMITTEES

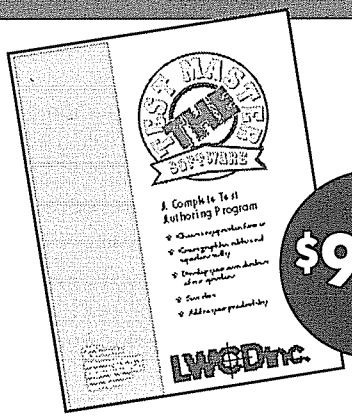
Early/Formative Years

The Junior Division Assessment document should be out by now: ... note the ordering information on page 16 in this *Gazette.* Following the success of the Transition Years booklet, this one is not to be missed.

Transition Years

The public gets very concerned when standards are mentioned and want to know whether students are "ready" for high school. What about a challenge exam? The committee is exploring this, in the light of one of the secondary reform initiatives, namely Prior Learning Assessments. If you want further information or have suggestions or concerns, contact your chapter rep.

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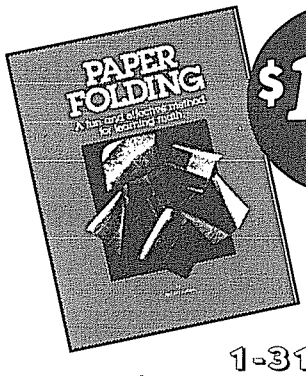
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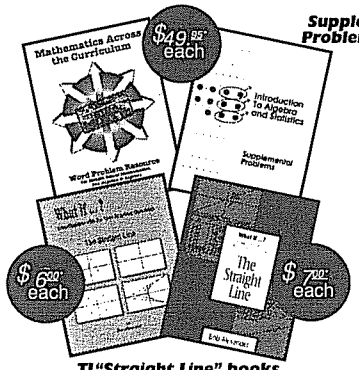
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Specialization Years

The OAME response to the Ministry Secondary Reform discussion papers was prepared by members of this committee and approved by the executive. A copy should be on the web site by now (or contact your chapter rep). The discussion papers and questionnaires provided many tantalizing opportunities for input, but on a very short timeline (as always). Several members also participated in the fall OMCA work group on restructuring the secondary program.

Equity

OAME has adopted, in principle, the NCTM Equity Guidelines to monitor our work and publications. A motion to specifically use NCTM's procedure for conducting equity reviews of all future publications was tabled until further information was available.

Publicity and Membership

Ties? Scarfs? Scrunchies? what would you like to have to promote your support for OAME? Let your chapter rep know before the next directors meeting when prototype models will be considered.

Publications

Please continue to support the *Gazette* advertisers, as their ad revenue greatly assists the work of the OAME.

***CHALLENGE! Why are 1996 pennies worth almost twenty dollars ($20)? *** First correct response wins a prize!!

CHAPTERS

One regret I have is that most of the wonderful conferences planned by chapters this fall, shared with the Directors at the Oct. 5 meeting, will be over by the time you read this in the December issue. Each chapter puts on some marvellous Professional Development. Keep in touch with your local chapter to be aware of their plans, before they happen: the reps names are on the back page!

CHAMP

135 attended MATHFEST. Plans for their Math Olympic playdowns are already underway for the week of April 7th in each of the four boards. The first newsletter was sent to ALL math teachers in CHAMP, while future issues will go to members only. Don't miss out, join now!

COMA

Tentative date for their mini-conference is Jan 27, 1997, plus plans for more Forums, T-shirts and participation in the OMO (of course! they are the founding chapter!). This group is also looking into an Olympics for Grade 9 and 10 students.

GVMA

Publications now include a grade 10 Advanced exam bank, with plans for grade 11 and 12 in the near future. GVMA also has a web site and they can be found at: http://www.golden.net/~gvma.

ISOMA

Or you can visit http://www.quinte.net/isoma and find out what the Independent Schools are up to. This group is really into the new technologies, ... just consider: their executive meeting this summer was by teleconference; for their fall conference key note speaker, they suggested participants check out her web page for further information; and their newsletter includes members email addresses and a pretty impressive list of interesting math sites on the Internet .

MAC²

This fledgling group continues to forge ahead with a fall conference, Math Leagues and by hosting next springs Student Math Olympics.

NOMA

Over 170 teachers from across Northern Ontario attended a most successful fall conference in Sudbury. The organizers especially encouraged student teachers to participate with lower rates (and dinner). Congratulations to the spark plugs (that's a computer replacement for organizer) for this Friday /Saturday event that kicked off with a special, well-received triumvirate high-lighting "Math from the Lighter Side".

NWOAME

MathFest '96 was held in mid November with workshops, publishers, dinner and Math Gems.

QSLMA

The fall saw two parallel conferences for elementary and secondary teachers. The spring conference will be held in conjunction with the OMO playdowns, providing teachers with an opportunity to see their students in action and get some PD themselves. A newsletter is coming out this fall.

Math-TEY²MS

A fall conference was held on Assessment, but most of the chapters energies are being put towards the 1997 OAME Annual conference (see info elsewhere).

WOMA

Fall conference, sponsoring teachers to the Leadership conference and Sleep-Time Math are still keeping this group busy after last years Annual conference.

Y⁴MA

Fall conference changed to December, for primary teachers. Next fall, with a new format, is already scheduled for Sept 23, 1997! Playdowns are planned for their Junior Olympics

See the notice, on page 31, for OAME director elections and consider running. If you have questions about the work of the directors or comments, contact me on the Internet at bb304@freenet.toronto.on.ca

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by the Awards Committee by January 31 of the calendar year in which the conference takes place.

Nominations received after that January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Awards Nomination Letter

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrations of leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contributions to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the **Nomination Cover Page** and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

Award Criteria

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In memory of the late Kenneth D. Fryer, OAME/AOEM in conjunction with the Grand Valley Mathematics Association established the Kenneth D. Fryer Award. This award is given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Mona MacGregor Award

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. The award is to recognize an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Morley MacGregor Award

In memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

Don Attridge Award

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

OAME/AOEM Award Nomination Cover Page

Please copy this form and attach it to the nomination letter.

Intended Award (please check one):

☐ Life Membership Award ☐ Mona MacGregor Award ☐ Don Attridge Award

☐ Morley MacGregor Award ☐ Kenneth D. Fryer Award

Nominee Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

School, Organization or Business Address

Telephone Number ( ) _____ Fax Number ( ) _____

Home Address _____

Telephone Number ( ) _____

Nominator Information (attach information for additional nominators)

Name of Nominator _____

_____

School, Organization or Business Address

Telephone Number ( ) _____ Fax Number ( ) _____

Home Address _____

Telephone Number ( ) _____

Send or fax the completed letter of recommendation and Nomination Cover Page to
David Zimmer, John Fraser Secondary School, 2665 Erin Centre Blvd,
Mississauga, ON L5M 5H6 FAX (905) 858-1973

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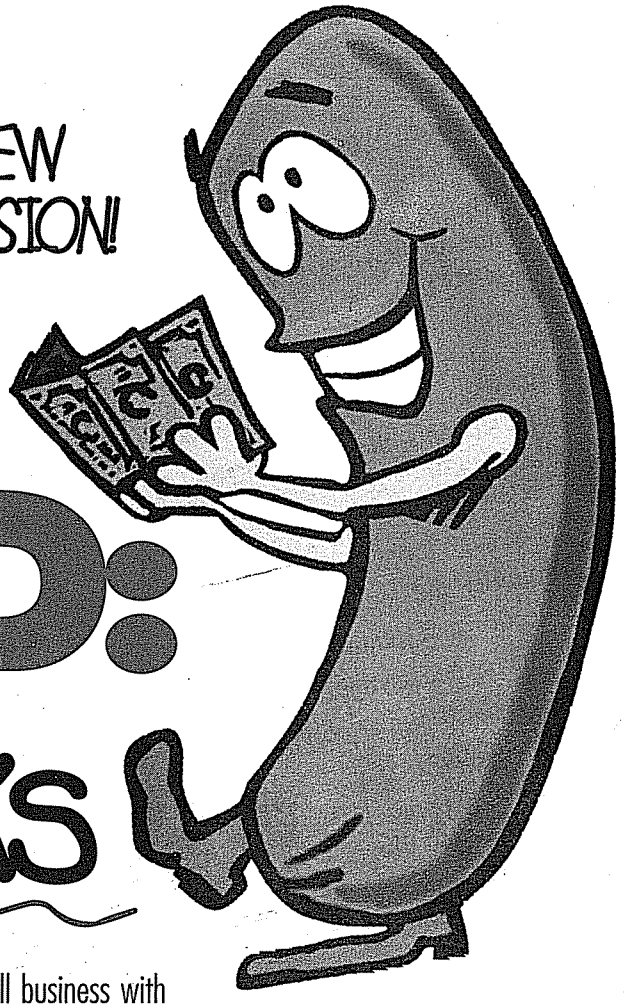
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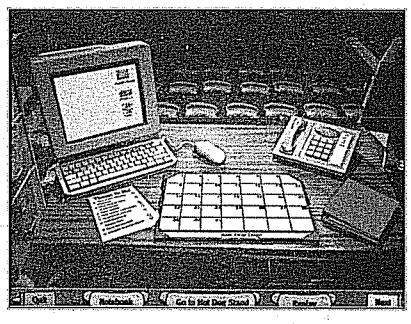
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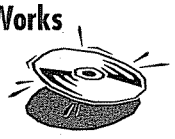


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